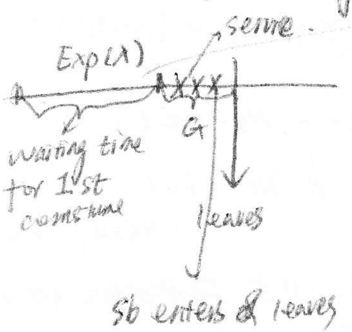
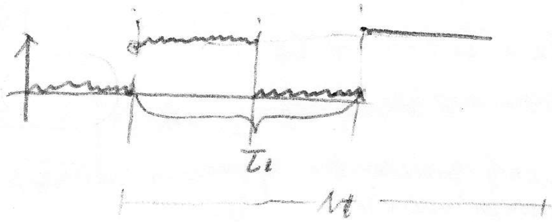
Single - Server Erlang Loss System $M/G/1/1$ Queue $\tilde{N}_t$ = # of arrivals up to time t

↓ service time
mean = m_G



we refuse more than one in the system
in poisson some may add



$$\frac{N_t}{t} \sim E[\tau_i]$$

$$\tilde{N}_t = N_{t-Exp(\lambda)} + 1 \Rightarrow \frac{\tilde{N}_t}{t} = \frac{N_{t-Exp(\lambda)} + 1}{t}$$

$$= \underbrace{\frac{1}{t}}_{\to 0} + \underbrace{\frac{N_{t-Exp(\lambda)}}{t - Exp(\lambda)}}_{\to \frac{1}{E[\tau_i]}} \cdot \underbrace{\frac{t - Exp(\lambda)}{t}}_{\to 1}$$

$\tau_i \sim G + Exp(\lambda)$.

$E[\tau_i] = \cancel{m_G + \frac{1}{\lambda}}\ m_G + \frac{1}{\lambda} \quad \to \frac{1}{E[\tau_i]}$.

mean time between renewals

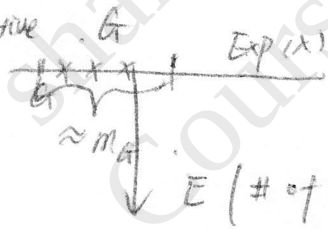
is $\mu = \frac{1}{\lambda} + m_G$

$\Rightarrow$ rate at which customers enter is $\frac{1}{\mu} = \frac{\lambda}{1 + \lambda m_G}$

Customers arrives at rate λ

$\Rightarrow$ proportion that enters the queue $= \frac{\text{entry rate}}{\text{arrival rate}} = \frac{\frac{\lambda}{1+\lambda m_G}}{\lambda} = \frac{1}{1+\lambda m_G}$

Another perspective



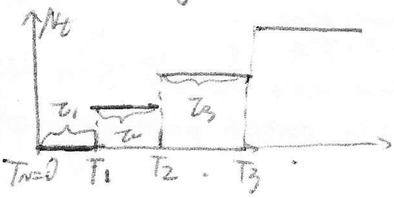
→ waiting time
no x

E(# of arrives) $\approx \lambda m_G$.

$\Rightarrow$ On avery for every 1 arrival, turn away λm_G Constomers

$\Rightarrow$ Proportion enters the queue $\frac{1}{1+\lambda m_G}$

Renewal theory



$T_K = \tau_1 + \cdots + \tau_K$.

$N_t = \max\{k : T_K \le t\}$

N_t counting process for $\tau_i \ge 0$ between successive events

If τ is NOT exponential, N_t is not Markov.

con

2 events

$A: \{N_t = k, N_{t-1} = k\}$

$B: \{N_t = k \quad N_{t-1} = k-1\}$.

A  $\Rightarrow \tau_{n+1} \ge 1$.

B:  $\tau_n < 1$.

$F_2(t) = P(\tau_1 + \tau_2 \le t)$

$= \int_0^t F_1(t-x) \cdot dF_1 x$.

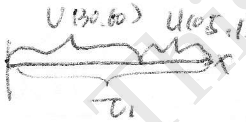
$P(\tau_1 + \cdots + \tau_n \le t) = E \quad P(\tau_1 \le t-x \mid \tau_2 + \cdots + \tau_n = x)$

Slide 26. 1) battery lifetime $\sim U(30, 60)$

then at what rate does Jenny has to change batterie ?

use Renew process. $\tau_i \sim U(30, 60)$ $\quad E[\tau_i] = \frac{1}{30}\int_{30}^{60} u\,du = 45$

2) What happen if she has only one rechargable battery $\sim U(0.5, 1)$



$E[\tau_i] = 45 + 0.75 = 45.75$.

$\frac{N_t}{t} \to \frac{1}{45.75}$