The University of Melbourne

Semester 2 Assessment 2010

Department:	CHEMICAL AND BIOMOLECULAR ENGINEERING CIVIL AND ENVIRONMENTAL ENGINEERING	
Subject Number:	ENGR30001 (400-306)	
Subject Title:	FLUID MECHANICS	PASS AND HONOURS

Reading Time: 15 minutes

Writing Time: 3 hours

This paper has eight pages consisting of five questions.

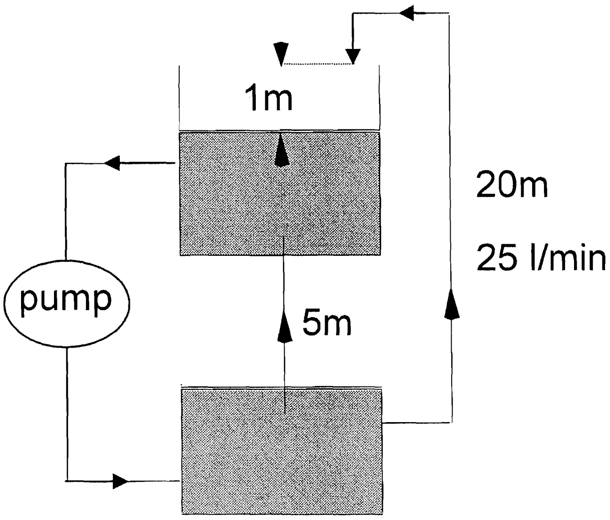
Authorized material:
Electronic calculators approved by the Melbourne School of Engineering may be used.
Two charts and a table of formulae are attached.

Instructions to Invigilators:
Handouts - 10 page script books.

Instructions to Students:
All questions are to be attempted.
Full marks will be awarded for obtaining 100 marks of a potential 110 marks.

This paper is to be held by the Baillieu Library

1. Liquid in an open tank is pumped through 10m of piping (with two elbows) into a closed tank below it. The closed tank is completely filled with liquid so pumping into it forces liquid to flow out through the two outlet pipes, as shown in the diagram. One outlet pipe (20m long with 3 elbows) discharges liquid freely at a height 1m above the liquid surface in the upper tank at a volumetric flow rate of 25 litres per minute. The other outlet pipe (5m long, no elbows) carries liquid from the bottom tank to the top tank directly. All piping has an internal diameter of 25mm with an absolute roughness of 0.05mm. Ignore pipe entry and pipe exit losses.



Density of the liquid	1100 kg m^{-3}
Viscosity of the liquid	0.0015 Pa s
Acceleration due to gravity	9.81 m s^{-2}
Equivalent length of one elbow	25 pipe diameters

(a) Calculate the volumetric flow rate through the 5m vertical outlet pipe

(18 marks)

(b) Hence calculate the volumetric flow rate of the liquid being pumped.

(2 marks)

(c) Use the results from (a) and (b) to calculate the power that the pump supplies to the liquid.

(8 marks)

(Total for Q1 = 28 marks)

2. The mechanical energy equation for horizontal, isothermal, ideal gas flow in a pipe of uniform cross-section is

$$\frac{P_2^2 - P_1^2}{2RT/M} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{2fL}{D}\left(\frac{G}{A}\right)^2 = 0$$

where all the variables have their usual meanings.

(a) If the pressure outside the pipe exit is reduced while the upstream pressure is fixed, give an expression for the maximum velocity attainable in the pipe, and state where it will occur.

(3 marks)

(b) If this maximum velocity occurs when the pressure at the pipe exit equals P_w, show that P_w satisfies the following equation:

$$\left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1 = \frac{4fL}{D}$$

(6 marks)

Now consider nitrogen gas flowing isothermally at a temperature of 100°C through a horizontal pipe having internal diameter 50 mm and length 20 m. The gas pressure at the pipe entrance is 800 kPa and the pressure in the gas outside the pipe exit is 200 kPa. The Fanning friction factor $f = 0.006$. Assume an ideal gas.

(c) Calculate P_w and hence determine whether or not the flow is choked. Give a reason for your conclusion.

(5 marks)

(d) Calculate the mass flow rate of nitrogen through the pipe.

(8 marks)

(e) Calculate the entry and exit gas velocities as percentages of the sonic velocity.

(6 marks)

(Question 2 continues on next page)

2. Continued

Data:

Gram molecular weight of nitrogen gas	28
Viscosity of Nitrogen gas	1.8×10^{-5} Pa s
Gas constant R	8.314 J mol^{-1} K^{-1}

(Total for Q2 = 28 marks)

3. (a) At its Best Efficiency Point, a centrifugal pump develops 11.75 m head of water (density 1000 kg m^{-3}) and pumps the water at 40 litres per second with an impeller rate of rotation equal to 850 revolutions per minute.

i) Calculate the power transferred to the water. **(2 marks)**

The same pump, delivering the same power, is now to be used to pump oil (density 886 kg m^{-3}) while again operating at the Best Efficiency Point.

ii) Calculate the head of oil that the pump will develop. **(4 marks)**

iii) Calculate the volumetric flow rate of oil in litres per second. **(2 marks)**

iv) Calculate the rate of rotation of the impeller in revolutions per minute when pumping the oil. **(1 mark)**

(b) (i) Explain what is meant by a Newtonian fluid, a shear thinning fluid, and a Bingham fluid. Make a correctly labelled sketch to illustrate. **(7 marks)**

(ii) Give the expression for the Power Law model of the relationship between shear stress and shear rate, defining all symbols used. **(3 marks)**

(iii) If the power law index $n = 0.2$, then is the fluid behaviour one of shear thickening, shear thinning or Newtonian? Give a reason for your answer. **(3 marks)**

(Total for Q3 = 22 marks)

4. Water is pumped from a large open tank through a smooth pipe of internal diameter 25 mm. The length of the piping on the suction side of the pump, including the equivalent length of any fittings, is 15 m and the height of the pump above the liquid surface in the tank is 3 m.

Density of water	1000 kg m^{-3}
Viscosity of water	0.001 Pa s
Atmospheric pressure	101.3 kPa
Vapour pressure of water	4.2 kPa absolute
Acceleration due to gravity	9.81 m s^{-2}
Required NPSH	4 m

If the mass flow rate is fixed at 1 kg s^{-1} :

b) Calculate the available NPSH. **(8 marks)**

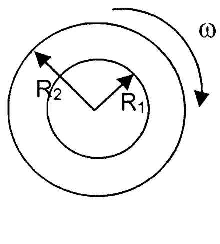
c) Do you have a cavitation problem? Give a reason. **(2 marks)**

d) If cavitation was a problem, you could vary the height of the pump above the liquid surface to eliminate the cavitation. Would you increase or decrease the height? Give a reason.

(2 marks)

(Total for Q4 = 12 mark)

5. An incompressible viscous Newtonian fluid is located in the space between two concentric cylinders having radii R_1 and R_2. The inner cylinder is fixed but the outer cylinder is rotating with an angular velocity ω, thus imparting a swirling motion to the fluid. The fluid is also being driven in the z-direction due to a constant applied pressure gradient $\frac{\partial p}{\partial z} = -B$. The flow is steady, fully developed and independent of the polar angle θ, where (r, θ, z) are cylindrical polar coordinates. The cross section of the two cylinders is shown in the diagram.



a) Find an expression for the axial velocity (v_z) profile. **(9 marks)**

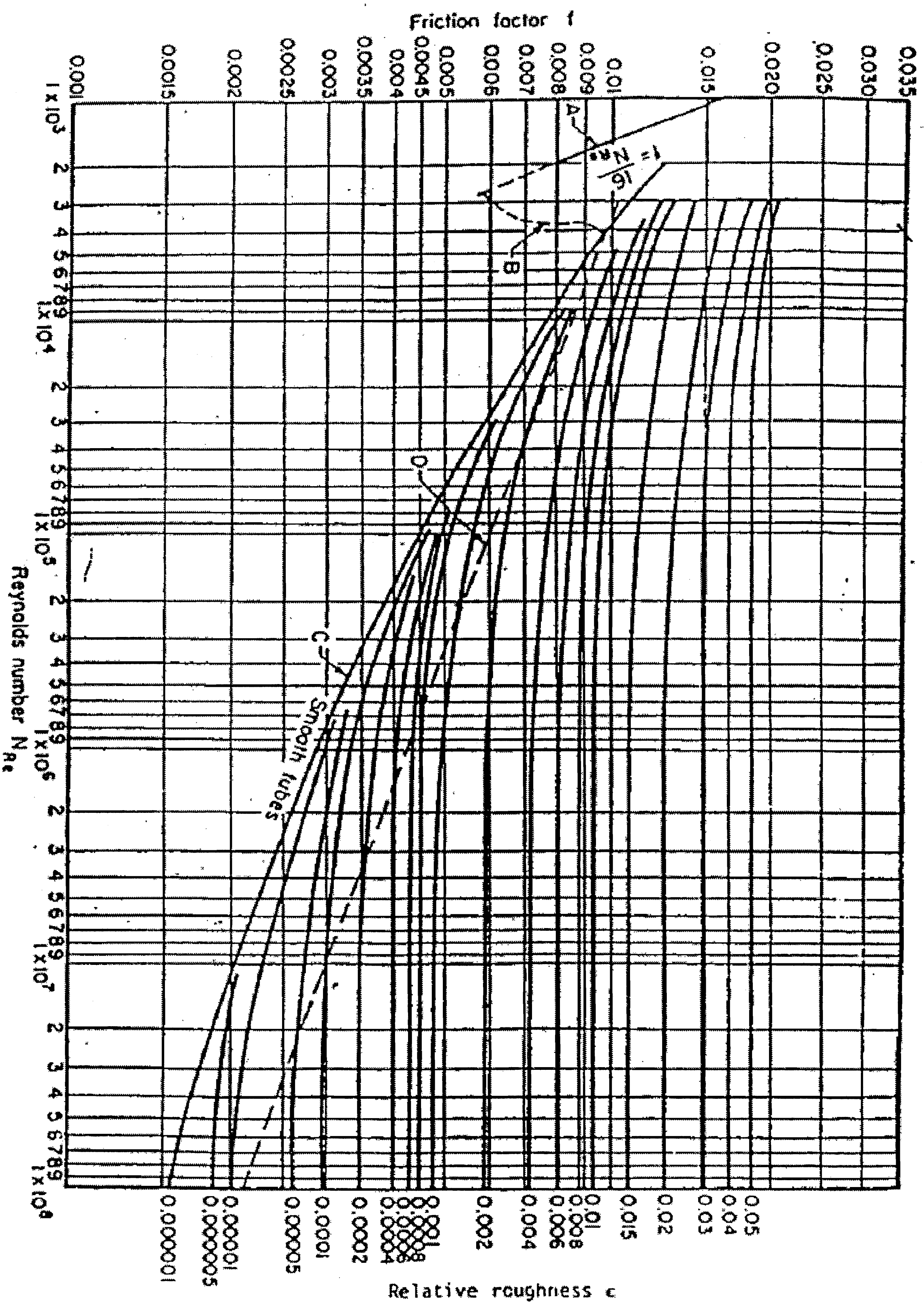
b) Find an expression for the tangential velocity (v_θ) profile **(9 marks)**

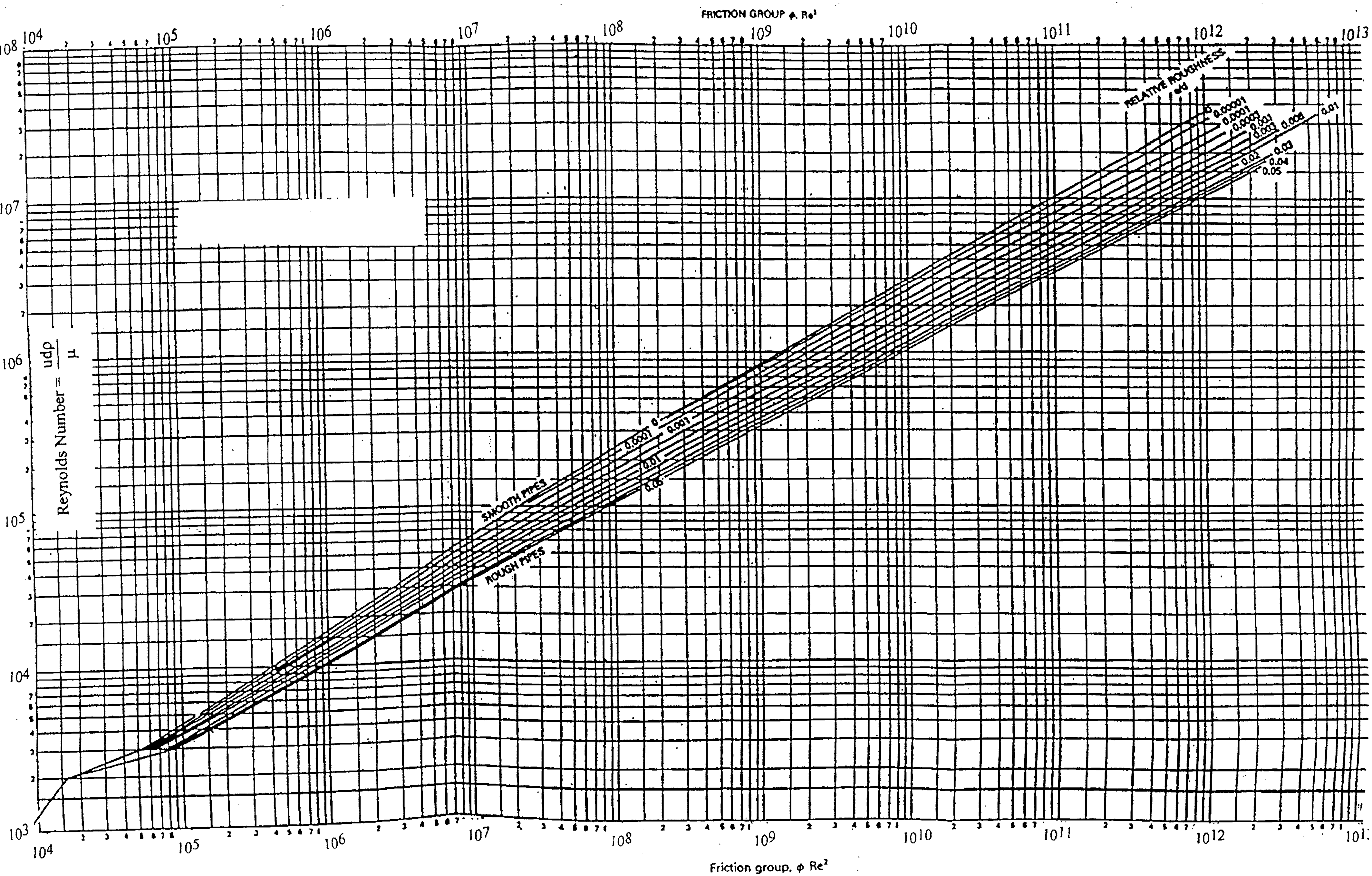
c) Find an expression for the radial pressure gradient. **(2 marks)**

(Total for Q5 = 20 marks)

(Total for paper = 110 marks)

End of Examination


Friction factor f
0.001
0.0015
0.002
0.0025
0.003
0.0035
0.004
0.0045
0.005
0.006
0.007
0.008
0.009
0.01
0.015
0.020
0.025
0.030
0.035
Reynolds number N_{Re}
$f = \frac{16}{N_{Re}}$
A
B
C
D
Smooth tubes
Relative roughness ε
0.000001
0.000005
0.00001
0.00005
0.0001
0.0002
0.0004
0.0006
0.0008
0.001
0.002
0.004
0.006
0.008
0.01
0.015
0.02
0.03
0.04
0.05


FRICTION GROUP ϕ Re^2
Reynolds Number = $\frac{ud\rho}{\mu}$
Friction group, ϕ Re^2
RELATIVE ROUGHNESS e/d
SMOOTH PIPES
ROUGH PIPES
0
0.00001
0.0001
0.0002
0.001
0.002
0.008
0.01
0.02
0.03
0.04
0.05

Continuity and Navier-Stokes equations for incompressible homogeneous fluids in Cartesian, cylindrical, and spherical coordinates

Cartesian	Cylindrical	Spherical
Continuity equation		
$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$	$\frac{1}{r}\frac{\partial (r v_r)}{\partial r}+\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta}\right)+\frac{\partial v_z}{\partial z}=0$	$\frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial (v_\theta \sin\theta)}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}=0$
Navier-Stokes equation		
$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)$ $=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)$ $=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)$ $=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_r}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}-\frac{2v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}-\frac{2v_\theta\cot\theta}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]$
$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)$ $=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)$ $=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)$ $=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_\theta}{\partial \theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta}{r^2\sin^2\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial \phi}\right]$
$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)$ $=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)$	$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$ $=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]$	$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r v_\phi}{r}+\frac{v_\theta v_\phi\cot\theta}{r}\right)$ $=-\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_\phi}{\partial \theta}\right)\right.$ $\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2}-\frac{v_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial \phi}\right]$