ENGR30002
Fluid Mechanics

Semester 1, 2020

What will we learn in Fluid Mechanics?

In this class we will study the physics of fluids

How fluids behave when static

How fluids behave when flowing

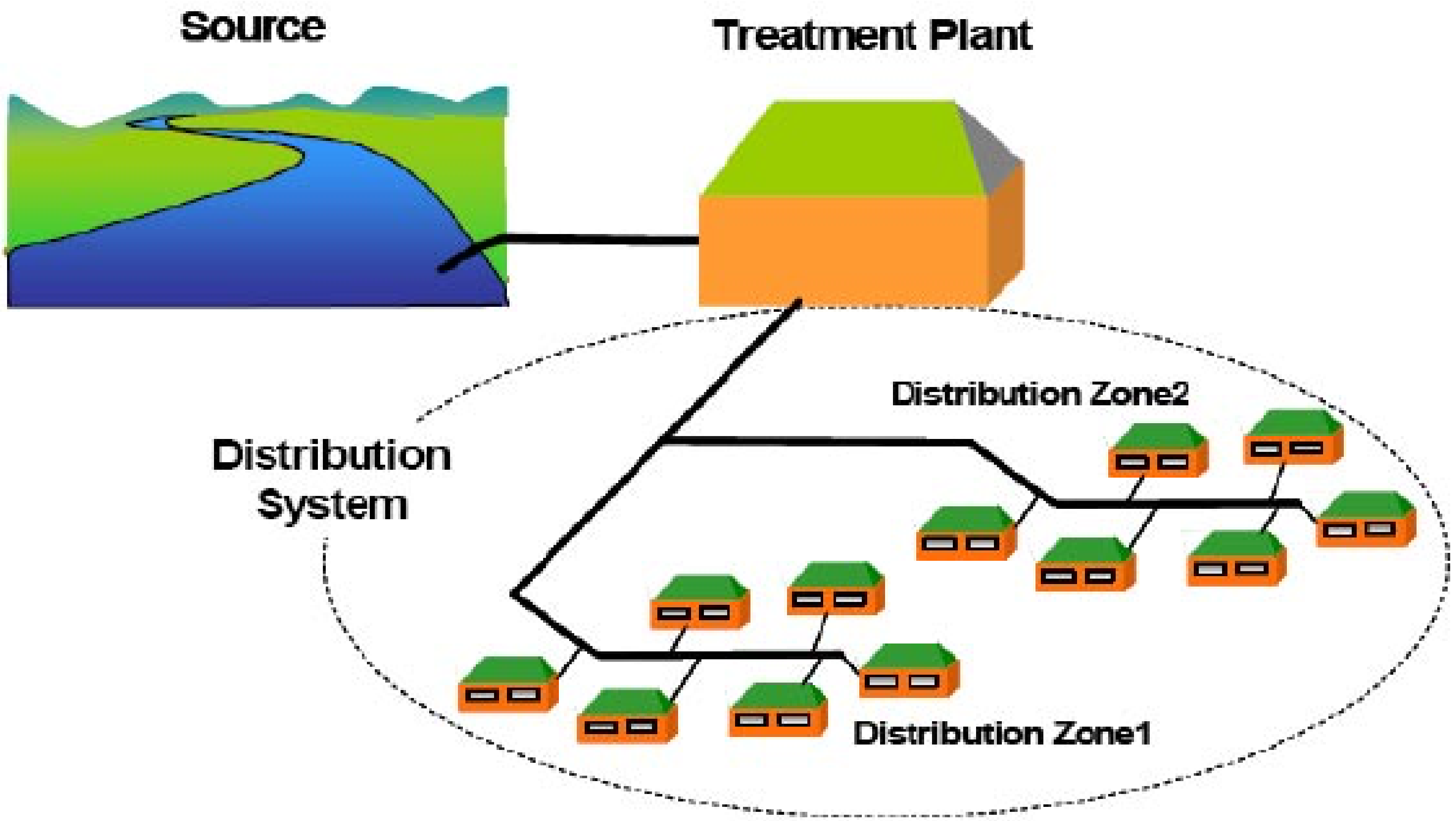
Why do we care about fluid flow?

Controlling fluid flow is critical for a practicing chemical or infrastructure engineer

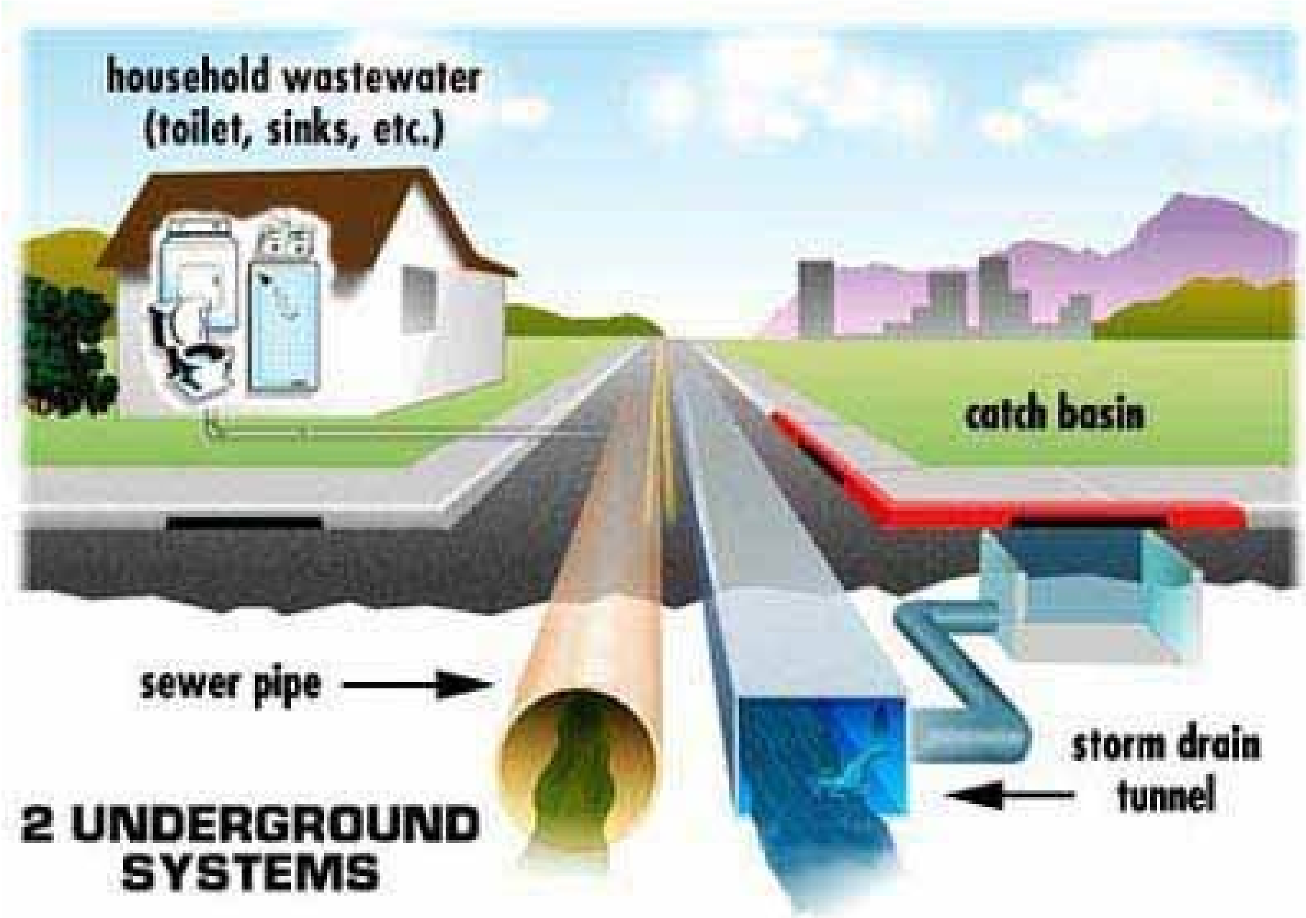
Essentially all projects you work on, will have some form of flowing liquid involved

In this class we will teach you how to achieve the desired flow

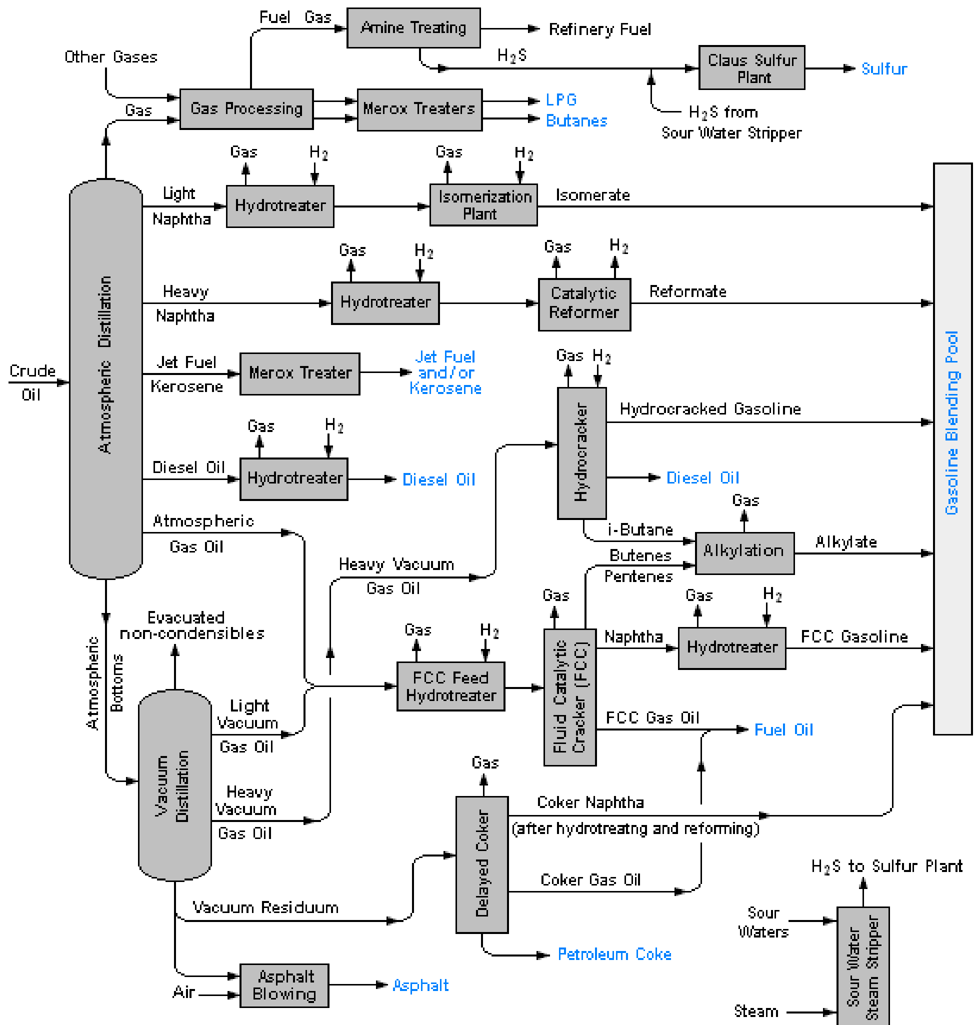
Water distribution system



Water drainage system



Process flow diagram for crude oil processing



- Finished products are shown in blue
- Sour waters are derived from various distillation tower reflux drums in the refinery
- The "other gases" entering the gas processing unit includes all the gas streams from the various process units

Topics we will cover in this subject

Fluid Mechanics

Fluid statics

Static pressure
Forces on walls
Buoyancy
Manometry

Fluid dynamics

Fluid flow (liquid and gas)
Mathematics of flow
- Conservation of mass
- Conservation of momentum
- Conservation of energy

3D modeling of flow

Equipment

Pipes
Open channels
Pumps
Mixing tanks

Fluid Mechanics

Topic 1

Hydrostatics

Learning objectives

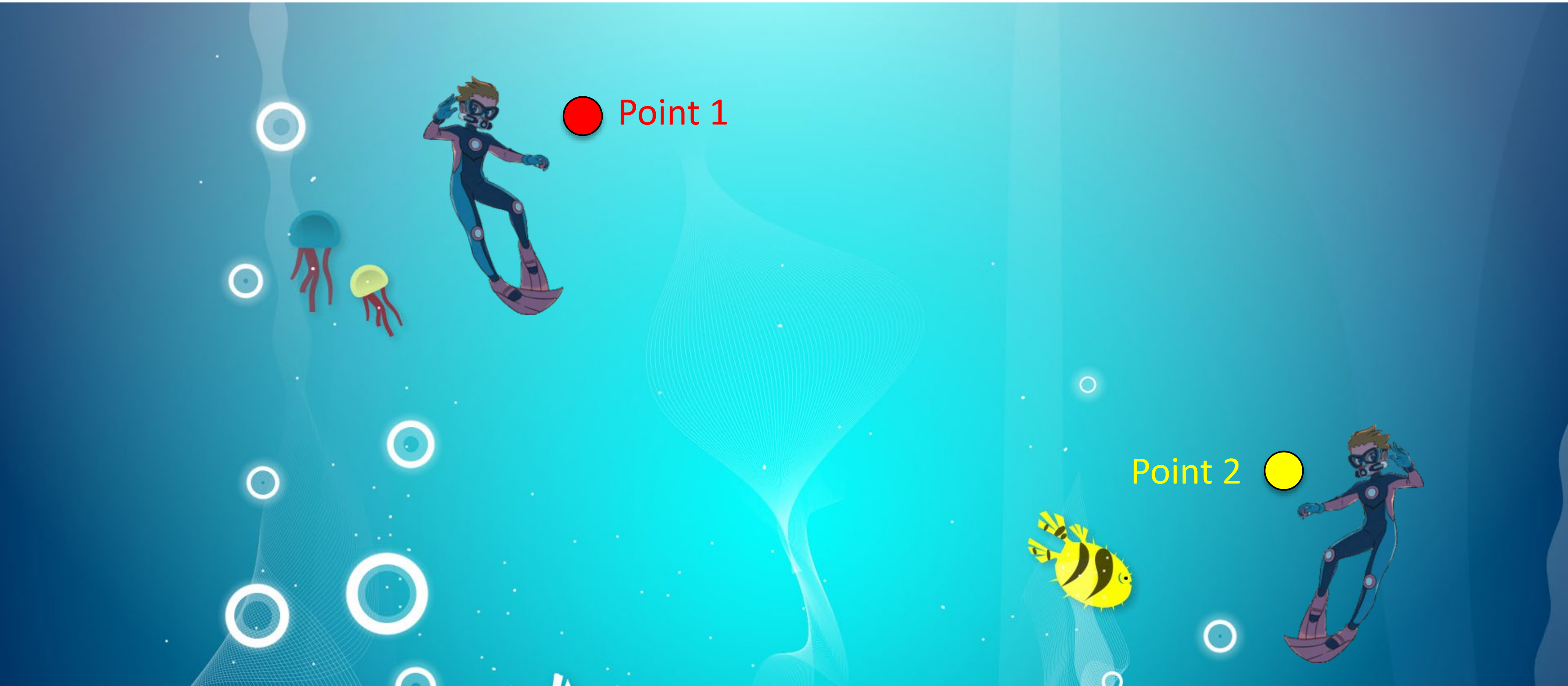
By the end of this lesson, students should be able to

- Explain the pressure gradients in a static fluid in the x-, y-, and z-directions
- Derive the pressure gradients in a static fluid in the x-, y-, and z-directions
- Explain the physics behind how a u-tube manometer and a differential manometer function
- Explain the difference (verbally and mathematically) between absolute pressure and gauge pressure
- Use manometers to calculate absolute pressure and/or the pressure difference in a system

Fluids at rest

Before we start talking about **fluid dynamics** (**fluid flow**), we'll first discuss **fluid statics** (**fluids at rest**)

The first concept we're going to introduce is **hydrostatic pressure**, or the pressure variations that occur within a stationary body of fluid



What is pressure?

The first question we need to answer is **what is pressure?**

Pressure is defined as the physical force exerted on an object

What causes pressure variations within a static fluid?

The variations of pressure with direction and depth in a fluid will be examined. These results are known as Pascal's law, which states that the pressure at a point in a static fluid is independent of direction. In other words, pressure is a scalar for fluids.

Is our diver experiencing **more**, **less**, or the **same** pressure when he is at point 2 compared to point 1?

MORE, because the driver at point 2 experience more depth (increase of direction) compare to where diver is at point 1.

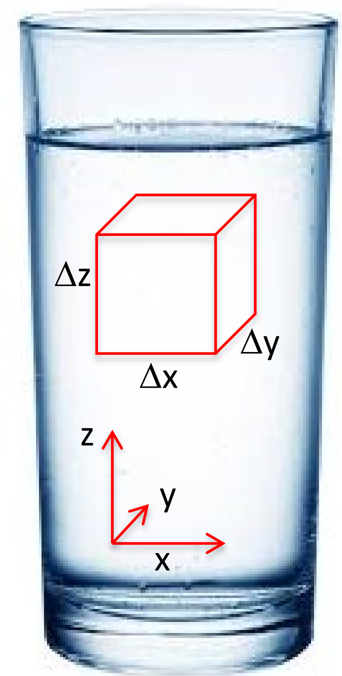
How does each component of pressure change as our diver moves from point 1 to point 2?

Z direction only, x and y are fixed points or remain the samen!

x-component of pressure:
y-component of pressure:
z-component of pressure:

We know this intuitively, now let's see if we can derive this mathematically

Force balances on static fluids



We have a glass of water

- The fluid within the glass is stationary
- Let's consider the fluid contained within a **control volume**
- The dimensions of the control volume are Δx, Δy, Δz

Let us perform a **force balance** on the fluid in the volume

- Since the fluid is stationary, the forces acting on opposing sides of the cube must be equal (Newton's first law of motion)
  —> Should it be from Newton's Third Law of Motion not first in that order!
- This means pressures acting on opposing sides of the cube must be equal

First, let us look at the **x-direction**

$$F_{x1} = F_{x2} \Rightarrow P_{x1}\Delta y \Delta z = P_{x2}\Delta y \Delta z \Rightarrow P_{x1} = P_{x2}$$

$$\Rightarrow P_{x2} - P_{x1} = 0 \Rightarrow \frac{P_{x2} - P_{x1}}{\Delta x} = 0 \xrightarrow[\Delta x \to 0]{\text{Limit}} \boxed{\frac{\delta P}{\delta x} = 0}$$

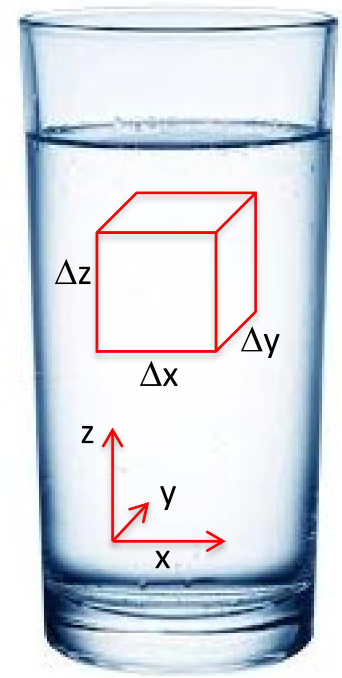
No pressure gradients in the x-direction

Force balances on static fluids

We can use the same logic in the **y-direction**

$$F_{y1} = F_{y2} \quad \Longrightarrow \quad \boxed{\frac{\delta P}{\delta y} = 0}$$

No pressure gradients in the y-direction



The **z-direction** is more complicated due to gravity

- Quantify the downward force acting on the fluid in the box

$$F_{downwards} = P_{z2}\Delta x \Delta y + \rho g \Delta x \Delta y \Delta z$$

- Quantify the upwards force acting on the fluid in the box

$$F_{upwards} = P_{z1}\Delta x \Delta y$$

- Again, since the fluid in the box is stationary, these forces must be equal

Delta depends on the magnitude of the direction where the ice is going!

$$P_{z2}\Delta x \Delta y + \rho \cancel{g \Delta x} \Delta y \Delta z = P_{z1}\Delta x \Delta y$$

This part does not matter

Force balances on static fluids

Let's simplify our force balance in the **z-direction**

$$P_{z2}\cancel{\Delta x\Delta y}+\rho g\cancel{\Delta x\Delta y}\Delta z=P_{z1}\cancel{\Delta x\Delta y}$$

$$P_{z2}+\rho g\Delta z=P_{z1}$$

$$\frac{P_{z2}-P_{z1}}{\Delta z}=-\rho g$$

Limit $\Delta z \rightarrow 0$

$$\frac{\delta P}{\delta z}=-\rho g$$

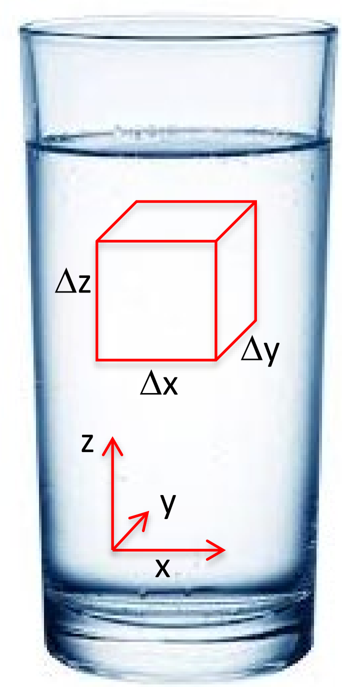
+ve

due to gravity!!

There is a pressure gradient in the z-direction and it is equal to -ρg

$\Delta \rho$

Taking absolute value



Calculating pressure differences in static fluids

Let's use these relationships to calculate the pressure difference experienced by our diver

- No pressure differences occur when he moves in the x-direction or y-direction

However, he does experience a difference in pressure as he descends deeper into the water

- We can quantify that change in pressure by integrating the dP/dz relationship we found on the previous page

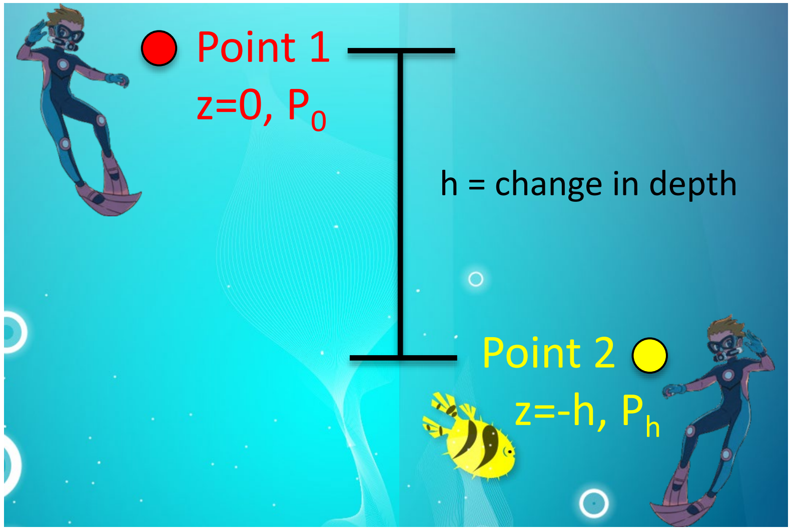
$$\frac{\delta P}{\delta z} = -\rho g \Rightarrow \int_{P_h}^{P_0} dP = -\rho g \int_{-h}^{0} dz$$

$$P_0 - P_h = -\rho g h$$

$$\boxed{\Delta P = \rho g h}$$

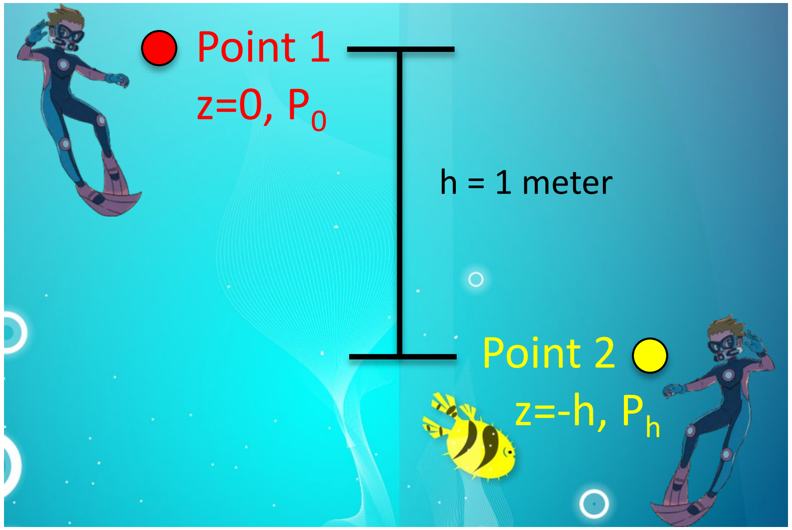
Pressure **increases** with depth

This is referred to as **static pressure**



Calculating pressure differences in static fluids

How much does the pressure increase for each additional meter that the diver descends?



$$\Delta P = \rho g h$$

ρ = 1000 kg/m^3
g = 9.8 m/s^2
h = 1 m

$$\Delta P = 9800\frac{kg}{ms^2} = 9800\boxed{\frac{N}{m^2}} = 9800\boxed{Pa} = 9.8kPa$$

Challenge question 1.1

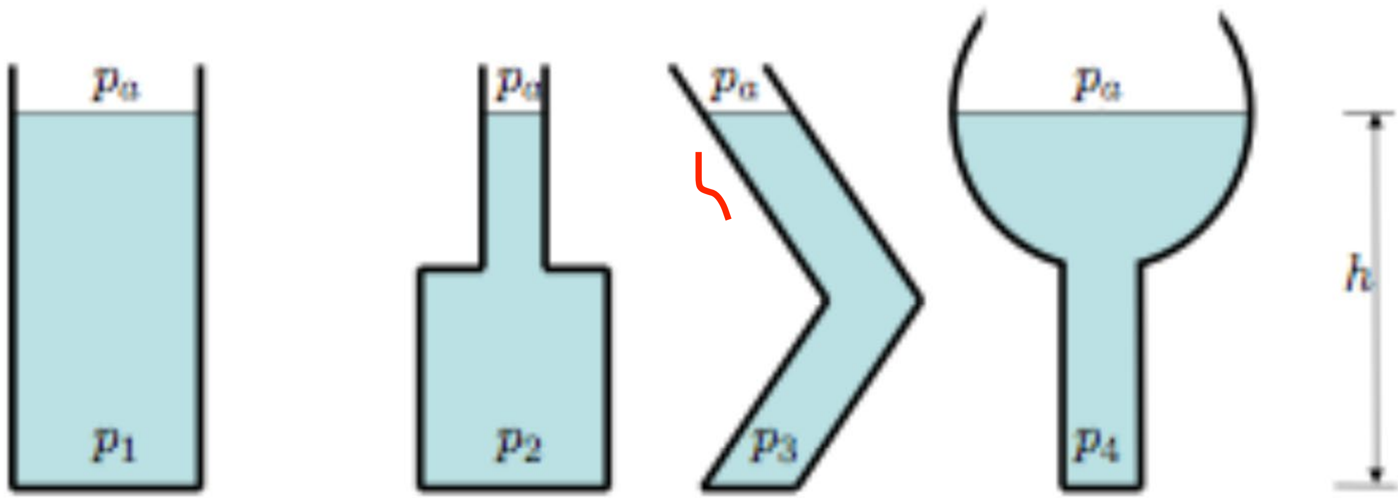
What absolute pressure does the diver experience if he is 3 meters below the surface of the water? Using Newtons 3rd Law

$P(0) - P(1) = 0 - pgh = - 1000 * 9.8 * 3 = -29400$

$\Delta P = 29400$Pascals

The hydrostatic paradox

We have four vessels of varying geometry that are all filled with water



Which pressure is highest: P_1, P_2, P_3, P_4?

From my memory, P1,P2,P4 below the surface of the water are the same in terms of their depth, same static pressure.
Not for P3 Because of the shape of the tube is not vertical compared to the others which involves using trigonometry!!

Do all fluids create the same of pressure?

Three parameters are needed to calculate pressure within a fluid

- Fluid **height** (h [=] m)
- Acceleration due to **gravity** ($g = 9.8\ m/s^2$) constant does not change (Earth's gravity)
- Fluid **density** (ρ [=] kg/m^3)

Therefore, if we have fluids with different densities, the amount of pressure they create will vary, for a given height of fluid

Challenge question 1.2

Each meter of water produces 9.8 kPa of pressure (ρ_{water} = 1000 kg/m^3). What height of mercury is need to produce the same amount of pressure (ρ_{Hg} = 13594 kg/m^3)?

9800 = 1000 * 9.8 * 1 (To Verify)

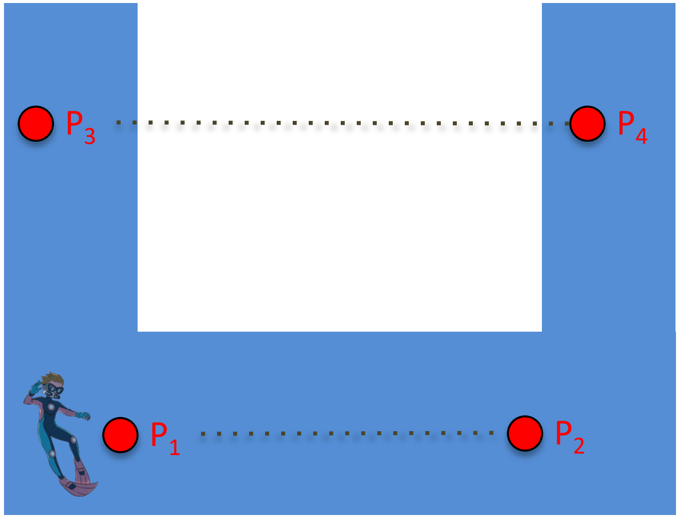
$$\Delta P = \rho g h$$

9800 = 13594*9.8*h

h = 0.0736m

Static pressure in complex shapes

Our diver is diving in a cave that contains static water. The cave has two exits to the atmosphere as shown below



- What is the pressure at each cave entrance?

Both 0Pa since it's top of the water surface!!
NOT 0Pa Assume there is atmospheric pressure above sea level is 101.325kPa?!

- What is the relationship between P_1 and P_2?

P1 and P2 Static Pressures are the same

- What is the relationship between P_3 and P_4?

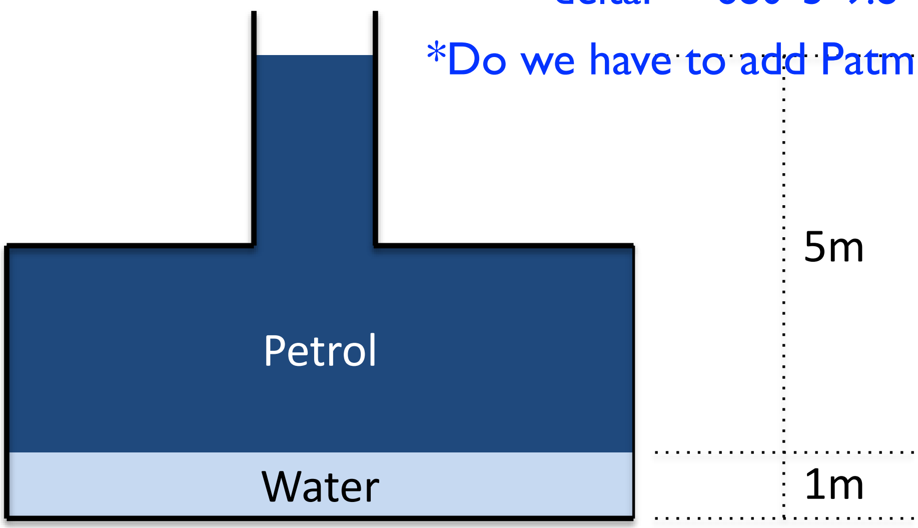
Both have the same Static Pressures

Example problem #1.1

Your company owns a buried petrol storage tank. Because of a leak, water has seeped into the tank as shown in the diagram below. Determine the gauge pressure at the petrol-water interface and at the bottom of the tank. The specific gravity of petrol is 0.68 and the density of water is 1000kg/m^3.

deltaP = 680*5*9.8 + 1000*1*9.8 = 43120Pa = 43.120kPa

*Do we have to add Patm since it's open at the top? to my final answer?



Mini-summary

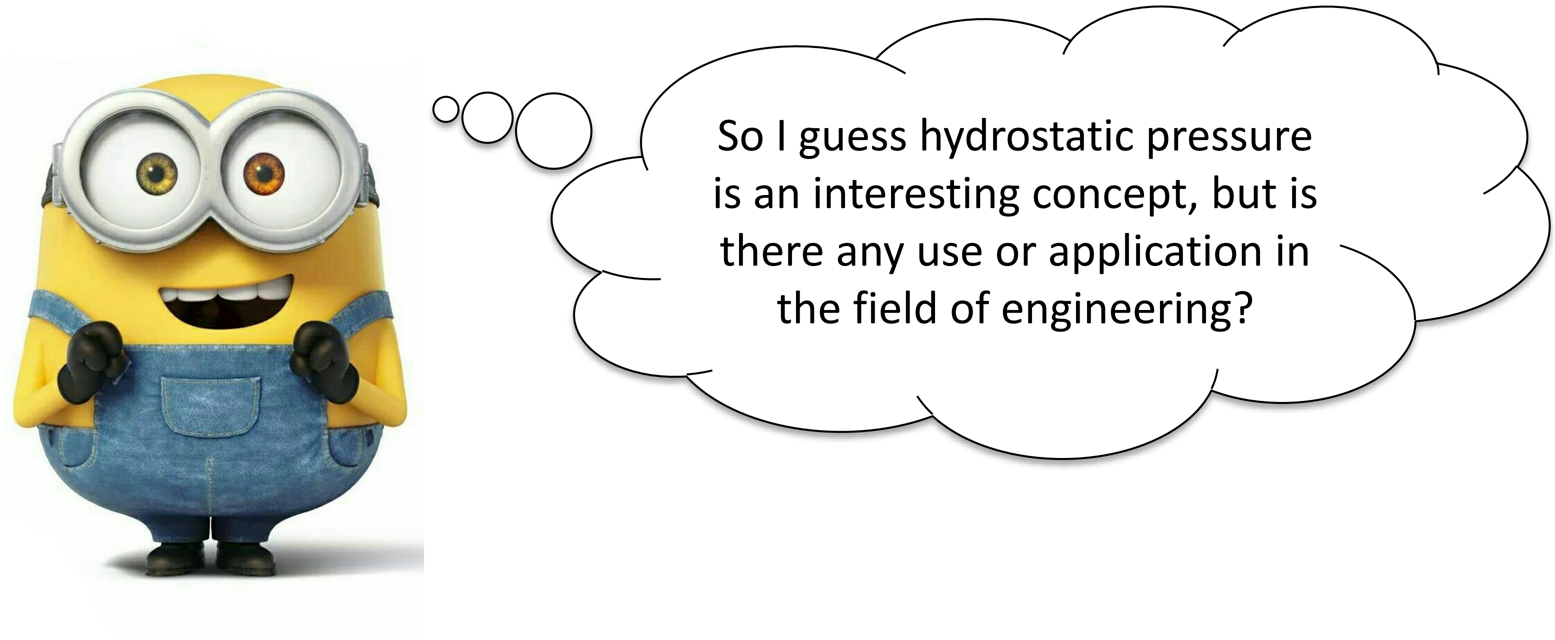
Pressure gradients exist within stationary fluids

- No gradient in the x-direction
- No gradient in the y-direction
- There is a gradient in the z-direction

The pressure gradient in the z-direction is due to gravity acting on the volume of fluid above you

- This is referred to as hydrostatic pressure
- The pressure change from a reference point is proportional to Δz
- The pressure change is also proportional to the density of the fluid
- The change in pressure isn't dependent on how much fluid is above you, only the height of that fluid

Applications of hydrostatic pressure?

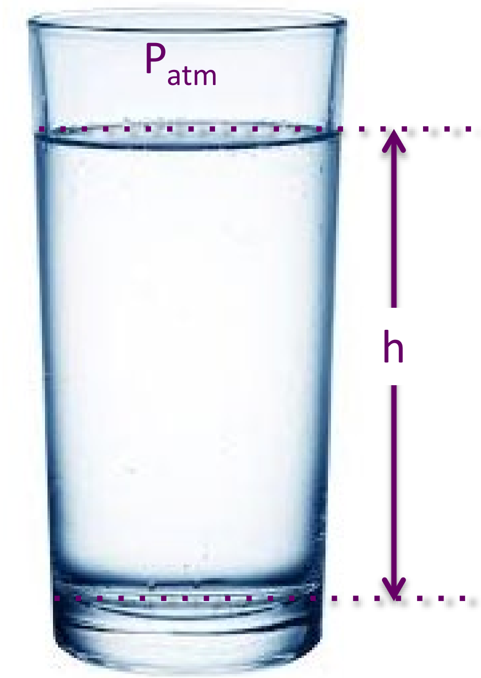


There are several useful applications of hydrostatics in engineering

- Forces on walls (dams/fluid storage vessels)
- Buoyancy
- **Manometry**
  - Manometry uses devices called **manometers** to measure pressure/pressure differences within a system
  - This is accomplished by looking at differences in height of a column of fluid and then using the concept of hydrostatic pressure to calculate a pressure difference

Absolute and gauge pressure

Sometimes we will talk about pressure in terms of **absolute pressure** and at other times we will talk about pressure in terms of **gauge pressure**



- **Gauge pressure** is the pressure **above** atmospheric pressure

$$P_g = \rho g h$$

- **Absolute pressure** is the actual pressure, so it is the gauge pressure plus atmospheric pressure

$$P_a = P_{atm} + \rho g h$$

Challenge problem 1.3

Our diver is now exploring an underground passage filled with static water. One entrance to the passage is open to the atmosphere and the other leads to a fully enclosed secret cave

- What is the relationship between the pressure in the atmosphere and the pressure in the cave?

One side is open and the other is closed! (i.e. the pressures are different top of the water surface!)

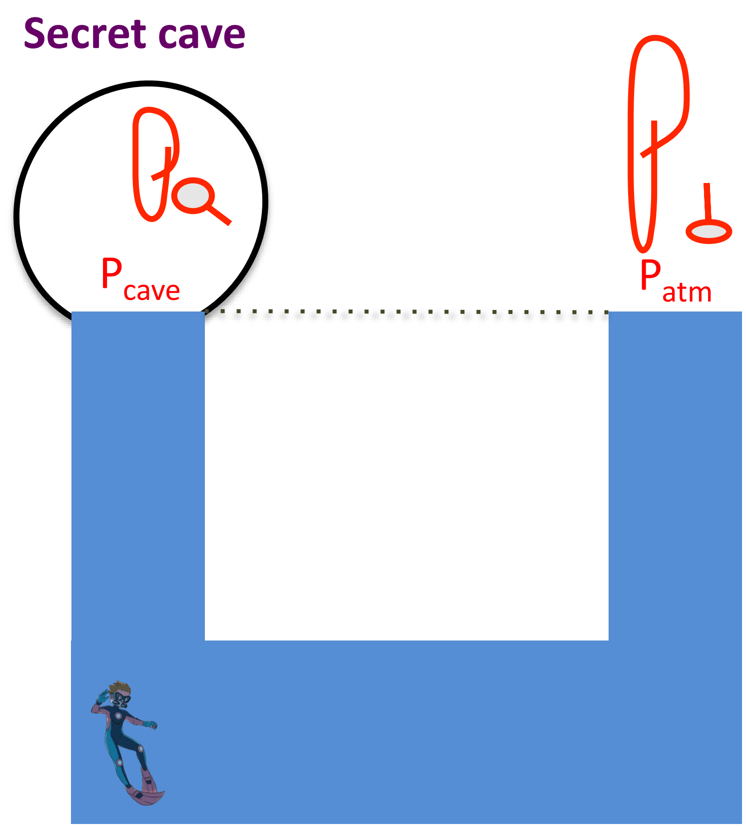
- What is the pressure in the cave?

$$P_a = P_{cave} + \rho g h$$

$$P_b = P_{atm} + \rho g h$$

Patm has more pressure than the Pcave!

$$P_b > P_a$$



Challenge problem 1.4

Our diver is now exploring an underground passage filled with static water. One entrance to the passage is open to the atmosphere and the other leads to a fully enclosed secret cave

- What is the relationship between the pressure in the atmosphere and the pressure in the cave?

  The Pressure is open (atm) 1.5m below the (Secret Save) which is closed!.
  —> Answer this better with the formula given in slide 24!

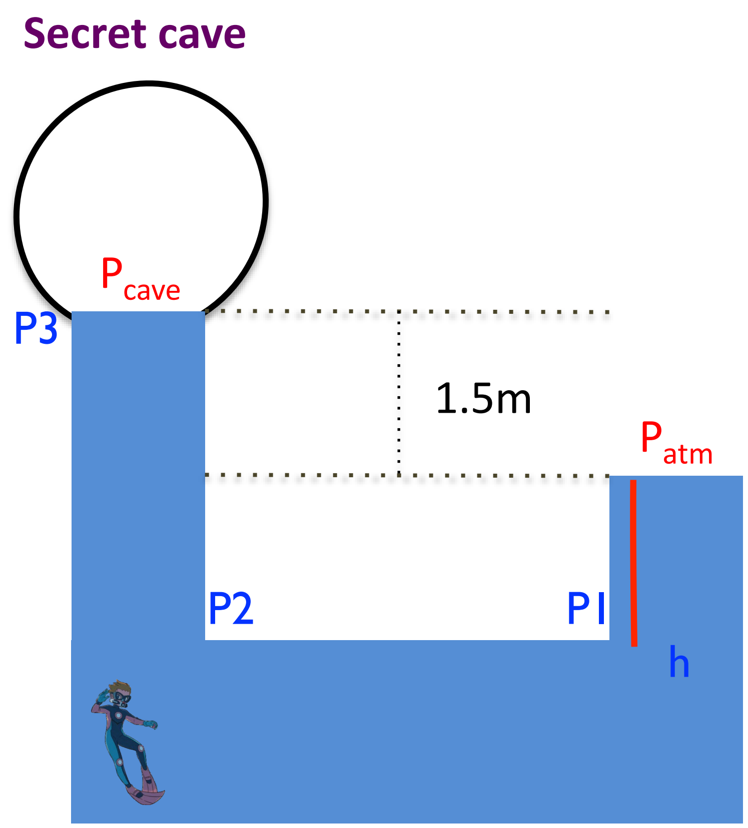
- What is the pressure in the cave?

-From the slide 24

Patm + pgh =P1=P2

—> Pcave = P2 - pg(h+1.5)

*Do we assume h= 1.5m?



Challenge problem 1.5

Our diver is now exploring an underground passage filled with static water. One entrance to the passage is open to the atmosphere and the other leads to a fully enclosed secret cave

- What is the relationship between the pressure in the atmosphere and the pressure in the cave?

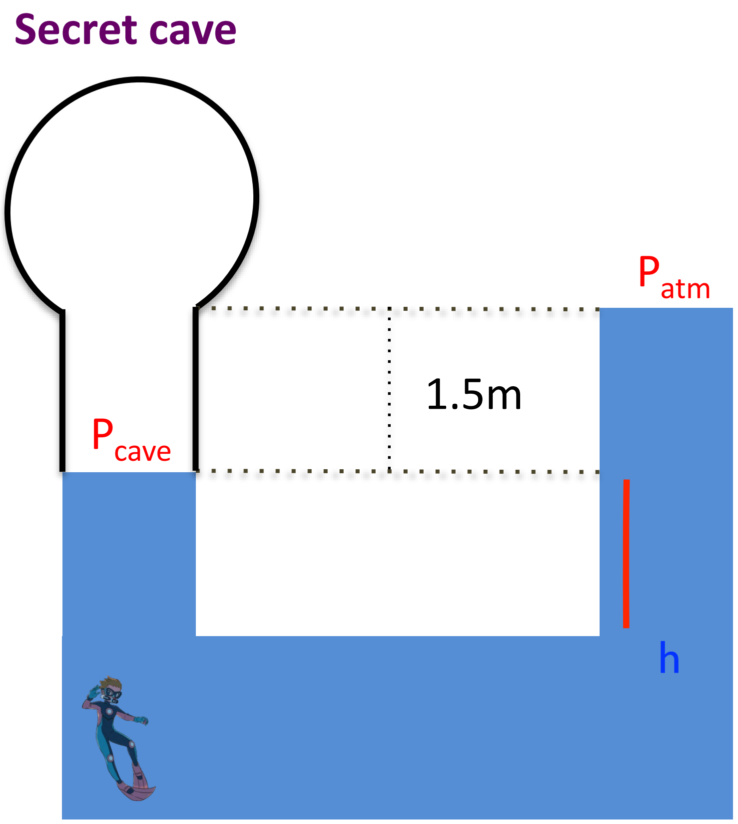
  The Pressure is open (atm) 1.5m above the which is closed (Secret Cave)!.
  —> Answer this better with the formula given in slide 24!

- What is the pressure in the cave?

  *Do we assume h is 1.5m?

  Patm + pg(h + 1.5) = Pbtm
  Pcave + pgh = Pbtm

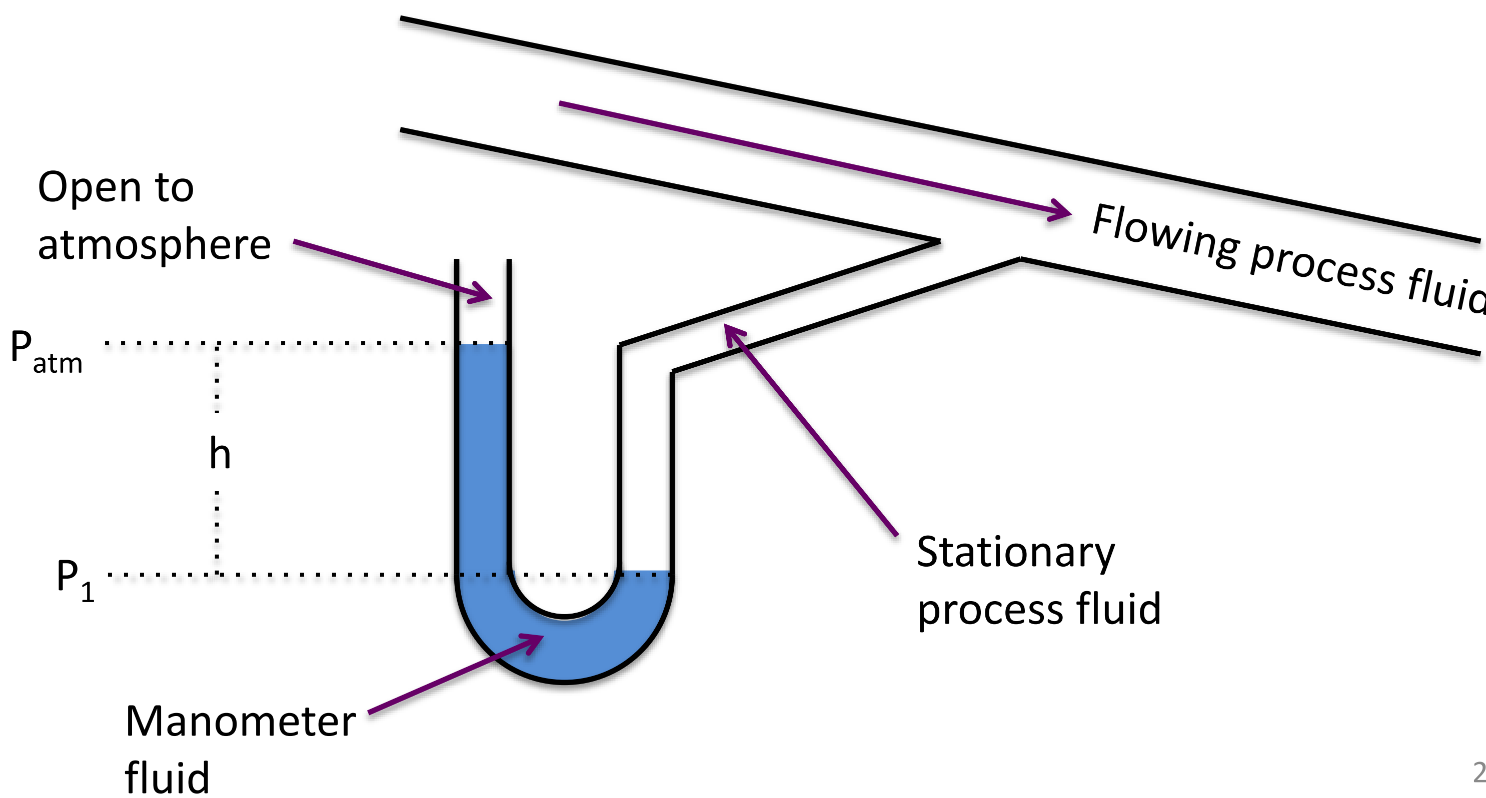
  —> Pcave = Pbtm - pgh



Manometry in engineering

Most of this course will talk about fluid flowing through pipes

- This flow is usually driven by pressure
- Manometry provides a method of calculating the pressure (or pressure drop) experienced by a flowing fluid
- In fact, manometers are one of the first pressure gauges



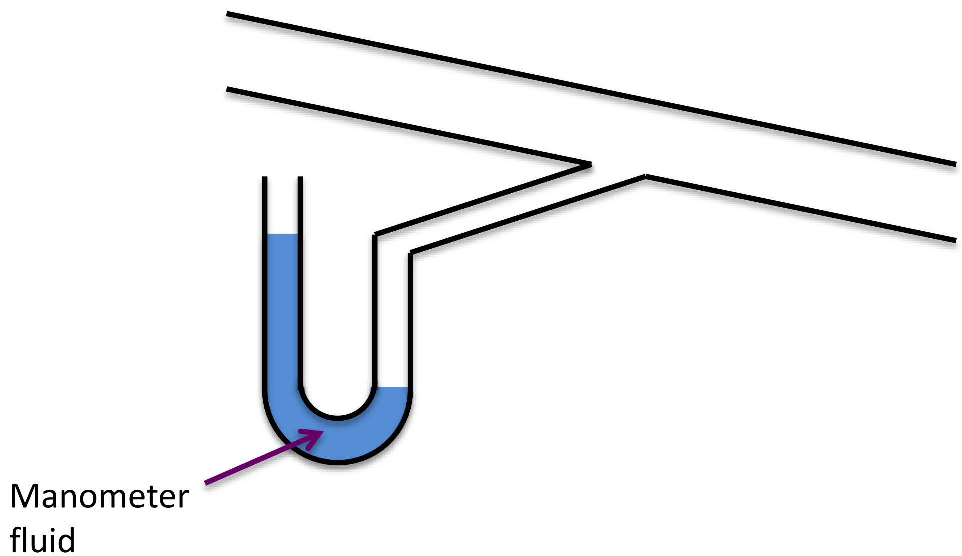
Manometer fluid

What characteristics are necessary for a manometer fluid?

- Immiscible with the process fluid
- Denser than the process fluid
- Incompressible (meaning density does not change with pressure)

What is a common manometer fluid?

- Mercury (Hg)
- ρ_{Hg} = 13,534 kg/m^3



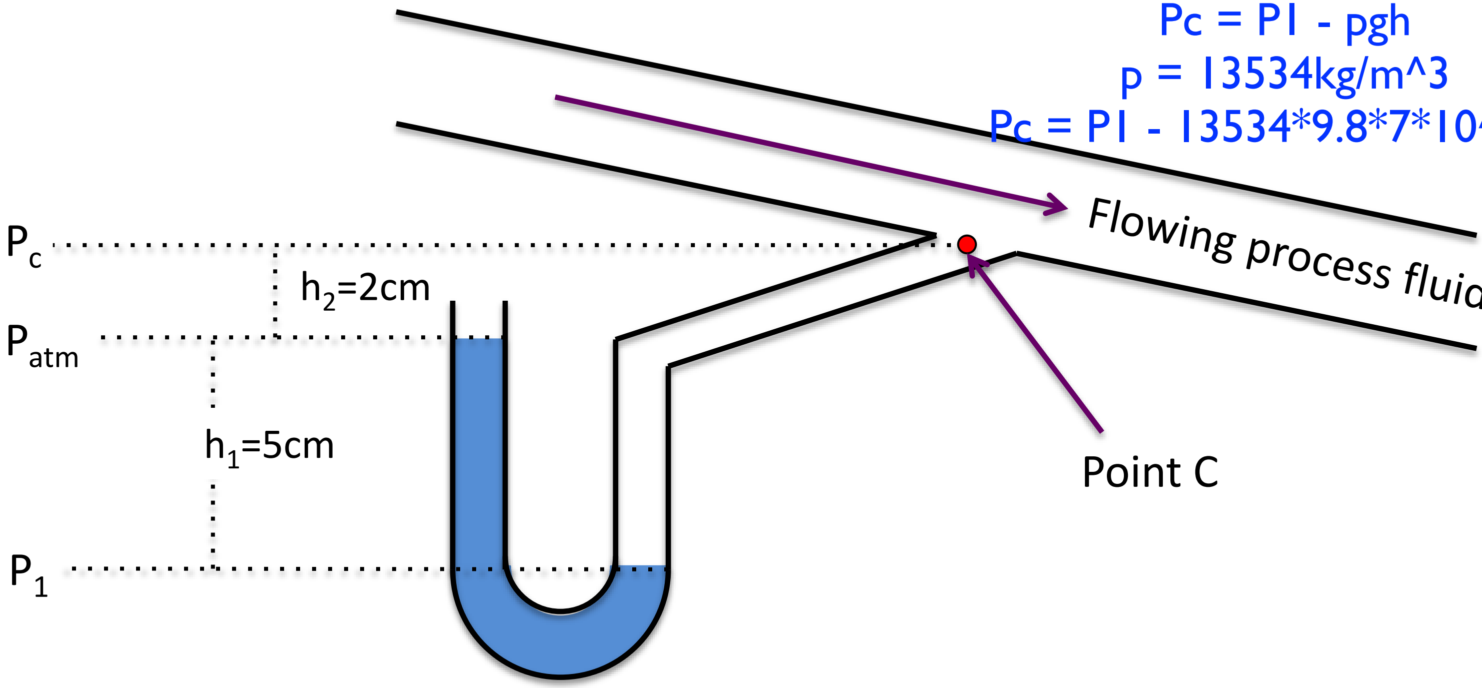
Manometers to calculate absolute pressure

You have water flowing through a pipe. The manometer fluid is mercury. What is the absolute pressure at **Point C**?

- ρ_{Hg} = 13,534 kg/m^3
- ρ_{water} = 1,000 kg/m^3

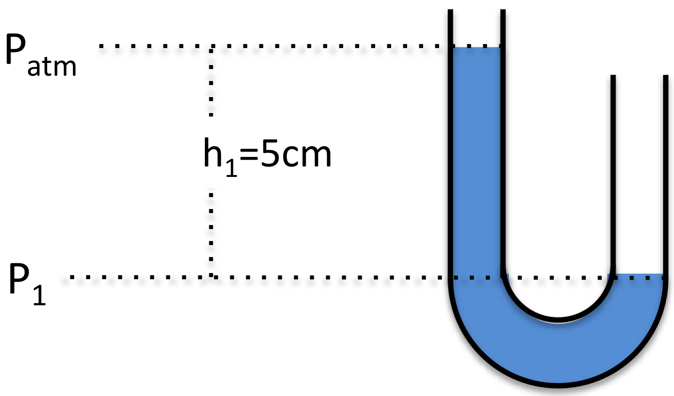
The Water
Patm = 101.325kpa
P1 = Patm + pgh
= 101.325kpa + 9.8*5*10^-2kPa

Pc = P1 - pgh
p = 13534kg/m^3
Pc = P1 - 13534*9.8*7*10^-2Pa



Challenge problem 1.6

What would the height of h_1 be if you were using mineral oil as your manometer fluid (ρ_{oil} = 800 kg/m^3) instead of mercury?

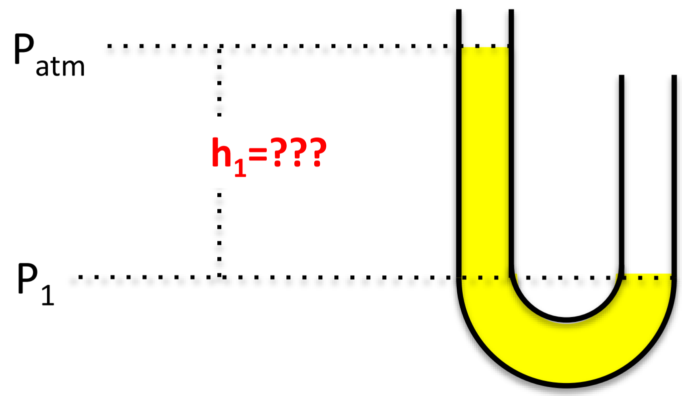


Mercury

Patm = 101325Pa

P1 = Patm + 13534*9.8*5*10^-2 Pa

*Do we assume the
Pressure at P1in
Mercury are the same
as the mineral Oil?!



Oil

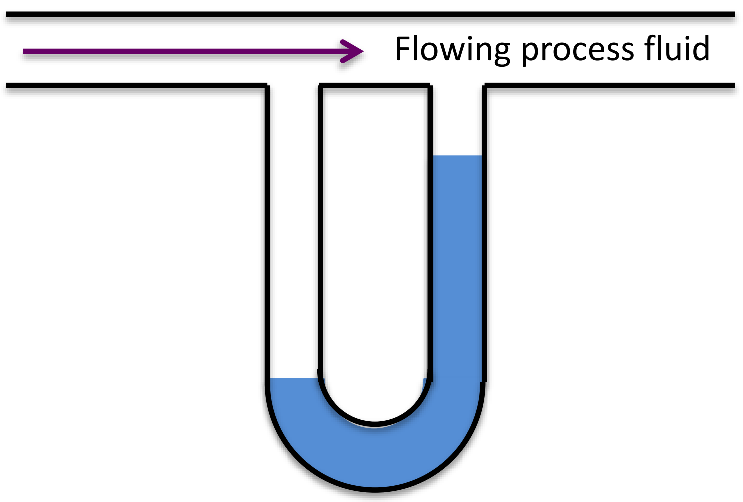
P1 = Patm + 800gh

—>h = (P1 - Patm)/(800*9.8)

Differential manometers

In the previous examples, we were able to calculate an **absolute pressure** for our process fluid because the manometer was open to the atmosphere, providing us a known reference pressure

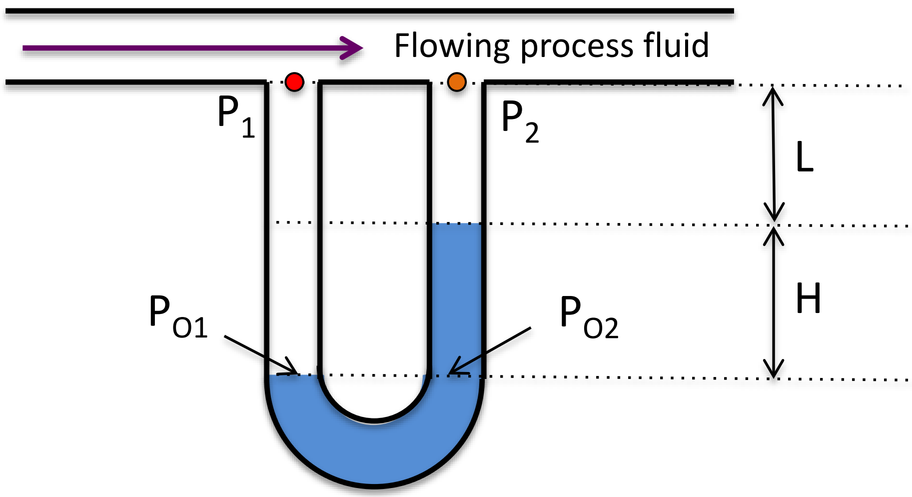
Sometimes we just want to measure the **pressure drop** along the length of a pipe, and for this we use a **differential manometer**



- Flow in a horizontal pipe is due to pressure.
- Pressure decreases down the length of the pipe in the flow direction.
- A differential manometer can be used to quantify that pressure drop down a length of pipe

Differential manometers

Find the pressure drop between **Point 1** and **Point 2**. The density of the process fluid is ρ_f and the density of the manometer fluid is ρ_m



$$P_{O1} = P_1 + \rho_f g(L+H)$$

$$P_{O2} = P_2 + \rho_f gL + \rho_m gH$$

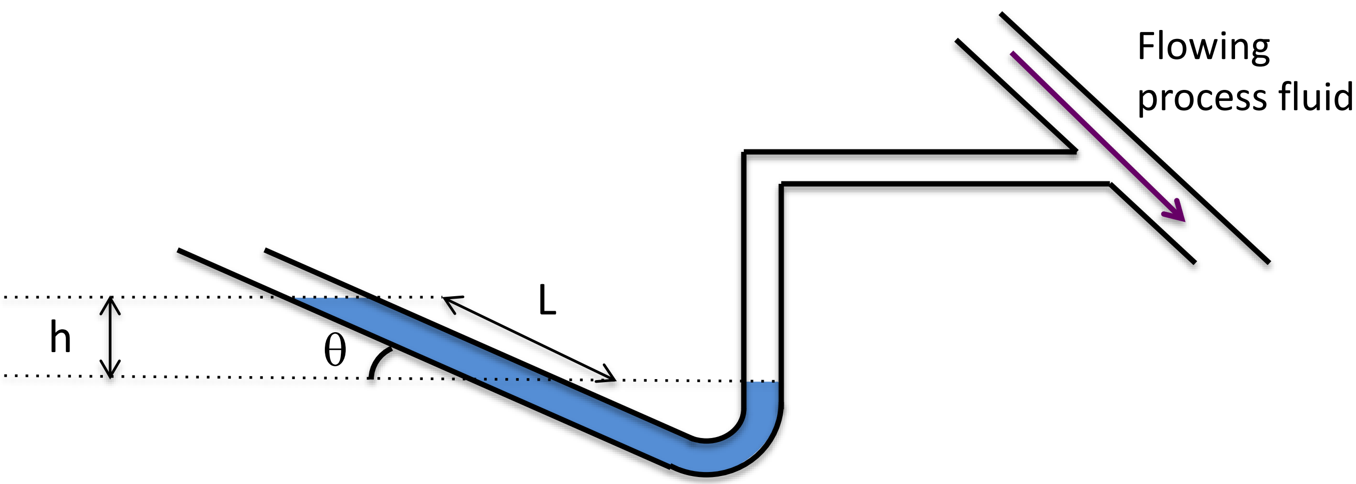
And $P_{O1} = P_{O2}$

$$\boxed{P_1 - P_2 = (\rho_m - \rho_f)gH}$$

Inclined manometers

Sometimes inclined manometers are used

- For some systems the pressure changes you are trying to measure are small, making it difficult to accurately measure the height of the fluid
- For small angles of θ, the change in length of the manometry fluid (L) will be large compared to the height (h), and this means getting an accurate measurement of L will be easier
- You can then use trigonometry to solve for the change in height

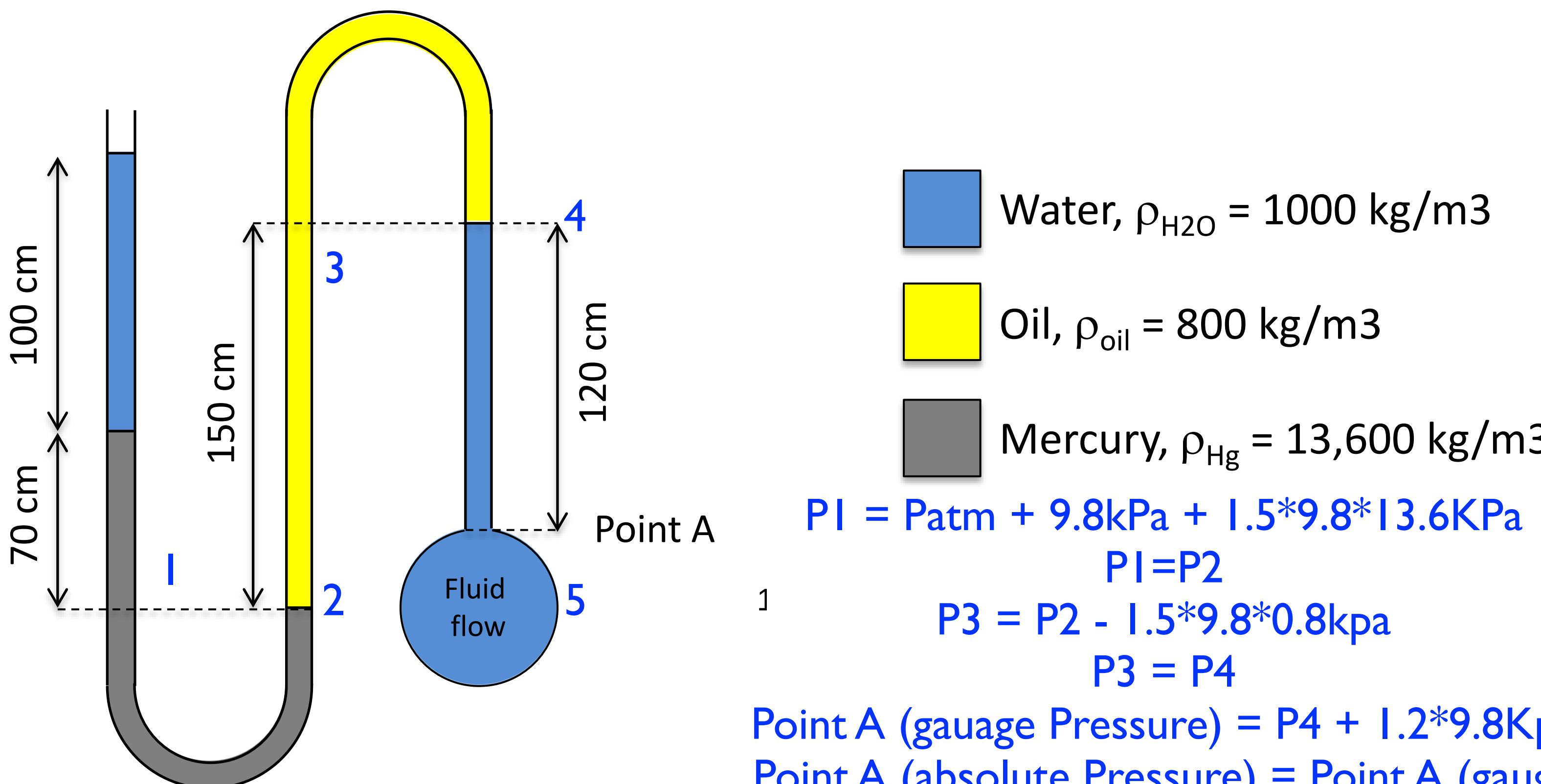


$$h = L*\sin\theta$$

$$L = h/\sin\theta$$

Example problem 1.2

A fluid is flowing through a pipe in the direction perpendicular to the page. The flow is connected to a manometer that contains three fluids. The water interface is open to atmospheric pressure. Calculate the gauge pressure and absolute pressure in the pipe at point A.



P1 = Patm + 9.8kPa + 1.5*9.8*13.6KPa

P1=P2

P3 = P2 - 1.5*9.8*0.8kpa

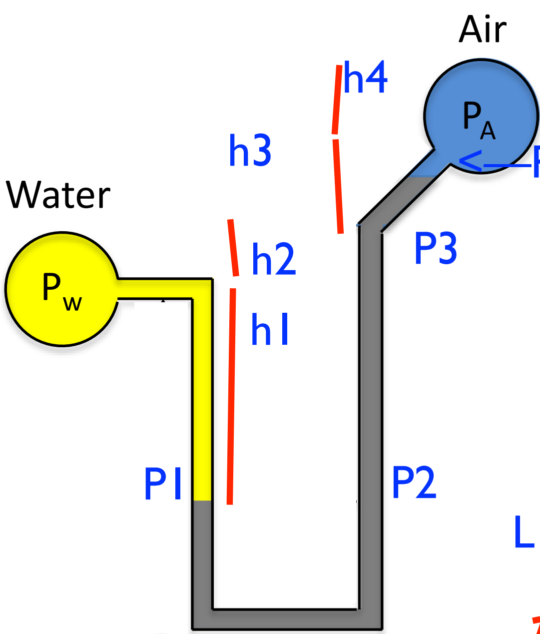
P3 = P4

Point A (gauage Pressure) = P4 + 1.2*9.8Kpa

Point A (absolute Pressure) = Point A (gauge Pressure) +9.8*0.3kPa (*i.e in the Fluid flow ball)

Example problem 1.3

Derive a relationship between the difference in pressure between the tank that contains water and the tank that contains air ($P_w - P_A$). The manometer fluid is mercury. Assume the density of air is constant, and that the small amount of air in the monometer tubing is negligible. This relationship will be in terms of variables, not numbers.

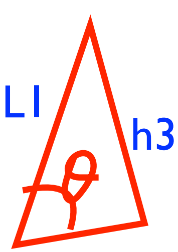


Do we assume Pw and Pa has no abs. pressure since they both closed?

PW = p(density for water)*g*h1

PA = p(density for Hg)g(h1+h2) + p(density for Hg)*h3sin(theta) + p(density for Hg)*g*h4sin(theta)

*Can I apply the same idea?



L1 = h3sin(theta)

I dunno what i did here?

PW - PA =p(density for water)*g*h1-(p(density for Hg)g(h1+h2) + p(density for Hg)*h3sin(theta) + p(density for Hg)*g*h4sin(theta)

Summary

The static pressure of fluids has many applications in engineering

- Forces on walls (dams)
- Buoyancy
- Manometry

We didn't talk about these concepts today, but Marco will talk about them thoroughly later in the course

Manometry uses the height of an immiscible fluid to determine pressure/pressure differences

- The basic concept behind manometry is that the pressure at equal z-values in a continuous fluid is constant
- Some manometers (those open to atmosphere or to a space with known pressure) can be used to determine the absolute pressure in a process stream
- Differential manometers can only be used to determine pressure differences
- Sometimes inclined manometers are used to make readings more accurate