MAST30013 – Techniques in Operations Research

Semester 1, 2021

Tutorial 10

1. Consider the constrained nonlinear program

$$\begin{aligned} \min \quad & \tfrac{1}{2}x_1^2 + \tfrac{1}{2}x_2^2 - x_1 + x_2 \\ \text{subject to} \quad & x_1, x_2 \leq 0. \end{aligned}$$

   (a) Write down the log barrier penalty function $P_k(\boldsymbol{x})$ with penalty parameter $\alpha_k = k$.

   (b) Write down $\boldsymbol{\nabla} P_k(\boldsymbol{x})$, and solve $\boldsymbol{\nabla} P_k(\boldsymbol{x}) = \mathbf{0}$ to find any stationary points $\boldsymbol{x}^k = (x_1^k, x_2^k)$ for $P_k(\boldsymbol{x})$.

   (c) Find all stationary points $\boldsymbol{x}^k = (x_1^k, x_2^k)$ for $P_k(\boldsymbol{x})$, and find the limit $\boldsymbol{x}^* = \lim\limits_{k\to\infty} \boldsymbol{x}^k$.

   (d) For each stationary point, write down an estimate $\boldsymbol{\lambda}^k$ of the optimal Lagrange multiplier vector, and find the limit $\boldsymbol{\lambda}^* = \lim\limits_{k\to\infty} \boldsymbol{\lambda}^k$.

2. Consider the constrained nonlinear program

$$\begin{aligned} \min \quad & x_1^2 + 2x_2^2 \\ \text{subject to} \quad & x_1^2 + x_2^2 \ \leq \ 1 \\ & x_1 + x_2 \ = \ 1. \end{aligned}$$

   (a) Find all KKT points and determine if any are minima.

   (b) Do the Lagrangian Saddle Point inequalities hold? That is, for feasible $\boldsymbol{x}$ and $\lambda \geq 0$, $\eta \in \mathbb{R}$,

$$L(\boldsymbol{x}^*, \lambda, \eta) \ \leq \ L(\boldsymbol{x}^*, \lambda^*, \eta^*) \ \leq \ L(\boldsymbol{x}, \lambda^*, \eta^*).$$

3. Consider the nonlinear program

$$\begin{aligned} \min \quad & \tfrac{1}{4}x_1^4 - \tfrac{1}{2}x_1^2 + x_2^2 \\ \text{subject to} \quad & x_1 \geq 0 \\ & x_2 \geq 2. \end{aligned}$$

   (i) Write down the log barrier penalty function $P_k(\boldsymbol{x})$ with penalty parameter $\alpha_k = k$.

   (ii) Write down $\boldsymbol{\nabla} P_k(\boldsymbol{x})$, and solve $\boldsymbol{\nabla} P_k(\boldsymbol{x}) = \mathbf{0}$ to find any stationary points $\boldsymbol{x}^k = (x_1^k, x_2^k)$ for $P_k(\boldsymbol{x})$.

   (iii) Find the limit $\boldsymbol{x}^* = \lim\limits_{k\to\infty} \boldsymbol{x}^k$.

   (iv) Find the limit $\boldsymbol{\lambda}^* = \lim\limits_{k\to\infty} \boldsymbol{\lambda}^k$.