HW 5.2

Find an equation for the superposed magnetic field.

Because of the symmetry above and below the R, ϕ_r plane, the $\hat{z}$ components cancel for the magnetic field vectors of any ring ~~above~~ of current bisected by the R, ϕ_r plane.

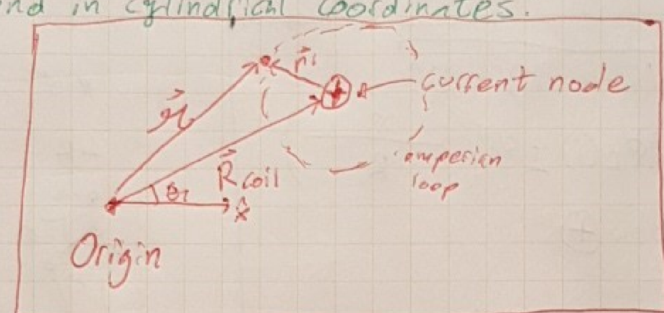
This 2-D plane configuration turns current coils into binary nodes of positive and negative current.

Finding $\vec{B}$ from a given node in cylindrical coordinates.

$\vec{B}$ is found with Ampère's law, noting that B is constant for a radius r' from the node of current.

$$\int_0^{2\pi} B\, r' d\ell = \mu_0 I \;\Rightarrow\; B = \frac{\mu_0 I}{2\pi r'} \;\underset{\text{with RHR}}{\Rightarrow}\; \boxed{\vec{B} = \frac{\mu_0 I}{2\pi r'}\,\hat{\theta}_B}$$

Next r' is found in cylindrical coordinates:

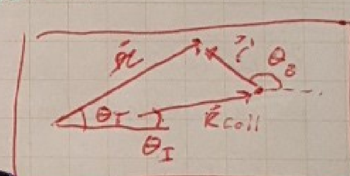


Geometry to be used

$$\vec{r}' = \vec{\mathscr{r}} - \vec{R}_{coil} \;\Rightarrow\; r' = (\mathscr{r}^2 + R_{coil}^2)^{1/2}$$

It is easier to express vectors in $\langle x, y, z\rangle$ form:

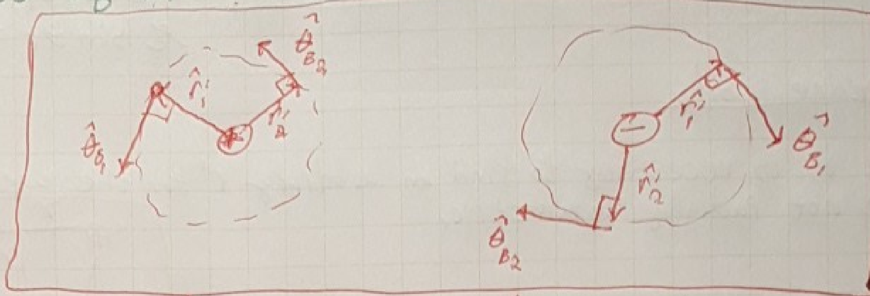
$$\vec{r}' = \langle \mathscr{r}\cos\theta_r, \mathscr{r}\sin\theta_r\rangle - \langle R_{coil}\cos\theta_I, R_{coil}\sin\theta_I\rangle$$



angles used

$$\boxed{|\vec{r}'| = \left((\mathscr{r}\cos\theta_r - R_{coil}\cos\theta_I)^2 + (\mathscr{r}\sin\theta_r - R_{coil}\sin\theta_I)^2\right)^{1/2}}$$

Now we express $\hat{\theta}_B$ in $\langle x, y \rangle$ form:



No matter what, $\hat{\theta}_B$ is $\pm 90°$ from $\hat{r}'$ depending on current

$\hat{r}'$ rotated $90° \rightarrow \frac{\langle -r'_y, r'_x \rangle}{r'}$ (+) or $\frac{\langle r'_y, -r'_x \rangle}{r'}$ (−)

$$\boxed{\hat{\theta}_{B\pm} = \frac{\langle \mp(r \sin\theta_T - R_{coil} \sin\theta_I), \pm(r\cos\theta_T - R_{coil}\cos\theta_I) \rangle}{|r'|}}$$

Now we can express the B vector anywhere on R, θ_T

Using the principle of superposition and symmetry to create a function for B_{total}

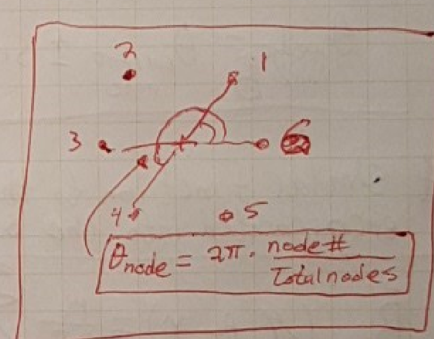
I set my positive node closest to the origin.

Since the positive nodes are radially identical, I group them together:

$$\boxed{R_{coil\,inner} \Rightarrow R_{maj} - a}$$

Same for outer coils

$$\boxed{R_{coil\,outer} = R_{maj} + a}$$



nodes are spaced at regular angular intervals

Their angles can be a function of the # of nodes

$$\boxed{\theta_I = \theta_{node} = 2\pi \frac{nodeIndex}{Total\ nodes}}$$

The superposition principle says: $\vec{B}_{12} = \vec{B}_1 + \vec{B}_2$

So my function for $\vec{B}_{tot}(r, \theta_T)$ is:

$$\boxed{\vec{B}_{tot}(r, \theta_T) = \frac{\mu_0 I}{2\pi} \sum_{i=1}^{\#coils} \frac{\hat{\theta}_{B\oplus}(R_{coilinner}, \theta_{node}(i))}{(r'(R_{coilinner}, \theta_{node}(i)))} + \frac{\hat{\theta}_{B\ominus}(R_{coilouter}, \theta_{node}(i))}{r'(R_{coilouter}, \theta_{node}(i))}}$$

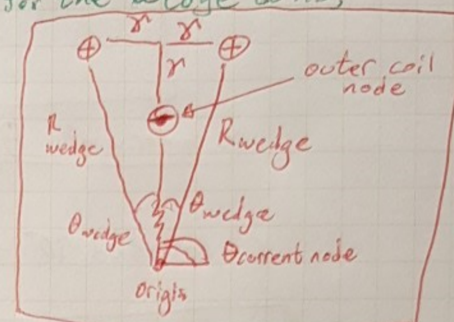
technically $(r, \theta_T, R_{coilinner}, R_{coilouter}, \theta_{node})$

SEE CODE FOR IMPLEMENTATION. $r\cos\theta_T = X$

$r\sin\theta_T = Y$

Adding wedge coils

As before, it is necessary to find an arbitrary r' for the wedge coils, which are not radially symmetric.



angles and radii for the outer wedge nodes

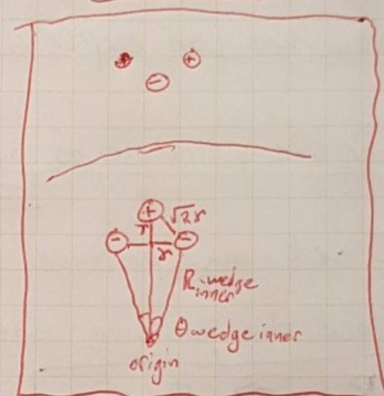
Rwedge & θwedge for the outer wedges (⊕ nodes)

I set the wedge coils to be r to the left & right of each coil and $2r$ longer.

$$\boxed{R_{\substack{wedge\\outer}} = \left((R_{maj} + a + r)^2 + r^2 \right)^{1/2}}$$

$$\boxed{\theta_{\substack{wedge\\outer}} = \pm \sin^{-1}\left(\frac{r}{R_{wedge\ outer}} \right) + \theta_{current\ node}}$$

for inner wedge nodes



Inner wedge nodes (⊖)

$$\boxed{R_{\substack{wedge\\inner}} = \left((R_{maj} - a - r)^2 + r^2 \right)^{1/2}}$$

$$\boxed{\theta_{\substack{wedge\\inner \oplus\ominus}} = \theta_{\substack{current\\node}} \pm \sin^{-1}\left(\frac{r}{R_{\substack{wedge\\inner}}} \right)}$$

A similar function to the one before may be used, since θwedge can be incorporated into the summation due to θcurrentnode.

$$\vec{B}_{\substack{Tot\\wedge}}(r, \theta_T, r) = \frac{\mu_0}{2\pi} \sum_{i=1}^{\#coils} I_{coil} \left[\frac{\hat{\theta}_{B\oplus}(R_{coilinner}, \theta_{node}(i))}{r'(R_{coilinner}, \theta_{node}(i))} + \frac{\hat{\theta}_{B\ominus}(R_{coilouter}, \theta_{node}(i))}{r'(R_{coilouter}, \theta_{node}(i))} \right]$$

$$+ I_{wedge} \left[\frac{\hat{\theta}_{B\oplus}(R_{wedgeouter}(r), \theta_{wedgeouter\oplus}(r,i))}{r'(R_{wedgeouter}(r), \theta_{wedgeouter\oplus}(r,i))} + \frac{\hat{\theta}_{B\oplus}(R_{wedgeouter}(r), \theta_{wedgeouter\ominus}(r,i))}{r'(R_{wedgeouter}(r), \theta_{wedgeouter\ominus}(r,i))} \right.$$

$$\left. + \frac{\hat{\theta}_{B\ominus}(R_{wi}, \theta_{wi\oplus})}{r'(R_{wi}, \theta_{wi\oplus})} + \frac{\hat{\theta}_{B\ominus}(R_{wi}, \theta_{wi\ominus})}{r'(R_{wi}, \theta_{wi\ominus})} \right]$$

Same idea, just abbreviated

The wedges accentuate the mirror ratio, allowing greater pitch angles – More particles remain trapped.

The EBT's Total Magnetic Field Vector Function in Cartesian Coordinates:

```
ElmoBumpyCart[X_, Y_, μ_, I_, NumCoils_, Rcoils_, a_] := μ I / (2 π) Sum[

   1 / ((X - (Rcoils - a) Cos[2 π i / NumCoils]) ^2 +
         (Y - (Rcoils - a) Sin[2 π i / NumCoils]) ^2)
     {(Rcoils - a) Sin[2 π i / NumCoils] - Y, X - (Rcoils - a) Cos[2 π i / NumCoils]}

    - 1 / ((X - (Rcoils + a) Cos[2 π i / NumCoils]) ^2 +
         (Y - (Rcoils + a) Sin[2 π i / NumCoils]) ^2)
     {(Rcoils + a) Sin[2 π i / NumCoils] - Y, X - (Rcoils + a) Cos[2 π i / NumCoils]}

   , {i, NumCoils}]
```

Here is the EBT's Total Magnetic Field Vector Function with Wedge Coils in Cartesian Coordinates:

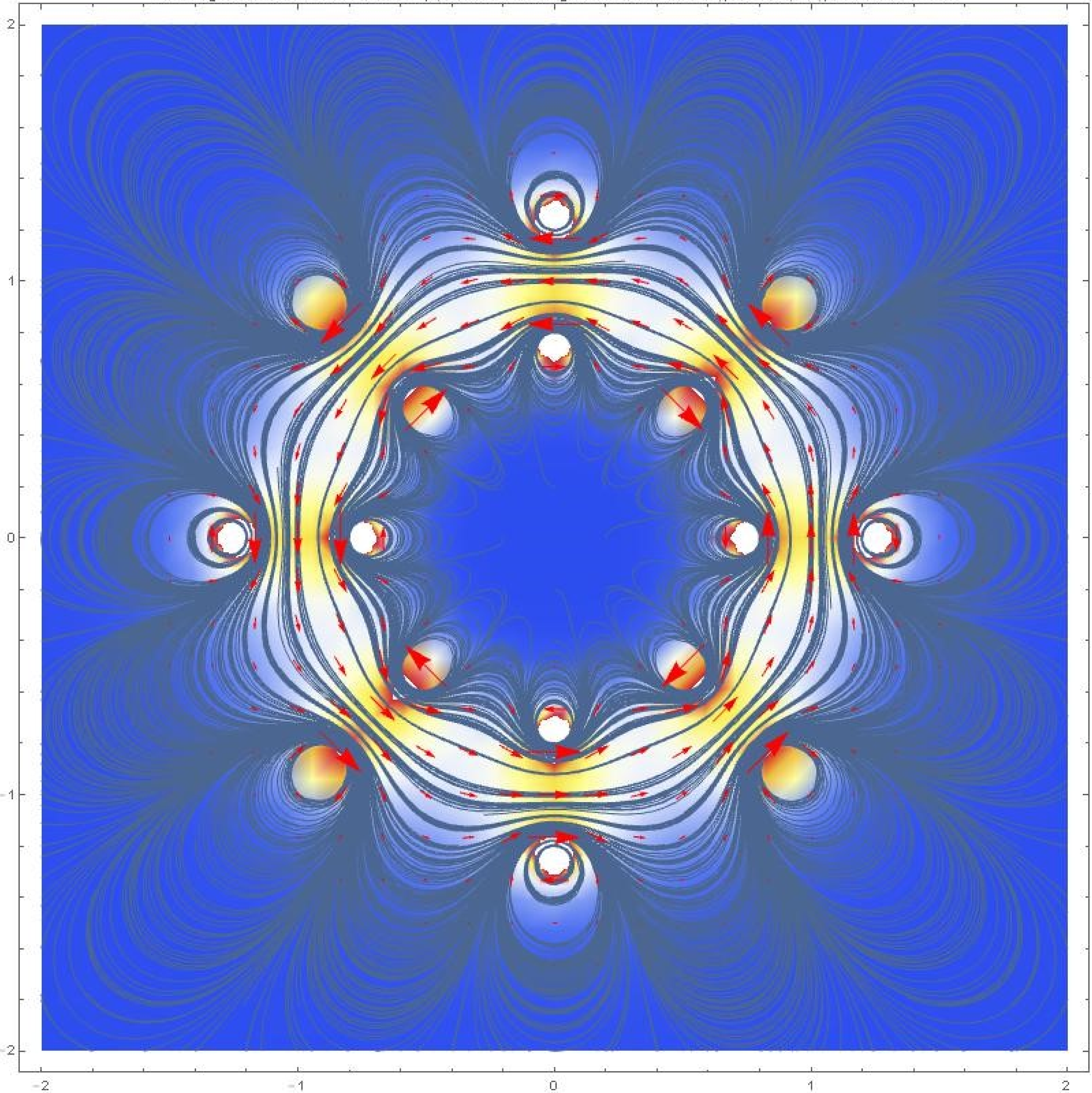
```
In[14]:= ElmoBumpyCartWedge[X_, Y_, μ_, I_, NumCoils_, Rcoils_, a_, γ_, Iwdg_] := μ / (2 π) Sum[

   I / ((X - (Rcoils - a) Cos[2 π i / NumCoils]) ^2 +
         (Y - (Rcoils - a) Sin[2 π i / NumCoils]) ^2)
     {(Rcoils - a) Sin[2 π i / NumCoils] - Y, X - (Rcoils - a) Cos[2 π i / NumCoils]}

    - I / ((X - (Rcoils + a) Cos[2 π i / NumCoils]) ^2 +
         (Y - (Rcoils + a) Sin[2 π i / NumCoils]) ^2)
     {(Rcoils + a) Sin[2 π i / NumCoils] - Y, X - (Rcoils + a) Cos[2 π i / NumCoils]}

    - Iwdg / ((X - Sqrt[(Rcoils + a + γ) ^2 + γ^2]
              Cos[(2 π i / NumCoils) + ArcSin[γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]]) ^2 +
         (Y - Sqrt[(Rcoils + a + γ) ^2 + γ^2] Sin[(2 π i / NumCoils) +
                ArcSin[γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]]) ^2)
     {Sqrt[(Rcoils + a + γ) ^2 + γ^2] Sin[(2 π i / NumCoils) + ArcSin[
             γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]] - Y, X - Sqrt[(Rcoils + a + γ) ^2 + γ^2]
         Cos[(2 π i / NumCoils) + ArcSin[γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]]}

    - Iwdg / ((X - Sqrt[(Rcoils + a + γ) ^2 + γ^2]
              Cos[(2 π i / NumCoils) - ArcSin[γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]]) ^2 +
         (Y - Sqrt[(Rcoils + a + γ) ^2 + γ^2] Sin[(2 π i / NumCoils) -
                ArcSin[γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]]) ^2)
     {Sqrt[(Rcoils + a + γ) ^2 + γ^2] Sin[(2 π i / NumCoils) - ArcSin[
             γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]] - Y, X - Sqrt[(Rcoils + a + γ) ^2 + γ^2]
         Cos[(2 π i / NumCoils) - ArcSin[γ / Sqrt[(Rcoils + a + γ) ^2 + γ^2]]]}

    + Iwdg / ((X - Sqrt[(Rcoils - a - γ) ^2 + γ^2]
```

```
            Cos[(2 π i / NumCoils) + ArcSin[γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]]) ^2 +
        (Y - Sqrt[(Rcoils - a - γ) ^2 + γ^2] Sin[(2 π i / NumCoils) +
              ArcSin[γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]]) ^2)
    {Sqrt[(Rcoils - a - γ) ^2 + γ^2] Sin[(2 π i / NumCoils) + ArcSin[
          γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]] - Y, X - Sqrt[(Rcoils - a - γ) ^2 + γ^2]
       Cos[(2 π i / NumCoils) + ArcSin[γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]]}

   + Iwdg / ((X - Sqrt[(Rcoils - a - γ) ^2 + γ^2]
            Cos[(2 π i / NumCoils) - ArcSin[γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]]) ^2 +
        (Y - Sqrt[(Rcoils - a - γ) ^2 + γ^2] Sin[(2 π i / NumCoils) -
              ArcSin[γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]]) ^2)
    {Sqrt[(Rcoils - a - γ) ^2 + γ^2] Sin[(2 π i / NumCoils) - ArcSin[
          γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]] - Y, X - Sqrt[(Rcoils - a - γ) ^2 + γ^2]
       Cos[(2 π i / NumCoils) - ArcSin[γ / Sqrt[(Rcoils - a - γ) ^2 + γ^2]]]}

  , {i, NumCoils}]
```

Plotting the wedge-less EBT with eight coils with I=10000A, Rmaj=1m, and a=0.25m gives:

```
In[22]:= Show[{DensityPlot[Norm[ElmoBumpyCart[X, Y, 1.2566*^-6, 10 000, 8, 1, 0.25]],
    {X, -2, 2}, {Y, -2, 2}, ColorFunction → "TemperatureMap",
    PlotPoints → 35, PlotLegends → Automatic,
    PlotLabel → "EBT Magnetic Field as Scalar Density (red to blue
       background and side-bar), Vectors(red), and Streamlines"],
   StreamPlot[ElmoBumpyCart[X, Y, 1.2566*^-6, 10 000, 8, 1, 0.25], {X, -2, 2},
    {Y, -2, 2}, StreamPoints → {Tuples[Range[-2, 2, 0.2], 2], Automatic, 10},
    StreamStyle → "Line", VectorPoints → Fine,
    VectorPoints → 10, VectorStyle → Red], ImageSize → 800}]
```

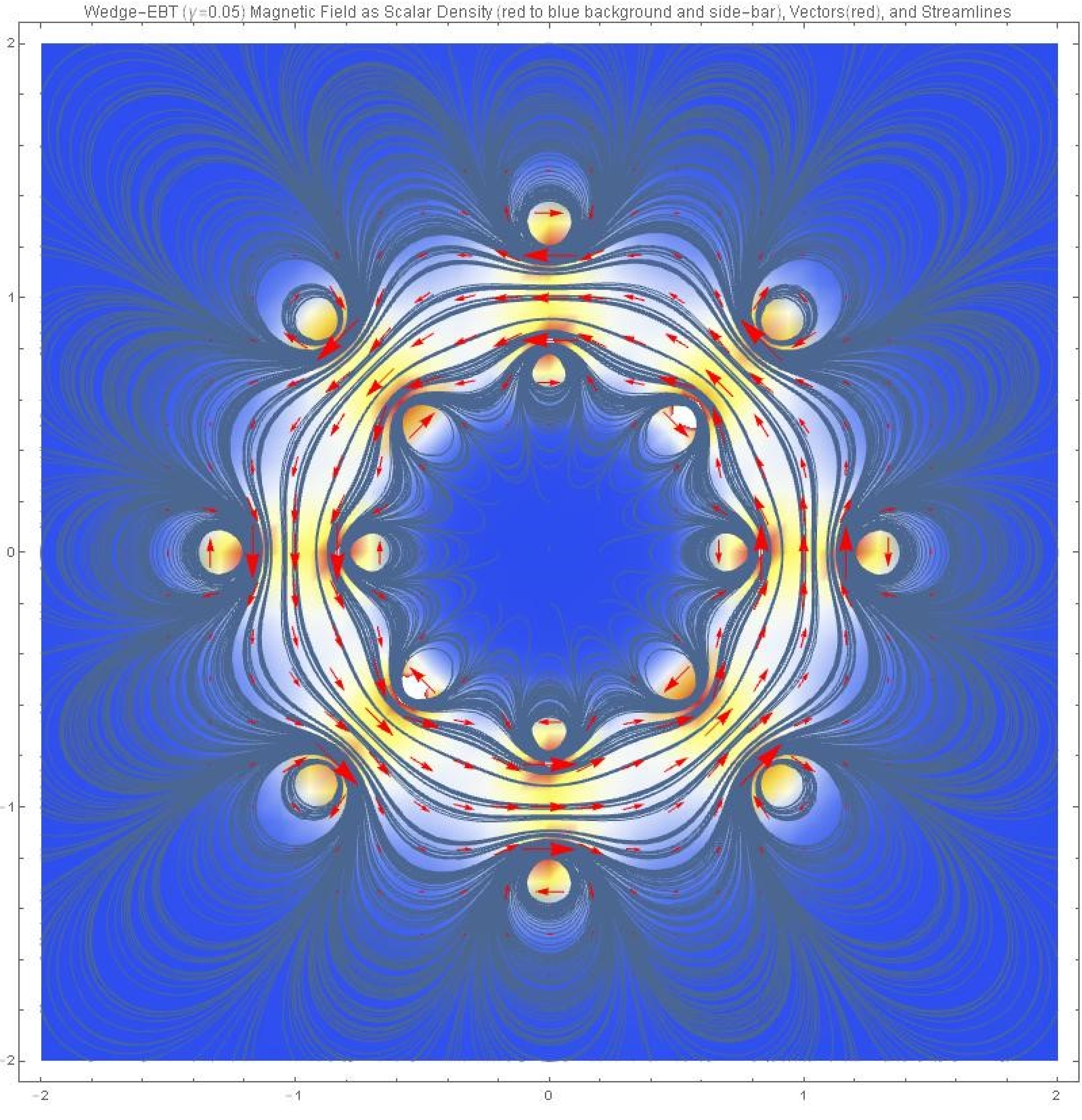
EBT Magnetic Field as Scalar Density (red to blue background and side-bar), Vectors(red), and Streamlines



Plotting the EBT with wedge coils for two values of γ with Iwedge=2500A, I=10000A, Rmaj=1m, and a=0.25m:

In[24]:=
```
Show[{DensityPlot[
   Norm[ElmoBumpyCartWedge[X, Y, 1.2566*^-6, 10 000, 8, 1, 0.25, 0.05, 2500]],
   {X, -2, 2}, {Y, -2, 2}, ColorFunction → "TemperatureMap",
   PlotPoints → 35, PlotLegends → Automatic,
   PlotLabel → "Wedge-EBT (γ=0.05) Magnetic Field as Scalar Density (red to
      blue background and side-bar), Vectors(red), and Streamlines"],
  StreamPlot[ElmoBumpyCartWedge[X, Y, 1.2566*^-6, 10 000, 8, 1, 0.25, 0.05, 2500],
   {X, -2, 2}, {Y, -2, 2},
   StreamPoints → {Tuples[Range[-2, 2, 0.2], 2], Automatic, 10},
   StreamStyle → "Line", VectorPoints → Fine,
   VectorPoints → 10, VectorStyle → Red], ImageSize → 800}]
```

In[23]:=
```
Show[{DensityPlot[
   Norm[ElmoBumpyCartWedge[X, Y, 1.2566*^-6, 10 000, 8, 1, 0.25, 0.3, 2500]],
   {X, -2, 2}, {Y, -2, 2}, ColorFunction → "TemperatureMap",
   PlotPoints → 35, PlotLegends → Automatic,
   PlotLabel → "Wedge-EBT (γ=0.3) Magnetic Field as Scalar Density (red to
      blue background and side-bar), Vectors(red), and Streamlines"],
  StreamPlot[ElmoBumpyCartWedge[X, Y, 1.2566*^-6, 10 000, 8, 1, 0.25, 0.3, 2500],
   {X, -2, 2}, {Y, -2, 2},
   StreamPoints → {Tuples[Range[-2, 2, 0.2], 2], Automatic, 10},
   StreamStyle → "Line", VectorPoints → Fine,
   VectorPoints → 10, VectorStyle → Red], ImageSize → 800}]
```


Wedge-EBT (γ=0.05) Magnetic Field as Scalar Density (red to blue background and side-bar), Vectors(red), and Streamlines
2
1
0
-1
-2
-2
-1
0
1
2

Wedge-EBT (γ=0.3) Magnetic Field as Scalar Density (red to blue background and side-bar), Vectors(red), and Streamlines