

# Learning How to Dynamically Decouple

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Current quantum computers suffer from noise that stems from interactions between the quantum system that constitutes the quantum device and its environment. These interactions can be suppressed through dynamical decoupling to reduce computational errors. However, the performance of dynamical decoupling depends on the type of the system-environment interactions that are present, which often lack an accurate model in quantum devices. We show that the performance of dynamical decoupling can be improved by optimizing its rotational gates to tailor them to the quantum hardware. We find that compared to canonical decoupling sequences, such as CPMG and XY4, the optimized dynamical decoupling sequences yield the best performance in suppressing noise in superconducting qubits. Our work thus enhances existing error suppression methods which helps increase circuit depth and result quality on noisy hardware.

## I. INTRODUCTION

Noise currently limits quantum computers from harnessing their full potential. In the long term, quantum error correction is expected to overcome this issue [1]. In the short term, noise mitigation and suppression techniques are critical to improve quantum device performance [2]. Error mitigation is designed to reduce noise, typically, in expectation values [2, 3]. The computation is executed on the noisy quantum computer multiple times to either extrapolate to a zero-noise limit [4], or to cancel the noise on average [5]. By contrast, error suppression methods, such as dynamical decoupling [6] and pulse-efficient transpilation [7], reduce the presence of noise directly in the quantum circuits.

In this work we focus on the well-established noise and error suppression technique dynamical decoupling (DD). Inspired by nuclear magnetic resonance spectroscopy (NMR) [8], the theory of DD was first developed by Lorenza Viola, Emanuel Knill, and Seth Lloyd in 1998 [6] as an open-loop control technique. DD suppresses errors by decoupling the system from its environment through the application of a sequence of pulses which, in the ideal case, compose to the identity. The utility of DD has been demonstrated in a wide range of quantum systems, such as coupled nuclear and electron spins [9], trapped ions [10], electron spins [11], and superconducting qubits [12]. Since the introduction of the DD framework in the 90's, DD has also become a viable method to suppress noise and errors in quantum computing [13]. For example, DD can suppress crosstalk [14, 15] and improve the performance of superconducting qubit based quantum devices [12, 16] in general. DD sequences can be designed from first principles [17] or with numerical simulations, leveraging tools such as genetic algorithms [18] and machine learning [19].

In recent years, DD has become a major error suppres-

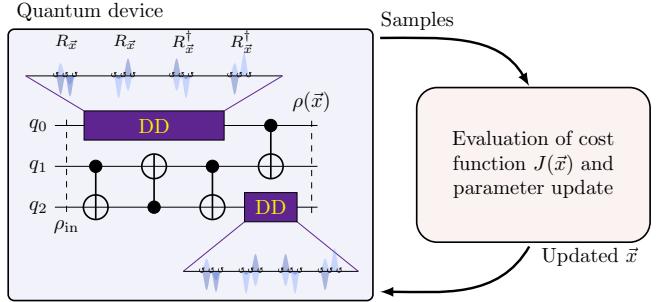


FIG. 1. **Illustration of LDD.** The left and the right block indicates a quantum device and a classical computer, respectively, working in tandem. A DD sequence parameterized by  $\vec{x}$  is added to idling qubits in a given quantum circuit, here exemplified by a SWAP gate on  $(q_1, q_2)$  followed by a CNOT on  $(q_0, q_1)$ . The arbitrary single-qubit rotations  $R_{\vec{x}}$  in the DD sequence are realized by three parameterized virtual  $Z$ -gates (circular arrows) and two  $\sqrt{X}$  gates (pulse with in-phase and quadrature shown in light and dark blue, respectively). The noisy output state  $\rho(\vec{x})$  is compared to the expected ideal state  $\rho_{\text{ideal}}$  through the cost function  $J(\vec{x})$ . A classical optimization algorithm minimizes  $J(\vec{x})$  to find the parameters of the gates in the DD sequence that best fit the circuit.

sion method for noisy superconducting quantum computers [16]. Indeed, the method is easy to apply as a simple transpilation pass that inserts delays and pulses into a quantum circuit. Furthermore, simple sequences such as  $X - X$ , where  $X$  is a  $\pi$ -rotation around the qubit's  $x$ -axis, already yield excellent results [20]. More elaborate sequences, such as staggered  $X - X$  [21] and staggered XY4, improve, for instance, the execution of dynamic circuits by cancelling cross-talk [22]. Crucially, the performance of a DD sequence depends on the interactions present in the quantum hardware. In superconducting qubits [23], a good model of these interactions is typically not known, a case familiar to optimal control [24–27] that can be overcome with closed-loop optimization [28, 29]. Similarly, it is possible to tailor DD sequences to the

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quantum hardware at hand by learning them through genetic algorithms [30]. However, in hardware, DD pulses are not perfect, potentially introducing additional noise and errors through the controls that implement the DD pulses, thereby diminishing the quality of the designed DD sequence.

To overcome these limitations, in this work we tailor the DD sequences to the hardware and quantum circuits to execute. This is achieved by optimizing the rotational angles of the gates in the DD sequence in a closed-loop with the quantum hardware. A classical optimizer is fed the cost function value that is reconstructed from quantum samples and that is sensitive to the quality of the DD pulses, see Fig. 1.

This manuscript is structured as follows. We introduce in Sec. II two commonly employed DD sequences CPMG and XY4. Next, in Sec. III we develop the theoretical framework of how optimal parameters in DD sequences are found on quantum hardware, which we refer to as learning dynamical decoupling (LDD). We demonstrate in Sec. IIIB the utility of LDD on IBM Quantum hardware by comparing the performance of LDD to CPMG and XY4 to suppress noise in two experiments. We show that LDD outperforms CPMG and XY4 for suppressing noise present during mid-circuit measurements and noise resulting from increasing the depth of a quantum circuit. We conclude in Sec. IV.

## II. BACKGROUND: DYNAMICAL DECOUPLING

DD can in general suppress generic interactions through pulses that rotate around multiple axis [31]. However, DD is most resource efficient when tailored to the specific type of interactions at hand [32, 33]. Furthermore, the most effective DD sequence depends on the noise type present in the physical system.

The spin echo [34] in NMR can be seen as a DD experiment where a single Pauli  $X$  gate refocuses coherent errors. The “Carr–Purcell–Meiboom–Gill” (CPMG) DD sequence is an extension of the spin echo with two symmetric insertions of an  $X$  gate [35, 36]. The resulting pulse sequence is

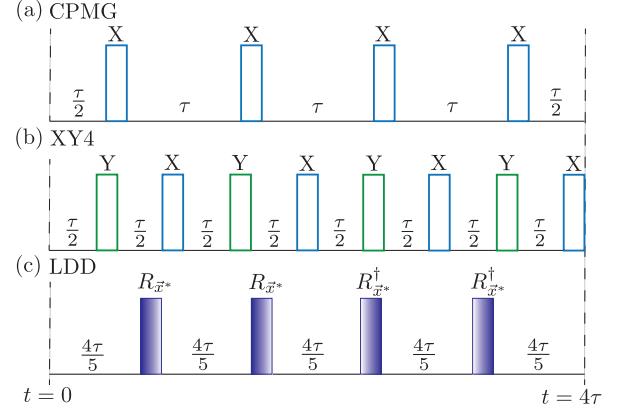
$$\text{CPMG} \equiv \tau/2 - X - \tau - X - \tau/2, \quad (1)$$

where  $\tau$  is the duration of the free evolution. Multiple CPMG sequences can be concatenated one after another, see Fig. 2(a). While the CPMG sequence can suppress homogeneous dephasing along one axis, it cannot suppress noise stemming from generic system-environment interactions. By contrast, the XY4 DD sequence [37], defined by

$$\text{XY4} \equiv Y - \tau - X - \tau - Y - \tau - X - \tau, \quad (2)$$

and shown in Fig. 2(b), is a universal DD sequence that can suppress generic system-environment interactions. It

employs  $\pi$ -rotations around the  $x$  and  $y$ -axis described by Pauli operators  $X$  and  $Y$ , respectively. However, the effectiveness of these DD sequences critically depends on the noise, i.e., the detrimental interactions that are present in the system, which is often challenging to infer in superconducting quantum devices [23]. As such, the performance of a DD sequence can vary substantially across different device architectures.



**FIG. 2. Schematic diagram of DD sequences.** (a) Two concatenated CPMG sequences. (b) Two concatenated XY4 sequences. (c) the LDD sequence we introduce in this work. All sequences are depicted for an idle period of  $4\tau$ . CPMG uses only  $X$  gates (blue rectangles) whereas XY4 uses both  $X$  and  $Y$  gates (green rectangles). LDD uses parameterized gates,  $R_{\vec{x}^*}$  (purple rectangles) where the optimal rotation angles  $\vec{x}^*$  are found by classical optimization routines performed in tandem with a quantum device. A single LDD sequence is defined by  $\tau - R_{\vec{x}^*} - \tau - R_{\vec{x}^*}^\dagger - \tau$ , where  $\tau$  is the duration of the free evolution.

## III. LEARNING HOW TO SUPPRESS NOISE

We use tools from closed-loop optimal control [24] and optimization [28–30, 38] to learn optimal DD sequences without precise knowledge of a noise model. Tong *et al.* [30] demonstrate the usefulness of this approach by optimizing with a genetic algorithm the placement of DD gates on idling qubits in a quantum circuit to achieve, for instance, a higher success probability in the Bernstein–Vazirani algorithm compared to canonical DD sequences. In their approach, they chose DD gates from the fixed set  $\{I_\pm, X_\pm, Y_\pm, Z_\pm\}$  where  $I_+$ ,  $X_+$ ,  $Y_+$ ,  $Z_+$  are the Pauli matrices and the minus sub-script indicates an added phase of  $\pi$ .

DD gates themselves are prone to errors, such as errors in the rotation angles. Furthermore, the optimal rotation axis of the gates in the DD sequence may depend on the type of noise in the system. We address these issues by adopting a different optimization approach from Tong *et al.* [30]. Instead of fixing a set of DD gates and optimizing over the DD sequence structure to find the best DD sequence [30], we optimize the rotational parameters

entering in the chosen DD sequence to improve performance. This approach is similar to finding noise resilient quantum circuits through machine learning [39]. After introducing the theory behind such a learning dynamical decoupling approach, we demonstrate on IBM Quantum hardware in two experiments that the optimized LDD sequences yield the best performance (compared to XY4 and CPMG) in suppressing errors.

### A. Theory

We consider a quantum circuit described by the ideal quantum channels  $\mathcal{U}_j$  applied in sequence,

$$\rho_{\text{ideal}} = \prod_j \mathcal{U}_j(\rho_0), \quad (3)$$

to an initial state  $\rho_0 = |\psi_0\rangle\langle\psi_0|$ , which we assume is pure, to create a desired target state  $\rho_{\text{ideal}} = |\psi_{\text{ideal}}\rangle\langle\psi_{\text{ideal}}|$ . Here, we concern ourselves with noise described by a collection of typically unknown quantum channels  $\mathcal{M}_j$ , in which we include potential mid-circuit measurements, that act between the unitary channels  $\mathcal{U}_j$  when qubits are idling. The noisy quantum circuit then takes the form  $\prod_j \mathcal{M}_j \mathcal{U}_j$ . Each noise channel  $\mathcal{M}_j$  can describe unitary and non-unitary errors. To suppress noise induced by  $\mathcal{M}_j$  we divide  $\mathcal{M}_j$  into  $N+1$  channels  $\mathcal{M}_{j,k}$  so that  $\prod_{k=1}^{N+1} \mathcal{M}_{j,k} = \mathcal{M}_j$ , and insert a LDD sequence of the form shown in Fig. 2(c). The LDD sequence is described by parameterized unitary quantum channels  $\mathcal{R}_{\vec{x}}(\cdot) = R_{\vec{x}}(\cdot)R_{\vec{x}}^\dagger$  that are applied between the  $\mathcal{M}_k$ 's where the LDD gates  $R_{\vec{x}}$  are parameterized by  $\vec{x} \in \mathbb{R}^M$ . For simplicity, we assume that the LDD parameters  $\vec{x}$  are the same for each LDD gate  $R_{\vec{x}}$ . If we include  $N/2$  LDD gates between the noise channels  $\mathcal{M}_{j,k}$ , followed by implementing  $N/2$  times the corresponding inverse  $R_{\vec{x}}^\dagger$  to insure that the total LDD sequence composes to the identity in the absence of noise, as depicted in Fig. 2, the noise channel  $\mathcal{M}_j$  becomes

$$\mathcal{M}_{j,\vec{x}} := \mathcal{M}_{j,N+1} \prod_{k=N/2}^N \mathcal{R}_{\vec{x}}^\dagger \mathcal{M}_{j,k} \prod_{k=1}^{N/2} \mathcal{R}_{\vec{x}} \mathcal{M}_{j,k}. \quad (4)$$

The state resulting from the noisy circuit with LDD is then given by

$$\rho(\vec{x}) = \prod_j \mathcal{M}_{j,\vec{x}} \mathcal{U}_j(\rho_0). \quad (5)$$

To optimize the parameters  $\vec{x}$  we need a cost function  $J(\vec{x})$  that is sensitive to the quality of the circuits in Eq. (3). For small circuits, a natural choice for a figure of merit or cost function is the fidelity error,

$$J(\vec{x}) = 1 - \langle \psi_{\text{ideal}} | \rho(\vec{x}) | \psi_{\text{ideal}} \rangle, \quad (6)$$

where  $F = \langle \psi_{\text{ideal}} | \rho(\vec{x}) | \psi_{\text{ideal}} \rangle$  is the fidelity with respect to the ideal state. However, state tomography scales exponentially with system size. For large circuits, a scalable cost function can be built in multiple ways. As done in Refs. [30, 40] we can invert the circuit with mirroring in which each  $\mathcal{U}_j^\dagger$  is applied in reverse such that  $\rho_{\text{ideal}} = \rho_0$ . Alternatively, one can reduce the original quantum circuit to a Clifford circuit as done, for instance, in Refs. [41, 42]. Here, the single-qubit gates in a circuit (with the CNOT as the two-qubit gate) are replaced by Clifford gates such that the whole circuit is a Clifford gate. In hardware where parameters are encoded in virtual-Z gates, this will preserve the structure and timing of the underlying pulses, thereby leaving most noise sources such as  $T_1$ ,  $T_2$  and cross-talk unchanged at the pulse-level. We can then compute  $\rho_{\text{ideal}}$  with efficient Clifford based simulators.

Solving the optimization problem

$$\min_{\vec{x} \in \mathbb{R}^M} J(\vec{x}), \quad (7)$$

yields the optimal parameter values  $\vec{x}^*$  of the LDD sequence that minimize  $J(\vec{x})$ . Since we do not know the noise processes described by  $\mathcal{M}_j$ , we minimize  $J$  in an iterative, variational quantum algorithm type fashion by using quantum and classical computing resources in tandem [27, 43]. By measuring the output at the end of the quantum circuit we estimate  $J$ , while a classical search routine is employed to update the parameters.

### B. Case studies on IBM hardware

We study the performance of LDD in two different experiments carried out on IBM Quantum hardware with Bell pairs; a valuable resource. For instance, Bell pairs enable quantum gate teleportation [44] and similar known states generated in a factory of resources to enable circuit cutting [22]. In the first experiment we suppress noise during mid-circuit measurements. In the second experiment we suppress noise resulting from an increasing circuit depth. In both cases we minimize fidelity loss due to noise by increasing the fidelity of preparing the Bell state  $|\psi_{\text{ideal}}\rangle = |\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ , on two qubits  $q_i$  and  $q_j$  within a  $n > 2$  qubit system, starting from  $\rho_0 = |0\rangle\langle 0|^{\otimes n}$ . To infer the value of  $J(\vec{x})$  in Eq. (6) for the Bell state, it suffices to measure the expectation values of the  $X_i X_j$ ,  $Y_i Y_j$  and  $Z_i Z_j$  Pauli operators with respect to  $\rho(\vec{x})$ , rather than performing full state tomography [45–51], i.e.,

$$J(\vec{x}) = 1 - \frac{1}{4} \langle \mathbf{1} + X_i X_j - Y_i Y_j + Z_i Z_j \rangle_{\rho(\vec{x})}. \quad (8)$$

We employ parameterized decoupling operations given by arbitrary single-qubit rotations

$$R_{\vec{x}} = R(\theta, \phi, \lambda) = e^{-i\frac{\theta}{2}Z} e^{-i\frac{\phi}{2}Y} e^{-i\frac{\lambda}{2}Z}, \quad (9)$$

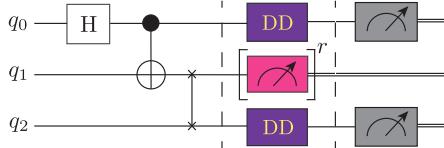
where in both experiments we repeat  $R(\theta, \phi, \lambda)$  and its inverse in total  $N = 4$  times, described by Eq. (4). Here,

we only consider three angles  $\vec{x} = (\theta, \phi, \lambda)$  to parameterize the full LDD sequence. All qubits with an LDD sequence thus share the same parameter values. This limits the size of the search space. On IBM Quantum hardware, these single-qubit rotations are implemented by three parameterized virtual- $Z$  rotations [52] and two  $\sqrt{X}$  pulses. Therefore, in LDD we optimize the angles in these virtual- $Z$  rotations. We pick the Simultaneous Perturbation Stochastic Approximation (SPSA) gradient descent method [53, 54] to solve the optimization problem in Eq. (7) starting with  $(\theta, \phi, \lambda) = (0, 0, 0)$ . We allow SPSA a total of 100 iterations. At each iteration SPSA requires only two estimations of the objective function, regardless of the number of optimization parameters. The hyperparameter ‘‘perturbation’’ in SPSA is set to the default values and the hyperparameter ‘‘learning rate’’ is calibrated by the optimizer [55].

Below we compare LDD with CPMG and XY4 where DD sequences are inserted when qubits are idling. Throughout the two experiments we concatenate two CPMG and XY4 sequences and adjust the duration  $\tau$  to match the idling times of the qubits.

### 1. Suppressing noise during mid-circuit measurements

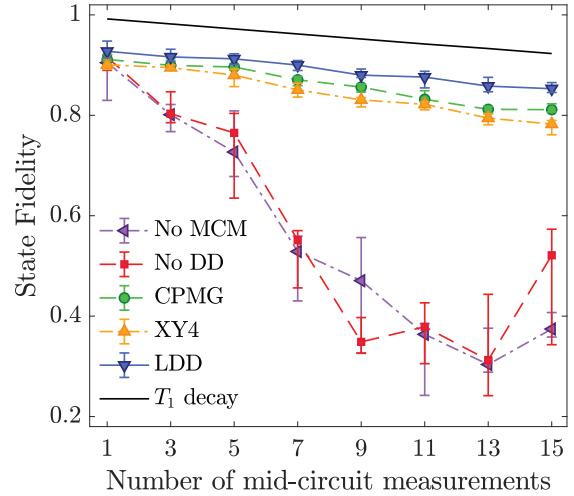
A mid-circuit measurement (MCM) involves measuring qubits at intermediate stages within a quantum circuit. MCMs have various applications including quantum error correction (QEC) [56], quantum teleportation [57], reducing the depth of a quantum circuit [58], circuit cutting [22], and analyzing complex quantum behaviour [59]. Unfortunately, MCMs may introduce noise on neighbouring qubits of the physical device [60].



**FIG. 3. Suppressing noise during MCMs through DD.** A Bell state is prepared on qubit  $q_0$  and  $q_2$ . The magenta block on qubit  $q_1$  is a MCM that is repeated  $r$  times. The purple blocks on qubit  $q_0$  and  $q_2$  indicate DD sequences inserted during the MCMs to mitigate noise on  $q_0$  and  $q_2$ . We vary  $r$  and study the performance of different DD sequences on a 27 qubit IBM Quantum Falcon device (*ibm\_hanoi*). The corresponding experimental results are shown in Fig. 4.

As depicted in Fig. 3, we consider the task of preparing the Bell state  $|\Phi_+\rangle$  between qubit  $q_0$  and qubit  $q_2$  while qubit  $q_1$  is subject to  $r$  repeated MCMs. Here, varying  $r \in \{1, \dots, 15\}$  allows us to amplify the amount of noise introduced in the quantum circuit.

First, we compute  $F$  without any DD, once with  $r$  MCMs and once with a delay equivalent to  $r$  MCMs. The experimental result are shown in Fig. 4 (red and purple



**FIG. 4. DD noise suppression during MCMs.** Fidelity for preparing a Bell state between qubit  $q_0$  and  $q_2$  in Fig. 3 as a function of the number of MCMs performed on qubit  $q_1$ . The dashed red curve shows the fidelity with MCMs but without DD and the dash-dotted purple curve shows the fidelity with a delay equivalent to MCMs. DD sequences are inserted on  $q_0$  and  $q_2$  when the qubits are idling. The dashed green curve corresponds to the CPMG sequence, the dash-dotted orange curve corresponds to the XY4 sequence and the solid blue curve corresponds to the optimized LDD sequence. The optimal LDD parameters are shown in Table I. For comparison, the solid black curve shows the  $T_1$  decay. The error bars show the interquartile range.

# MCMs	Idle time ( $\mu$ s)	Optimal LDD parameters		
		$\theta^*$	$\phi^*$	$\lambda^*$
1	0.82	$0.37\pi$	$0.22\pi$	$0.05\pi$
3	2.45	$-0.32\pi$	$-0.35\pi$	$0.13\pi$
5	4.09	$-0.66\pi$	$0.74\pi$	$-0.66\pi$
7	5.72	$-0.67\pi$	$0.40\pi$	$-0.54\pi$
9	7.35	$-0.28\pi$	$-0.53\pi$	$-0.29\pi$
11	8.99	$-1.13\pi$	$1.49\pi$	$-1.34\pi$
13	10.62	$0.95\pi$	$-1.07\pi$	$-0.68\pi$
15	12.26	$-0.99\pi$	$-0.79\pi$	$0.50\pi$

**TABLE I. Optimal LDD parameters to suppress noise during MCMs.** The angles in the LDD gate given in Eq. (9) are optimized to find the optimal rotational (Euler) angles  $\vec{x}^* = (\theta^*, \phi^*, \lambda^*)$ . The optimal angles are shown for a different number of MCMs applied on  $q_1$  that correspond to the idling times of qubit  $q_0$  and  $q_2$ . For reference, the Euler angles for the Pauli  $X$ , and Pauli  $Y$  gates used in the XY4 sequence are  $(\pi, -0.5\pi, 0.5\pi)$  and  $(\pi, 0, 0)$ , respectively.

curves) where we report the median and the lower and upper quartiles of ten measurements. In the presence of MCMs we observe a large drop in  $F$  from  $0.916^{+0.004}_{-0.026}$  at  $r = 1$  to  $0.447^{+0.052}_{-0.178}$  at  $r = 15$ . Without MCMs  $F$  drops from  $0.905^{+0.015}_{-0.075}$  at  $r = 1$  to  $0.386^{+0.033}_{-0.016}$  at  $r = 15$ . This implies that a MCM on  $q_1$  does not have a large impact on  $q_0$  and  $q_2$  aside from adding a 820 ns delay.

Crucially, the fidelity decrease due to the long delays induced by  $r$  MCMs on  $q_1$  can be mitigated by DD sequences inserted on neighbouring qubits  $q_0$  and  $q_2$  during the MCM measurement, see Fig. 3. In Fig. 4 we compare the performance of LDD (blue), CPMG (green), and XY4 (orange). The LDD sequence, whose corresponding optimal parameters are shown in Table I, yields the best performance, resulting in e.g., a fidelity of  $0.853_{-0.007}^{+0.012}$  at  $r = 15$  while CPMG and XY4 result in  $0.814_{-0.005}^{+0.012}$  and  $0.774_{-0.021}^{+0.007}$ , respectively. To evaluate the reliability of the LDD sequence we compute  $F(\vec{x}^*)$  ten times after the learning and report the lower quartile, median, and upper quartile of these ten runs. Due to queuing, these circuits were executed on the hardware two days after the optimal parameters  $\vec{x}^*$  were learnt. This indicates that the learned parameters  $\vec{x}^*$  are stable in time. We attribute the residual decay to  $T_1$  shown for comparison in Fig. 4 as a solid black curve. This curve is computed as  $e^{-t/T_1}$  where  $t$  is the idling time and  $T_1 = 167 \mu\text{s}$  is the average  $T_1$  time of qubits  $q_0$  and  $q_2$  of *ibm\_hanoi*.

## 2. Suppressing deep circuit noise

Next, we consider suppressing noise through LDD that is introduced due to increasing the depth of a quantum circuit. In particular, we consider the task of preparing the Bell state  $|\Phi_+\rangle$  between two qubits located at the edges of a qubit chain with nearest neighbour interactions.

The corresponding coupling graph of the IBM Quantum device is shown in Fig. 5(a) where the considered qubit chain is highlighted in the dashed grey box. In Fig. 5(b) we show the quantum circuit that prepares a Bell state between qubits  $q_0$  and  $q_{12}$  by bringing them into proximity with a ladder of SWAP gates. Since the ancilla qubits are in their ground state  $|0\rangle$ , we can implement each SWAP gate with two CNOT gates instead of three. Similar gate ladders with a single CNOT at each rung often occur in quantum simulation algorithms to create unitaries generated by Pauli strings [61, 62]. In Fig. 5(c) we plot the fidelity  $F$  for preparing the Bell state between the two edge qubits of the chain in Fig. 5(a) as a function of the intermediate qubits (IQ) shown by the dashed red curve. We see a fidelity decrease from  $0.948_{-0.002}^{+0.002}$  for a chain consisting of 2 qubits (i.e., 0 IQ) to  $0.720_{-0.013}^{+0.024}$  for a chain consisting of 10 qubits (i.e., 8 IQ).

To mitigate the Bell state fidelity decrease, as depicted in Fig. 5(b), we insert DD sequences on idling qubits and compare their performance. The results shown in Fig. 5(c) for the CPMG sequence (green), the XY4 sequence (orange), and the LDD sequence (blue) suggest that the best performance is obtained for LDD where the corresponding optimal parameters are given in Table II.

In fact, since the fidelity obtained through inserting XY4 and CPMG can even be below the fidelity obtained without DD, observing a significant fidelity drop for XY4 at 1 IQ, we conclude that DD can increase the noise instead of suppressing it. This situation is avoided by

LDD as the DD sequence is tailored to the device.

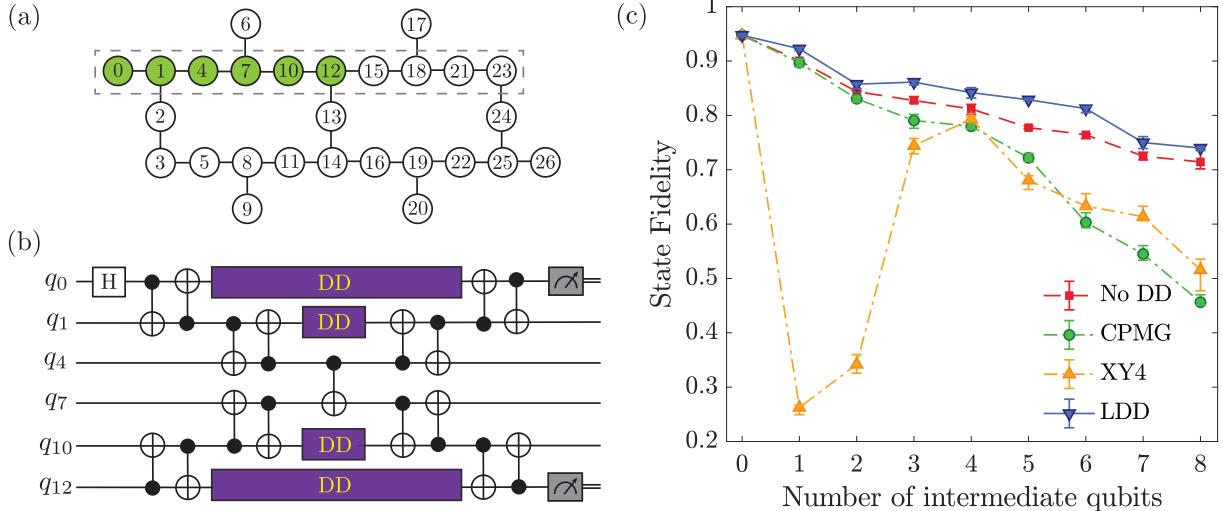
# IQs	Optimal LDD parameters		
	$\theta^*$	$\phi^*$	$\lambda^*$
1	$-0.41\pi$	$0.47\pi$	$-0.28\pi$
2	$-0.03\pi$	$0.72\pi$	$0.03\pi$
3	$-0.02\pi$	$-0.45\pi$	$0.02\pi$
4	$-0.07\pi$	$-0.47\pi$	$-0.25\pi$
5	$-0.03\pi$	$-0.48\pi$	$-0.17\pi$
6	$-0.02\pi$	$0.24\pi$	$1.02\pi$
7	$1.94\pi$	$0.85\pi$	$-0.42\pi$
8	$-0.01\pi$	$1.85\pi$	$-0.33\pi$

TABLE II. **Optimal LDD parameters for suppressing deep circuit noise.** The angles in the LDD gate given in Eq. (9) are optimized to find the optimal rotational (Euler) angles  $(\theta^*, \phi^*, \lambda^*)$ . The optimal  $\vec{x}^* = (\theta^*, \phi^*, \lambda^*)$  are shown for different number of intermediate qubits (IQ) in the chain configuration in Fig. 5(a).

## IV. CONCLUSION

Dynamical decoupling (DD) is a powerful noise suppression strategy that averages out detrimental processes by applying properly designed pulses to the system. We introduced the framework of “learning dynamical decoupling” (LDD). Instead of considering a DD sequence with fixed rotational gates, LDD optimizes directly on quantum hardware the rotational parameters in the DD gates. We compared the performance of such optimized DD sequences with the known DD sequences CPMG and XY4 on IBM Quantum hardware. We found that LDD outperforms both sequences in suppressing noise that occurs during mid-circuit measurements and noise that stems from increasing the depth of a quantum circuit.

The LDD sequences that we studied have by design a small number of single-qubit gates and a fixed number of rotational parameters. While we believe that performance can be increased even further by adding more optimization parameters, the results shown in Fig. 5 for different system sizes (i.e., a different number of intermediate qubits) suggest that the number of LDD parameters can remain constant while achieving a similar performance when the system is scaled. As such, the classical optimization overhead does not need to increase when the system size increases. Therefore, the LDD approach considered here is scalable by design. Furthermore, inserting only a small number of single-qubit gates in LDD or DD on idling qubits to suppress noise is important for current quantum devices. Idle times in quantum circuits typically occur when a subset of all qubits undergo two-qubit gates. Therefore, the shorter the two-qubit gate duration is on a device, the more compact the DD sequence needs to be. To illustrate this consider *ibm\_torino* and *ibm\_sherbrooke* which have a median two-qubit gate duration of 84 ns and 533 ns, respectively [63]. The duration of both the single-



**FIG. 5. Suppressing deep circuit noise through DD.** (a) Qubit coupling graph of the 27 qubit IBM Falcon device *ibm\_cairo*. Qubits are shown as circles labeled by numbers. The green circles highlight the qubits in the quantum circuit in (b). The grey dashed box shows the chain of qubits 0, 1, 4, 7, 10, 12, 15, 18, 21, and 23 used in the experiment. (b) Quantum circuit to prepare a Bell state between qubit  $q_0$  (one edge of the chain) and qubit  $q_i$  (here shown for  $i = 12$ ) using a ladder of CNOT gates. The insertion of DD sequences is shown as purple boxes. (c) Fidelity of preparing a Bell state between qubit  $q_0$  and  $q_i$  in the chain in the coupling graph (a) as a function of the number of intermediate qubits. The dashed red curve shows the fidelity without DD, the dashed-dotted orange curve corresponds to the XY4 sequence, the dashed-dotted green curve corresponds to the CPMG sequence, and the solid blue curve correspond to the optimized LDD sequence. The optimal LDD parameters are shown in Table II. The error bars show the interquartile range.

qubit  $X$  and  $\sqrt{X}$  gates are 32 ns and 57 ns for *ibm\_torino* and *ibm\_sherbrooke*, respectively. As such, on *ibm\_torino* we can insert up to two  $X$  or  $Y$  gates or one arbitrary single-qubit rotation during the median two-qubit gate duration. By contrast, on *ibm\_sherbrooke* these numbers become eight and four, respectively. Consequently, as the two-qubit gate duration become comparable with the single-qubit gate duration, short DD sequences become more important.

In summary, DD is crucial to suppressing errors in noisy hardware. As DD sequences improve – becoming more tailored to the hardware – so do the hardware results. This motivates the strong interest in DD. Future work may include optimizing the spacing of the DD pulses in LDD. Furthermore, one could explore how to protect circuit cutting resources, consumed in teleportation circuits as demonstrated in Ref. [22]. Indeed, these resources are less

costly to generate simultaneously. However, this has the draw-back that they idle until they are consumed. Finally, we optimized virtual- $Z$  rotations that sandwich  $\sqrt{X}$  gates. Future work may thus elect to directly optimize the pulses that implement the DD sequence, e.g., similar to pulse-level variational quantum algorithms [27, 64].

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