The University of Melbourne

School of Engineering

Semester 2 Assessment 2014

ENGR30002 – Fluid Mechanics

Exam Duration: 3 hours

This paper has TEN (10) pages consisting of SIX (6) questions.

Authorized material:

Electronic calculators approved by the School of Engineering may be used.
Two Charts and one Table of Formulae are attached.

Instructions to Invigilators:

Script books to be provided.

Instructions to Students:

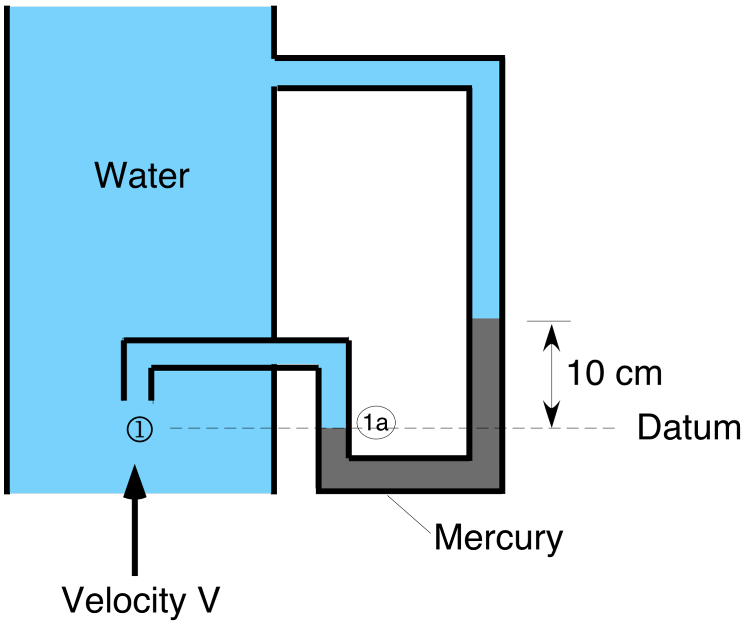
All questions are to be attempted.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

Water is flowing upwards through a vertical pipe. When a manometer is attached to the pipe in the manner shown in the diagram, the manometer fluid (mercury) is displaced by 10 cm as illustrated.

Ignore friction in the pipe.



Density of water	1000 kg m^{-3}
Specific gravity of mercury	13.6
Gravitational acceleration	9.81 m s^{-2}

(i) Express the pressure at point (1a) in terms of pressure and velocity at point (1)

(2 marks)

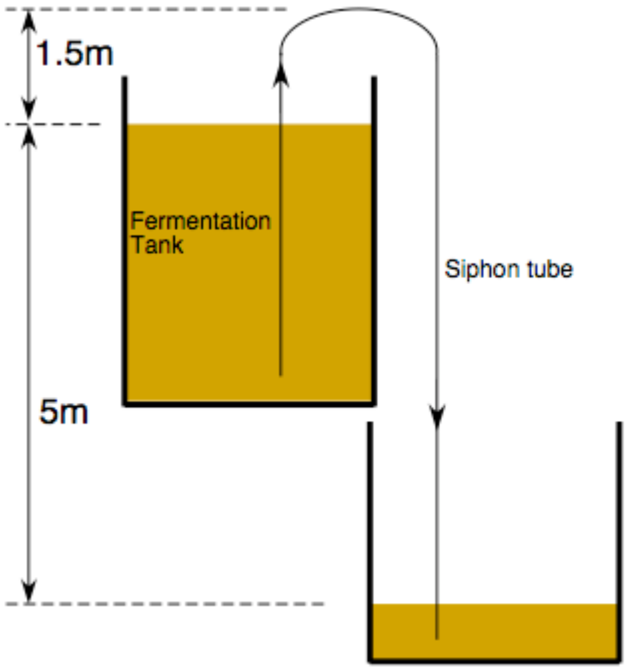
(ii) Calculate the velocity V of the water in the pipe.

(10 marks)

(Total for Question 1 = 12 marks)

Question 2

A fermented broth is decanted from a fermentation tank to a receiving tank by siphon as shown in the diagram below. Both tanks are open to the atmosphere. The siphon tube is smooth and 10m long with an internal diameter of 25.4 mm. The surface of the liquid in the receiving tank is 5m below the surface in the fermentation tank. Assume that the flow through the siphon tube has been started. Ignore tube entry and exit losses.



Density of the fermented broth	990 kg m^{-3}
Viscosity of the fermented broth	0.00495 Pa s
Acceleration due to gravity	9.81 m s^{-2}
Atmospheric pressure	101.3 kPa

(i) Calculate the volumetric flow rate.

(10 marks)

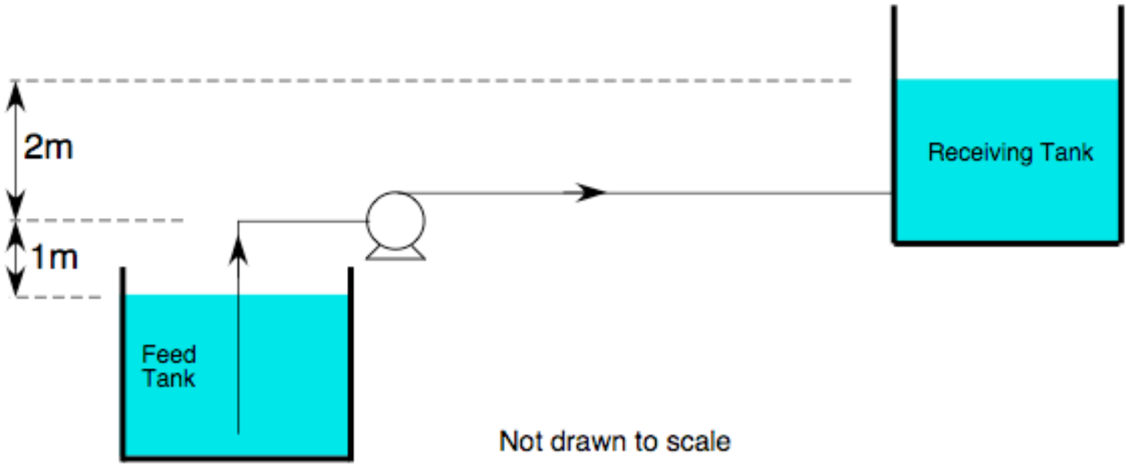
(ii) Calculate the pressure at the highest point of the siphon tube which is 1.5m above the liquid level in the fermentation tank; the tube length from the tube entrance to the highest point is 4m.

(7 marks)

(Total for Question 2 = 17 marks)

Question 3

Water is pumped from one large, open tank to a second large, open tank as shown in the diagram below. The pipe internal diameter throughout is 10 cm, and the total length of the pipe between the pipe entrance and pipe exit is 165 m. The center line of the pump is 1m above the water level in the feed tank and 2m below the level in the receiving tank. Resistance coefficients for the pipe entrance and exit, as well as an equivalent length for the elbow, are given in the data below. The Fanning friction factor is taken to equal 0.015.



Density of water	1000 kg m^{-3}
Viscosity of water	0.001 Pa s
Acceleration due to gravity	9.81 m s^{-2}
Resistance coefficient of pipe entry	0.5
Resistance coefficient of pipe exit	1.0
Equivalent length of the elbow	35 pipe diameters
Fanning friction factor	0.015
Atmospheric pressure	101.3 kPa
Vapour pressure	3 kPa
Required NPSH	4m

(Question 3 continues on next page)

Question 3 (continued)

(i) If the characteristic curve for the pump head h_p (m) versus volumetric flow rate Q (m^3/s) is given by

$$h_p = 12 - 70Q - 4300Q^2$$

then calculate the volumetric flow rate of water between the two tanks.

(10 marks)

(ii) Calculate the brake power of the pump, if its mechanical efficiency is 70 per cent.

(4 marks)

(iii) Determine the absolute roughness of the pipe.

(3 marks)

(iv) Calculate the Available NPSH, if the length of the piping on the suction side of the pump is 5m.

(5 marks)

(v) Considering your answer in part (iv), is there a cavitation problem? You must give a correct reason for your conclusion.

(1 mark)

(Total for Question 3 = 23 marks)

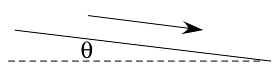
Question 4

A sewer pipe of circular cross section is made of concrete with a roughness of 0.045 mm. It is used to convey water at uniform, turbulent flow conditions such that the pipe is exactly half full (so this is channel flow). The slope of the pipe is such that there is a vertical fall of 1 m for every 3000 m in the horizontal direction. The volumetric flow rate is given by

$$Q = 2AD_e^{2/3} e^{-1/6} \sqrt{g \sin\theta}$$

g = 9.81 m/s^2

tan θ = 1/3000



(i) Calculate the pipe diameter given that the volumetric flow rate is 1.75 m^3/s.

(6 marks)

(ii) Hence calculate the water velocity in the pipe.

(2 marks)

(Total for Question 4 = 8 marks)

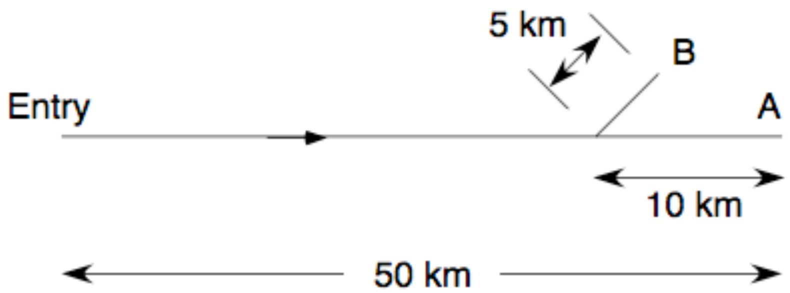
Question 5

The mechanical energy equation for horizontal, isothermal, compressible, ideal gas flow in a pipe of uniform cross-section is

$$\frac{P_2^2 - P_1^2}{2(RT/M)} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{2fL}{D}\left(\frac{G}{A}\right)^2 = 0$$

where all symbols have their usual meaning. You may ignore the kinetic energy term when using this equation to calculate pressure. Otherwise you should retain the kinetic energy term.

Natural gas flows through a pipeline 50 km long to a receiving station A. At a point 10 km before A, a branch leads off from the main pipeline and runs 5 km to a receiving station B. For all pipes, the internal diameter is 20 cm and the Fanning friction factor is $f = 0.005$. At the entry to the pipeline, the pressure is 1000 kPa and the mass flow rate is 1.125 kg/s. The pressure at station A is 400 kPa. Assume that the flow is isothermal at 30°C and is not choked. Ignore energy losses at the branch point.



Gram molecular weight of natural gas	16
Gas constant R	8.314 J mol^{-1} K^{-1}

(i) Calculate the pressure where the pipe branches.

(6 marks)

(ii) Calculate the mass flow rate of gas entering station B

(7 marks)

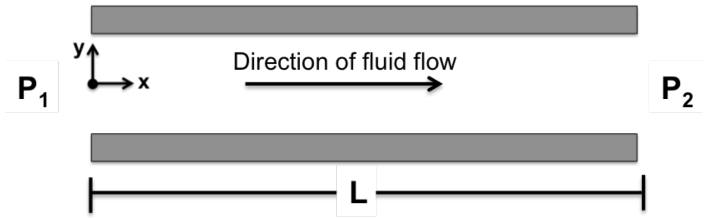
(iii) Calculate the velocity of the gas entering station B

(7 marks)

(Total for Question 5 = 20 marks)

Question 6

We have steady, fully developed pressure driven flow of a Newtonian fluid between two stationary, horizontal, flat, parallel plates. Consider plates of length L with pressure P_1 and P_2 at the ends as shown in the Figure. The fluid flows only in the positive x-direction. The z-dimension is perpendicular to the plane of the paper, and velocity and pressure are independent of the z-coordinate. Ignore gravity in your analysis.



(i) Show that the fluid velocity v_x depends only on the y -coordinate

(2 marks)

(ii) Show that the pressure depends only on x

(2 marks)

(iii) Show that $\frac{dp}{dx}$ is constant and write down an expression for it

(2 marks)

(iv) Derive an expression that describes the fluid velocity $v_x(y)$

(7 marks)

(v) Qualitatively sketch the velocity profile of the fluid.

(2 marks)

(vi) Suppose the top plate is now moving in the positive x-direction with a velocity slower than the maximum velocity of the fluid. Qualitatively sketch the velocity profile of the fluid.

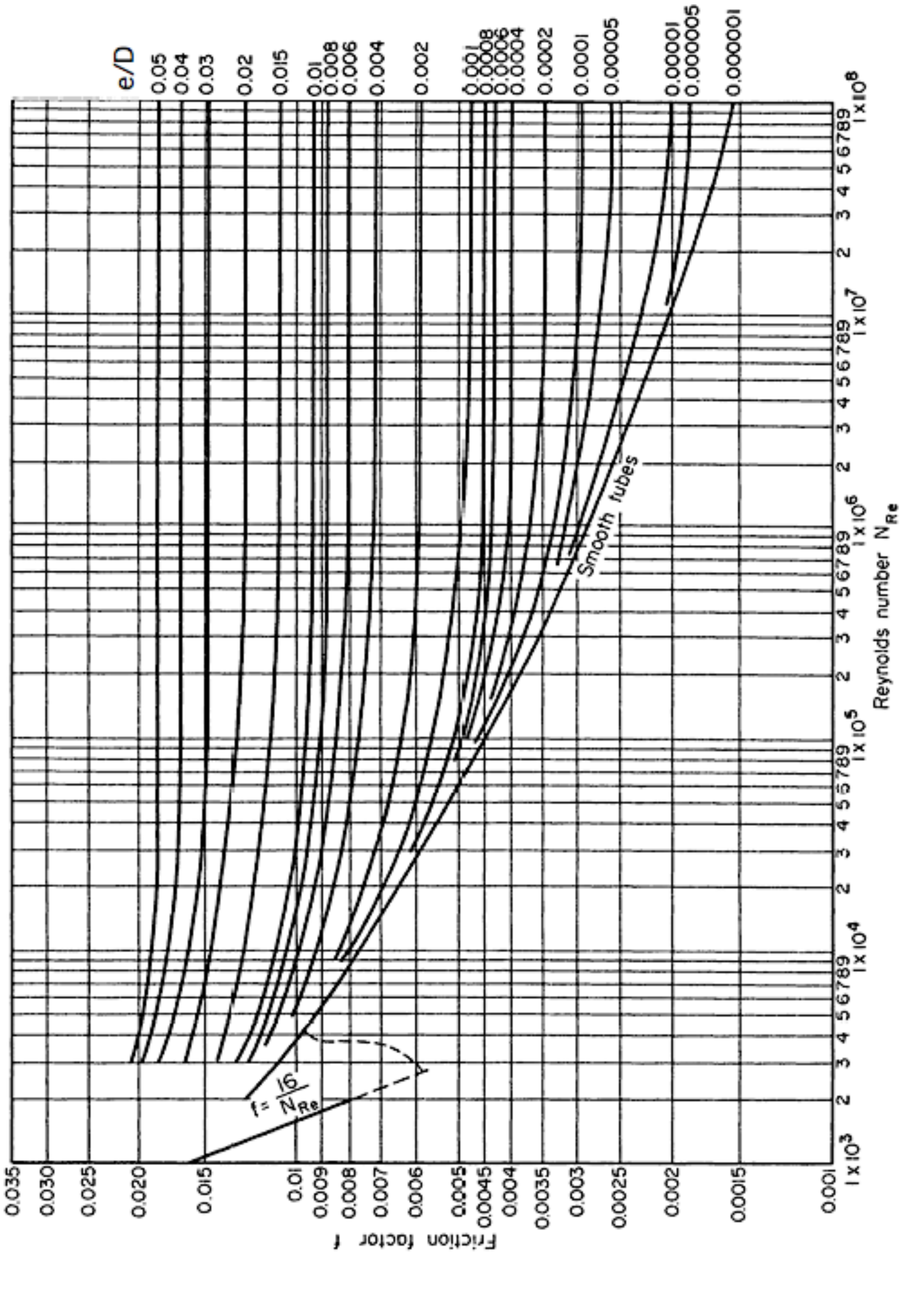
(2 marks)

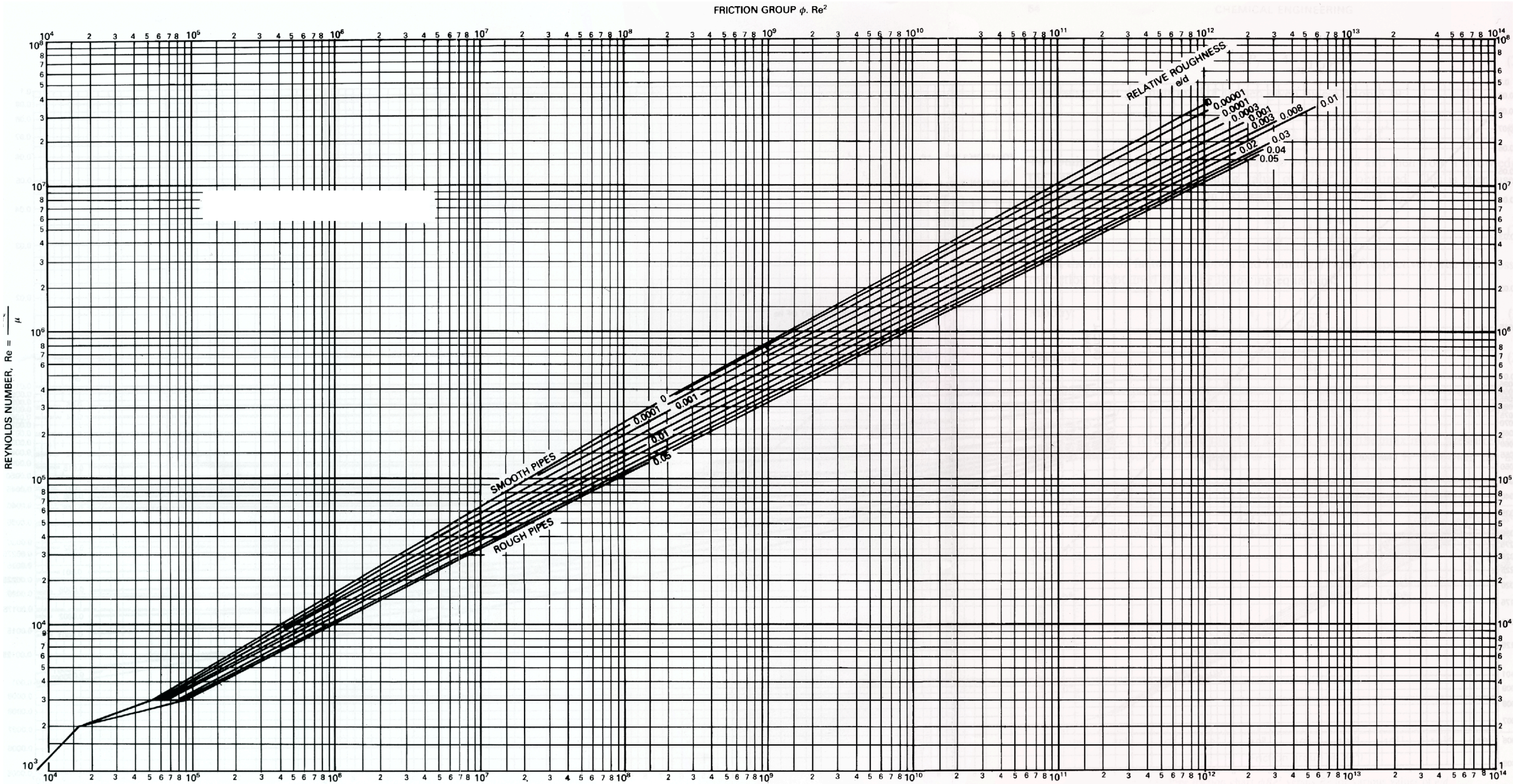
(vii) Now suppose that both plates are moving in the negative x-direction with the same velocity so that the net flow rate is zero. Qualitatively sketch the velocity profile of the fluid in this case

(3 marks)

(Total for Question 6 = 20 marks)

(Total for paper = 100 marks)


e/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
0.000005
0.000001
Smooth tubes
$f = \frac{16}{N_{Re}}$
Friction factor f
0.035
0.030
0.025
0.020
0.015
0.01
0.009
0.008
0.007
0.006
0.005
0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
Reynolds number N_{Re}
1×10^3
1×10^4
1×10^5
1×10^6
1×10^7
1×10^8


FRICTION GROUP ϕ. Re^2
REYNOLDS NUMBER, Re =
RELATIVE ROUGHNESS e/d
SMOOTH PIPES
ROUGH PIPES
0.00001
0.0001
0.0003
0.001
0.003
0.008
0.01
0.02
0.03
0.04
0.05

Continuity and Navier-Stokes equations for incompressible homogeneous fluids in Cartesian, cylindrical, and spherical coordinates

Cartesian	Cylindrical	Spherical
Continuity equation		
$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$	$\frac{1}{r}\frac{\partial (rv_r)}{\partial r}+\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta}\right)+\frac{\partial v_z}{\partial z}=0$	$\frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial (v_\theta \sin\theta)}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}=0$
Navier-Stokes equation		
$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)$ $=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)$ $=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)$ $=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_r}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}-\frac{2v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}-\frac{2v_\theta\cot\theta}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]$
$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)$ $=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)$ $=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)$ $=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_\theta}{\partial \theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta}{r^2\sin^2\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial \phi}\right]$
$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)$ $=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$ $=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r v_\phi}{r}+\frac{v_\theta v_\phi\cot\theta}{r}\right)$ $=-\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_\phi}{\partial \theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2}-\frac{v_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial \phi}\right]$