assignment

October 19, 2023

1 Answer 1

(a)The code is below.

[ ]:
```
library(mvtnorm)
Y_obs0 <- Y_obs<- read.csv("CitySurvey.csv")
Y_obs[,1]<-log(Y_obs0[,1])
rinwish <- function(n, nu0, iS0) {
    sL0 <- chol(iS0)
    S <- array(dim = c(dim(iS0), n))
    for (i in 1:n)
    {
        Z <- matrix(rnorm(nu0 * dim(iS0)[1]), nu0, dim(iS0)[1]) %*% sL0
        S[, , i] <- solve(t(Z) %*% Z)
    }
    return(S[, , 1:n])
}
Y_obs.mean <- mu_0 <- apply(Y_obs,2, mean)
n<-nrow(Y_obs)
S0 <- Rambda0 <- Sigma <- matrix(cov(Y_obs),nrow = 2)
nu0<-dim(Y_obs)[2]+2
nu0
```

4

[ ]:
```
THETA<-NULL
S<-10000
SIGMA<- array(dim = c(dim(S0),S/2 ))
Y_syn<-NULL
set.seed(101)

for (s in 1:S){
  Rambda_n<-solve(solve(Rambda0)+n*solve(Sigma))
  mu_n<-Rambda_n%*%(solve(Rambda0)%*%mu_0+n*solve(Sigma)%*%Y_obs.mean)
  theta<-rmvnorm(1,mu_n,Rambda_n)

  Sn<-S0+(t(Y_obs)-c(theta))%*%t(t(Y_obs)-c(theta))
  Sigma<-rinwish(1,nu0+n,solve(Sn))
```

```
  if(s >S/2){
    y<-rmvnorm(1,theta,Sigma)
    y[1]<-exp(y[1])
    Y_syn<-rbind(Y_syn,y)
    THETA<-rbind(THETA,theta)
    SIGMA[,,s-(S/2)]<-Sigma
  }
}
```

(b)From the below statistics we can see the assumed synthetic data generation have relatively close values in mean ,standard deviation,5th,25th,50th(median),75th and 95th percentile.The difference between the 2 groups on the minimum and the maximum are shown relatively large in this case.Besides, the minimum value of age is negative which is not possible.

[ ]:
```
sqrt(cov(Y_obs0)[1,1])
sqrt(cov(Y_syn)[1,1])
sqrt(cov(Y_obs0)[2,2])
sqrt(cov(Y_syn)[2,2])
```

37870.5506244862

35811.0168154886

13.262828164961

13.1638784119039

[ ]:
```
quantile(Y_obs0[,1],c(0,0.05,0.25,0.5,0.75,0.95,1))
quantile(Y_syn[,1],c(0,0.05,0.25,0.5,0.75,0.95,1))
```

0\% 24600 **5\%** 32640 **25\%** 46000 **50\%** 60000 **75\%** 90000 **95\%** 150000 **100\%** 214500

0\% 10615.8049865264 **5\%** 30526.962948629 **25\%** 47237.5482359176 **50\%** 63951.7985366351 **75\%** 87781.7351094319 **95\%** 139558.688279509 **100\%** 364586.389378802

[ ]:
```
quantile(Y_obs0[,2],c(0,0.05,0.25,0.5,0.75,0.95,1))
quantile(Y_syn[,2],c(0,0.05,0.25,0.5,0.75,0.95,1))
```

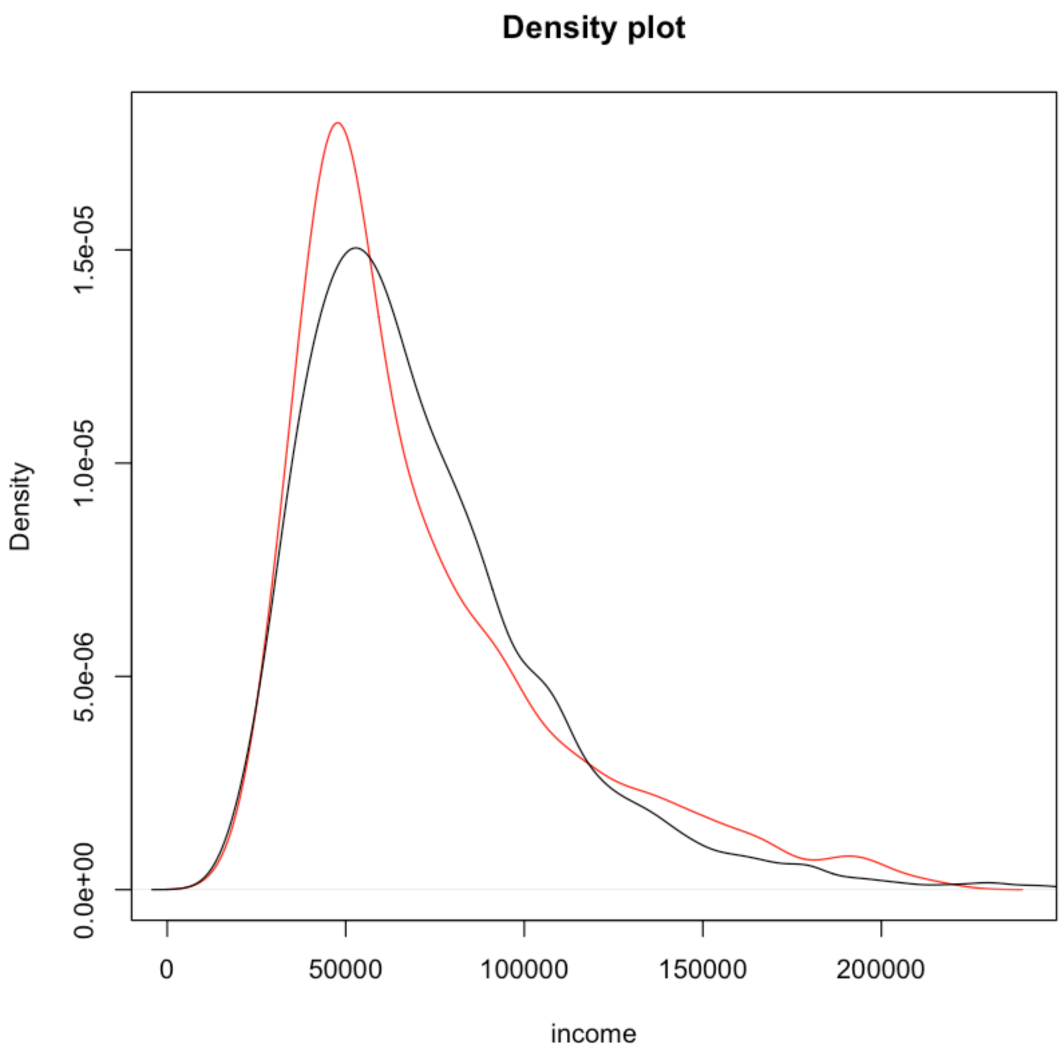
0\% 18 **5\%** 24 **25\%** 33 **50\%** 46 **75\%** 54 **95\%** 64.8 **100\%** 84

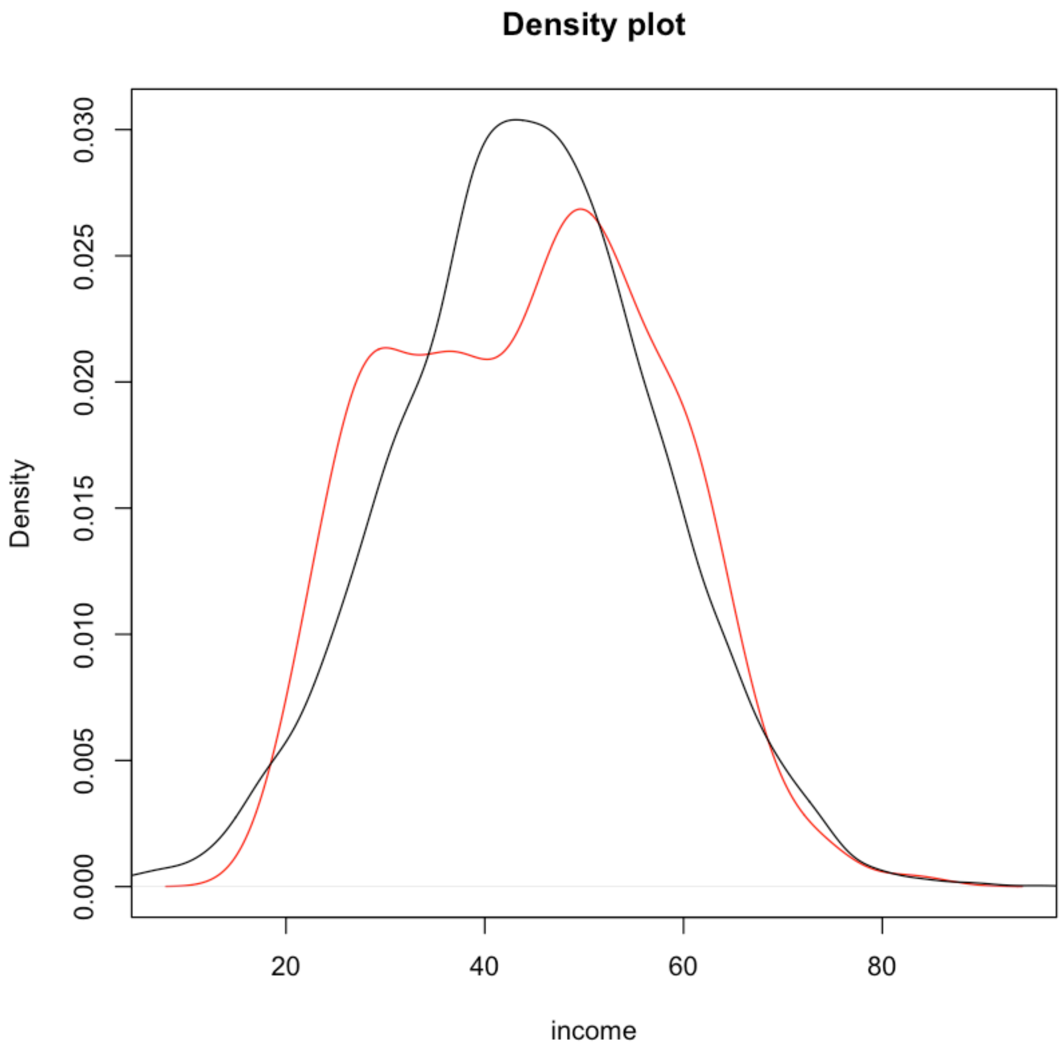
0\% -7.55255185365368 **5\%** 22.5089890842929 **25\%** 35.7554805613738 **50\%** 44.349860442718 **75\%** 53.0758998185895 **95\%** 65.8645229797877 **100\%** 95.8674065425699

(c)The plot are shown below. We see that the distrubtion of the Y_sys share smiliar figure especialy on income and age espeically the middle part whcih may be a 90% interval .However , when it comes to extreme value, it may cause people to derive poor conculsion because the large difference.

[ ]:
```
par(bg='white')
plot(density(Y_obs0[,1]),xlab='income',col='red',main='Density plot')
lines(density(Y_syn[,1]))
plot(density(Y_obs0[,2]),xlab='income',ylim=c(0,max(density(Y_syn[,2])[[2]])),col='red',main='
  ↪plot')
```

```
lines(density(Y_syn[,2]))
```





2 Answer 2

(a) The logistic model can be presented as below

$$Pr(Y_i = 1|\theta_i) = \frac{exp(\theta_i)}{1 + exp(\theta_i)}$$

where θ is the linear model shown as

$$\theta_i = X_i\bar{\beta} + \beta_0$$

. X_i is a vector of covariates and $\bar{\beta}$ is a vector of coefficiens except intercept which is β_0.

When we make a model selection the linear part turns into

$$\theta_i = X_i(\gamma\bar{\beta}) + \beta_0$$

where γ is a diagonal matrix with 2 elements $\{1, 0\}$ or we can view whole $\gamma\bar{\beta}$ as a vector with i th elements as $\gamma_i\beta_i$ where indicator $\gamma_i = 1$ or 0 means whether to eliminate coefficient $\beta_i (i \neq 0)$.

(b) First , I make the covarites to minus mean and divided by variance so that different covariates have the same scale.Since they have the same scale .Then, we can assume that the $\beta_i (i \neq 0)$ coefficients have independent identical piror distribution $Norm(0, 2^2)$ .Since β_0 is intercept , I assume it follows $Norm(0, 4^2)$.And $\gamma_i \sim Bern(1/2)$.

(c) The main steps of Metropolis-Hastings algorithm in this case is in every iteration first generating new γ_i one by one with Gibbs sampler since γ_i is the Bern distribution.Then we generate the $\beta = (\beta_0, \bar{\beta}^T)^T$ with Metropolis-Hastings algorithm.To make a easier sampling (can also generate the whole β)here I use $J(\beta^*|\beta^{(s)}) \sim MVN(\beta^{(s)}, \nu\Sigma)$ which is a symmetric distribution meaning do no contribution to acceptance rates r.

(d)I use $\nu = 0.5$ as my tuning parameter value ,after about 3000 interations most β_i comes to a relatively stationary status and acceptance rates is 32 after we burn in the first 3000 iterations.When $\nu = 2$,steps may be too big ,the acceptacne is only about 14.

[ ]:
```
p2<-read.csv('Recidivism.csv')
set.seed(101)
p2$X...Gender<- ifelse(p2$X...Gender=='M' ,1,0)
p2$Race<- ifelse(p2$Race=='WHITE' ,1,0)
p2$Recidivism_Arrest_Year1<- ifelse(p2$Recidivism_Arrest_Year1=='TRUE',1,0)
# VAR<-var(p2)
for(i in 1:(ncol(p2)-1)){
    p2[,i]<-(p2[,i]-mean(p2[,i]))/var(p2[,i])
}
x<-as.matrix(p2[,1:10])
y<-as.matrix(p2[,11])
# p2$X...Gender <-factor(p2$X...Gender)
# p2$Race <-factor(p2$Race)
# p2$Recidivism_Arrest_Year1 <-factor(p2$Recidivism_Arrest_Year1)
attach(p2)
model.fit<-glm(y ~ 1+x ,family = binomial(link=logit))
#glm(Recidivism_Arrest_Year1 ~ X...
 ↪Gender+Race+Age_at_Release+Supervision_Level+Educ_HighSchool+Educ_College+Dependents+Prison
 ↪= binomial(link=logit))
summary.coe<-summary(model.fit)$coefficients
detach(p2)
```

[ ]:
```
summary.coe
p<-nrow(summary.coe)
p
```

A matrix: 11 x 4 of type dbl

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-0.91133221	0.014337840	-63.561332	0.000000e+0
xX...Gender	0.04881845	0.005144721	9.489039	2.331743e-21
xRace	-0.02204699	0.007126481	-3.093671	1.976965e-03
xAge_at_Release	-3.86139141	0.190553337	-20.264098	2.668070e-91
xSupervision_Level	0.03973130	0.007427567	5.349168	8.835967e-08
xEduc_HighSchool	0.01291468	0.007511832	1.719245	8.556979e-02
xEduc_College	-0.02703406	0.006360042	-4.250610	2.131892e-05
xDependents	-0.01311463	0.006482841	-2.022975	4.307569e-02
xPrison_Years_plus2	-0.08373527	0.007183829	-11.656077	2.136610e-31
xPrior_Conviction_Felony	0.07069621	0.006924299	10.209873	1.790996e-24
xPrior_Conviction_Misd	0.10918468	0.007098911	15.380482	2.212925e-53

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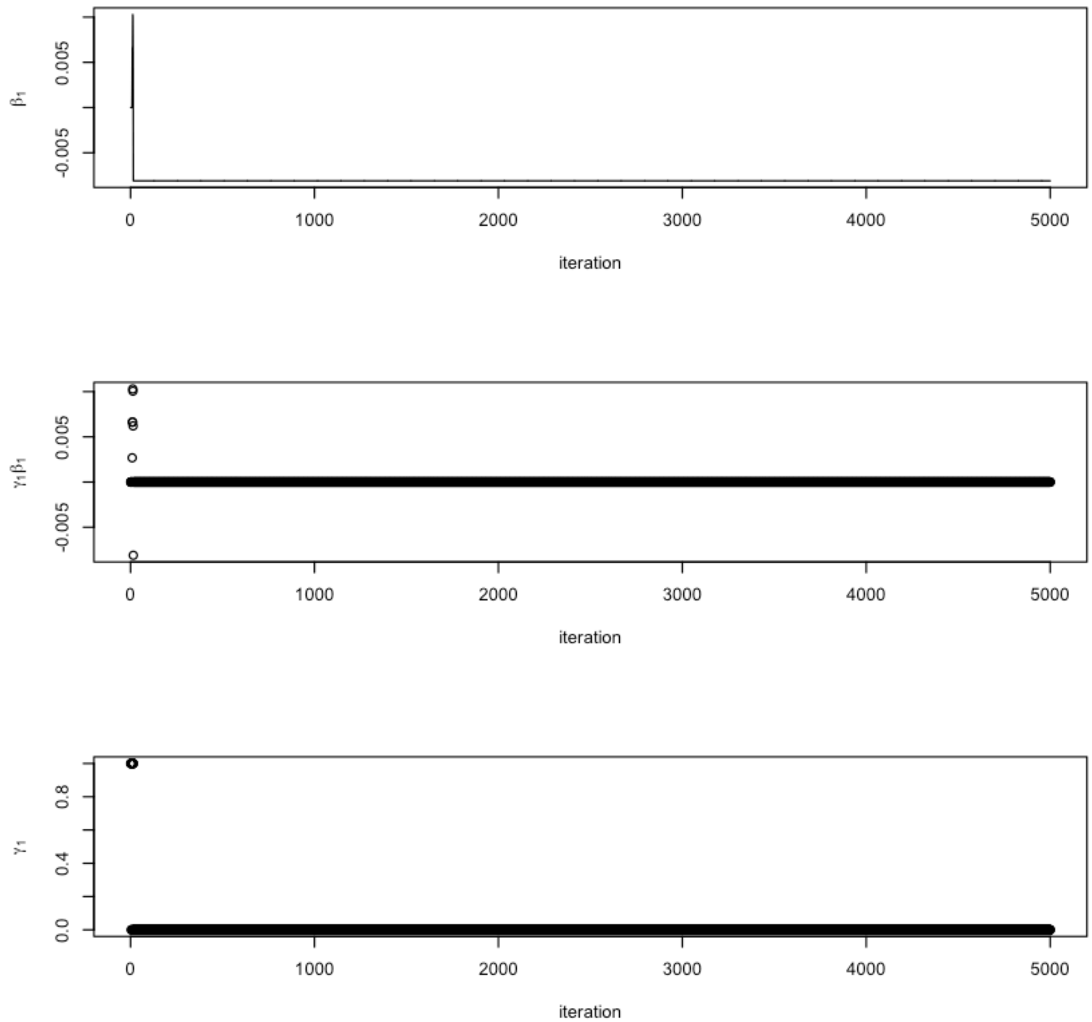
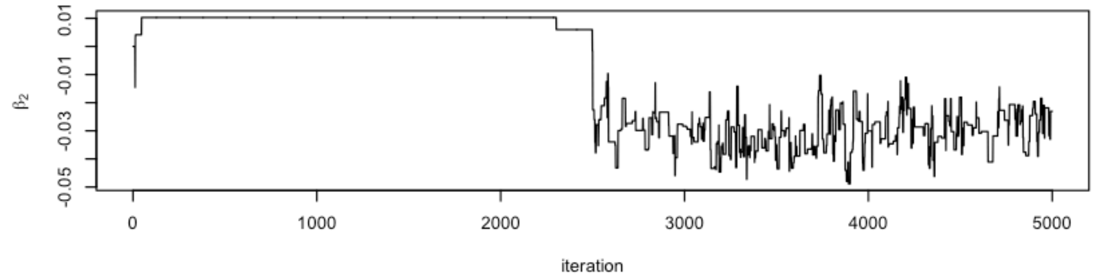
[ ]:
```
ilogit<-function(theta){
  out<-exp(theta)/(1+exp(theta))
  return(out)
}
```

[ ]:
```
beta.pm<-rep(0,p)
beta.psd<-c(4,rep(2,p-1))
gamma<- rbinom(p-1,1,0.5)
beta<- rep(0,p)#summary.coe[,1]
beta.var<-summary(model.fit)$cov.unscaled
S2<- 1000*5
Beta<-P<-NULL
Gamma<-NULL
nu<-2
acc<- NULL
for(i in 1:S2){
    for(j in 1:(p-1)) {
        new_gamma<- gamma
        new_gamma[j]<- 1-gamma[j]
        p0<-ilogit(x%*%(matrix(beta[-1]*gamma))+beta[1])
        p1<-ilogit(x%*%(matrix(beta[-1]*new_gamma))+beta[1])
        logp <- sum((dbinom(y,1,p1,log=T)-dbinom(y,1,p0,log=T)))
        pj<-rbinom(1,1,1/(1+exp(-logp)))
        gamma[j]<- pj*new_gamma[j]+(1-pj)*gamma[j]
        }
    new_beta<- rmvnorm(1,beta,nu*beta.var)
    p0<-ilogit(x%*%(matrix(beta[-1]*gamma))+beta[1])
    p1<-ilogit(x%*%(matrix(new_beta[-1]*gamma))+new_beta[1])
    # jbeta<-dmvnorm(beta,new_beta,nu*beta.var,log=T)
    # new_jbeta<- dmvnorm(new_beta,beta,nu*beta.var,log=T)
```

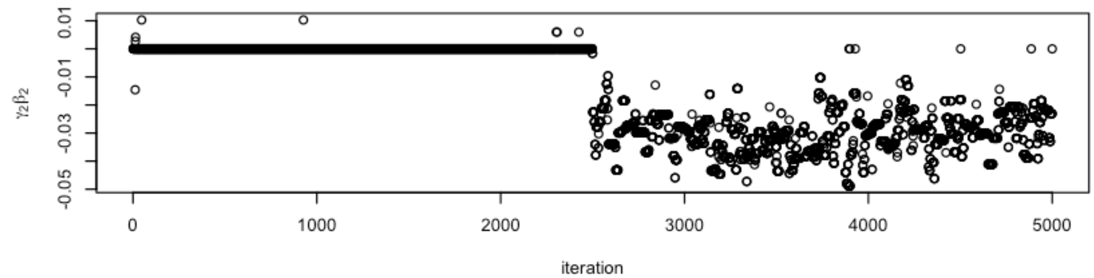
```
    logp <-␣
↪sum(dbinom(y,1,p1,log=T)-dbinom(y,1,p0,log=T))+sum((dnorm(new_beta,beta.
↪pm,beta.psd,log=T)-dnorm(beta,beta.pm,beta.
↪psd,log=T))*c(1,gamma))#-new_jbeta+jbeta
    if(log(runif(1))<logp){
        beta<-new_beta*c(1,gamma)+beta*c(0,1-gamma)
        acc <- c(acc,1)
    }
    else{
        acc<-c(acc,0)
    }
    Beta<-rbind(Beta,beta)
    Gamma<-rbind(Gamma,gamma)
}

mean(acc[3000:S2])
par(bg='white',mfrow=c(3,1))
for(i in 1:(p-1)){
    plot(Beta[,i+1],type='l',ylab=bquote(beta[.(i)]),xlab='iteration')
    plot(Gamma[,i]*Beta[,i+1],ylab=bquote(gamma[.(i)]*beta[.
↪(i)]),xlab='iteration')
    plot(Gamma[,i],ylab=bquote(gamma[.(i)]),xlab='iteration')
}
plot(Beta[,0+1],type='l',ylab=bquote(beta[.(0)]),xlab='iteration')
```

0.144427786106947

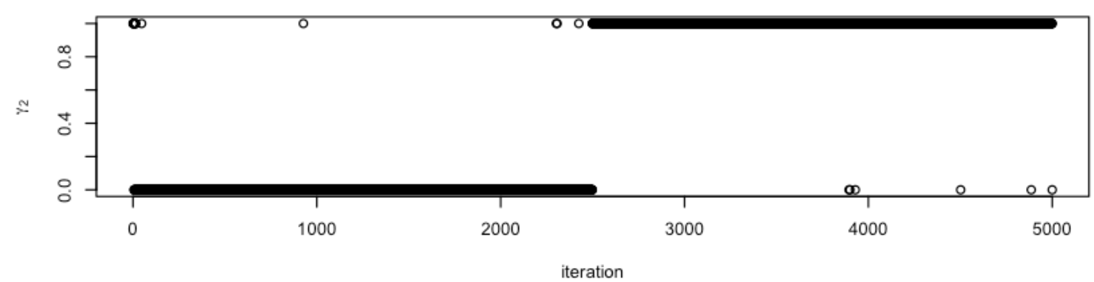

β_1
0.005
-0.005
0
1000
2000
3000
4000
5000
iteration
$\gamma_1\beta_1$
0.005
-0.005
0
1000
2000
3000
4000
5000
iteration
γ_1
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



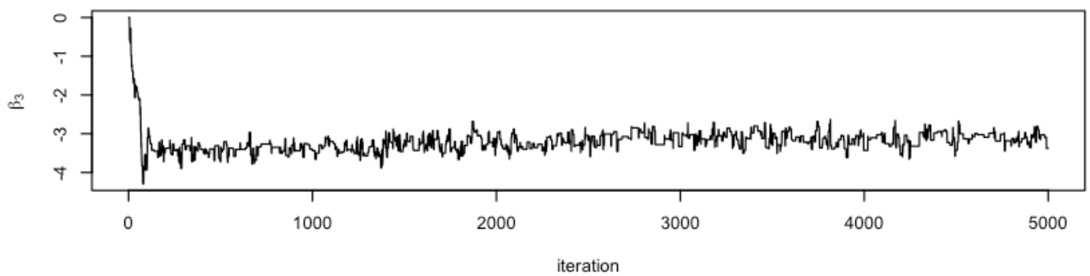
β2
0.01
-0.01
-0.03
-0.05
0
1000
2000
3000
4000
5000
iteration



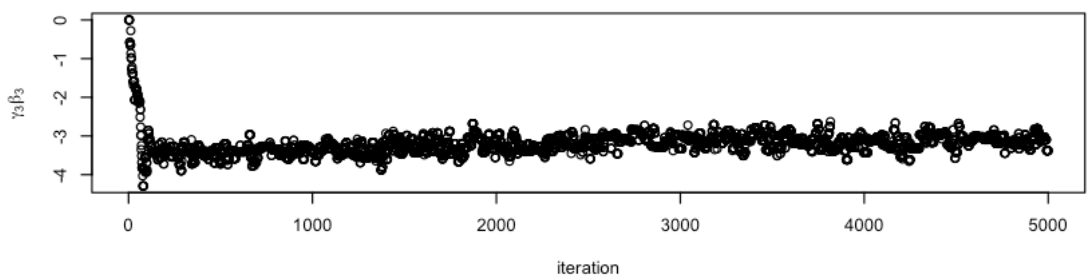
γ2β2
0.01
-0.01
-0.03
-0.05
0
1000
2000
3000
4000
5000
iteration



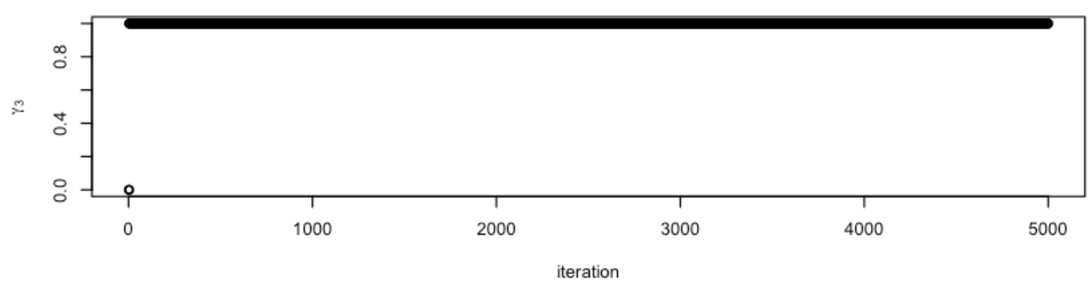
γ2
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



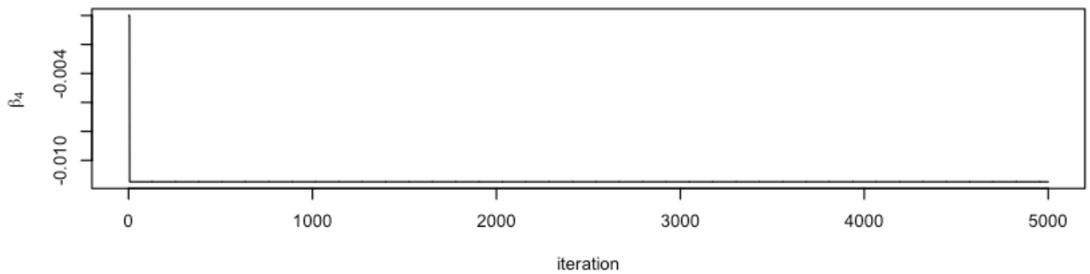
0
-1
-2
-3
-4
β3
0
1000
2000
3000
4000
5000
iteration



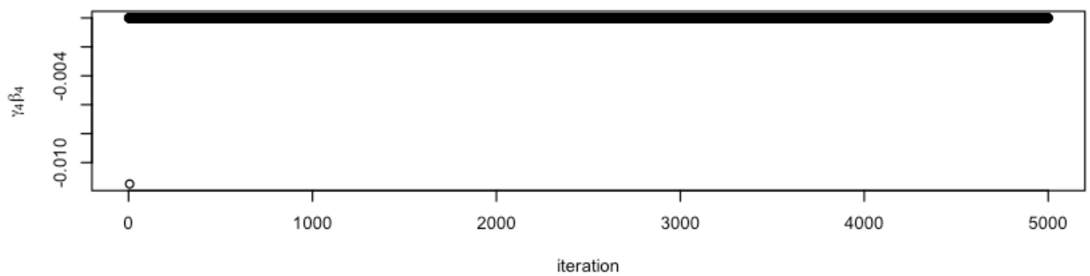
0
-1
-2
-3
-4
γ3β3
0
1000
2000
3000
4000
5000
iteration



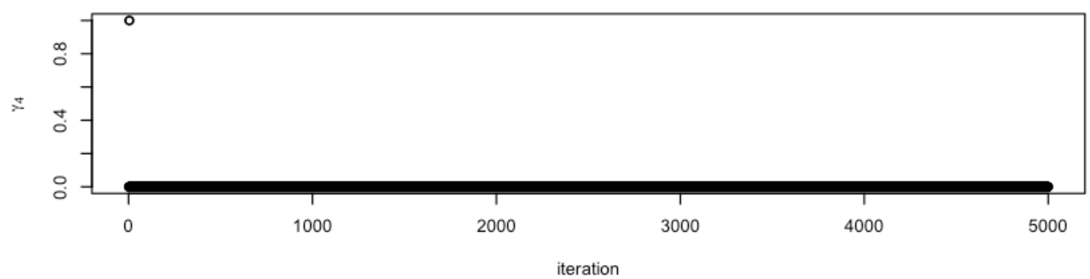
0.0
0.4
0.8
γ3
0
1000
2000
3000
4000
5000
iteration



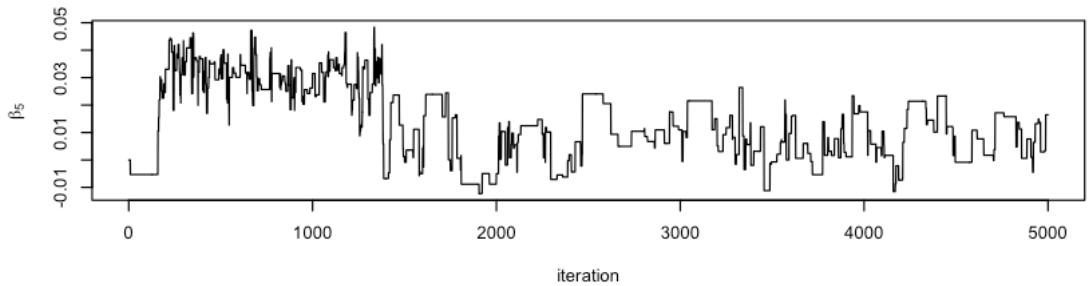
β_4
-0.004
-0.010
0
1000
2000
3000
4000
5000
iteration

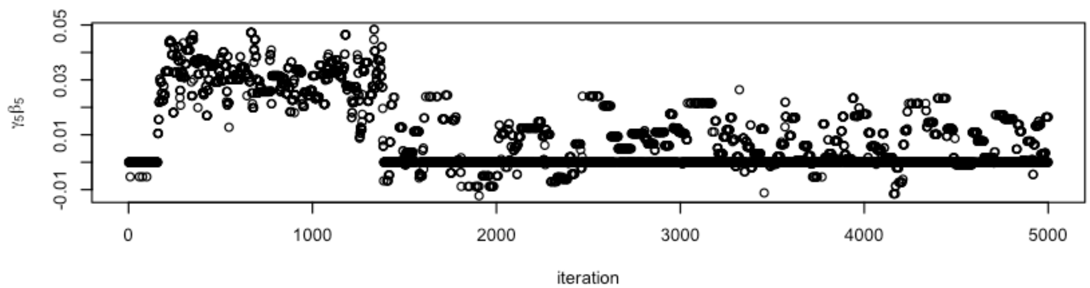


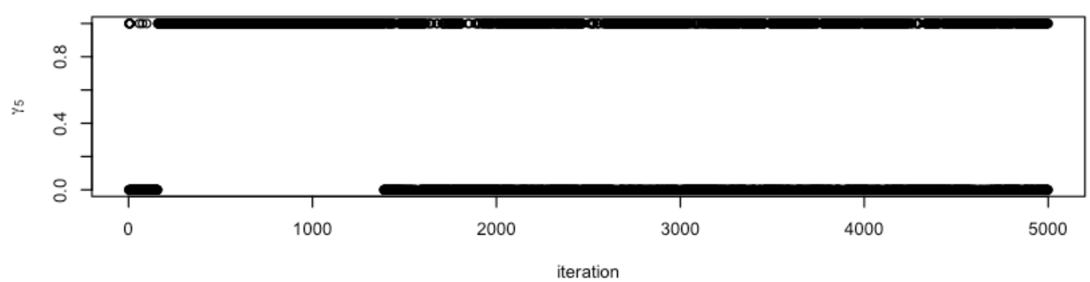
$\gamma_4\beta_4$
-0.004
-0.010
0
1000
2000
3000
4000
5000
iteration

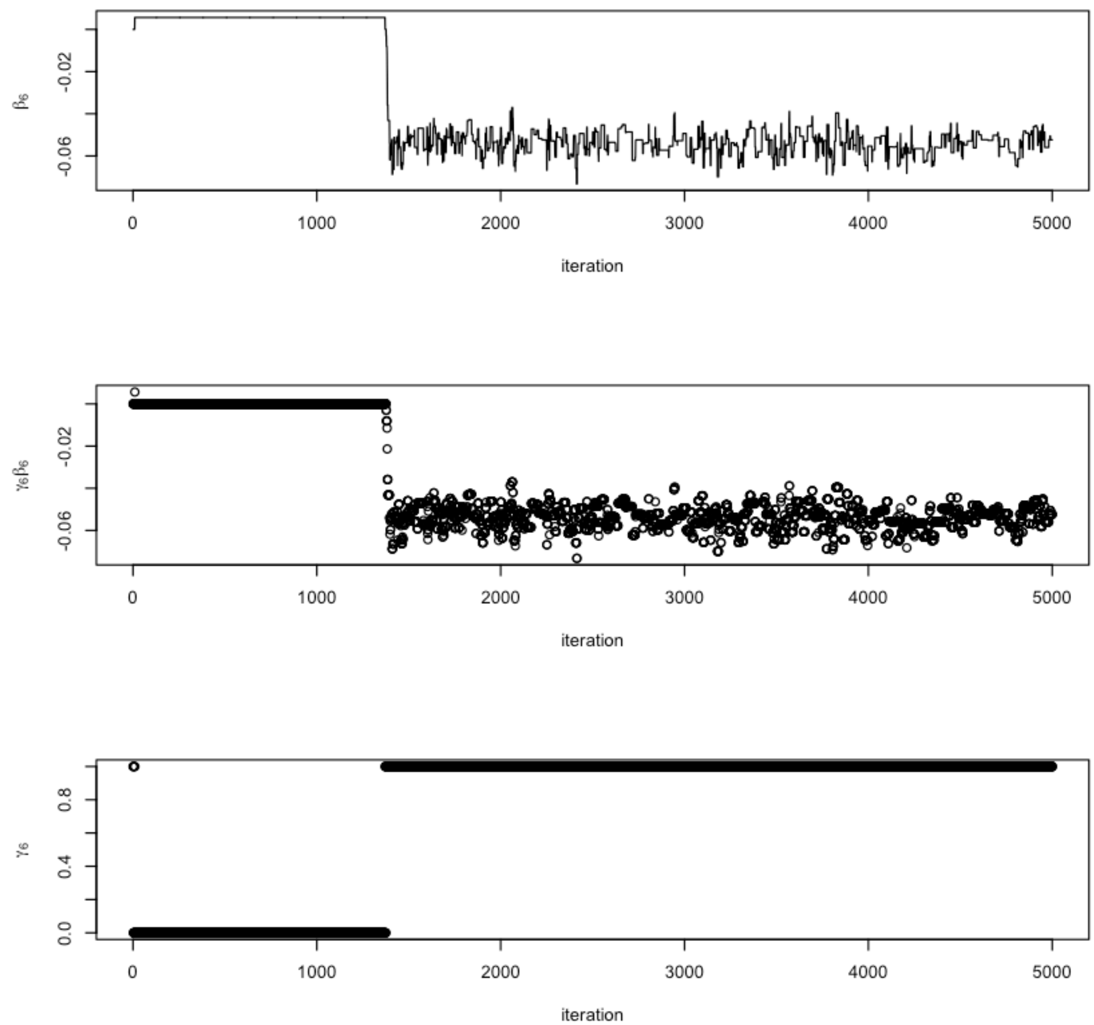


γ_4
0.8
0.4
0.0
0
1000
2000
3000
4000
5000
iteration

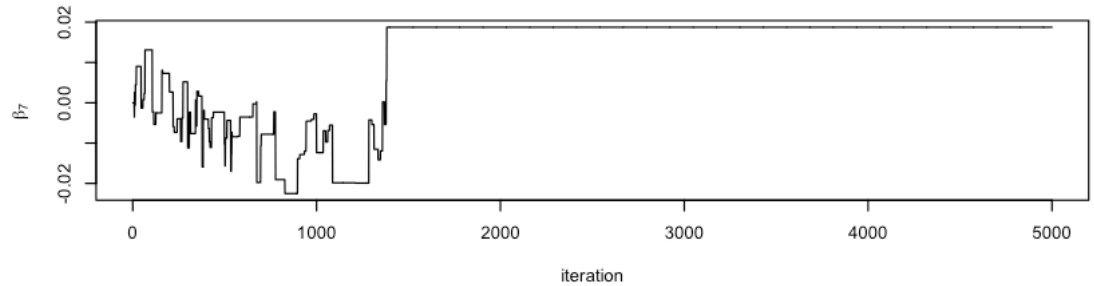

β_5
-0.01
0.01
0.03
0.05
0
1000
2000
3000
4000
5000
iteration


$\gamma_5\beta_5$
-0.01
0.01
0.03
0.05
0
1000
2000
3000
4000
5000
iteration

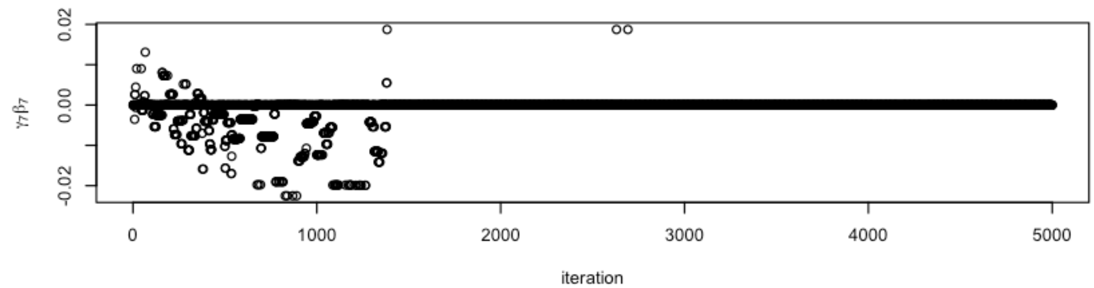

γ_5
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



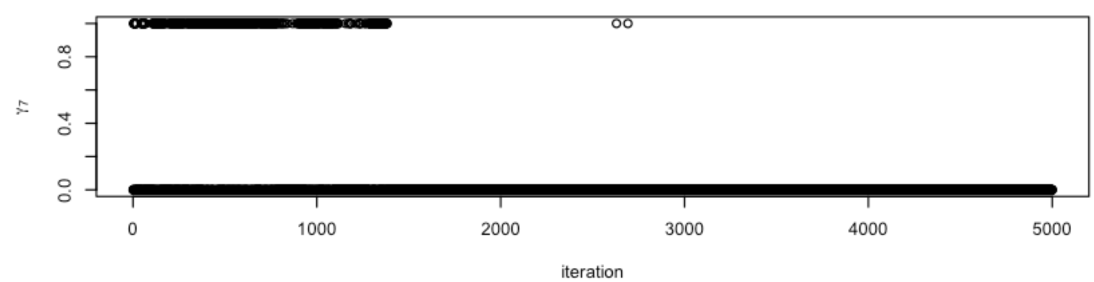
β_6
-0.06
-0.02
0
1000
2000
3000
4000
5000
iteration
$\gamma_6\beta_6$
-0.06
-0.02
0
1000
2000
3000
4000
5000
iteration
γ_6
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



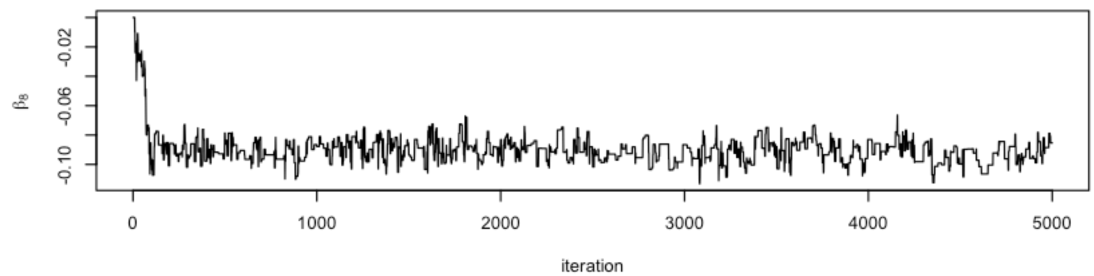
β_7
iteration

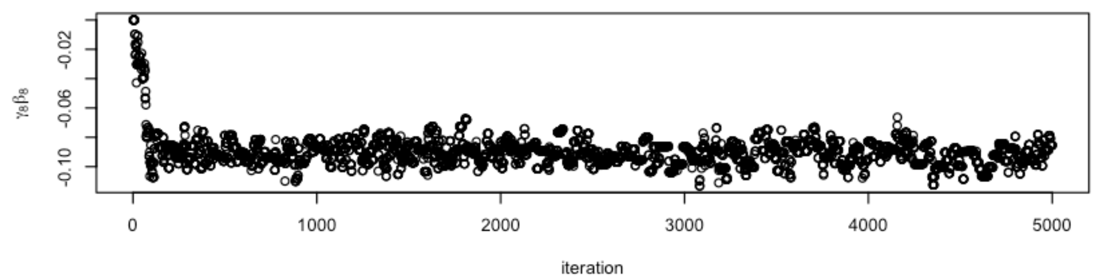


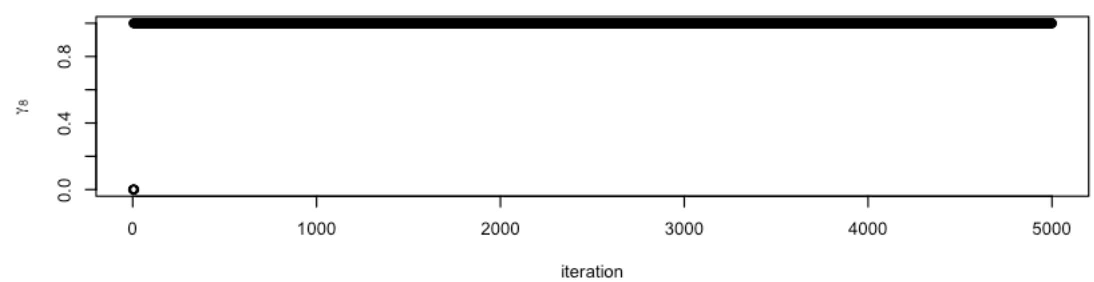
$\gamma_7\beta_7$
iteration

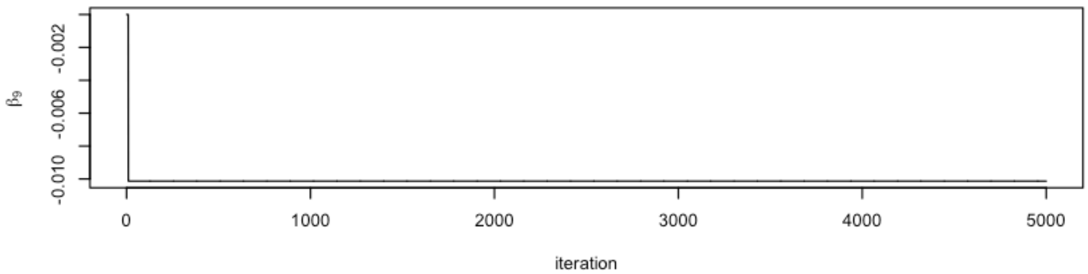


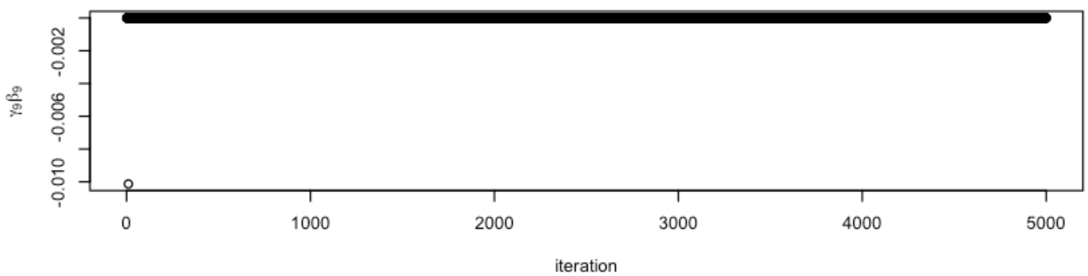
γ_7
iteration

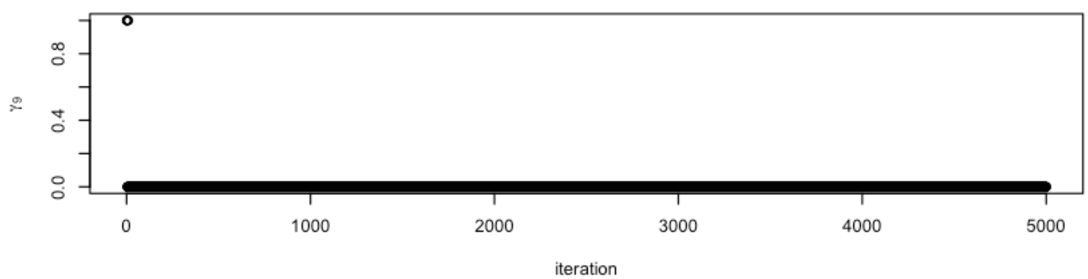

β_8
-0.02
-0.06
-0.10
0
1000
2000
3000
4000
5000
iteration

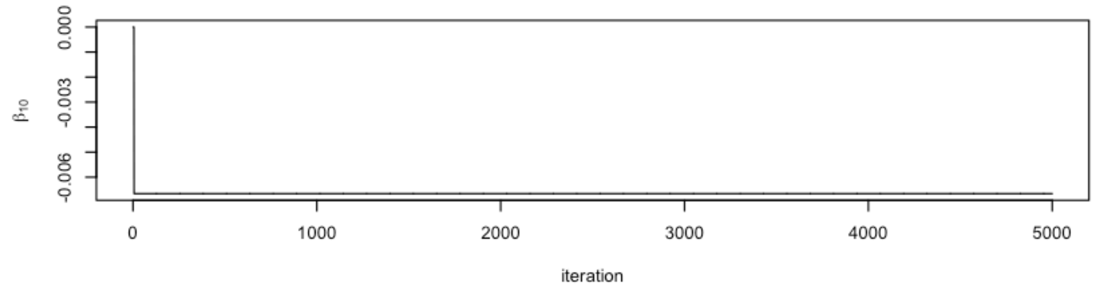

$\gamma_8\beta_8$
-0.02
-0.06
-0.10
0
1000
2000
3000
4000
5000
iteration

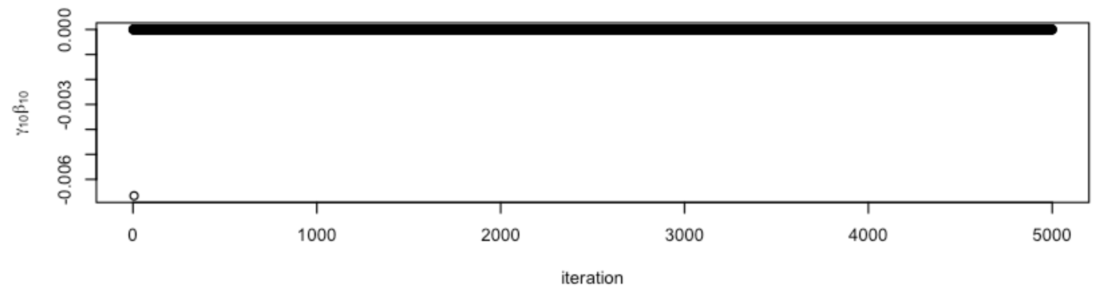

γ_8
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration

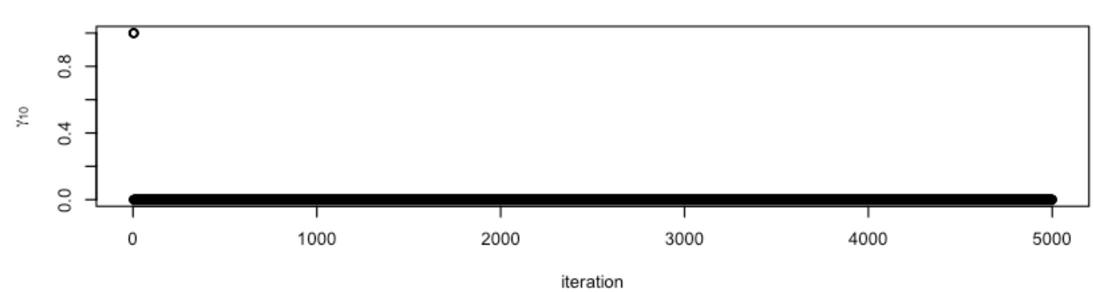

β9
-0.002
-0.006
-0.010
0
1000
2000
3000
4000
5000
iteration

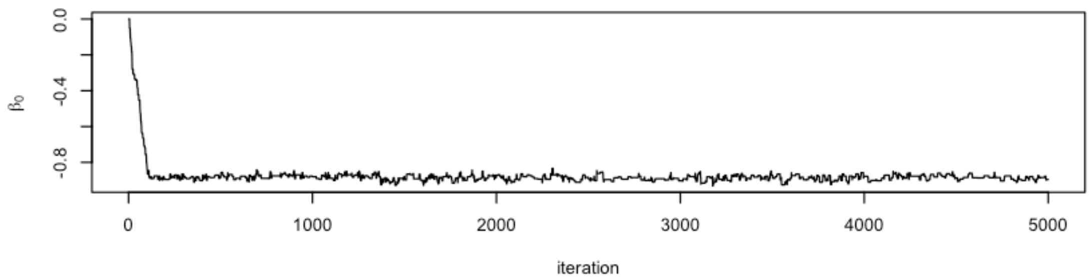

γ9β9
-0.002
-0.006
-0.010
0
1000
2000
3000
4000
5000
iteration


γ9
0.8
0.4
0.0
0
1000
2000
3000
4000
5000
iteration


0.000
-0.003
-0.006
β_{10}
0
1000
2000
3000
4000
5000
iteration


0.000
-0.003
-0.006
$\gamma_{10}\beta_{10}$
0
1000
2000
3000
4000
5000
iteration

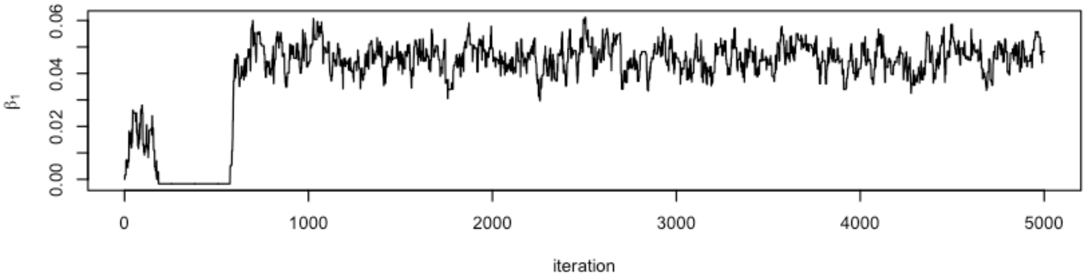

0.0
0.4
0.8
γ_{10}
0
1000
2000
3000
4000
5000
iteration

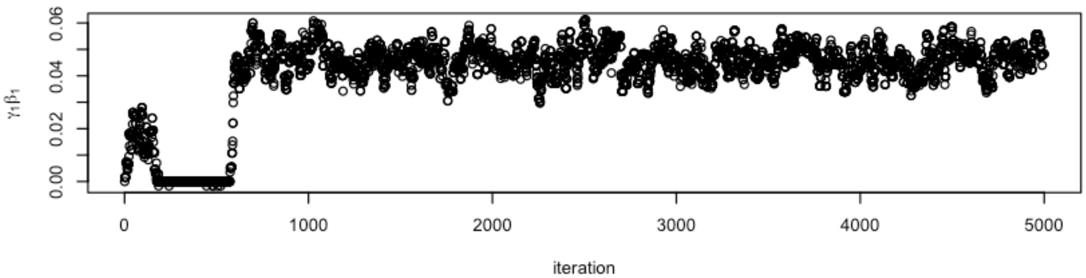


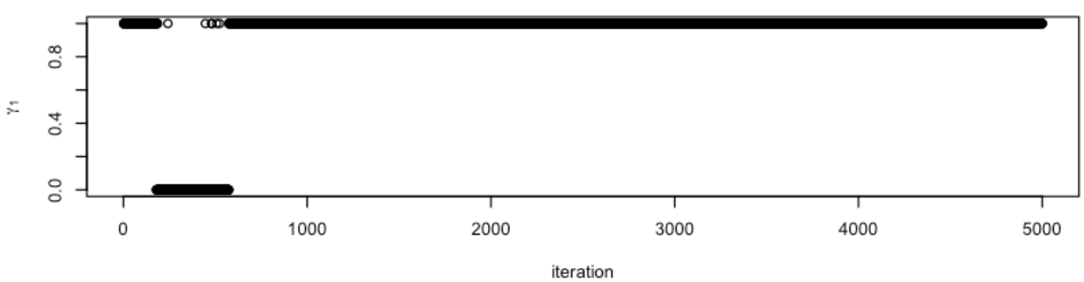
[ ]:
```
beta.pm<-rep(0,p)
beta.psd<-c(4,rep(2,p-1))
gamma<- rbinom(p-1,1,0.5)
beta<- rep(0,p)#summary.coe[,1]
beta.var<-summary(model.fit)$cov.unscaled
S2<- 1000*5
Beta<-P<-NULL
Gamma<-NULL
nu<-0.5
acc<- NULL
for(i in 1:S2){
    for(j in 1:(p-1)) {
        new_gamma<- gamma
```

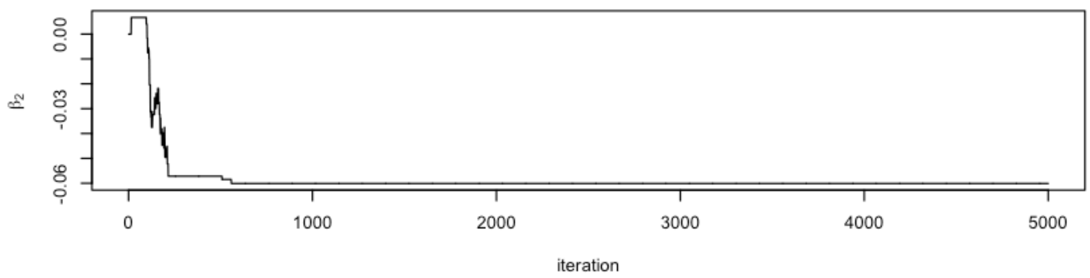
```
        new_gamma[j]<- 1-gamma[j]
        p0<-ilogit(x%*%(matrix(beta[-1]*gamma))+beta[1])
        p1<-ilogit(x%*%(matrix(beta[-1]*new_gamma))+beta[1])
        logp <- sum((dbinom(y,1,p1,log=T)-dbinom(y,1,p0,log=T)))
        pj<-rbinom(1,1,1/(1+exp(-logp)))
        gamma[j]<- pj*new_gamma[j]+(1-pj)*gamma[j]
        }
    new_beta<- rmvnorm(1,beta,nu*beta.var)
    p0<-ilogit(x%*%(matrix(beta[-1]*gamma))+beta[1])
    p1<-ilogit(x%*%(matrix(new_beta[-1]*gamma))+new_beta[1])
    # jbeta<-dmvnorm(beta,new_beta,nu*beta.var,log=T)
    # new_jbeta<- dmvnorm(new_beta,beta,nu*beta.var,log=T)
    logp <-␣
↪sum(dbinom(y,1,p1,log=T)-dbinom(y,1,p0,log=T))+sum((dnorm(new_beta,beta.
↪pm,beta.psd,log=T)-dnorm(beta,beta.pm,beta.
↪psd,log=T))*c(1,gamma))#-new_jbeta+jbeta
    if(log(runif(1))<logp){
        beta<-new_beta*c(1,gamma)+beta*c(0,1-gamma)
        acc <- c(acc,1)
    }
    else{
        acc<-c(acc,0)
    }
    Beta<-rbind(Beta,beta)
    Gamma<-rbind(Gamma,gamma)
}

mean(acc[3000:S2])
par(bg='white',mfrow=c(3,1))
for(i in 1:(p-1)){
    plot(Beta[,i+1],type='l',ylab=bquote(beta[.(i)]),xlab='iteration')
    plot(Gamma[,i]*Beta[,i+1],ylab=bquote(gamma[.(i)]*beta[.
↪(i)]),xlab='iteration')
    plot(Gamma[,i],ylab=bquote(gamma[.(i)]),xlab='iteration')
}
plot(Beta[,0+1],type='l',ylab=bquote(beta[.(0)]),xlab='iteration')
```

0.32983508245877

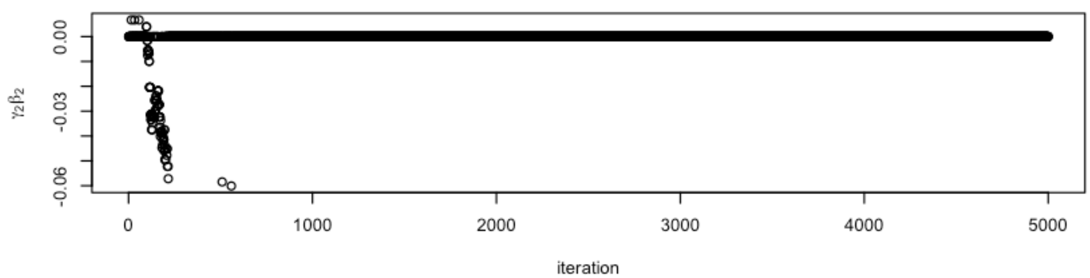

β_1
0.00 0.02 0.04 0.06
0 1000 2000 3000 4000 5000
iteration


$\gamma_1\beta_1$
0.00 0.02 0.04 0.06
0 1000 2000 3000 4000 5000
iteration

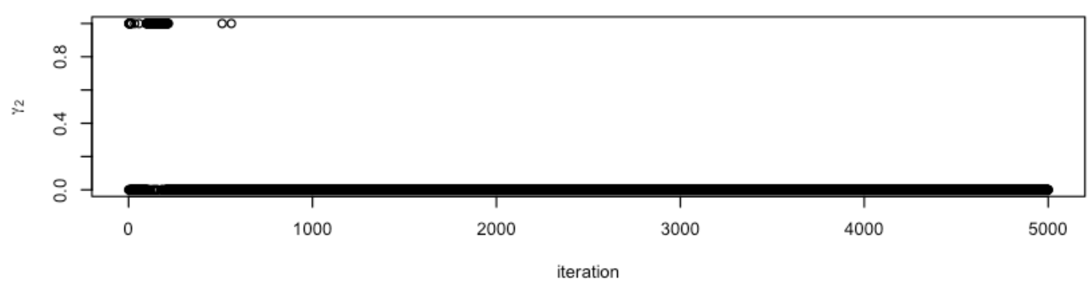

γ_1
0.0 0.4 0.8
0 1000 2000 3000 4000 5000
iteration



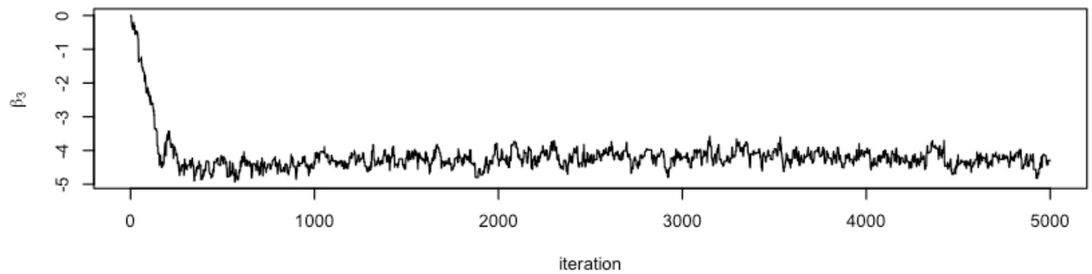
β_2
0.00
-0.03
-0.06
0
1000
2000
3000
4000
5000
iteration



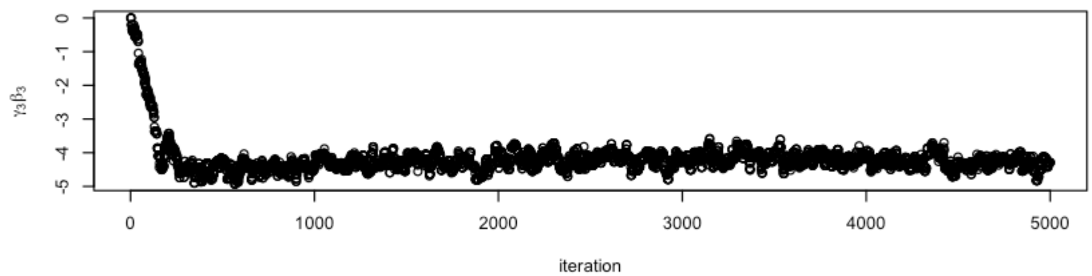
$\gamma_2\beta_2$
0.00
-0.03
-0.06
0
1000
2000
3000
4000
5000
iteration



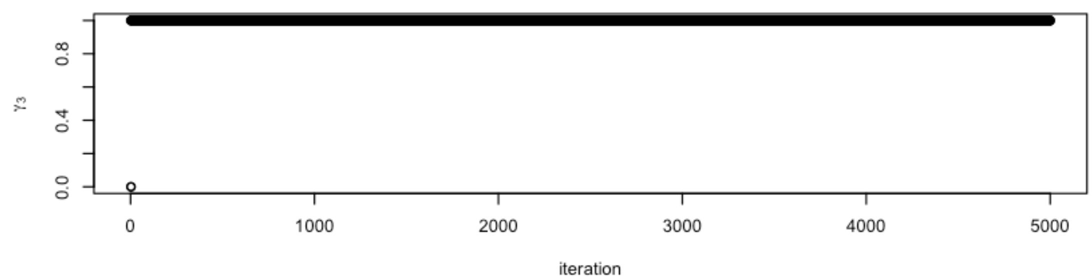
γ_2
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



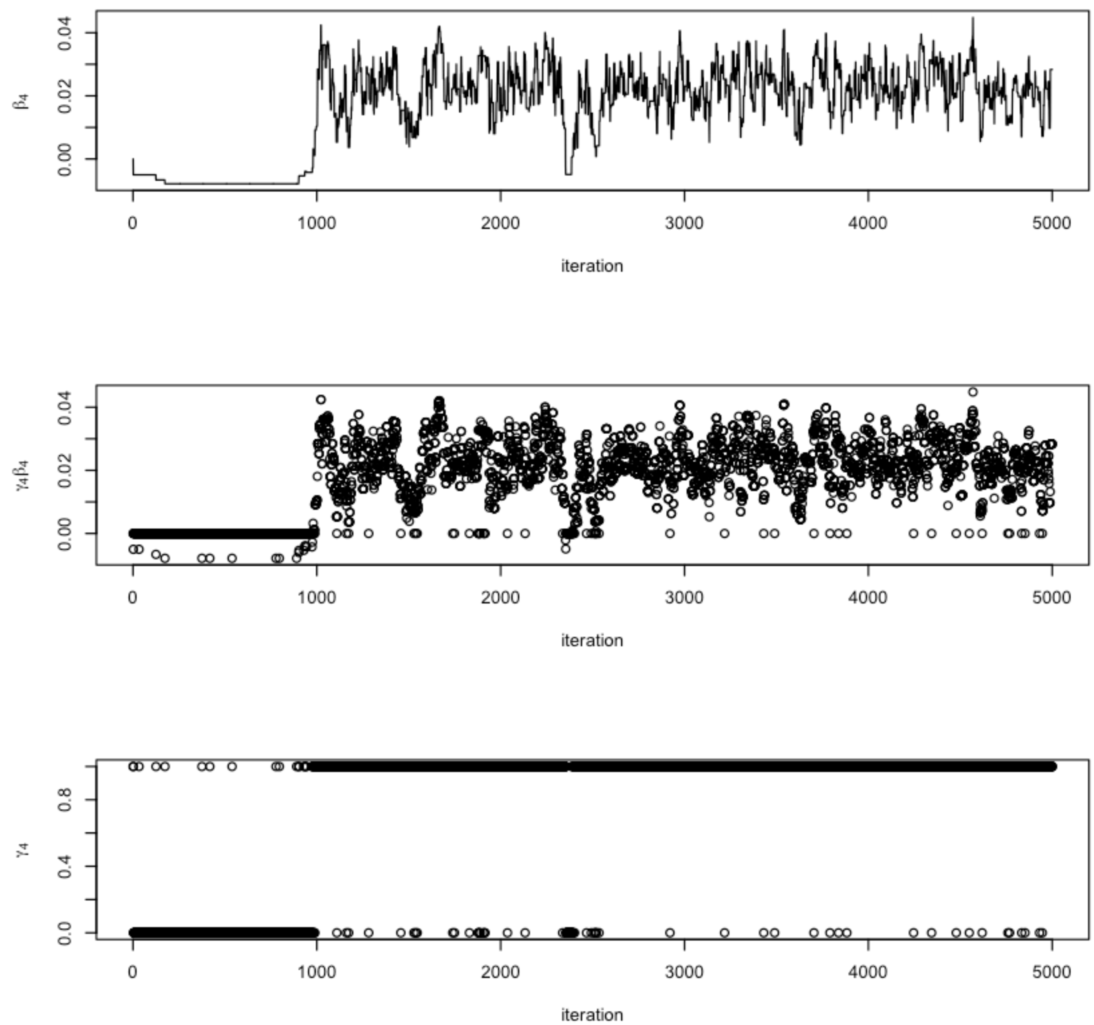
β_3
0
-1
-2
-3
-4
-5
0
1000
2000
3000
4000
5000
iteration



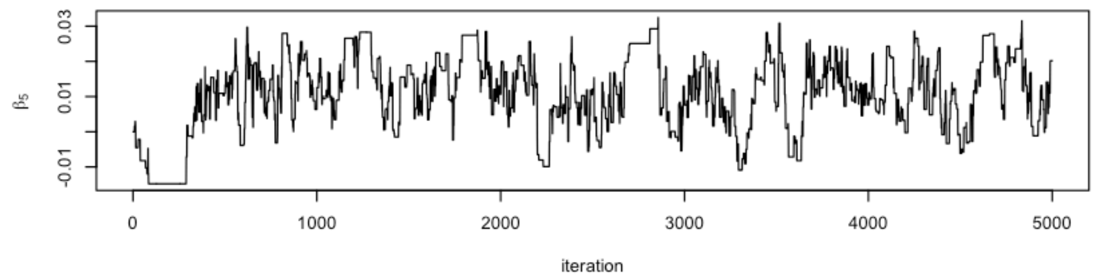
$\gamma_3\beta_3$
0
-1
-2
-3
-4
-5
0
1000
2000
3000
4000
5000
iteration



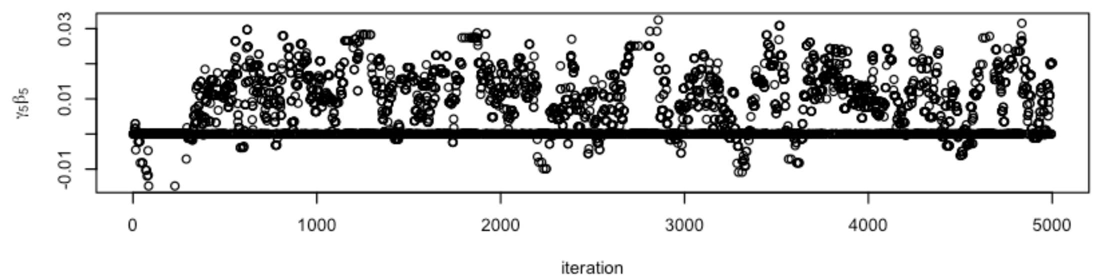
γ_3
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



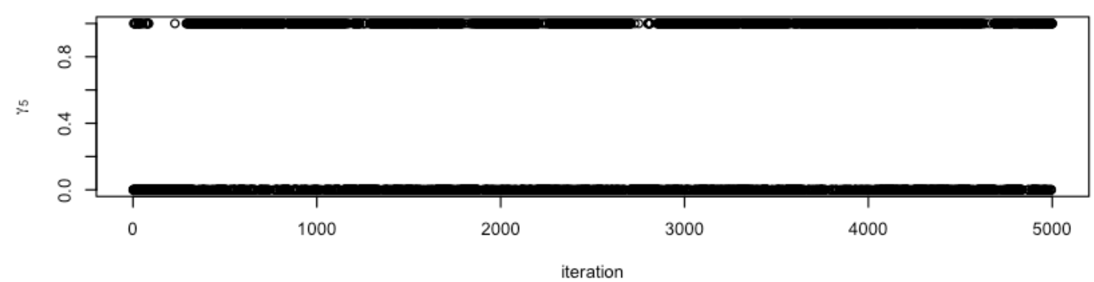
β_4
0.00
0.02
0.04
0
1000
2000
3000
4000
5000
iteration
$\gamma_4\beta_4$
0.00
0.02
0.04
0
1000
2000
3000
4000
5000
iteration
γ_4
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



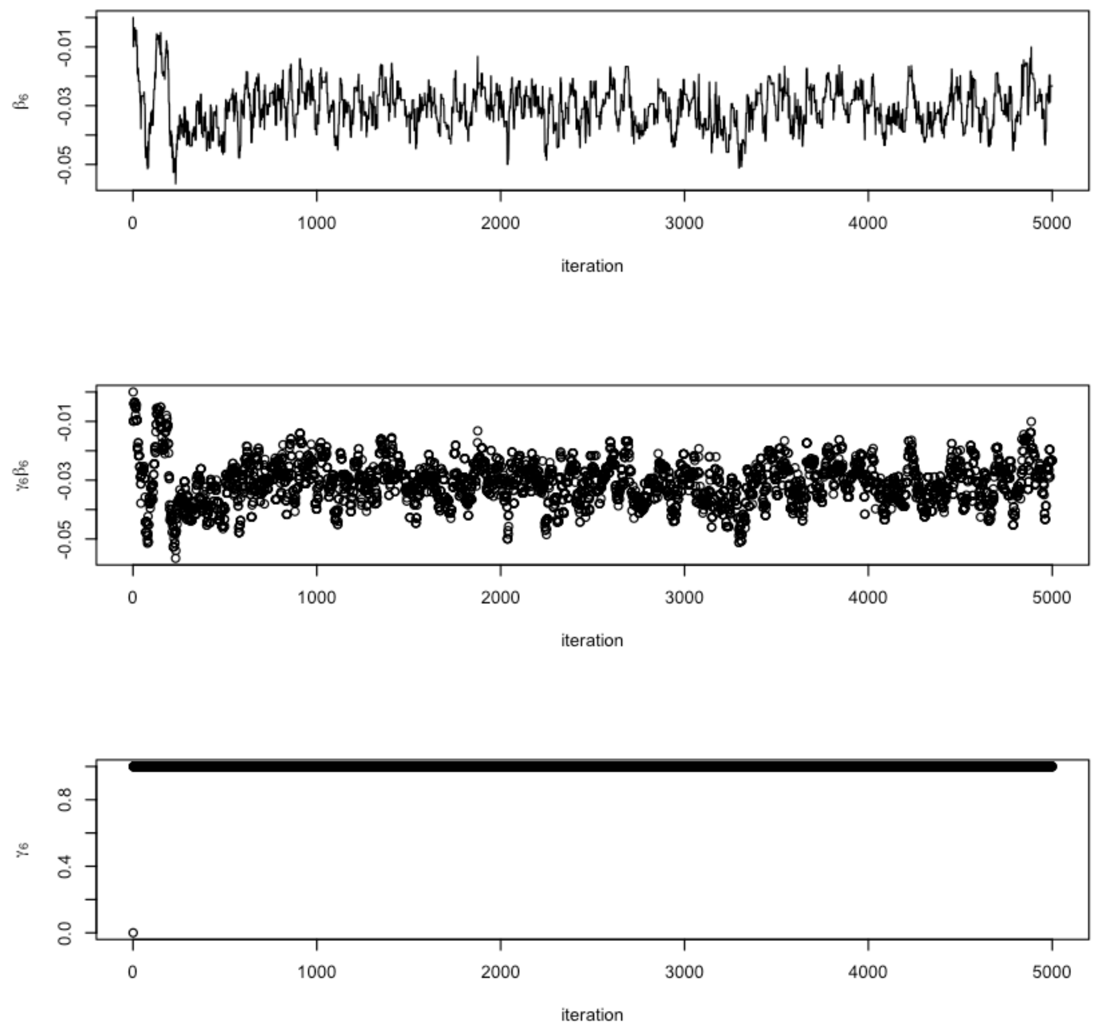
0.03
0.01
-0.01
β_5
0
1000
2000
3000
4000
5000
iteration



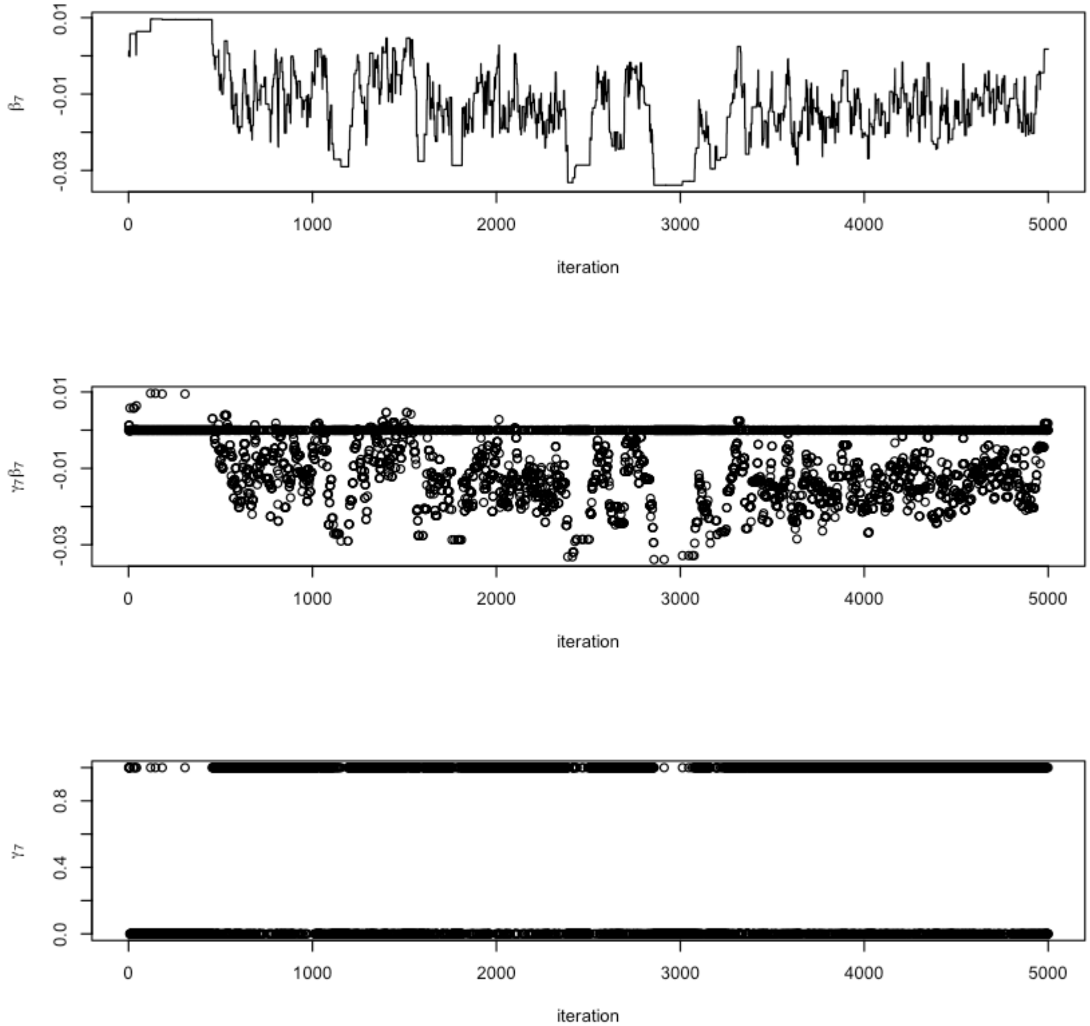
0.03
0.01
-0.01
$\gamma_5\beta_5$
0
1000
2000
3000
4000
5000
iteration



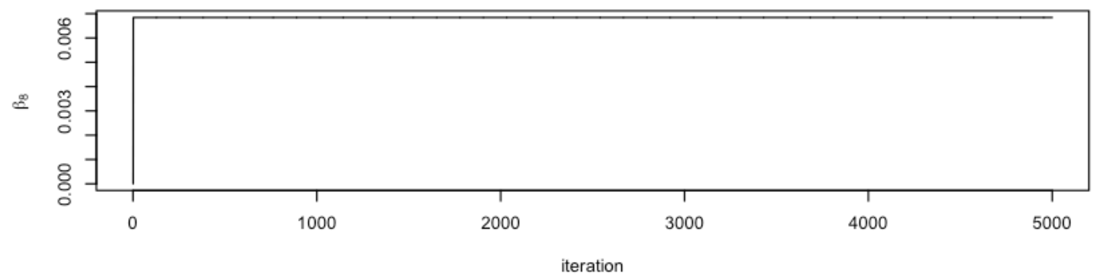
0.8
0.4
0.0
γ_5
0
1000
2000
3000
4000
5000
iteration



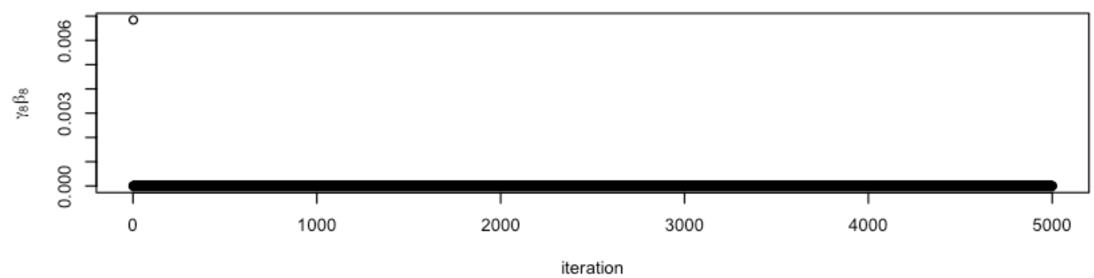
β_6
-0.05
-0.03
-0.01
0
1000
2000
3000
4000
5000
iteration
$\gamma_6\beta_6$
-0.05
-0.03
-0.01
0
1000
2000
3000
4000
5000
iteration
γ_6
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



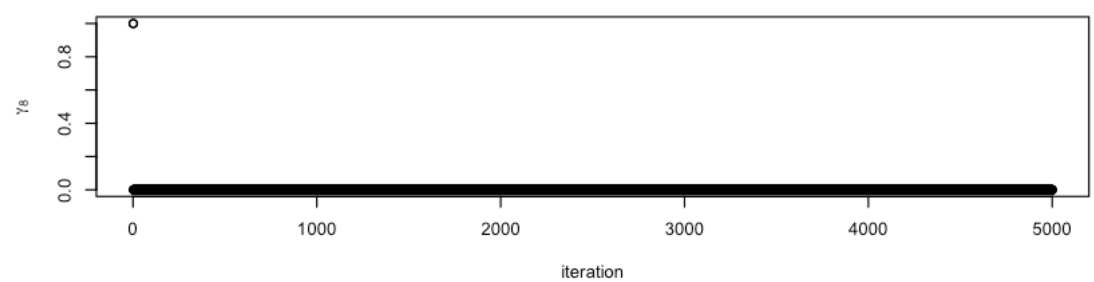
0.01
-0.01
-0.03
β7
0
1000
2000
3000
4000
5000
iteration
0.01
-0.01
-0.03
γ7β7
0
1000
2000
3000
4000
5000
iteration
0.0
0.4
0.8
γ7
0
1000
2000
3000
4000
5000
iteration



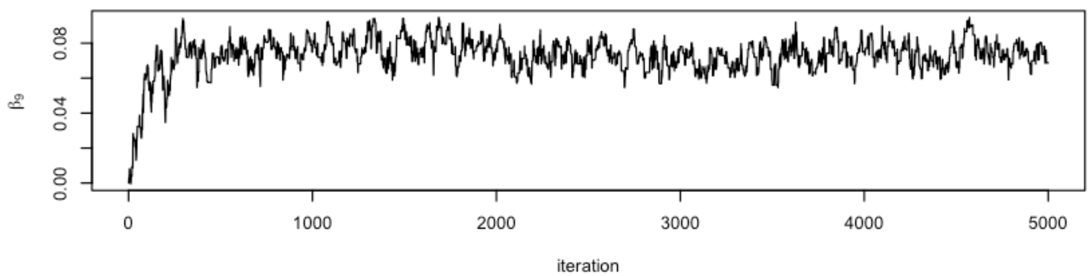
β8
0.000
0.003
0.006
0
1000
2000
3000
4000
5000
iteration



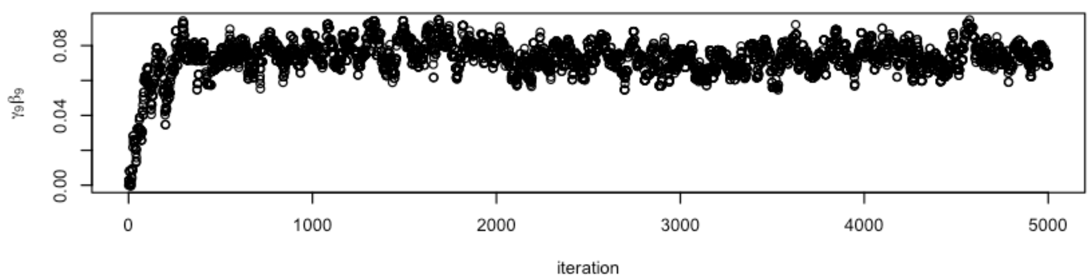
γ8β8
0.000
0.003
0.006
0
1000
2000
3000
4000
5000
iteration



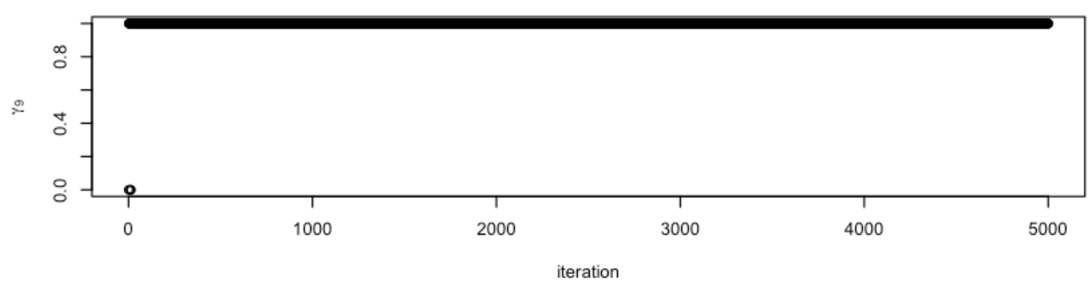
γ8
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



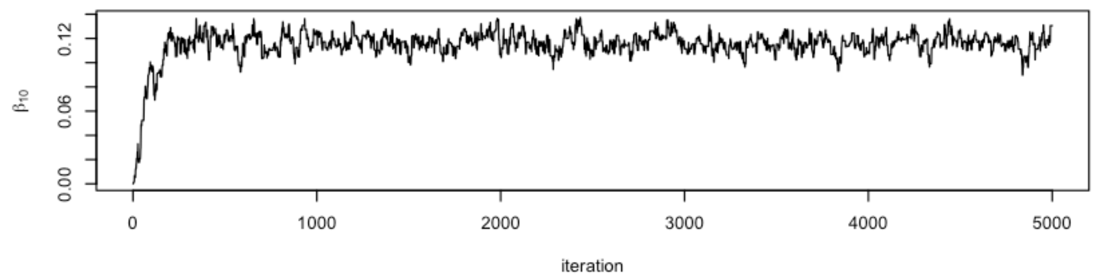
β_9
0.00
0.04
0.08
0
1000
2000
3000
4000
5000
iteration

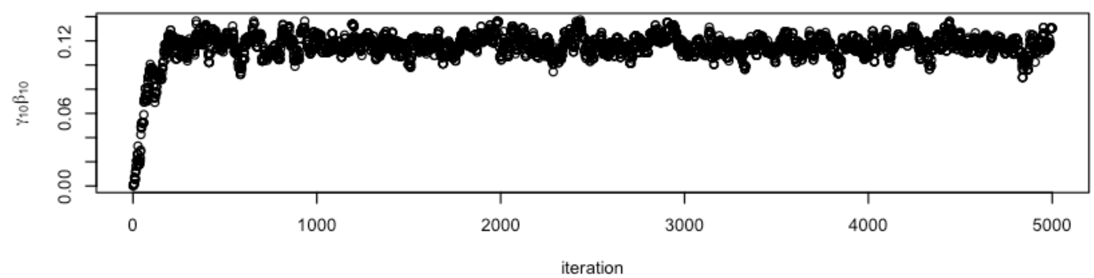


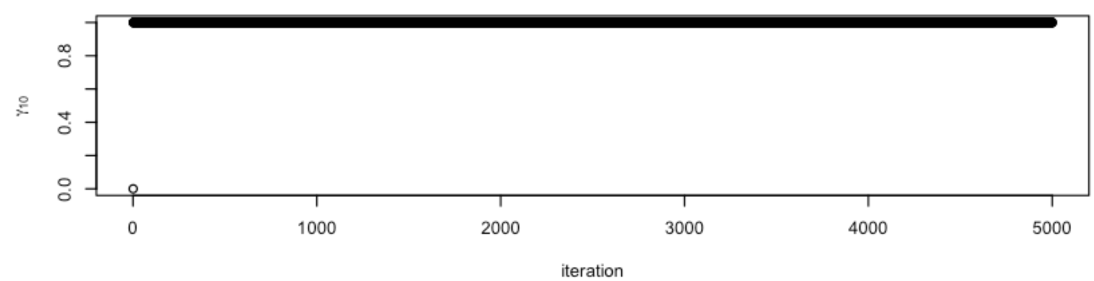
$\gamma_9\beta_9$
0.00
0.04
0.08
0
1000
2000
3000
4000
5000
iteration

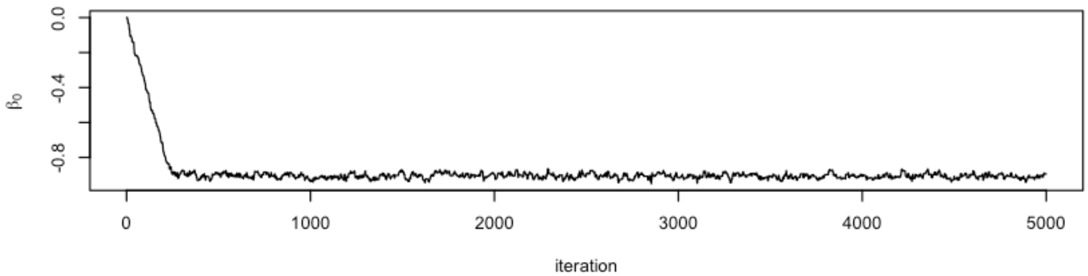


γ_9
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration


β10
0.00
0.06
0.12
0
1000
2000
3000
4000
5000
iteration


γ10β10
0.00
0.06
0.12
0
1000
2000
3000
4000
5000
iteration


γ10
0.0
0.4
0.8
0
1000
2000
3000
4000
5000
iteration



(e) $\beta_{0,1,3,6,9,10}$ are important which means Gender,Age_at_Release,Educ_College,Prior_Conviction_Felony,Pri are important variables.The mean and interval are shown in the below print.We may remove Gender variable because it include zero in its interval.

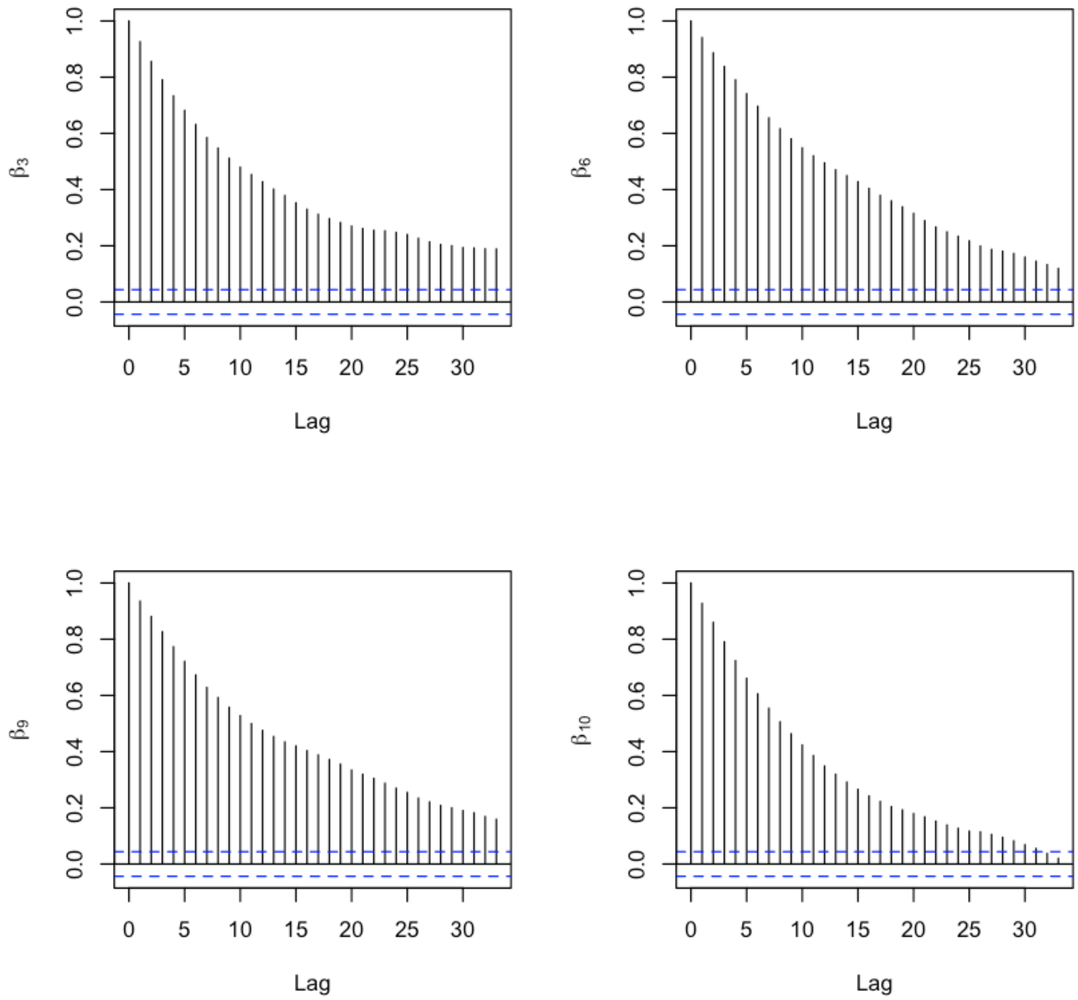
[ ]:
```
print('Index       Mean          Interval')
for(i in c(1,3,6,9,10)){
    print(c(i,mean(Beta[,i+1]),quantile(Beta[,i+1],c(0.025,0.975))))
}
```

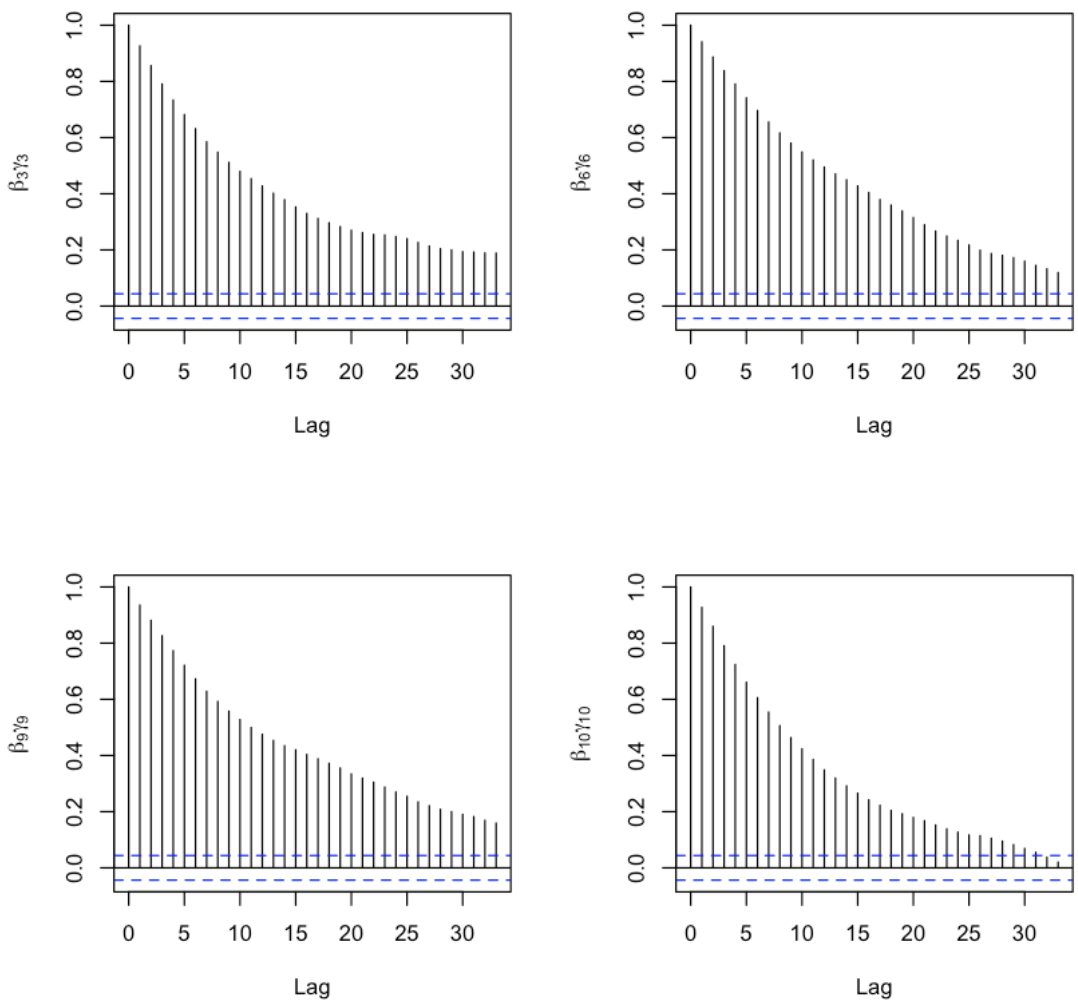
```
[1] "Index       Mean          Interval"
                                   2.5%        97.5%
 1.000000000  0.040957782 -0.001685456  0.055632396
                          2.5%     97.5%
 3.000000 -4.186117 -4.718956 -2.793553
```

```
                                  2.5%       97.5%
 6.00000000 -0.03072141 -0.04424430 -0.01664913
                               2.5%      97.5%
9.00000000 0.07289874 0.05401909 0.08936938
                                  2.5%       97.5%
10.00000000  0.11493954  0.09143464  0.13071354
```

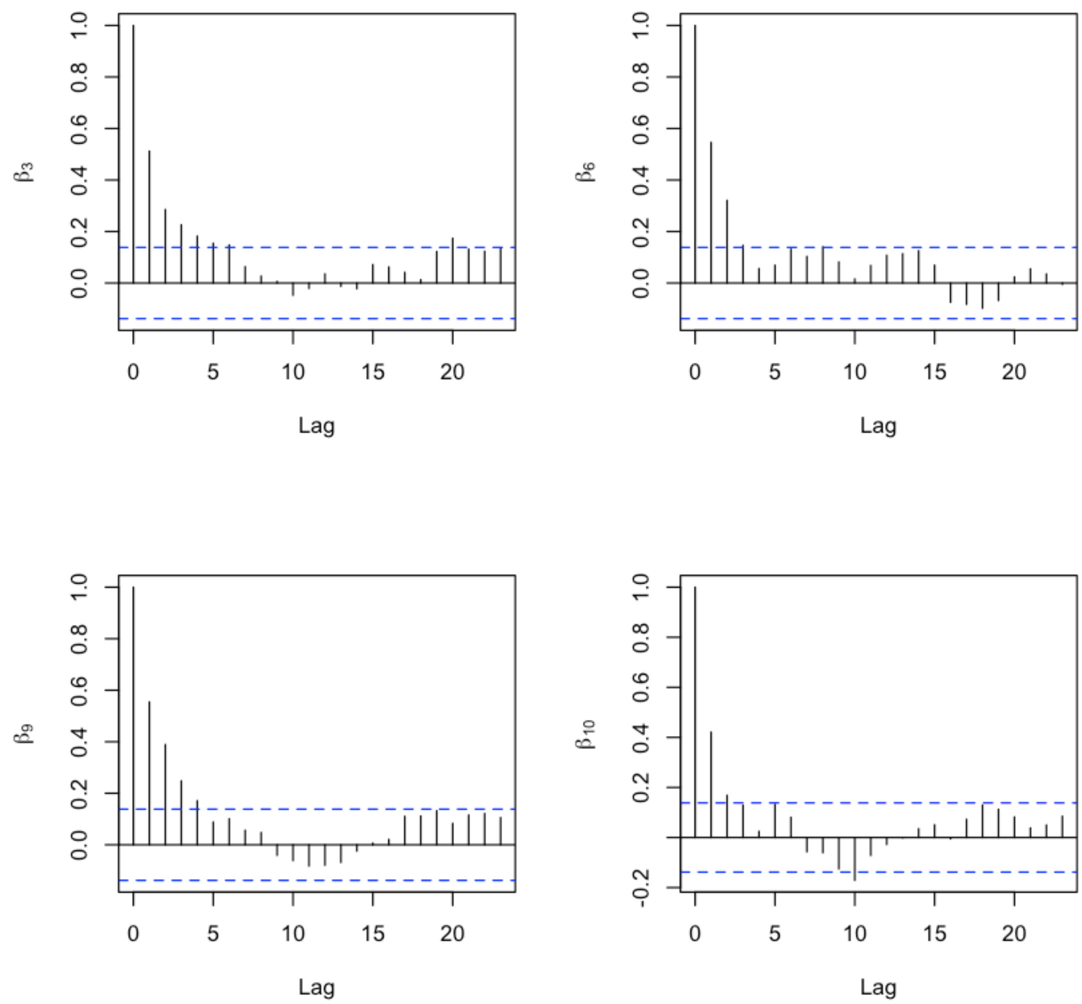
(f) It seems still high auto-correlation which may suggest us select data with a rate such as 1 in 10. When we run the iteration, we only record the data only the numbers of iteration mod 10 equals to zero.

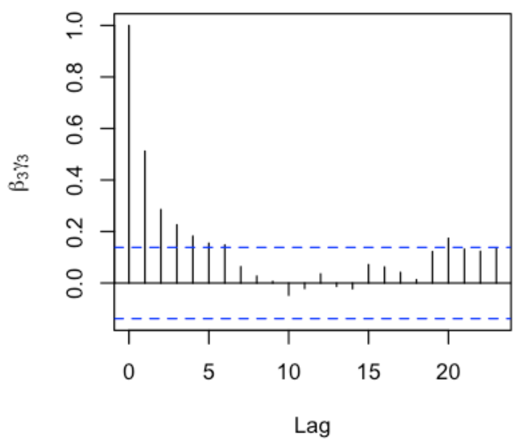
[ ]:
```
par(bg='white',mfrow=c(2,2))
for(i in c(3,6,9,10)){
    acf(Beta[3000:S2,i+1],main="",ylab=bquote(beta[.(i)]))
}
for(i in c(3,6,9,10)){
    acf(Beta[3000:S2,i+1]*Gamma[3000:S2,i],main="",ylab=bquote(beta[.
 ↪(i)]*gamma[.(i)]))
}
for(i in c(3,6,9,10)){
    acf(Beta[seq(3000,S2,by=10),i+1],main="",ylab=bquote(beta[.(i)]))
}
for(i in c(3,6,9,10)){
   ␣
 ↪acf(Beta[seq(3000,S2,by=10),i+1]*Gamma[seq(3000,S2,by=10),i],main="",ylab=bquote(beta[.
 ↪(i)]*gamma[.(i)]))
}
```

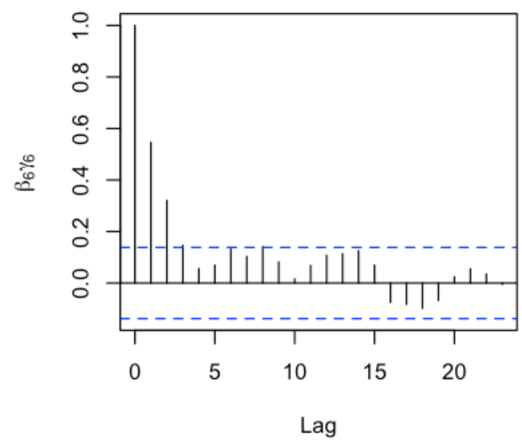

β_3
β_6
β_9
β_{10}
Lag
Lag
Lag
Lag

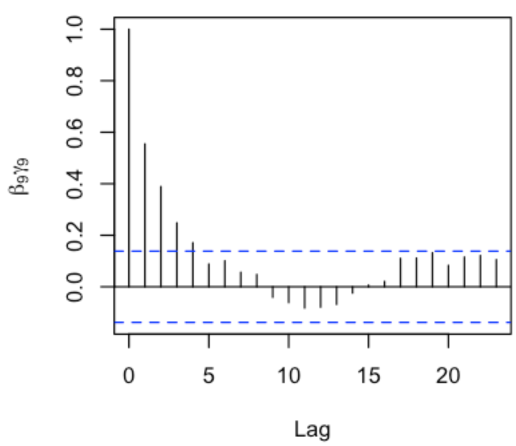


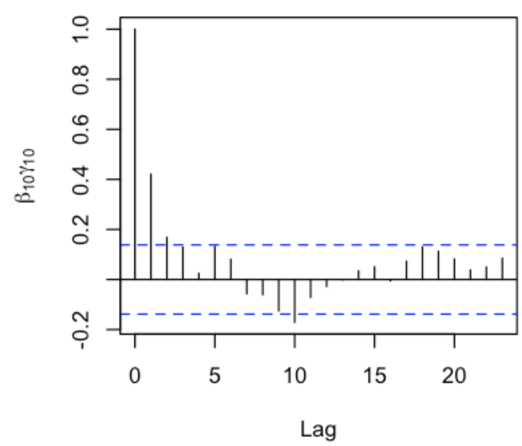
$\beta_3\gamma_3$
Lag
$\beta_6\gamma_6$
Lag
$\beta_9\gamma_9$
Lag
$\beta_{10}\gamma_{10}$
Lag


β3
Lag
β6
Lag
β9
Lag
β10
Lag









3 Answer 3

There are some missing data so we may ingnore them due to the Hierarchical Normal Modeling but not the MVN modeling.

(a) $\mu_0 = \bar{y}$ while others need to be small to be small except γ_0^2 which should be big so that the priors are weakly informative.

(b) The code was shown below

(c) It seems 30000 is enough because the chain shows stationary.

```
[ ]: #q3
q3<-read.csv('formula1.csv')
```

```
n3<-nrow(q3)
nc<-ncol(q3)-1
y_sumsq<-0
Y<-NULL
M<-0
Y_bar <- NULL
for (i in 1:n3) {
    y<-(q3[i,-1])[FALSE==is.na((q3[i,-1]))]
    Y<-c(Y,y)
    M<-c(M,M[i]+length(y))
    Y_bar <- c(Y_bar,mean(y))
    theta <- Y_bar
    y_sumsq <- sum((y-theta[i])^2)+y_sumsq
}
# M
# Y[(M[20]+1):M[21]]
# Y_bar
mu<-mu0 <-mean(Y)
S3<-10000*3
lambda0_sq <- 100
tao_sq <- tao0_sq <- 1
eta0 <- 1/2
sigma_sq <- sigma0_sq <- 1
nu0 <- 1/2
THETA <- MST<-SIGMA_sq <- NULL
n<-M[21]
theta_sumsq<-sum((theta-mu)^2)
post_norm_arg <- function(n,y_bar,mu0,sigma_sq,tao_sq){
    return(c(((n*y_bar/sigma_sq)+(mu0/tao_sq))/(n/sigma_sq+1/tao_sq),1/(n/
 ↪sigma_sq+1/tao_sq)))
}
post_gamma_arg <- function(m,sum_sq,nu_0,tao0_sq){
    return(c((nu_0+m)/2,(nu_0*tao0_sq+sum_sq)/2))
}
```

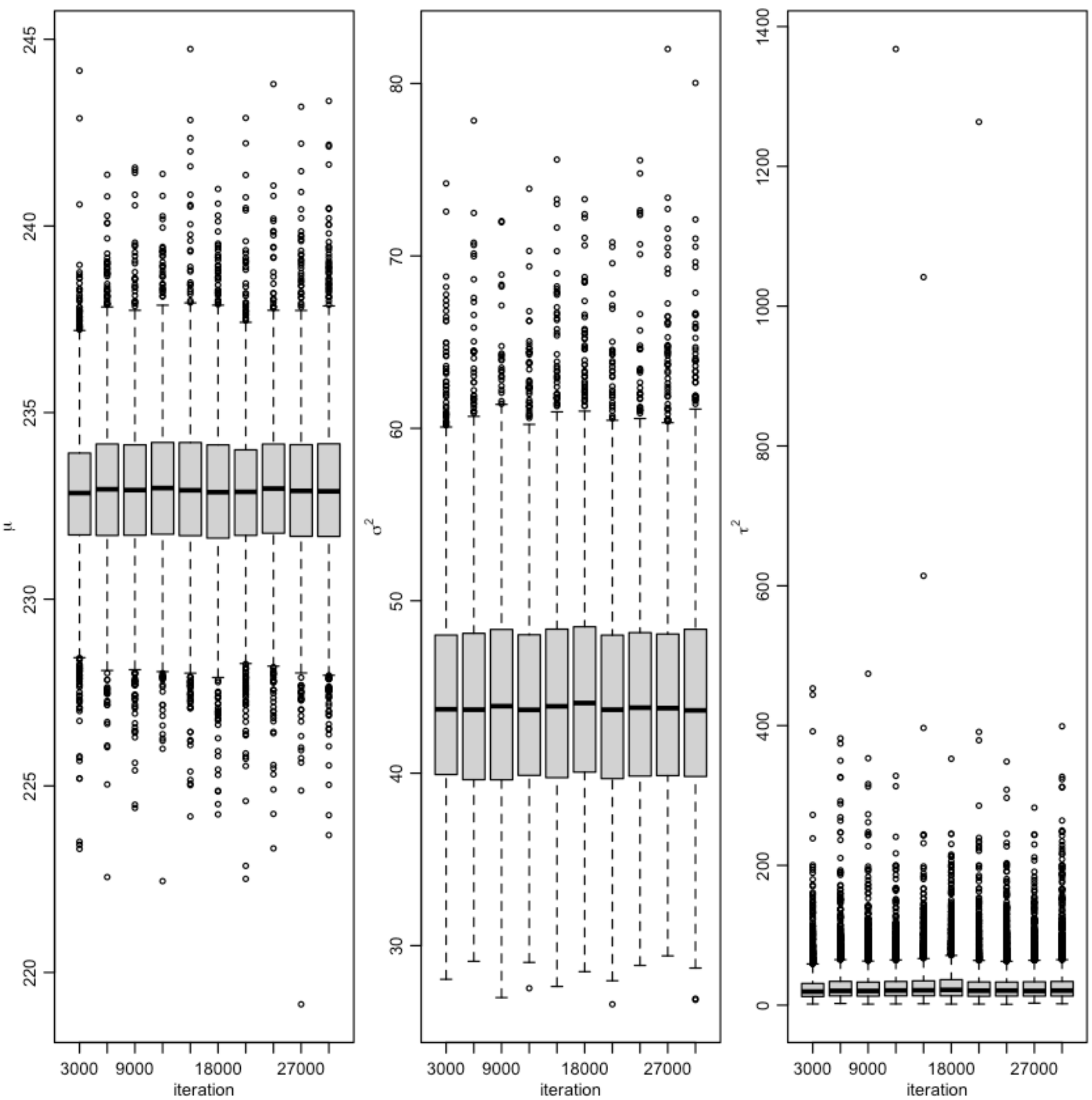
[ ]:

```
for(i in 1:S3){
    arg <- post_norm_arg(8,mean(theta),mu0,tao_sq,lambda0_sq)
    mu <- rnorm(1,arg[1],sqrt(arg[2]))
    arg <- post_gamma_arg(8,theta_sumsq,eta0,tao0_sq)
    tao_sq <- 1/rgamma(1,arg[1],arg[2])
    arg <- post_gamma_arg(n,y_sumsq,nu0,sigma0_sq)
    sigma_sq <- 1/rgamma(1,arg[1],arg[2])
    y_sumsq<-0
    for (j in 1:n3){
        m<- M[j+1]-M[j]
        y_bar<- Y_bar[j]
        y<- Y[(M[j]+1):M[j+1]]
```

```
        arg<-post_norm_arg(m,y_bar,mu,sigma_sq,tao_sq)
        theta[j] <- rnorm(1,arg[1],sqrt(arg[2]))
        y_sumsq <- sum((y-theta[j])^2)+y_sumsq
    }
    theta_sumsq<-sum((theta-mu)^2)
    THETA<-rbind(THETA,c(theta))
    MST<-rbind(MST,c(mu,sigma_sq,tao_sq))

}
```

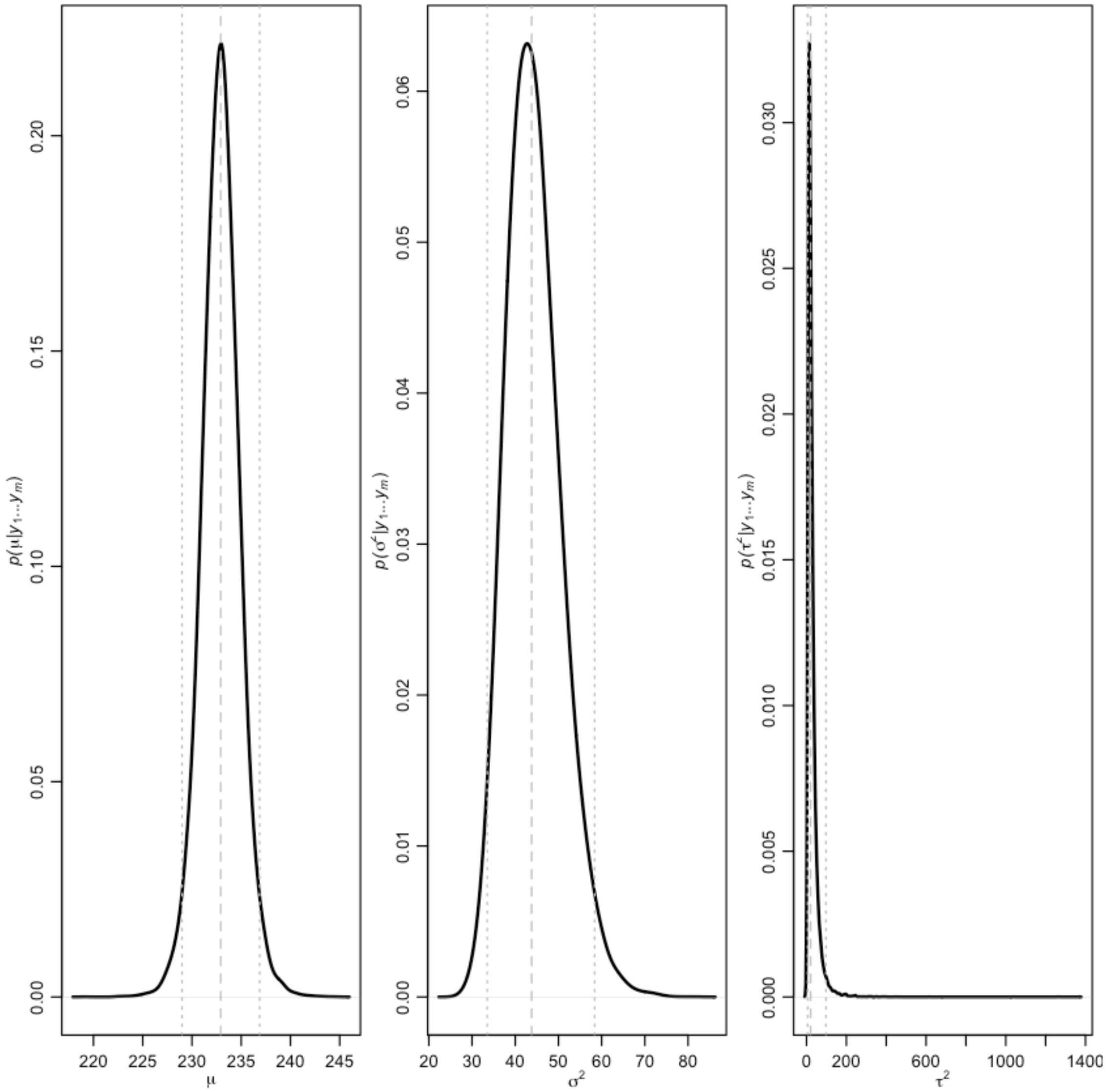
[ ]:

```
stationarity.plot<-function(x,...){

  S<-length(x)
  scan<-1:S
  ng<-min( round(S/100),10)
  group<-S*ceiling( ng*scan/S) /ng

  boxplot(x~group,...)               }
par(bg='white')
par(mfrow=c(1,3),mar=c(2.75,2.75,.5,.5),mgp=c(1.7,.7,0))
stationarity.plot(MST[,1],xlab="iteration",ylab=expression(mu))
stationarity.plot(MST[,2],xlab="iteration",ylab=expression(sigma^2))
stationarity.plot(MST[,3],xlab="iteration",ylab=expression(tau^2))
```




```
[ ]: par(bg='white',mfrow=c(1,3),mar=c(2.75,2.75,.5,.5),mgp=c(1.7,.7,0))
     plot(density(MST[,1],adj=2),xlab=expression(mu),main="",lwd=2,
          ylab=expression(paste(italic("p("),mu,"|",italic(y[1]),"...
       ↪",italic(y[m]),")")))
     abline( v=quantile(MST[,1],c(.025,.5,.975)),col="gray",lty=c(3,2,3) )
     plot(density(MST[,2],adj=2),xlab=expression(sigma^2),main="", lwd=2,
          ylab=expression(paste(italic("p("),sigma^2,"|",italic(y[1]),"...
       ↪",italic(y[m]),")")))
     abline( v=quantile(MST[,2],c(.025,.5,.975)),col="gray",lty=c(3,2,3) )
     plot(density(MST[,3],adj=2),xlab=expression(tau^2),main="",lwd=2,
          ylab=expression(paste(italic("p("),tau^2,"|",italic(y[1]),"...
       ↪",italic(y[m]),")")))
```

```
abline( v=quantile(MST[,3],c(.025,.5,.975)),col="gray",lty=c(3,2,3) )
```



(d)

(e)

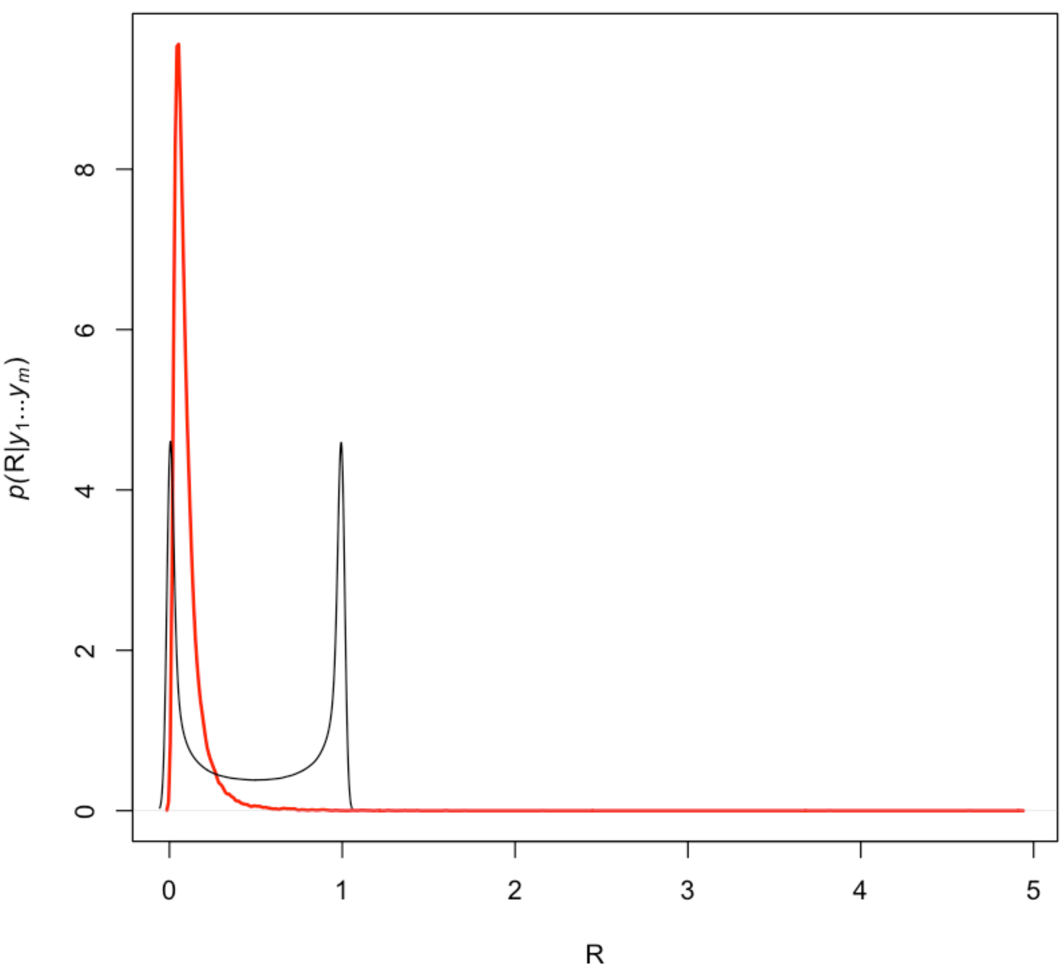
(f)$$

$$

```
[ ]: R<-MST[,3]/(MST[,2]+MST[3])
par(bg='white')
plot(density(R),col='red',xlab=expression(R),main="",lwd=2,ylab=expression(paste(italic("p("),
  ↪..",italic(y[m]),")")))
l<-length(MST[,3])*100
```

```
S0<-1/rgamma(1,nu0/2,nu0*sigma0_sq/2)
T0<-1/rgamma(1,eta0/2,eta0*tao0_sq/2)
R0<-T0/(S0+T0)
lines(density(R0))
quantile(R0,c(0.025,0.975))
```

2.5\% 4.48242970855877e-06 **97.5\%** 0.999995276099022



[ ]:
```
theta==max(theta)
```

1. FALSE 2. FALSE 3. FALSE 4. FALSE 5. FALSE 6. FALSE 7. FALSE 8. FALSE 9. FALSE 10. FALSE 11. FALSE 12. FALSE 13. FALSE 14. FALSE 15. FALSE 16. TRUE 17. FALSE 18. FALSE 19. FALSE 20. FALSE