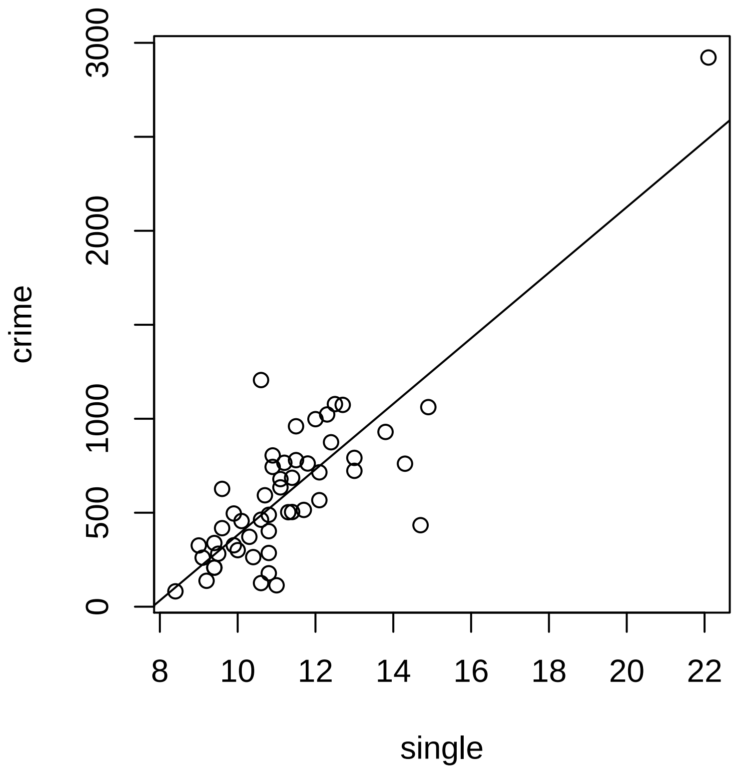
# MAST20005/MAST90058: Week 6 Lab Solutions

```r
library(foreign)
cdata <- read.dta("crime.dta")
```

1. (a)

```r
m1 <- lm(crime ~ single, cdata)
par(mar = c(4, 4, 1, 1))  # tighter figure margins
plot(crime ~ single, cdata)
abline(m1)
```

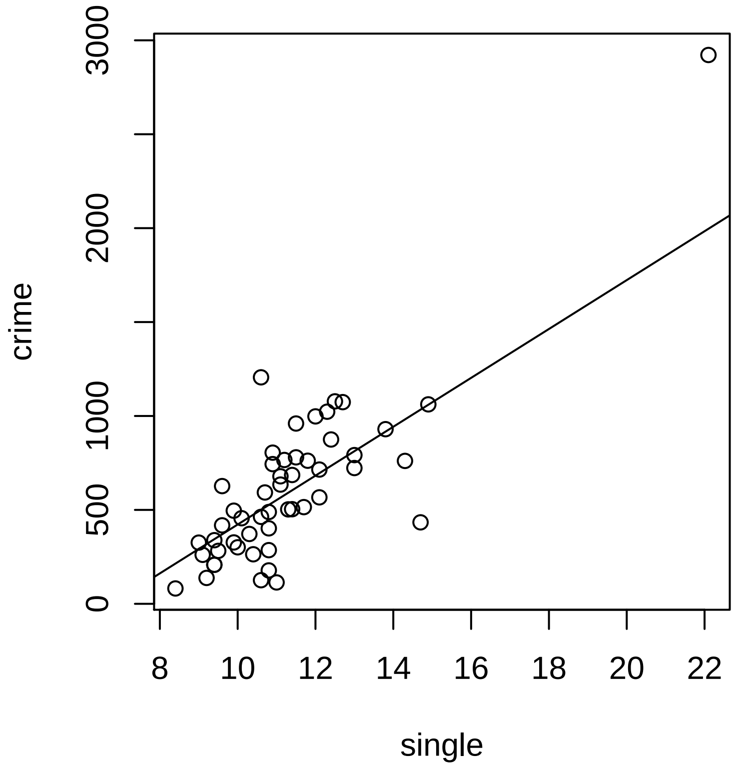



(b) The point in the top-right corner of the plot. It corresponds to the state `dc` (Washington, D.C.).

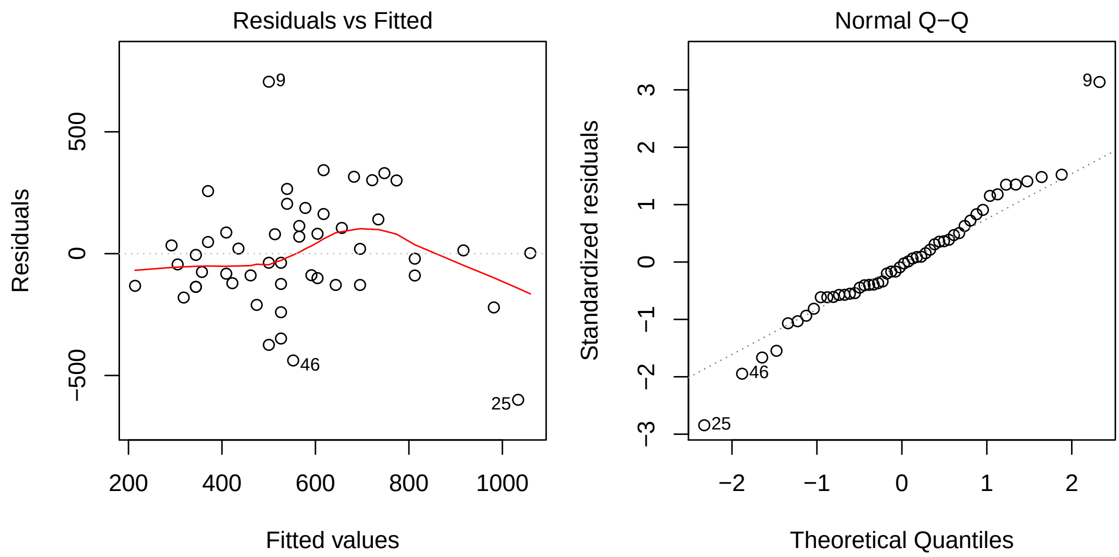
(c) An informal geometric argument: it seems like it might be quite influential because it is all by itself and will pull the best-fit line to be closer to itself while 'pivoting' it around the cluster of points in the bottom-left. (Indeed, this point does indeed have a strong influence on the parameter estimates. You will learn more about this in more advanced subjects that cover regression.)

(d) We could do this by copying the data frame and removing the row corresponding to this outlying point. A neater way is to use the `subset` argument in `lm()`:

```r
m1a <- lm(crime ~ single, cdata, subset = -51)
par(mar = c(4, 4, 1, 1))  # tighter figure margins
plot(crime ~ single, cdata)
abline(m1a)
```




```r
par(mfrow = c(1, 2), mar = c(4, 4, 2, 1))
plot(m1a, 1:2)
```



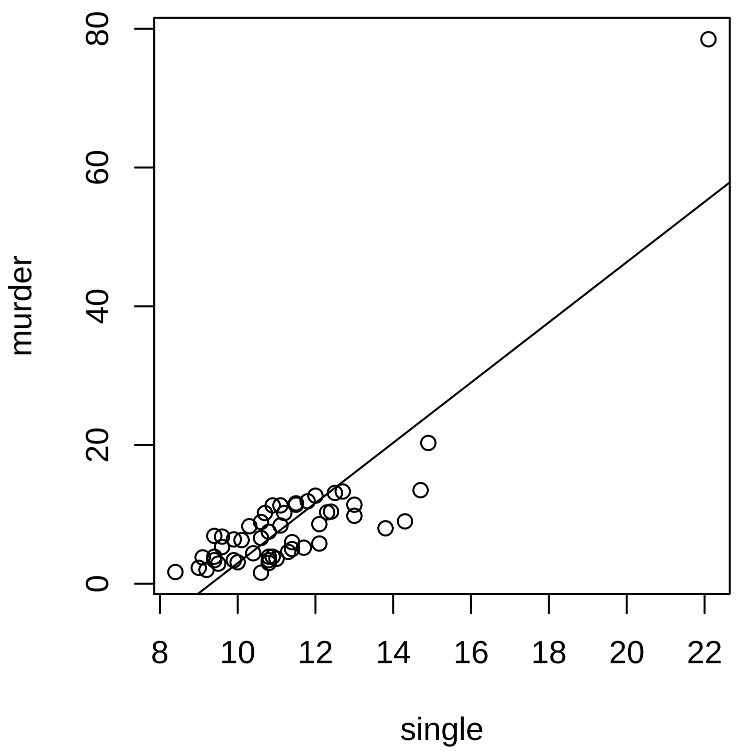
```r
coef(summary(m1))

##                Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -1362.5324  186.23306 -7.316276 2.150037e-09
## single        174.4186   16.16796 10.787910 1.529137e-14

coef(summary(m1a))

##               Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -878.8612  246.89537 -3.559650 8.491666e-04
## single       130.1099   22.03329  5.905152 3.498831e-07

confint(m1)

##                  2.5 %    97.5 %
## (Intercept) -1736.7818 -988.2831
## single        141.9278  206.9093

confint(m1a)

##                   2.5 %    97.5 %
## (Intercept) -1375.27761 -382.4448
## single        85.80902  174.4108
```

The estimates have shifted substantially!

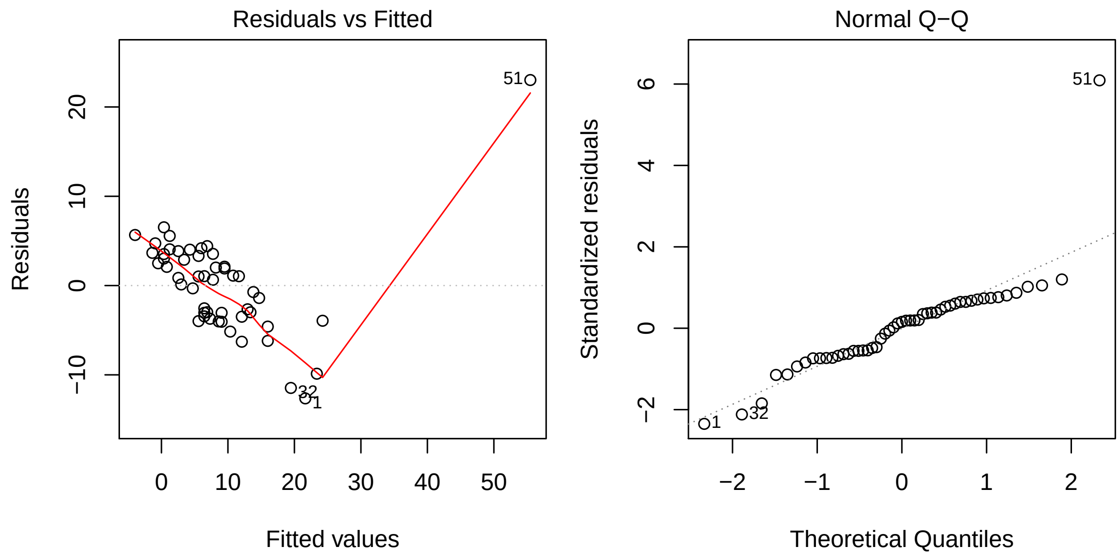
Which model we prefer to use will depend on to what extent we think that `dc` is representative or different to the other states. Given its special status within the USA and its very small size, there is a good argument for excluding it.

2. (a)
```r
m2 <- lm(murder ~ single, cdata)
```

(b)
```r
par(mar = c(4, 4, 1, 1))  # tighter figure margins
plot(murder ~ single, cdata)
abline(m2)
```



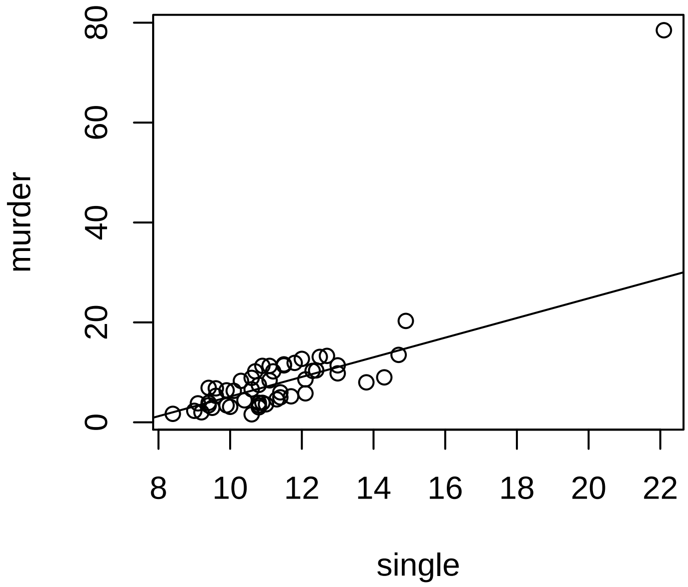

```r
par(mfrow = c(1, 2), mar = c(4, 4, 2, 1))
plot(m2, 1:2)
```



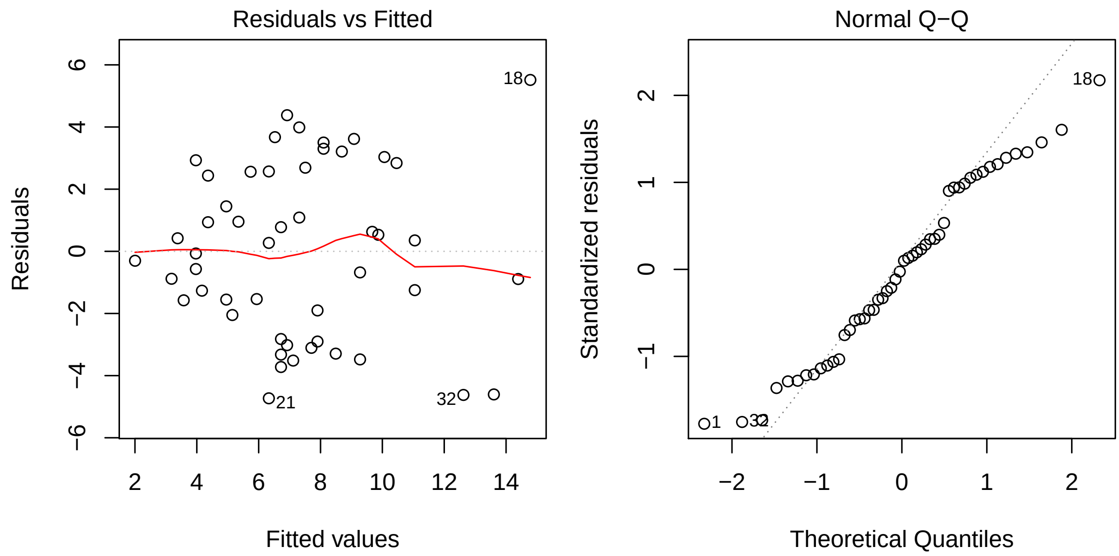

Outlying point (`dc`) is causing a poor model fit.

(c) Removing `dc` on the same basis as before:

```
m2a <- lm(murder ~ single, cdata, subset = -51)
plot(murder ~ single, cdata)
abline(m2a)
```




```
par(mfrow = c(1, 2), mar = c(4, 4, 2, 1))
plot(m2a, 1:2)
```



```r
coef(summary(m2))
```

```
##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -40.41533   4.257303 -9.493177 1.099312e-12
## single        4.33913   0.369601 11.740041 7.536045e-16
```

```r
coef(summary(m2a))
```

```
##               Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) -14.514349   2.994406 -4.847155 1.354009e-05
## single        1.966368   0.267225  7.358474 2.078957e-09
```

```r
confint(m2)
```

```
##                  2.5 %     97.5 %
## (Intercept) -48.970698 -31.859958
## single        3.596389   5.081871
```

```r
confint(m2a)
```

```
##                  2.5 %    97.5 %
## (Intercept) -20.535005 -8.493693
## single        1.429076  2.503660
```

(d)
```r
data12 <- data.frame(single = 12)
predict(m2a, newdata = data12, interval = "prediction", level = 0.9)
```

```
##        fit     lwr      upr
## 1 9.082068 4.39098 13.77316
```

(e)
```r
data8 <- data.frame(single = 8)
predict(m2a, newdata = data8, interval = "prediction", level = 0.9)
```

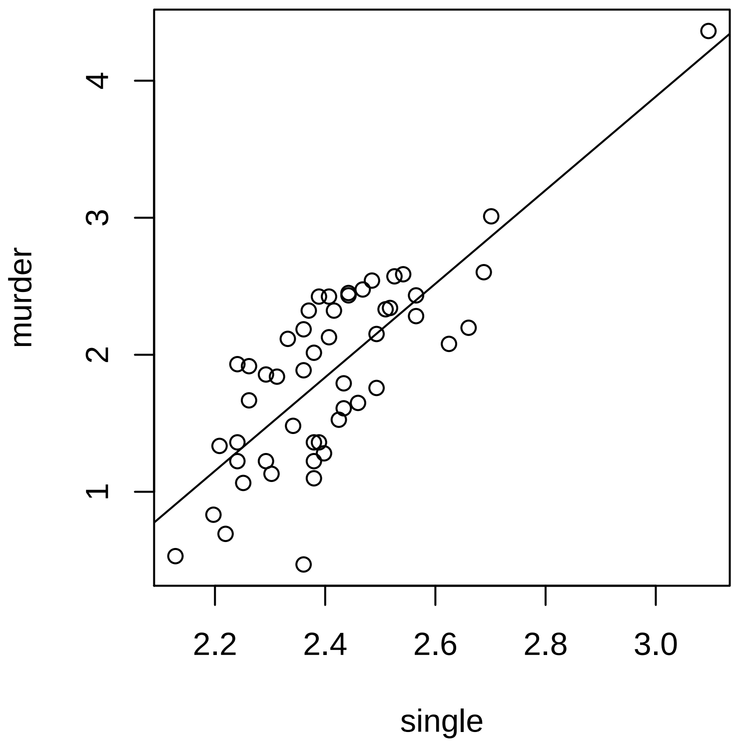
```
##        fit       lwr      upr
## 1 1.216595 -3.660916 6.094107
```

Negative estimates (lower endpoint of interval)!

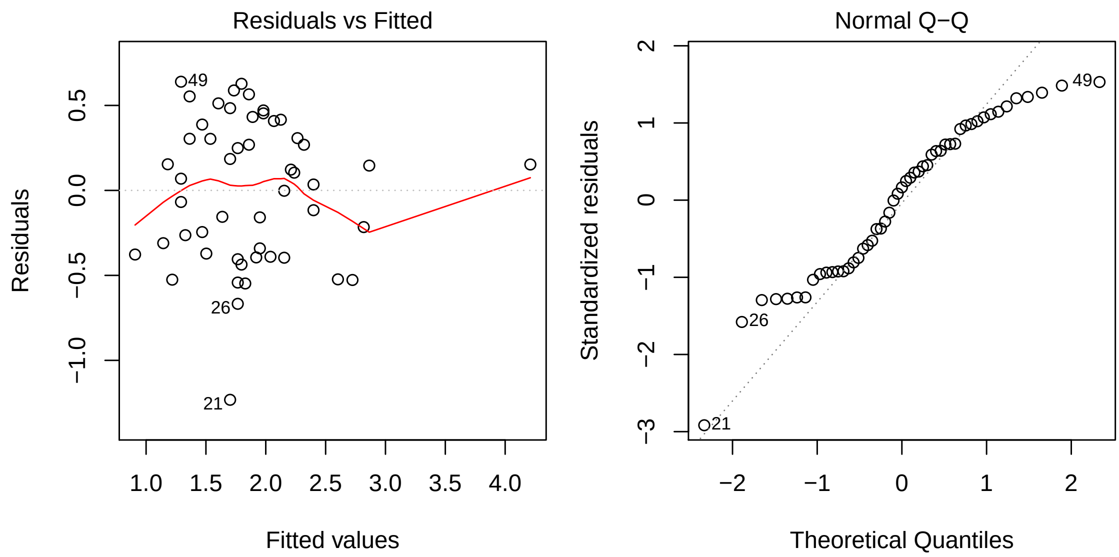
(f)
```r
cdata2 <- log(cdata[, c("murder", "single")])
str(cdata2)
```

```
## 'data.frame': 51 obs. of  2 variables:
##  $ murder: num  2.2 2.45 2.32 2.15 2.57 ...
##  $ single: num  2.66 2.44 2.37 2.49 2.53 ...
```

```r
m3 <- lm(murder ~ single, cdata2)
par(mar = c(4, 4, 1, 1))  # tighter figure margins
plot(murder ~ single, cdata2)
abline(m3)
```




```
par(mfrow = c(1, 2), mar = c(4, 4, 2, 1))
plot(m3, 1:2)
```



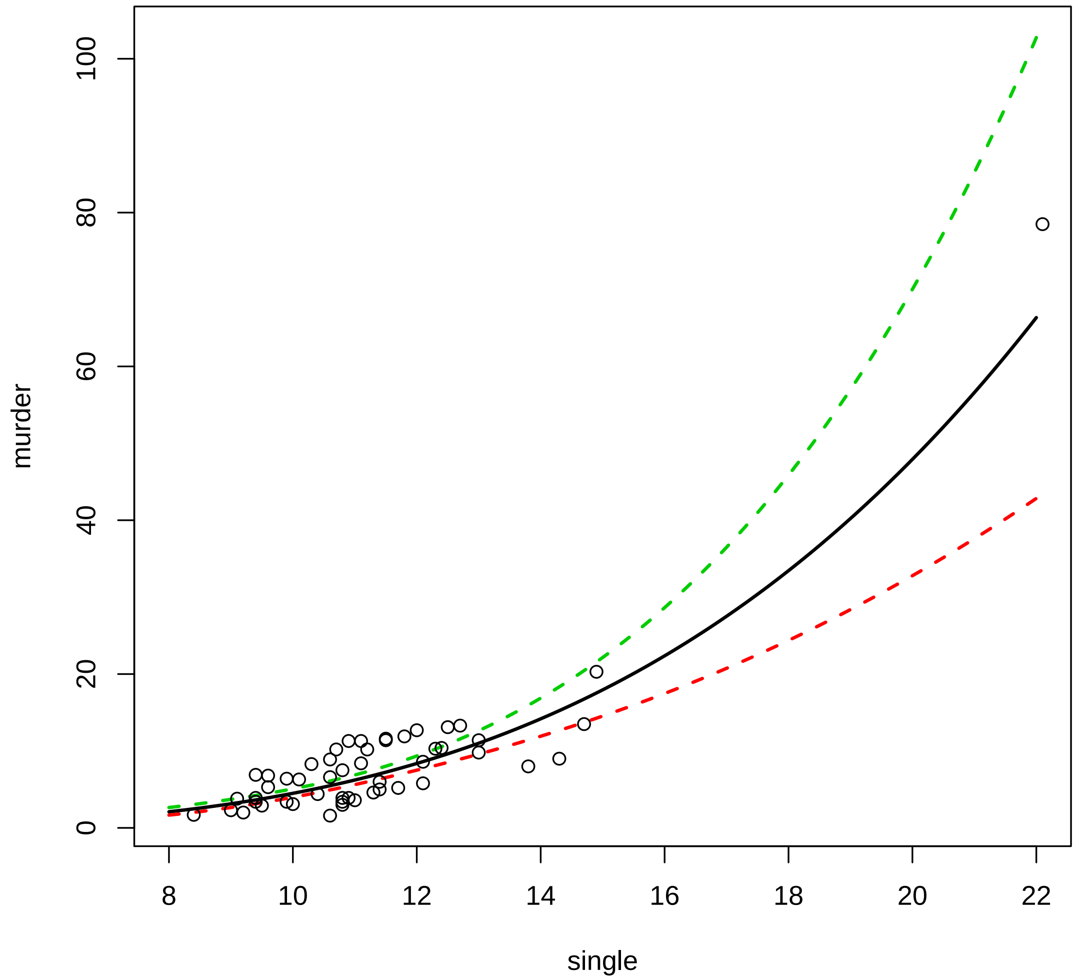

```
data3 <- data.frame(single = log(8))
predint <- predict(m3, newdata = data3, interval = "prediction", level = 0.9)
predint

##        fit         lwr     upr
## 1 0.741214 -0.01148196 1.49391

exp(predint)

##        fit       lwr      upr
## 1 2.098482 0.9885837 4.454479
```

(g) The model equation is $\log(\text{murder}) = \alpha + \beta \log(\text{single})$, which can be expressed as:

$$\text{murder} = A \times \text{single}^{\beta}$$

The 'slope' parameter $\beta$ has become an exponent. The model defines a power-law between the two variables, with $\beta$ measuring the strength of this relationship.

(h)
```
data4 <- data.frame(single = log(seq(8, 22, 0.05)))
pred <- predict(m3, newdata = data4, interval = "confidence", level = 0.9)
matplot(exp(data4$single), exp(pred), type = "l", lty = c(1, 2, 2),
        lwd = 2, xlab = "single", ylab = "murder")
points(murder ~ single, cdata)
```




3.
```
x <- c(8, 8, 8, 11, 17, 17, 20, 20, 20, 23, 26, 26, 26)
y <- c(14.8,  9.0, 11.0, 17.3, 20.8, 23.7, 24.4,
       28.9, 27.8, 29.3, 35.0, 33.4, 37.8)
m1 <- lm(y ~ x)
summary(m1)

##
```

```
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.6821 -1.5884  0.2703  1.7767  3.1179
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.43868    1.69846   0.847    0.415
## x            1.28042    0.08982  14.255 1.95e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.161 on 11 degrees of freedom
## Multiple R-squared:  0.9486,Adjusted R-squared:  0.944
## F-statistic: 203.2 on 1 and 11 DF,  p-value: 1.946e-08
```

(a)
```
coef(m1)

## (Intercept)           x
##    1.438676    1.280423

sigma(m1)

## [1] 2.160964
```

(b)
```
coef(summary(m1))

##             Estimate Std. Error    t value     Pr(>|t|)
## (Intercept) 1.438676 1.69846317  0.8470459 4.150186e-01
## x           1.280423 0.08982449 14.2547169 1.946385e-08
```

(c)
```
data2 <- data.frame(x = 18)
predict(m1, data2, interval = "confidence")

##        fit      lwr      upr
## 1 24.48628 23.16574 25.80683
```

(d)
```
predict(m1, data2, interval = "prediction")

##        fit      lwr      upr
## 1 24.48628 19.55012 29.42245
```