The University of Melbourne

School of Engineering

Semester 2 Assessment 2016

ENGR30002 – Fluid Mechanics

Exam Duration: 3 hours

Reading Time: 15 minutes

This paper has TWELVE (12) pages consisting of SIX (6) questions.

Authorized material:
Only electronic calculators approved by the School of Engineering may be used.
Two (2) pages of equations and two (2) charts are attached.

Instructions to Invigilators:
Script books to be provided.
Charts and equation sheets can be detached.

Instructions to Students:
All Six questions are to be attempted.
Total marks for the exam = 100.
Charts and equation sheets can be detached.

This paper is to be held by the Baillieu Library

Question 1

Provide short answers to the following questions:

(a) Shear stress is written using the symbol τ_{ij}. What information do the subscripts i and j provide to you?

(2 marks)

(b) You have a gas flowing through a cylindrical pipe. The pressure is changed upstream. A pressure wave will flow down the length of the pipe. At what speed does this pressure wave travel?

(1 marks)

(c) What is the physical meaning when you assume Stokes flow?

(1 marks)

(d) What five types of head did we describe when discussing the mechanical energy balance?

(2 marks)

(e) In pipe flow, explain why friction factors asymptote to a constant value at high Reynolds numbers.

(1 marks)

(f) Why is volumetric flow rate not constant down the length of a pipe during pressure driven gas flow?

(1 marks)

(g) Why does the amount of pressure you experience not change when you move horizontally through a stationary fluid?

(1 marks)

(Question 1 continued on next page)

Question 1 continued

(h) Why do the Navier Stokes Equations consist of three equations?

(1 marks)

(i) Water is flowing through a horizontal pipe with a constant diameter of 0.*5 cm* at a rate of 0.25 *m/s*. Could you use the Navier Stokes Equations to develop an expression for the velocity profile of the fluid? Explain your answer. The density of water is 1000 kg/m^3 and the viscosity of water is 8.9 x 10^{-4} *Pa s* at the flow conditions.

(2 marks)

(j) What is the unique characteristic of a Bingham fluid. Describe this characteristic.

(2 marks)

(k) What information does the impeller Reynolds number provide you about a mixing system?

(1 marks)

(l) What is the velocity of a gas that is flowing at choked flow conditions?

(1 marks)

(m) When deriving the Navier Stokes equations, what are the two mechanisms by which momentum is transported into/out of the control volume?

(2 marks)

(n) Define $NPSH_A$ and $NPSH_R$. What is the significance of these values?

(2 marks)

(Total for Question 1 = 20 marks)

Question 2

Hydrogen gas is flowing isothermally at $25^{o}C$ from one storage vessel to another through a horizontal pipe with a length of 400 *m* and uniform cross section. The pressure at the pipe inlet is 25 *bar* and the pressure in the pipe outlet is 20 *bar*. The Fanning friction factor is f_F = 0.005 and other constants are provided below:

Molecular weight of hydrogen	2
1 bar	10^5 *Pa*
Ideal gas constant	8.314 $J\ mol^{-1}\ K^{-1}$

Assuming kinetic energy is negligible, answer the following questions.

(a) Calculate the diameter of the pipe required for a mass flow rate of 0.2 *kg/s*.

(8 marks)

(b) Calculate the gas velocity at the entrance of the pipe and the exit of the pipe.

(7 marks)

(c) If the pipe length is increased and the pressure at the pipe inlet is fixed, will P_w increase, decrease, or stay the same?

(2 marks)

(d) If pressure at the inlet and outlet of the pipe are fixed, can reducing the length of the pipe result in choked flow? Explain your answer.

(1 marks)

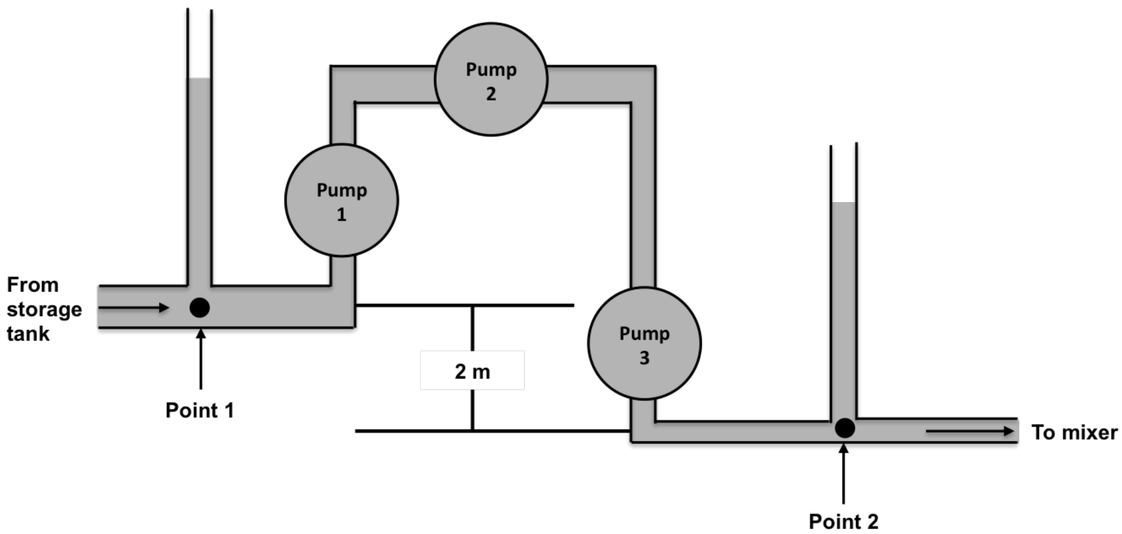
(e) Calculate the velocity at the pipe exit when the flow is choked.

(2 marks)

(Total for Question 2 = 20 marks)

Question 3

You are working in a food production plant. Olive oil is being pumped from a storage tank to a mixer through a series of pipes and pumps, as seen in the schematic below. At two points in the flow (point 1 and point 2) the pressure is measured through the use of manometers that are open to the atmosphere. Mechanical energy is being added to the flow by the three pumps and mechanical energy is being removed from the flow by friction losses. At point 1 the diameter of the pipe is 30 *cm* and the velocity of the fluid is 2.12 *m/s*. At point 2 the pipe diameter is 15 *cm*. The height of olive oil in both manometers is 1.5 *m*, with point 1 being located 2 *m* higher than point 2. The density of olive oil is 800 *kg/m3* and the viscosity is 0.81 *Pa s*.



(a) What is the sum of shaft work and friction ($W_s + F$) that occurs between point 1 and point 2, where W_s represents the combined shaft work of the three pumps and F represents the frictional losses.

(13 marks)

(b) Is there a net gain or loss of energy in the flow between point 1 and point 2? Explain your answer.

(2 marks)

(Total for Question 3 = 15 marks)

Question 4

An incompressible Newtonian fluid is located in the annular space between concentric cylinders having radii R_1 and R_2. The inner cylinder is fixed, but the outer cylinder is rotating with an angular velocity ω. The fluid is also being driven in the axial direction due to a constant applied pressure gradient $\frac{dp}{dz} = -B$. The flow is steady and fully developed.

(a) Find an expression that describes the velocity profile in the axial direction.

(9 marks)

(b) Find an expression that describes the velocity profile in the θ direction.

(9 marks)

(c) Find an expression for the radial pressure gradient.

(2 marks)

(Total for Question 4 = 20 marks)

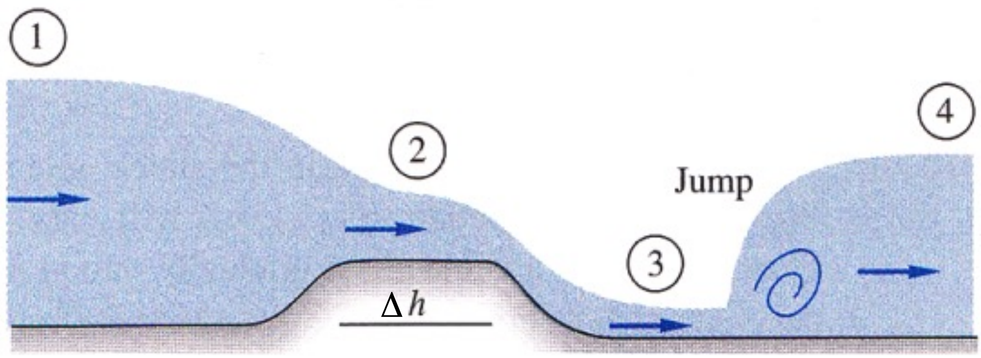
Question 5

The figure below shows channel flow over a bump, which results in a hydraulic jump downstream. The flowrate per unit width in the channel is 1.6 m^2s^{-1}. If the depth at point 1 is 100 cm, determine:

(a) the depths at points 2, 3 and 4 **(7 marks)**

(b) the bump height, Δh **(4 marks)**

(c) the fraction of flow energy dissipated in the hydraulic jump **(4 marks)**



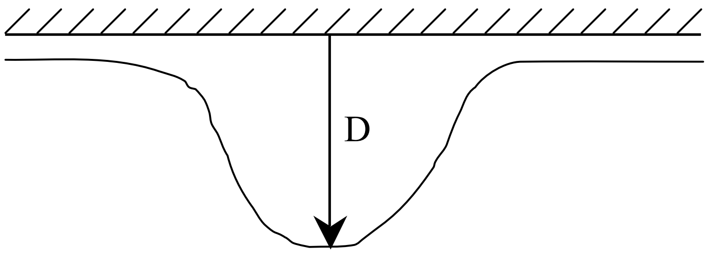
(Total for Question 5 = 15 marks)

Question 6

Use your understanding of dimensionless parameters in fluid mechanics to answer the following questions.

(a) Consider the flow in Question 5. To study this system, you create a dynamically-similar 1:10 model, with water as the working fluid. What should the flowrate per unit width be in the model? **(5 marks)**

(b) When fluid coats the underside of a horizontal plate (e.g. when painting a ceiling), it can accumulate in what are known as pendant drops. Surface tension is the restoring force here, acting upwards against the weight of the drop. Given that the surface tension of the air-water interface is roughly 0.07 *N/m*, provide an estimate of the length scale (*D*) of the largest pendant water drop that can form. **(5 marks)**



(Total for Question 6 = 10 marks)

Reference material only beyond this point

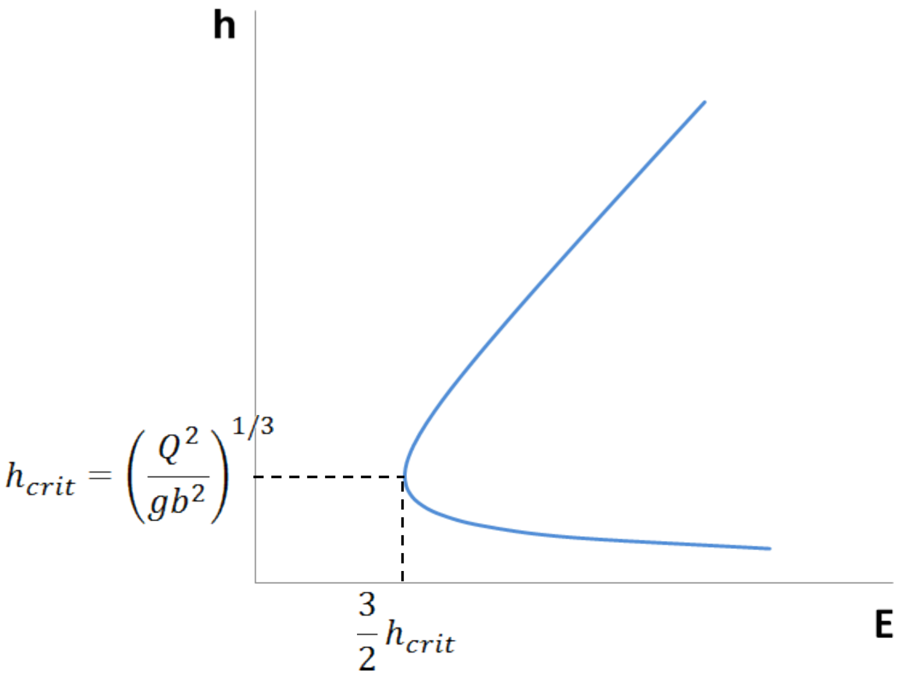
EQUATION SHEET

$$\frac{P_2^2 - P_1^2}{2(RT/M)} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{2f_F L}{D}\left(\frac{G}{A}\right)^2 = 0$$

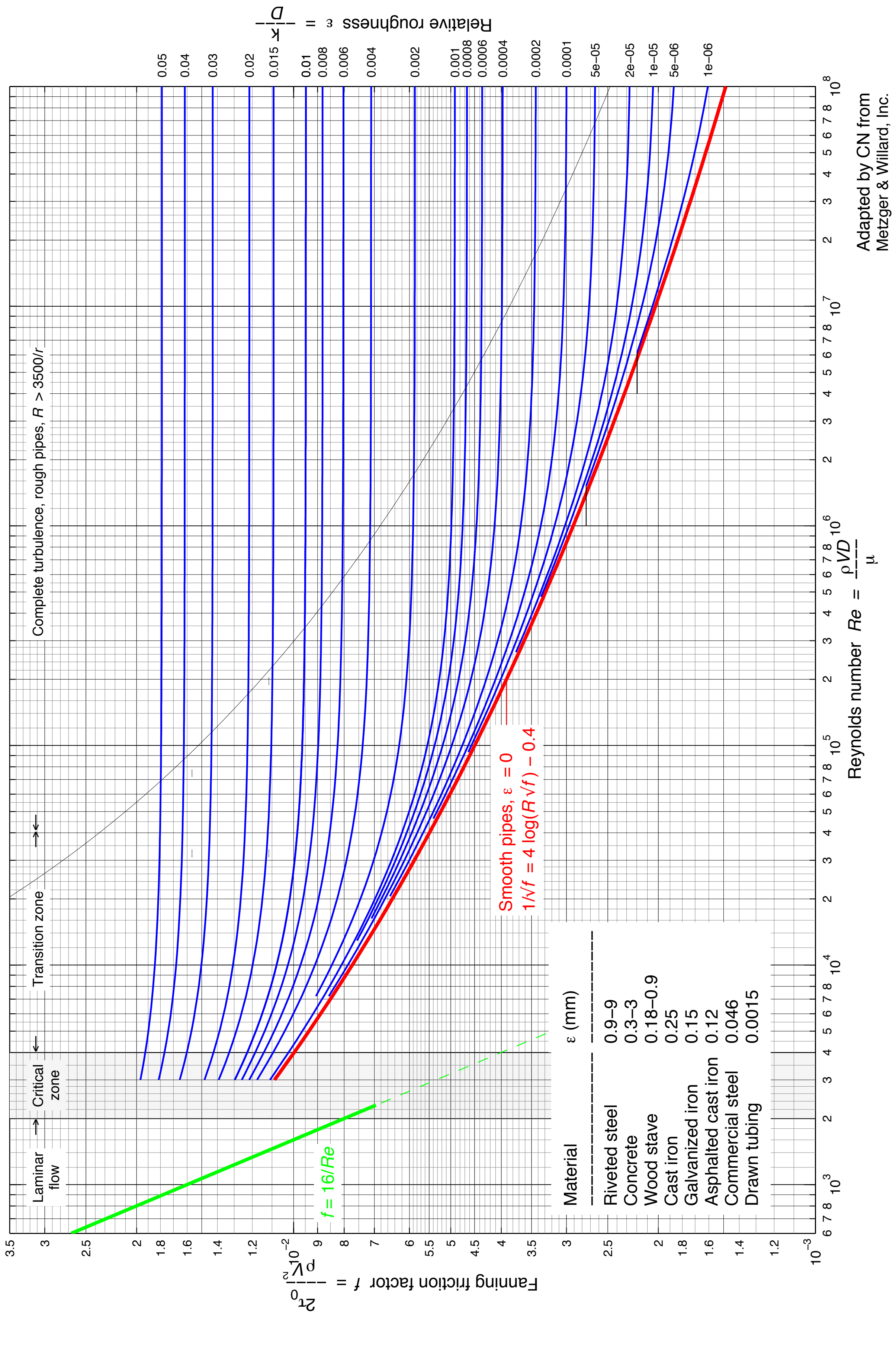
$$\frac{4f_F L_{crit}}{D} = \left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1$$

$$U = \frac{1}{n} R_h^{2/3} S^{1/2}$$

$$E(h) = \frac{Q^2}{2gb^2h^2} + h$$



$$\frac{h_2}{h_1} = \frac{-1 + \sqrt{1 + 8Fr_1^2}}{2}$$


Fanning friction factor $f = \frac{2\tau_0}{\rho V^2}$
Reynolds number $Re = \frac{\rho V D}{\mu}$
Relative roughness $\varepsilon = \frac{k}{D}$
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes, $R > 3500/r$
$f = 16/Re$
Smooth pipes, $\varepsilon = 0$
$1/\sqrt{f} = 4\log(R\sqrt{f}) - 0.4$
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
5e−05
2e−05
1e−05
5e−06
1e−06
Material
Riveted steel
Concrete
Wood stave
Cast iron
Galvanized iron
Asphalted cast iron
Commercial steel
Drawn tubing
ε (mm)
0.9–9
0.3–3
0.18–0.9
0.25
0.15
0.12
0.046
0.0015
Adapted by CN from
Metzger & Willard, Inc.

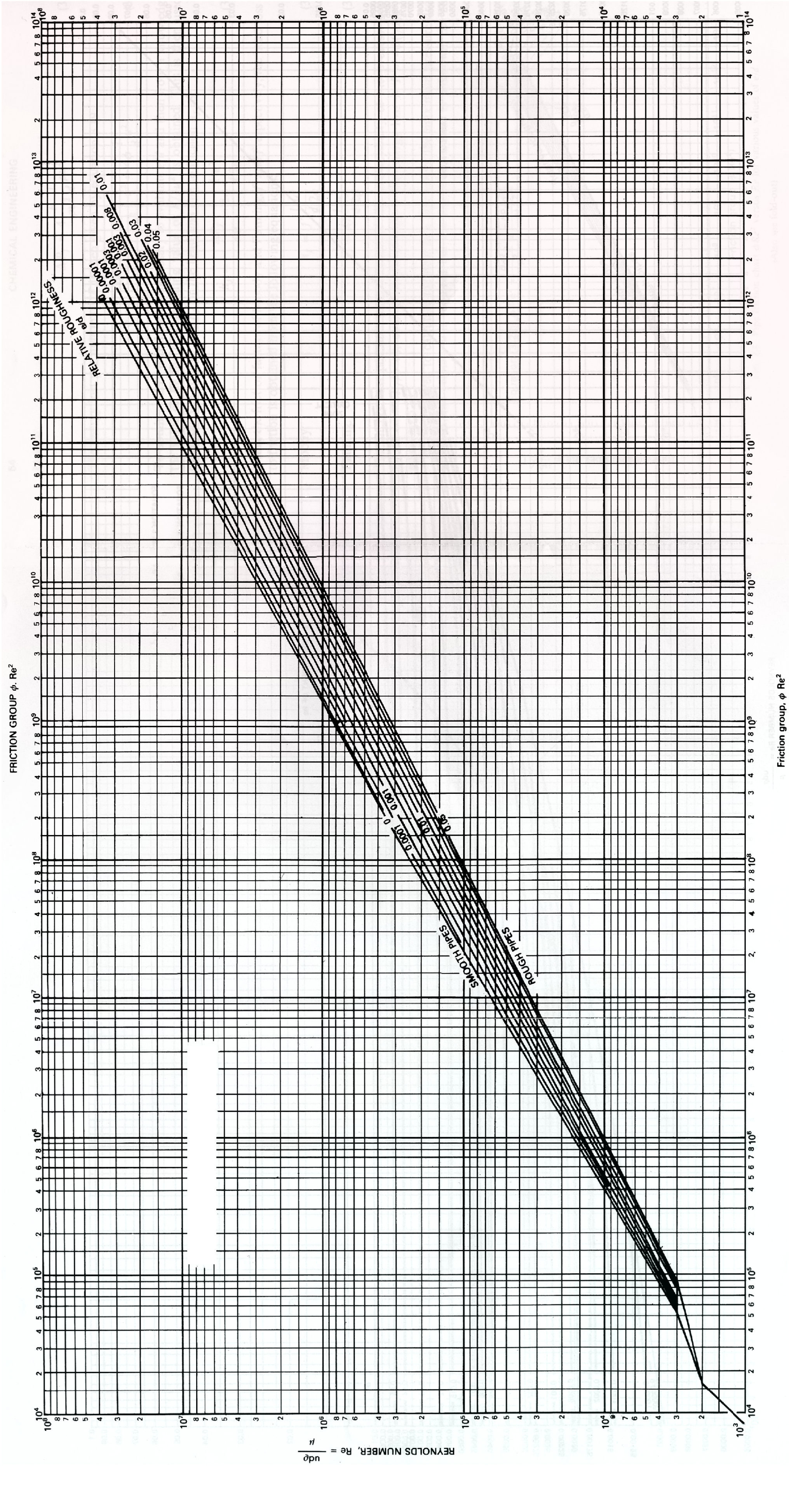


Fig. 3.8. Pipe friction chart ϕRe^2 versus Re for various values of e/d.

Continuity and Navier-Stokes equations for incompressible homogeneous fluids in Cartesian, cylindrical, and spherical coordinates

Cartesian	Cylindrical	Spherical
Continuity equation		
$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$	$\frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta}\right)+\frac{\partial v_z}{\partial z}=0$	$\frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(v_\theta \sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}=0$
Navier-Stokes equation		
$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)$ $=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)$ $=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial^2 v_r}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial\phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)$ $=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_r}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}-\frac{2v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}-\frac{2v_\theta\cot\theta}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right]$
$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)$ $=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)$ $=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)$ $=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_\theta}{\partial\theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta}{r^2\sin^2\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial\phi}\right]$
$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)$ $=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}\right)$ $=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}+\frac{v_r v_\phi}{r}+\frac{v_\theta v_\phi\cot\theta}{r}\right)$ $=-\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_\phi}{\partial\theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2}-\frac{v_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial\phi}\right]$

END OF EXAM