

SCHOOL OF MATHEMATICS AND STATISTICS

MAST30013 Techniques in Operations Research Semester 1, 2021

Assignment 3

Due: 5pm, Monday, 17 May

- All submissions must be through Canvas and typeset in LaTeX.
- One of the questions will be picked randomly for marking.
- Show all necessary working.

1. Consider the program

$$\begin{array}{ll}\min & -x_1 - 3x_2 \\ \text{s.t.} & x_1 + x_2 \geq 1 \\ & x_2 = 2x_1 + 1 \\ & x_1 \leq 2\end{array}$$

Perform the following steps of the KKT method:

- (a) Determine the solution graphically.
- (b) Write the problem in the standard form presented in lectures.
- (c) State the KKT conditions of the program.
- (d) Determine that candidate points that should be tested for optimality.

2. Consider the nonlinear program

$$\begin{array}{ll}\min & (x_1 - 2)^4/4 + x_2^4 + 4 \\ \text{s.t.} & x_1 - x_2 \leq 8 \\ & x_1 - x_2^2 \geq 4.\end{array}$$

- (a) Write down the KKT conditions and find all points that satisfy the conditions.

- (b) Check that one of the constraint qualifications holds.
- (c) Check whether or not the point(s) in (a) is a local minimizer.
- (d) Make a graphical illustration of the NLP. Verify that there is one minimizer and one active constraint. (Hint: sketch level curves at 8, 100, 500 and curves $x_1 - x_2 - 8 = 0$, $x_1 - x_2^2 - 4 = 0$.)
- (e) Show that the objective function is convex for all x in the constraint set.