

MAST30001 Stochastic Modelling

Tutorial Sheet 9

1. A rent-a-car washing facility can wash one car at a time. Cars arrive to be washed according to a Poisson process with rate 3 per day and the service time to wash a car is exponential with mean $7/24$ days. It costs the company \$150 per day to operate the facility and the company loses \$10 per day for each car tied up in the washing facility. The company can increase the rate of washing to get down to a mean service time of $1/4$ days at the additional cost of $\$C$ per day. What's the largest C can be for this upgrade to make economic sense?
2. Customers arrive at a bank according to a Poisson process rate λ . The bank's service policy is that
 - if there are fewer than 4 customers in the bank, then there is 1 teller,
 - if there are 4 – 9 customers, there are 2 tellers,
 - if there are more than 9 customers, there are 3 tellers.

Tellers' service times are independent and exponentially distributed with rate μ . Model the number of customers in the bank as a birth and death chain and determine for what values of λ and μ there is stable long run behavior and for these parameters compute the steady state distribution. [*Hint: This is similar to the analysis of the $M/M/a$ queue done in lecture.*]

3. ($M/M/\infty$ queue) Assume that in a queuing system customers arrive according to a rate λ Poisson process, customers are always served immediately (for example, customers making purchases on the internet), and the service time of a customer is exponential with rate μ , independent of arrival times and other service times.
 - (a) Model this queue as a birth-death chain and write down its generator.
 - (b) Describe the long run behaviour of the chain.
 - (c) When the queue is in stationary (i.e., after its been running a long time), what is the expected number of customers in the system, number of customers in the queue, number of busy servers, and service time for an arriving customer?
 - (d) Let X_t be the number of customers in the system (including those being served) at time t and set $X_0 = 0$. What is $E[X_t]$? [*Hint: if $m(t) = E[X_t]$, consider $m'(t)$.*] You should check your formula makes sense as t tends to infinity.
4. ($M/G/\infty$ queue) In a certain communications system, information packets arrive according to a Poisson process with rate λ per second and each packet is processed in one second with probability p and in two seconds with probability $1-p$, independent of the arrival times and other service times. Let N_t be the number of packets that have entered the system up to time t and X_t be the number of packets in the system (including those being served) at time t .
 - (a) Is $(X_t)_{t \geq 0}$ a Markov chain? (No detailed argument is necessary here, just think about it heuristically.)
 - (b) If $X_0 = 0$, what is the distribution of X_2 ?

- (c) If $X_0 = 0$, is there a “stationary” limiting distribution $\pi_k = \lim_{t \rightarrow \infty} P(X_t = k)$?
If so, what is it?
- (d) If $X_0 = N_0 = 0$, what is the joint distribution of X_t and N_t ?