MAT4MDS — Practice 8

FUNCTIONS of MORE THAN ONE VARIABLE

For functions of two or more variables, we can form **partial derivatives** by differentiating with respect to one variable only, whilst treating any others as constant.

We use notation $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ to indicate that we are treating a function of several variables.

Question 1. A family of functions which arises in economics are the Cobb-Douglas utility functions, which have the form $U(x, y) = x^a y^{1-a}$. The first order partial derivatives of U are called the marginal utilities. Find the marginal utilities.

Question 2. Let $g(x, y) = \frac{ax+by}{cx+dy}$, where $ad - bc = 0$. Show that $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$.

Question 3. Consider the function $f(x, y) = x^2 e^{2y+x} - \frac{x}{y}$.

(a) Find the two first partial derivatives: $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) Differentiate each derivative found in (a) with respect to both x and y.

(c) What do you notice about the mixed second derivatives?

The second order Taylor polynomial of f about (a, b):

$$T^2_{(a,b)}f(x, y) = f(a, b) + (x - a)\frac{\partial f}{\partial x}(a, b) + (y - b)\frac{\partial f}{\partial y}(a, b)$$

$$+\frac{1}{2}\left\{(x - a)^2\frac{\partial^2 f}{\partial x^2}(a, b) + 2(x - a)(y - b)\frac{\partial^2 f}{\partial x\,\partial y}(a, b) + (y - b)^2\frac{\partial^2 f}{\partial y^2}(a, b)\right\}.$$

Question 4. Using your answers to Question 3, find the second order Taylor polynomial of $f(x, y) = x^2 e^{2y+x} - \frac{x}{y}$ about $(1, -1)$.

Question 5.

(a) Consider the partial derivatives of third order, for a function of two variables. How many are there? How many of these are distinct?

(b) What additional terms are in $T^3_{(a,b)}f(x, y)$ (compared to $T^2_{(a,b)}f(x, y)$)?

(c) For a function of three variables, $g(x, y, z)$ how many partial derivatives are there of first order, and of second order? What would its linear approximation be?