Practice Exam for MAT4MDS

Question 1. Total: 8 marks

(a) Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

2 marks

(b) Hence find the inverse of the matrix $K = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

2 marks

(c) Find the three eigenvalues of K.

3 marks

(d) Is K of full rank? Why or why not?

1 mark

Question 2. Total: 10 marks

- (a) In the following, you may not use the identity matrix, the matrix of ones or the zero matrix as your examples.
 - (i) Write down an example of a 2×2 matrix which does not have an inverse. 1 mark
 - (ii) Write down an example of a 3×3 matrix N such that $N = N^T$. 1 mark

- (iii) Write down an example of a 2×2 matrix M such that trace(M) = 0. 1 mark
- (b) Find the least squares line $y = \alpha x + \beta$ for the following data:

| y | 4 | 5 | 7 | 10 |
|----------------|----|----|---|----|
| \overline{x} | -2 | -1 | 2 | 3 |

| (c) | Calculate the average (mean) least squares line to calculate | | use the 2 marks |
|-----|--|--|-----------------|
| | | | |

Question 3. Total: 12 marks

Consider the function

$$h: \mathbb{R} \to \mathbb{R}, \ h(x) = x^2 - e^{-x}$$

(a) Find the x coordinate of the point of inflection of h(x).

3 marks

(b) What is the slope of the graph at x = 0? What is the value of the function at x = 0?

2 marks

(c) You are given that h(x) has no local maximum or minimum points. In principle, does this function possess an inverse? (Note: Do not attempt to find such an inverse, if it exists.)

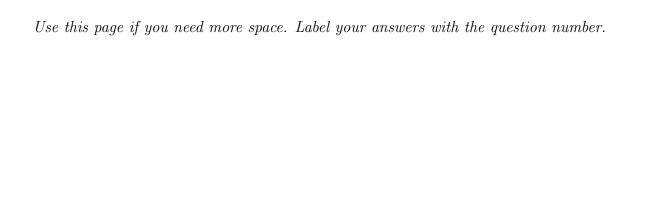
1 mark

- (d) Sketch the graph of h, showing clearly the features found in (a) and (b).
- $3\ marks$

(e) Find the area under part of the graph of h given by

3 marks

 $\int_{1}^{4} h(x)dx$



Question 4. Total: 7 marks

(a) Evaluate 3 marks

$$\int_{1}^{9} \sqrt{x} \log_{e}(x) dx$$

(b) Using substitution, find the following

(i)
$$\int_0^\infty x^2 e^{-x^3} dx$$

2 marks

(ii)
$$\int_0^\infty x^4 e^{-x^3} dx$$

Question 5. Total: 12 marks

Consider the function of two variables

$$f(x,y) = x^3y - 2e^{y^2}$$

(a) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

2 marks

(b) Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial y \partial x}$

3 marks

(c) Hence find the second order Taylor polynomial for f(x,y) near (1,0).

(d) Let
$$g(x, y) = x^2 y - x \log_e(y)$$
.

Calculate:

$$\int_1^e \int_0^3 g(x,y) \ dx \ dy$$

| Question 6. | Total: | 6 | marks |
|-------------|--------|---|-------|
|-------------|--------|---|-------|

(a) Show that

$$xB(1,x) = 1$$

2 marks

(b) A student plots data using a log-log plot, and observes that it falls on a line with slope 3. If the variables plotted are x on the horizontal axis and T on the vertical axis, what is the form of the relationship between T and x?

2 marks

(c) Another student plots different data using a linear-log plot, and observes that it falls on a line with slope -3. If the variables plotted are x on the horizontal axis and z on the vertical axis, what is the form of the relationship between z and x?

2 marks

**** End of Questions ****