

School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 4.
Streamlines, particle paths and streamfunctions

Question 1

At time t , the velocity field $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$ is given by

$$u(x, y, t) = \frac{\alpha x}{1 + \alpha t}; v(x, y, t) = \beta$$

where α and β are constants.

- i. Find the streamlines.
- ii. Find the path of the particle that was located at (X, Y) at $t = 0$.
- iii. Verify from the particle path that

$$\frac{\partial \mathbf{u}}{\partial t} \Big|_{\mathbf{R}} = \frac{D\mathbf{u}}{Dt}$$

Question 2

For the the following two steady 2-dimensional flows

$$u(x, y) = y; v(x, y) = x$$

$$u(x, y) = y; v(x, y) = -x$$

- i. Find the curves describing the shape of the streamlines.
- ii. Find the particle paths in terms of (X, Y) .
- iii. Verify that

$$\frac{\partial \mathbf{u}}{\partial t} \Big|_{\mathbf{R}} = \frac{D\mathbf{u}}{Dt}$$

For the second flow, find $\nabla \times \mathbf{u}$ and describe the flow.

Question 3

Which of the flows above are incompressible?

For those that are plane flows, construct the streamfunction and hence sketch the streamlines.

Question 4

Show that any 2D flow of the form

$$u_\sigma = 0, u_\phi = f(\sigma)$$

is a possible incompressible flow and find the streamlines in plane polar coordinates for the special cases

- i. $f(\sigma) = C\sigma$
- ii. $f(\sigma) = D/\sigma$

In each case, calculate $\nabla \times \mathbf{u}$. Describe the flows.

Question 5

For the 2D flow represented by the streamfunction in plane polar coordinates

$$\psi = U\sigma \sin \phi - Ua^2 \sin \phi / \sigma$$

sketch the streamlines. In particular, describe the flow at large σ , find any stagnation points in the flow (points where $\mathbf{u} = \mathbf{0}$), describe the streamline $\psi = 0$, calculate $\nabla \times \mathbf{u}$ and describe what sort of flow it could possibly represent.

There is no need to consider the case $\sigma < a$.