1. The last column of $Df(\mathbf{w}, b)$ is $\frac{\partial f}{\partial b}$, and since the variable b does not appear in f we have $\frac{\partial f}{\partial b} = 0$.

   For the other columns, we have

$$\frac{\partial f}{\partial w_i} = \frac{1}{2}\frac{\partial}{\partial w_i}\left(w_1^2 + w_2^2 + \ldots + w_n^2\right) = w_i,$$

   so $Df(\mathbf{w}, b) = \begin{pmatrix}\mathbf{w} & 0\end{pmatrix}$.

2.
```
syms w [2 1] real
syms b real
Df = [w' 0]
```

3.
```
p = [2 3 1 3; 4 3 2 1];
l = [1 1 -1 -1]';
```

4. The last column of $Dg_i(\mathbf{w}, b)$ is equal to

$$\frac{\partial g_i}{\partial b} = \frac{\partial}{\partial b}\left(1 - l_i(\mathbf{w}^T\mathbf{p}_i + b)\right) = -l_i.$$

   Differentiating with respect to $\mathbf{w}$, we have

$$Dg_i(\mathbf{w}) = -l_i D(\mathbf{w}^T\mathbf{p}_i)(\mathbf{w}) = -l_i\mathbf{p}_i.$$

   Hence,

$$D\mathbf{g}(\mathbf{w}, b) = \begin{pmatrix} Dg_1(\mathbf{w}, b) \\ Dg_2(\mathbf{w}, b) \\ Dg_3(\mathbf{w}, b) \\ Dg_4(\mathbf{w}, b) \end{pmatrix} = \begin{pmatrix} -l_1\mathbf{p}_1^T & -l_1 \\ -l_2\mathbf{p}_2^T & -l_2 \\ -l_3\mathbf{p}_3^T & -l_3 \\ -l_4\mathbf{p}_4^T & -l_4 \end{pmatrix}.$$

5.
```
Dg = [-l.*p' -l];
```

6.
```
syms mu [4 1] real
```

7.
```
DL = Df + mu'*Dg;
```

8.
```
g = 1 - l.*(w'*p + b)';
mug = mu'.*g';
```

9.
```
KKT = [DL mug];
```

10.
```
vars = [w' b mu'];
sol = solve(KKT, vars);
```

    Running `subs(KKT, sol)` verifies that everything evaluates to zero, as required.

11. By running `subs(mu', sol)` we can see that row 2 includes a negative value of μ_1; this will not result in a minimiser.

12. Feasible solutions require $\mathbf{g}(\mathbf{w}, b) \leqslant \mathbf{0}$. We can see from `subs(g', sol)` that only rows 1, 2 and 6 have feasible solutions.

13. `r1` is a vector where row i contains 1 if the i-th solution is feasible.

    `r2` is a vector where row i contains 1 if the i-th solution for $\boldsymbol{\mu}$ has $\boldsymbol{\mu} \geqslant \mathbf{0}$, which is required by the KKT theorem.

14. Running `find(r1 & r2)` finds the row numbers of potential minimisers.

15. The previous answer shows that the only valid solution is in row 6.

    Running the following will give the coefficients for the support vector machine:

    ```
    vals = subs([w' b], sol);
    result = vals(6,:);
    ```

    This shows that $\mathbf{w} = \begin{pmatrix}\frac{1}{2} & 1\end{pmatrix}^T$ and $b = -\frac{7}{2}$.

    So the optimal separating hyperplane is given by

    $$\tfrac{1}{2}x_1 + x_2 - \tfrac{7}{2} = 0 \iff x_2 = \tfrac{7}{2} - \tfrac{1}{2}x_1$$

16. ```
    hold on
    scatter(p(1,1:2),p(2,1:2),50,'+')
    scatter(p(1,3:4),p(2,3:4),50,'*')
    X=0:4;
    plot(X,(7-X)/2)
    ```

17. ```
    w = result(1:2);
    b = result(3);
    q = [1/2 2 7/2; 3 3 2];
    sign(w*q + b)
    ```

    The labels are -1, 1 and 1.