Events and sample spaces

Given a random variable X, there is a *sample space*. Each *element* of the sample space is called a *simple event*. Contrast a simple event to just an *event*, which can be made up of several simple events.

Example

Suppose we roll a six-sided die and record the number shown on the face-up side; call this result X. Then X is a random variable, and:

- $\Omega_X =$
- The simple events are:
- Some examples of events are:

Note that X is a discrete random variable.

Probability mass functions

A probability mass function (PMF) is a function that tells you the probability for each simple event.

One way of presenting a PMF is by using a table, e.g.,

x	1	2	3	4	5	6
$P(X = x)$	1/6	1/6	1/6	1/6	1/6	1/6

These are useful when the simple events are *not* all equally likely. We will see this now in the context of expected value and variance.

Expected value and variance

Expected value and variance are ways of measuring *location and spread*. Here we will consider a modified die with faces 2, 3, 4, 4, 5, 5.

Set operations and probability

Set operations

There are three standard set operations used in mathematics: union, intersection and complement. These have important interpretations in probability.

- The union of two events A and B is denoted by $A \cup B$, and corresponds to the event that one of A or B occurs.
- The intersection of two events A and B is denoted by $A \cap B$, and corresponds to the event that both A and B occur.
- The complement of an event A is denoted by A^c, and corresponds to the event that A does not occur.

Example

Consider the standard six-sided die, let A denote the event of rolling an even number, let B denote the event of rolling an odd number, and let C denote the event of rolling a number greater than or equal to 4.

Then $A =$, $B =$ and $C =$.

Describe each of the following in words, and then write the event as a set of simple events.

- $A \cup B$
- $A \cap B$
- $A \cap C$
- $B \cup C$
- A^c
- C^c

Calculating probabilities

It is essential to remember, and know how to use, the following properties:

- The complement rule:

$$P(A^c) = 1 - P(A)$$

- The inclusion-exclusion principle (IEP):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Note: the IEP can be rearranged to calculate other probabilities, e.g.:

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Refer to Example 1.6.5 from the reading notes for some examples.

A worked example

Example

A random variable X has the following probability mass function:

x	1	3	6	7
$P(X=x)$	0.121	0.606	0.161	0.112

For this question, use 3 decimal places of accuracy while performing all calculations, and give all final answers to 3 decimal places.

- Write down the set Ω_X.
- Determine $E(X)$, $\text{Var}(X)$ and $\text{SD}(X)$.
- Let A denote the event "$X \leqslant 6$", let B denote the event "$X \geqslant 3$", and let C denote the event "$X = 3$".
  Determine each of the following:
  - $P(A)$, $P(B)$ and $P(C)$
  - $P(A^c)$, $P(B^c)$ and $P(C^c)$
  - $P(A \cup B)$ and $P(A \cup C)$
  - $P(A \cap B)$ and $P(A \cap C)$