

STM4PSD – Workshop 1

- Decide whether each of the following would be best described by a continuous random variable or a discrete random variable.
 - The time you spend talking on your next phonecall.
 - The number of weddings in the next six months.
 - The genre of a randomly chosen movie.
 - The average temperature on Earth on a randomly chosen day.
 - The number of different places visited in one day by a random person.
 - The amount of distance travelled in one day by a random person.
- Calculate each of the following binomial coefficients by hand, showing working.
 - $\binom{10}{9}$
 - $\binom{10}{1}$
 - $\binom{9}{4}$
 - $\binom{11}{8}$
- Eighteen people walk into a room with 23 seats. How many different ways can they choose their seating arrangement?
- Suppose that a four-sided die with numbers 1, 2, 3 and 4 on the faces is rolled once, and let X denote the random variable associated to the outcome of the die roll.
 - Write down the probability mass function for X , in table form. Give each entry in the table exactly.
 - Use your answer to (a) to determine each of the following, giving answers exactly:
 - $P(X = 3)$
 - $P(X \neq 3)$
 - $P(X \leq 3)$
 - $P(X < 3)$
 - $P(X \geq 2)$
 - Calculate $E(X)$ exactly. Show all working.
 - Calculate $\text{Var}(X)$ and $\text{SD}(X)$. Show all working, and give your final answer to at least 2 decimal places.
- Suppose that a fair six-sided die is to be rolled *twice*. Let (i, j) denote the (simple) event that the first roll resulted in the number i , and the second roll resulted in the number j . For example, $(5, 2)$ would mean that the first number was a 5 and the second was a 2. Then let Y denote the random variable associated to this pair of rolls.
 - Write down all 36 elements of the sample space Ω_Y . For later parts, it may be useful to write your answer in a 6×6 grid.

Now let A denote the event that the sum of the two numbers is greater than or equal to 9, let B denote the event that the numbers rolled are equal to one another, and let C denote the event that both numbers rolled are odd.

 - Before doing any calculations, based on your intuition, which of the events A , B and C do you think is most likely and which do you think is least likely?
 - Write down the elements of the events A , B and C , and then determine $P(A)$, $P(B)$ and $P(C)$. After doing this, revisit your answer to part (b).
 - By writing out the elements of each event, determine the following, giving exact answers:
 - $P(A \cup B)$
 - $P(A \cap B)$
 - $P(B \cap C)$
 - $P((A \cup B) \cap C)$
- As in Question 5, suppose a fair six-sided die is to be rolled twice. But this time, let Z denote the random variable obtained by adding the two numbers together. For example, if the first number was 5 and the second was 3, then the variable Z would take on the value 8.
 - Write down the sample space Ω_Z .
 - Determine the probability mass function for Z , giving your answer in table form. Your answer to 5(a) might be helpful.
 - The event A from Question 5 can be represented as " $Z \geq 9$ ". Use your answer to (b) to determine $P(A)$. Make sure you get the same answer.
 - Determine $E(Z)$, $\text{Var}(Z)$ and $\text{SD}(Z)$. Give $E(Z)$ exactly, and $\text{Var}(Z)$ and $\text{SD}(Z)$ to three decimal places.

7. For each of the scenarios described below,
- Define an appropriate random variable for the scenario by describing it in a sentence.
 - State which distribution this random variable would follow.
 - State a calculation that would be needed to answer the given question.

Note: you are not being asked to perform any calculations for this question. The task of the question is to interpret the scenario using probability terminology. An example of the expected answer format is shown below.

- Each day, a computer server has a 0.3% chance of crashing. What is the expected number of days the server would run before crashing?
- In the popular video game Minecraft, the character (you) can tame a wolf by giving it a bone. The probability of taming a wolf with each attempt is fixed at $1/3$. You encounter a wolf, and you have 6 bones available. You want to tame the wolf, and will use all bones if necessary. What are the chances the wolf will be tamed?

Note: there is more than one way to approach this problem. Try to describe them both.

- Consider the same scenario as in part (b), but you now have an unlimited supply of bones. On average, how many bones will be needed to tame the wolf?
- On average, a high risk traffic area has 20 days with car accidents per year. What is the probability that there will be fewer than 10 days with car accidents in one year?
(Estimate the probability of “success” by using the number of days in a year.)
- You have a 10% chance of winning a prize by spinning a wheel. On average, how many spins would be needed to win three prizes?
- Consider the same scenario as in part (e), but this time you are only allowed five attempts. What is the probability you win at least one prize?
- At a grocery store, on average, 1 in every 100 cans of baked beans has a dent in it. If the store has ordered 1250 cans of baked beans, what is the probability that at most 20 cans will have a dent?

Example: Based on historical data, 1.5% of all GPUs produced by a certain manufacturer are defective. After producing 1,000 GPUs, what is the probability that more than 10 are defective?

- Let X denote the number of defective GPUs.
- Then $X \sim \text{Bin}(1000, 0.015)$.
- We need to determine $P(X > 10)$.

8. Let $X \sim \text{Bin}(10, 0.3)$. Referring to the reading materials if necessary, find each of the following:

- $P(X = 3)$
- $P(X \leq 1)$
- $P(X \geq 2)$
- $P(X > 8)$
- $P(X \leq 9)$
- $P(2 \leq X \leq 9)$

Determine the binomial coefficients exactly, and then give each final answer to 3 decimal places.

9. Let $Y \sim \text{NB}(3, 0.15)$. Referring to the reading materials if necessary, find each of the following:

- $P(Y = 3)$
- $P(Y \leq 1)$
- $P(Y \geq 2)$
- $P(1 < Y \leq 3)$

Determine the binomial coefficients exactly, and then give each final answer to 3 decimal places.

10. Perform the calculations described by Questions 7(a), (b), (c), (e) and (f).

(Parts (d) and (g) are not reasonable to compute by hand, but soon we will see how to compute them using R.)