Fluid Mechanics (ENGR30002)
Laboratory Report 2

Elements and Evaluation of a Centrifugal Pump

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ABSTRACT

In this report, the pressure difference across a centrifugal pump for three different flow rate settings is measured through an experiment and used to calculate the pump head generated by each setting. The pump head is then plotted against the volumetric flow rate. Approximations of the pump curve equations are plotted using the Solver function of Excel to obtain values of parameter. The purpose and implication of throttling valve is discussed in terms of Net Positive Suction Head (NPSH). A theoretical system head curve for the centrifugal pump is estimated with estimation of minor losses in the system, which is then compared with the empirical system head curve for discrepancies.

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1 AIMS

The aims of the experiment are to better understand mechanism of centrifugal pump by disassembling pumps into its component and inspecting them, determine the performance curve of a centrifugal pump at different settings, and analyse discrepancies between theoretical system head calculated in consideration of friction and minor losses with the empirical system head.

2 RESULTS AND DISCUSSION

2.1 SCHEMATIC DIAGRAM OF EXPERIMENTAL SET-UP

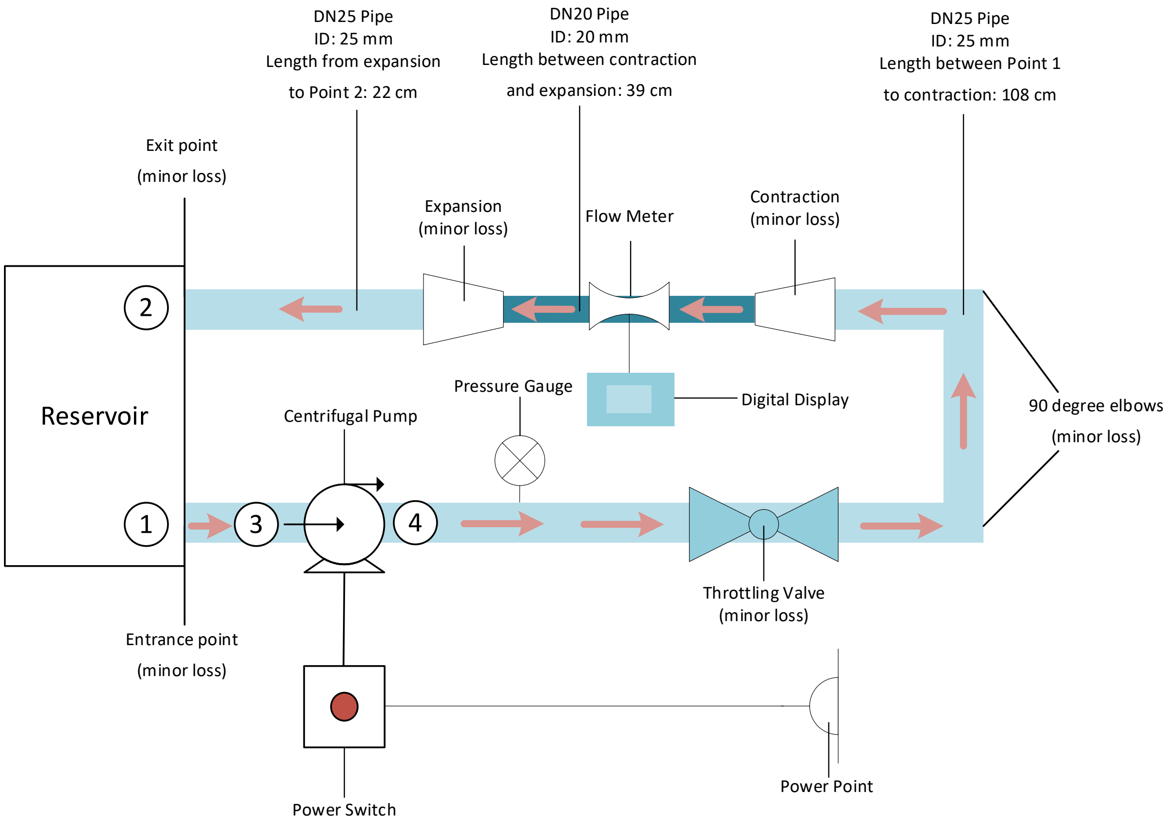

Figure 1. Schematic Diagram of Experimental Set-Up

The entry (Point 1) and exit (Point 2) of the pipe from and to the reservoir are at the same height which is the free surface height. Point 3 is the suction side of the pump, while Point 4 is the discharge side of the pump. The schematic diagram shown in **Figure 1** is drawn based on the top view of the experimental set-up.

2.2 PUMP PRIMING

Priming of a centrifugal pump involves filling the liquid at the suction pipe and impeller to put pump into working order. Priming is required because the pressure developed by the impeller of the pump is proportional to the density of the fluid in the impeller. If running in air, it will produce a negligible pressure and will not suck liquid from source through the suction pipe. As centrifugal pump used in the experiment is designed to operate in a fluid-immersed condition, operation without fluid inside may cause cavitation and overheating, which leads to damage of the pump and system.

2.3 PLOT OF H_P VERSUS Q

See Appendix A: Tabulated Results and Sample Calculation for Section 2.3 Plot of h_p Versus Q for full tabulated data.

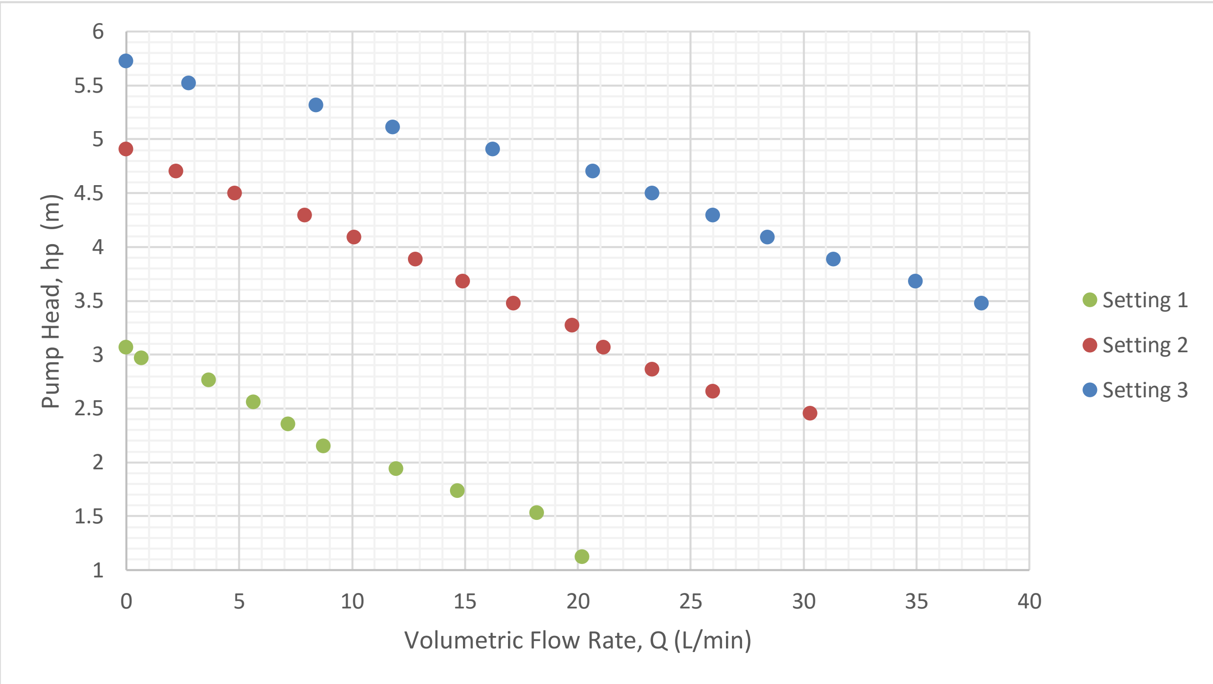

Figure 2. Plot of h_p Versus Q

2.4 PUMP CURVE EQUATIONS

See Appendix B: Tabulated Results for Section 2.4 Pump Curve Fittings for full tabulated data.

See Appendix C: MATLAB Script for Fitted Pump Curves for MATLAB Script used.

The fitted pump data follows the form of Equation 1:

$$h_{p,fit} = a + bQ^c$$

Equation 1

where $h_{p,fit}$ is the fitted pump head in m.

a, b, and c, are non-dimensional constants,

and Q is the flow rate in L/min.

Excel Solver is used to find estimates of the parameters a, b, and c of **Equation 1**. Initial guesses of the parameter values have to be inputted and these guesses are made so that the fitted values are similar to the empirical values. The squared difference between each empirical pump head and fitted pump head is obtained and totalled. With Solver, the sum of the squared difference/error (SSE) is set to 0 by changing the variable cells containing initial guesses of a, b, c. The solving method should be GRG Nonlinear, with the box "make unconstrained variables non-negative" unticked. Although Solver does not find a feasible solution for SSE to be equal to 0, it will minimise SSE and return values of a, b, c. The smaller the SSE value, the better the overall fit.

The Excel Solver analysis results are tabulated below for each setting.

Table 1. Fitted Curve Parameter Values

Parameter	Setting 1	Setting 2	Setting 3
a	3.061	4.908	5.667
b	-0.104	-0.080	-0.024
c	0.956	1.015	1.242
SSE	0.043	0.032	0.019

The fitted curves are shown below.

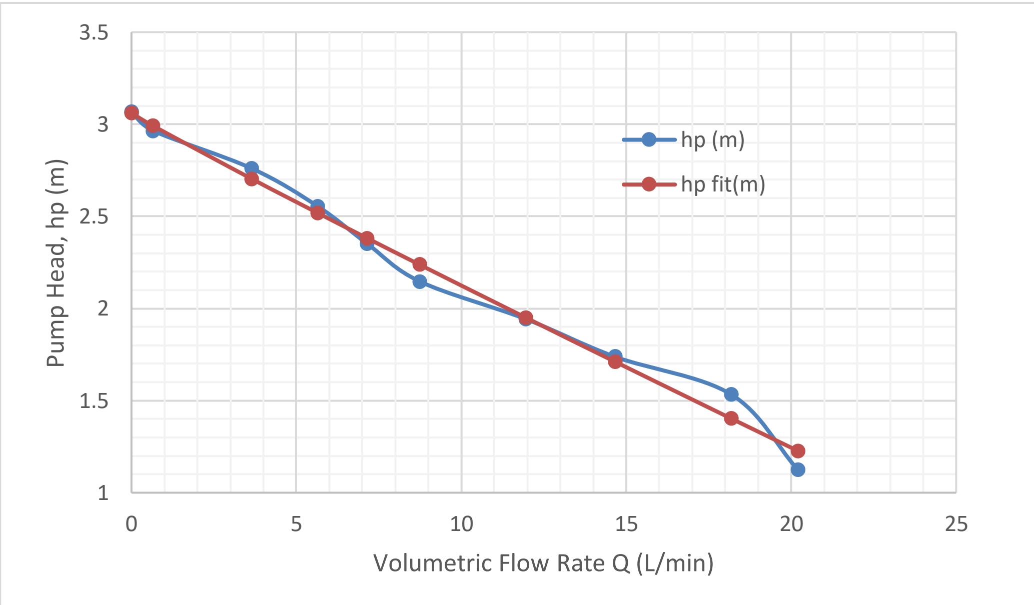

Figure 3. Pump Curve Fitting: Setting 1

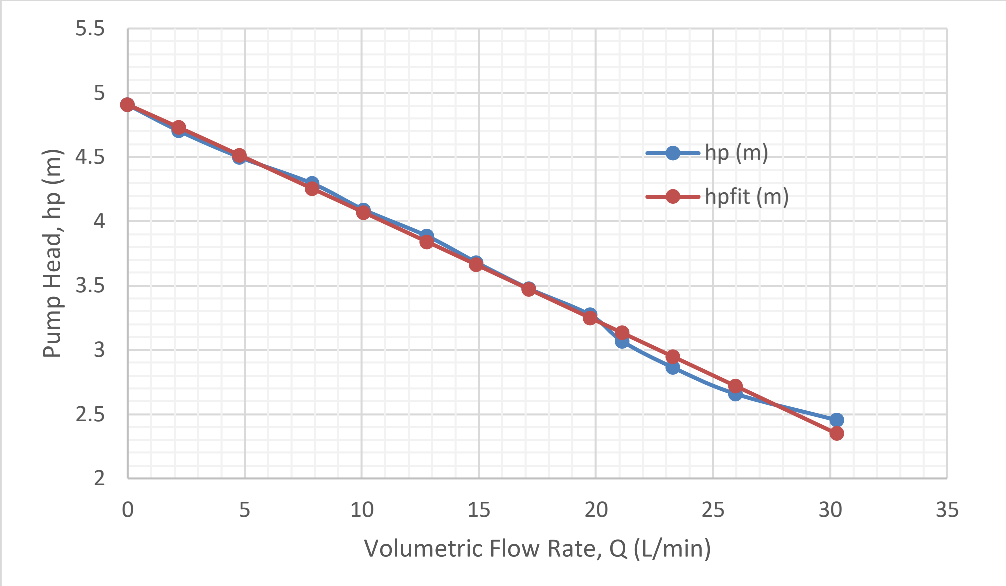

Figure 4. Pump Curve Fitting: Setting 2

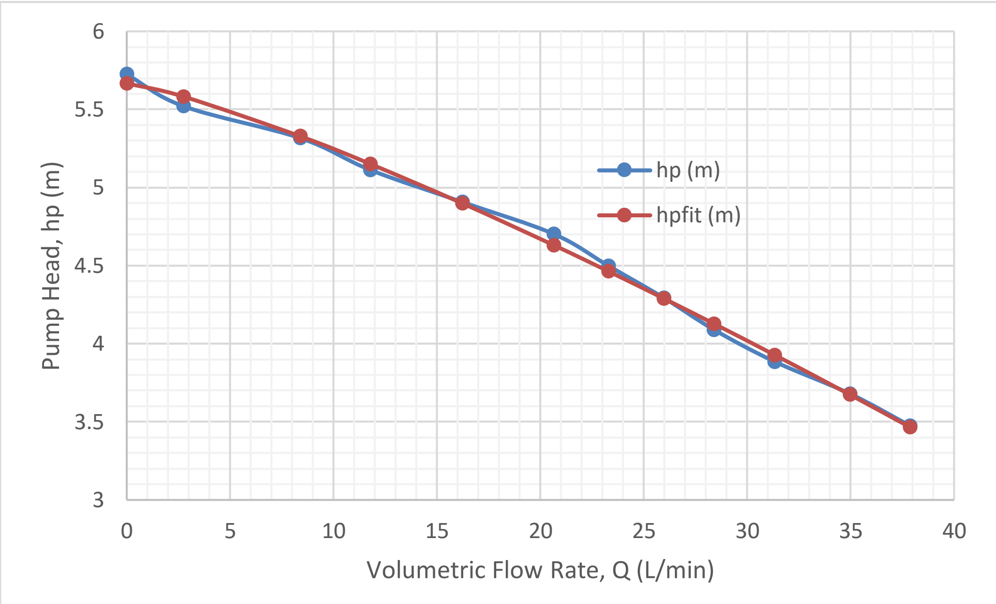

Figure 5. Pump Curve Fitting: Setting 3

Based on these results, MATLAB programme is utilised to construct the pump curves with the parameters obtained in **Table 1. Fitted Curve Parameter Values**.

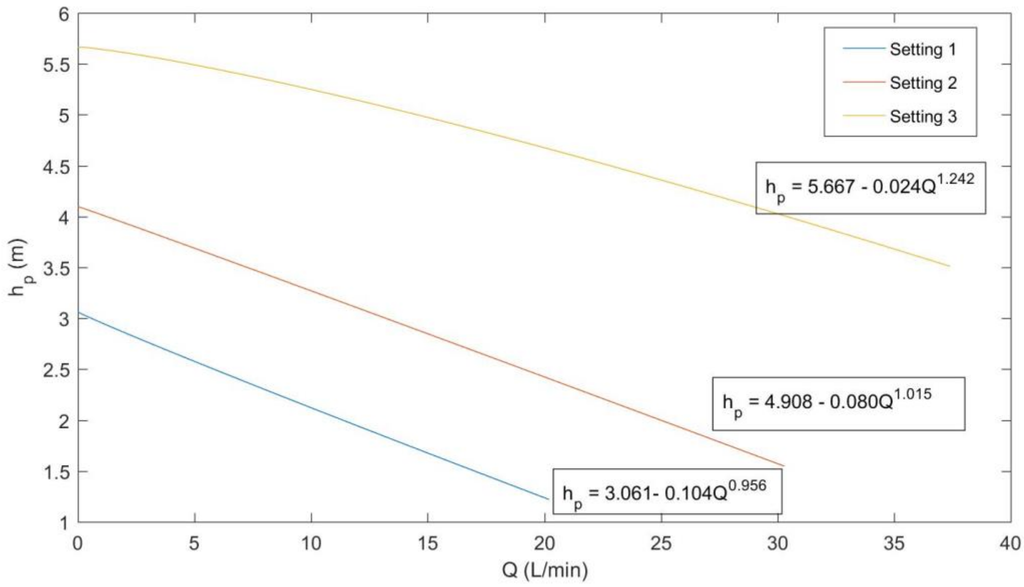

Figure 6. Pump Curves for Each Settings Based on Approximate Parameters

From the comparison between empirical and fitted values in **Figure 3** to **Figure 5**, it is shown that the results are good fit. However, the constructed pump curves for Setting 1 and Setting 2 in **Figure 6** show deviation from the conventional shape of a pump curve. This is due to the limitation of the Solver function which minimises SSE but does not take into account the form of the equation. A larger set of data can be used to construct a more accurate pump curve

2.5. THROTTLING VALVE PURPOSE AND IMPLICATIONS

Throttling valve is a method of controlling flow rate through introducing additional friction loss into the system, shifting the system curve to the left so it intersects with the pump curve at the appropriate operating point to achieve the desired flow rate. This is due to system curve being made of static components (elevation and pressure head) which are independent of flow velocity, and the variable component i.e. the friction head which is dependent of velocity. Therefore, minor losses by fittings and valves play affects the system curve. By adjusting the opening of the valve, the minor losses in the system can be varied.

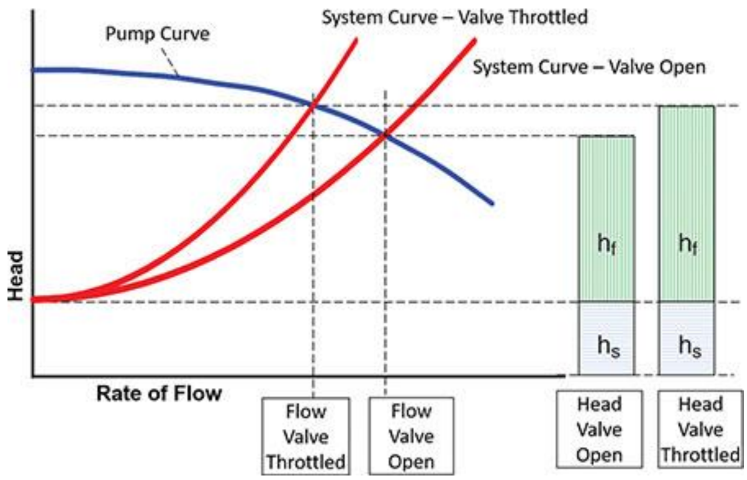

Figure 7. Throttling Valve Adjustment of Flow Rate

As seen in **Figure 7**, the flow rate with the valve open is higher than the flow rate with the valve throttled or partially close. By throttling the valve, both the operating flow rate, Q, and average flow velocity, $\bar{V}$, decreases within a pipe of constant cross-sectional area A. In consequence, h_{fs} decreases as a function of $\bar{V}$ as shown in the following equation:

$$h_{fs} = \frac{2f_F L\bar{V}^2}{Dg}$$

Equation 2

where h_{fs} is the friction head loss from suction side of the pump in m,

f_F is the Fanning friction factor,

L is the length of pipe m,

D is the diameter of the pipe in m, and

g is the gravitational acceleration in m/s^2.

The Available Net Positive Suction Head ($NPSH_A$) can be calculated by the following formula:

$$NPSH_A = \frac{P_1 - P_{vp}}{\rho g} - z_1 - h_{fs}$$

Equation 3

where P_1 is the pressure at the start of the system (usually free surface) in Pa,

P_{vp} is the vapour pressure of the liquid in Pa,

ρ is the density of water in kg/m^3, and

z_1 is the elevation from a certain datum point in m.

As shown through **Equation 2** and **Equation 3**, when h_{fs} decreases, $NPSH_A$ increases.

The system operates in a permissible operating region of Q where $NPSH_A$ is greater than $NPSH_R$. When h_{fs} decreases and $NPSH_A$ increases, the $NPSH_A$ curve is shifted upwards and the system now has greater range of Q where it can operate safely without risk of cavitation.

Minimum pressure of the system happens in the inlet of the pump/suction side. A valve prior to the pump will increase suction side losses and decrease the $NPSH_A$. As a result, cavitation may occur when valve is nearly closed.

2.6 THEORETICAL SYSTEM HEAD CURVE FOR FULLY OPEN THROTTLE VALVE POSITION

See Appendix D: Sample Calculations for Section 2.5. Theoretical System Head Curve for Fully Open Throttle Valve Position for full tabulated data and sample calculations.

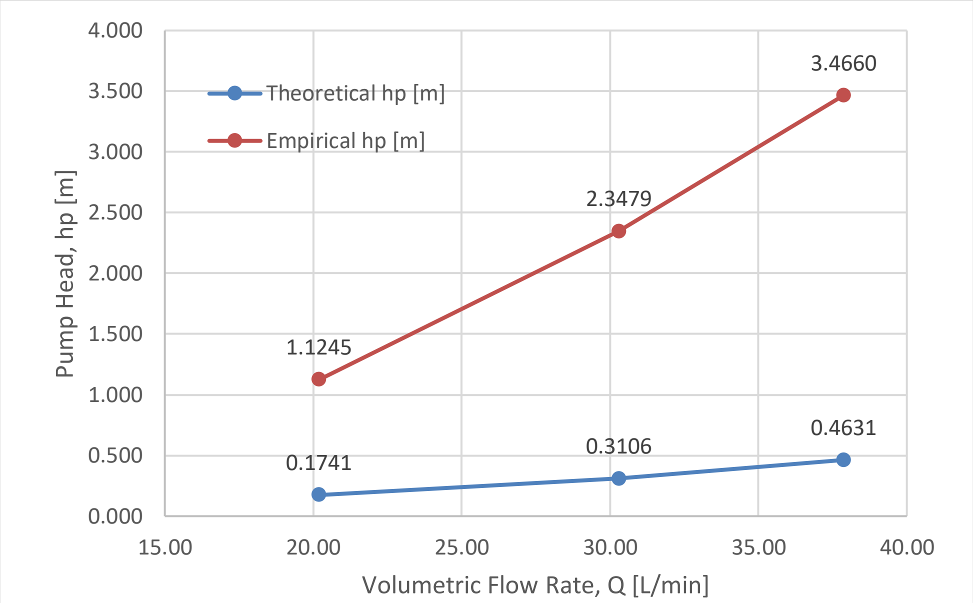

Figure 8. Theoretical and Empirical System Curves

2.7 DISCREPANCY BETWEEN THEORETICAL AND EMPIRICAL SYSTEM HEAD CURVES

Theoretical head losses are lower than the empirical system head losses.

Percentage differences between the head losses are calculated through the formula,

$$\%\,difference = \left(\frac{h_{p,empirical} - h_{p,theoretical}}{h_{p,empirical}}\right) x\ 100\%$$

Equation 4

Using the values shown in **Figure 8** in the previous section, for Setting 1,

$$\%\,difference = \left(\frac{1.1245 - 0.1741}{1.1245}\right) x\ 100\%$$

$$\approx 84.5\%$$

For Setting 2,

$$\% \, difference = \left(\frac{2.3479 - 0.3106}{2.3479}\right) x \, 100\%$$

$$\approx 86.7\%$$

For Setting 3,

$$\% \, difference = \left(\frac{3.4460 - 0.4631}{3.4460}\right) x \, 100\%$$

$$\approx 86.6\%$$

The discrepancies between the theoretical and empirical values can be explained by the assumptions and simplifications taken when obtaining the theoretical values.

Calculation of system head from the empirical data uses the following equation:

$$h_p = \frac{P_4 - P_3}{\rho g}$$

Equation 5

where h_p is the pump head in m,

P_4 is the pressure of the discharge side of the pump in Pa,

P_3 is the pressure of the suction side of the pump in Pa.

The following assumptions made to obtain this equation may be inaccurate:

1. Flow is assumed to be steady-state and fully developed; however, the small system might mean that flow does not have time to be fully developed in each section, causing inaccurate estimation of pipe friction.
2. The reservoir is assumed to have a large enough cross-sectional area that the fluid velocity on the surface is approximately zero.
3. There is no elevation difference between point 1 and 2 (refer to **Figure 1**).
4. The pressure at the suction side of the pump is assumed to be negligible.
5. The temperature and density of water are constant.
6. The gravitational acceleration is 9.8 m/s^2.
7. The flow rate estimation may be inaccurate as the flow meter is a fan and may have nonlinear pressure-flow profile.

Calculation of theoretical system head uses the following equation:

$$h_p = \frac{2f_F(L_{pipe} + \sum L_{eq})\,\bar{V}^2}{Dg} + \frac{1}{2g}\sum K\bar{V}^2$$

Equation 6

where L_{eq} is the equivalent length of fittings in m, and

K is the resistance coefficient of fittings.

The following approximations are made when using this equation, which may be underestimated:

1. L_{eq}/D ratios for the gate valve and 90° degree elbows.
2. K values for exit point, entry point, as well contraction and expansions.

As well as the following assumptions, which may be inaccurate:

1. The temperature and density of water are constant.
2. The gravitational acceleration is 9.8 m/s^2.
3. The wetted perimeter is assumed to the perimeter of the pipe cross-sectional area throughout the pipe.

3 CONCLUSION

Fitted values of the empirical results give the following pump curve equations for Setting 1, 2, and 3 respectively:

$$h_{p,1} = 3.061 - 0.104Q^{0.956}$$

Equation 7

$$h_{p,2} = 4.908 - 0.080Q^{1.015}$$

Equation 8

$$h_{p,3} = 5.667 - 0.024Q^{1.242}$$

Equation 9

The theoretical system head obtained using Fanning friction factor and minor losses coefficients are 84.5-86.7% lower than the empirical system head recorded from the experiment as a result of inaccurate assumptions and underestimation of minor loss coefficients which gives inherent uncertainty.

4 REFERENCES

Pumps and Systems Magazine. 2020. *How Discharge Piping Affects Pump Performance*. [online] Available at: <https://www.pumpsandsystems.com/how-discharge-piping-affects-pump-performance> [Accessed 13 October 2020].

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5 APPENDICES

APPENDIX A: TABULATED RESULTS AND SAMPLE CALCULATION FOR SECTION 2.3 PLOT OF H_P VERSUS Q

Table A 1. Tabulated Results for Setting 1

P [kPa]	P [Pa]	Q [L/min]	h_p [m]
11	11000	20.20	1.124
15	15000	18.18	1.533
17	17000	14.66	1.738
19	19000	11.96	1.942
21	21000	8.74	2.147
23	23000	7.15	2.351
25	25000	5.64	2.556
27	27000	3.65	2.760
29	29000	0.66	2.964
30	30000	0.00	3.067

Table A 2. Tabulated Results for Setting 2

P [kPa]	P [Pa]	Q [L/min]	h_p [m]
24	24000	30.30	2.453
26	26000	25.97	2.658
28	28000	23.30	2.862
30	30000	21.13	3.067
32	32000	19.76	3.271
34	34000	17.15	3.476
36	36000	14.90	3.680
38	38000	12.80	3.885
40	40000	10.10	4.089
42	42000	7.90	4.293
44	44000	4.80	4.498
46	46000	2.20	4.702
48	48000	0.00	4.907

Table A 3. Tabulated Results for Setting 3

P [kPa]	P [Pa]	Q [L/min]	h_p [m]
34	34000	37.87	3.476
36	36000	34.96	3.680
38	38000	31.34	3.885
40	40000	28.4	4.089
42	42000	25.97	4.293
44	44000	23.3	4.498
46	46000	20.65	4.702
48	48000	16.23	4.907
50	50000	11.8	5.111
52	52000	8.41	5.316
54	54000	2.75	5.520
56	56000	0	5.725

Sample Calculation: Run 1, Setting 1

In the experiment, the Bernoulli equation is simplified to **Equation 5**:

$$h_p = \frac{P_4 - P_3}{\rho g}$$

P_3 is relatively small and negligible, thus value can be approximated to zero and the equation becomes:

$$h_p \approx \frac{P_4}{\rho g}$$

Equation 10

Gravitational acceleration is assumed to be constant at g = 9.8 m/s^2 and water temperature is assumed to be constant at 20°C, thus the corresponding density at that temperature is 998.21 kg/m^3 (Water - Density, Specific Weight and Thermal Expansion Coefficient, 2020).

For Run 1, Setting 1, the pump head is calculated using **Equation 10** as below,

$$h_p = \frac{11 \,*\, 10^3\ Pa}{998.21\ kg/m^3 * \ 9.8\ m/s^2}$$

$$\approx \ 1.124\ m$$

APPENDIX B: TABULATED RESULTS FOR SECTION 2.4 PUMP CURVE FITTINGS

Table B 1. Tabulated Fitting Results for Setting 1

Q [L/min]	h_p [m]	$h_{p,fit}$ [m]	SSE
20.20	1.124	1.227	0.011
18.18	1.533	1.403	0.017
14.66	1.738	1.711	0.001
11.96	1.942	1.950	0.000
8.74	2.147	2.237	0.008
7.15	2.351	2.381	0.001
5.64	2.556	2.519	0.001
3.65	2.760	2.703	0.003
0.66	2.964	2.991	0.001
0.00	3.067	3.061	0.000
		Total SSE	0.043

Table B 2. Tabulated Fitting Results for Setting 2

Q [L/min]	h_p [m]	$h_{p,fit}$ [m]	SSE
30.30	2.453	2.348	0.011
25.97	2.658	2.719	0.004
23.30	2.862	2.947	0.007
21.13	3.067	3.132	0.004
19.76	3.271	3.249	0.000
17.15	3.476	3.471	0.000
14.90	3.680	3.662	0.000
12.80	3.885	3.840	0.002
10.10	4.089	4.068	0.000
7.90	4.293	4.254	0.002
4.80	4.498	4.513	0.000
2.20	4.702	4.729	0.001
0.00	4.907	4.908	0.000
		Total SSE	0.032

Table B 3. Tabulated Fitting Results for Setting 3

Q [L/min]	h_p [m]	$h_{p,fit}$ [m]	SSE
37.87	3.476	3.466	0.000
34.96	3.680	3.674	0.000
31.34	3.885	3.927	0.002
28.40	4.089	4.127	0.001

25.97	4.293	4.289	0.000
23.30	4.498	4.463	0.001
20.65	4.702	4.631	0.005
16.23	4.907	4.898	0.000
11.80	5.111	5.150	0.001
8.41	5.316	5.327	0.000
2.75	5.520	5.582	0.004
0.00	5.725	5.667	0.003
		Total SSE	0.019

APPENDIX C: MATLAB SCRIPT FOR FITTED PUMP CURVES

```
%Pump Curve: Setting 1
Q1=0:0.1:20.2
a1=3.061
b1=-0.104
c1=0.956

hpfit1=a1+b1.*Q1.^c1;
plot(Q1,hpfit1)
title('Pump Curves')
xlabel('Q (L/min)')
ylabel('h_p (m)')
hold on

%Pump Curve: Setting 2
Q2=0:0.1:30.3
a2=4.098
b2=-0.080
c2=1.015

hpfit2=a2+b2.*Q2.^c2;
plot(Q2,hpfit2)
hold on

%Pump Curve: Setting 2
Q3=0:0.1:37.4
a3=5.667
b3=-0.024
c3=1.242

hpfit3=a3+b3.*Q3.^c3;
plot(Q3,hpfit3)
hold on
```

APPENDIX D: SAMPLE CALCULATIONS FOR SECTION 2.5. THEORETICAL SYSTEM HEAD CURVE FOR FULLY OPEN THROTTLE VALVE POSITION

Table D 1. Summary of Results

Setting	Q [L/min]	Q [m3/s]	V_1 [m/s]	Re_1	f_1	V_2 [m/s]	Re_2	f_2	$h_{p,1}$	$h_{p,2}$	Total h_p [m]
1	20.20	0.0003	0.6859	17116	0.0068	1.0716	21395	0.0064	0.1267	0.0474	0.1741
2	30.30	0.0005	1.0288	25673	0.0061	1.6075	32092	0.0058	0.2641	0.1005	0.3106
3	37.87	0.0006	1.2858	32088	0.0058	2.0091	40109	0.0053	0.3985	0.1489	0.4631

Sample Calculation: Setting 1

The Laboratory Manual derives the relationship of pump head with friction head, involving friction factors and minor loss coefficients as **Equation 6**,

$$h_p = h_f$$

$$= \frac{2f_F(L_{pipe} + \sum L_{eq})\bar{V}^2}{Dg} + \frac{1}{2g}\sum K\bar{V}^2$$

The average flow velocity has to be calculated from flow rate and the respective cross-sectional areas of the DN25 and DN20 pipes by the following equation,

$$\bar{V} = \frac{Q\left(\frac{L}{\text{min}}\right) * \frac{1\,m^3}{1000\,L} * \frac{1\,min}{60\,s}}{\pi * \frac{D^2}{4}}$$

Equation 11

For the large diameter pipe, DN25, the flow velocity is calculated as below,

$$\bar{V}_1 = \frac{20.20\left(\frac{L}{\text{min}}\right) * \frac{1\,m^3}{1000\,L} * \frac{1\,min}{60\,s}}{\pi * \frac{(0.025)^2}{4}}$$

$$= \frac{3.37\ x\ 10^{-4}\ m^3/s}{4.91\ x\ 10^{-4}\ m^2}$$

$$\approx 6.86\ x\ 10^{-1}\ m/s$$

For the smaller diameter pipe, DN20, the flow velocity is calculated as below,

$$\bar{V}_2 = \frac{20.20\left(\frac{L}{\text{min}}\right) * \frac{1\,m^3}{1000\,L} * \frac{1\,min}{60\,s}}{\pi * \frac{(0.020)^2}{4}}$$

$$= \frac{3.37\ x\ 10^{-4}\ m^3/s}{3.14\ x\ 10^{-4}\ m^2}$$

$$\approx 1.07\ m/s$$

As the temperature and density assumption is the same as that in Appendix A, the Reynolds number can be obtained from the following formula,

$$Re = \frac{p\bar{V}D}{\mu}$$

Equation 12

where μ is the kinematic viscosity of water in Pa s.

At 20°C, the kinematic viscosity of water is 0.001 Pa s.

For the larger diameter pipe, DN25, the Reynolds number is,

$$Re_1 = \frac{998.21\ kg/m^3\ x\ 6.86\ x\ 10^{-1}\ m/s\ x\ 0.025\ m}{0.001\ Pa\ s}$$

$$\approx 17116$$

While for the smaller diameter pipe, DN20, the corresponding Reynolds number is,

$$Re_2 = \frac{998.21\ kg/m^3\ x\ 1.07\ m/s\ \ x\ 0.020\ m}{0.001\ Pa\ s}$$

$$\approx 21395$$

To obtain the Fanning friction factor that corresponds to the system, both the Reynolds number and the relative roughness of the pipe for each section is needed. The absolute roughness and the diameter of pipe is required to calculate the relative roughness of the pipe. For a PVC pipe, the absolute roughness is estimated to be 0.0015 mm (Roughness & Surface Coefficients, 2020).

Therefore, the large diameter pipe, DN25, has a relative roughness as below,

$$\frac{\varepsilon}{D_1} = \frac{0.0015\ mm}{25\ mm}$$

$$\approx 6\ x\ 10^{-5}$$

And the small diameter pipe, DN20, has a relative roughness as below,

$$\frac{\varepsilon}{D_2} = \frac{0.0015\ mm}{20\ mm}$$

$$\approx 7.5\ x\ 10^{-5}$$

From the Moody diagram, the Fanning friction factors are $6.8\ x\ 10^{-3}$ for the DN25 pipe and $6.4\ x\ 10^{-3}$ for the DN20 pipe.

As minor losses from fittings are considered in this system, there are equivalent lengths and resistance coefficients that can be used to represent these minor losses in the equation.

The equivalent lengths of some fittings in the system are shown in **Table D 1**.

Table D 2. Equivalent Lengths of Fittings

	Le/d	Le (m)
Ball Valve	7	0.175
90 degree elbow	40	2

There are two 90° elbows and 1 gate valve within the DN25 pipe section.

$$L_{eq,90°\,elbow} = 2\ x\ 40\ x\ 0.025\ m$$

$$= \ 2\ m$$

$$L_{eq,gate\ valve} = 7\ x\ 0.025\ m$$

$$= \ 0.175\ m$$

The total length of the DN25 pipe is therefore (1.08 + 0.22 + 2 + 0.175 m) = 3.475 m.

The resistance coefficients (K) for some of the fittings in the system are:

Table D 3. Resistance Coefficient of Fittings

Fittings	K
Sharp-edged entrance	0.5
Sharp-edged exit	1
Contraction	0.18
Expansion	0.1296

Coefficients for exit and entry points are obtained from Lecture Notes Topic 3. Both corresponds to the DN25 pipe section and its velocity.

Calculations for the coefficients for contraction and expansion are elaborated below. Both corresponds to the DN20 pipe section and its velocity.

The resistance coefficient due to contraction is obtained using the formula

$$K_{con} = 0.5 * (1 - \left(\frac{D_s}{D_L}\right)^2)$$

Equation 13

$$= \ 0.5 * (1 - \left(\frac{0.02}{0.025}\right)^2)$$

$$= 0.18$$

$$K_{exp} = (1 - \left(\frac{D_s}{D_L}\right)^2)^2$$

Equation 14

$$= (1 - \left(\frac{0.02}{0.025}\right)^2)^2$$

$$= 0.1296$$

The head loss due to friction and fittings in the DN25 pipe is calculated as below,

$$h_{p,1} = \frac{2f_1(L_{DN\,25\,pipe} + \sum L_{eq})\bar{V}_1{}^2}{D_1 g} + \frac{1}{2g}(K_{entrance} + K_{exit})\bar{V}_1{}^2$$

Equation 15

$$= \frac{2 * 6.8\ x\ 10^{-3} * 3.475\ \mathrm{m} * (6.86\ x\ 10^{-1}\ m/s\ )^2}{0.025\ m * 9.8\ m/s^2} + \frac{1.5 * (6.86\ x\ 10^{-1}\ m/s)^2}{2 *\ 9.8\ m/s^2}$$

$$\approx\ 0.1267\ m$$

The head loss due to friction and fittings in the DN20 pipe is calculated as below,

$$h_{p,2} = \frac{2f_2(L_{DN\,20\,pipe})\bar{V}_2{}^2}{D_2 g} + \frac{1}{2g}(K_{con} + K_{exp})\bar{V}_2{}^2$$

Equation 16

$$= \frac{2 * 6.4\ x\ 10^{-3} * 0.39\ \mathrm{m} * (1.07\ m/s\ )^2}{0.020\ m * 9.8\ m/s^2} + \frac{0.3096\ * (1.07\ m/s)^2}{2 *\ 9.8\ m/s^2}$$

$$\approx\ 0.0474\ m$$

The total head loss is,

$$h_p = h_{p,1} + h_{p,2}$$

Equation 17

$$\approx\ 0.1741\ m$$