Week 5

Empirical covariance functions

[icon] Overview

Welcome to Week 5 of Spatial Analysis.

This week we will cover three topics:

- Stationary random fields

- Modelling and estimating covariance functions

You will start this week learning about several theoretical models of stationarity random fields that generalise weak stationarity time series. We also consider several popular theoretical models and examples of covariance functions. You will study key properties of covariance functions that are important to describe dependencies in spatial data.

This week we will also consider several estimating approaches to obtain empirical covariance functions using real data. These estimators will be used in the following weeks as the main statistical tools for the analysis and extrapolation of spatial data.

Finally, you will participate in a workshop, in which you can practise conducting an analysis of real spatial data.

By the end of this week, you will learn:

Topic 1: Stationary random fields

- Several different models of stationary fields.
- Isotropy.
- Mean functions.
- Some properties and attributes of covariance functions.
- Nugget effect.
- Gaussian random fields.
- Examples of isotropic models.

Topic 2: Modelling and estimating covariance functions

- Sample covariance functions.
- Computing sample variograms in R.
- Computing directional variograms.
- Studying spatial dependencies in the s100 data.

By completing this module, you will be working towards the following subject-intended learning outcomes:

1. Formulate purposeful questions to explore new statistical ideas and subsequently design valid statistical experiments.
2. Present clear, well-structured analysis of important statistical model results.
3. Creatively find solutions to real-world problems consistent with those commonly faced by practising statisticians.

Topic 1: Stationarity

This topic discusses the basic theoretical concepts of stationarity for random fields. These models can be applied to spatial data that have the same properties regardless of their location, i.e. properties that are invariant under shifts, rotations or similar spatial transformations. We consider several specific classes of covariance functions for such fields. Finally, you will learn about some properties and examples of covariance functions that are important in applied spatial data analysis.

Stationary field

In this section, you will learn three concepts of stationarity:

- Strict stationarity
- Weak stationarity
- Intrinsic stationarity

The concept of strict stationarity deals with joint probability distributions. It requires their dependence only on the relative positions of the observations' locations. While it is a very important theoretical concept and is used in various theoretical results, it is very difficult to check it for real data. Therefore, weak and intrinsic stationarity concepts are often used in practical spatial data analysis. These concepts consider only second-order properties of random fields and are based on mean, variogram and covariance functions.

The mean functions are used to describe spatial non-random trends. Usually, a simple parametric form of mean functions is used. For example, polynomials with respect to spatial coordinates, elevation, or other covariates. Often it is easy to estimate a mean function (see, the previous section about fitting spatial trends). After that, it is usually subtracted from the data and one uses more advanced methods to analyse the remaining information. In the following, we mainly concentrate on these advanced methods.

Read

First, in the reading Stationarity and covariance functions, you will learn the definitions of the three concepts of stationarity and overview mean functions.

From the folder, open the document titled **Week_5_Topic_1** and read **slides 1–4**.

Properties and attributes of covariance functions.

In this part, we introduce 3 properties of covariance functions that are useful in applications

- Isotropy
- Geometric anisotropy
- Separability

Then, we use the Matern class of covariance functions to illustrate various attributes of covariance functions that are important for interpreting different types of spatial behaviour.

Next, we consider the nugget effect which can be evident in the covariance function via its discontinuity at the origin. For example, this effect can be used to describe errors due to measuring devices.

Finally, you recall several properties of covariance functions (positive definite functions) that are useful to construct new covariance models from the known ones.

Read

You will read the second part of Stationarity and covariance functions and will about several important properties of covariance functions.

From the folder, open the document titled **Week_5_Topic_1** and read **slides 5–11**.

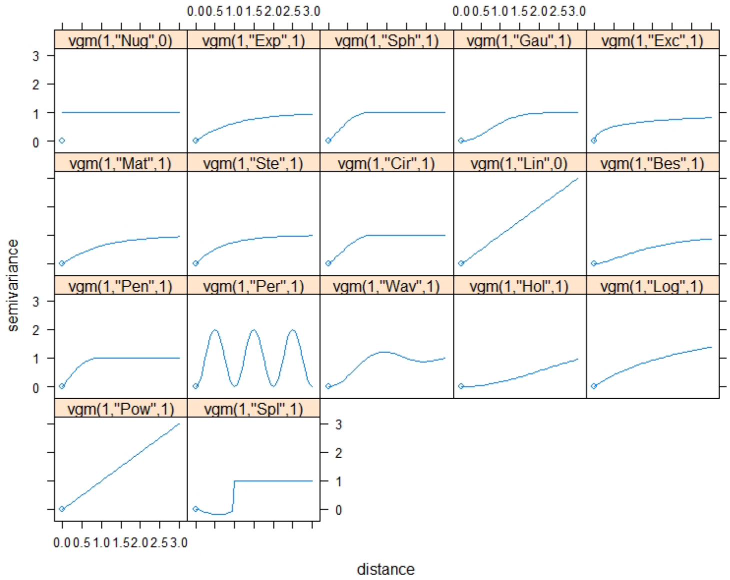
Gaussian random fields and isotropic covariances

In this section, we introduce a class of Gaussian random fields. These fields have the property that any non-degenerated linear combination of their values is Gaussian. This is one of the most popular statistical models of random fields in geostatistic.

Then, we will consider several models of isotropic semivariogram functions that are realised in R and often used in spatial data analysis, see Fig 3.3. We provide a mathematical formula for each of these functions and encourage students to plot them and investigate changes in their plots by changing their parameters.

Figure 3.3

Isotropic semivariograms



Read

In the third part of the reading Stationarity and covariance functions, you will learn about different isotropic variogram models.

From the folder, open the document titled **Week_5_Topic_1** and read **slides 12–19**.

Topic 2: Sample covariance and variogram in R

In this topic, you will start with learning four approaches to estimate covariance and variogram functions for spatial data. Then, you will use them to investigate spatial dependencies in the s100 data with R. We will use the obtained results and these approaches in the following weeks.

Sample covariances and variograms

To check whether dependencies are present or not one usually produces plots of observations grouped according to their separation distance. In this topic we consider four such methods:

- Variogram cloud
- Sample semivariogram
- Sample covariance
- Directional variograms

We provide the mathematical formulae for computing and plotting results for these methods.

Read

First, in the reading Sample covariance and variogram in R, you will learn four methods to compute empirical covariance and variogram functions.

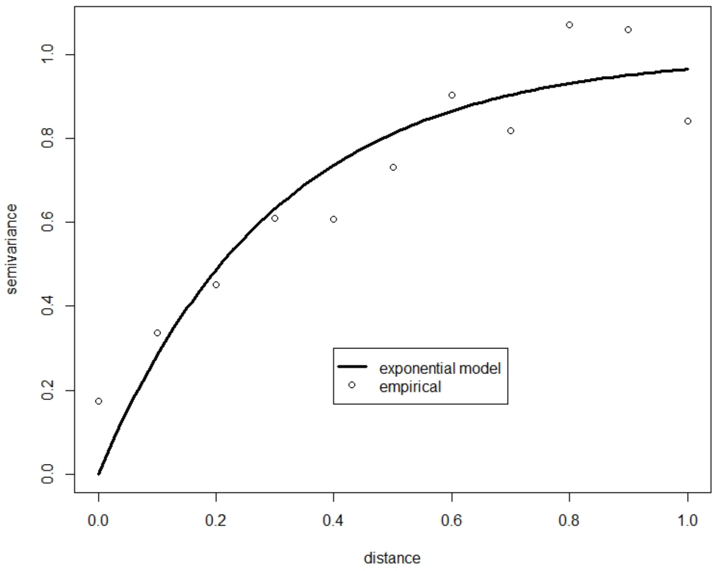
From the folder, open the document titled **Week_5_Topic_2** and read **slides 1–3**.

Computing sample covariance and variogram in R

We will use the data set s100 and the function variog(…) from the R package geoR to illustrate computations of different sample covariance and variogram functions in R. First, we consider computing these functions for isotropic models. As the data set s100 was simulated from a specific statistical model, we can compare the obtained empirical results with the expected function for this model. Then, we illustrate how these functions can be computed in a specified direction or a set of given directions.

Figure 3.4

Theoretical and empirical semivariograms



Read
In the second part of the reading Sample covariance and variogram in R, you will use the geoR package to practice with empirical estimation methods for covariance and variogram functions. From the folder, open the document titled **Week_5_Topic_2** and read **slides 4–8**.

Revise key R commands used in this week's materials.

Read
In the reading Key R commands, you will revise some of the key R commands that were used in this topic's materials. From the folder, open the document titled **Week_5_Topic_2** and read **slide 9**.

Workshop

Activity

Workshop

This activity will be completed in R. Repeat the R programming content covered in Week 5. Modify the code and understand the impact of different R parameters on changes in results.

Your task

- Repeat R commands learnt in Week 5 prior to the two-hour workshop session.
- Try to modify the code and understand the impact and meaning of different R function parameters. Interpret the observed changes in plots and analysis results.
- Feel free to discuss questions with other students as you go in the forum, and please also take the time to help others. It is amazing how much we all can learn from each other's questions, and how in helping others we strengthen our own understanding.
- Revisit these problems in later weeks and challenge yourself to get a deeper understanding to build on what you learn later.

Guidelines

- This activity is not graded but is an essential part of your learning. It will be held synchronously and facilitated by your instructor.
- You don't need to submit your R code; however, to be successful in this subject it is necessary to work through all R coding materials from this week and understand how to apply the corresponding R commands.
- You should repeat all R commands in this week's materials before the workshop. This will give you an opportunity to efficiently work with the facilitator during the workshop and get your questions answered.
- You should spend around two hours on this activity.

Summary

This week, we studied several models of stationarity that simplify general random fields to simple classes, that can be easier handled. We provided several examples of such isotropic models that are realised in R. Finally, we learned how to estimate covariance and variogram functions and practised it with the s100 data.

Next week, we will continue learning about estimating covariance and variogram functions and applications of the obtained results for several problems in spatial data analysis.

Here's a list of tasks that you should be working on or have completed:

- **Required readings**
- **Workshop**

The following resources provide you with this week's references and additional suggested readings.

[icon] Additional suggested readings and resources

While these readings and resources are not essential, they provide greater insight into the concepts covered in the week's lectures and give you the choice to enhance your learning or pursue an area of interest in greater detail.

Software and data:

- CRAN documentation for the geoR package: https://cran.r-project.org/web/packages/geoR/index.html

Theoretical concepts:

- Variograms: https://help.rockware.com/rockworks17/WebHelp/gridding_krig_variogram.htm

Books:

- Bivand, R. S., Pebesma, E., & Gomez-Rubio, V. (2013). Applied spatial data analysis with R (2nd ed.). Springer. https://doi.org/10.1007/s12061-014-9118-y Avalilable on-line in La Trobe EBL ebook Library
- Cressie, N.A.C (1993) Statistics for spatial data. Wiley. https://onlinelibrary.wiley.com/doi/book/10.1002/9781119115151 Avalilable on-line in La Trobe EBL ebook Library