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1) Momentum equation

$$F = \rho_w A_2 V_2^2$$

$$70 = 1000 \times \frac{\pi \times 0.03^2}{4} \times V_2^2$$

$$V_2 = 9.95 \, m/s$$

continuity

$$A_1 V_1 = A_2 V_2$$

$$\frac{\cancel{\pi} \times 0.1^2}{\cancel{4}} \times V_1 = \frac{\cancel{\pi} \times 0.03^2}{\cancel{4}} \times 9.95$$

$$V_1 = \frac{0.03^2}{0.1^2} \times 9.95$$

$$= 0.896 \, m/s$$

MEB between 1 and 2

$$\frac{\Delta P}{\rho} + \overset{0}{\cancel{g \Delta z}} + \frac{\Delta v^2}{2} + \overset{0}{\cancel{Ws}} + \overset{0}{\cancel{F}} = 0$$

$$\frac{\Delta P}{\rho} = \frac{1}{2}(V_1^2 - V_2^2)$$

$$P_2 - P_1 = \frac{1}{2}(0.896^2 - 9.95^2) \times 1000$$

$$P_1 = 101325 + 49100$$

$$= 150.425 \, kPa$$

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2) height of spillway.

assume frictionless bump

$$E_1 = E_2$$

$$\frac{Q^2}{2gb^2(4)^2} + 4 = \frac{Q^2}{2gb^2(0.6)^2} + 0.6$$

let $\frac{Q}{b} = q$

$$\frac{q^2}{2 \times 9.81 \times 16} + 4 = \frac{q^2}{2 \times 9.8 \times 0.36} + 0.6$$

$$0.139\, q^2 = 3.4$$

$$q^2 = \frac{3.4}{0.139}$$

$$q = \sqrt{\frac{3.4}{0.139}}$$

$$= 4.95 \, m^2/s$$

$$h_{crit} = \left(\frac{q^2}{9.8}\right)^{1/3}$$

$$= 1.36 \text{ m}$$

$$h_{bump} = 4 - 1.36$$

$$= \underline{2.64 \text{ m}} \,//$$

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3) $Q = 200 L/min$

$= \frac{200}{1000} \times \frac{1}{60} \; m^3/s$

$= 3.33 \times 10^{-3} m^3/s$

$h_{f_1} = h_{f_2} = h_{f_3}.$

$$\frac{2 \times f_F \times 2 \times V_1^2}{D_1} = \frac{2 \times f_F \times 2 \times V_2^2}{D_2} = \frac{2 \times f_F \times 2 \times V_3^2}{D_3}.$$

$D_1 = D_3$ same, thus f_F and V also same.

assume f_F constant for all pipes.

$$\frac{2 \times f_F \times 2 \times Q_1^2}{(D_1)^5 \pi^2} = \frac{2 \times f_F \times 2 \times Q_2^2}{D_2^5 \pi^2}$$

$$\frac{Q_1^2}{(18)^5} = \frac{Q_2^2}{(24)^5}.$$

$$Q_1 = \left(\frac{18}{24}\right)^{5/2} Q_2.$$

$Q_1 + Q_2 + Q_3 = Q$ total

$$\left(2 \times \left(\frac{18}{24}\right)^{5/2} + 1\right) Q_2 = Q\text{total}$$

$Q_2 = 1.69 \; m^3/s.$

$= 101.3 \; L/min$

$Q_1 = Q_3 = \left(\frac{18}{24}\right)^{5/2} \times 101.3$

$= 49.35 \; L/min$

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4) (1) $\tau_{xz}\Big|_{x=h} = 0$

$\left.\begin{array}{l} \dfrac{dV_z}{dx} = 0 \quad " \end{array}\right\}$ b.c

$V_z(x=0) = W_{belt}$

using equation of motion in the z-component

$$\rho g_z = \mu \frac{\partial^2 V_z}{dx^2}$$

$$\frac{\partial^2 V_z}{\partial x^2} = \frac{\rho g_z}{\mu}$$

$$\frac{\partial V_z}{\partial x} = \frac{\rho g_z}{\mu} x + C_1$$

using B.C (1), $C_1 = -\dfrac{\rho g_z h}{\mu}$

$$\frac{\partial V_z}{\partial x} = \frac{\rho g_z}{\mu}(x-h)$$

$$V_z(x) = \frac{\rho g_z}{\mu}\left(\frac{x^2}{2} - hx\right) + C_2$$

at $x=0$, $V_z = W_{belt}$

$$V_z(x) = \frac{\rho g_z}{\mu}\left(\frac{x^2}{2} - hx\right) + W_{belt}$$

no net flow

$$\int_0^h V_z(x)\, dx = 0$$

$$\int_0^h \frac{\rho g_z}{\mu}(x^2/2 - hx)\, dx + \int_0^h W_{belt}\, dx = 0$$

$$\frac{\rho g_z}{\mu}\left(\frac{x^3}{6} - \frac{hx^2}{2}\right)_0^h + W_{belt}(h) = 0$$

$$\frac{\rho g_z}{\mu}\left(-\frac{h^3}{3}\right) + W_{belt}h = 0$$

$$W_{belt} = \frac{\rho g_z h^2}{3\mu}$$

$$V_z(x) = \frac{\rho g_z}{\mu}\left(\frac{x^2}{2} - hx + \frac{h^2}{3}\right)$$

6 (a) diagphragm pump → avoid corrosion of the pump by liquid

(b) peristaltic pump → biological isolation.

(c) Cavitation ⇒ bubbles form inside pump and cause physical damage

(d). $\Delta z = -W_s/g$

$= h_p$

$= 15m.$

head is 15 m of water.

$$\text{head oil} = 15m \times \frac{\rho_{water}}{\rho_{oil}}$$

$$= 15m \times 1000 \times 0.00116$$

$$= 17.4\ m.$$

(e) (i) 0010, 0011, 0012, 0013, 0014

(ii) 0013

(iii) 31.5 L/min

(iv) use throttling valve that ↑ system head and will ↓ Q as Friction term increases.

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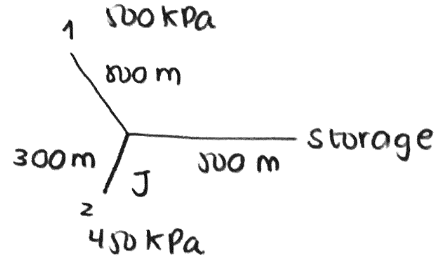
7)



Find P_J

$$\frac{P_J^2-(500\times10^3)^2}{2\times\frac{8.314\times298}{19\times10^{-3}}}+\frac{2\times500\times0.005}{0.15}\times\left(\frac{1.5}{\frac{\pi\times0.15^2}{4}}\right)^2=0$$

$$P_J = \underline{432856\ Pa}$$

Find G_2

$$\frac{432856^2-(450\times10^3)^2}{2\times\frac{8.314\times298}{19\times10^{-3}}}+G^2\ \frac{\times2\times300\times0.005}{\frac{\pi^2\times0.15^5}{16}}=0$$

$$G^2 = 0.906\ kg^2/s^2$$

$$G = 0.952\ kg/s.$$

$$G_R = G_1 + G_2$$
$$= 2.45\ kg/s.$$

Find P_R

$$\frac{P_R^2-(432856)^2}{2\times\frac{8.314\times298}{19\times10^{-3}}}+\frac{2.45^2\times2\times500\times0.005}{\frac{\pi^2\times0.15^5}{16}}=0$$

$$P_R = 345120\ Pa$$

$$\rho_R = \frac{P_R}{RT/M} = \frac{345120}{\frac{8314\times298}{19\times10^{-3}}} = 2.65\ kg/m^3$$

$$\frac{G}{\rho_R A} = V_R$$

$$V_R = \frac{2.45}{\frac{\pi\times0.15^2}{4}\times2.65} = \underline{52.4\ m/s}$$