Tut5

August 28, 2023

1 Tutorial 5

1.1 6.1

(a)

$$\begin{aligned}&P(\theta_A, \theta_B)\\=&P(\theta, \gamma)|\frac{\partial(\theta_A, \theta_B)}{\partial(\theta, \gamma)}|\\=&P(\theta)P(\gamma)\theta\\=&P_\theta(\theta_A)P_\gamma(\frac{\theta_B}{\theta_A})\theta_A\end{aligned}$$

$$\begin{aligned}&P(\theta_A)P(\theta_B)\\=&P_\theta(\theta_A)\int_0^\infty \frac{1}{\theta}P_\gamma(\frac{\theta_B}{\theta})P_\theta(\theta)d\theta\end{aligned}$$

In most circumstance,the part $P_\gamma(\frac{\theta_B}{\theta_A})\theta_A$ is a fuction of θ_A and θ_B since the $P_\gamma(\frac{\theta_B}{\theta_A})$ is the density of Gamma distribution. Thus the part $P_\gamma(\frac{\theta_B}{\theta_A})\theta_A$ is unequal to the part $P(\theta_B)$ which is only function of θ_B

So in most circumstance

$$P(\theta_A, \theta_B) \neq P(\theta_A)P(\theta_B)$$

which means the θ_A and θ_B are dependent.But with some restrictions to hyperparameters we may get the independent θ_A and θ_B.The restriction can be derive as below.

The keneral of $P_\gamma(\frac{\theta_B}{\theta_A})\theta_A$ is $(\frac{\theta_B}{\theta_A})^{(a_\gamma-1)}e^{-(b_\gamma\frac{\theta_B}{\theta_A})}\theta_A$. To make it become only a function of θ_B , we need $a_\gamma = 2, b_\gamma = 0$.

(b)

$$\begin{aligned}&P(\theta|\mathbf{y}_A, \mathbf{y}_B, \gamma)\\\propto&P(\theta)P(\mathbf{y}_A|\theta)P(\mathbf{y}_B|\theta, \gamma)\\\propto&\theta^{a_\theta-1}e^{-(b_\theta\theta)}\theta^{\sum_A y_A+\sum_B y_B}e^{-(n_A\theta+n_B\theta\gamma)}\\\propto&\theta^{[(a_\theta+\sum_A y_A+\sum_B y_B)-1]}e^{-\theta(b_\theta+n_A+n_B\gamma)}\end{aligned}$$

So the form is

$$P(\theta|\mathbf{y}_A,\mathbf{y}_B,\gamma)\sim\Gamma(a_\theta+\sum_{i=1}^{58}y_{Ai}+\sum_{j=1}^{218}y_{Bj},b_\theta+58+218\gamma)$$

[ ]:
```
y_A<-read.table('menchild30bach.txt')
y_B<-read.table('menchild30nobach.txt')
nA<-nrow(y_A)
nB<-nrow(y_B)
c(nA,nB)
```

1. 58 2. 218

(c)

$$\begin{aligned}
&P(\gamma|\mathbf{y}_A,\mathbf{y}_B,\theta)\\
\propto&P(\gamma)P(\mathbf{y}_B|\theta,\gamma)\\
\propto&\gamma^{a_\gamma-1}e^{-(b_\gamma\gamma)}\gamma^{\sum_B y_B}e^{-n_B\theta\gamma}\\
\propto&\gamma^{a_\gamma+\sum_{j=1}^{218}y_{Bj}-1}e^{-(b_\gamma+n_B\theta)\gamma}
\end{aligned}$$

So the form is

$$P(\gamma|\mathbf{y}_A,\mathbf{y}_B,\theta)\sim\Gamma(a_\gamma+\sum_{j=1}^{218}y_{Bj},b_\gamma+218\theta)$$

(d)

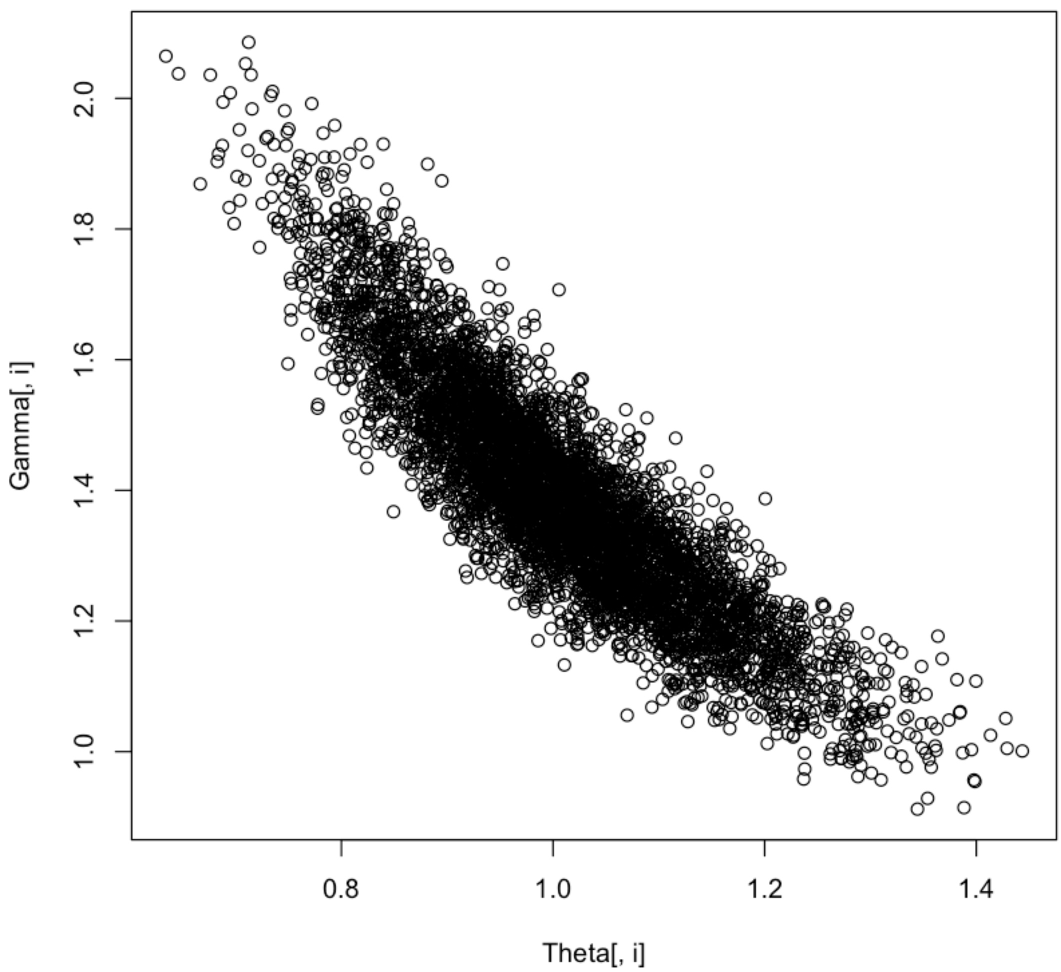
The bigger parameters are, the smaller $E[\theta_B-\theta_A|\mathbf{y}_A,\mathbf{y}_B]$ is.

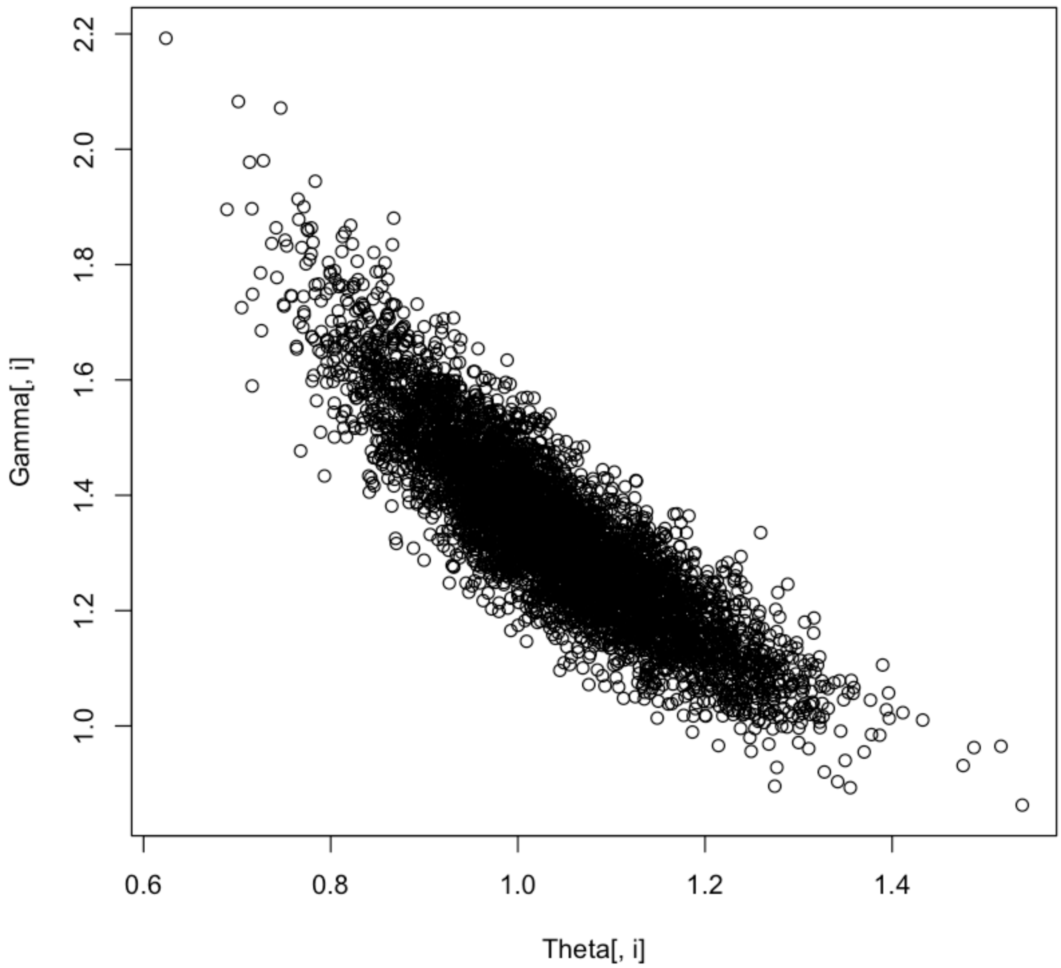
[ ]:
```
a_theta<-2
b_theta<-1
a_gamma<-c(8,16,32,64,128)
b_gamma<-a_gamma
sum_yA<-sum(y_A)
sum_yB<-sum(y_B)
n_ite<-5000
par_initial<-1
theta<-par_initial
gamma<-par_initial
Theta<-matrix(nrow=n_ite,ncol=length(a_gamma))
Gamma<-Theta
E<-NULL
for (j in 1:length(a_gamma)){
    a_g<-a_gamma[j]
    b_g<-a_g
    for (i in 1:n_ite){
        theta<-rgamma(1,shape=a_theta+sum_yA+sum_yB,rate=b_theta+nA+nB*gamma)
        gamma<-rgamma(1,shape=a_g+sum_yB,rate=b_g+nB*theta)
        Theta[i,j]<-theta
        Gamma[i,j]<-gamma
```

```
    }
}
# E<-rowSums(Theta*Gamma-Theta)
E<-colSums(Theta*Gamma-Theta)/n_ite
E
```

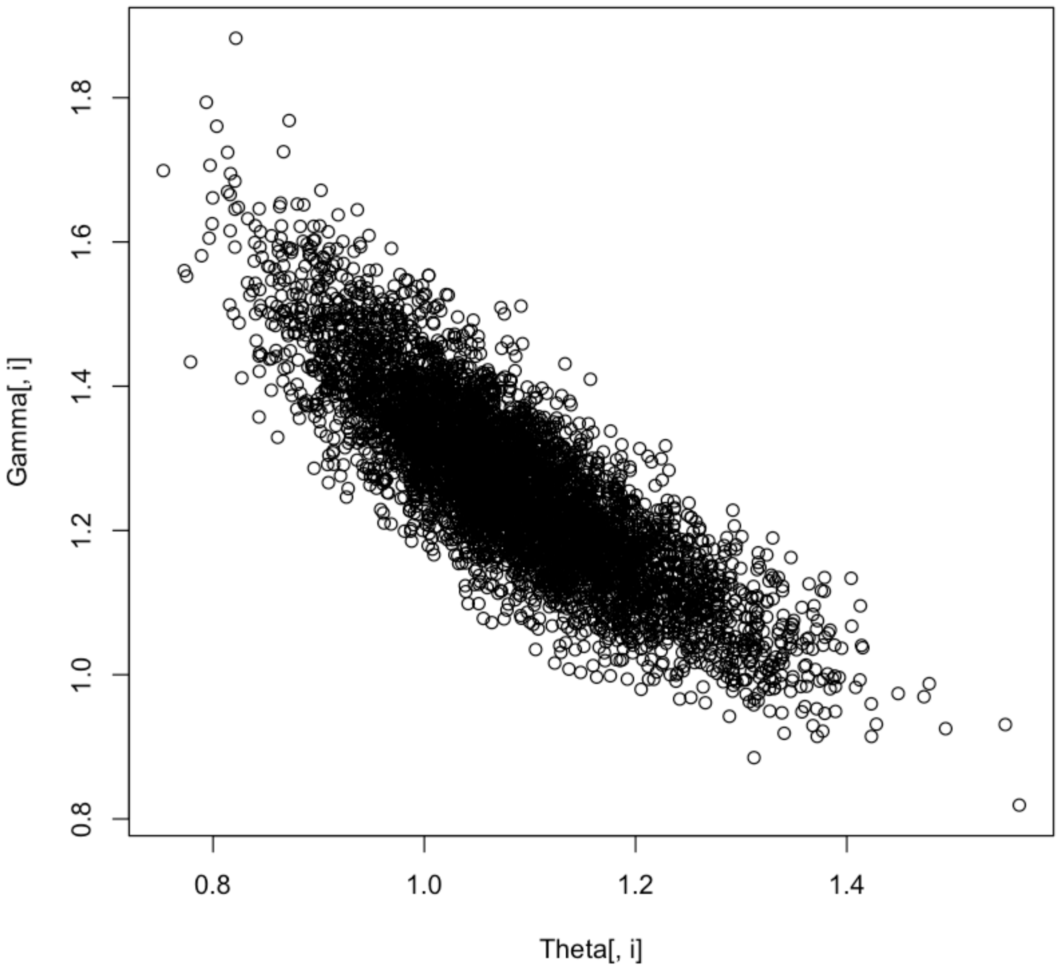
1. 0.37764605403732 2. 0.334444596751819 3. 0.273652883952121 4. 0.20051808232011 5. 0.133719958562493

[ ]:
```
par(bg='white')
for (i in 1:5){
    plot(Theta[,i],Gamma[,i])
}
```

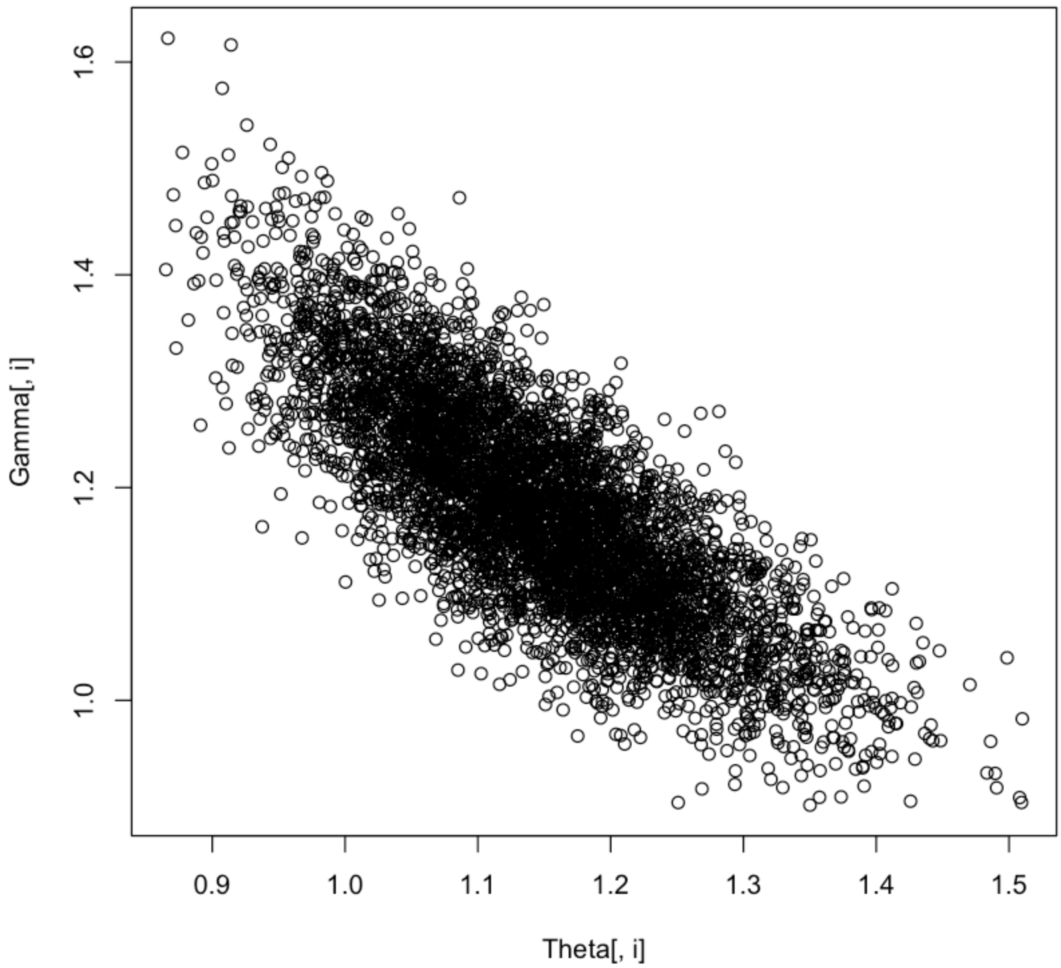




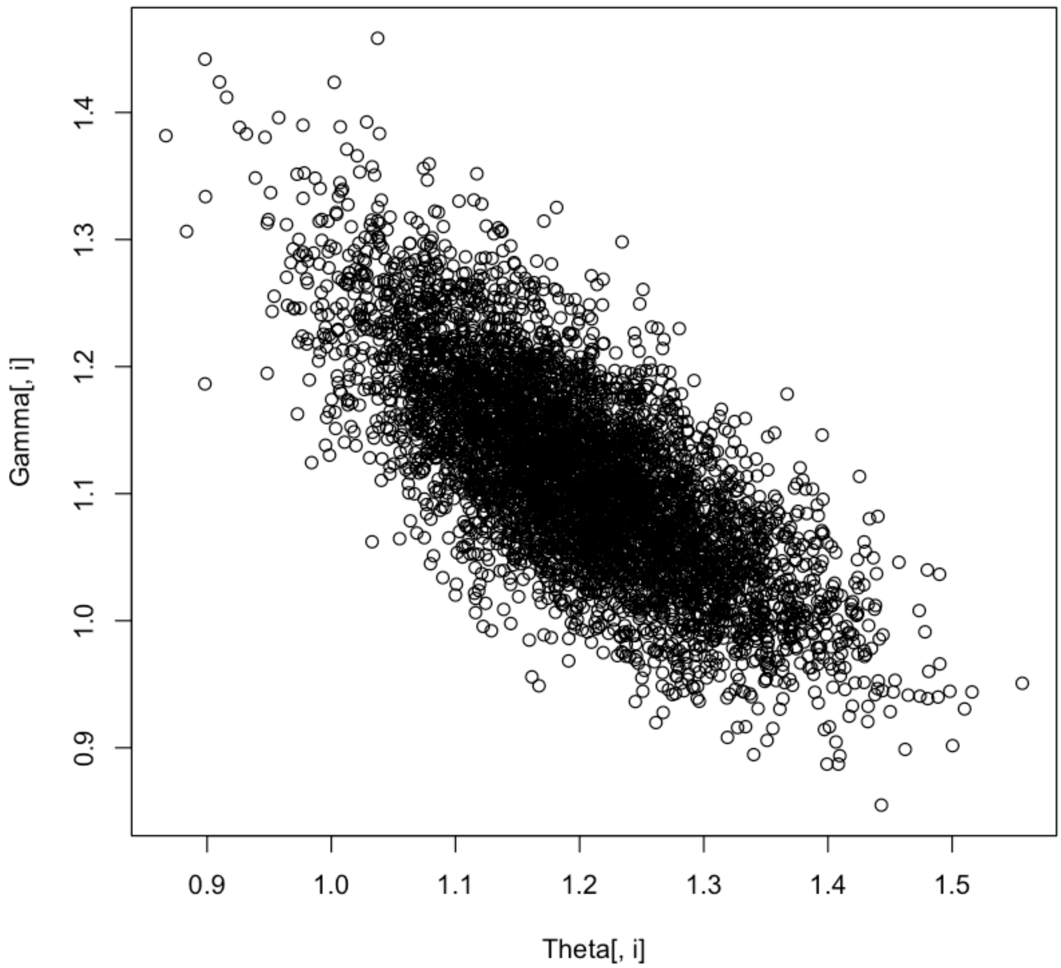
Gamma[, i]
Theta[, i]
0.6
0.8
1.0
1.2
1.4
1.0
1.2
1.4
1.6
1.8
2.0
2.2



Gamma[, i]
Theta[, i]
0.8
1.0
1.2
1.4
1.6
1.8



Gamma[, i]
1.6
1.4
1.2
1.0
0.9
1.0
1.1
1.2
1.3
1.4
1.5
Theta[, i]



Gamma[, i]
0.9
1.0
1.1
1.2
1.3
1.4
0.9
1.0
1.1
1.2
1.3
1.4
1.5
Theta[, i]