Part 2 Model and application

Mike

Basic idea

Applications of different loss functions in making decisions under BHM , especially in spatio-temporal related process eg. decisions or predictions on floods in environmental science.

- Prediction is a decision
- Prediction of multivariate spatio-temporal processes
- Loss functions based on displacement
- Other classes of loss functions

Prediction is a decision

Consider BHM model on sequence Y which is a latent random process , (a_i) is a sequence of decisions and $L(a, y)$ is loss function. Therefore the risk function can be defined as follows

$$R(a_i, y, z) := \int_y \int_z L(a, y)\ p(y, z)\ dz\ dy$$

And minimizing the risk function above which is equal to minimizing the following risk function according to lecture 10a

$$R(a_i, y) := \int_y L(a, y)\ p(y|z)\ dy = E[L(a, Y)|z]$$

And the best soluation a_i is given by $\delta^*(z)$ which means it depends on data z. If the loss function depends on the function of Y eg. $g(Y)$.The risk function is given by

$$R(a_i, y) := E[L(a, g(Y)|z]$$

Predicion of multivariate spatio-temporal processes

Statistical model & estimation procedure

The random latent process Y may be multivariate spatio-temporal processes.

$$\mathbf{Y} := \{Y_i(\mathbf{s}, t) : \mathbf{s} \in D_s, t \in D_t, i = 1, \ldots, n\}$$

Thus, there is a correlation matrix of $\mathbf{Y}$ noted as $\mathbf{R}$ which can be decomposed as follows

$$\mathbf{R} = \mathbf{P}\Lambda\mathbf{P}' = \sum_{i=1}^{n} \lambda_i \mathbf{P}_i \mathbf{P}_i'$$

The above equation can be derived because $\mathbf{R}$ can be conducted by the noise of random process meanwhile the correlation matrix is usually positive-definite .

It may looks like to view $\mathbf{Y}$ as Gaussian process. Therefore, consider a kind of general loss function as

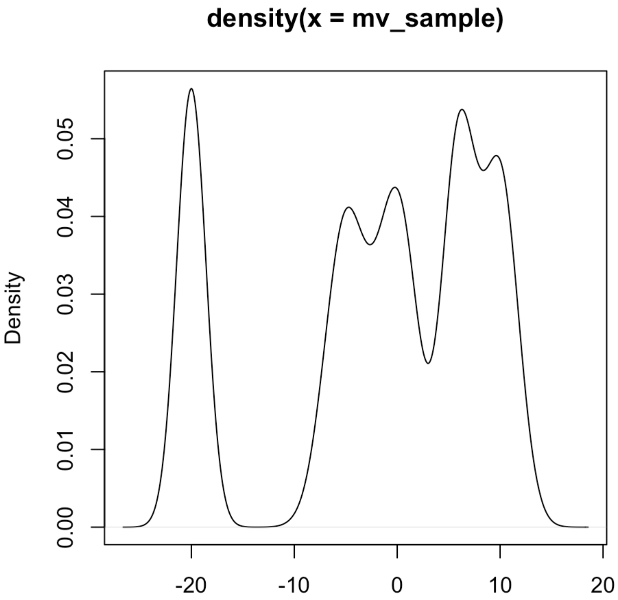
$$\mathcal{L}(a, \mathbf{Y}) = \sum_{i=1}^{n} L(\mathbf{P}_i'\mathbf{a}(s,t), \mathbf{P}_i'\mathbf{Y}(s,t))$$

Consider loss fucntion $||\mathbf{P}'\mathbf{a} - \mathbf{P}'\mathbf{Y}||_2$ The loss function turns out to be

$$(\mathbf{a} - \mathbf{Y})'\mathbf{P}\mathbf{P}'(\mathbf{a} - \mathbf{Y})' = \sum_{i=1}^{n}(a_i - y_i)^2$$

which is ordinary square error function.

Extension on mvn mean estimation



$ optim(par=intp,fn=plos,data=mv_sample,N=N,method="L-BFGS-B")$par

[1] 10.007719467 0.008847759 -4.977106627 5.995543637
-20.005492237

$ optim(par=intp,fn=sqe,data=mv_sample,N=N,method="L-BFGS-B")$par

[1] 10.007719463 0.008847757 -4.977106624 5.995543640
-20.005492236

$ apply(mv_sample,2, mean)

[1] 10.007719463 0.008847757 -4.977106623 5.995543639
-20.005492236

Loss function based on displacement :symmetric loss function

$$L(a, y) = ||a - y||_p$$

When $p = 0, 1, 2$,loss function are represented as 0-1 loss , absolute-deviation loss and squared-error loss respectively.

Loss function based on displacement :asymmetric loss function

$$L(a, y) = (a - y)[\mathbb{I}_{(0,\infty)}(a - y) - q]; q \in (0, 1)$$

LINEX computation

$$L(a, y) = exp\{\psi(a - y)\} - \psi(a - y) - 1; \psi \in (-\infty, \infty)$$

The optimal predictor of Y:

$$\delta^* = \frac{1}{\psi} \log(E[exp\{\psi y\}|z])$$

Potential function

$$L(a, Y) = -\log(f(a - y; \omega)) + \log(f(0; \omega)); \omega \in \Omega$$

Custome different loss function in Xgboost predicition on flood

Data record of the monthly rainfall index of Kerela from 1900-2018 while telling weather a flood took place that month or not and can be used to predict floods by observing the rainfall pattern.
https://www.kaggle.com/datasets/mukulthakur177/kerela-flood

data structure

head(data)

	SUBDIVISION	YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	ANNUAL.RAINFALL	FLOODS
8	KERALA	2008	0.8	30.3	217.2	108.4	81.2	469.9	505.1	349.0	347.0	343.4	55.4	17.0	2524.5	NO
.0	KERALA	2010	18.6	1.0	31.4	138.9	190.6	667.5	629.0	356.0	275.6	441.4	335.1	46.8	3131.8	YES
[illegible]	KERALA	1934	74.5	1.7	47.7	92.4	106.7	852.9	415.0	337.2	48.4	335.9	93.4	4.9	2410.7	NO
[illegible]	KERALA	1996	2.8	9.1	14.4	124.3	74.3	572.4	696.0	327.4	342.7	294.1	89.9	62.5	2610.0	NO
[illegible]	KERALA	1974	1.6	5.4	16.0	128.0	221.5	266.9	1004.2	533.6	383.6	142.1	61.0	3.6	2767.4	NO
[illegible]	KERALA	1959	3.0	21.4	6.3	150.7	347.2	872.8	1155.7	397.3	405.5	200.4	151.9	34.0	3746.0	YES

tuning parameter

$q = 0.1, h = 0.1$

$\psi = 10$

[1] "trian-error= 0"

[1] "test-error= 0.0212765957446809"