LSM Assignment 1

Linear Statistical Models (University of Melbourne)

MAST30025: Linear Statistical Models

Assignment 1, 2018

Due: 5pm Friday, March 23 (week 4)

This assignment is worth 6% of your total mark.

You may use R for this assignment, but for matrix calculations only (you may not use the `lm` function). If you do, include your R commands and output.

1. Prove that if a symmetric matrix A has eigenvalues which are all either 0 or 1, it is idempotent.
2. Prove (without using Theorem 2.5) that if A and B are symmetric matrices, $A+B$ is idempotent and $AB = BA = 0$, then both A and B are idempotent. (*Hint*: Use Theorem 2.4. Then derive two relations between the diagonalisations of A and B.)
3. Let $\mathbf{y}$ be a 3-dimensional multivariate normal random vector with mean and variance
$$\boldsymbol{\mu} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad V = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$
Let
$$A = \frac{1}{3}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$
   (a) Describe the distribution of $A\mathbf{y}$.
   (b) Find $E[\mathbf{y}^T A\mathbf{y}]$.
   (c) Describe the distribution of $\mathbf{y}^T A\mathbf{y}$.
   (d) Find a matrix B such that $\mathbf{y}^T B\mathbf{y}$ is independent of $\mathbf{y}^T A\mathbf{y}$.
4. Let $\mathbf{y} \sim MVN(\boldsymbol{\mu}, V)$ be a $n \times 1$ random vector and suppose V is nonsingular. Find A and $\mathbf{b}$ such that $A\mathbf{y} + \mathbf{b}$ is an n-length vector of independent standard normals.
5. A study is conducted to determine if (and how) the fuel mileage of a car is dependent on its weight, and the speed at which it is driven. A linear model is assumed, and the following data is obtained:

Weight (tons)	Speed (km/hr)	Mileage (km/litre)
1.35	50	8.5
1.33	55	8
2	60	7.5
1.4	52	10
1.43	47	11
1.2	45	15
1.3	49	13.5
1.28	63	14

   (a) Write down the linear model as a matrix equation, writing out the matrices in full.
   (b) Calculate the least squares estimator of the parameters.
   (c) Calculate the residual sum of squares SS_{Res} and sample variance s^2.
   (d) Predict (using a point estimate) the average fuel mileage of a car which weighs 1.8 tons and is driven at 59 km/hr.
6. Let A be a symmetric and idempotent matrix with entries a_{ij}. Prove that $0 \le a_{ii} \le 1$. Use this to derive limits on the leverage of a point in the full rank model.