ENGR30002 Fluid Mechanics

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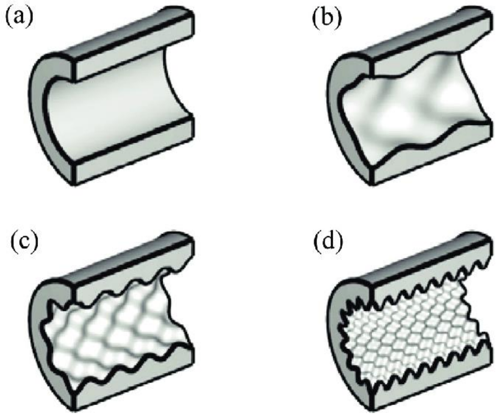
# 4 Pipe Flow

# What we did in the last module

- We applied conservation laws to fluid flow:
  - Conservation of mass: $G_1 = G_2$, $Q_1 = Q_2$, $A_1V_1 = A_2V_2$
  - Conservation of momentum
  - Conservation of energy → Bernoulli equation
- Although the Bernoulli equation is a nice way to think about fluid flow, it is an incomplete model!





# Module 4.1:

## Friction factors

# Beyond Bernoulli

- Although the Bernoulli equation is a nice way to think about fluid flow, it is incomplete (not fully correct):
    - We need to include a kinetic energy correction factor
    - We cannot neglect friction in the real world
- When deriving Bernoulli, we averaged kinetic energy over the cross-section by using the average velocity
    - This is not exactly correct, not does it accurately reflect experimental data
    - We need to add a **kinetic energy correction factor, α**

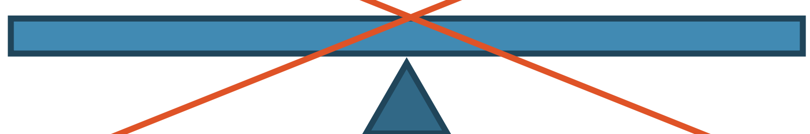
$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z = 0$$

        - $\alpha = 1$ for turbulent flow
        - $\alpha = 0.5$ for fully-developed laminar flow in circular pipes

# Friction exists

- According to Bernoulli, the mechanical energy of a fluid is constant down the length of your pipe
- However, this is not true in real systems!

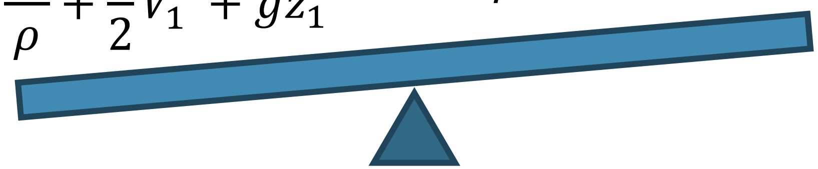
$$\frac{P_1}{\rho} + \frac{1}{2}\bar{V}_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}\bar{V}_2^2 + gz_2$$



# Friction exists

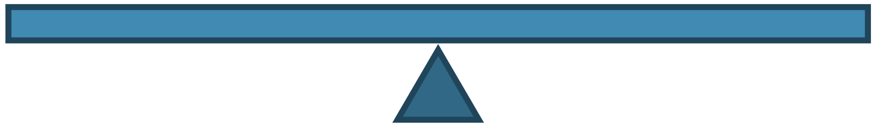
- Instead, energy contained in the flow is greater at Point 1 than Point 2

$$\frac{P_1}{\rho} + \frac{1}{2}\bar{V}_1^2 + gz_1 \neq \frac{P_2}{\rho} + \frac{1}{2}\bar{V}_2^2 + gz_2$$



- To balance energy, we must include energy that is lost due to friction

$$\frac{P_1}{\rho} + \frac{1}{2}\bar{V}_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}\bar{V}_2^2 + gz_2 + F$$



# The mechanical energy balance

- The mechanical energy balance includes frictional losses:

$$\frac{P_1}{\rho} + \frac{1}{2}\bar{V}_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}\bar{V}_2^2 + gz_2 + F$$

$$\frac{P_1}{\rho g} + \frac{1}{2g}\bar{V}_1^2 + z_1 = \frac{P_2}{\rho g} + \frac{1}{2g}\bar{V}_2^2 + z_2 + h_l$$

Bernoulli equation written in terms of energy:

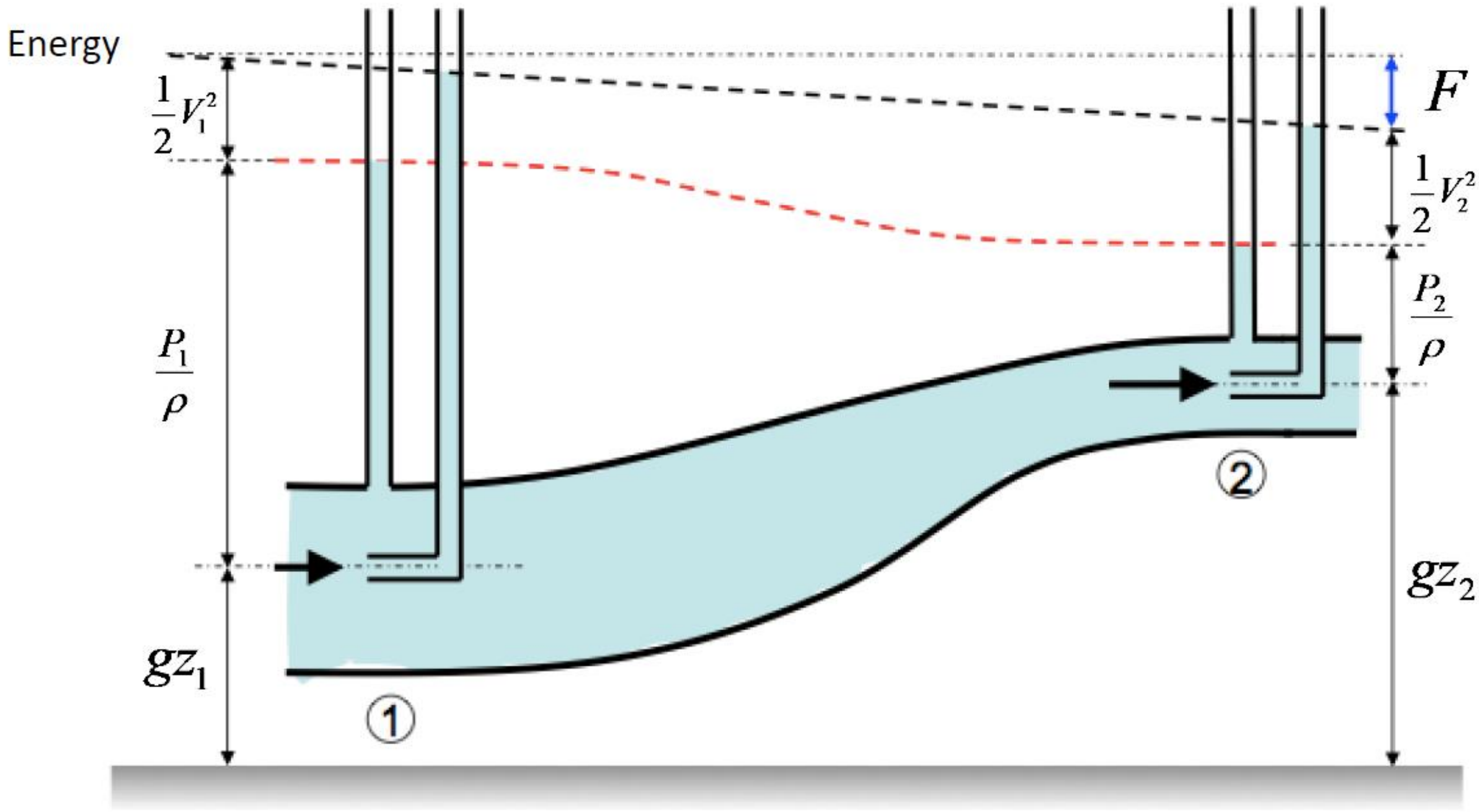
- Pressure energy
- Kinetic energy
- Potential energy
- Friction

Bernoulli equation written in terms of head:

- Pressure head
- Velocity head
- Gravitational head
- Head loss

# The mechanical energy balance

- We can illustrate the contribution of friction schematically:



# Calculating frictional losses

- Friction occurs with the pipe walls as the fluid flows
- This friction can be experimentally measured

$$\left(\frac{P_2}{\rho}+\frac{\bar{V}_2^2}{2\alpha}+gz_2\right)-\left(\frac{P_1}{\rho}+\frac{\bar{V}_1^2}{2\alpha}+gz_1\right)+W_S+F=0$$

- For uniform pipe flow with constant cross-section, $\bar{V}_1 = \bar{V}_2$

$$\left(\frac{P_2}{\rho}+gz_2\right)-\left(\frac{P_1}{\rho}+gz_1\right)+W_S+F=0$$

- For systems with no shaft work

$$\left(\frac{P_2}{\rho}+gz_2\right)-\left(\frac{P_1}{\rho}+gz_1\right)+F=0$$

# Calculating frictional losses

- Rearranging the previous expression to make F the subject:

$$F = \frac{P_1 - P_2}{\rho} + g(z_1 - z_2)$$

  - Using this expression, we could calculate the frictional losses that occurs during flow by measuring the pressures and heights of the pipe
  - The units of F correspond to **frictional losses per unit mass**
- Although it’s nice that we can measure frictional losses within a piping system, what we really need is a way of **predicting frictional losses**
- If we can do this, we can correctly size pumps (see later) to overcome friction and gravity before building a pipe network

# Friction factors

- In order to predict the frictional losses that our piping system will experience, we use friction factors
- The frictional term in the mechanical energy balance is commonly recast in terms of friction factors:

$$F = \frac{\text{work done against friction}}{\text{mass}}$$

$$F = \frac{\text{work done against friction/time}}{\text{mass/time}}$$

$$F = \frac{\text{force } \times \text{distance/time}}{\text{mass/time}}$$

$$\boldsymbol{F = \frac{\tau_w 2\pi R L \times \bar{V}}{\rho \bar{V} \pi R^2}}$$

$\tau_w$ is the shear stress at the wall

# Friction factors

- We can massage this expression a little more…

$$F = \frac{\tau_w 2\pi RL \times \bar{V}}{\rho \bar{V} \pi R^2} = \tau_w \frac{2L}{\rho R}$$

cancelling terms…

$$F = \tau_w \frac{4L}{\rho D}$$

letting D=2R

$$F = \frac{\tau_w}{\rho \bar{V}^2} \times \frac{4L\bar{V}^2}{D}$$

factor out $1/\bar{V}^2$, rearrange

$$\boldsymbol{F = \frac{4\phi L\bar{V}^2}{D}}$$

define $\phi = \tau_w/\rho\bar{V}^2$, which is dimensionless

- Plugging this back into our mechanical energy balance:

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_s + \boldsymbol{\frac{4\phi L\bar{V}^2}{D}} = 0$$

# So many friction factors!

- In fluid mechanics, two different friction factors are used:
  - Fanning friction factor, $f_F$
  - Darcy-Weisbach friction factor, $f_D$

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_S + \frac{\mathbf{4\phi L\bar{V}^2}}{\mathbf{D}} = 0$$

**Fanning friction factor, $f_F$**

$$f_F = 2\phi$$

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_S + \frac{\mathbf{2f_F L\bar{V}^2}}{\mathbf{D}} = 0$$

**Darcy-Weisbach friction factor, $f_D$**

$$f_D = 8\phi$$

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_S + \frac{\mathbf{f_D L\bar{V}^2}}{\mathbf{D}} = 0$$

# Fanning friction factor

- Most chemical engineering textbooks use the Fanning friction factor, while most civil engineering textbooks use the Darcy-Weisbach friction factor
- In this class, for totally historic and definitely not biased reasons, we'll mostly use the **Fanning friction factor, $f_F$**

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_S + \frac{\boldsymbol{2 f_F L \bar{V}^2}}{\boldsymbol{D}} = 0$$


Yea, cool cool cool, but how the f does f actually work?!

# Using friction factors

- Earlier in this module, we said that frictional losses (F) could be experimentally determined using the following equation:

$$F = \frac{P_1 - P_2}{\rho} + g(z_1 - z_2)$$

- Researchers have performed a huge number of experiments to see how this value changes with different aspects of a flow system:
  - Pipe diameter
  - Flow velocity
  - Fluid viscosity
  - Pipe roughness
  - Etc.

# Using friction factors

- To use the mechanical energy balances that contain friction factors, we need to know the value for friction factor
- Through experimentation, researches discovered that the **value of the friction factor** depends on two main criteria of the fluid flow system:

1. The **flow regime** of the fluid, which is quantified using the **Reynolds number, Re**
    - I.e. whether the fluid flow is **laminar** or **turbulent** significantly affects the value of the friction factor
    - For pipe flow, recall that:
        - Re<2000 → laminar flow
        - Re=2000-3000 → transitional flow
        - Re>3000 → turbulent flow
2. The **relative roughness of the pipe** that the fluid is flowing through

# Relative roughness

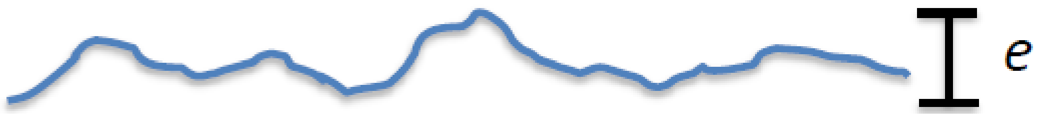
- What is this **relative roughness?**
  - Friction arises when fluid flows over the walls of the pipe
  - It turns out that the material the pipe is made from is not a big contributor to friction but…
  - What does greatly affect friction is the **roughness** of that surface
- The **relative (surface) roughness** is quantified as follows:

$$\text{Relative roughness} = \frac{e}{D}$$

  - e: height of roughness element [m]
  - D: pipe diameter [m]

# Relative roughness

- On a macroscopic level, a pipe's surface may look quite smooth
- But if you zoom in, the surface will have topography
  - The increase in surface area can increase frictional losses of energy
  - The parameter "e" is the average height of this roughness

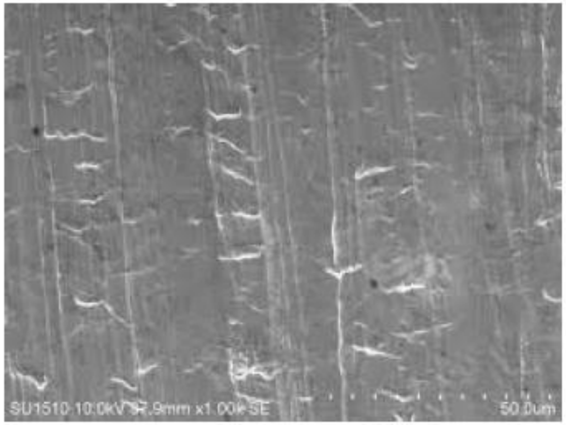



- Values of "e" must be experimentally determined
  - The value of "e" for commercially-purchased pipe will be reported to you by the supplier

# Relative roughness

- Here are some values of “e” for a variety of pipe types:

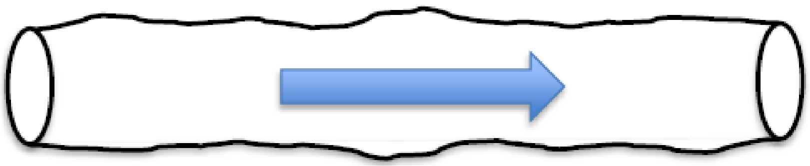
| Pipe material | Roughness, e [mm] |
|---|---|
| Riveted steel | 0.9-9 |
| Concrete | 0.3-3 |
| Wood stave | 0.2-0.9 |
| Cast iron | 0.26 |
| Galvanised iron | 0.15 |
| Asphalted cast iron | 0.12 |
| Commercial steel/wrought iron | 0.046 |
| Drawn tubing | 0.0015 |



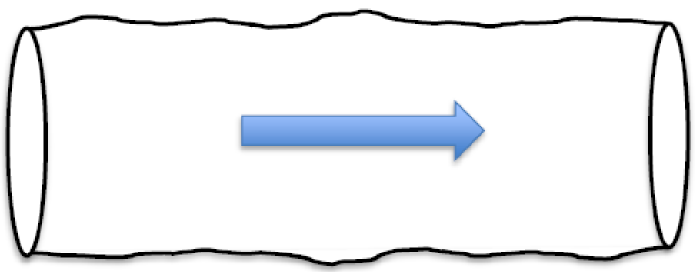

Scanning electron microscope image of brass surface

# Relative roughness: why does D come into play?

- In a pipe of smaller diameter:
  - A larger percentage of the fluid is in contact with the walls
  - A larger portion of the fluid is experiencing wall friction



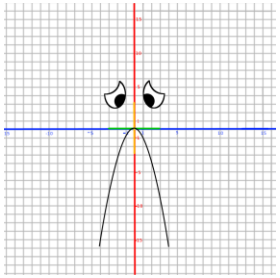
- In a pipe with the same roughness but a larger diameter:
  - A smaller percentage of the fluid is in contact with the walls
  - A smaller portion of the fluid is experiencing wall friction



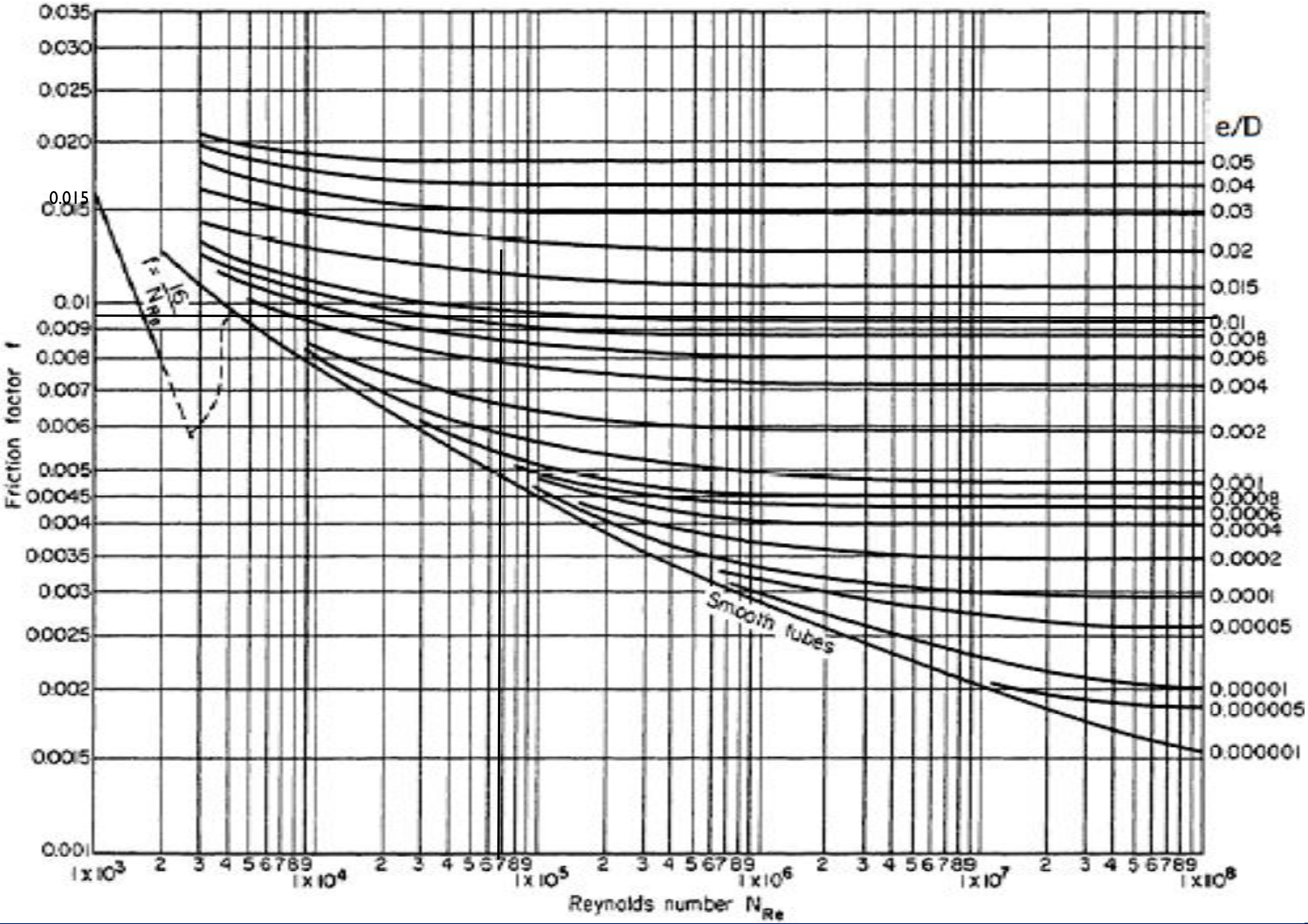
- That's why relative roughness is presented as e/D

# The Moody diagram

- Researchers have performed experiments in order to generate plots that visually relate the **(Fanning) friction factor, $f_F$** to the **Reynolds number** and the **relative roughness** of the pipe – these plots are referred to as Moody diagrams
- A Moody diagram:
  - Is a log-log plot
  - Has two y-axes and one x-axis
    - The x-axis is the Reynolds number of the flow system, Re
    - The y-axis on the right is the relative roughness of the pipe, e/D
    - The y-axis on the left is the Fanning friction factor, $f_F$
  - Each line on the diagram corresponds to a certain relative roughness
  - There is a line for "smooth pipe" that assumes no surface roughness
  - Most of the plot accounts for turbulent flow, representing that this is the realm we often operate within



# The Moody diagram



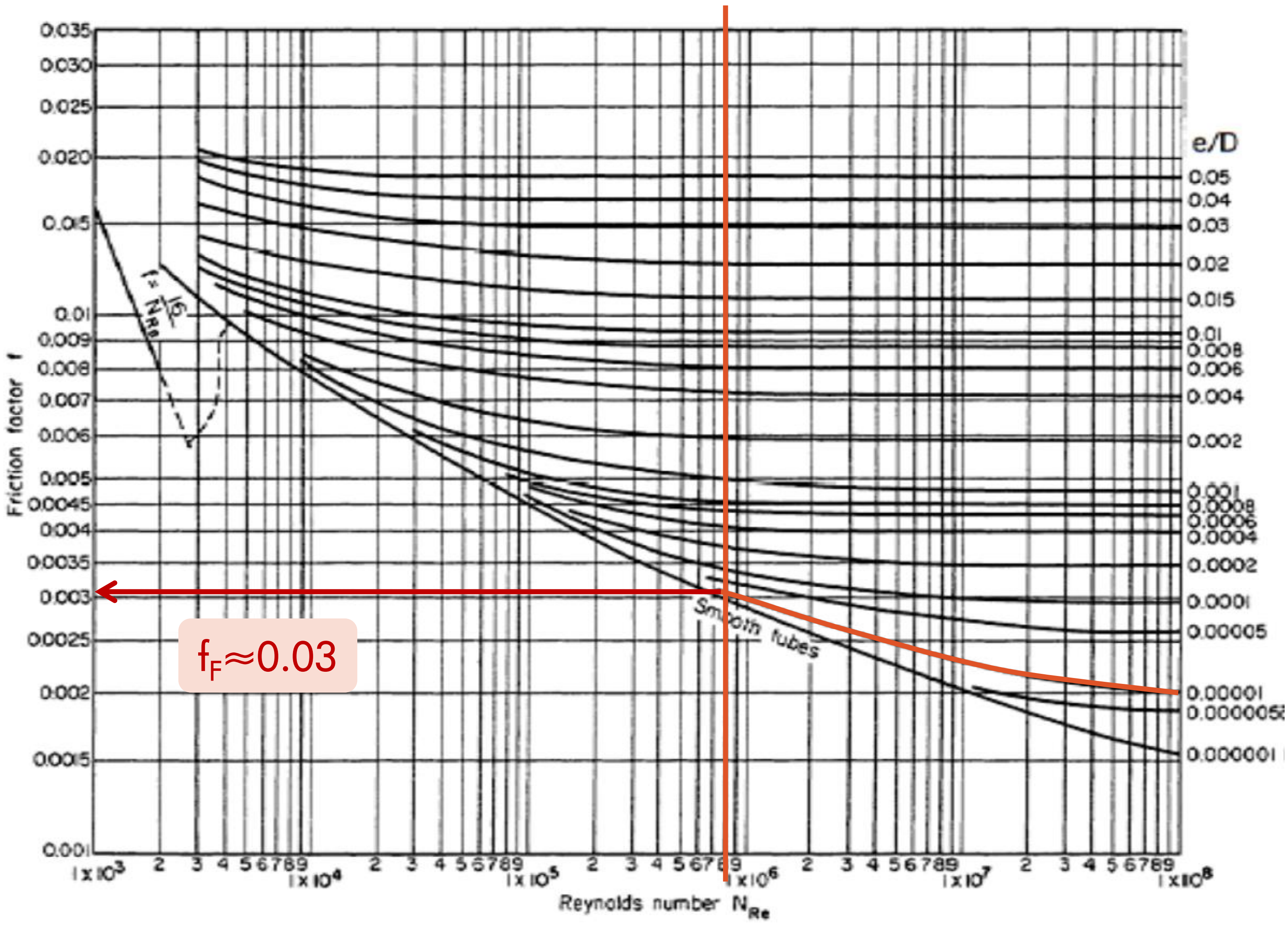
# The Moody diagram

- The Moody diagram allows you to determine the friction factor
- Strategy:
  1. Calculate the Reynolds number for your flow
  2. Calculate the relative roughness of your pipe
  3. Identify where those two points cross on the Moody diagram
  4. Read the value on the left y-axis to determine your friction factor

- Note: the version of the Moody diagram shown here provides you with the **Fanning friction factor, $f_F$**. There are other versions that provide you with the Darcy-Weisbach friction factor, $f_D$.

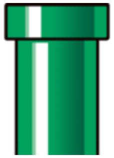
# Example 4.1

- You are pumping seawater from the ocean to a desalination plant. The fluid is flowing with an average velocity of 3 [m/s] through a pipe with a diameter of 250 [mm] and a roughness of 0.004 [mm]. The density and viscosity of seawater is 1029 [kg/m$^3$] and 0.96 [cP] respectively.
  - Determine the Fanning friction factor of this system.

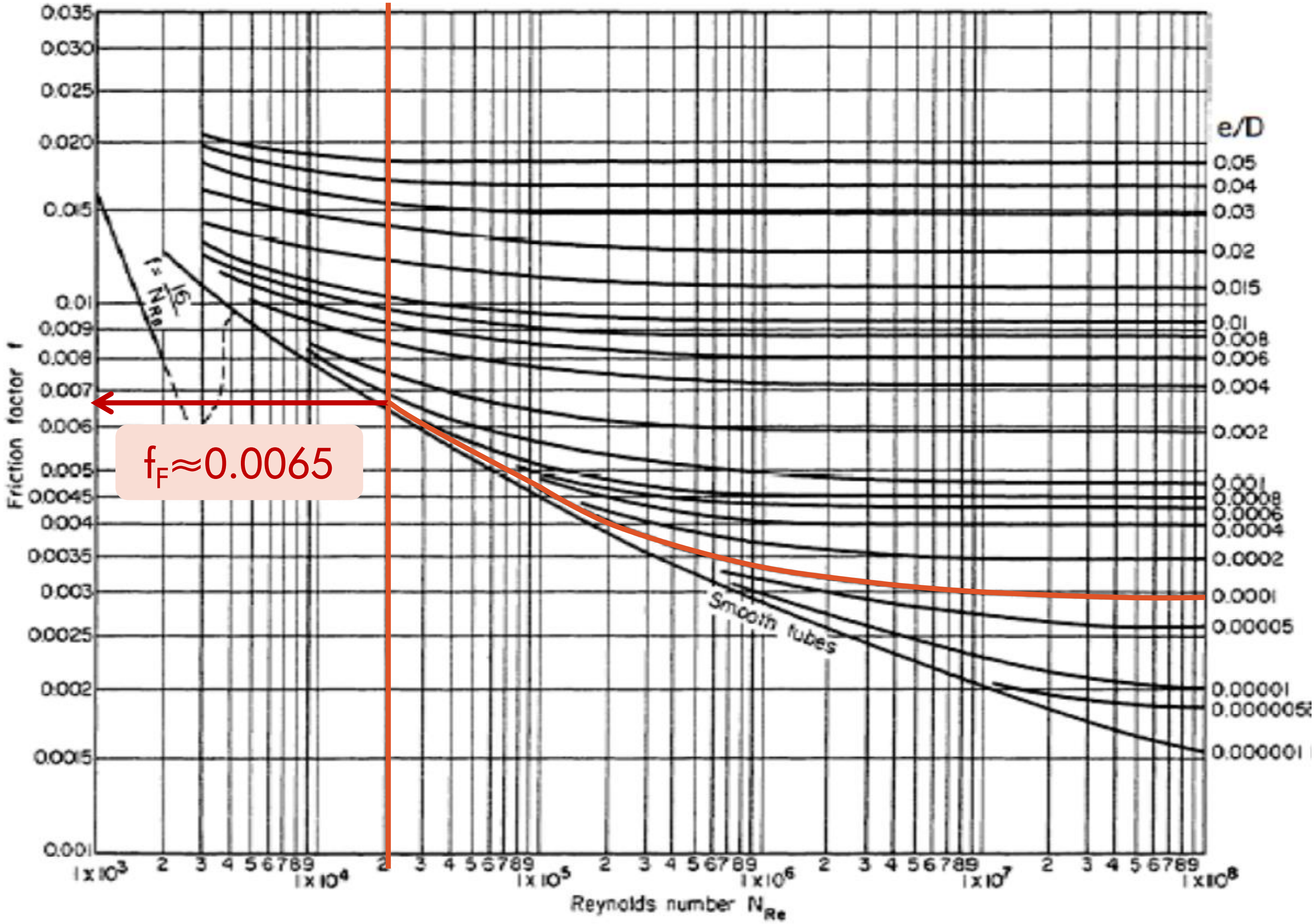


Friction factor f
Reynolds number $N_{Re}$
e/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
0.000001
$f = \frac{16}{N_{Re}}$
Smooth tubes
$f_F \approx 0.03$

# Example 4.2



- We have a 10-metre straight pipe through which 100 litres per minute of water is being pumped 1 metre uphill. The diameter of the pipe is 10 [cm], the roughness of the pipe is 0.001 [cm], the density of the water is 1000 [kg/m$^3$], and it viscosity is 0.001 [Pa.s]. The pressure at the exit of the pipe is $P_{atm}$.
  - What is the gauge pressure at the entrance of the pipe?

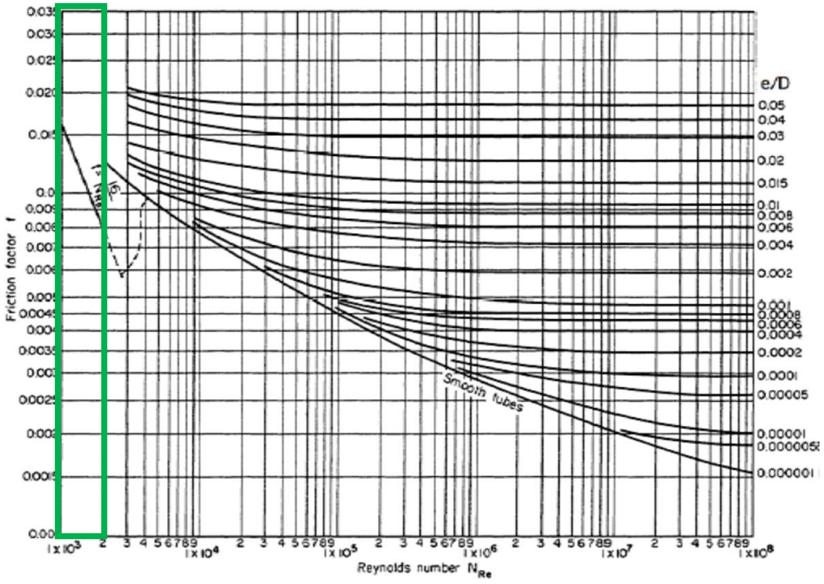


Friction factor f
Reynolds number $N_{Re}$
e/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
0.000005
0.000001
$f = \frac{16}{N_{Re}}$
Smooth tubes
$f_F \approx 0.0065$

# The Moody diagram

- If we look more closely at the Moody diagram, we can learn some interesting things about fluid flow in pipes
- Specifically, the diagram can be divided into four zones:
  - **Zone 1:** laminar zone (Re<2000)
  - **Zone 2:** transitional zone (2000<Re<3000)
  - **Zone 3:** low turbulence zone
  - **Zone 4:** high Reynolds number zone

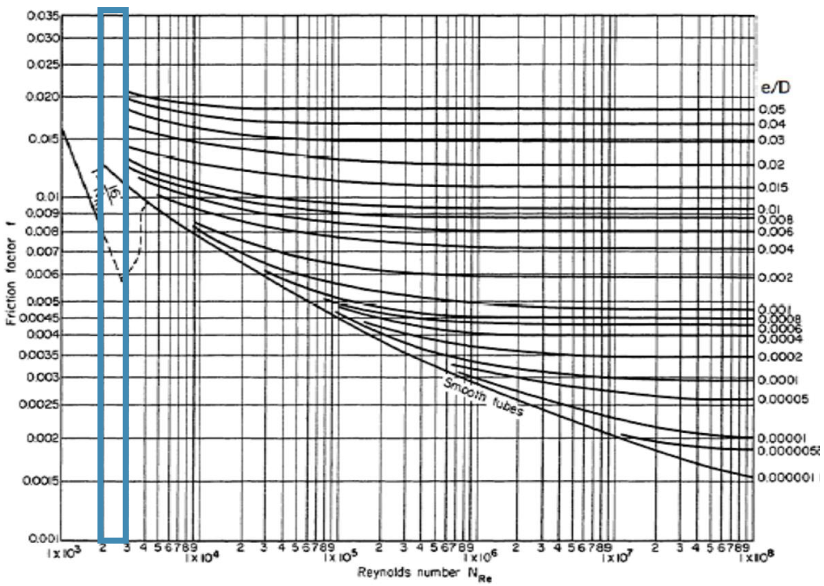
# The Moody diagram: Zone 1




- **Zone 1** is laminar flow (Re<2000)
- On the Moody diagram, there is only one curve in Zone 1:
  - This one curve changes linearly with Reynolds number
  - This implies that the **friction in Zone 1 is independent of surface roughness** – it is only a function of Reynolds number
- We can calculate $f_F$ in this region using:

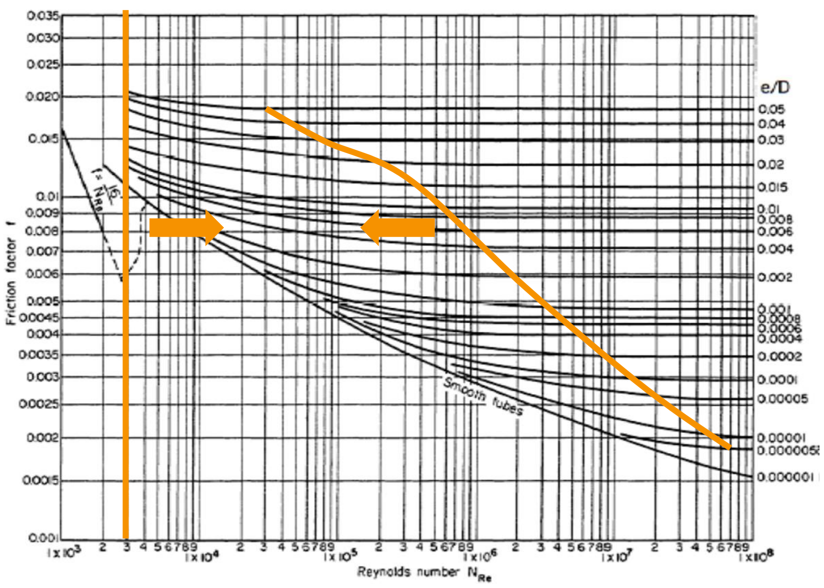
$$f_F = \frac{16}{Re}$$

# The Moody diagram: Zone 2



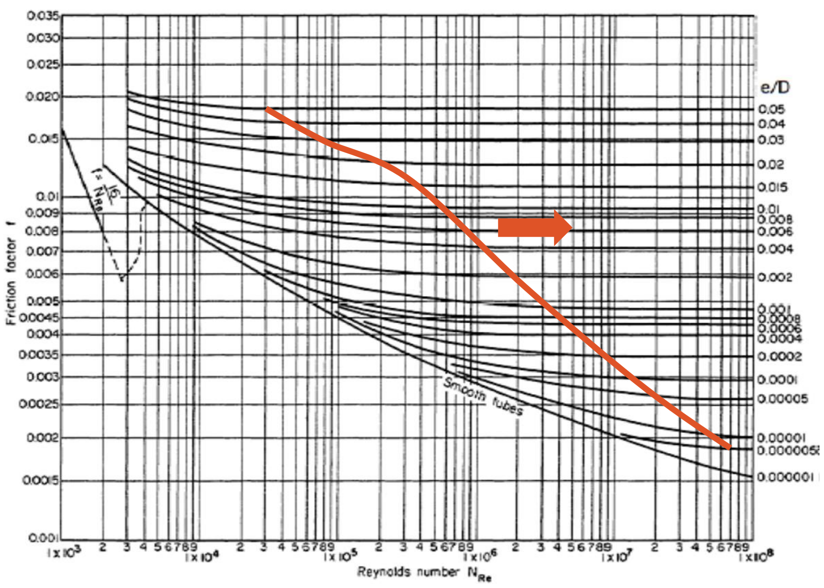

- **Zone 1** is the transitional zone (2000<Re<3000)
- On the Moody diagram, there are two curves in Zone 2:
  - The first is a continuation of the laminar flow line
  - The second is the beginning of the turbulent flow line
  - In this zone, flow can either behave as laminar or turbulent – it will actually switch between the two
  - This makes experimentally measuring reproducible $f_F$ values very difficult

# The Moody diagram: Zone 3




- **Zone 3** is the low turbulent flow regime
- On the Moody diagram, there are many curves in Zone 4:
  - Each curve represents a pipe with a certain relative roughness
  - In Zone 3, these lines are curved
  - This means that in this region, frictional losses in the system are strong functions of both the Reynolds number and pipe roughness

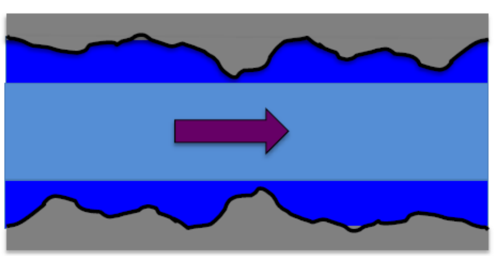
# The Moody diagram: Zone 4



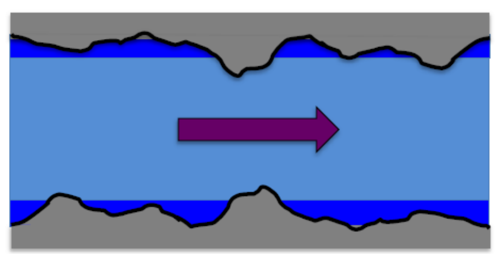

- **Zone 4** is high Reynolds number flow
- On the Moody diagram, there are many curves in Zone 4:
  - Each curve represents a pipe with a certain relative roughness
  - In Zone 4, these lines are essentially horizontal
  - This means that the friction factor **no longer changes with the Reynolds number of the flow**
  - **Surface roughness** is the main parameter determining frictional losses

# Why do these zones exist?

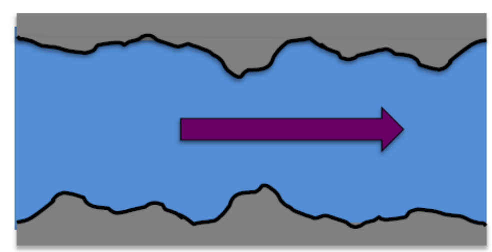
- Researchers have tried to rationalise why we observe this behaviour
  - At **low Re**, a layer of fluid covers the roughness elements of the pipe, shielding their impact. Therefore roughness has no impact on laminar flow
  - As **Re increases**, this boundary layer thins, and the exposed roughness increases friction
  - Eventually, the whole roughness element is exposed and friction becomes **independent of Re**



Zone 1



Zone 3



Zone 4

# The Moody diagram can only do so much…

- The Moody diagram has one major limitation
  - In order to use it, you need to know the Reynolds number
  - In order to calculate the Reynolds number, you need to know the average velocity
  - This means you can use the Moody diagram to calculate $\Delta P$ for a given flow rate (Example 4.2)
- However, you **cannot** use the Moody diagram to calculate the flow rate for a given $\Delta P$
- In order to accomplish this task, you need a different plot known as the **friction factor plot** (also known as the **Re vs. $\phi Re^2$ plot**)

**Warning:** The analysis we're about to do is going to be weird. It's even weirder than seeing the Moody diagram for the first time.

# Re vs. $\phi Re^2$

- To use the Re vs. $\phi Re^2$ chart to find the flow rate, you must:
  1. Use the mechanical energy balance to solve for friction, F
  2. Use this value of F to calculate $\phi Re^2$
  3. Use the Re vs. $\phi Re^2$ chart to determine the value of the Reynolds number
  4. Use this Reynolds number to solve for the velocity and flow rate

# Re vs. ϕRe$^2$

- We can use the mechanical energy balance to solve for F – we previously derived the following relationship for F:

$$F = \frac{4\phi L\bar{V}^2}{D}$$

remember: we don't know $\bar{V}$ or $\phi$ here

- We can massage this expression:

$$\frac{FD}{4L} = \phi\bar{V}^2$$

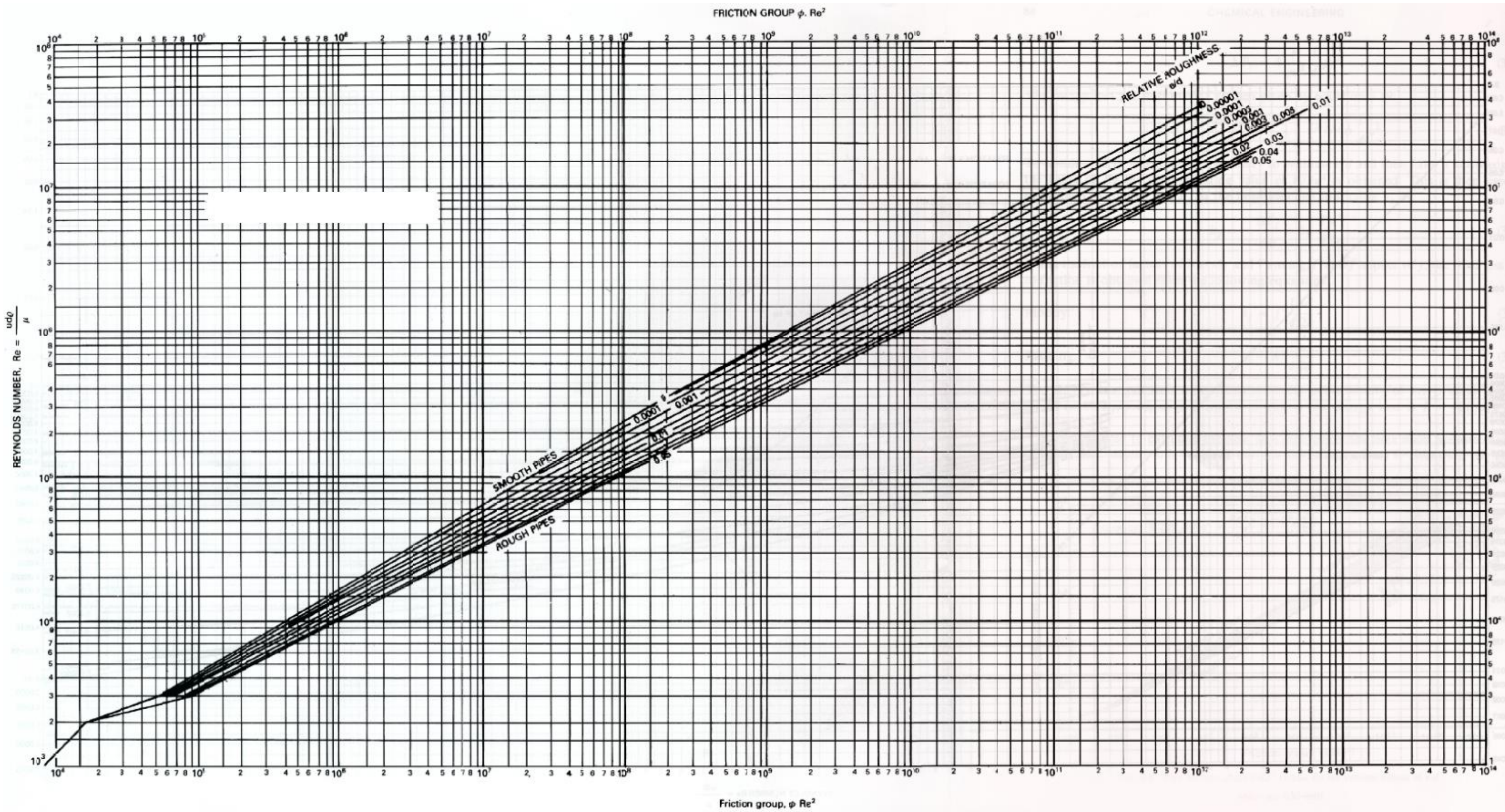
- Now, multiply both sides with by the same expression:

$$\left(\frac{\rho^2 D^2}{\mu^2}\right)\frac{FD}{4L} = \phi\bar{V}^2\left(\frac{\rho^2 D^2}{\mu^2}\right) \Rightarrow \frac{FD^3\rho^2}{4L\mu^2} = \phi\left(\frac{\rho\bar{V}D}{\mu}\right)^2$$

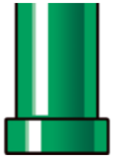
$$\therefore \boldsymbol{\frac{FD^3\rho^2}{4L\mu^2} = \phi Re^2}$$

this quantity $\phi Re^2$ is called the "friction group"

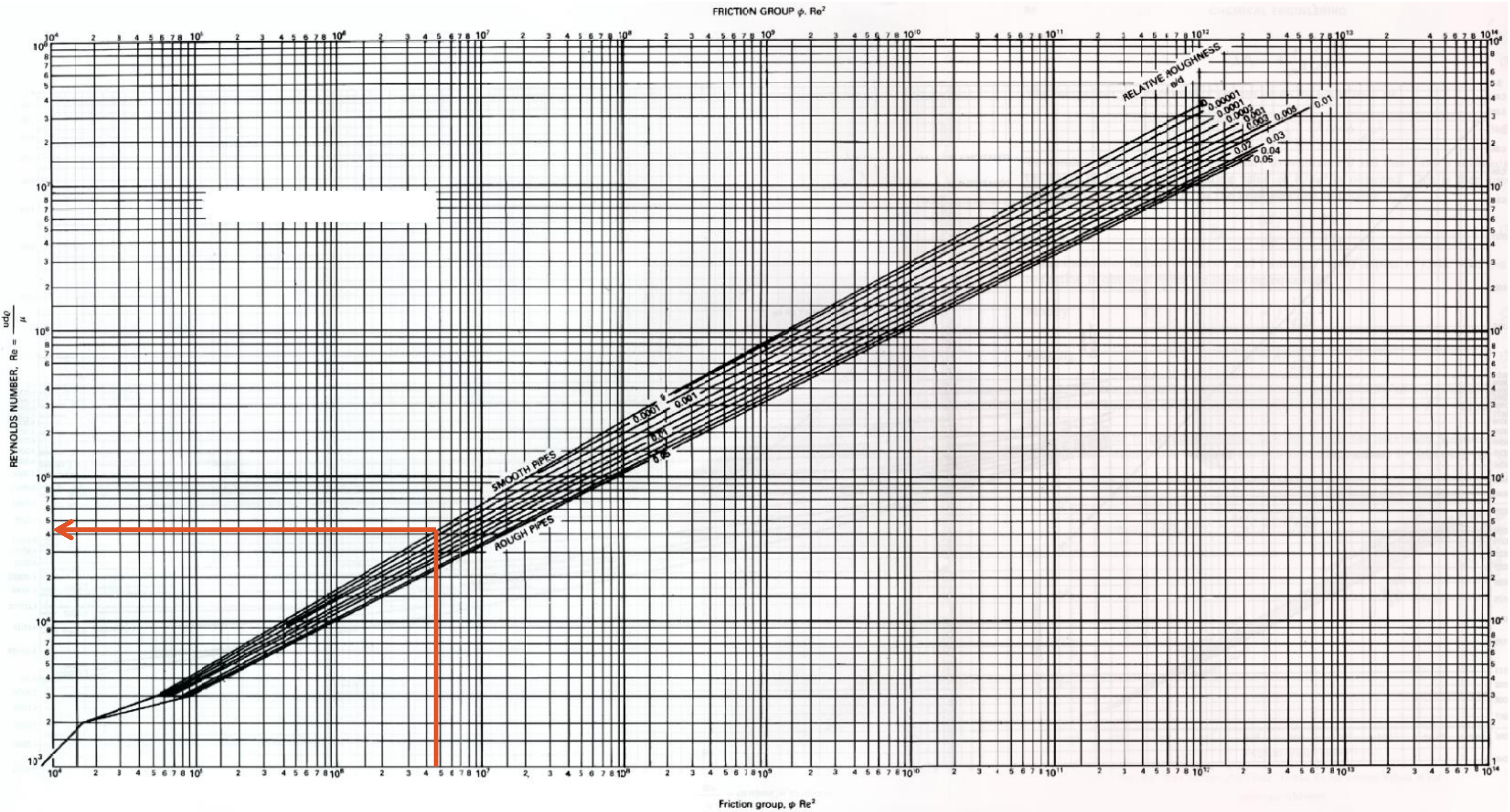
# Re vs. $\phi Re^2$


FRICTION GROUP φ. Re²
RELATIVE ROUGHNESS e/d
0.00001
0.0001
0.0002
0.001
0.002
0.005
0.01
0.02
0.03
0.04
0.05
SMOOTH PIPES
ROUGH PIPES
REYNOLDS NUMBER, Re = udρ/μ
Friction group, φ Re²

# Example 4.3

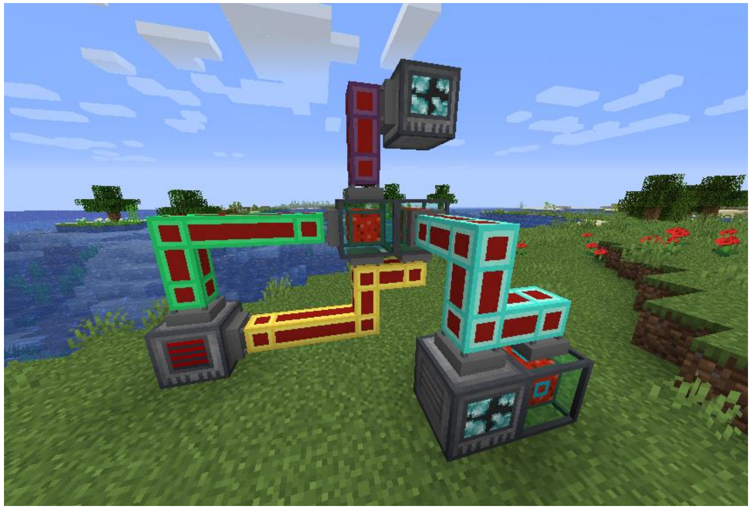


- We have a 10-metre straight pipe through which water is being pumped 1 metre uphill. The diameter of the pipe is 10 [cm], the length of the pipe is 10 [m], the roughness of the pipe is 0.001 [cm], the density of the water is 1000 [kg/m$^3$] and its viscosity is 0.001 [Pa.s]. The pressure at the exit of the pipe is $P_{atm}$, and the pressure at the inlet of the pipe is $P_{atm}$+10,000 [Pa].
    - What is the flow rate of water through the pipe?


FRICTION GROUP φ. Re²
REYNOLDS NUMBER, Re = udρ/μ
RELATIVE ROUGHNESS e/d
SMOOTH PIPES
ROUGH PIPES
Friction group, φ Re²

# Mini-summary

- In this module, we are focusing solely on pipe flow as this is (arguably?) what we are most interested in as engineers
- Although the Bernoulli equation is great to conceptually think about fluid flow, it is not applicable in most engineering systems as it neglects friction
  - To address this, we manipulated the mechanical energy balance to include **friction factors**
  - To determine these friction factors, we use the **Moody diagram**, which requires that you first calculate two parameters of your flow system:
    - The Reynolds number, Re
    - The relative roughness, e/D
  - However, if you know the pressure drop and are asked to determine the flow rate, you will need to use the **Re vs. $\phi Re^2$ plot**

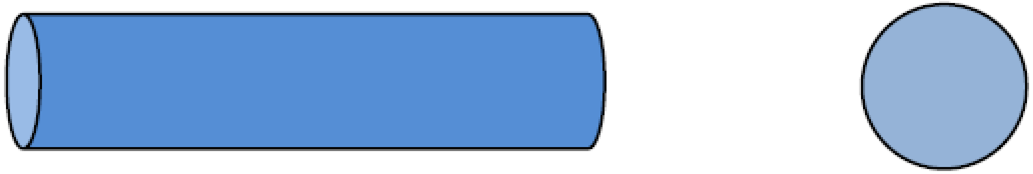


# Module 4.2

## Flow through pipes with non-circular cross-sections

# Assuming circular cross-section

- Until now, we have assumed the conduit through which our fluid is flowing is a cylindrical pipe with circular cross section



- However, sometimes our fluid may be flowing through conduits with other geometries, e.g.:
    - Annuli
    - Ducts

# Modifying our mechanical energy balance

- Currently, our mechanical energy balance is in terms of pipe diameter:

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_S + \frac{2 f_F L \bar{V}^2}{D} = 0$$

  - D: the diameter of a **circular** pipe
- For non-circular conduits, we replace "D" with the **hydraulic mean diameter, $D_e$**:

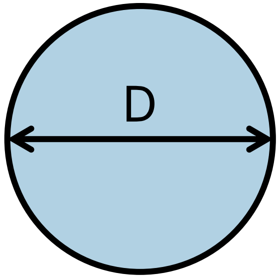
$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_S + \frac{2 f_F L \bar{V}^2}{D_e} = 0$$

  - Really, we just need to solve for the hydraulic mean diameter and then solve the mechanical energy balance as usual
- The **hydraulic mean diameter, $D_e$** is an empirical relationship:

$$D_e = \frac{4 \times \text{cross}-\text{sectional area of flow}}{\text{wetted perimeter}}$$

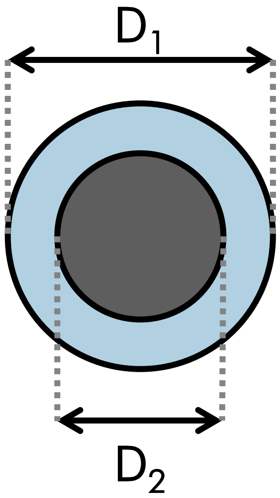
# Hydraulic mean diameter

- We shall now calculate the hydraulic mean diameters for three conduits of different cross-sections:
    1. What is the hydraulic mean diameter for a pipe with circular cross-section?



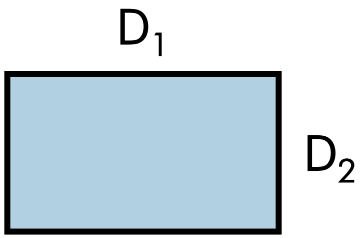
# Hydraulic mean diameter

- We shall now calculate the hydraulic mean diameters for three conduits of different cross-sections:
   2. What is the hydraulic mean diameter for a pipe with annular cross-section?



# Hydraulic mean diameter

- We shall now calculate the hydraulic mean diameters for three conduits of different cross-sections:
  3. What is the hydraulic mean diameter for a rectangular duct?



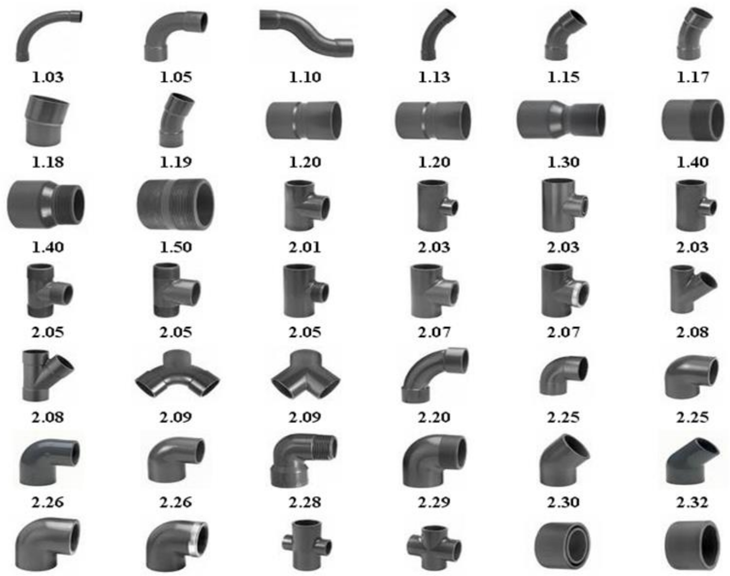
# Mini-summary

- In this section, we discussed how to modify the mechanical energy balance in order to model flow through conduits with non-circular cross-sections
- This is accomplished by replacing pipe diameter, D with the hydraulic mean diameter of the system, $D_e$:

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\bar{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_S + \frac{2 f_F L \bar{V}^2}{D_e} = 0$$

- The hydraulic mean diameter can be calculated using:

$$D_e = \frac{4 \times \text{cross}-\text{sectional area of flow}}{\text{wetted perimeter}}$$




# Module 4.3:

## Minor losses from fittings

# Limitations of the friction factor

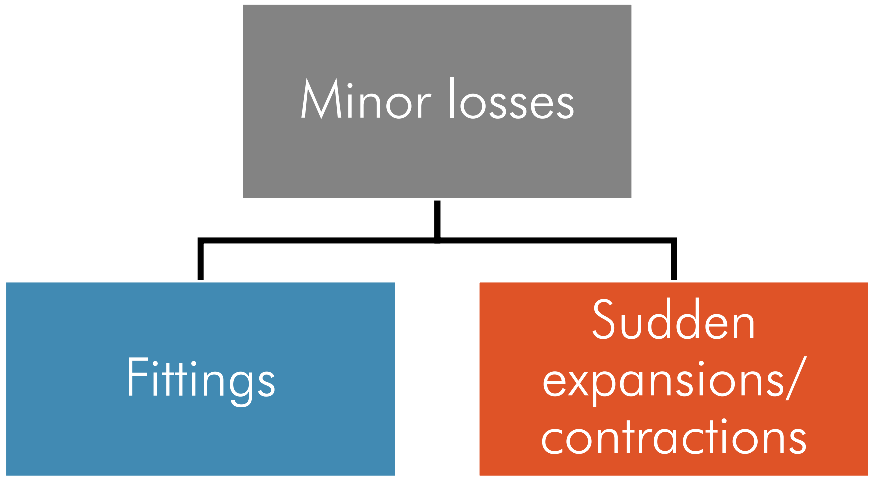
- So we've covered friction factors, the Moody diagram, the Re vs. $\phi Re^2$ chart, and non-circular conduits… We can now calculate the frictional losses in any pipe system!!

… Yeah, nah.

# Limitations of the friction factor

- Friction factors only quantify frictional losses over a straight (or very gently curved) length of pipe
- Significant frictional losses occur around fittings such as:
  - L-bends
  - Valves
  - T-junctions
  - Etc.
- Significant frictional losses also occur around sudden expansions/contractions in the flow system
- Together, these frictional losses are often referred to as **minor losses**
  - They are "minor" losses because usually they are smaller in magnitude to the "major" losses we discussed previously

# Types of minor frictional losses



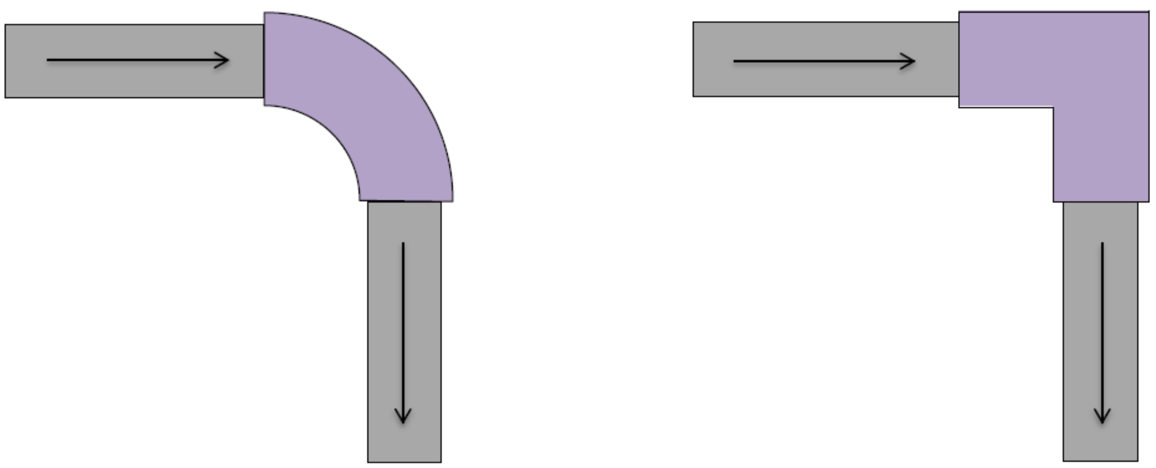

Accounted for by:
1. Equivalent length or
2. Resistance coefficient

Accounted for by:
1. Resistance coefficient

- First we'll focus on how to quantify losses due to **fittings**

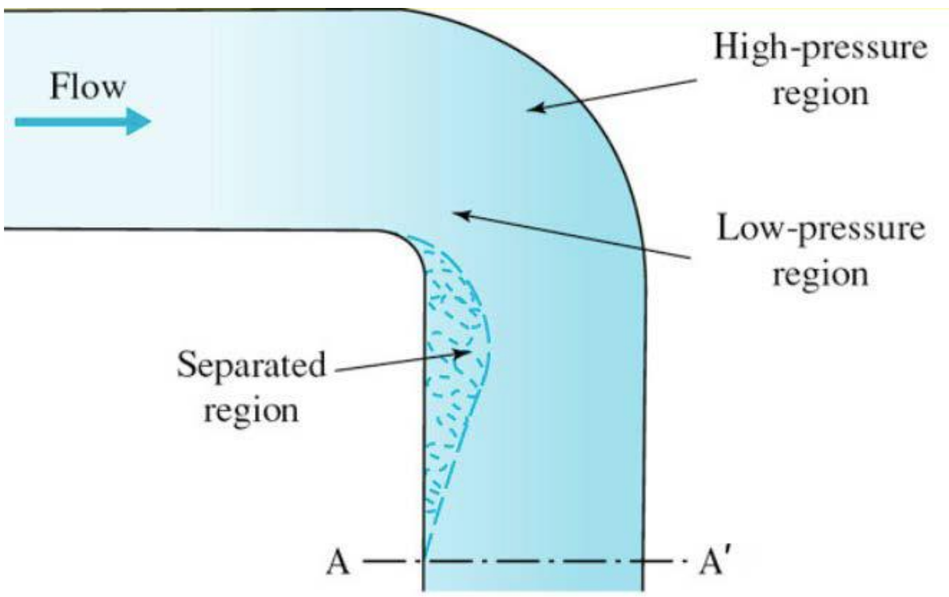
# Why do fittings result in minor losses?

- Fluid must flow through an elbow. Which elbow will result in a lower frictional loss and why?



# Why do fittings result in minor losses?

- The presence of fittings causes disturbances in the flow, such as separation regions
- It is these disturbances that result in frictional losses



# Quantifying minor losses due to fittings

- There are two methods that can be used to account for the frictional losses that occur due to fittings:
  1. Equivalent length, $L_{eq}$
  2. Resistance coefficient, K
- Researchers have performed experiments, and developed empirical methods that allow us to predict frictional losses
  - This is highly analogous to how the Moody diagram was developed
  - To use either of the above methods, you need to be provided with a table of quantities

# Method 1: Equivalent length, $L_{eq}$

- Frictional losses through a fitting were experimentally measured
- Researchers then calculated how long a piece of pipe the fluid would have to flow through in order to experience the same frictional loss
- Essentially, the researchers are saying:

*"The fluid flowing through this fitting experiences the same loss due to friction as the fluid would if it had travelled through an extra ___ metres of pipe."*

- These values of equivalent pipe length are then tabulated for common types of fittings

# Method 1: Equivalent length, $L_{eq}$

- The table below lists the equivalent length for common fittings:
  - These are listed as multiples of D, the pipe diameter

| Fittings | | $L_{eq}$ |
|---|---|---|
| 45° elbow | | 15D |
| 90° elbow | | 30-40D |
| 45° elbow (square) | | 60D |
| Entry from leg of T-piece | | 60D |
| Entry into leg of T-piece | | 90D |
| Unions and couplings | | Very small |
| Gate valve | Fully open | 7D |
| | Half open | 200D |
| | Quarter open | 500D |

# Method 1: Equivalent length, $L_{eq}$

- To use this method:
  1. Calculate an equivalent length, $L_{eq}$ for each fitting in your system
  2. Add these equivalent lengths to the actual length of your pipe
  3. Calculate F
  4. Solve the mechanical energy balance as per normal

$$F = \frac{2 f_F L \bar{V}^2}{D} \quad \Rightarrow \quad F = \frac{2 f_F \left(L + \sum L_{eq}\right) \bar{V}^2}{D}$$

# Method 2: Resistance coefficient, K

- To use **resistance coefficients**, you add an additional frictional loss term to the mechanical energy balance for each fitting
- For each fitting, you add a $\frac{1}{2}K\bar{V}^2$ term to the energy balance, where K values are found through a tables of K-values

$$F = \frac{2f_F L\bar{V}^2}{D} \quad \Rightarrow \quad F = \frac{2f_F L\bar{V}^2}{D} + \frac{1}{2}\sum K\bar{V}^2$$

- If necessary, you can even use equivalent lengths and resistance coefficients together in the same energy balance – just be sure to only account for the frictional loss of a given fitting once (don't double count!)

$$F = \frac{2f_F\left(L + \sum L_{eq}\right)\bar{V}^2}{D} + \frac{1}{2}\sum K\bar{V}^2$$

# If these losses are minor, why are they important?

- You might be wondering, if these losses are minor, why can't we just treat them as negligible?
  - It turns out the frictional losses that occur due to a single fitting are usually much smaller than what occurs in the pipe, especially for long pipes…
  - But there are often many fittings within a piping system, and the sum of their individual contributions can be quite significant

# Mini-summary

- Additional frictional losses arise from fittings within a piping network
- These minor losses must be accounted for in the mechanical energy balance as they can add up to be quite significant
- There are two methods to quantify these minor losses:
  1. Equivalent length, $L_{eq}$
  2. Resistance coefficient, K
- Either method is fine, and both methods can even be used together if necessary
  - However, make sure you only account for the minor loss due to each fitting just once

# Module 4.4:

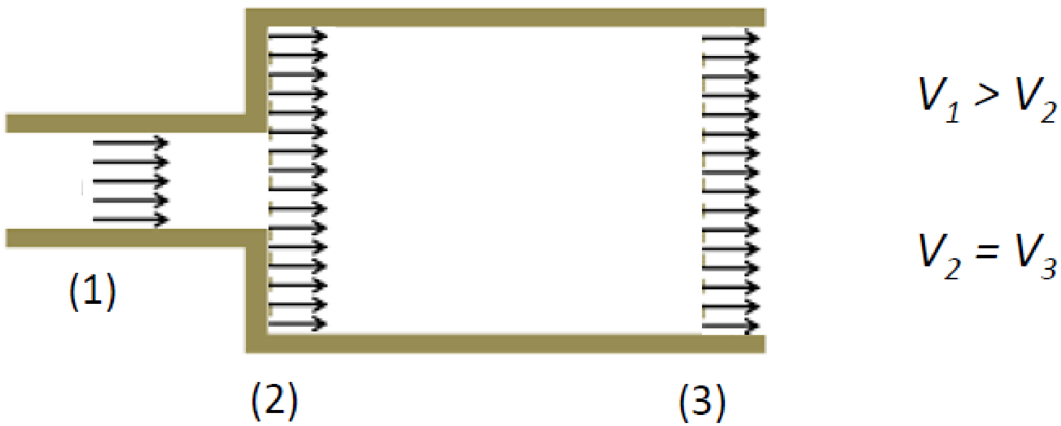
## Minor losses from sudden expansions/contractions

# Other sources of minor losses

- Besides fittings, minor losses also occur when
  - A fluid flows through a sudden expansion
  - A fluid flows through a sudden contraction
- Each of these geometries results in disturbances in the flow field, and subsequent loss of energy from the flow due to friction

# Flow through sudden expansions

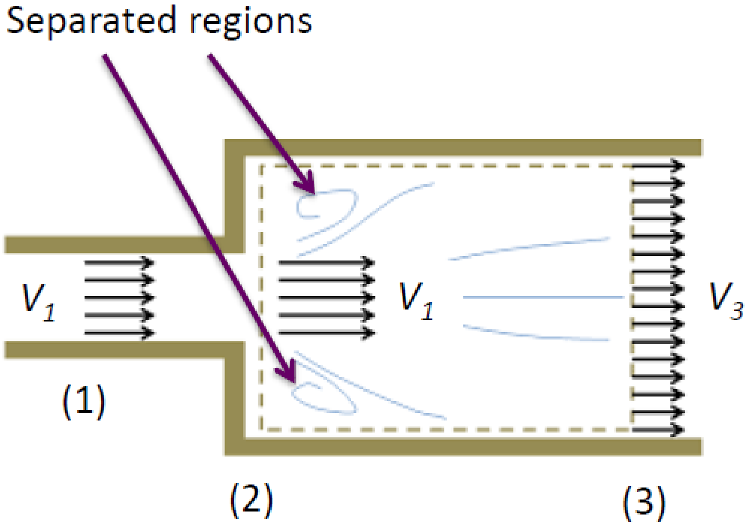
- According to our relationship derived from the conservation of mass:
$$V_1 A_1 = V_2 A_2$$
- If we strictly apply that relationship to a sudden expansion, we would predict the following flow behaviour:




- However, as your intuition probably tells you, this is not the experimentally-observed flow behaviour

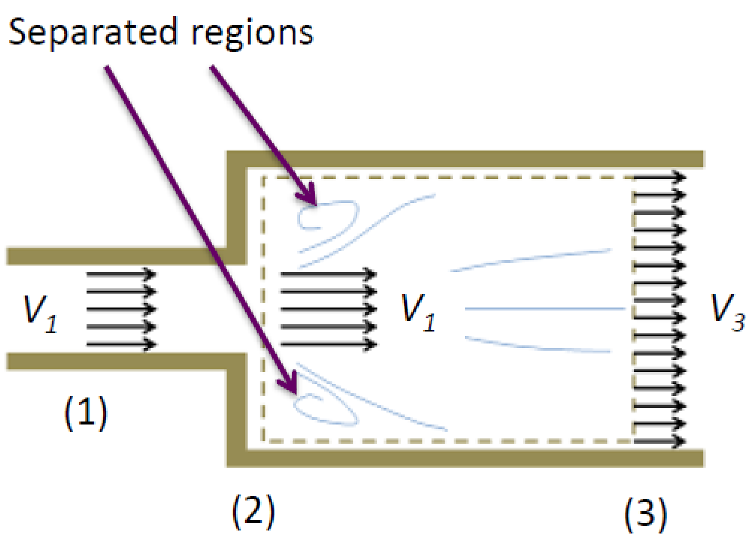
# Flow through sudden expansions

- As seen with flow through fittings, sudden expansions result in a separated region in the flow
- In reality, the flow profile will look more like the following:
  - For a certain duration into the expansion, the velocity remains constant
  - There are separated regions around the expansion
  - Velocity eventually decreases to what is expected in order to maintain the flow rate



# Quantifying losses through expansions

- Frictional losses through expansions are:
  - Quantified using a resistance coefficient, $K_{ex}$, as done for fittings
  - The value of the coefficient is determined by the geometry of the expansion




Friction due to expansion

$$F_{ex} = \frac{1}{2} K_{ex} V_S^2$$

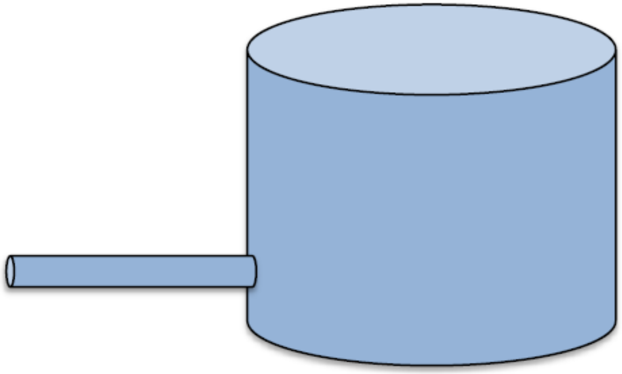
Resistance coefficient

$$K_{ex} = \left(1 - \frac{A_S}{A_L}\right)^2$$

S: small pipe
L: large pipe

# Quantifying losses through expansions

- What if your pipe is flowing into a large liquid storage tank?
  - This is modelled the same way
  - However, in this case $A_L >> A_S$
  - This allows us to assume that $K_{ex} = 1$



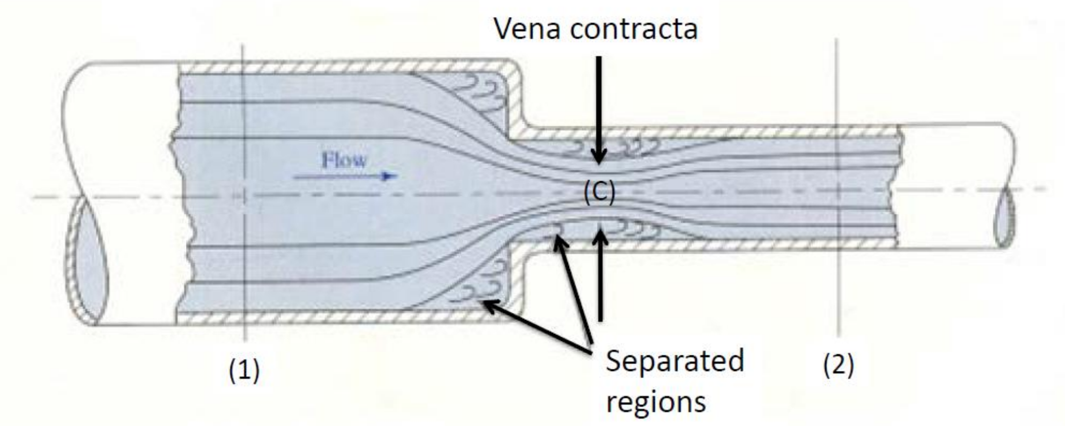
Resistance coefficient

$$K_{ex} = \left(1 - \frac{A_S}{A_L}\right)^2$$

$$\frac{A_S}{A_L} \to 0 \Rightarrow K_{ex} \to 1$$

# Flow through sudden contractions

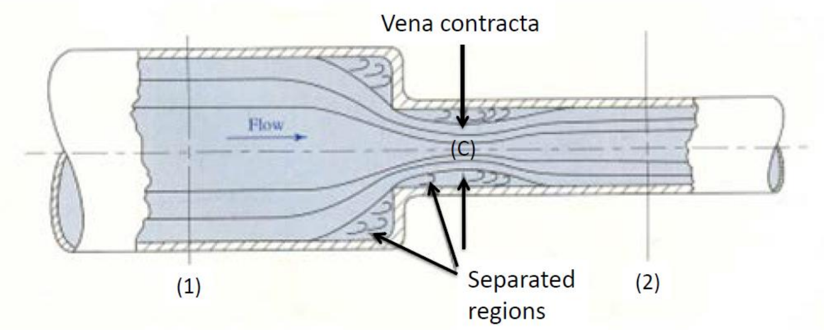
- Separated regions will also exist at sudden contractions




- In this flow geometry, there are two separated regions:
  - The area immediately before the contraction
  - A region along the walls within the contraction (*vena contracta*)
    - The point of minimum cross-section within the *vena contracta* is Point C
- Experimental results indicate that most of the frictional losses occur during the re-expansion of flow between Point C and Point 2
  - As with expansions, this is quantified using a resistance coefficient, $K_{con}$

# Quantifying losses through contractions

- Frictional losses through contractions are:
  - Quantified using a resistance coefficient, $K_{con}$, as done for fittings
  - The value of the coefficient is determined by the geometry of the expansion




Friction due to contraction

$$F_{con} = \frac{1}{2} K_{con} V_2^2$$

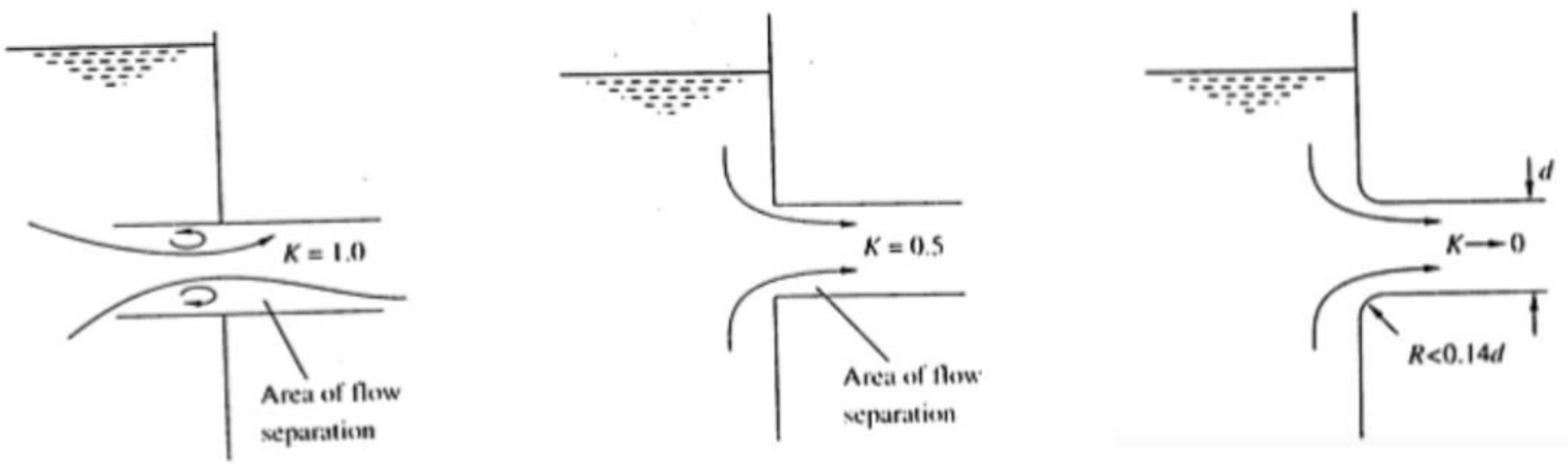
Resistance coefficient

$$K_{con} = \left(\frac{A_2}{A_C} - 1\right)^2$$

2: small pipe
C: minimal cross-section (*vena contracta*)
con: contraction

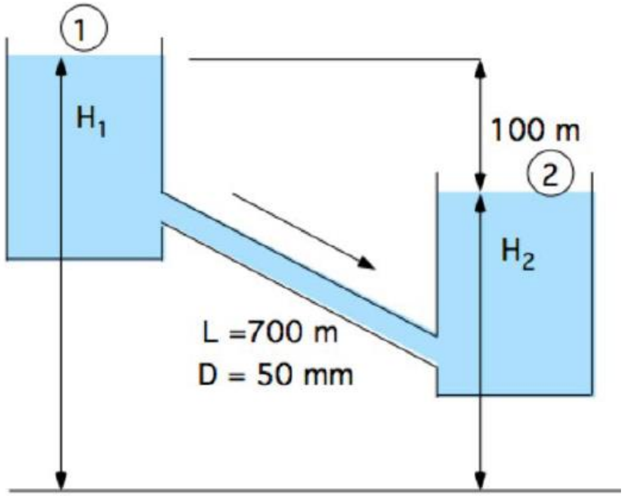
# Quantifying losses through contractions

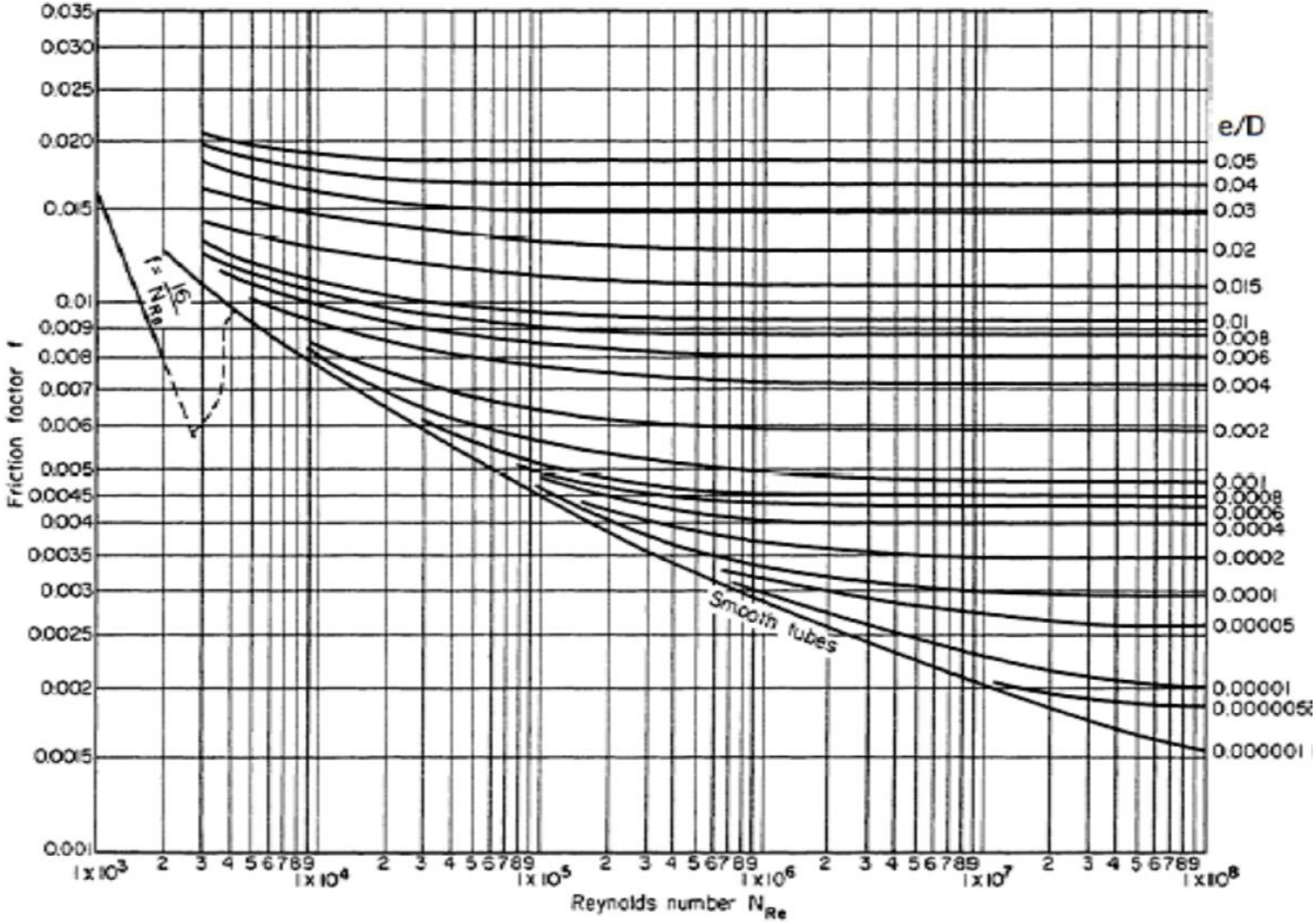
- The previous discussion dealt with contractions within pipes
- If you have fluid entering a pipe from a tank where the fluid can be considered as stagnant, the value for the resistance coefficient is different and depends on the geometry of the system

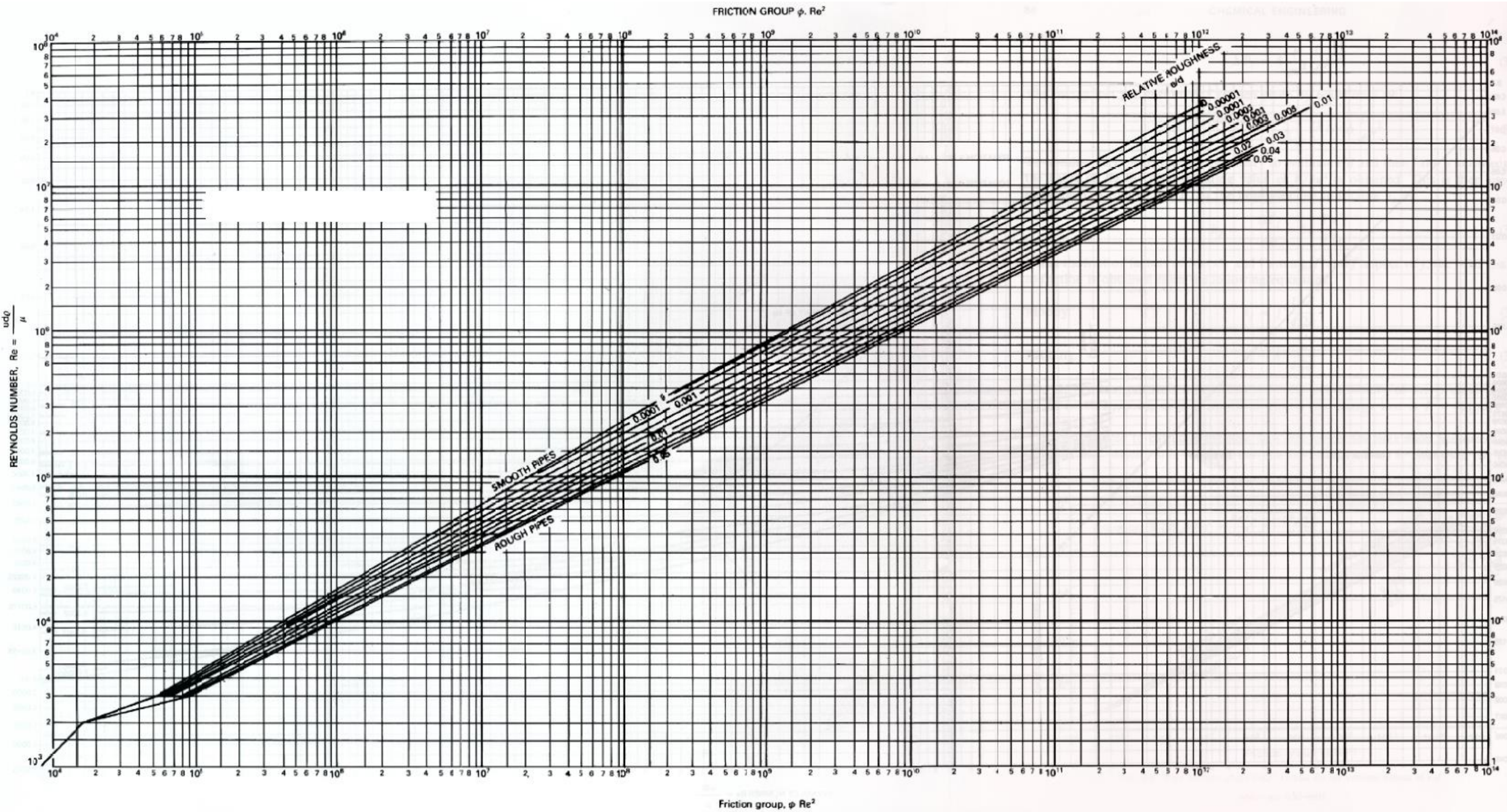


# Example 4.4

- Two large reservoirs are open to the atmosphere. They are connected by a smooth pipe, as shown in the figure. Calculate the average velocity within the pipe if $K_{entry}=0.5$, $K_{exit}=1.0$, the density of the fluid is 1000 [kg/m$^3$] and the viscosity is 0.001 [Pa.s].

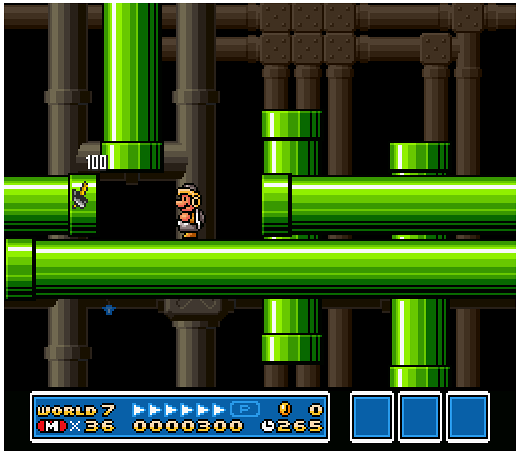



Friction factor f
Reynolds number $N_{Re}$
$f = \frac{16}{N_{Re}}$
Smooth tubes
e/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
0.000005
0.000001
0.035
0.030
0.025
0.020
0.015
0.01
0.009
0.008
0.007
0.006
0.005
0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
1 x 10³
1 x 10⁴
1 x 10⁵
1 x 10⁶
1 x 10⁷
1 x 10⁸


FRICTION GROUP $\phi Re^2$
REYNOLDS NUMBER, $Re = \frac{ud\rho}{\mu}$
RELATIVE ROUGHNESS e/d
0.00001
0.0001
0.0002
0.001
0.002
0.003
0.005
0.01
0.02
0.03
0.04
0.05
SMOOTH PIPES
ROUGH PIPES
Friction group, $\phi Re^2$

# Mini-big-summary

- In Modules 4.2-4.4, we looked at how to expand the applicability of the mechanical energy balance to a wider variety of flow problems
- To do so, we introduced three modifications:
  - The use of the hydraulic mean diameter for modelling flow through conduits of non-circular cross-section
  - Accounting for minor losses due to various fittings within the pipe
    - Equivalent length
    - Resistance coefficient
  - Accounting for minor losses due to sudden expansions/contractions in the pipe
    - Resistance coefficient




# Module 4.5:

## Branched pipes and pipe networks

# What does this section cover?

- Thus far, we have assumed that our fluid is flowing through a single pipe
- However, often we will have **branched pipes** or the fluid will be flowing through a **network of pipes**
- The mechanical energy balance will allow us to model the fluid flow through these systems; however, it is not as straightforward as the examples we have discussed thus far

# Working in terms of "head"

- When discussing flow in pipe networks, it is more useful to work with the "head" version of the mechanical energy balance (divide through by g):

$$\frac{P_1}{\rho} + \frac{\bar{V}_1^2}{2\alpha} + gz_1 = \frac{P_2}{\rho} + \frac{\bar{V}_2^2}{2\alpha} + gz_2 + F$$

each term has units of energy per unit mass

$$\frac{P_1}{\rho g} + \frac{\bar{V}_1^2}{2\alpha g} + z_1 = \frac{P_2}{\rho g} + \frac{\bar{V}_2^2}{2\alpha g} + z_2 + \frac{F}{g}$$

each term has units of length

pressure head

velocity head

gravitational head

head loss

# Working in terms of "head"

- The sum of the pressure head, velocity head, and gravitational head is referred to as the **total head** at a given point in the flow system

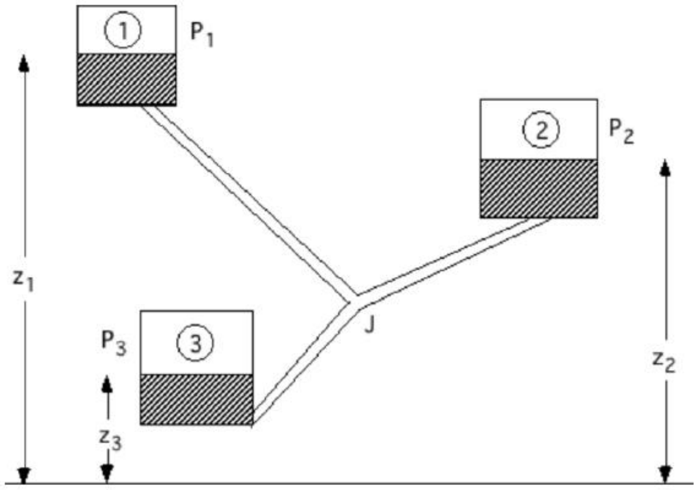
$$\underbrace{\frac{P_1}{\rho g} + \frac{\bar{V}_1^2}{2\alpha g} + z_1}_{\text{total head at Point 1}} = \underbrace{\frac{P_2}{\rho g} + \frac{\bar{V}_2^2}{2\alpha g} + z_2}_{\text{total head at Point 2}} + \frac{F}{g}$$

head loss between Points 1 & 2

$$\boxed{h_1 = h_2 + h_L}$$

# Thought experiment: flow in branched pipes

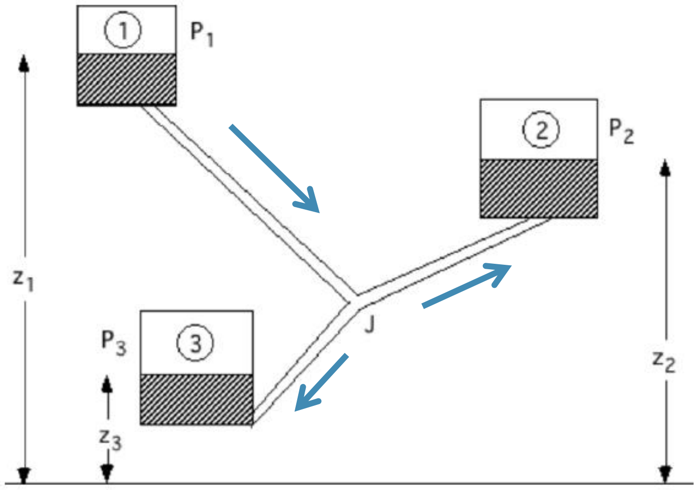
- Consider three large tanks at various heights and under various pressures, interconnected at Point J:
  - Which tanks will fill and which will empty?
  - What are the flow rates into/out of each tank?



# Thought experiment: flow in branched pipes

- General considerations about branch points:
  1. **Mass is conserved at a branch point:** the mass flow rate into a branch has to equal the mass flow rate out of a branch

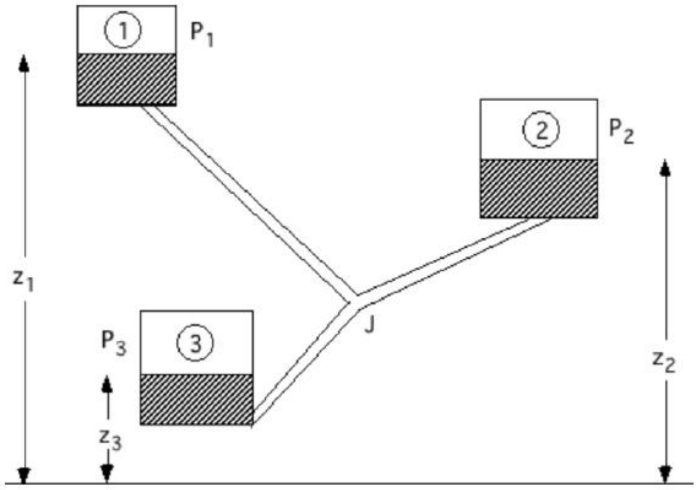
$$Q_1 = Q_2 + Q_3$$




  2. The head is **common** to all branches at a junction. The energy may be partitioned differently, but the total heads are equal:
     - Flow in Pipe 1 ends up with a total head of $h_J$
     - Flow in Pipes 2 and 3 begin with a total head of $h_J$
     - Said another way, the total head of the system **cannot be discontinuous** anywhere in the flow

# Thought experiment: flow in branched pipes

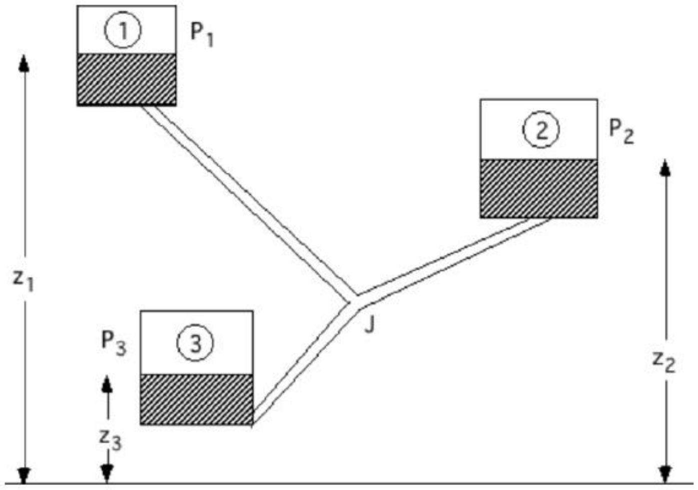
- If these tanks were open to the atmosphere, the pressure at each fluid surface would be $P_{atm}$
- In that scenario, gravity head would be the main driver of fluid flow, and all the fluid would drain down into the third tank




- However, in this scenario, the tanks are sealed, providing the opportunity to adjust the pressure head in each tank
- This could result in a scenario where the pressure head in Tank 3 is large enough to push fluid against gravity into Tanks 1 and 2

# Thought experiment: flow in branched pipes

- The fluid flow is governed by the mechanical energy at each of the three free surfaces
  - Said another way, the fluid flow is governed by the head of fluid at each of the three surfaces
- Let us calculate the total head at the free surface in Tank 1:



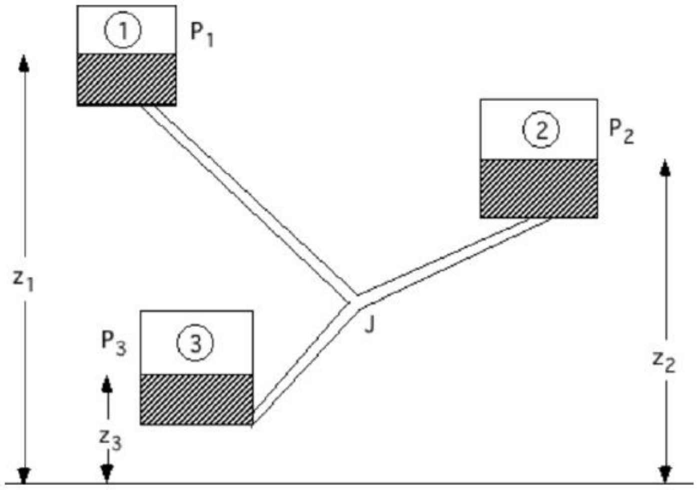

$$h_1 = \frac{P_1}{\rho g} + \frac{\bar{V}_1^2}{2\alpha g} + z_1$$

since the tanks are large, we can assume that the kinetic energy at the surface is negligible

$$h_1 = \frac{P_1}{\rho g} + z_1$$

# Thought experiment: flow in branched pipes

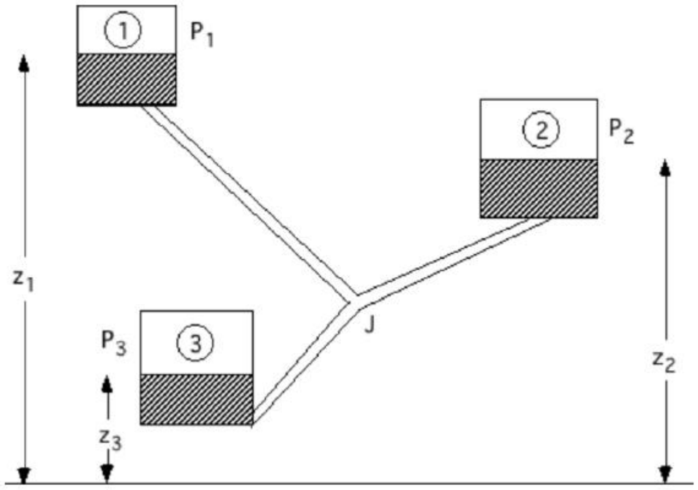
- Using the same analysis, we can write down the head at each of the free surfaces:
  - Point 1: $h_1 = \frac{P_1}{\rho g} + z_1$
  - Point 2: $h_2 = \frac{P_2}{\rho g} + z_2$
  - Point 3: $h_3 = \frac{P_3}{\rho g} + z_3$




- This analysis confirms our previous rationalisation, that flow will be driven both by the height of the tanks and the pressure within the tanks
  - Said another way, there is a contribution to flow by both the gravitational head and the pressure head

# Thought experiment: flow in branched pipes

- Given: $h_1 > h_J$
  - With this condition, multiple scenarios we can consider




1. $h_J > h_2$ and $h_J > h_3$
   - Fluid will flow from Tank 1 to the joint
   - From the joint, a portion will go to Tank 2 and a portion will go to Tank 3

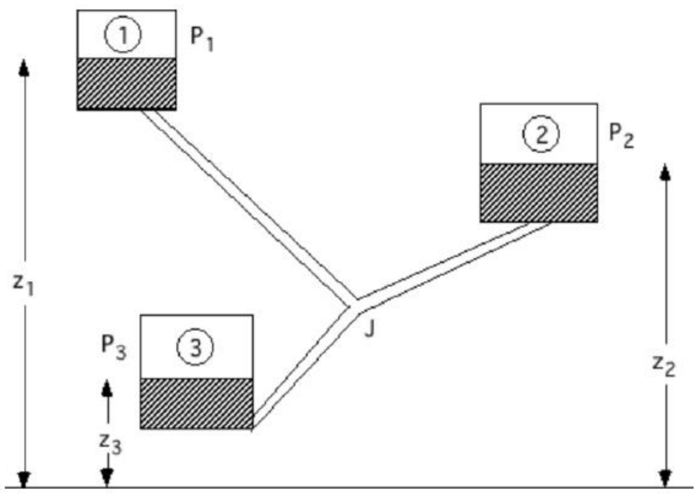
$$Q_1 = Q_2 + Q_3$$

2. $h_J < h_2$ and $h_J > h_3$
   - Fluid will flow from Tanks 1 and 2 through the joint to Tank 3

$$Q_1 + Q_2 = Q_3$$

# Thought experiment: flow in branched pipes

- Given: $h_1 > h_J$
  - With this condition, multiple scenarios we can consider



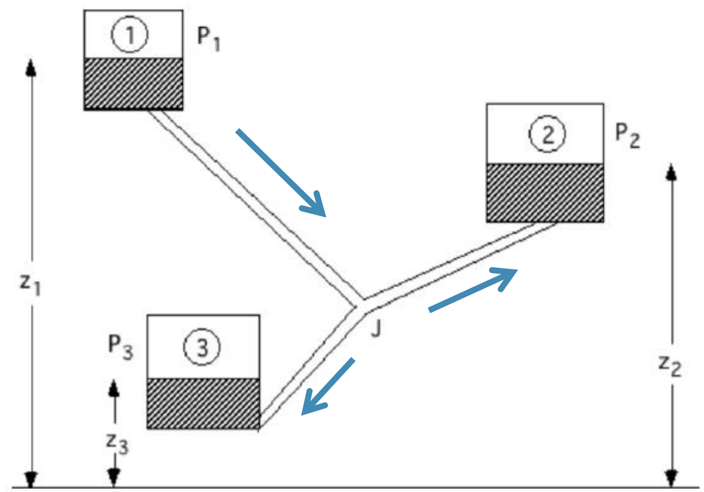

3. $h_J > h_2$ and $h_J < h_3$
   - Fluid will flow from Tanks 1 and 3 through the joint to Tank 2

$$Q_1 + Q_3 = Q_2$$

4. $h_J < h_2$ and $h_J < h_3$
   - **Impossible!** Under these conditions, all the fluid would flow to the joint but because it is incompressible, it would have nowhere to go.

# Flow in branched pipes

- Let's assume we have Scenario 1:
  - $h_1 > h_j$
  - $h_J > h_2$
  - $h_J > h_3$
- To calculate the flow, we must determine the head loss between each free surface and the joining point



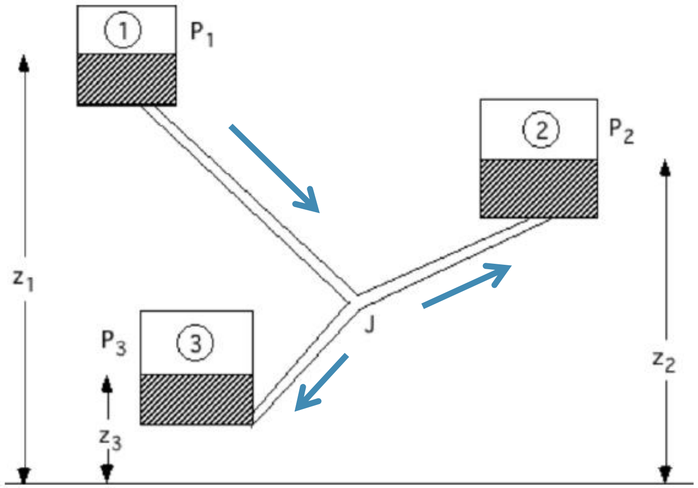

- **Pipe 1 to J**

$$\frac{P_1}{\rho g} + \frac{\bar{V}_1^2}{2\alpha g} + z_1 = \frac{P_{1J}}{\rho g} + \frac{\bar{V}_{1J}^2}{2\alpha g} + z_J + \frac{F_1}{g}$$

$$\underbrace{\frac{P_1}{\rho g} + z_1}_{\text{total head at Point 1}} = \underbrace{\frac{P_{1J}}{\rho g} + \frac{\bar{V}_{1J}^2}{2\alpha g} + z_J}_{\text{total head at Point 2}} + \underbrace{\frac{F_1}{g}}_{\text{head loss between 1 \& J}} \Rightarrow \boldsymbol{h_1 - h_J = \frac{F_1}{g}}$$

# Flow in branched pipes

- The same analysis can be performed for the other two pipe segments
- However, fluid is flowing from J to these tanks, to J is now Point 1 and either Tank 2 or 3 is now Point 2
- J to Point 2

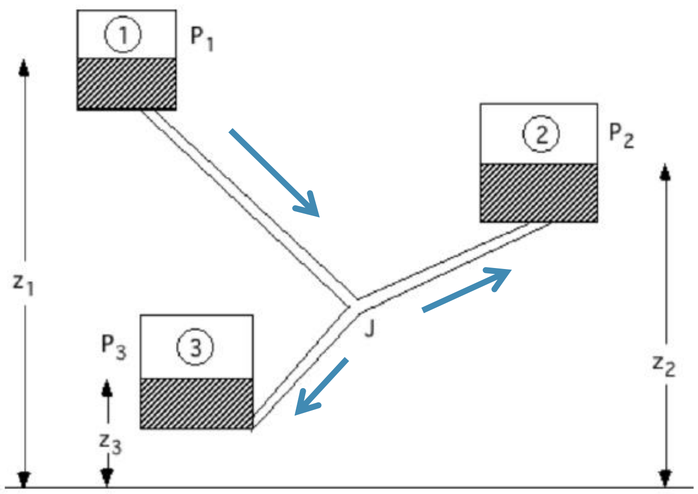



$$\frac{P_{2J}}{\rho g}+\frac{\bar{V}_{2J}^2}{2\alpha g}+z_J=\frac{P_2}{\rho g}+\frac{\bar{V}_2^2}{2\alpha g}+z_2+\frac{F_2}{g}$$

$$\frac{P_{2J}}{\rho g}+\frac{\bar{V}_{2J}^2}{2\alpha g}+z_J=\frac{P_2}{\rho g}+z_2+\frac{F_2}{g}\Rightarrow \boldsymbol{h_J-h_2=\frac{F_2}{g}}$$

# Flow in branched pipes

- The same analysis can be performed between the Joint and Tank 3
- This produces the following three energy balances:
  - $h_1 - h_J = \frac{F_1}{g}$
  - $h_J - h_2 = \frac{F_2}{g}$
  - $h_J - h_3 = \frac{F_3}{g}$

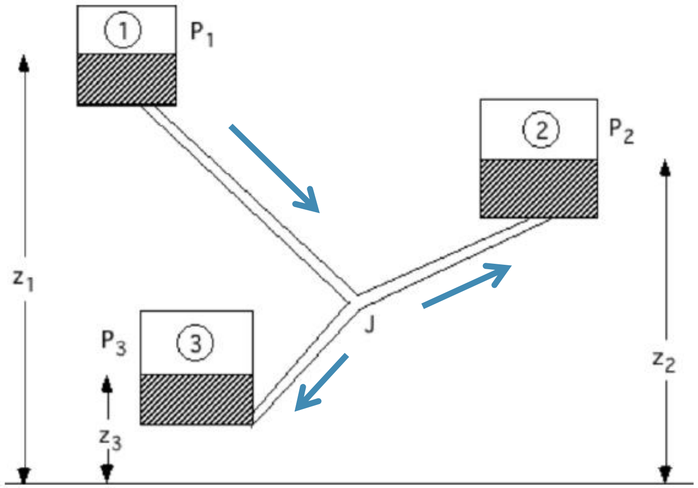



- Even with the three energy balances, more information is needed before you can solve the problem
  - The pressure in each tank
  - The height of the free surfaces
  - Pipe parameters to calculate friction (length, diameter, roughness)

# Flow in branched pipes

- If you are provided with all of the necessary information, you can solve for the flow rates and velocities of the fluid through each pipe

- However, it is necessary to use an iterative approach in order to solve the problem

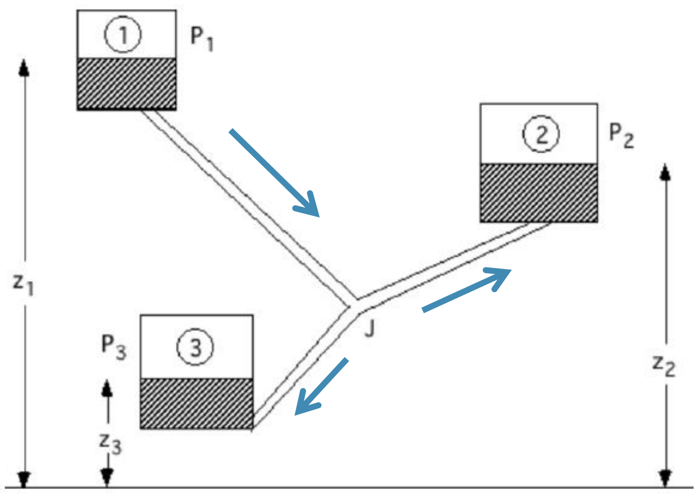



- With the tank pressures, heights, and pipe parameters provided, you can calculate the head in each of the tanks

- However, we still do not know the value of $h_J$ and we cannot calculate friction without knowing the velocity of the fluid in the pipes

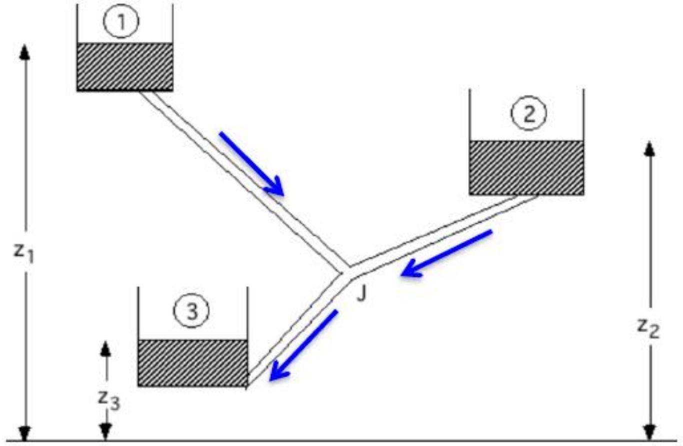
# Flow in branched pipes

- In order to solve for the velocities and flow rates in each pipe, you must use the following iterative algorithm:
    1. Calculate the head at each free surface
    2. Estimate a reasonable value of $h_J$
    3. Calculate estimated values of frictional losses: $F_1$, $F_2$, and $F_3$
    4. Calculate $\phi Re^2$ values for each pipe to determine the estimated Re for each pipe
    5. Using the estimated Re values, calculate velocities and volumetric flow rates in each pipe: $Q_1$, $Q_2$, and $Q_3$
    6. Check to see if $Q_1$, $Q_2$, and $Q_3$ fulfill the equation of continuity
    7. If continuity is violated, guess a new value of $h_J$ and re-iterate



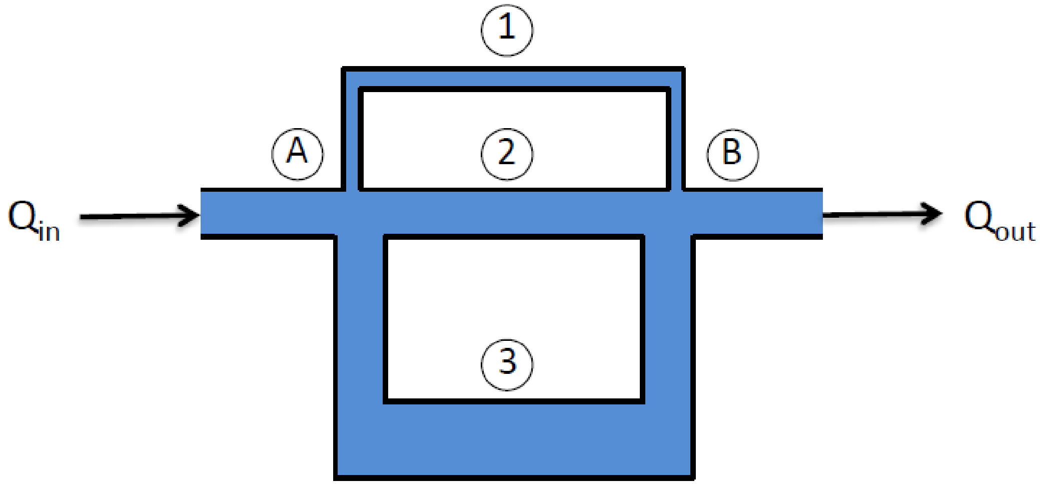
# Example 4.5

- Now we will solve a (slightly easier) version of the three reservoirs problem: three large and open tanks are interconnected as shown in the diagram. Determine the **volumetric flow rate in each pipe** using the following information:
  - The flow directions in the system are indicated
  - The lengths of the pipes are $L_1=150$ [m], $L_2=125$ [m], $L_3=100$ [m]
  - The diameter of all the pipes is 0.1 [m]
  - The value of the friction factor in each pipe is constant ($f=0.005$)
  - The tanks are open to the atmosphere



# Flow in pipe networks

- Sometimes we will also have flow through pipe networks such as this:




- This particular network has two joints and three branches
  - How much fluid will go through each branch?
  - What is the velocity of fluid in each branch?

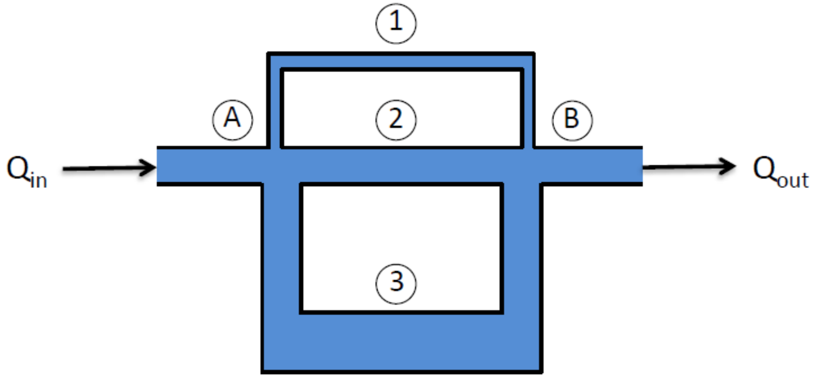
# Thought experiment: flow in branched pipes

- We had two general rules when considering flow in branched pipes
- For flow in pipe networks, we have three general rules:
  1. Mass is conserved at a branch point: the mass flow rate into a branch has to equal the mass flow rate out of a branch
  2. The head is common to all streams entering or leaving the branch point
  3. The frictional losses between two points must be equal regardless of the path the fluid takes

# Flow in pipe networks

- From our overall mass balance, we know that the mass into the system has to equal the mass out of the system:

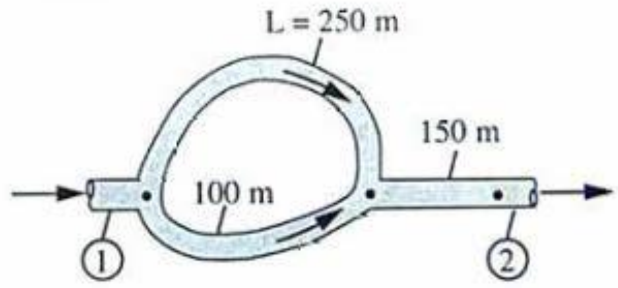
$$Q_{in} = Q_{out}$$




  - From **Rule 1**, the mass into a joint must equal to the mass out of a joint
  - From **Rule 2**, the head entering/leaving a node from each branch must be common
  - From **Rule 3**, the frictional losses between any two points must be the same, regardless of the path taken by the fluid

# Example 4.6

- A pipe network transports water from Point 1 to Point 2. All pipes are cast iron (e=0.12 [mm]) and have a diameter of 8 [cm]. The total pressure drop between Points 1 and 2 is 750 [kPa]. Find the resulting flow rates. Neglect minor losses.



# Summary

- Pipe flow is often modelled through an application of the **equation of continuity** and the **mechanical energy balance**
  - The equation of continuity arises from a mass balance
  - The mechanical energy balance arises from an energy balance
- The mechanical energy balance can be simplified to the Bernoulli equation, which assumes:
  - No frictional losses
  - No shaft work

# Summary

- But friction does exist, and we need to properly quantify it to predict flow behaviour:
    - Viscous friction & friction with pipe walls → **friction factor**
        - Moody diagram
        - Re vs. $\phi Re^2$ chart
    - Friction due to fittings, valves, expansions, contractions
        - Equivalent length
        - Resistance coefficient
- Use hydraulic mean diameter for conduits with non-circular cross-sections
- For flow in pipe networks, three rules must be maintained:
    1. Conservation of mass through a joint
    2. Head is common to each stream entering/leaving a joint
    3. Frictional losses between any two points are equal, regardless of path