

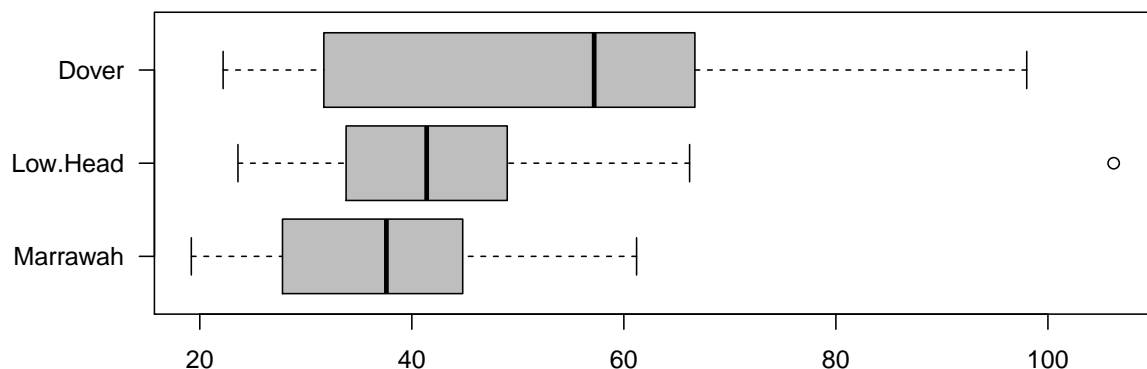
# MAST20005/MAST90058: Week 3 Lab Solutions

```
library(MASS)
library(evd)
tasmania <- read.csv("tasmania.csv")
```

1. `summary(tasmania[, c(7, 9, 30)])`

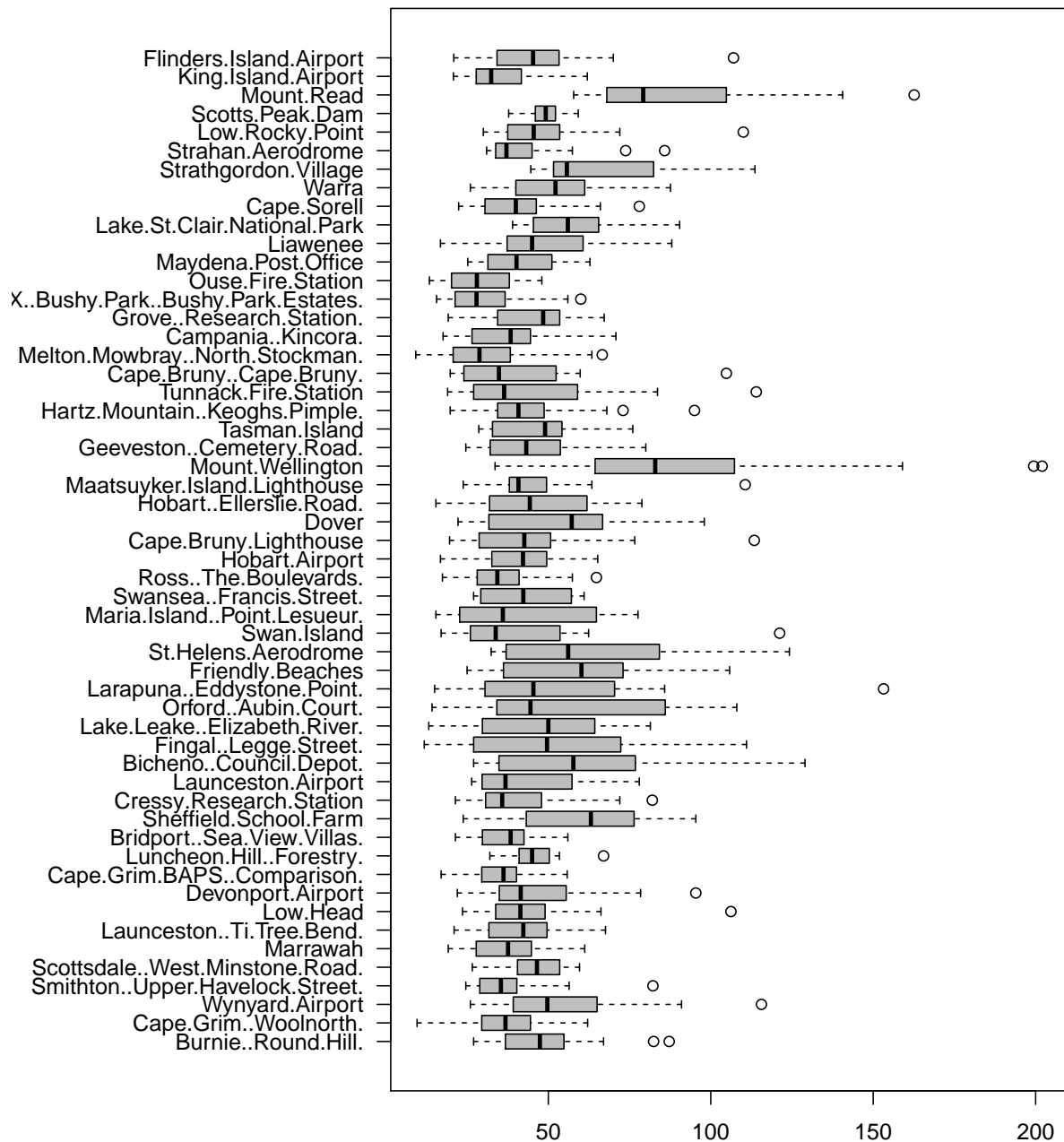
```
##      Marrawah      Low.Head      Dover
## Min.   :19.20   Min.    : 23.60   Min.    :22.20
## 1st Qu.:28.50   1st Qu.: 33.80   1st Qu.:33.55
## Median :37.60   Median : 41.40   Median :57.20
## Mean   :37.85   Mean    : 45.31   Mean    :52.73
## 3rd Qu.:42.10   3rd Qu.: 49.00   3rd Qu.:66.65
## Max.   :61.20   Max.    :106.20   Max.    :98.00
##                                     NA's   :3
```

```
par(mar = c(4, 6, 1, 1)) # adjust margins to fit axis labels
boxplot(tasmania[, c(7, 9, 30)], col = 8, horizontal = TRUE, las = 1)
```



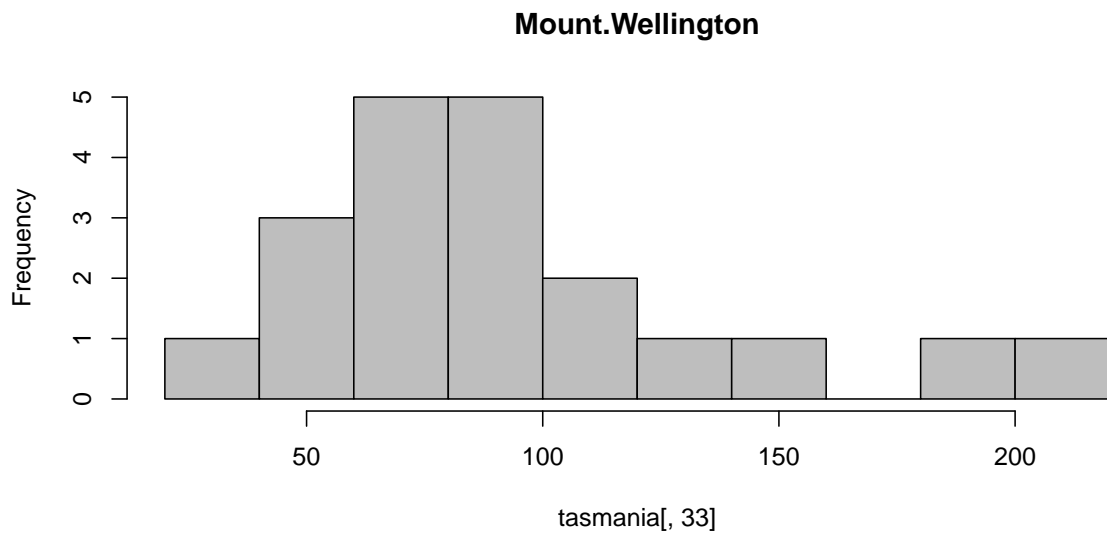
2. The data frame `tasmania` is also a list, which means we can pass it directly to `boxplot()`. However, we should omit the first column, since these are dates rather than rainfall measurements.

```
par(mar = c(4, 14, 1, 1))
boxplot(tasmania[, -1], col = 8, horizontal = TRUE, las = 1)
```

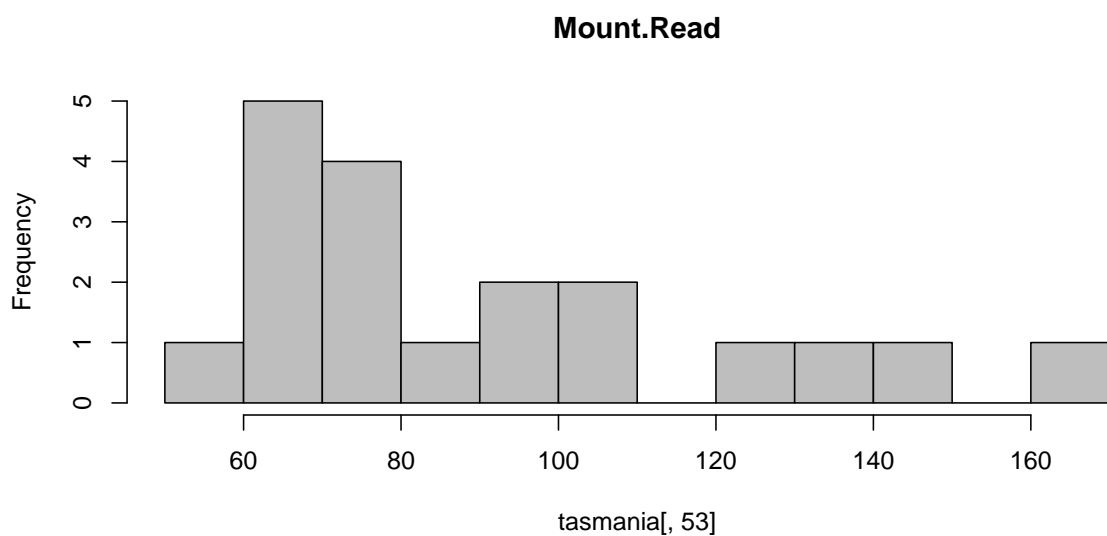


3. Mount Wellington and Mount Read have much higher maximum rainfall than the other stations. Perhaps these are the only two mountainous stations in this dataset?

```
hist(tasmania[, 33], 10, col = 8, main = names(tasmania)[33])
```



```
hist(tasmania[, 53], 10, col = 8, main = names(tasmania)[53])
```



```
dgumbel <- function(x, mu, sigma)
  exp((mu - x) / sigma - exp((mu - x) / sigma)) / sigma
start1 <- list(mu = 50, sigma = 10)
fitdistr(tasmania[, 33], densfun = dgumbel, start = start1)

##      mu      sigma
## 74.389990 32.104613
## ( 7.519231) ( 5.825075)

fitdistr(tasmania[, 53], densfun = dgumbel, start = start1)

## Error in fitdistr(tasmania[, 53], densfun = dgumbel, start = start1): 'x'
## contains missing or infinite values
```

The Mount Read data is missing the first value. Let's just omit it and do the MLE calculation:

```
fitdistr(tasmania[-1, 53], densfun = dgumbel, start = start1)

## Warning in log(dens(parm, ...)): NaNs produced

##      mu      sigma
## 78.659729 20.726543
## ( 4.976391) ( 4.026766)
```

```
4. a <- 0.577215
   b <- 1.978
   sigma.mm <- function(x)
     sqrt((mean(x^2) - mean(x)^2) / (b - a^2))
   mu.mm <- function(x)
     mean(x) - a * sigma.mm(x)

   B <- 10000 # number of simulated samples

   # Initialise vectors to hold estimates from the simulations.
   sigma.mms <- numeric(B)
   mu.mms <- numeric(B)

   # Run simulations.
   for (i in 1:B) {
     x <- rgumbel(20, 50, 10)
     sigma.mms[i] <- sigma.mm(x)
     mu.mms[i] <- mu.mm(x)
   }

   # Evaluate simulation results.
   mm.mean <- c(mean(mu.mms), mean(sigma.mms))
   mm.var <- c(var(mu.mms), var(sigma.mms))
   true <- c(50, 10)
   bias.estimate <- mm.mean - true
   mm.mean

## [1] 50.235427 9.477394

   bias.estimate

## [1] 0.2354274 -0.5226064

   mm.var

## [1] 5.699199 4.629722
```

```

5. d <- rep(0:6, c(5, 7, 12, 9, 5, 1, 1)) # create vector with the data
d

## [1] 0 0 0 0 0 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3
## [34] 4 4 4 4 4 5 6

table(d) # table of counts

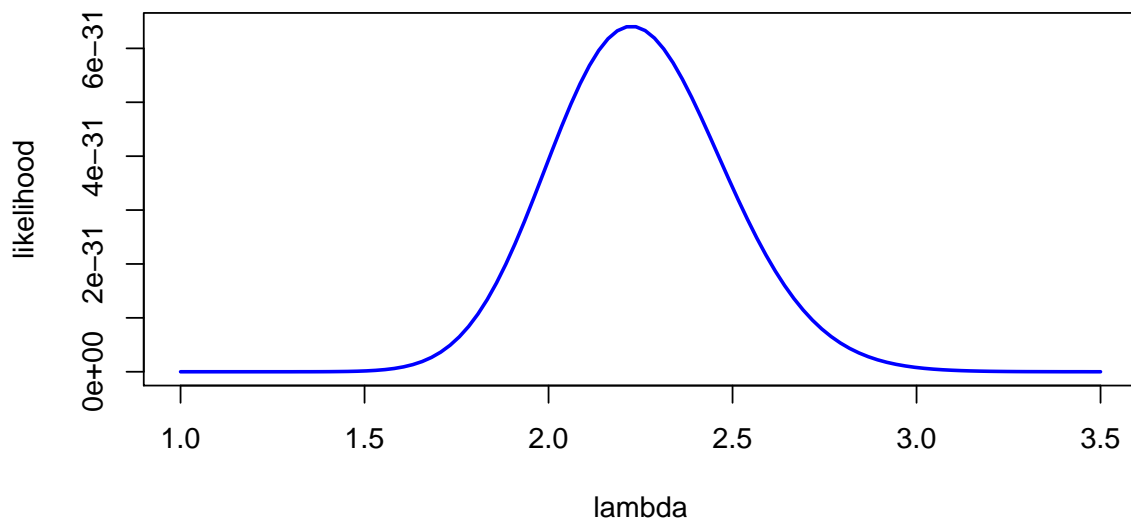
## d
##  0  1  2  3  4  5  6
##  5  7 12  9  5  1  1

mean(d) # MLE

## [1] 2.225

lambda <- seq(1, 3.5, length.out = 100)
likelihood <- numeric(100)
for (i in 1:100)
  likelihood[i] <- prod(dpois(d, lambda[i]))
plot(lambda, likelihood, type = "l", lwd = 2, col = 4)

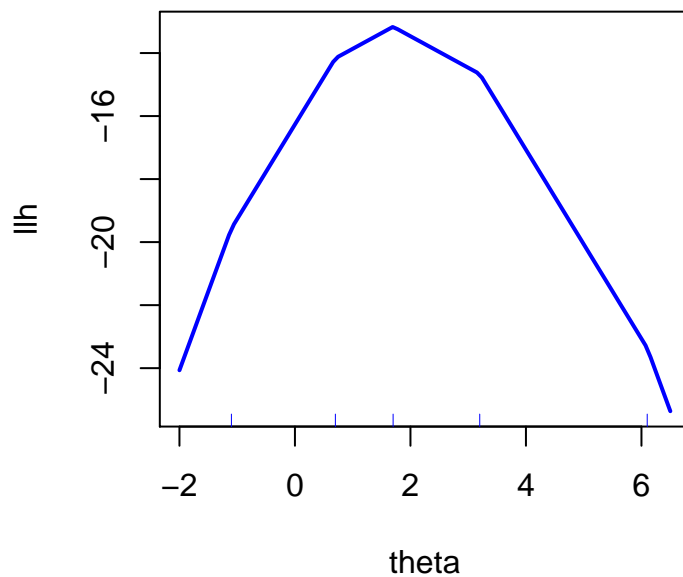
```



```

6. d <- c(6.1, -1.1, 3.2, 0.7, 1.7) # data from the hint
theta <- seq(-2, 6.5, length.out = 100)
llh <- numeric(100)
for (i in 1:100)
  llh[i] <- -length(d) * log(2) - sum(abs(d - theta[i]))
plot(theta, llh, type = "l", lwd = 2, col = 4)
rug(d, col = 4) # plot the location of data points, on the x-axis

```



```

7. x <- c(0.0256, 0.3051, 0.0278, 0.8971, 0.0739,
          0.3191, 0.7379, 0.3671, 0.9763, 0.0102)
sum(log(x))

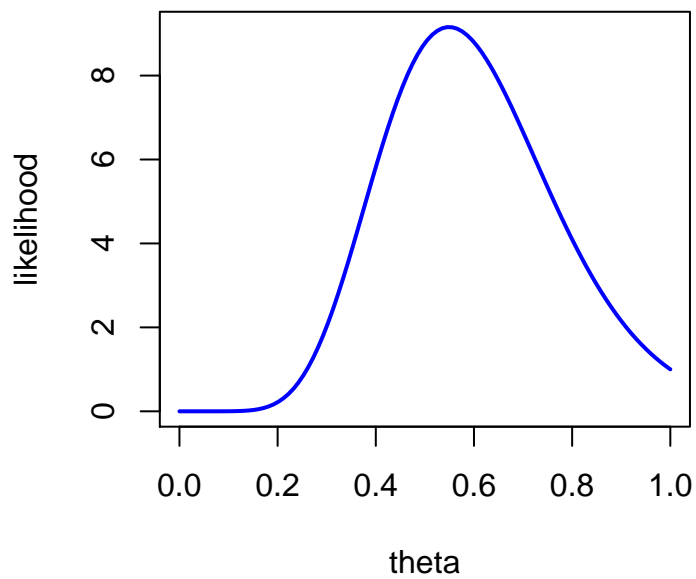
## [1] -18.2063

sum(x)

## [1] 3.7401

ddistr <- function(x, theta)
  theta * x^(theta - 1)
theta <- seq(0, 1, length.out = 100)
likelihood <- numeric(100)
for (i in 1:100)
  likelihood[i] <- prod(ddistr(x, theta[i]))
plot(theta, likelihood, type = "l", lwd = 2, col = 4)

```



```
-length(x) / sum(log(x)) # MLE using formula

## [1] 0.5492604

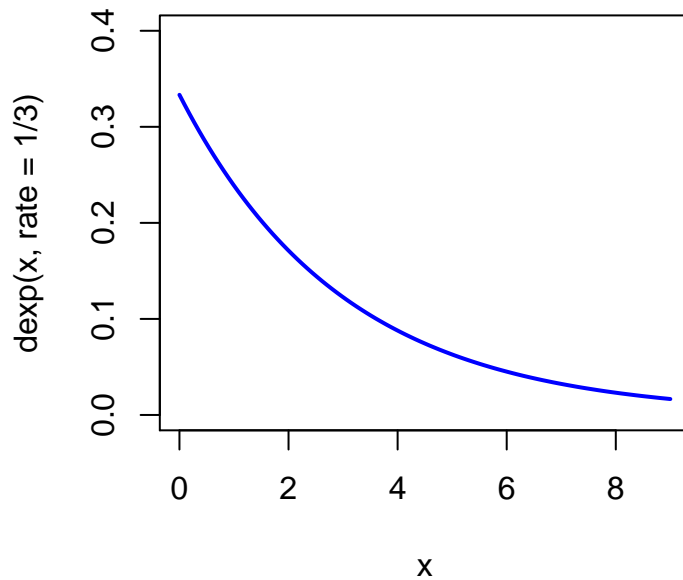
fitdistr(x, densfun = ddistr, start = list(theta = 0.5))

## Warning in stats::optim(x = c(0.0256, 0.3051, 0.0278, 0.8971, 0.0739, 0.3191,
: one-dimensional optimization by Nelder-Mead is unreliable:
## use "Brent" or optimize() directly

##      theta
## 0.5492188
## (0.1736776)

# Don't worry about the above warning.
# The optimisation is working fine in this case.
```

8. `curve(dexp(x, rate = 1/3), 0, 9, ylim = c(0, 0.4), lwd = 2, col = 4)`



```
B <- 10000
xbars <- numeric(B)
for (i in 1:B) {
  x <- rexp(20, rate = 1/3)
  xbars[i] <- mean(x)
}
c(mean(xbars), var(xbars)) # estimates from simulations

## [1] 2.9987859 0.4577881

c(3, 3^2 / 20) # true values

## [1] 3.00 0.45

fitdistr(c(3.5, 8.1, 0.9, 4.4, 0.5), "exponential")

##      rate
## 0.2873563
## (0.1285097)

1 / 0.2873563 # MLE for theta rather than the rate parameter

## [1] 3.48
```