Fluid Mechanics

Topic 10.1

Applications of hydrostatics

Goals for this class

Take understandings of hydrostatics and apply them to calculate:

- Forces
- Pressures
- Moments

In fluidic systems at rest

Pressure variation in a fluid at rest

In the vertical direction, we have already derived the law of hydrostatic pressure from the conservation of momentum. This law states that:

$$\frac{\partial P}{\partial z} = -\rho g$$

In the horizontal direction, the conservation of momentum shows that, in a fluid at rest:

$$\frac{\partial P}{\partial x} = 0$$

This leads to the result that the pressure is identical at all points in a horizontal plane connected by the same fluid.

Pressure variation in a fluid at rest

The equality of pressure at equal elevations is exploited in hydraulic jacks, lifts and brakes, where a small force can be applied over a small area to generate a much larger force over a larger area.

Example Problem 10.1

Suppose you need to lift your 1500 kg car with a hydraulic jack to change a tire. For the jack shown in the diagram below, what force (F) must you apply to the handle if the smaller diameter (d) is 1/5th the diameter of the larger diameter? The handle lever is 40 cm long, and the piston anchor is placed 2 cm from the lever pivot.

Example Problem 10.1

Example Problem 10.1

Hydrostatic forces of vertical surfaces

(e.g. gates, walls, dams)

A submerged object is subjected to a distribution of fluid pressures over its surface. We want to describe this distributed force as a single resultant force, for which we need to determine the **magnitude**, the **direction** and the **point of application** (this is termed the "centre of pressure")

Note: The effects of atmospheric pressure often cancel out, se we tend to use gauge pressure in these circumstances.

Hydrostatic forces of vertical surfaces

magnitude

Hydrostatic forces of vertical surfaces

This gives us the result that **the magnitude of the force acting on a submerged plane is equal to the product of the pressure at the centre of the surface (Pc) and the area of the surface**.

Direction: The force acts perpendicular to the surface

Point of application: The centre of pressure of a submerged plate is not necessarily the same as the centre of the plate. It will act where there is no net moment on the plate.

To determine the point of application, we equate the moment of the resultant force and the moment of the distributed pressure force.

Hydrostatic forces of vertical surfaces

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Hydrostatic forces of vertical surfaces

Steps:

- Examine moments about z = 0
- Moment of resultant force =
- Moment of distribute pressure forces (M_p)

Hydrostatic forces of vertical surfaces

The moment created by the pressure has to be equal to the moment from the resultant force:

Example Problem 10.1

A 3 m wide, 8 m high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water. The gate is hinged at its base and is held closed by a horizontal force, F_H, located at the centre of the gate. The maximum value of this force is 3,500 kN.

a) Determine the maximum water depth (h) above the centre of the gate that can exist without the gate opening.
b) Does your answer change if the gate is hinged at the top, rather than at the bottom?

Example Problem 10.2

Example Problem 10.2

Example Problem 10.2

Special case: vertical surfaces that protrude from the surface

F_R	
h_R	

BUOYANCY, FLOTATION AND STABILITY

When and object is completely submerged in, or floating on, a fluid, the fluid pressure exerts a force on the object in the z-direction known as the **buoyant force (F_B)**. This force is caused by the increase of pressure with depth, such that the pressure forces acting on the object from below exceed those acting from above.