ENGR30002 SM1 2021 Assignment 2

Michael Le

TOTAL POINTS

94 / 100

QUESTION 1

1 Q1 **57 / 60**

✓ **- 0 pts** **Correct answers for (a) - (c)**

a)

- 0 pts a) correct G, $$v_1$$, and $$v_2$$

- 3 pts Incorrect G, $$v_1$$, and $$v_2$$ (all calculations are consistent)

- 2 pts Incorrect $$v_1$$

- 2 pts Incorrect $$v_2$$

- 3 pts No work shown

- 5 pts Only equations (no numerical expressions)

- 22 pts No velocities

- 11 pts No $$v_1$$

- 6 pts all incorrect answers

- 10 pts Pages not included

b)

- 0 pts Correct $$v_{max}$$ and location

- 2 pts Incorrect $$v_{max}$$

- 4 pts No $$v_{max}$$ or incorrect value

- 1 pts No specified location but provide reasonable explanation

- 2 pts Incorrect location

- 3 pts No location

- 8 pts No calculation

c)

- 0 pts Correct expression

- 2 pts Incorrect but minor mistakes

- 4 pts Incorrect expression

- 6 pts Not enough work shown

- 6 pts Incorrect and not enough work shown

- 14 pts No attempt or no work shown

d)

- 0 pts correct description

- 1 pts Partially correct (not full description)

- 3 pts Not enough explanation

✓ **- 3 pts** **Incorrect (or not relevant) explanation**

- 5 pts Incorrect and not enough explanation

- 6 pts No attempt

QUESTION 2

2 Q2 **37 / 40**

- 0 pts all correct answers for (a), (b), and (c)

✓ **- 0 pts** **Correct answers for (a) and (b)**

(a)

- 0 pts Correct velocity profile ($$v_x$$)

- 1 pts Incorrect but minor mistake

- 24 pts incomplete work

- 3 pts Incorrect velocity profile

(b)

- 0 pts Correct expression for the film thickness

- 4 pts incorrect h

- 2 pts Not enough work shown

- 8 pts No attempt

(c)

- 0 pts Correct velocity profile

- 1 pts No sketch but showed all work

- 1 pts Sketches but incorrect velocities

✓ **- 3 pts** **Sketches but no velocities at y=0 and y=h**

- 6 pts No attempt

- 3 pts Incorrect velocity profile

- 3 pts No sketches

- 1 pts incorrect sketch but minor error

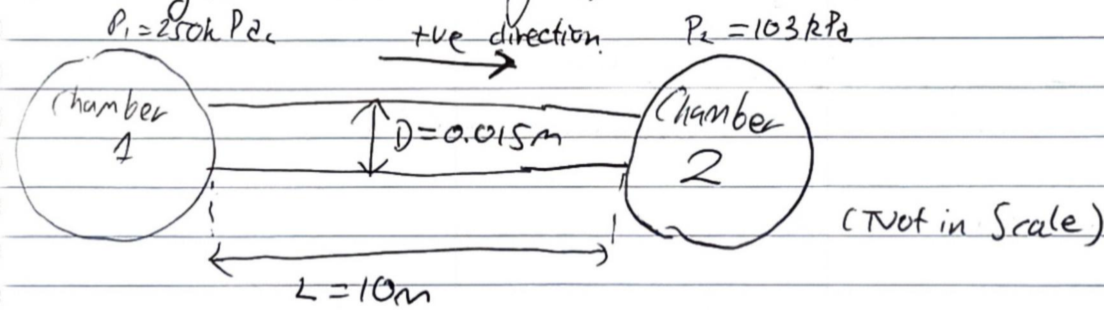
- 4 pts no sketches (or incorrect) and no velocities at y=0 and y=h

Q1a) ENGR30002 Assignment 2, Michael Le (998211)

$L = 10m$
$\Delta z = 0m$ (horizontal pipe).
$ID = D = 0.015m$.
$M = 40 \times 10^{-3}$ kg/mol
$L = 10m$
$P_1 = 250kPa$
$P_2 = 103kPa$
$T = (50 + 273)K = 323K$
$A = \pi \frac{(D^2)}{4} m^2 = \frac{\pi}{4}(0.015)^2 m^2 = 0.0001767 1458 \, m^2$

$\phi = 0.001$, then our fanning friction factor is.
$f_F = 2\phi = 0.002$.

Assuming it is in isothermal flow,



$$\left(\frac{G}{A}\right)^2 = \frac{(P_1^2 - P_2^2)}{\frac{2RT}{M}\left(\ln\left(\frac{P_1}{P_2}\right) + 2 \times \frac{f_F L}{D}\right)}$$

note, $2\phi = f_F$

$$= \frac{(250 \times 10^3)^2 - (103 \times 10^3)^2}{\frac{2 \times 8.314 \times 323}{40 \times 10^{-3}}\left(\ln\left(\frac{250}{103}\right) + \frac{2 \times 2 \times 0.001 \times 10}{0.015}\right)}$$

$= 108759.0904, \ (kg/(m^2 s))^2$

Take the square root,

$\frac{G}{A} = 329.7864315 \ kg/(m^2 s)$

$G = (329.7864315 \times 0.00017671458)$ kg/s
$= 0.05827807297$ kg/s. (mass flow rate).

Now finding the densities at the exit & entry points;

$$P_2 = \frac{\rho_2 RT}{M} \Rightarrow \rho_2 = \frac{P_2 M}{RT} = \frac{103 \times 40 \times (10^{-3} \times 10^3)}{8.314 \times 323}$$

$\rho_2 = 1.534209521$ kg/m^3

$$P_1 = \frac{\rho_1 RT}{M} \Rightarrow \rho_1 = \frac{P_1 M}{RT} = \frac{250 \times 40 \times (10^{-3} \times 10^3)}{8.314 \times 323}$$

$= 3.723809517$ kg/m^3

Now finding the velocities (v_1 & v_2) at entry and exit points respectively.

$$V_1 = \frac{G}{\rho_1 A} = \frac{0.05827807297}{3.723809517 \times 0.00017671458}$$

$= 88.56$ m/s

$$V_2 = \frac{G}{\rho_2 A} = \frac{0.05827807297}{1.534209521 \times 0.00017671458}$$

$= 214.9552846$ m/s ≈ 214.96 m/s.

Q1b)

Sonic velocity, V_{max} velocity. (gas velocity) $= \sqrt{\frac{RT}{M}} = 259.105\ m/s$

At max flow rate (choked flow) the velocity occurs at the end of the pipe in sonic velocity, which is ~~the~~ less than the speed of sound (290 m/s).

Q1c)

$$\frac{P_2^2 - P_1^2}{2(RT/M)} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + 2f_F\frac{L}{D}\left(\frac{G}{A}\right)^2 = 0$$

Imply, $\left(\frac{G}{A}\right)^2 = \frac{P_2^2}{RT/M}$, then,

$$\frac{P_2^2 - P_1^2}{2\cancel{(RT/M)}} + \frac{P_2^2}{\cancel{RT/M}} \ln\left(\frac{P_1}{P_2}\right) + 2f_F\frac{L}{D}\frac{P_2^2}{\cancel{RT/M}} = 0$$

(multiply 2 both sides)

$$P_2^2 - P_1^2 + 2P_2^2 \ln\left(\frac{P_1}{P_2}\right) + 4f_F\frac{L}{D} P_2^2 = 0$$

$\boxed{f_F = 2\phi}$ NOTE

Divide P_2^2 Both sides,

$$\frac{P_2^2}{P_2^2} - \frac{P_1^2}{P_2^2} + \ln\left(\frac{P_1}{P_2}\right)^2 + 8\phi\frac{L}{D} = 0$$

$$\Rightarrow \boxed{\frac{8\phi L}{D} = \left(\frac{P_1}{P_2}\right)^2 - \ln\left(\frac{P_1}{P_2}\right)^2 - 1}$$

Q1(d) Compressible fluids have theoretical maximum velocity, which is the sound speed. (mentioned in Q1(b)).
This means that applying the M.E.B is said to be higher than the speed of sound which is false.
The gas will reach the speed of sound which is described as the choked flow. if the down stream pressure is reduced, When the applicable (seen in part c/proven already) the gas velocity, $\sqrt{\frac{RT}{M}} = V_{gas}$ will reach sonic speed before leaving the pipe results in choked flow.

1 Q1 **57 / 60**

✓ **- 0 pts** **Correct answers for (a) - (c)**

a)

- **- 0 pts** a) correct G, v_1, and v_2
- **- 3 pts** Incorrect G, v_1, and v_2 (all calculations are consistent)
- **- 2 pts** Incorrect v_1
- **- 2 pts** Incorrect v_2
- **- 3 pts** No work shown
- **- 5 pts** Only equations (no numerical expressions)
- **- 22 pts** No velocities
- **- 11 pts** No v_1
- **- 6 pts** all incorrect answers
- **- 10 pts** Pages not included

b)

- **- 0 pts** Correct v_{max} and location
- **- 2 pts** Incorrect v_{max}
- **- 4 pts** No v_{max} or incorrect value
- **- 1 pts** No specified location but provide reasonable explanation
- **- 2 pts** Incorrect location
- **- 3 pts** No location
- **- 8 pts** No calculation

c)

- **- 0 pts** Correct expression
- **- 2 pts** Incorrect but minor mistakes
- **- 4 pts** Incorrect expression
- **- 6 pts** Not enough work shown
- **- 6 pts** Incorrect and not enough work shown
- **- 14 pts** No attempt or no work shown

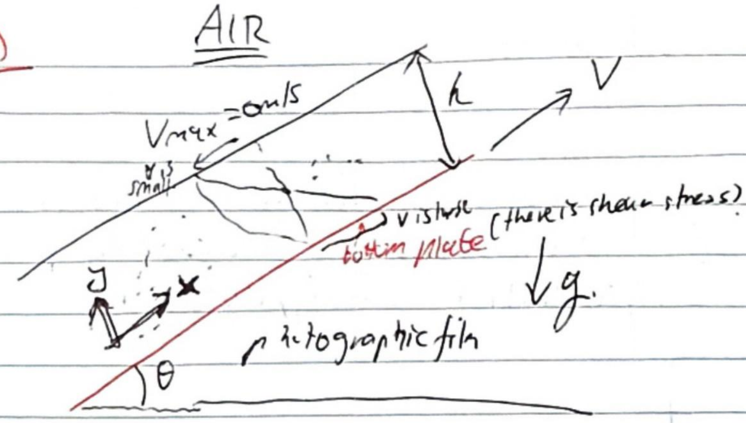
d)

- **- 0 pts** correct description
- **- 1 pts** Partially correct (not full description)
- **- 3 pts** Not enough explanation

✓ **- 3 pts** **Incorrect (or not relevant) explanation**

- **- 5 pts** Incorrect and not enough explanation
- **- 6 pts** No attempt

Q2a)



Our Assumption is that we use cartesian to be our coordinate system, in the x-direction. The driving force is gravity and shear in which direction is the velocity changing the y-direction.

$$\frac{\delta \rho}{\delta t} + \frac{\delta}{\delta x}(\rho V_x) + \frac{\delta}{\delta y}(\rho V_y) + \frac{\delta}{\delta z}(\rho V_z) = 0$$

$$\rightarrow \frac{\delta V_x}{\delta x} = 0 \quad — (1) \quad \text{Equation of continuity,}$$

Navier-Stokes Equation,

$$\rho\left(\cancel{\frac{\delta V_x}{\delta t}} + \cancel{V_x \frac{\delta V_x}{\delta x}} + \cancel{V_y \frac{\delta V_x}{\delta y}} + \cancel{V_z \frac{\delta V_x}{\delta z}}\right) = -\frac{\delta P}{\delta x} + \mu\left[\cancel{\frac{\delta^2 V_x}{\delta x^2}} + \frac{\delta^2 V_x}{\delta y^2} + \cancel{\frac{\delta^2 V_x}{\delta z^2}}\right] + \rho g, \quad — (2)$$

$$\rho\left(\cancel{\frac{\delta V_y}{\delta t}} + \cancel{V_x \frac{\delta V_y}{\delta x}} + \cancel{V_y \frac{\delta V_y}{\delta y}} + \cancel{V_z \frac{\delta V_y}{\delta z}}\right) = -\frac{\delta P}{\delta y} + \mu\left[\cancel{\frac{\delta^2 V_y}{\delta x^2}} + \cancel{\frac{\delta^2 V_y}{\delta y^2}} + \cancel{\frac{\delta^2 V_y}{\delta z^2}}\right] + \rho g, \quad — (3).$$

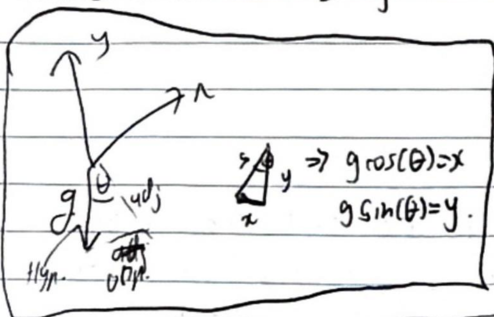
$$\rho\left(\cancel{\frac{\delta V_z}{\delta t}} + \cancel{V_x \frac{\delta V_z}{\delta x}} + \cancel{V_y \frac{\delta V_z}{\delta y}} + \cancel{V_z \frac{\delta V_z}{\delta z}}\right) = \cancel{-\frac{\delta P}{\delta z}} + \mu\left[\cancel{\frac{\delta^2 V_z}{\delta x^2}} + \cancel{\frac{\delta^2 V_z}{\delta y^2}} + \cancel{\frac{\delta^2 V_z}{\delta z^2}}\right] + \cancel{\rho g},$$

Simplied ~~NS~~ NSE,

$$\frac{\delta P}{\delta x} = \mu \frac{\delta^2 V_x}{\delta y^2} + \rho g_x \quad — (5). \text{ dervived from (2).}$$

$\frac{\delta p}{\delta y} = \rho g y$ —(6) Derived from (3).

Our x-axis is not a horizontal / ~~...~~ incline with gravity it is tilted by the angle θ. Therefore we use geometry to solve the x- & y- components of gravity.



$\boxed{\frac{\delta P}{\delta x} = \mu \frac{\delta^2 V_x}{\delta y^2} - \rho g \sin\theta}$ —(7)

2D equations in $\mathbb{R}^2$

$\boxed{\frac{\delta P}{\delta y} = -\rho g \cos\theta}$ —(8)

from (8) Integrate,

$\frac{\delta P}{\delta y} = -\rho g\cos\theta \rightarrow P = \int -\rho g\cos\theta \, dy$

$\Rightarrow \boxed{P = -(\rho g\cos\theta)y + f(x)}$ —(9) Apply vector calculus.

we have derived from pressure w.r.t x (with respect to x).

$\frac{\delta P}{\delta x} = 0$, If $f(x)=C$, $C \in \mathbb{R}$, c is arbitrary constant.

where, $P = P_{atm}$ & $y=h$ (shear stress is 0).

$\boxed{P_{atm} = -(\rho g\cos\theta)h + f(x)}$ —(10).

Now,

$$\cancel{\frac{\delta P}{\delta x}} + \rho g \sin\theta = \mu \frac{\delta^2 V_x}{\delta y^2}$$

$$\boxed{\alpha = \frac{\rho g \sin\theta}{\mu}, \text{ is constant } \alpha \in \mathbb{R}.}$$

$$\frac{\delta^2 V_x}{\delta y^2} = \alpha, \Rightarrow \frac{\delta V_x}{\delta y} = \alpha y + c_1, \quad c_1 \in \mathbb{R}.$$

Because, at $y = H$, $\frac{\delta V_x}{\delta y} = 0$, (Initial conditions).

shear stress is proportional to shear rate for Newtonian fluids. Since the air is constant with the upper surface of the film has a smaller viscosity than the fluid, Assume there is no shear stress at the interface, which is proportional to shear rate, shear rate is 0. $(\delta V_x / \delta y = 0)$

We integrate again,

$$V_x = \alpha\left(\frac{1}{2}y^2 - Hy\right) + c_2 \qquad (y = 0, V_x = V)$$

Initial conditions.

$$V_x = V - \alpha y\left(H - \frac{1}{2}y\right)$$

Q2b)

The flow rate of the fluid being pulled down by gravity is equal to the flow rate being dragged up by the velocity of the coated surface.

We must find the volumetric flow rate (Q) & find the value of H where the Q is 0 m³/s.

Finding the expression for Q we need to integrate over the cross sectional area of the flow.

$$Q = B\int_0^H V_x \, dy$$, "B" is the width of the surface being coated in the z-direction.

Plug in V_z $Q = B\int_0^H \left(V - \alpha H y + \frac{\alpha}{2} y^2\right) dy.$

$$Q = B\left[Vy - \alpha\frac{Hy^2}{2} + \frac{\alpha}{6}y^3\right]_{y=0}^{y=H}$$

$$= B\left[VH - \alpha\frac{H^3}{2} + \frac{\alpha H^3}{6} - \cancel{0+0-0}\right]$$

$$= B\left[VH - \alpha H^3\left(\frac{1}{3}\right)\right].$$

solve for H,

$$H = \sqrt{\frac{3V}{\alpha}} = \sqrt{\frac{3V\mu}{\rho g \sin\theta}}.$$

Q2c)

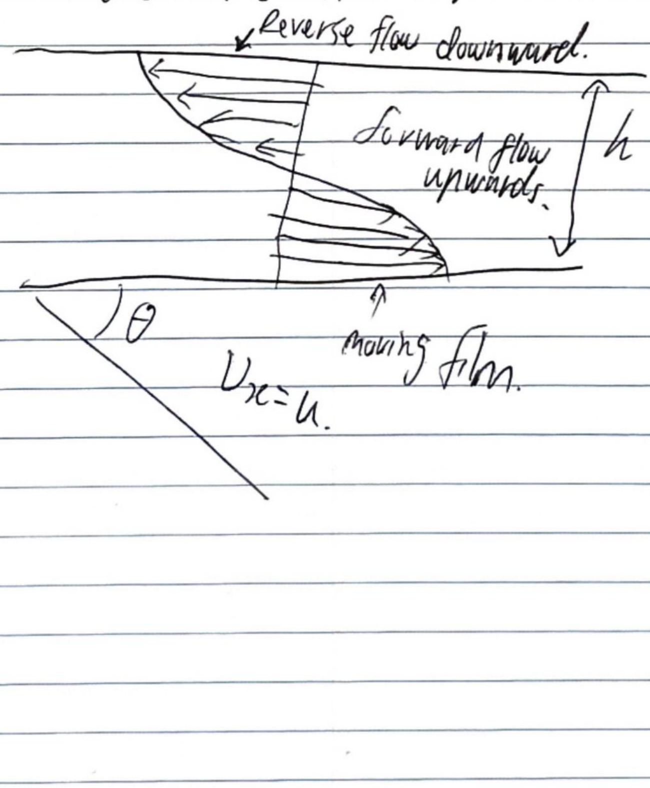
$Q = \int_0^h v_x \, dy,$ $\int_0^h (u-\alpha y)(h - y/2)\,dy$

$\boxed{h=H}$

$uh - \frac{1}{3}\alpha h^3 = 0 \Rightarrow h = \sqrt{3u/\alpha}.$

$\Rightarrow \alpha$ is constant, $v=u$ steady state.

Here is a better illustration of the velocity profile.



2 Q2 **37 / 40**

- **0 pts** all correct answers for (a), (b), and (c)

✓ - **0 pts** **Correct answers for (a) and (b)**

(a)

- **0 pts** Correct velocity profile ($$v_x$$)
- **1 pts** Incorrect but minor mistake
- **24 pts** incomplete work
- **3 pts** Incorrect velocity profile

(b)

- **0 pts** Correct expression for the film thickness
- **4 pts** incorrect h
- **2 pts** Not enough work shown
- **8 pts** No attempt

(c)

- **0 pts** Correct velocity profile
- **1 pts** No sketch but showed all work
- **1 pts** Sketches but incorrect velocities

✓ - **3 pts** **Sketches but no velocities at y=0 and y=h**

- **6 pts** No attempt
- **3 pts** Incorrect velocity profile
- **3 pts** No sketches
- **1 pts** incorrect sketch but minor error
- **4 pts** no sketches (or incorrect) and no velocities at y=0 and y=h