~~FM~~ S1 2011

4 (a) small diameter pipe | large diameter pipe

$Q = 70 \times 10^{-3} m^3/s$ for both

$V = \frac{Q}{A}$

$V_1 = \frac{70 \times 10^{-3} \ m^3/s}{\frac{\pi \times 0.1^2}{4} \ m^2}$

$= 8.91 \ m/s$

$V_2 = \frac{70 \times 10^{-3} \ m^3/s}{\pi \times 0.2^2/4 \ m^2}$

$= 2.23 \ m/s$

$Re = \frac{\rho V d}{\mu}$

$Re_1 = \frac{1000 \ kg m^{-3} \times 8.91 \ m/s \times 0.1 m}{1.31 \times 10^{-3} \ Ns/m^2}$

$= 680153$

$Re_2 = \frac{1000 \ kg/m^{-3} \times 8.91 \ m/s \times 0.2 m}{4 \times 1.31 \times 10^{-3} \ Ns/m^2}$

$= 340179$

$\frac{e}{D_1} = \frac{0.0002 \ m}{0.1 \quad m}$

$= 0.002$

$f_1 = 0.006$

$\frac{e}{D_2} = \frac{0.0002 \ m}{0.2 \ m}$

$= 0.001$

$f_2 = 0.005$

(b) $\frac{2 \times 0.005 \times (304.5 + 100) \times 2.23^2}{0.2 \times 9.8} + \frac{1}{2 \times 9.8} (0.5) \times 2.23^2$

$+ \frac{2 \times 0.006 \ (65) \times 8.91^2}{0.1 \times 9.8} + \frac{1}{2 \times 9.8} (1 + 0.375) \times 8.91^2 = 79.15 \ m.$

$K_{contraction} = \frac{1}{2}\left(1 - \left(\frac{0.1}{0.2}\right)^2\right)$

$= 0.375$

(d) suction side of the pump.

(c) MEB [balance between 1 and 2]

$\frac{\cancel{\Delta P}^{0}}{\rho} + \frac{1}{\cancel{2}} \cancel{\Delta v^2}^{0} + g\Delta z + W_s + F = 0$

Patm | free surface velocity

$9.8 \times 30 + W_s + 79.15 \times 9.8 = 0$

$-W_s = 1069.67 \ J/kg$

Power $= -W_s G$

$= 1069.67 J/kg \times 0.07 \ m^3/s \times 1000 \ kg/m^3 = 74876.9 \ W$

2011

(a) $\Rightarrow Q = VA$

$V = Q/A$

o free surface ↙ between ① and ②

$$h_{sys} = \cancel{\frac{\Delta p}{\rho g}}^{0} + \cancel{\frac{\Delta V^2}{2\alpha g}}^{0} + \Delta z + h_f$$

$\alpha = 1$

Patm

$$= 49\text{ m} + \frac{2 \times 0.008 \times}{9.8 \times 0.4}\left(\frac{Q}{\frac{\pi \times 0.4^2}{4}}\right)^2 \times 464.74$$

$$= 49 + 120.122Q^2 \,//$$

(b) $h_{sys} = h_p$ at operating point

$$49 + 120.122\,Q^2 = 149 - 20Q^2$$

$$140.122Q^2 = 100$$

$$Q^2 = 0.714\ m^6/s^2$$

$$Q = 0.845\ m^3/s$$ vv above $0.5\,m^3/s$

(c) $$NPSHA = \frac{P_1 - P_{vap}}{\rho g} + z_1 - h_{fs}$$

$$= \frac{101325 - 2333}{1000 \times 9.8} + 10 - \frac{2 \times 0.008 \times 50 \times}{0.4 \times 9.8}\left(\frac{0.845}{\pi \times 0.4^2/4}\right)^2$$

$$= 10.87\text{ m}$$

as NPSHA > NPSHR, pump is within the permissable operating range

(d). $$Re = \frac{\rho v d}{\mu} = \frac{1000 \times \frac{0.845}{\frac{\pi \times 0.4^2}{4}} \times 0.4}{1 \times 10^{-3}} = 2689719$$

$f_F = 0.008$

$$\frac{e}{D} = 0.00625$$

$e = 0.00625 \times 0.4\text{ m}$

$= 0.0025\text{ m}$

2011

(a) Flow when pressure drop along the pipe has resulted in maximum flowrate of gas and further pressure drop will not ↑ G anymore.

(b) Pw is the critical downstream pressure when choked flow will begin to occur.

(c) $\frac{4fL_{min}}{D} = \left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1$

$\frac{4 \times 0.005 \times L_{min}}{55 \times 10^{-3}} = \left(\frac{6}{4}\right)^2 - \ln\left(\frac{6}{4}\right)^2 - 1$

$L_{min} = \frac{55 \times 10^{-3}}{4 \times 0.005}\left[0.439\right]$

$= 1.21\,m$

as Lpipe > Lmin, flow inside the pipe is not choked.

(d) $\frac{(400 \times 10^3)^2 - (600 \times 10^3)^2}{2 \times \frac{8.314 \times (273+150)}{28 \times 10^{-3}}} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{600}{400}\right) + \frac{2 \times 0.005 \times 10}{55 \times 10^{-3}}\left(\frac{G}{A}\right)^2 = 0$

$\left(\frac{G}{A}\right)^2\left[\ln(3/2) + 20/11\right] = 796173$

$\left(\frac{G}{A}\right)^2 = 358048$

$\frac{G}{A} = 598\ kg/m^2s$

$G = 598 \times \pi \times \frac{(55 \times 10^{-3})^2}{4}$

$= 1.42\ kg/s$

(e) $\rho_2 = \frac{P_2}{RT/M} = \frac{400 \times 10^3}{\frac{8.314 \times 423}{28 \times 10^{-3}}} = 3.18\ kg/m^3$

$G = \rho_2 V_2 A$

$V_2 = \frac{1.42\ kg/s}{3.18\ kg/m^3 \times \pi \times \frac{(55 \times 10^{-3})^2}{4}\ m^2} = 187.8\ m/s$

sonic $v = \sqrt{\frac{RT}{M}} = \left(\frac{8.314 \times 423}{28 \times 10^{-3}}\right)^{1/2} = 354.4\ m/s$

$V_2 = \frac{187.8}{354.4} \times 100\% = 53\%$ //

2011

7) $Re = \frac{\rho u L}{\mu}$ $Fr = \frac{u}{\sqrt{gL}}$

Fr

$$\frac{u_{model}}{\sqrt{g L_{model}}} = \frac{u_{real}}{\sqrt{g L_{real}}}$$

$L_{real} = 5 L_{model}$

$$u_{model} \times \sqrt{5} = u_{real}$$

Re

$$\frac{\rho_m u_m L_m}{\mu_m} = \frac{\rho_R u_R L_R}{\mu_R}$$

$$\frac{\rho_m \cancel{u_m} \cancel{L_m}}{\mu_m} = \frac{900 \times \sqrt{5} \cancel{u_M} \times 5 \cancel{L_m}}{0.005}$$

$$\frac{\mu_m}{\rho_m} = \nu_m = \frac{0.005}{900 \times 5\sqrt{5}}$$

$$= 5 \times 10^{-7} \text{ Pa s}$$

2011

3 (a) $V_x \neq V_x(x)$ as flow is steady therefore it does not change along the length of the pipe

$V_x \neq V_x(z)$ as it has no dependence on the coordinate z //

$V_x = V_x(y)$ otherwise no flow

(b) equations of motion : conservation of momentum

in x-direction

$$\rho\left(\cancel{\frac{\partial V_x}{\partial t}}^{0} + V_x \frac{\partial V_x}{\cancel{\partial x}}^{0} + \cancel{V_y}^{0}\frac{\partial V_x}{\partial y} + \cancel{V_z}^{0}\frac{\partial V_x}{\cancel{\partial z}}^{0}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 V_x}{dx^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2}\right)$$

Steady flow

x change with time

$V_y = V_z = 0$

$$0 = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 V_x}{\partial y^2}$$

$$\frac{\partial P}{\partial x} = \mu\frac{\partial^2 V_x}{\partial y^2}$$

(c)

$$\frac{P_2 - P_1}{L} = \mu\frac{\partial^2 V_x}{\partial y^2}$$

$$\frac{\partial V_x}{\partial y} = \frac{P_2 - P_1}{\mu L} y + C_1$$

$$V_x(y) = \frac{P_2 - P_1}{2\mu L} y^2 + C_1 y + C_2$$

boundary conditions

$V_x(y=0) = 0$ and $V_x(y=h) = V_{plate}$

$\hookrightarrow C_2 \Rightarrow 0$

$$V_{plate} = \frac{P_2 - P_1}{2\mu L} h^2 + C_1 h$$

$$C_1 = \frac{V_{plate} - \left(\frac{P_2 - P_1}{2\mu L}\right)h^2}{h}$$

$$= \frac{V_{plate}}{h} - \left(\frac{P_2 - P_1}{2\mu L}\right)h$$

$$V_x(y) = \frac{P_2 - P_1}{2\mu L} y^2 + \left[\frac{V_{plate}}{h} - \left(\frac{P_2 - P_1}{2\mu L}\right)h\right]y \quad //$$