MAST30001 Stochastic Modelling – 2016

Assignment 2

If you didn't already hand in a completed and signed Plagiarism Declaration Form (available from the LMS or the department's webpage), please do so and attach it to the front of this assignment.

Don't forget to staple your solutions and to print your name, student ID, and the subject name and code on the first page (not doing so will forfeit marks). The submission deadline is **Friday, 21 October by 4pm** in the appropriate assignment box at the north end of Richard Berry Building (near Wilson Lab).

There are 2 questions, both of which will be marked. No marks will be given for answers without clear and concise explanations. Clarity, neatness and style count.

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1. Assume that rainstorms arrive in Melbourne according to a Poisson process with rate 1/2 per day, the amount of rain that falls during each storm is exponentially distributed with *mean* 4 mm, and the amount of rain that falls in separate storms are independent of each other and the times of arrivals of the storms. Let $(N_t)_{t\geq 0}$ be the number of storms between now and t days from now and $(R_t)_{t\geq 0}$ be the amount of rainfall between now and t days from now.

   (a) What is the expected amount of rainfall in Melbourne over the 5 days?

   (b) What is the probability that over the next 5 days, there are exactly 3 storms, 2 of which have rainfall amounts less than 4 mm, and 1 with a rainfall amount more than 4 mm?

   (c) What is the probability that over the next 5 days, there are exactly 3 storms, 2 of which have rainfall amounts less than 4 mm, and 1 with a rainfall amount more than 6 mm?

   (d) Given that there is exactly 1 storm with a rainfall amount more than 6 mm in the next 5 days, what is the expected number of storms with a rainfall amount less than 7 mm over this same time period?

2. Customers arrive to a queuing system according to a Poisson process with rate 4 per hour. If there are fewer than 3 people in the queue, then an arriving customer will join the queue, and otherwise will leave the system. At exponential rate 2 (per hour) times, a server arrives and instantaneously serves all customers in the queue (if there is no one in the system, the server does nothing and exits the system).

   (a) What is the long run proportion of time there is no one in the queue?

   (b) What is the average number of customers in the system?

   (c) What is the expected amount of time that customers that enter the system have to wait for service?

   (d) What is the long run proportion of arriving customers that enter the system?

   (e) Given an arriving server finds customers in the system, what is the expected number of customers served?

   (f) If X_t denotes the number of customers in the system at time $t \geq 0$, find $P(X_t = 0|X_0 = 0)$.