Math 366: Assignment 7

Due: April 9, 2025

Instructions: Show your work and explain your reasoning. Incorrect or incomplete answers may receive partial credit for demonstrated progress. If you collaborate with your peers, please indicate who you collaborated with at the top of your submission

1. Adapt your code for RK4 to solve a system of ODEs:

$$\begin{cases} \frac{dx}{dt} = f(x,y,t), \\ \frac{dy}{dt} = g(x,y,t), \\ x(0) = x_0, \\ y(0) = y_0 \end{cases}$$

   I find that Notepad++ is useful for this. Set the language equal to Matlab to get nice formatting and print out your code.

2. Consider the system of ODEs

$$\begin{cases} x'(t) = -x(t) + 2y(t), \ t \geq 0 \\ y'(t) = -2y(t), \ t \geq 0 \\ x(0) = 2, \\ y(0) = 1. \end{cases} \tag{1}$$

   (a) Compute the solution of this system.

   (b) Test your code from problem 1 by using it to solve the above system and compare it to the true answer on the time interval $[0,2]$ with a time step of 0.001. Plot the true and approximate solutions.

3. Consider the 1D heat equation with periodic boundary data:

$$\begin{cases} u_t - u_{xx} = 0, \ t \geq 0, \ -\pi \leq x \leq \pi, \\ u(t,-\pi) = u(t,\pi), \ t \geq 0 \\ u(0,x) = u_0(x), \ -\pi \leq x \leq \pi. \end{cases}$$

   Define the energy as

$$E(t) = \int_{-\pi}^{\pi} |u(t,x)|^2 dx.$$

   Show that $E(t) \leq E(0)$. *Hint: To do this, multiply the PDE by $u(t,x)$ and integrate from $-\pi$ to π. Use the fact that $\frac{d}{dt}(f(t))^2 = 2f(t)f'(t)$.*