Fluid Mechanics

Topic 10.1

Rheology

Goals for this class

Define key rheological terms

•ex. shear, shear stress, shear rate, viscosity…

Describe the key differences between Newtonian fluids and non-Newtonian fluids under flow conditions

Quantitatively model behavior of Newtonian and non-Newtonian fluids under flow conditions using constitutive equations

•Power law, Bingham plastic, etc…

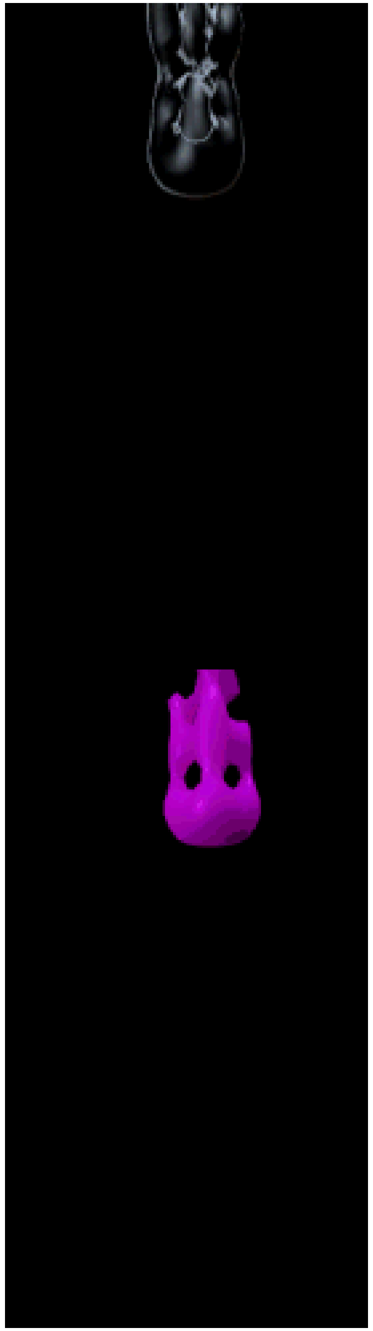
Goals for this class

What is viscosity?

Is viscosity a constant?

What factors affect viscosity?

How will viscosity affect fluid flow?



Basic terminology and principles

When a force is applied to a body, the body can respond by translating, rotating, or deforming

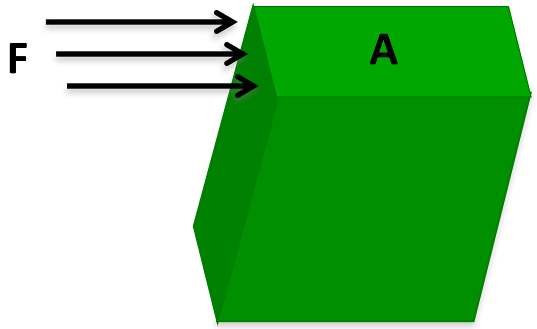
- In this course we are focused only on forces that deform
- When a body is deformed (or **strained**) the force that causes the deformation is referred to as a **stress**

The force can act perpendicular to a surface or tangential to a surface (or both)

- Perpendicular forces are called **normal stress**
- Tangential forces are called **shear stress**

Shear stress and deformation

When a material is exposed to a shear stress the material will skew

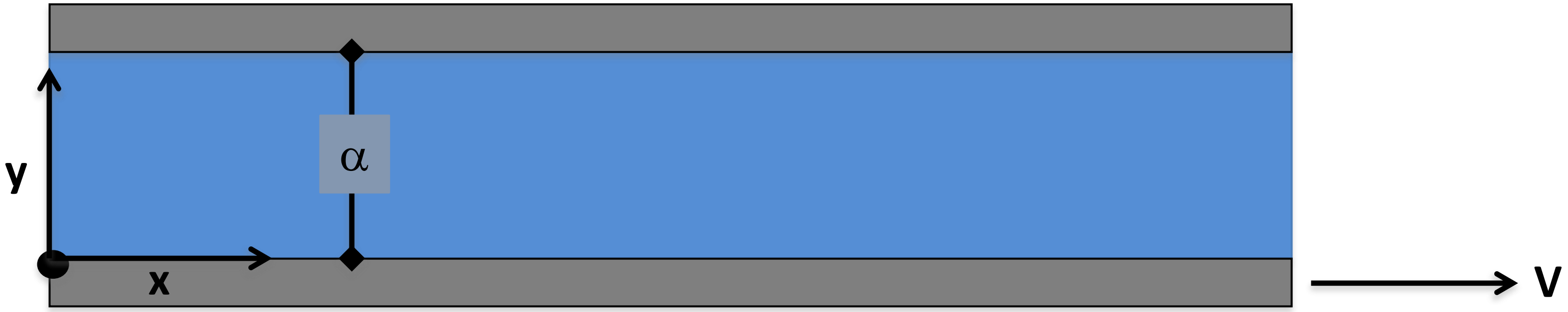


- We can easily measure the force we’re applying (F)
- We can also measure the area (A) on which the force is acting, in this case the top face of the cube
- The ratio of this tangential force to the area (F/A) is the shear stress (τ)

$$\boxed{t = \frac{F}{A}}$$

Shearing a liquid

Let us confine fluid between two solid plates that are infinitely long



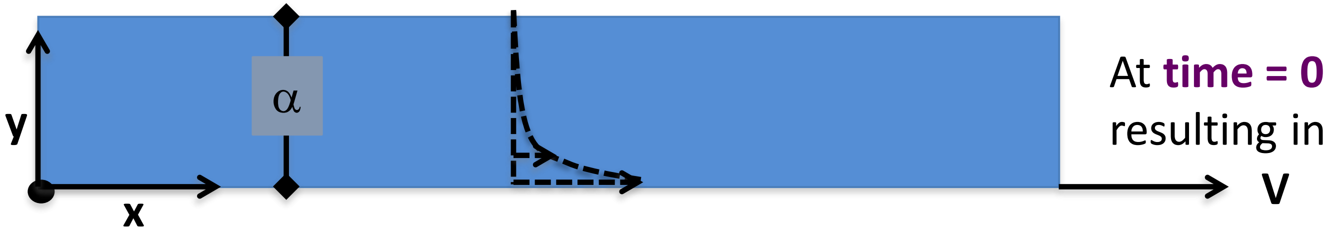
- We can shear the fluid by moving the bottom plate at a velocity (V)
- We can measure the force (F) that we apply to move the plate at given velocity
- We can measure the area of the plate (A)
- We can calculate the shear stress (τ) we are applying to the fluid

$$\boxed{t = \frac{F}{A}}$$

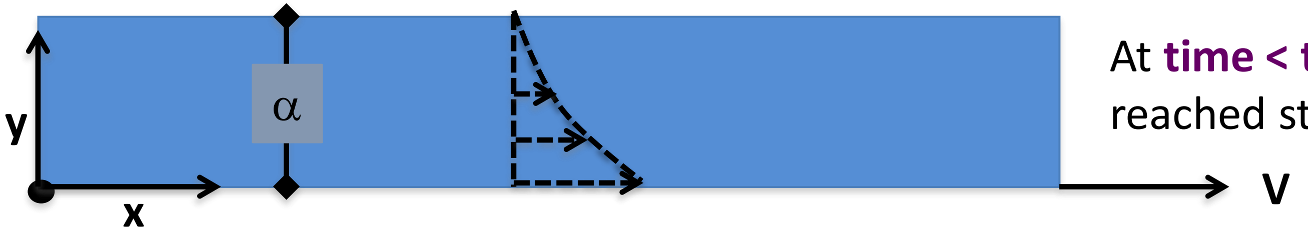
This flow geometry is referred to as **simple shear flow**

Development of SS flow takes time

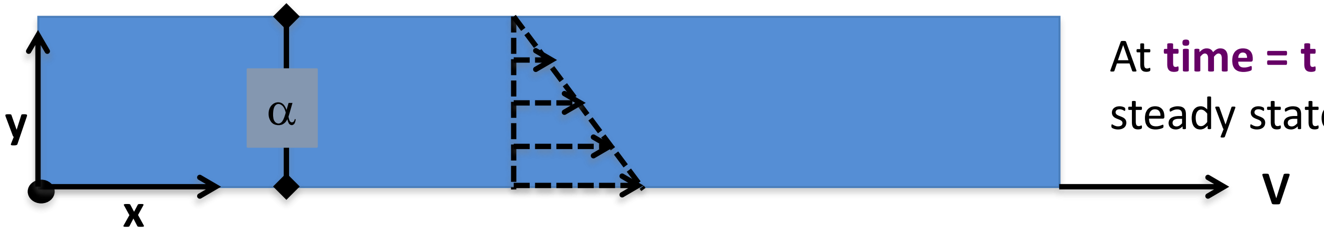
Let us look how the fluid flow develops over time



At **time = 0** plate begins to move resulting in flow near the surface



At **time < t** the fluid still has not reached steady state



At **time = t** the fluid has reached steady state flow conditions

Development of SS flow takes time

The **no slip boundary condition** states that fluid in contact with a solid surface always has the same velocity as the surface

- Fluid touching the upper plate (the stationary plate) has a velocity of zero
- Fluid touching the bottom plate has a velocity equal to the velocity of the plate (V)

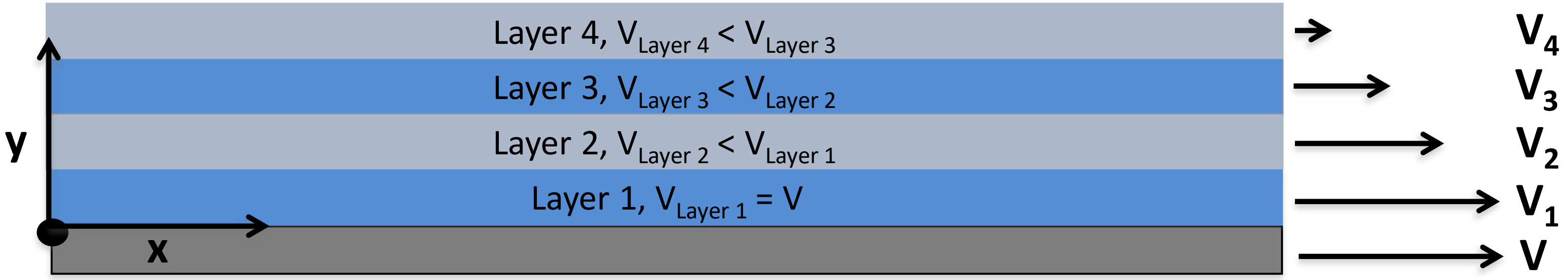
At time = 0 only the fluid touching the lower plate is moving; however, as time progresses the flow will continue to develop and eventually at time = t a steady state flow profile will be reached

In this simple shear flow geometry the velocity profile of the fully developed flow is linear in the y-direction

Momentum transfer

Why does this occur?

- You can think of the fluid as a series of layers. Friction between layers causes mechanical energy (and thus momentum) to be transferred between layers
- If this seems odd to you, you can produce the same phenomenon by shearing a deck of playing cards between your two hands.



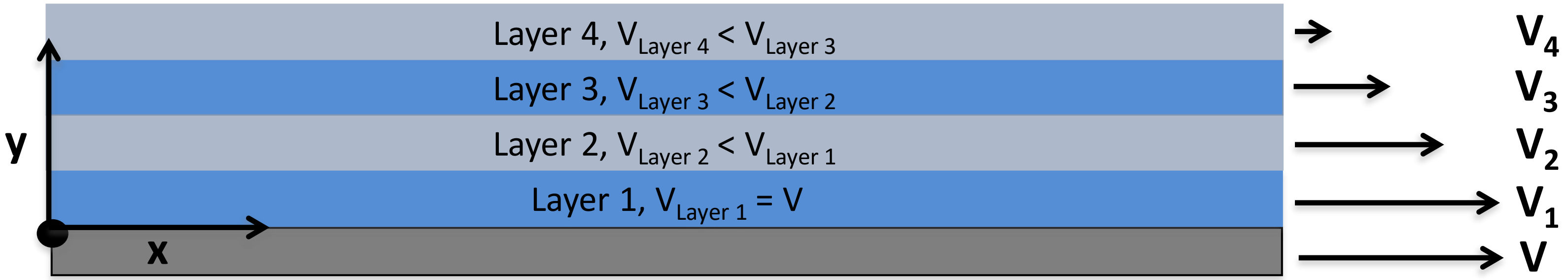
The fluid has momentum/velocity in the x-direction, but due to friction between fluid layers, that momentum is transferring (or diffusing) to adjacent fluid layers in the y-direction. This why fluid mechanics is sometimes referred to as momentum transfer

Connecting viscosity to shear stress

Until now, we have discussed how different fluids have different viscosities, or resistances to flow

However, we have not mathematically defined viscosity

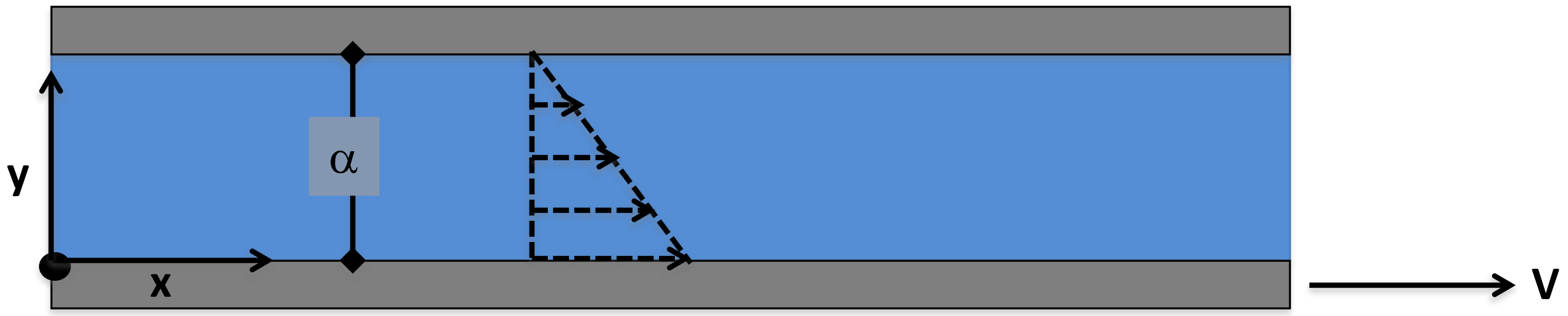
A fluid's viscosity is experimentally determined by applying a certain shear stress to the fluid and measuring how quickly the fluid deforms (flows)



Connecting viscosity to shear stress

A relationship develops if you study multiple fluids under simple shear

- Shear stress applied to the fluid is proportional to the velocity of the lower plate and inversely proportional to the gap size between plates
- The term V/α is the **shear rate** of the fluid and it is a measure of how quickly the fluid is being deformed



$$\frac{F}{A} \mu \frac{V}{a} \quad \Rightarrow \quad t \, \mu \frac{V}{a}$$

Connecting viscosity to shear stress

There are several different classes of fluids that each exhibit a unique relationship between viscosity and shear stress

- Newtonian fluids
- Power law fluids (shear thinning and shear thickening)
- Bingham fluids

We will discuss all these different fluids today

We will focus mostly on Newtonian fluids

Newton’s Law of Viscosity

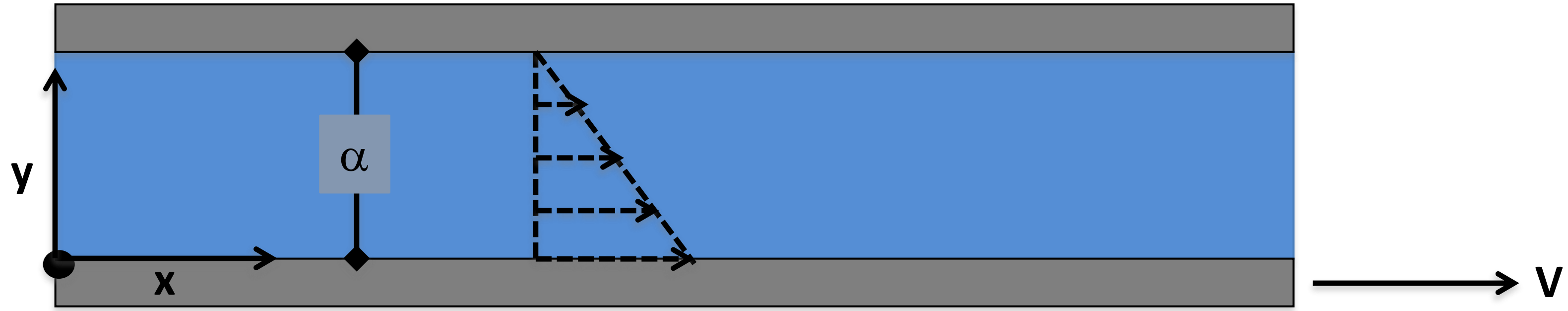
Newton’s law of viscosity

•Taking the limit as $\alpha \rightarrow 0$ yields a relationship between shear stress (applied force) and shear rate (how quickly the fluid is deforming)

•This relationship accurately describes a wide variety of fluid, i.e. **Newtonian fluids**

•The proportionality constant is the fluid’s viscosity

- Thicker fluids are more resistant to flow, therefore they have larger viscosities
- Thinner fluids are less resistant to flow, therefefor they have smaller viscosities



$$t \mu \frac{V}{a} \quad \longrightarrow \quad \boxed{t_{yx} = - m \frac{dV_x}{dy}}$$

Fill in the table

Fluid	Shear stress (Pa)	Viscosity (cP or mPa*s)	Shear rate (s^{-1})
Air		0.0186	268800
Benzene		0.6076	16460
Water	20		22000
Olive oil	10		120
Glycerol	5	1,200	
Honey	20	5,000	

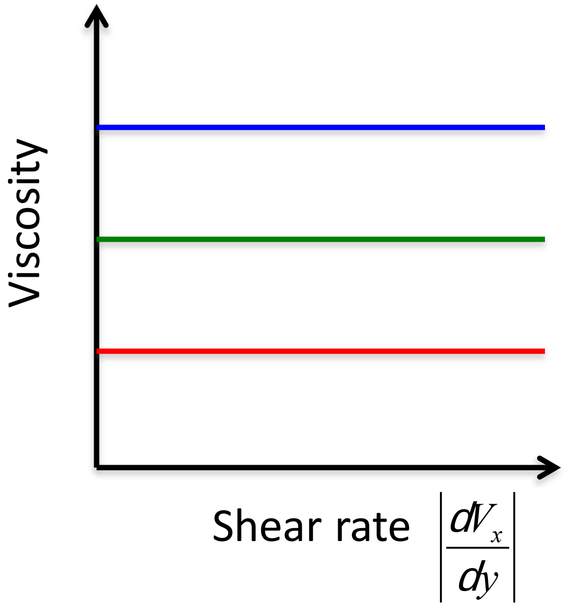
Viscosity of Newtonian fluids

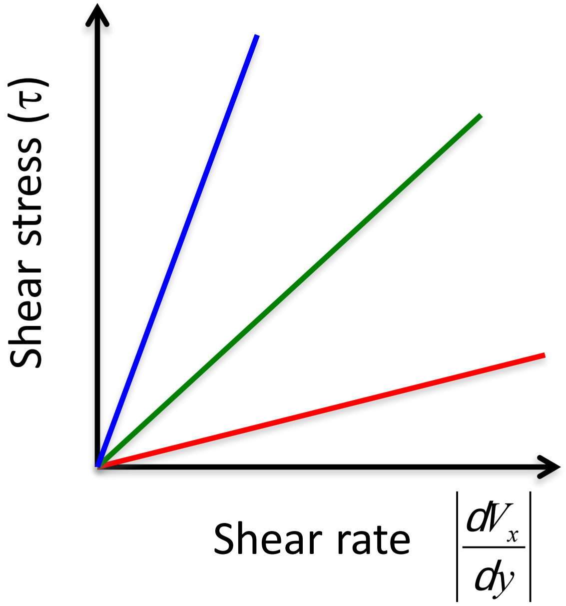
- Newtonian fluids are usually simple systems
  - Gases: air, argon, helium, oxygen
  - Low molecular weight liquids: water (18kDa), glycerol (92kDa)
  - Simple solutions: honey, salt water, tea

- Newtonian fluids are simple in that their viscosity is **not** a function of shear stress
  - However, that does not mean that the viscosity is a constant; viscosity is a strong function of other variables such as temperature

Viscosity of Newtonian fluids

- Qualitative graphical representations of Newtonian fluids under shear





Low viscosity fluid, medium viscosity fluid, high viscosity fluid

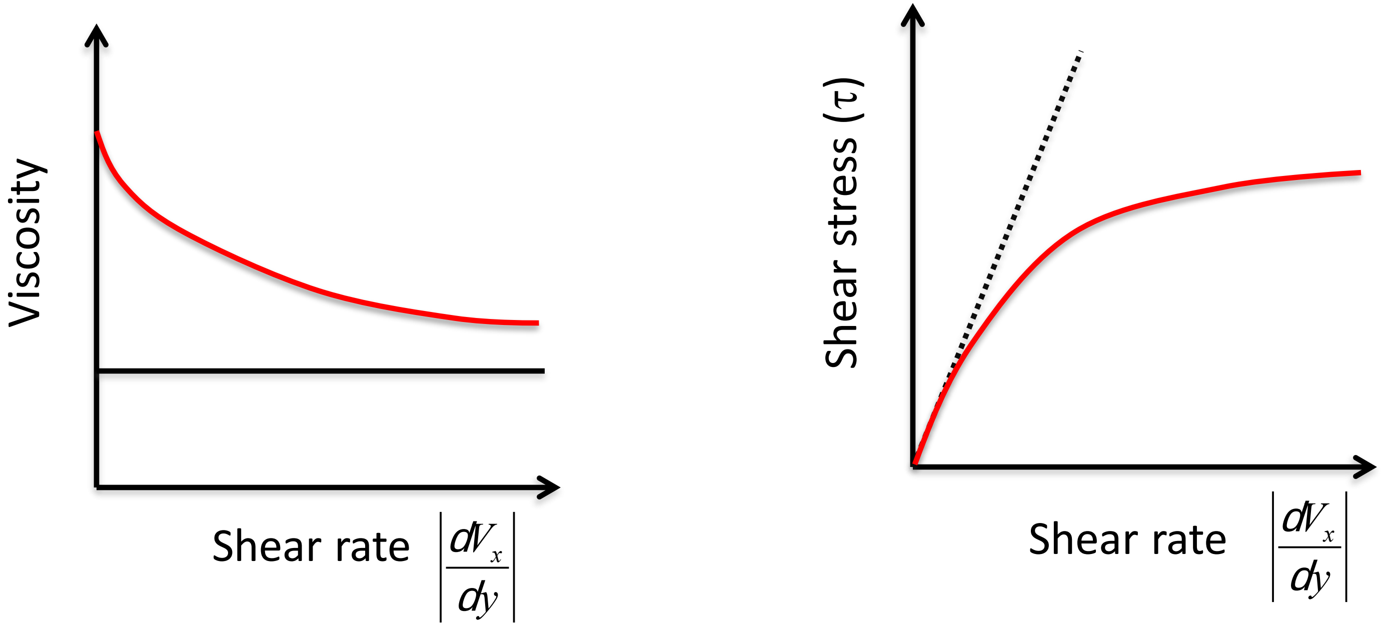
Non-Newtonian fluids

- Non-Newtonian fluids exhibit more complex flow behavior, specifically the viscosity of the fluid ***is*** a function of shear stress

- We will talk about 4 different types of non-Newtonian fluids in this course
  - Shear thinning fluids (pseudoplastics)
  - Shear thickening fluids (dilatants)
  - Bingham plastics
  - Viscoelastic fluids

Non-Newtonian fluids: shear thinning

As the name suggests, shear thinning fluids exhibit a drop in viscosity as shear rate increases



Newtonian fluid, Shear thinning fluid

Non-Newtonian fluids: shear thinning

There is always a physical reason why fluids behave the way they do

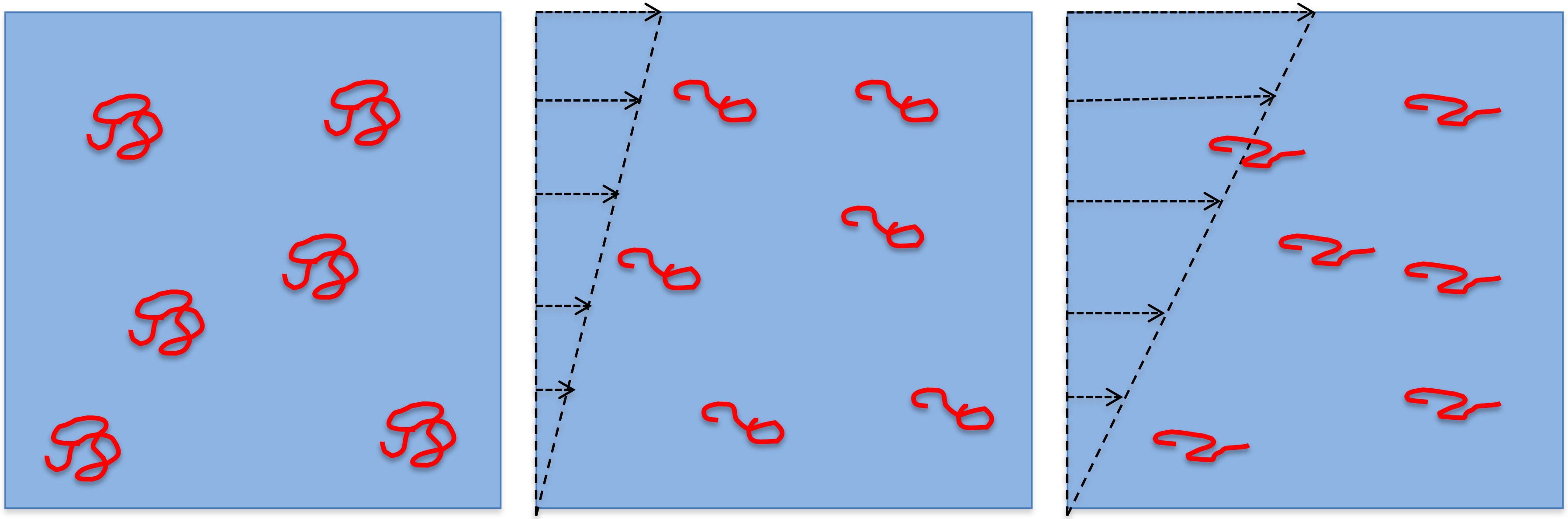
Many fluids exhibit shear thinning behavior

- Ex. lava, ketchup, whipped cream, blood, paint, nail polish, polymer solutions

Usually these materials have suspended particles or dissolved molecules that will align in the direction of flow as shear increases, thus decreasing the viscosity

Non-Newtonian fluids: shear thinning

A solution of polymer molecules in a low molecular weight solvent expose it to varying degrees of shear

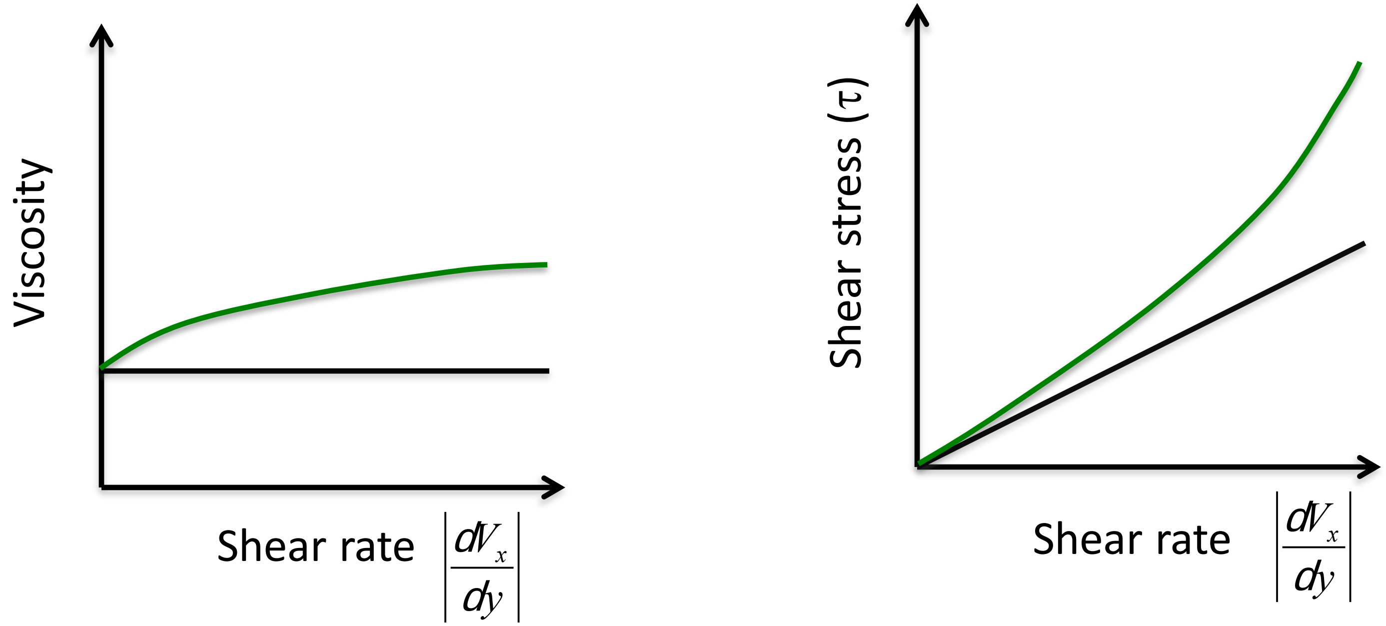


Stagnant conditions **Low shear** **High shear**

As shear increases, molecules align more and thus flow past one another more easily, this is macroscopically observed as a decrease in viscosity

Non-Newtonian fluids: shear thickening

As the name suggests, shear thickening fluids exhibit an increase in viscosity as shear rate increases

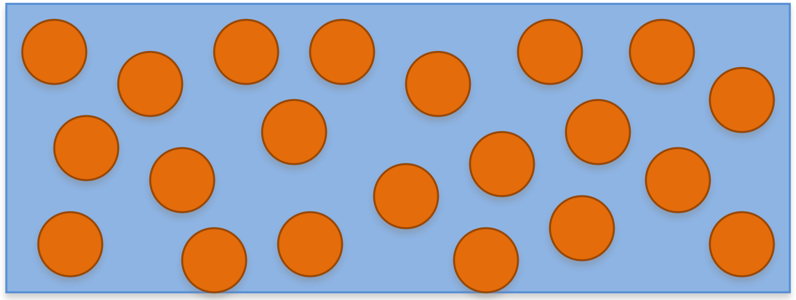


Newtonian fluid, Shear thickening fluid

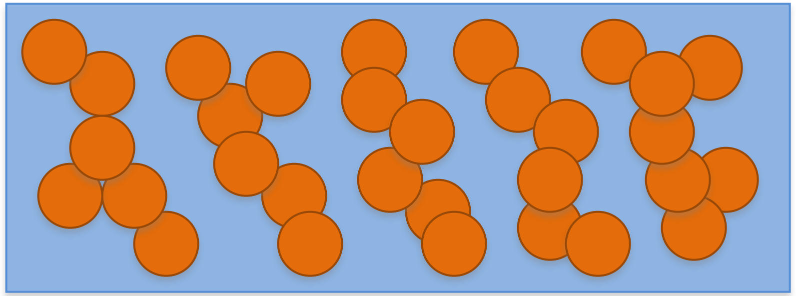
Non-Newtonian fluids: shear thickening

Shear thickening is often observed in suspensions of small, solid particles due to the shear-induced flocculation (or clustering) of particles

- Ex. Starch in water (Oobleck), beach sand, concentrated slurries



Before impact



After impact

Power Law model

The mathematical relation that describes Newton's law of viscosity cannot be used on shear thinning or shear thickening fluids

Instead the **Power Law model** is most often used

$$t_{yx} = - m \frac{dV_x}{dy}$$

Newton's Law of Viscosity

$$\left| t_{yx} \right| = k \left| \frac{dV_x}{dy} \right|^n$$

Power Law model

Now the viscosity depends on two parameters

- The consistency of a fluid (k)
- The power law index of a fluid (n)

Power law model

The Power Law model can be re-written in the following form

$$\left|t_{yx}\right| = k\left|\frac{dV_x}{dy}\right|^{n} \quad \Longrightarrow \quad \left|t_{yx}\right| = \underline{k\left|\frac{dV_x}{dy}\right|^{n-1}}\left|\frac{dV_x}{dy}\right|$$

The underlined quantity called the **apparent viscosity** (μ_a) and can be used to calculate a viscosity for a shear thinning/thickening fluid at a given shear rate

Calculate the apparent viscosity

Calculate the apparent viscosity under each of the following cases

- Note: The units of consistency depend on the power law index. Assume all values below are appropriate

Consistency	Power law index	Shear rate	Apparent viscosity
2	0.2	5	
2	0.2	10	
5	1	5	
5	1	10	
10	2.5	5	
10	2.5	10	

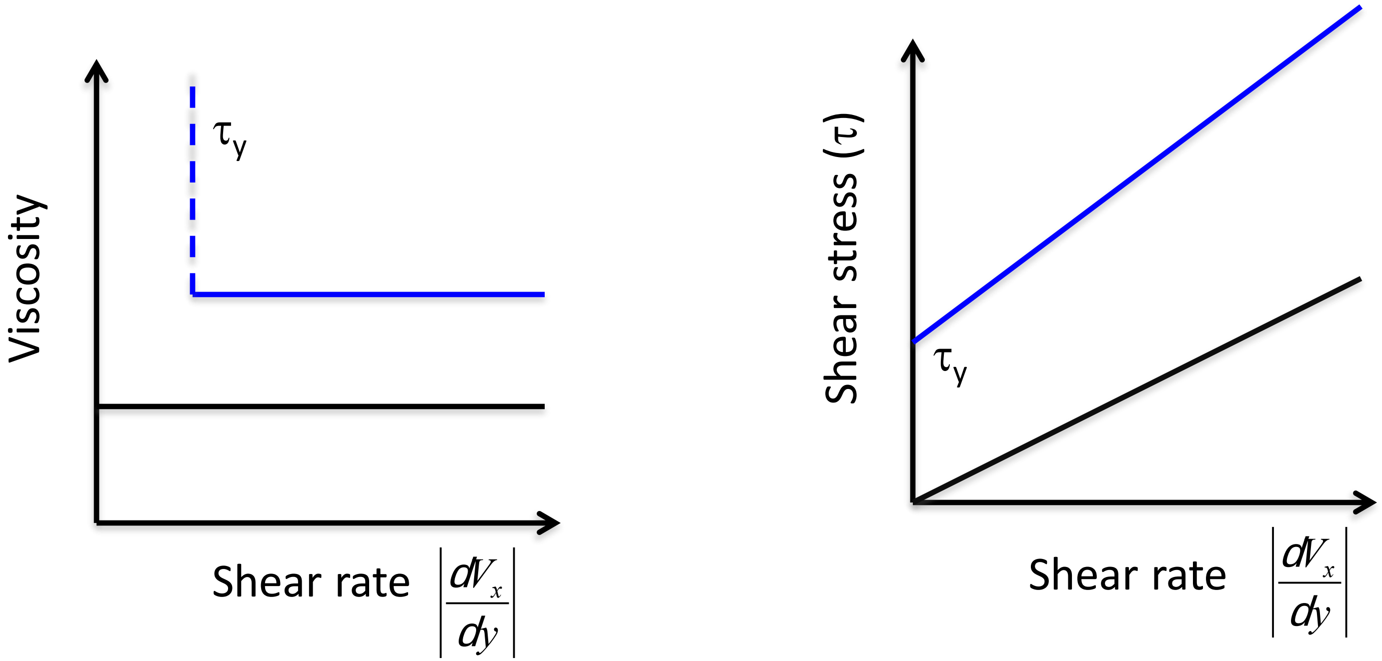
Impact of the power law index?

- What fluids have a power law index less than one?
- What fluids have a power law index of one?
- What fluids have a power law index greater than one?

Non-Newtonian fluids: Bingham fluid

Bingham plastic fluids are fluids that exhibit a **yield stress** meaning a threshold stress must be applied before the fluid will begin to flow

Below the yield stress, the viscosity of the fluid can be thought of as infinite



Newtonian fluid, Bingham plastic fluid

Bingham plastic fluids

Bingham plastic behavior is often observed in suspensions such as whipped cream, mud, toothpaste, and ketchup

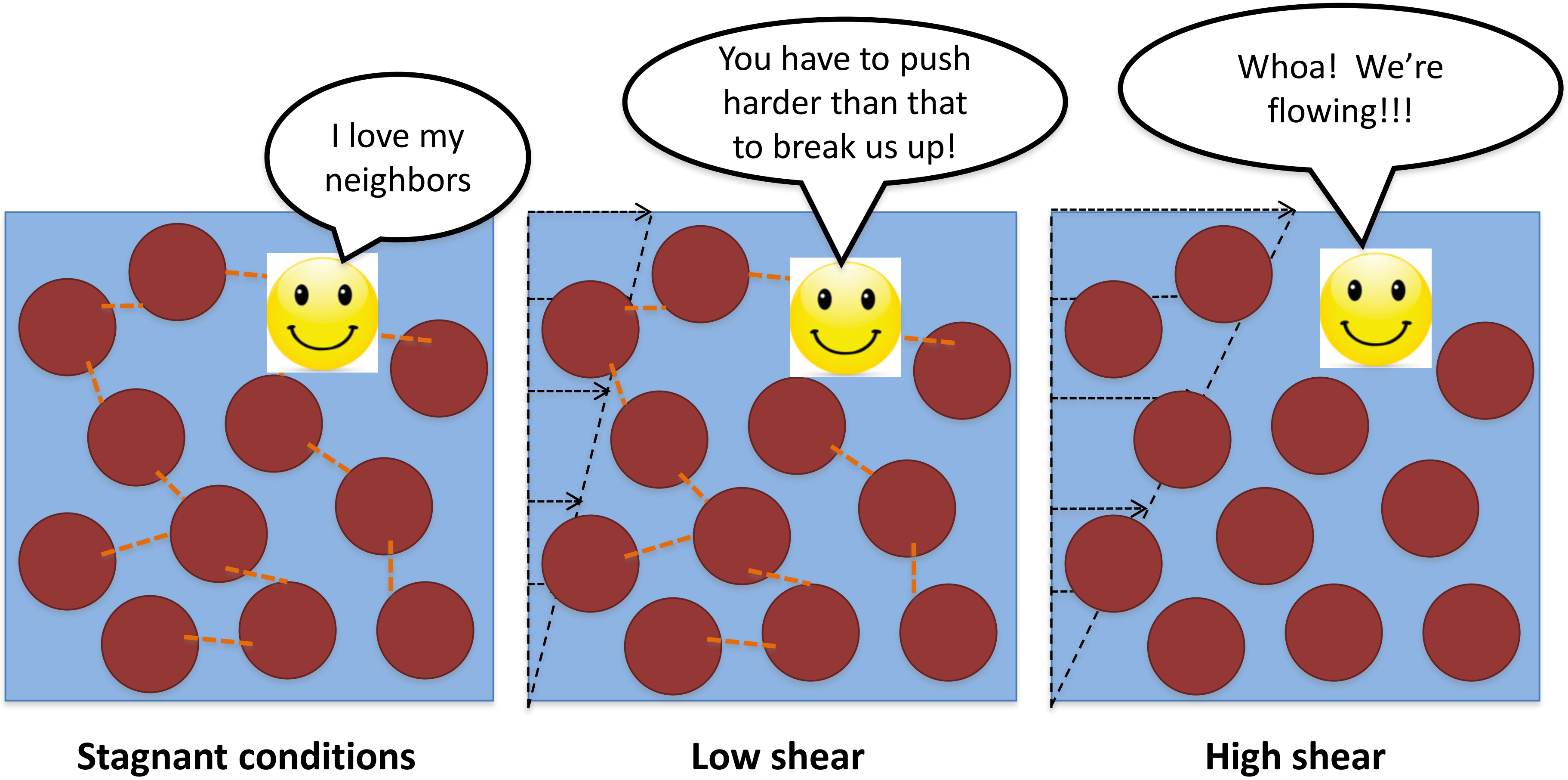
Behavior is usually rationalized as arising from “structure” within the fluid that must be overcome before flow begins

- For example, structure could be attractive van der Waals forces between particles. These attractive forces must be overcome before the fluid can flow

Bingham plastic fluids

Cartoon illustration of Bingham plastic flow

•Orange dotted lines represent attractive interactions between particles, also referred to as "structure" within the system



Bingham plastic model

Neither Newton's Law of Viscosity nor the Power Law model can be used to mathematically describe the shear behavior of a Bingham plastic

Instead the **Bingham plastic model** is most often used

$$\tau_{yx} = -\mu \frac{dV_x}{dy}$$

Newton's Law of Viscosity

Two parameter model

- Yield stress (τ_y)
- Plastic viscosity (μ_p)

$$\left|\tau_{yx}\right| = \tau_y + \mu_p \left|\frac{dV_x}{dy}\right| \quad \text{when} \quad \left|\tau_{yx}\right| > \tau_y$$

$$\left|\frac{dV_x}{dy}\right| = 0 \quad \text{when} \quad \left|\tau_{yx}\right| \le \tau_y$$

Bingham plastic model

Generalized Bingham equation

Or sometimes a Bingham plastic fluid will also exhibit shear dependent behavior in which case the Generalized Bingham equation (or the Herschel-Bulkley equation) is used

$$t_{yx} = - m\frac{dV_x}{dy}$$

Newton's Law of Viscosity

$$\left|t_{yx}\right| = t_y + k_p\left|\frac{dV_x}{dy}\right|^n \quad \text{when} \quad \left|t\right| > t_y$$

Generalized Bingham Equation

Three parameter model

- Yield stress (τ_y)
- Plastic consistency (k_p)
- Power law index of a fluid (n)

Viscoelastic materials

We have discussed previously how the hallmark of an ideal solid is finite deformation for a given stress followed by total recoil after the stress is removed

For an ideal liquid the hallmark is continuous deformation (flow) while a force is applied and no recoil once a force is removed

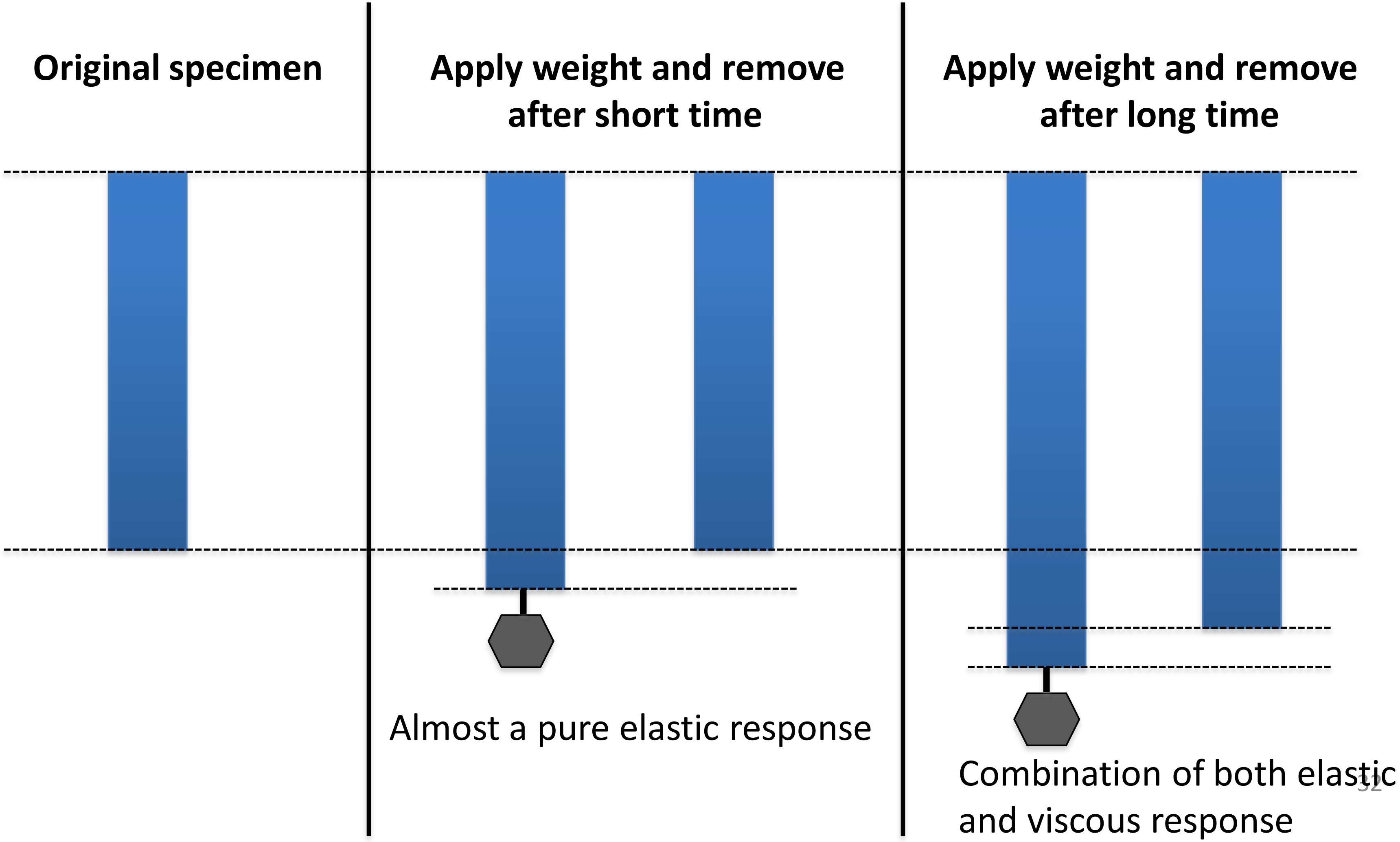
Very few true materials are either ideal solids or ideal fluids; instead most real substances exhibit both solid-like and liquid-like behavior

These materials are termed **viscoelastic**

Viscoelastic material

Example of viscoelastic behavior: creep

- Initial elastic deformation followed by time dependent flow behavior



What you should be able to do

Define key rheological terms

- ex. shear, shear stress, shear rate, viscosity…

Describe the key differences between Newtonian fluids and non-Newtonian fluids under flow conditions

Quantitatively model behavior of Newtonian and non-Newtonian fluids under flow conditions using constitutive equations

- Power law, Bingham plastic, etc…