# tut3

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## 1

A 2.6: an unbaised estimator $T(X)$ has expected value $g\prime(\theta)$ CR lower bound is given by $\frac{g\prime(\theta)^2}{\mathcal{I}(\theta)}$ where $\mathcal{I}(\theta)$ is fisher information of $\theta$ is given by

$$\mathcal{I}(\theta) = E[(\frac{\partial}{\partial\theta}\log f(X;\theta))^2] = -E[\frac{\partial^2}{\partial\theta^2}\log f(X;\theta)]$$

a. $g(\theta) = \theta$,for each individual independent Bernouilli trial we have

$$\mathcal{I}(\theta) = -E[\frac{\partial^2}{\partial\theta^2}\log f(X;\theta)] = -E[\frac{\partial^2}{\partial\theta^2}(X\log\theta + (1-X)\log(1-X))] = \frac{1}{\theta(1-\theta)}$$

So for n trials, we have $\mathcal{I}(\theta) = \frac{n}{\theta(1-\theta)}$. Therefore, CR lowerbound is $\frac{\theta(1-\theta)}{n}$ b.According to a,the only difference is $g(\theta) = \theta^2$ We can easily get CR lower bound is $\frac{4\theta^3(1-\theta)}{n}$ A 2.8 a. According to A 2.6 and the property of normal distribution,$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ which have expected value of $n-1$.Therefore,

$$g(\sigma^2) = E(S^2) = \sigma^2$$

while in one sample

$$\mathcal{I}(\sigma^2) = -E[\frac{\partial}{\partial(\sigma^2)}(-\frac{1}{2\sigma^2} + \frac{(X-\mu)^2}{2\sigma^4})] = E[-\frac{1}{2\sigma^4} + \frac{(X-\mu)^2}{\sigma^6})] = \frac{1}{2\sigma^4}$$

For n samples ,we have $\mathcal{I}(\sigma^2) = \frac{n}{2\sigma^4}$ So CR lower bound is $\frac{2\sigma^4}{n}$.

$$Var(S^2) = \frac{\sigma^4 Var(\chi^2(n-1))}{(n-1)^2} = \frac{2\sigma^4}{n-1}$$

The conclusion is obvious. b.

$$MSE(\hat{\theta}) = E[(\hat{\theta}-\theta)^2] = Var(\hat{\theta}) + Bais(\hat{\theta})$$

Let $\hat{\sigma^2} = c\sum_{i=1}^{n}(X_i - \bar{X})$

$MSE(\hat{\sigma^2}) = Var(\hat{\sigma^2}) + Bais(\hat{\sigma^2}) = c^2(n-1)^2Var(S) + [c(n-1)-1]^2\sigma^4 = \sigma^4[(n-1)(n+1)c^2 - 2(n-1)c + 1]$

Therefore, we get min MSE when $c = \frac{1}{n+1}$. A 2.10 $g(\theta) = \theta$,$\mathcal{I}(\theta) = \frac{n}{\sigma^2}$ Therefore, CR bound is $\frac{\sigma^2}{n}$

## 2.

$$\int_0^1 cos(2\pi x)dx = 0$$

```r
#numeric method
n=100
x=seq(0+1/n,1,1/n)
y=cos(x*2*pi)
intcos2pix=sum(y)/n
intcos2pix
```

```
## [1] 1.776357e-17
```

```r
n=1000
x=seq(0+1/n,1,1/n)
y=cos(x*2*pi)
intcos2pix=sum(y)/n
intcos2pix
```

```
## [1] 6.328271e-18
```

```r
n=100
x=runif(n)
y=1-2*runif(n)
cos2pix=cos(2*pi*x)
sum(cos2pix)/n
```

```
## [1] -0.1781335
```

```r
n=1000
x=runif(n)
y=1-2*runif(n)
cos2pix=cos(2*pi*x)
sum(cos2pix)/n
```

```
## [1] -0.005799709
```

```r
#numeric method
n=100
x=seq(0+1/n,1,1/n)
y=cos(2*pi*(x)^2)
intcos2pix2=sum(y)/n
intcos2pix2
```

```
## [1] 0.2441267
```

```r
n=1000
x=seq(0+1/n,1,1/n)
y=cos(2*pi*(x)^2)
intcos2pix2=sum(y)/n
intcos2pix2
```

```
## [1] 0.2441267
```

```r
n=100
x=runif(n)
y=1-2*runif(n)
cos2pix2=cos(2*pi*x^2)
sum(cos2pix2)/n
```

```
## [1] 0.28582
```

```r
n=1000
x=runif(n)
y=1-2*runif(n)
cos2pix2=cos(2*pi*x^2)
sum(cos2pix2)/n
```

```
## [1] 0.2409883
```