MAST30013 – Techniques in Operations Research

Semester 1, 2021

Tutorial 2 Solutions

1. **Method of False Position**

   First, we need to calculate $g(x) = f'(x) = x \cdot (1/x) + 1 \cdot \log x - 1 = \log x$. We note that $\log x = 0$ when $x = 1$, thus $x_{\min} = 1$.

   We have that $g(0.5) = \log(0.5) = -0.6931 < 0$ and $g(1.2) = \log(1.2) = 0.1823 > 0$.

Step 1 $k = 1$

$$\begin{aligned} p &= 0.5 + \frac{(1.2 - 0.5)\log(0.5)}{\log(0.5) - \log(1.2)} = 1.0542 \\ g(1.0542) &= 0.0528 \end{aligned}$$

Step 2 $k = 2$ and $g(1.0542) > 0$, therefore

$$\begin{aligned} b &= 1.0542 \qquad (a = 0.5) \\ p &= 0.5 + \frac{(1.0542 - 0.5)\log(0.5)}{\log(0.5) - \log(1.0542)} = 1.0150 \\ g(1.0150) &= 0.0149 \end{aligned}$$

Step 2 $k = 3$ and $g(1.0150) > 0$, therefore

$$\begin{aligned} b &= 1.0150 \qquad (a = 0.5) \\ p &= 0.5 + \frac{(1.0150 - 0.5)\log(0.5)}{\log(0.5) - \log(1.0150)} = 1.0042 \\ g(1.0042) &= 0.0042 \end{aligned}$$

Since $|g(1.0042)| < 0.01$ we stop and conclude that $x_{\min} = 1.0042$. 5 g-calculations were required for convergence.

Newton's Method

First we need to calculate $g'(x) = 1/x$.

We start with $a = 0.5$.

Step 1 $k = 1$ and $g'(0.5) = 2 > 0.001$, therefore

$$p = 0.5 - \frac{\log(0.5)}{1/0.5} = 0.8466$$

$|g(0.8466)| = 0.1665$.

Step 2 $k = 2$

$$a = 0.8466$$

$g'(0.8466) = 1.1812 > 0.001$, therefore

$$p = 0.8466 - \frac{\log(0.8466)}{1/0.8466} = 0.9876$$

$|g(0.9876)| = 0.0125$.

Step 2 $k = 3$

$$a = 0.9876$$

$g'(0.9876) = 1.0126 > 0.001$, therefore

$$p = 0.9876 - \frac{\log(0.9876)}{1/0.9876} = 0.9999$$

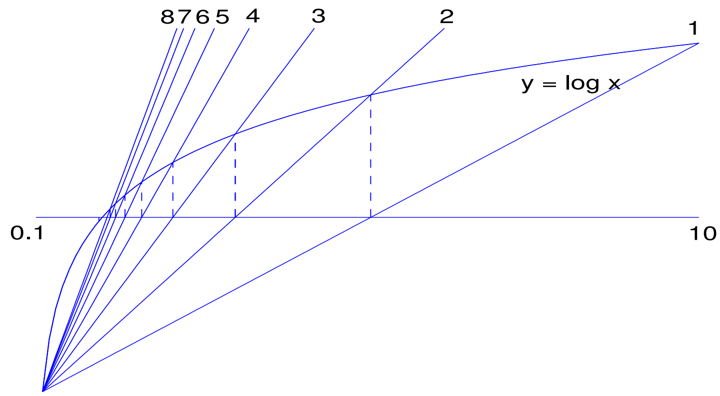
$|g(0.9999)| = 0.0001$.

Since $|g(0.9999)| = 0.0001 < 0.01$ we stop and conclude that $x_{\text{min}} = 0.9999$. 4 g-calculations and 3 g'-calculations were required for convergence.

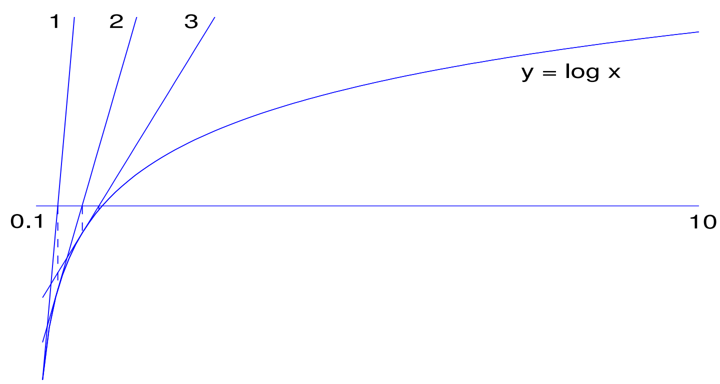
While both the Method of False Position and Newton's Method converged in 3 iterations, Newton's Method was the most accurate but required two more calculations.

Different starting points in $[0.5, 1.2]$ for Newton's Method will yield different results but convergence should be quite quick.

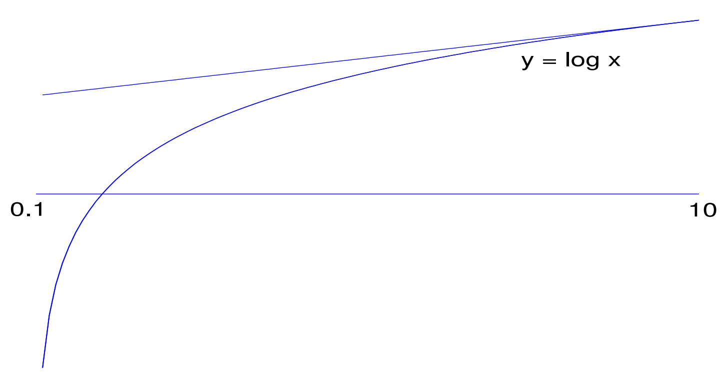
2. For an increasing function g where convergence is slow for the Method of False Position try $g(x) = \log x$ on the interval $[0.1, 10]$. 8 iterations gets reasonably close.



Newton's Method fares better taking only 3 iterations if we start at 0.1 to achieve a better result, at least visually.



However, if we start Newton's Method at 10 we do not get convergence. The tangent line through $(10, \log 10)$ cuts the x-axis at a negative value where log is not defined.



3. $f(x) = (x-3)^2$, $f'(x) = 2(x-3)$, $f(0) = 9$, $f'(0) = -6$. Note that $x_{\min} = 3$.

Step 1

$$\begin{aligned} t_{\text{low}} &= 0 \\ t_{\text{high}} &= \infty \\ t &= 10 \end{aligned}$$

Step 2 $f(10) = 49 > f(0) + t\sigma f'(0) = 9 - 10.\frac{1}{2}.6 = -21$ (the Armijo-Goldstein condition is not satisfied), therefore

$$\begin{aligned} t_{\text{high}} &= 10 \\ t &= \tfrac{1}{2}(0+10) \;=\; 5 \end{aligned}$$

$f(5) = 4 > f(0) + t\sigma f'(0) = 9 - 5.\frac{1}{2}.6 = -6$ (the Armijo-Goldstein condition is not satisfied), therefore,

$$\begin{aligned} t_{\text{high}} &= 5 \\ t &= \tfrac{1}{2}(0+5) \;=\; 2.5 \end{aligned}$$

$f(2.5) = 0.25 < f(0) + t\sigma f'(0) = 9 - 2.5.\frac{1}{2}.6 = 1.5$. The Armijo-Goldstein condition is satisfied.

$f'(2.5) = -1$ and $\mu f'(0) = -\frac{3}{4}.6 = -\frac{9}{2}$. The Wolfe condition is also satisfied.

Therefore, the step size is $t = 2.5$.

We now find an interval on which to find the minimum of f.

Step 1 We have $T = 2.5$. $k = 1$

$$\begin{aligned} p &= 0 \\ q &= 2.5 \end{aligned}$$

$f(0) = 9 > f(2.5) = 0.25$

Step 2 $k = 2$

$$\begin{aligned} p &= 2.5 \\ q &= 5 \end{aligned}$$

$f(2.5) = 0.25 < f(5) = 4$, therefore the interval we can start searching on is $[0, 5]$.

If instead, the algorithm on Slides 99–100 was used to find the interval, in Step 2 $q = 7.5$, $f(2.5) = 0.25 < f(7.5) = 20.25$, and the interval we can start searching on is $[0, 7.5]$.