Introduction to Bayesian Data Analysis
Tutorial 9

(1) Problem 10.4 (Hoff) Gibbs sampling: Consider the general Gibbs sampler for a vector of parameters ϕ. Suppose $\phi^{(s)}$ is sampled from the target distribution $p(\phi)$ and then $\phi^{(s+1)}$ is generated using the Gibbs sampler by iteratively updating each component of the parameter vector. Show that the marginal probability $Pr(\phi^{(s+1)} \in A)$ equals the target distribution $\int_A p(\phi)d\phi$.

(2) Problem 10.5 (Hoff) Logistic regression variable selection: Consider a logistic regression model for predicting diabetes as a function of x_1=number of pregnancies, x_2=blood pressure, x_3=body mass index, x_4=diabetes pedigree and x_5=age. Using the data in `azdiabetes.dat` , centre and scale each of the x-variables by subtracting the sample average and dividing by the sample standard deviation for each variable. Consider a logistic regression model of the form $Pr(Y_i = 1|\mathbf{x_i}, \boldsymbol{\gamma}, \boldsymbol{\beta}) = \exp(\theta_i)/(1 + \exp(\theta_i)$ where

$$\theta_i = \beta_0 + \beta_1\gamma_1 x_{i,1} + \beta_2\gamma_2 x_{i,2} + \beta_3\gamma_3 x_{i,3} + \beta_4\gamma_4 x_{i,4} + \beta_5\gamma_5 x_{i,5}$$

In this model, each γ_i is either 0 or 1, indicating whether or not variable j is a predictor of diabetes. For example, if it were the case that $\gamma = (1, 1, 0, 0, 0)$, then $\theta_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2}$. Obtain the posterior distributions for $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, using independent prior distributions for the parameters, such that $\gamma_j \sim$ Bern(1/2), $\beta_0 \sim$ normal$(0, 16)$ and $\beta_j \sim$ normal$(0, 4)$ for each $j > 0$.

(a) Implement a Metropolis-Hastings algorithm for approximating the posterior distribution of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. Examine the sequences $\beta_j^{(s)}$ and $\beta_j^{(s)} \times \gamma_j^{(s)}$ for each j and discuss the mixing of the chain.

(b) Obtain $Pr(\gamma_j = 1|\mathbf{x}, \mathbf{y})$ for each j. How good do you think the MCMC estimates of these posterior probabilities are?

(c) For each j, plot posterior densities and obtain posterior means for $\beta_j\gamma_j$.