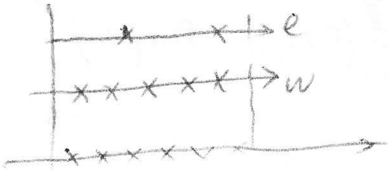
The Poisson process

ep. Slide 162. A shop with east & west entrance. constiomers independent. Poisson process with rate 0.5 & 1.5 per min.



q1) What is the chance the 1st. enter from west entrance

$\Rightarrow P(T_w < T_e) = \int_0^\infty \int_x^\infty \frac{3}{2} e^{-\frac{3}{2}x} \cdot \frac{1}{2} e^{-\frac{1}{2}y} \, dy \, dx$

$= \int_0^\infty \frac{3}{2} e^{-\frac{3}{2}x} \cdot e^{-\frac{1}{2}x} \, dx = \frac{3}{4}$

If T_u is $Exp(u)$. T_λ is $Exp(\lambda)$

$\Rightarrow P(T_u < T_\lambda) = \frac{u}{\lambda + u}$

q2) What's the chance that after time t, the 1st constiomer enter from the west entrance?

After time t → still exponiential → answer same

The averge # of comiser $\begin{cases} 0.5 & e \\ 1.5 & w \end{cases}$

N_t. Possion λ. M_t each independent probability p → $P_o(\lambda p)$

$N_t - M_t \sim$ Poisson $\lambda(1-p)$.

ep

The flow of cumstome. Poisson rate 25/h

Each customer $p = 0.8$. making a pchudir

1) What's the probalitiry all clastoms during 11:00 - 11:15 make a purchase

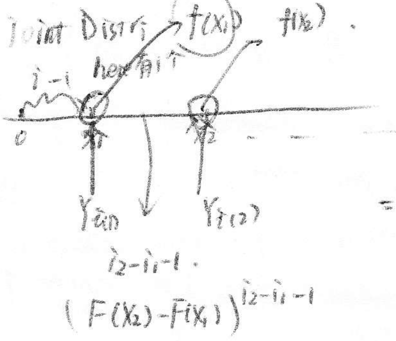
$M_t \sim P_o(20/hr)$

$N_t - M_t \sim P_o(5/hr)$

$(11, 11.25) \rightarrow P\big((N_{11.25} - M_{11.25}) - (N_{11} - M_{11}) \neq 0 \big)$

$\underset{P_o(\frac{5}{4})}{\Downarrow}$ $= 1 - e^{-\frac{5}{4}}$

2) the probabily that, condititional on there being 2 constomer make a purchase, all purchase?



$$f_{Y(i_1)\cdots Y(i_r)}(x_1,\dots,x_r) = \binom{k}{i_1-1,\,1,\,i_2-i_1-1,\,1,\,\dots,\,1,\,k-i_r-1} \times \prod_{j=1}^{r} f(x_j)\cdot\prod_{j=0}^{r}\left(F(x_{j+1})-F(x_j)\right)^{i_{j+1}-i_j-1}$$

for $r=k$. $x_1<\cdots<x_r$.

$$f_{Y(i_1)\cdots Y(i_k)}(x_1,\dots,x_k) = k!\prod_{j=1}^{k} f(x_j)$$

$$P(N_{t+h}-N_t \geq 1) = 1-P(N_{t+h}-N_t=0) = 1-e^{-\lambda h} \approx h\lambda \quad (h \text{ small enough})$$

$$\frac{1-e^{-\lambda h}}{h} \xrightarrow[h\to 0]{} \lambda.$$

- ~~Density of~~ If $\tau_1 \dots$ are iid exponential variables

$$\Rightarrow \left(\frac{\tau_1}{\sum_{j=1}^{n+1}\tau_j},\ \frac{\tau_1+\tau_2}{\sum_{j=1}^{n+1}\tau_j},\ \dots,\ \frac{\sum_{j=1}^{n}\tau_j}{\sum_{j=1}^{n+1}\tau_j}\right)$$

have the same distribution as uniform order statistics