TUTORIAL 4

1. Recall that the Simplex Algorithm generally has two phases: the first phase is concerned with obtaining a basic feasible solution and the second is concerned with finding an optimal basic feasible solution. The first phase is trivial if we have a slack variable in each constraint of our canonical form. However, if we have artificial variables then additional work is required in order to force the the artificial variables to become zero. We remove the artificial variables from the solution by performing the Simplex Algorithm with objective function $\min w = \sum_i y_i$, where the y_i are the artificial variables.

   Consider the following linear program:

$$\begin{aligned}\min z &= -x_1 - x_2\\ x_1 + x_2 &\geq 1\\ x_1 + x_2 &\leq 2\\ x_1, x_2 &\geq 0.\end{aligned}$$

   (a) Transform this problem to canonical form.
   (b) Obtain canonical form for the Phase 1 problem.
   (c) Solve Phase 1 for this problem.
   (d) Once Phase 1 is complete, you substitute the reduced costs from z into the bottom row. Do this, and explain the immediate problem with the tableau and how to fix it.

2. Rewrite the following linear programming problems in canonical form:

   (a)

$$\begin{aligned}\min \qquad & z = 4x_1 + 4x_2 + x_3\\ \text{subject to} \qquad & x_1 + x_2 + x_3 \leq 2\\ & 2x_1 + x_2 \leq 3\\ & 2x_1 + x_2 + 3x_3 \geq 3\\ & x_1 \geq 0, x_2 \geq 0\ x_3 \geq 0.\end{aligned}$$

   (b)

$$\begin{aligned}\min \qquad & z = 4x_1 + 4x_2 + x_3\\ \text{subject to} \qquad & x_1 + x_2 + x_3 = 2\\ & 2x_1 + x_2 \leq 3\\ & 2x_1 + x_2 + 3x_3 \geq 3\\ & x_1 \geq 0, x_2 \geq 0\ x_3 \geq 0.\end{aligned}$$

(c)

$$\min \qquad z = 4x_1 + 2x_2 + 3x_3$$

$$\begin{aligned} \text{subject to} \qquad x_1 + x_2 + x_3 &= 11 \\ 2x_1 + 3x_2 + x_3 &\leq -20 \\ x_1 + 3x_2 + 2x_3 &\geq 10 \\ x_1 \in \mathbb{R},\ x_2 \geq 0\ x_3 &\geq 0. \end{aligned}$$

3. Consider the following LP problem:

$$\min \qquad z = x_1 + x_2 - x_4$$

$$\begin{array}{llll} 2x_1 + x_2 - x_3 & & = & 14 \\ x_1 - x_2 & -x_4 & = & 6 \\ x_1 + x_2 & & \leq & 10 \\ x_1, x_2, x_3, x_4 & & \geq & 0 \end{array}$$

(a) Transform this problem to canonical form.

(b) Complete Phase 1 for the two-phase method.

(c) Set up the first canonical tableau for Phase 2.