

**School of Mathematics and Statistics**  
**MAST30030**  
**Applied Mathematical Modelling**

**Problem Sheet 3. Some answers**

**Question 1**

- (a) Linear, homogeneous, 2nd order.
- (b) Nonlinear, homogeneous, 2nd order.
- (c) Nonlinear, inhomogeneous, 2nd order.
- (d) Nonlinear, homogeneous, 3rd order.
- (e) Linear, homogeneous, 2nd order.
- (f) Linear, inhomogeneous, 4th order.
- (g) Nonlinear, homogeneous, 1st order.
- (h) Nonlinear, homogeneous, 2nd order.
- (i) Nonlinear, homogeneous, 3rd order.
- (j) Linear, homogeneous, 2nd order.

**Question 2**

$$\phi(x, t) = \begin{cases} 1, & x < 3t \\ 0, & x > 3t \end{cases}$$

**Question 3**

$$u(x, t) = g(xe^{-t})e^{-t}$$

**Question 4**

$$u(x, t) = -1 - t + [u_0(x - 2t) + 1] e^t$$

**Question 5**

$$\phi(x, t) = \begin{cases} B_0, & x < t \\ A_0, & x > t \end{cases}, \quad \phi(3, -5) = A_0$$

**Question 6**

$$(a) \phi(x, t) = \begin{cases} 1, & t > 2x \\ 0, & t < 2x \end{cases}$$

(b) If  $t < 2$ :

$$\phi(x, t) = \begin{cases} 0, & x < 0 \\ \frac{x}{t}, & 0 < x < t \\ 1, & t < x < \frac{1}{2}t + 1 \\ 0, & x > \frac{1}{2}t + 1 \end{cases}$$

If  $t > 2$ :

$$\phi(x, t) = \begin{cases} 0, & x < 0 \\ \frac{x}{t}, & 0 < x < \sqrt{2t} \\ 0, & x > \sqrt{2t} \end{cases}$$

### Question 7

$$\phi(x, t) = \begin{cases} 0, & x < 0 \\ \sqrt{\frac{x}{t}}, & 0 < x < 4t \\ 2, & x > 4t \end{cases}, \quad \phi(1, 2) = \frac{1}{\sqrt{2}}$$