

MAST20005/MAST90058: Week 4 Solutions

- (a) We use the sample mean and sample median as estimators for the population mean.
From the R output, $\bar{x} = 10.29$ and $\hat{\pi}_{0.5} = 10.52$.
(b) From problem 7 in week 3, we know that:

$$\begin{aligned}\text{sd}(\bar{X}) &= \sqrt{\text{var}(\bar{X})} = \frac{\sigma}{\sqrt{n}} \\ \text{sd}(\hat{\pi}_{0.5}) &= \sqrt{\text{var}(\hat{\pi}_{0.5})} = \sqrt{\frac{\pi}{2}} \frac{\sigma}{\sqrt{n}}\end{aligned}$$

We can approximate both of these by substituting $s = 1.159$ for σ (recall that S is an estimator for σ). This gives standard errors for each of the two estimates:

$$\begin{aligned}\text{se}(\bar{x}) &= \frac{s}{\sqrt{n}} = 0.367 \\ \text{se}(\hat{\pi}_{0.5}) &= \sqrt{\frac{\pi}{2}} \frac{s}{\sqrt{n}} = 0.459\end{aligned}$$

- $73.8 \pm 1.96 \times 5/4 = [71.35, 76.25]$
- \bar{X} is approximately normally distributed. Using this gives:
 $2.09 \pm 1.96 \times 0.12/4 = [2.03, 2.15]$
- \bar{X} is approximately normally distributed. Using this gives:
 $11.95 \pm 1.96 \times 11.8/\sqrt{37} = [8.148, 15.75]$
Some people prefer to use the t -distribution approximation, which is more conservative. Since the necessary quantiles were provided, let's try it:
 $11.95 \pm 2.028 \times 11.8/\sqrt{37} = [8.016, 15.88]$
Note that the two are fairly similar.
- $20.9 \pm 2.306 \times 1.858/3 = [19.47, 22.33]$.
One sensible interpretation of the claim is that the *average* weight of a '22 kg' wheel is 22 kg (rather than claiming that *every* wheel is *exactly* 22 kg in weight). Since 22 is within our confidence interval for the mean, and the interval is relatively narrow, this claim seems to be reasonable given our data.
- $937.4 - 988.9 \pm 1.96\sqrt{784/56 + 627/57} = [-61.3, -41.7]$.
This confidence interval is very far from zero, meaning that our data show fairly strong evidence that the mean lifetimes are *not* the same.