MAST30013 – Techniques in Operations Research

Semester 1, 2021

Group Project

Submissions: Solutions are to be submitted before the (strict) due date: 12pm Monday of Week 12. A complete submission will consist of a LaTex typeset report in pdf form and the Matlab files of your implementation.

Groups: must be composed of a minimum of 3 and a maximum of 4 members. It is your responsibility to talk to your colleagues and create/find a group, as early as possible in the semester.

Evaluation:

Reporting: You should present a coherent and self-contained report presenting your algorithms and results for the questions below.

Presentation: In the last week of class, groups will present their work in 5 minutes + 2 minutes for questions during the usual lecture times (Mon, Thu, Fri). The presentations will be online via Zoom, and will NOT take place on campus. You are NOT expected to explain the problem in detail during the presentation. Instead, focus on the innovative aspects that your group has found. Which solution algorithm did you implement? What conclusions did you get? Not all group members are expected to speak, but all must contribute to the structure of the presentation and in particular the slides.

Report Structure:

Introduction (Motivation, applications)
Background (Mathematical preliminaries)
Algorithms (Pseudo-code and descriptions)
Experimental setup (What instances are you testing on and how will you evaluate the results? Which algorithms will you compare and how? What hardware are you using?)
Experimental results (Graphs and tables)
Discussion and results (What do you conclude?)

Project Description

Power Optimisation in Wireless Sensor Networks:

Wireless sensor networks (WSNs) are autonomous systems that measure aspects of an environment. Each sensor transmits information to a central relay which aggregates the local sensed data before transmitting it to a base station for processing. Sensors and relays are generally battery operated and therefore the optimisation of power consumption in wireless networks is an important research problem in the mathematical and engineering sciences.

The power consumed by a relay or sensor in an ideal environment (eg. a vacuum) can be expressed as $P = kr^2$, where r is the transmission distance and k is come constant. With this definition, we can now formally express a central optimisation problem in WSN design: in the *WSN relay location problem* we are given a set X of n sensor locations in the plane $\mathbb{R}^2$. The problem is to find the optimal location of a relay in the plane so that the power of the network is minimised. Each sensor only transmits at the minimum distance needed to reach the relay and the relay transmits at the minimum distance needed to reach all sensors. More formally, we are required to find a location $s \in \mathbb{R}^2$ to place the relay so that the following objective is minimised:

$$P(s) = \sum_{x \in X} \|s - x\|^2 + \max_{x \in X} \|s - x\|^2$$

Tasks:

1. Prove that the unconstrained problem, as formulated above, is NOT C^1 but is convex. Illustrate an example consisting of three sensors in the plane by drawing the lines consisting of all points at which the problem is not differentiable.
2. Remodel the problem as a constrained optimisation problem with a smooth objective. Prove that the problem is still convex. You should explain each constraint and variable.
3. Derive the KKT conditions for the model in (2).
4. Implement an algorithm for solving your model in (2). Do a computational study (different initial solutions, parameter settings, etc.) for instances of different values of n and various random sets X of n points. Compare your algorithm with other algorithms or pre-existing optimisation functions in Matlab.

Important Notes

- You will be marked on the quality of your report. Make sure it is self contained and follows the structure of a scientific report.
- It is the group's responsibility to ensure that everyone contributes their fair share to the project. All members of the group will receive the same mark.
- It is recommended that members of the group share a Dropbox folder for their work or use Overleaf. This is so that all group members can have access to all parts of the project at any time.
- This project contains an extensive programming component. It is assumed that all students are able to program in Matlab. The lecturer will **not** assist in issues relating to the debugging of code.
- Students may research ideas on the web, but full credit to the relevant authors must be given if the students use any of these ideas. Code may NOT be copied from existing sources. Collaboration between groups is NOT allowed.
- All code must be in Matlab. Code will not be marked directly, but marks may be deducted for messy code or code that is not well commented. Code must be in a form so that the lecturer can easily test your algorithm on an independent set of instances.