ECOM20001: Econometrics 1

Tutorial 3: Distributions, Law of Large Numbers, Central Limit Theorem, Conditional Probability Practice Problems

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Part 1: Normal, Chi-Square, t, F, Distributions; LLN, and CLT in R

1. The pnorm() command returns the CDF of a normal distribution at a given value x, F(x). **`pnorm(-1.65,mean=0,sd=1)=1-pnorm(1.65,mean=0,sd=1)`** holds because the normal distribution is symmetric about the mean. This implies that for a given value x, F(-x)=1-F(x), which is true, for example, for x=1.65.

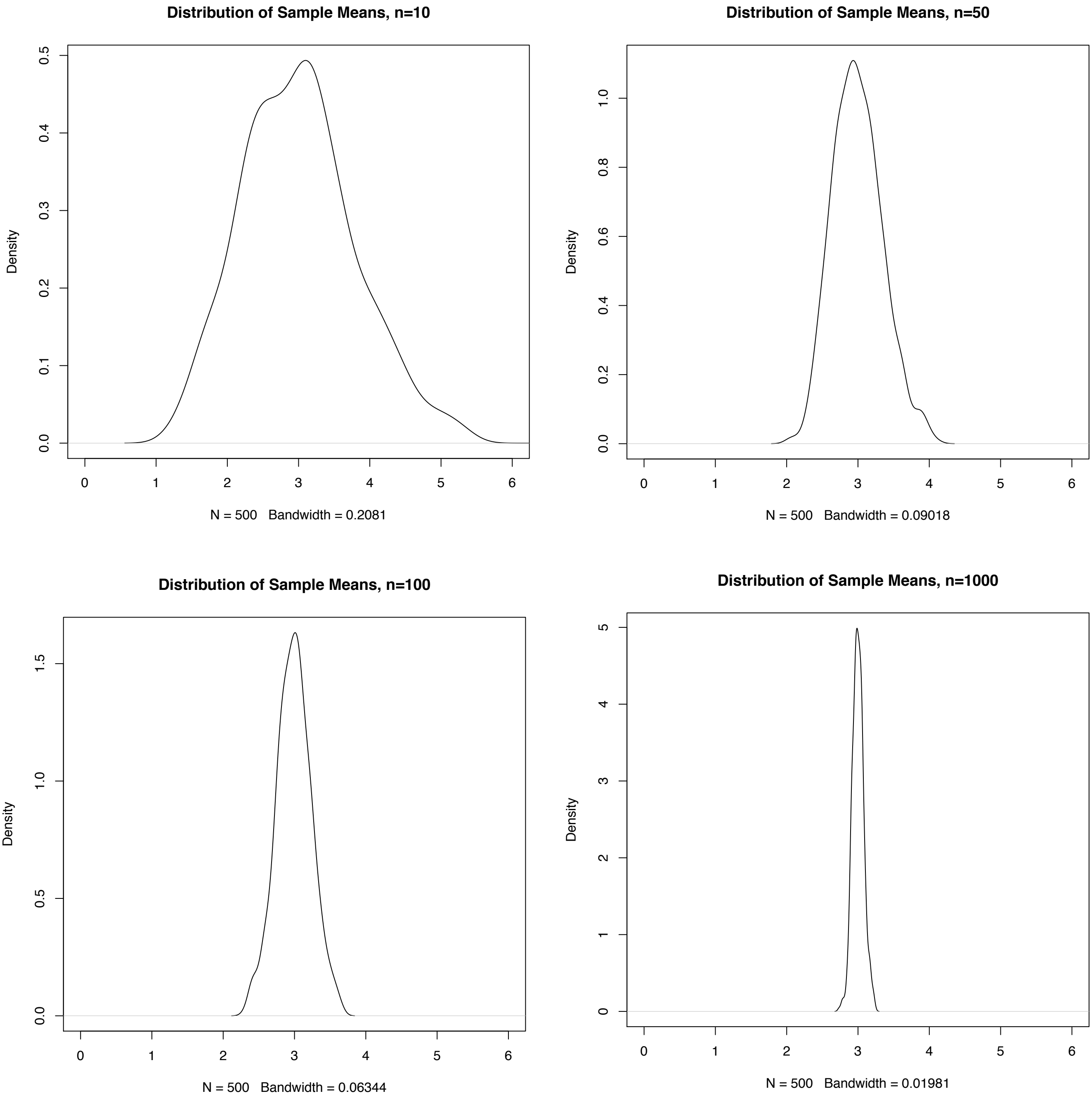
2. The pnorm() command returns the CDF of a normal distribution at a given value of x, F(x). The qnorm() command takes an argument p, and returns the value of a normally distributed random variable, x, that yields the CDF value of p. That is, p=F(x). So, **`pnorm(1.96,mean=0,sd=1)`** returns the CDF of a standard normal random variable, which is 0.975. And **`qnorm(0.975,mean=0,sd=1)`** returns the standard normally distributed value that yields of CDF value of 0.975, which is 1.96. Therefore, the commands are related as inverses of each other for a given value of a normally distributed random variable, **`x`**, and its CDF value, **`p`**: **`pnorm(x,mean=0,sd=1)=p`** and **`x=qnorm(p,mean=0,sd=1)`**

3. The normal and t distributions are symmetric; the Chi-Square and F distributions are right skewed. The mean is greater than the median for the latter 2 distributions because they have long right tails, which causes the mean to grow, but not the median.

4. Answers from the tute3.R code are as follows:

   - Variance of the sampling distribution of the mean for nobs=10, 50, 100, 1000 is approximately 0.60, 0.12, 0.06, and 0.006
   - Percentage of sample means lying within 0.3 of the true value of the mean of 3 is approximately 30%, 65%, 80% and 100%
   - The LLN says that the sample average will be more likely to be close to the true value of the mean as n becomes large. The LLN is illustrated here: as nobs increases, the fraction of sample means that you compute from random sampling is more likely to be close (e.g., within 0.3 as our "close" rule) to the true value of the mean.

5. The CLT is illustrated by the following 4 graphs. These show that the distribution of the sample mean becomes more symmetric and closer to a normal distribution as nobs grows. The fact that the variance of the sample mean falls as nobs grows is further revealing of the LLN in action as nobs grows.



Part 2: Conditional Distribution Practice Problems

1. Joint probability distribution for all combinations of studying and performance:

	High Grade	Medium Grade	Low Grade	Total
Study Hard	0.20	0.10	0.02	0.32
Study Sometimes	0.07	0.30	0.10	0.47
Study Never	0.01	0.05	0.15	0.21
Total	0.28	0.45	0.27	1

a. Marginal distribution of Studying:

- P(Study Hard)=0.32
- P(Study Sometimes)=0.47
- P(Study Never)=0.21

b. Marginal distribution of Performance:

- P(High Grade)=0.28
- P(Medium Grade)=0.45
- P(Low Grade)=0.27

c. Conditional probability distribution of Performance, conditional on Studying Hard:

- P(High Grade | Studying Hard)=0.20/0.32=0.625
- P(Medium Grade | Studying Hard)=0.10/0.32=0.3125
- P(Low Grade | Studying Hard)=0.02/0.32=0.0625

d. Conditional probability distribution of Performance, conditional on Studying Sometimes:

- P(High Grade | Studying Sometimes)=0.07/0.47=0.1489
- P(Medium Grade | Studying Sometimes)=0.30/0.47=0.6383
- P(Low Grade | Studying Sometimes)=0.10/0.47=0.2128

e. Conditional probability distribution of Studying, conditional on Medium Grade:

- P(Study Hard | Medium Grade)=0.10/0.45=0.2222
- P(Study Sometimes | Medium Grade)=0.30/0.45=0.6666
- P(Study Never | Medium Grade)=0.05/0.45=0.1111

f. Conditional probability distribution of Studying, conditional on Low Grade:

- P(Study Hard | Low Grade)=0.02/0.27=0.0741
- P(Study Sometimes | Low Grade)=0.10/0.27=0.3704
- P(Study Never | Low Grade)=0.15/0.27=0.5555

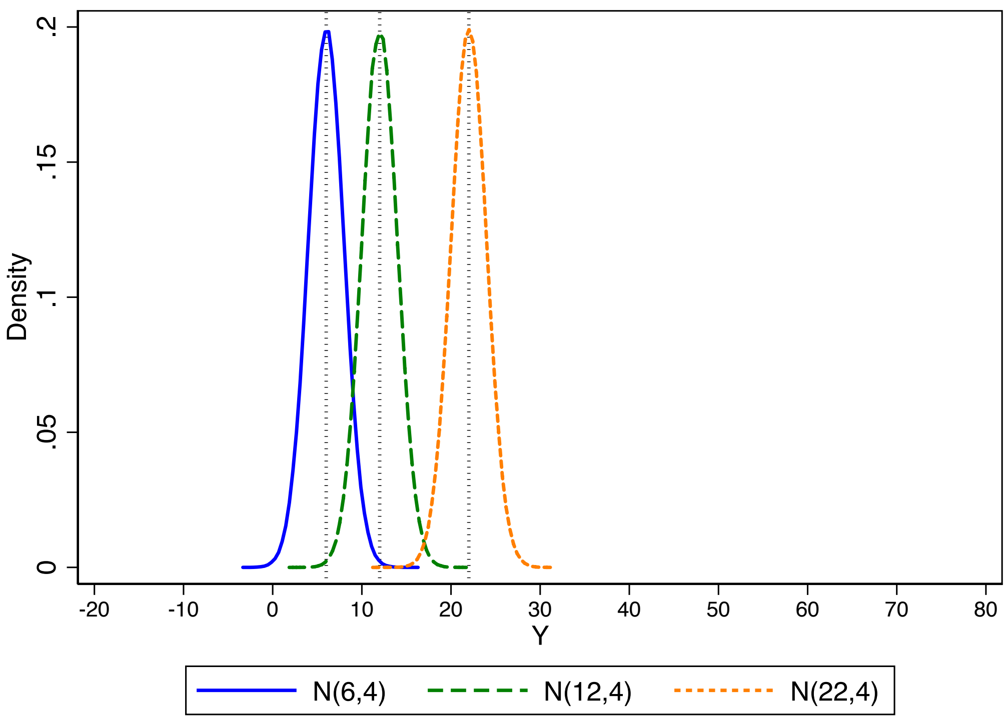
g. If, for example, Studying and Performance were independent, then the joint probability of P(Study Hard, High Grade) would equal the product of the marginal probabilities of Study Hard and High Grade: P(Study Hard) x P(High Grade). Computing this product we obtain 0.32 x 0.28 = 0.0896 which is not equal to the joint probability of P(Study Hard, High Grade) in the table of 0.20. Therefore, Studying and Performance are not independent.

2. Suppose you have a random variable X that is i.i.d distributed from a N(mu**x**,1) distribution, and a separate random variable Y that is defined as follows:

$$Y=2+2X$$

a. The distribution of Y is N(2+2mu**x**,4).

b. If mu**x**=2, 5, or 10, then the distribution of Y is N(6,4), N(12,4), and N(22,4), respectively. The following graph plots the distributions of Y, conditional on the three mu**x** values. Larger values of X shift the distribution of Y to the right.



c. The distribution of Y is now N(2+4mu$_X$,16). If mu$_X$=2, 5, or 10, then the distribution of Y is now N(10,16), N(22,16), and N(42,16). There are two key changes in the results from part b. from changing the definition of Y to Y=2+4X. The variance of Y increases, and the shifts in the mean in the distribution for different X values become larger in magnitude. That is, the conditional mean of Y given X becomes more sensitive to changes in X.