Introduction to Bayesian Data Analysis
Tutorial 1

(1) Redo example I from class (estimating the probability of a rare event) but try different prior distributions and assess how your posterior inference changes. For example, you might try a beta(40,400). Also try a beta(2,2) prior. What about a Uniform(0,1) prior? How about a Uniform(0.05,0.20) prior? Discuss and interpret your findings.

(2) Conditional probability: suppose that if $\theta = 1$, the y has a normal distribution with mean 1 and standard deviation σ, and if $\theta = 2$, then y has a normal distribution with mean 2 and standard deviation σ. Also, suppose $\Pr(\theta = 1) = 0.5$ and $\Pr(\theta = 2) = 0.5$

  (a) For $\sigma = 2$, write the formula for the marginal probability density for y and sketch it.

  (b) What is $\Pr(\theta = 1 | y = 1)$, again supposing $\sigma = 2$?

  (c) Describe how the posterior density $\Pr(\theta = 1 | y = 1)$ changes in shape as σ is increased and as it is decreased

(3) An airline company uses past data to estimate the probability that a passenger who is scheduled to take a particular flight, fails to show up. Let p denote this probability. You are given the following prior distribution on p:

p	0	0.025	0.05	0.075	0.10
$g(p)$	0.80	0.10	0.05	0.035	0.015

  (a) Suppose the for the next flight, 60 tickets have been sold, but only 55 passengers turned up. What is the posterior probability that all passengers show up? What is the posterior probability that 10% of passengers don't show up?

  (b) Suppose for the next ten flights, 60 tickets have been sold on each flight, and the number of passengers who actually turned up on each flight is shown in the table below:

Flight	1	2	3	4	5	6	7	8	9	10
No. passengers on flight	58	55	59	54	56	57	57	50	52	60

  What is the posterior mode?

  (c) Suppose the airline does not update its assumptions on boarding rates of sold tickets. Furthermore, assume that the airline engages in the practice of overbooking seats to make up for passengers who do not show up, to ensure that the flight is as close to full capacity as possible and thus maximise profits. Discuss the implications on profitability of the company in light of your Bayesian analysis.