CS 7/5382 – F23 – HW 3

Name: Michael Doherty

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1. (34 points) Given the following two points A and B in 4D homogeneous coordinates,

   A: [1 1 1 1]   B [2 -1 3 1]

   a) Define a ray,.*r*, that passes through A in the direction of B.

$$r = \overrightarrow{AB} = B - A = [2-1,\ -1-1,\ 3-1,\ 1-1]$$

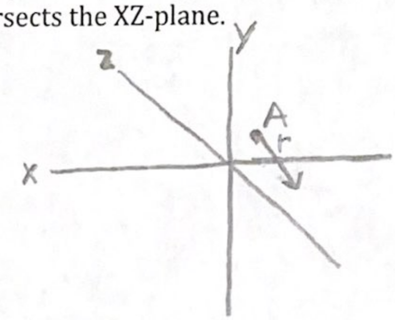
$$\boxed{r = [1\ \ -2\ \ 2\ \ 0]}$$

   b) Express the line that passes through A and B in parametric form.

$$\boxed{L(\alpha) = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \alpha\begin{bmatrix}1\\-2\\2\\0\end{bmatrix} = \begin{bmatrix}1+\alpha\\1-2\alpha\\1+2\alpha\\1\end{bmatrix}}$$

$$x(\alpha) = 1+\alpha$$
$$y(\alpha) = 1-2\alpha$$
$$z(\alpha) = 1+2\alpha$$
$$w(\alpha) = 1$$

   c) Sketch point A and ray, *r*, and show(generally) where the line that includes it intersects the XZ-plane.



   d) Express the point where the line intersects the XZ plan in homogeneous coordinates. Line intersects XZ plane at $y=0$.

$$y(\alpha) = 1-2\alpha = 0$$
$$2\alpha = 1$$
$$\alpha = 1/2$$

$$\begin{bmatrix}1+\alpha\\1-2\alpha\\1+2\alpha\\1\end{bmatrix} \rightarrow \begin{bmatrix}1+1/2\\1-2(1/2)\\1+2(1/2)\\1\end{bmatrix} = \boxed{\begin{bmatrix}3/2\\0\\2\\1\end{bmatrix}}$$

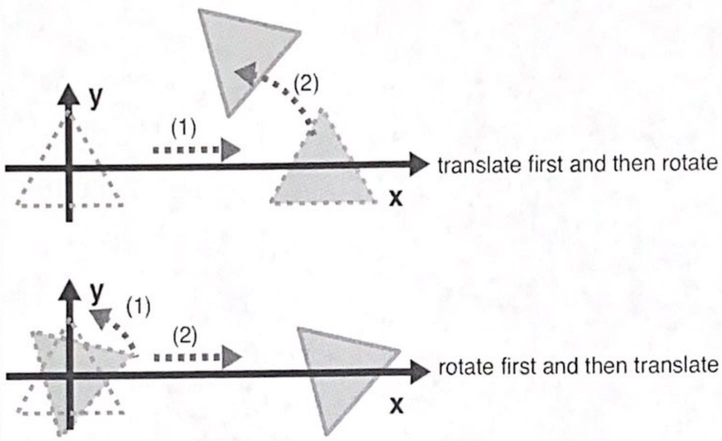
2. (16 points) Here, you see a triangle centered about the origin and the effect of applying transformations to the original vertices. For each (the top and the bottom) construct a <u>single transformation matrix</u> to achieve each of the following using 4D homogenous coordinates.
Use a **horizontal translation of 8** and a rotation about the **Z-axis of 30 degrees to construct the matrics M_A and M_B.**

A) Vertices transformed as

$v' = M_A v$

B) Vertices transformed as

$v' = M_B v$



a. Construct the single matrix M_A

$$T(8,0,0) = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_z(30) = \begin{bmatrix} \cos(30) & -\sin(30) & 0 & 0 \\ \sin(30) & \cos(30) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_A = R_z(30)T(8,0,0) = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 4\sqrt{3} \\ 1/2 & \sqrt{3}/2 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_A}$$

b. Construct the single matrix M_B

$$M_B = T(8,0,0)R_z(30) = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 0 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 8 \\ 1/2 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_B}$$

3. (50 points) Compute the transformed location, of the following 2 points and the following vector under the following series of transformations, expressed as functions.

   a. Rewrite the following series of transformations as concatenated 4x4 matrix multiplication using homogenous coordinates.

q' = $R_z(90)$ S(2, 4,1) q

$$R_z(90) = \begin{bmatrix} \cos(90) & -\sin(90) & 0 & 0 \\ \sin(90) & \cos(90) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S(2,4,1) = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(90) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -4 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \boxed{q' = \begin{bmatrix} 0 & -4 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} q}$$

   b. The points and vector are expressed below as two-element vectors representing x and y. Represent as 4-element homogeneous coordinates.

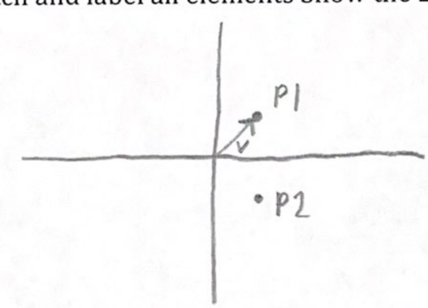
Express as homogeneous coordinate *q*

P1 Points: (0.5, 0.5) ⟶ P1 = [0.5 0.5 0 1]

P2 Point. (0.5, -0.5) ⟶ P2 = [0.5 -0.5 0 1]

V Vector (0.5, 0.5) ⟶ V = [0.5 0.5 0 0]

   c. Sketch and label all elements Show the 2D coordinate axes



Line = P2 − P1

= [0.5−0.5, −0.5−0.5, 0−0, 1−1]

= [0, −1, 0, 0]

Is the vector normal to the line? **Verify using the proper vector operation.** No, the vector is not normal to the line

$$\overset{n}{\begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix}} \cdot \overset{line}{\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}} = 0.5(0) + 0.5(-1) + 0(0) = -0.5$$

Since dot product isn't 0, vector isn't normal to line

d. Apply the above transformations to each of the following points and vectors, replacing the values for q in the expression above to compute q'.

Transformed homogeneous coordinate q'

Points: (0.5, 0.5)

$$q' = \begin{bmatrix} 0 & -4 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad P1'$$

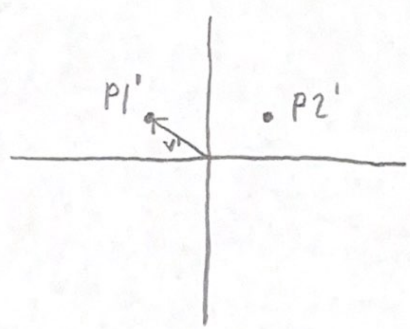
Point(0.5, -0.5)

$$q' = \begin{bmatrix} 0 & -4 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ -0.5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad P2'$$

Vector (0.5, 0.5)

$$q' = \begin{bmatrix} 0 & -4 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad V'$$

e. Sketch and label transformed elements q', Show the 2D coordinate axes



f. Is the vector normal to the line? Verify using the proper vector operation.

no, the vector isn't normal to the line

Line = P2'-P1'

= [2-(-2), 1-1, 0-0, 1-1]

= [4, 0, 0, 0]

$$\underset{n}{\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}} \cdot \underset{line}{\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}} = -2(4) + 1(0) + 0(0) = -8$$

Since dot product isn't 0, the vector isn't normal to the line