MAST30013 – Techniques in Operations Research

Semester 1, 2021

Tutorial 5

1. Consider the function $f : \mathbb{R}^3 \to \mathbb{R}$

$$f(x_1, x_2, x_3) \;=\; x_1^2 + x_2^2 + x_3^2 - 4x_1 + x_2x_3.$$

   Starting at $\boldsymbol{x}^0 = (1,1,1)^T$, find the local minimum of f using Newton's method and the BFGS method.

   For the BFGS method use $\boldsymbol{H}_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$.

   Stop when $\| \boldsymbol{\nabla} f(x_1, x_2) \| < 0.01$.

2. Show that, for $k \geq 0$, $\boldsymbol{H}_{k+1}$ is symmetric if $\boldsymbol{H}_0$ is symmetric.
   (Hint: Use induction.)

3. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x_1, x_2) \;=\; x_1^2 - x_1x_2 + x_2^3 + x_1 - x_2.$$

   Starting at $\boldsymbol{x}^0 = (0,0)$, find the local minimum of f using Newton's method.

   Stop when $\| \boldsymbol{\nabla} f(x_1, x_2) \| < 0.01$.

   Compare Newton's method with the steepest descent method used to solve the same problem in Tutorial 4.

4. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x_1, x_2) \;=\; x_1^2 - x_1x_2 + x_2^3 + x_1 - x_2.$$

   Starting at $\boldsymbol{x}^0 = (0,0)$ with $\boldsymbol{H}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the local minimum of f using the BFGS quasi-Newton method.

   Stop when $\| \boldsymbol{\nabla} f(x_1, x_2) \| < 0.01$.

   Compare the BFGS quasi-Newton method with the steepest descent method and Newton's method.