Midterm

Mike

2024-04-18

#4 a.

$$pmf(w) = \frac{\theta^w}{w!}\frac{e^{-\theta}}{(1-e^{-\theta})}$$

b. 1.

$$E_\theta(W) = \frac{\theta}{1-e^{-\theta}}$$

Thus,

$$\frac{\partial E_\theta(W)}{\partial \theta} = \frac{1-e^{-\theta}+\theta e^{-\theta}}{(1-e^{-\theta})^2}$$

is decreasing postive function since $\frac{1-e^{-\theta}}{(1-e^{-\theta})^2}$ and $\frac{\theta e^{-\theta}}{(1-e^{-\theta})^2}$ are decreasing when $\theta > 0$. Thus we could use Newton-Raphson approach to calculate $\frac{\theta}{1-e^{-\theta}} - \bar{w}$

2.

$$0 = \frac{\partial \mathcal{L}(\theta, W)}{\partial \theta} = \sum_{i=1}^{n} w_i \frac{1}{\theta} - 1 - \frac{e^{-\theta}}{1-e^{-\theta}}$$

Thus, $\frac{\theta}{1-e^{-\theta}} - \bar{w} = 0$

3.

```r
da<-read.csv(file ="data.csv")
wbar<-mean(da[,1])
n<-1000
theta=100
err=1
while(err>1e-3){
f<-theta/(1-exp(-theta))-wbar
fdash<-(1-exp(-theta)+theta*exp(-theta))/(1-exp(-theta))^2
err<-f/fdash
theta<- theta-err
}
theta
```

```
## [1] 3.137351
```

4.

```r
fn<-function(w,theta){
  (-sum(w*log(theta)-theta-log(factorial(w))-log(1-exp(-theta))))
}
optim(par=100,fn=fn,w=da[,1],method = "BFGS")
```

```
## Warning in log(theta): NaNs produced
```

```
## Warning in log(1 - exp(-theta)): NaNs produced

## Warning in log(theta): NaNs produced

## Warning in log(1 - exp(-theta)): NaNs produced

## $par
## [1] 3.137227
##
## $value
## [1] 179.5827
##
## $counts
## function gradient
##       34        8
##
## $convergence
## [1] 0
##
## $message
## NULL
```