Week 6

Further studies of covariance functions

[icon] Overview

Welcome to Week 6 of Spatial Analysis.

This week we will cover two topics:

- Estimation of covariance functions and variograms

- Properties of covariance functions

You will start this week learning about several methods of estimating covariance functions and variograms. In particular, you will consider the ordinary least squares and maximum likelihood estimation methods. You will use the geoR package and get experience in fitting covariance functions of real spatial data. You will learn how to model anisotropic data. You will continue practising with R by fitting the variograms for the Meuse dataset.

Finally, you will revise the known and learn new approaches to check whether a given function is a valid covariance function. You will use these approaches to construct new covariance models.

Finally, you will participate in a workshop, in which you can practise conducting an analysis of real spatial data.

By the end of this week, you will learn about:

Topic 1: Further studies of covariance functions and variograms

- Variogram's parameter estimation.
- Example of fitting variograms to simulated data.
- Example of fitting variograms to the Meuse data.
- Anisotropy.

Topic 2: Some properties of covariance functions

- Properties of covariance functions.
- Methods to obtain basic covariance functions.

By completing this module, you will be working towards the following subject-intended learning outcomes:

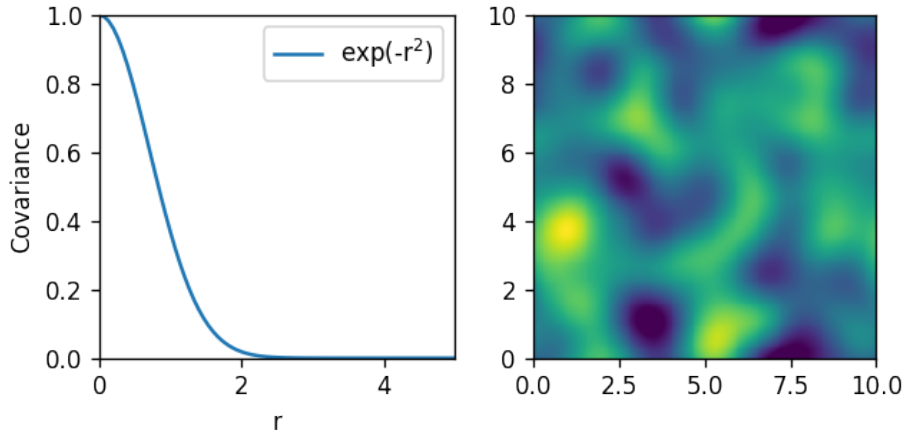
1. Formulate purposeful questions to explore new statistical ideas and subsequently design valid statistical experiments.
2. Present clear, well-structured analysis of important statistical model results.
3. Creatively find solutions to real-world problems consistent with those commonly faced by practising statisticians.
4. Professionally defend or question the validity of existing statistical analyses and associated evidence-based conclusions that are derived via application of sound spatial statistical methodology.

Topic 1: Further studies of covariance functions and variograms

You will learn and practise several methods of estimating covariance functions. We will use the geoR package and the s100 and Meuse data. Then, you will learn how to construct new complex theoretical models of covariance functions by using basic covariance functions and properties of the class of positive definite functions.

Figure 4.1

2D random field realisation and its covariance function



https://commons.wikimedia.org/wiki/File:Gaussian_process_2D_squared_exp.png

You will read the content, watch videos and complete activities to test your understanding of this topic.

Upon completion of this topic, you will be able to construct new covariance models and estimate parameters of covariance functions using actual spatial datasets. You will develop your spatial computational skills with the geoR package.

Variogram's parameter estimation

We will begin this topic by discussing how to fit parametric models to sample covariance functions or variograms. Parametric models of covariance functions are completely defined by a finite number of parameters. Usually, when one analyses real spatial data, these parameters are unknown values and must be estimated. We will explore the ordinary least squares and maximum likelihood estimation methods. Often, the ordinary least squares method is easier to use, but, in many cases, it was shown that the maximum likelihood methods provide better estimators of the parameters than the ordinary least squares.

Then, we consider how these methods can be applied to spatial data in R via the functions variofit, likfit and fit.variogam. We will use the simulated data s100 first. As this dataset is simulated from a known statistical model, it provides a convenient way to check whether the parameters of the fitted and theoretical variograms are similar or substantially different. Then, we will estimate the variogram for residuals of the detrended Meuse data. We will also consider fitting anisotropic models for the Meuse data.

Read

In the reading Covariance functions and variograms modelling, you will explore several methods to estimate parameters of covariance functions. You will apply them to the s100 data and the Meuse data using the geoR package.

From the folder, open the document titled **Week_6_Topic_1** and read **slides 1–20**.

Revise key R commands used in the first topic.

Read

In the reading Key R commands, you will revise some of the key R commands that were used in this topic's materials.

From the folder, open the document titled **Week_6_Topic_1** and read **slide 21**.

Topic 2: Some properties of covariance functions

We consider three properties of positive definite(covariance) functions. If a function fails one of these properties for particular values of its arguments, then it is not a covariance function.

However, if a function satisfies these properties, it is not necessarily a covariance function. There may be other properties that fail. So, one of the possible ways to demonstrate that a function is a covariance one is to construct it using some rules. We provide some basic rules on how to obtain valid covariance models.

For each of the above scenarios, we give simple examples to illustrate them.

Read

In the second part of the reading Covariance functions and variograms modelling, you will learn some basic theoretical properties of covariance functions and check them using simple examples. You will see how to construct new classes of covariance models using a few simple rules.

From the folder, open the document titled **Week_6_Topic_2** and read **slides 1–5**.

Workshop

Activity

Workshop

This activity will be completed in R. Repeat the R programming content covered in Week 6. Modify the code and understand the impact of different R parameters on changes in results.

Your task

- Repeat R commands learnt in Week 6 prior to the two-hour workshop session.
- Try to modify the code and understand the impact and meaning of different R function parameters. Interpret the observed changes in plots and analysis results.
- Feel free to discuss questions with other students as you go in the forum, and please also take the time to help others. It is amazing how much we all can learn from each other's questions, and how in helping others we strengthen our own understanding.
- Revisit these problems in later weeks and challenge yourself to get a deeper understanding to build on what you learn later.

Guidelines

- This activity is not graded but is an essential part of your learning. It will be held synchronously and facilitated by your instructor.
- You don't need to submit your R code; however, to be successful in this subject it is necessary to work through all R coding materials from this week and understand how to apply the corresponding R commands.
- You should spend around two hours on this activity.

Summary

This week, we started with further studies of covariance functions and variograms. We learned two methods to estimate their parameters using real spatial data. We also considered some properties of covariance functions that are used to reject not valid dependency models or built new ones.

Here's a list of tasks that you should be working on or have completed:

- **Required readings**
- **Workshop**

The following resources provide you with this week's references and additional suggested readings.

Additional suggested readings and resources

While these readings and resources are not essential, they provide greater insight into the concepts covered in the week's lectures and give you the choice to enhance your learning or pursue an area of interest in greater detail.

Software and data:

- CRAN documentation for the geoR package: https://cran.r-project.org/web/packages/geoR/index.html
- CRAN documentation for the gstat package: https://cran.r-project.org/web/packages/gstat/index.html
- Meuse river data set https://rsbivand.github.io/sp/reference/meuse.html

Theoretical concepts:

- Variograms: https://help.rockware.com/rockworks17/WebHelp/gridding_krig_variogram.htm

Books:

- Bivand, R. S., Pebesma, E., & Gomez-Rubio, V. (2013). Applied spatial data analysis with R (2nd ed.). Springer. https://doi.org/10.1007/s12061-014-9118-y Avalilable on-line in La Trobe EBL ebook Library
- Cressie, N.A.C (1993) Statistics for spatial data. Wiley. https://onlinelibrary.wiley.com/doi/book/10.1002/9781119115151 Avalilable on-line in La Trobe EBL ebook Library