ECOM20001: Econometrics 1

Tutorial 9: Suggested Solutions

Testing Joint Hypotheses About Regression Coefficients

1. F-statistic from waldtest() and linearHypothesis() commands is 23.88, with df1=10 and df2=2989 and p-value less than 0.00001.
   - df1=10 because there are 10 regressors whose coefficients we restrict to 0 under the null for the test
   - df2=n-k-1=3000-10-1=2989

waldtest() output:

```
> waldtest(reg, vcov = vcovHC(reg, "HC1"))
Wald test

Model 1: birthweight ~ smoker + alcohol + drinks + nprevisit + tripre1 +
    tripre2 + tripre3 + unmarried + educ + age
Model 2: birthweight ~ 1
  Res.Df  Df      F    Pr(>F)
1   2989
2   2999 -10 23.88 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

linearHypothesis() output:

```
Linear hypothesis test

Hypothesis:
smoker = 0
alcohol = 0
drinks = 0
nprevisit = 0
tripre1 = 0
tripre2 = 0
tripre3 = 0
unmarried = 0
educ = 0
age = 0

Model 1: restricted model
Model 2: birthweight ~ smoker + alcohol + drinks + nprevisit + tripre1 +
    tripre2 + tripre3 + unmarried + educ + age

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F    Pr(>F)
1   2999
2   2989 10 23.88 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

F-statistic from summary() command is 30.94 with df1=10 and df2=2989, and p-value less than 0.000001. Not accounting for heteroskedasticity yields a larger F-statistic, which makes it easier to reject the null under the test of the overall regression model (although it does not make a difference in this case).

summary() output:

```
Call:
lm(formula = birthweight ~ smoker + alcohol + drinks + nprevisit +
    tripre1 + tripre2 + tripre3 + unmarried + educ + age, data = mydata1)

Residuals:
    Min      1Q  Median      3Q     Max
-2788.6  -302.5    21.4   360.4  2309.1

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 2923.515    131.859  22.172  < 2e-16 ***
smoker      -178.207     27.436  -6.495 9.67e-11 ***
alcohol        3.942     94.675   0.042  0.96679
drinks        -3.027     18.861  -0.160  0.87252
nprevisit     32.087      3.406   9.421  < 2e-16 ***
tripre1      209.527    112.336   1.865  0.06226 .
tripre2      268.819    110.849   2.425  0.01536 *
tripre3      385.345    119.054   3.237  0.00122 **
unmarried   -206.856     28.795  -7.184 8.53e-13 ***
educ           1.828      5.562   0.329  0.74242
age           -2.143      2.270  -0.944  0.34533
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 564.6 on 2989 degrees of freedom
Multiple R-squared:  0.09381,    Adjusted R-squared:  0.09078
F-statistic: 30.94 on 10 and 2989 DF,  p-value: < 2.2e-16
```

2. Reporting the joint test results in order:

    - Test joint null that the coefficients on smoker and alcohol both equal 0: F-statistic=21.618 with df1=2 and df2=2989, and p-value<0.00001. In words, we reject the null that smoking and/or drinking alcohol during pregnancy has no effect on birth weight.

    - Test joint null that the coefficient on smoker equals -200 and that the coefficient on alcohol equals -50: F-statistic=0.547 with df1=2 and df2=2989, and p-value=0.547. In words, we fail to reject the null at the 5% level that smoking during pregnancy reduces baby weight by 200 grams AND that drinking alcohol reduces baby weight by 50 grams.

    - Test joint null that the coefficients on tripre1, tripre2 and tripre3 all equal 0: F-statistic=3.681 with df1=3 and df2=2989, and p-value=0.0116. In words, we reject the null at the 5% level (but not 1% level) that having prenatal care in any trimester has no effect on birth weight.

- Test joint null that the coefficients on tripre1, and tripre2 equals 0: F-statistic=3.037 with df1=2 and df2=2989, and p-value=0.0481. In words, we reject the null at the 5% level (but not 1% level) that having prenatal care in either the first or second trimester has no effect on birth weight.
  - Comparing these test results to the test results for the null about that the coefficients on tripre1, tripre2 and tripre3 all being 0 we find the F-statistic becomes smaller and the p-value rises. The reason for this is that the individual statistical significance of tripre3 (and its individual larger t-statistic for the null that its coefficient in the regression equals 0) pushes the F-statistic up and the corresponding p-value down for the test of the null that the coefficients on tripre1, tripre2 and tripre3 all equal 0 relative to the F-statistic up and the corresponding p-value for the test of the null that the coefficients on tripre1, tripre2 all equal 0.
    - This relates back to the intuition from the formula for the F-statistic for q=2 restrictions from slide 14 of Lecture Note 7, which highlights how an F-statistic for a joint hypothesis test rises as the t-statistics for the individual hypothesis tests that constitute the joint hypothesis test rise.
- Test joint null that the coefficients on tripre1=200, tripre2=300 and tripre3=400: F-statistic=0.484 with df1=3 and df2=2989, and p-value=0.694. In words, we fail reject to the null at the 5% level that having prenatal care for the first time in the first, second, and third trimester increases birth weight by 200, 300, and 400 grams, respectively.
  - Further interpreting the test result, we fail to reject the null that baby weight rises linearly by 100 grams as a function of the first trimester in which the baby first has prenatal care. Babies that have prenatal care in later semesters tend to have have greater birth weight.

3. Computing the homoskedasticity-only F-statistic by hand in steps:
   - Unadjusted (raw) R-squared from the unrestricted regression that does not impose any constraints is 0.0938 (called “R2u” in the tute9.R code).
   - Unadjusted (raw) R-squared from the restricted regression imposing the constraint that tripre1, tripre2 and tripre3 all equal 0 is 0.0890 (called “R2r” in the tute9.R code)
   - Computing the homoskedasticity-only F-statistic from the R-Squared-based formulae from page 34 in the text we obtain F=(R2u-R2r)/q)/((1-R2u)/(n-k-1))=(0.0938-0.0890)/3)/((1-0.0938)/2989)=5.244.

- Computing the p-value for the F-statistic requires we use the cumulative density for the F-distribution with df1=3 and df2=2989. From tutorial 2, the R command for computing this cumulative density is pf(). Using this command, and the definition of the p-value for F-statistics, we obtain a p-value=1-pf(5.244,df1=3,df2=2989)=0.0013, which implies we reject the null implied by the restrictions under the restricted model at the 1% level of significance.
  - In other words, given our sample, there is a sufficiently large drop in the R-Squared from 0.0938 under the unrestricted model to 0.0890 under the restricted model from imposing the joint constraint that the coefficients on tripre1, tripre2 and tripre3 all equal 0 such that we are able to reject the hypothesis that the data were generated under these restrictions. The drop in model fit when the restrictions are relaxed is too large to rationalise these restrictions statistically.

<u>Testing Joint Restrictions Involving Multiple Regression Coefficients</u>

4. Reporting the joint test results in order:
   - Test joint null that the coefficient on smoker equals the coefficient on alcohol: F-statistic=3.515, df1=1, df2=2989, p-value=0.061. Fail to reject the null at the 5% level of significance, meaning we cannot reject the null that smoking and drinking alcohol have the same impact on birthweight.
     - Note here while we find individually statistically significant effects for smoker but not alcohol on birthweight, we are unable to detect a statistically significant difference between the effects of smoker and alcohol on birthweight.
   - Test joint null that the coefficient on smoker is twice the coefficient alcohol: F-statistic=1.000, df1=1, df2=2989, p-value=0.317. Fail to reject the null at the 5% level of significance, meaning we cannot reject the null that smoking has twice as large an impact on birthweight relative to drinking.
   - Test joint null that the sum of the coefficients on smoker and alcohol equals -200: F-statistic=0.078, df1=1, df2=2989, p-value=0.780. Fail to reject the null at the 5% level of significance, meaning we cannot reject the null that smoking and drinking alcohol together yields a 200 gram reduction in birthweight.
   - Test joint null that the sum of the coefficients on alcohol and unmarried equals the coefficient on smoker: F-statistic=0.058, df1=1, df2=2989, p-value=0.809. Fail to reject the null at the 5% level of significance, meaning

we cannot reject the null that drinking alcohol and being unmarried together has the same impact on birthweight as smoking.

- Test joint null that the coefficient on tripre1 equals the coefficient on tripre2 and that the coefficient on tripre2 equals the coefficient on tripre3: F-statistic=3.511, df1=2, df2=2989, p-value=0.030. Reject the null at the 5% level of significance, meaning we reject the null that first having prenatal care in the first, second, or third semester has the same impact on birthweight.
    - Looking the individual regression coefficients for tripre1, tripre2 and tripre3 in the regression, they have values of 209, 268, and 385, respectively. So this joint test result formally confirms intuition about differential effects on birthweight, depending on when a baby first receives prenatal care. The large jump in the coefficient on tripre3 relative to tripre1 and tripre2 is preliminary evidence against the joint null hypothesis, which is confirmed by the test's F-statistic and p-value.
- Test joint null that the on tripre2 equals 2 times the coefficient on tripre1 and that the coefficient on tripre3 equals 2 times the coefficient on tripre2: F-statistic=0.4754, df1=2, df2=2989, p-value=0.6217. Fail to reject the null at the 5% level of significance, meaning we fail to reject the null that the effect of when a baby first receives prenatal care on birthweight doubles with each successive trimester.

5. The following set of calculations develops a transformed regression that allows us to use an individual hypothesis test and t-statistic to test the joint null hypothesis that the sum of the coefficients on alcohol and unmarried equals the coefficient on smoker against the alternative that the equality does not hold. These calculations follow the same strategy used in Lecture Note 7, slides 38-41. In undertaking the calculations, for the moment we omit all the other control variables for simplicity as the calculations for the altered regression is the same whether they are included or not:

$$
\begin{aligned}
Birthweight_i &= \beta_0 + \beta_1 Smoker_i + \beta_2 Alcohol_i + \beta_3 Unmarried_i + u_i \\
&= \beta_0 + \beta_1 Smoker_i + \beta_2 Alcohol_i + \beta_3 Unmarried_i + u_i + \beta_2 Smoker_i - \beta_2 Smoker_i + \beta_3 Smoker_i - \beta_3 Smoker_i \\
&= \beta_0 + (\beta_1 - \beta_2 - \beta_3) Smoker_i + \beta_2 Alcohol_i + \beta_3 Unmarried_i + u_i + \beta_2 Smoker_i + \beta_3 Smoker_i \\
&= \beta_0 + \underbrace{(\beta_1 - \beta_2 - \beta_3)}_{\gamma} Smoker_i + \beta_2 \underbrace{(Alcohol_i + Smoker_i)}_{W_i} + \beta_3 \underbrace{(Unmarried_i + Smoker_i)}_{Z_i} + u_i \\
&= \beta_0 + \gamma Smoker_i + \beta_2 W_i + \beta_3 Z_i + u_i
\end{aligned}
$$

When we include all the other controls in the transformed regression, we obtain the following regression, which we use to test the joint null hypothesis based on the smoker coefficient (gamma) in the tute9.R code:

$$Birthweight_i = \beta_0 + \gamma Smoker_i + \beta_2 W_i + \beta_3 Z_i + \beta_4 Drinks_i + \beta_5 Nprevisit_i + \beta_6 Tripre1_i + \beta_7 Tripre2_i + \beta_8 Tripre3_i + \beta_9 Educ_i + \beta_{10} Age_i + u_i$$

Running this transformed regression using the coeftest() command we obtain a coefficient estimate on smoker of 24.72 with a standard error of 102.46, and a corresponding **p-value=0.8094** for the test of the null that the coefficient on smoker is equal to 0. For reference, here is the regression output, which displays this p-value for the smoker coefficient:

```
t test of coefficients:

              Estimate Std. Error t value  Pr(>|t|)    
(Intercept) 2923.5151   159.5741 18.3207 < 2.2e-16 ***
smoker        24.7080   102.4462  0.2412   0.80943    
W              3.9417    90.7529  0.0434   0.96536    
Z           -206.8565    31.2952 -6.6099 4.539e-11 ***
drinks        -3.0267    16.4270 -0.1842   0.85383    
nprevisit     32.0871     4.2500  7.5498 5.753e-14 ***
tripre1      209.5266   148.8734  1.4074   0.15941    
tripre2      268.8187   146.6549  1.8330   0.06690 .  
tripre3      385.3451   155.4357  2.4791   0.01323 *  
educ           1.8282     5.5373  0.3302   0.74130    
age           -2.1426     2.4573 -0.8720   0.38330    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Based on the transformation calculations in the regression equation, testing whether the coefficient on smoker equals 0 is equivalent to testing the null that the effect of smoker on birthweight is the same as the sum of the effects of alcohol and unmarried on birthweight. Indeed, if we estimate our original (untransformed) regression model:

$$Birthweight_i = \beta_0 + \beta_1 Smoker_i + \beta_2 Alcohol_i + \beta_3 Drinks_i + \beta_4 Nprevisit_i + \beta_5 Tripre1_i + \beta_6 Tripre2_i + \beta_7 Tripre3_i + \beta_8 Unmarried_i + \beta_9 Educ_i + \beta_{10} Age_i + u_i$$

and use the linearHypothesis() command to jointly test the null that the sum of coefficients on alcohol and unmarried equals the coefficient on smoker we obtain an F-statistic=0.0582, with df1=1, df2=2989, and **p-value=0.8094**, where notice the p-value is exactly the same as what we obtained for transformed regression and the individual hypothesis test that the coefficient on smoker equals 0. The p-values for the individual and joint tests are exactly same because they are testing the exactly the same hypothesis.

Similarly, notice how the square of the t-statistic on smoker in the transformed regression is **0.2412*0.2412=0.0582**, which is exactly equal to the F-statistic computed for the joint hypothesis test from the untransformed regression. This corresponds to the result stated in Lecture Note 7 one slide 37 that the square of a t-statistic on a regression coefficient for a transformed regression that tests a single restriction involving multiple coefficients equals the F-statistic from the corresponding untransformed regression that jointly tests the same restriction involving multiple coefficients.

For reference, here's the linearHypothesis() output which also highlights the p-value=0.8094 for the joint test based on the untransformed regression:

```
Linear hypothesis test

Hypothesis:
- smoker  + alcohol  + unmarried = 0

Model 1: restricted model
Model 2: birthweight ~ smoker + alcohol + drinks + nprevisit + tripre1 +
    tripre2 + tripre3 + unmarried + educ + age

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F Pr(>F)
1   2990
2   2989  1 0.0582 0.8094
```