THE UNIVERSITY OF MELBOURNE

Student number	

Semester 2 Assignment 4, 2021

School of Mathematics and Statistics

MAST30022 Decision Making

Submission deadline: **4pm (Melbourne time), Friday 15 October**

This assignment consists of 12 pages (including this page)

Instructions to Students

- If you have a printer, print the assignment one-sided.

Writing

- There are 4 questions, of which 2 randomly chosen questions will be marked. Note you are expected to submit answers to all questions, otherwise **mark penalties will apply**.
- Working and reasoning **must** be given to obtain full credit. Give clear and concise explanations. Clarity, neatness, and style count.
- Write your answers in the boxes provided on the assignment that you have printed. If you need more space, you can use blank paper. Note this in the answer box, so the marker knows. The extra pages can be added to the end of the assignment to scan.
- If you have been unable to print the assignment write your answers on A4 paper. The first page should contain only your student number, the subject code and the subject name. Write on one side of each sheet only. Start each question on a new page and include the question number at the top of each page.

Scanning

- Put the pages in number order and the correct way up. Add any extra pages to the end. Use a scanning app to scan all pages to PDF. Scan directly from above. Crop pages to A4. Make sure that you upload the correct PDF file and that your PDF file is readable.

Submitting

- Go to the Gradescope window. Choose the Canvas assignment for this assignment. Submit your file. Get Gradescope confirmation on email.

Question 1

Consider the following lotteries

$$
\begin{aligned}
L_1 &= \langle 0.5, L_{1a}; 0.5, L_{1b} \rangle \\
L_{1a} &= \langle 0.25, 3; 0.75, 0 \rangle \\
L_{1b} &= \langle 0.1, 10; 0.4, 3; 0.5, -2 \rangle \\
L_2 &= \langle 0.2, 10; 0.55, 0; 0.25, -5 \rangle \\
L_3 &= \langle 0.8, L_{3a}; 0.1, -2; 0.1, -5 \rangle \\
L_{3a} &= \langle 0.6, 10; 0.4, -5 \rangle.
\end{aligned}
$$

Suppose Paul holds the utility function $u(x) = \sqrt{x+5}$ for a monetary reward $x \in [-5, \infty)$. In this question we assume that Paul agrees with the von Neumann-Morgenstern axioms.

(a) Determine the preferences of Alice over $\mathcal{L} = \{L_1, L_2, L_3\}$.

(b) Determine the risk premium of L_1.

(c) Is Paul risk-averse, risk-neutral, or risk-seeking? Justify your answer.

Question 2

The decision science department is trying to determine which of two copying machines to purchase. Both machines will satisfy the department's need for the next ten years. The department has a special bank account for the expenses in purchase and maintenance of the copying machine, on which there is currently \$5500.

Machine 1 costs \$2000 and has a maintenance agreement, which, for annual fee of \$150, covers all repairs. Machine 2 costs \$3000, and its annual maintenance cost is a random variable. At present, the decision science department believes there is a 40% chance that the annual maintenance cost for machine 2 will be \$0, a 40% chance it will be \$100, and a 20% chance it will be \$200.

Before the purchase decision is made, the department can pay $\$x$ to ask a trained repairer to evaluate the quality of machine 2. If the repairer believes that machine 2 is satisfactory, there is a 60% chance that its annual maintenance cost will be \$0, and a 40% chance it will be \$100. If the repairer believes that machine 2 is unsatisfactory, there is a 20% chance that the annual maintenance cost will be \$0, a 40% chance it will be \$100, and a 40% chance it will be \$200. There is a 50% chance that the repairer will give a satisfactory report.

(a) Provide a decision tree that models this problem. You are required to indicate for each vertex whether it is a decision vertex or an event vertex, for each arc to which decision or event it corresponds (in the latter case, also indicate the corresponding probability), and for each leaf the corresponding amount of money left on the department's special account after ten years.

(b) If the goal of the department is to maximize the total expected amount of money left on its special account, which strategy should be adopted if the repairer does not charge any money (that is, $x = \$0$)?

Answer the same question if the repairer charges $x = \$100$.

(c) Assume that the repairer does not charge any money, and that the department's utility function for the possible amounts of money left on the special account after ten years is given by

$$u(500) = 0, \quad u(1500) = 0.7, \quad u(2000) = 0.9, \quad u(2500) = 1.$$

If the department's goal is to maximize the expected utility of the money left on the special account, which strategy should be adopted?

Question 3

Suppose that you want to invest \$10,000 in the stock market by buying shares in one of two companies: X and Y. Shares in company X are risky but could yield a 50% return during the next year if the stock market conditions are favorable (i.e. "bull" market). However, if the stock market conditions are unfavorable (i.e. "bear" market), the stock may lose 20% of its value. Company Y provides safe investment with 15% return in a "bull" market and only 5% in a "bear" market. All the publications that you have consulted predict a 60% chance for a "bull" market and 40% chance for a "bear" market. The following decision table summarizes the information about one year returns together with the corresponding probabilities for each decision alternative.

Decision alternative	"Bull" market (\$)	"Bear" market (\$)
Company X	5000	-2000
Company Y	1500	500
Probability of occurence	0.6	0.4

(a) Use Wald's maximin, Hurwicz's maximax, Savage's minimax regret, and Laplace's expected value criteria to determine the stock you would like to invest your money in.

(b) Let us view each decision alternative above as a lottery as shown in Figure 1.

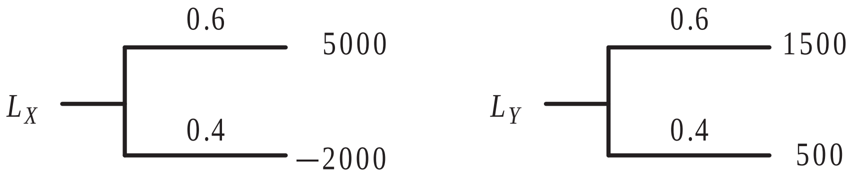

Figure 1: Question 3(b)

Suppose that your utility function gives the following utilities: $u(-2000) = 0$, $u(500) = 0.2$, $u(1500) = 0.5$, $u(5000) = 1$. Assuming that you agree with von Neumann-Morgenstern Axioms, determine your preferable stock for investment by using the *expected utility criterion.*

(c) Suppose that, besides the alternatives X and Y above, another alternative is to deposit your \$10,000 capital in a bank with a fixed interest rate 12%. Thus, for this option you will get \$1,200 return with certainty during the next year. Assume that you are risk-neutral and so your aim is to maximize your expected return. In which alternative should you choose to invest your money? Solve this problem by using the decision tree technique. Draw a decision tree and show all your work clearly.

Question 4

In the network in Figure 2 each arc is associated with a weight whose coordinates measure the cost and the distance from the head to the tail respectively. Note that the weight from E to F is (a, b), where $a \geq 0$ and $b \geq 0$ are real numbers.

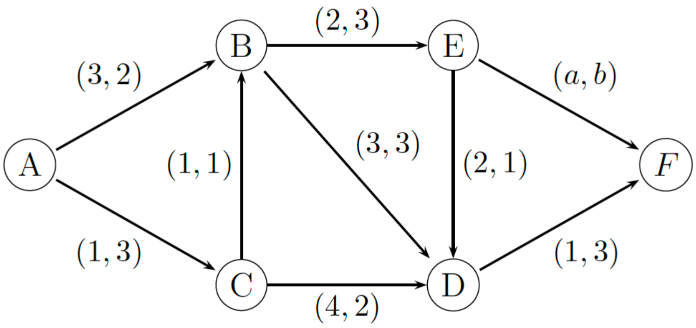

Figure 2: Network for Question 4

(a) Find a proper labelling for this network.

(b) Based on your proper labelling, determine the range of a and b such that there exists a unique Pareto minimal path between A and F.

(c) Based on your proper labelling, determine all values of a and b such that there exist at least two lexicographic shortest paths between A and F.

End of Assignment