Student ID __________

THE UNIVERSITY OF MELBOURNE

**Department of Biomedical Engineering**

# ENGR30002 Fluid Mechanics

**Semester 1 2019**

**Reading Time: 15 minutes – no writing or annotating allowed anywhere**

**Writing time: 180 minutes**

**This paper has 14 pages including this page and any Appendices.**

**Authorised Materials**
**Calculators:** Casio fx82 or fx100 calculators are permitted
**Notes:** One A4 sheet of handwritten notes (front and back) is permitted

**Instructions to Invigilators**

- The examination paper IS TO REMAIN in the examination room
- Students should include detached pages from question paper in the script book for collection
- Students are to be provided with one script book
- Provide extra script books on request

**Instructions to Students**

- Total marks for this paper is 180
- Ensure your student number is written on all script books and answer sheets during writing time. No annotating is allowed in reading time or after the end of writing time
- Attempt all 8 questions, which are of unequal marks value
- Answer all questions on the right-hand lined pages of the script book
- Start the answer to each question on a new page in the script book and write the question number in the top right hand corner
- The left-hand unlined pages of script books are for draft working and notes and will not be marked
- State clearly and justify any assumptions made
- Write legibly in blue or black pen
- Show all working for each problem
- Pages 12 to 14 contains information and formulas that may be useful in answering the questions
- Mobile phones, tablets, laptops, and other electronic devices, wallets and purses must be placed beneath your desk
- All electronic devices (including mobile phones and phone alarms) must be switched off and remain under your desk until you leave the examination venue. No items may be taken to the toilet

**Question 1 [33 marks]**

Briefly answer the following short questions

a) Should we use a pipe's internal diameter or external diameter when calculating mechanical energy balances in pipe flow?

**[2 mark]**

b) In a situation without cavitation, which is typically greater, $NPSH_r$ or $NPSH_a$?

**[2 mark]**

c) We have a pump moving fluid from a tank. Describe in one to two sentences how to find this system's operating point.

**[4 marks]**

d) Which of these parameters will remain constant along the length of a constant diameter pipe in incompressible flow?
   i. velocity
   ii. density
   iii. mass flow rate
   iv. pressure

**[2 marks]**

e) Which of these parameters will remain constant along the length of a constant diameter pipe in compressible flow?
   i. velocity
   ii. density
   iii. mass flow rate
   iv. pressure

**[2 marks]**

f) A fully insulated pipe with a friction factor of 0.005 is used as a conduit for air from a pressurized tank. Choose the answer that best describes this flow and explain in one sentence why.
   i. isentropic
   ii. adiabatic
   iii. isothermal
   iv. isobaric

**[4 marks]**

g) What is the molecular weight of gaseous ethanol ($C_2H_6O$) in g/mol?

**[3 marks]**

**Question 1 continued on next page**

h) At the start of a pipe with isentropic flow of nitrogen (28 g/mol) we have a pressure of 200 kPa and a density of 2.3 kg/m^3, and at the outlet we have a pressure of 100 kPa. The ratio of specific heat capacities (gamma) for diatomic gases is 1.4. Calculate the density at the outlet.

**[5 marks]**

i) What is the equivalent hydraulic diameter for a flow through an annulus, with an inner pipe diameter of 10 cm and an outer diameter of 20 cm?

**[5 marks]**

j) For each of these equations, write whether they are most appropriate for (1) Newtonian fluids, (2) Shear-thinning fluids, (3) Shear-thickening fluids or (4) Bingham plastics.

A. $|\tau_{xy}| = \tau_y + k_p \left|\frac{\delta V_x}{\delta y}\right|^n, |\tau| > \tau_y$

B. $\tau_{xy} = -\mu \frac{\delta V_x}{\delta y}$

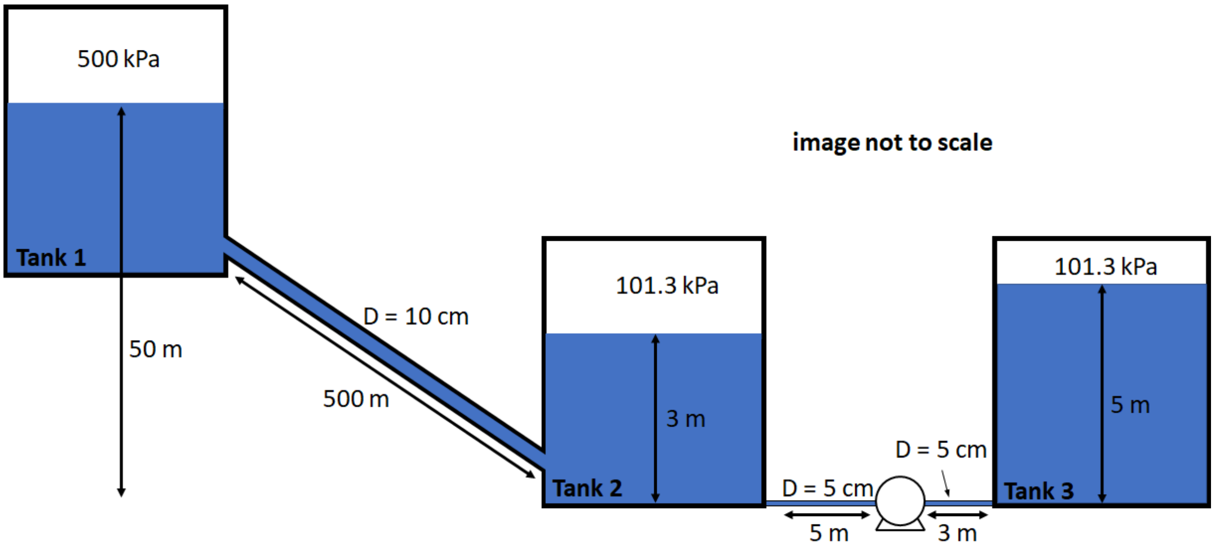
C. $|\tau_{xy}| = k \left|\frac{\delta V_x}{\delta y}\right|^n, n < 1$

D. $|\tau_{xy}| = k \left|\frac{\delta V_x}{\delta y}\right|^n, n > 1$

**[4 marks]**

**Question 2 [31 marks]**

We have set up a base on mars, where we want to transport water we've mined from surface ice. This water travels downhill from a tank at the mining site (Tank 1) to a settling and distribution tank (Tank 2) before it is pumped to a tank in our settlement. To make sure the water remains liquid, we have heated and pressurized our storage system. The pressures, diameters and free surface heights in each of our tanks are given in the image below. This image is not to scale.
The gravitational acceleration on mars is given by $\mathbf{g_{mars} = 3.7\ m/s^2}$, and the surface roughness of the iron pipes we're using is 1 mm. You can use values of ρ = 1000 kg/m$^3$ and μ = 0.001 Pa·s for water. Ignore minor losses.




a) Calculate the flow rate (in m$^3$/s) and average velocity (in m/s) in the pipe from Tank 1 to Tank 2.

**[9 marks]**

b) Imagine we wanted to regulate the flow through the pipe by changing the pressure in Tank 2. What would the pressure in Tank 2 have to be to stop the flow through the pipe? Give your answer in kPa.

**[6 marks]**

c) We have a centrifugal pump moving the water from Tank 2 to Tank 3. The vapour pressure of our water is 2 kPa. If our pump is moving 0.01 m$^3$/s from Tank 2, calculate the net positive suction head available ($NPSH_a$, in meters) of this pump. You can assume a fanning friction factor of 0.01 here.

**[8 marks]**

d) What is the suction head and discharge head of this pump at the this flow rate (0.01 m$^3$/s)?

**[8 marks]**

**Question 2 continued on next page**

e) If the pump has a mechanical efficiency of 65% at this flow rate, how much (brake) power do we need to supply to the pump? Assume your discharge head is equal to the pump head.

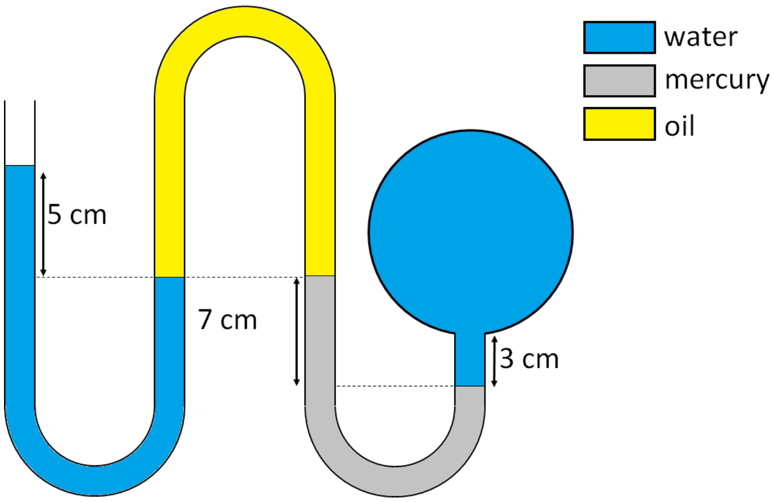
**[5 marks]**

**Question 3 [18 marks]**

Examine the use of manometers in the following scenarios [on earth, with g = 9.8 m/s]. The open ends of the manometers are exposed to the atmosphere (101.3 kPa). Use the following values for fluid densities: the density of water is 1000 $kg/m^3$, the density of oil is 800 $kg/m^3$ and the density of mercury is 13600 $kg/m^3$.

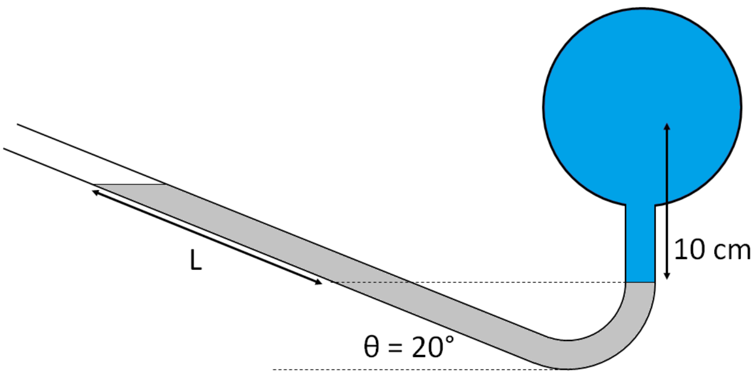
a) For the manometer below, find the absolute pressure at the end of the manometer connected to the pipe (the flow through the pipe is projected into the page).

**[9 marks]**




b) In the case of the inclined manometer below, the pressure at the centre of the pipe is 110 kPa, and our manometer fluid is mercury. In this case, the flow through the pipe is stopped. Find the length *L* (in cm) as per the diagram.

**[9 marks]**



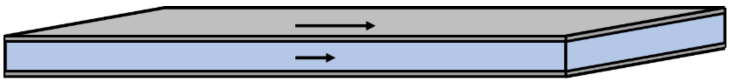
**Question 4 [18 marks]**

Examine the flow scenarios in the following statements and answer the questions.

a) Consider laminar Poiseuille flow in a wide and flat enclosed channel, which has a width of 20 cm and a height of 2 mm. The water flow in this channel is moving in the +x direction. The pressure at the channel inlet is 500 kPa and the pressure at the channel outlet is 200 kPa. Determine the force that's acting on the channel if it is kept stationary (in Newtons).

**[10 marks]**

b) Now consider laminar Couette flow between two parallel plates; the lower plate is stationary and the upper plate is moving at 0.1 m/s. The motion of the upper plate drives creates fluid flow between the plates. What is the average fluid velocity relative to the stationary plate? What is the minimum fluid velocity across the channel cross section? What is the maximum?



**[8 marks]**

**Question 5 [16 marks]**

Examine the following questions regarding mixing tanks and provide a response to each question.

a) After testing in a pilot-scale mixer we want to increase production in a full scale mixer. We will increase the volume of a mixing tank by a factor of 9. If our pilot scale mixer has a diameter of 1 meter, what should the diameter of the full-scale tank be?

**[6 marks]**

b) Describe why swirling is undesired in a mixing tank and two ways to avoid swirling in a mixing tank with a low viscosity fluid.

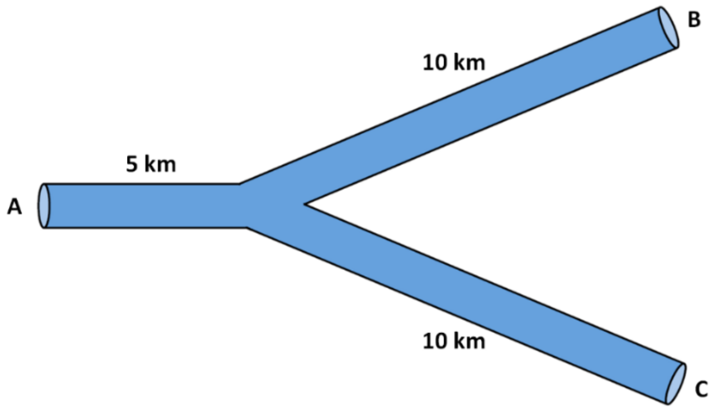
**[6 marks]**

c) What impeller type would be most appropriate for (i) a low viscosity fluid (water) and (ii) a very high viscosity fluid (melted plastic)?

**[4 marks]**

**Question 6 [28 marks]**

Back on mars, we're processing methane (16 g/mol) for later use on our rockets. A single stream of gaseous methane (flow at point A) diverges into two gas streams (flows to points B and C), with the values relevant to this system given in the table next to the diagram below. We have lost contact with our sensors at point C, however, so we want to reconstruct what the flow properties there are. All flow is horizontal and isothermal, and kinetic energy contributions can be ignored. Ignore minor losses.




| | | |
|---|---|---|
| pipe diameter | D | 20 cm |
| temperature | T | 20°C |
| mass flow rate (A) | $G_A$ | 2 kg/s |
| mass flow rate (B) | $G_B$ | 1 kg/s |
| pressure (A) | $P_A$ | 1000 kPa |
| pressure (B) | $P_B$ | 733 kPa |
| ideal gas constant | R | 8.314 J/(mol·K) |
| fanning friction factor | $f_F$ | 0.005 |
| molecular weight (methane) | M | 16 g/mol |

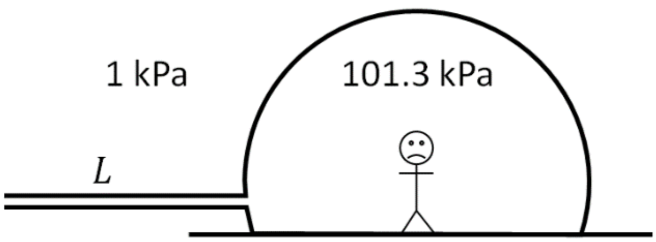
a) What is the pressure where the pipes join?

**[10 marks]**

b) What is the mass flow rate, pressure and velocity at the exit of flow C?

**[10 marks]**

In our martian base, a horizontal pipe connected to our habitat has been broken clean off, causing air (29 g/mol) to exit from our habitat in isothermal flow along the pipe. The pipe has a diameter of 1 cm, and a fanning friction factor of 0.01. The pressure inside of the habitat is 101.3 kPa, the temperature along the pipe is 20°C, and the pressure in the Martian atmosphere is 1 kPa.




c) What is the minimum length ($L$) the pipe would have to be for the flow to be choked?

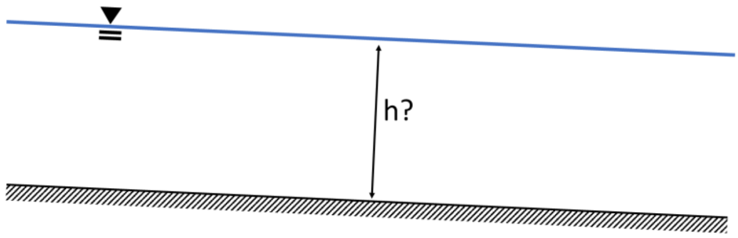
**[8 marks]**

**Question 7 [22 marks]**

Examine and answer the following questions regarding an open channel flow (on earth, with g = 9.8 m/s and atmospheric pressure of 101.3 kPa).

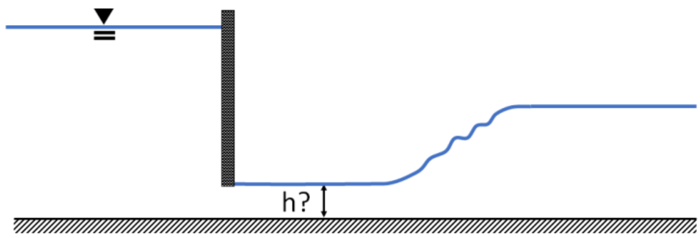
a) Water with a flow rate of 0.007 $m^3/s$ is moving along a channel that is 10 m wide, has a Manning's n of 0.05 and a slope of 1:200. To your best approximation, how deep is the water in this channel? You can assume that the height of the water is much less than the channel width.

**[8 marks]**




b) Water with a flow rate of 0.007 $m^3/s$ passes under a sluice gate which creates a rapid flow with a Froude number of 6 as the flow passes underneath it. This is followed by a hydraulic jump. All sections of this channel system (the upstream flow, the rapid flow, the hydraulic jump and the downstream flow) are 20 cm wide. Calculate the height of the rapid flow immediately after the sluice gate and calculate the head loss across the hydraulic jump.

**[10 marks]**




c) If we took this entire channel system and placed it in a pressurized tank in which the ambient pressure was doubled, would any of the water surface heights be altered? If yes, explain which ones. If not, explain in one to two sentences why.

**[4 marks]**

**Question 8 [14 marks]**

Examine and appropriately address the following questions.

a) You're running a space tourism company that sends people to space on rockets, and you have a customer who wants to walk on water just like a water strider does on earth, and have accordingly set up a swimming pool in your rocket. If ratio of surface tension forces to gravitational forces has to be greater than one for this to occur, what is the maximum value of your rocket's acceleration that will make your customer happy? She has a mass of 50 kg, her feet are each 0.2 m long, and the surface tension of water is 0.07 N/m. Express your answer in m/s$^2$.
**[8 marks]**

b) You want to build a model for a submarine that replicates the physics of the full scale submarine. If your model has dimensions that are 1/20$^{th}$ of the actual submarine, what should the flow velocity in your model system be relative to the flow velocity in the actual submarine (i.e., what is $v_{\text{model}}/v_{\text{actual}}$)?
**[6 marks]**

**Supplemental materials only beyond this point**

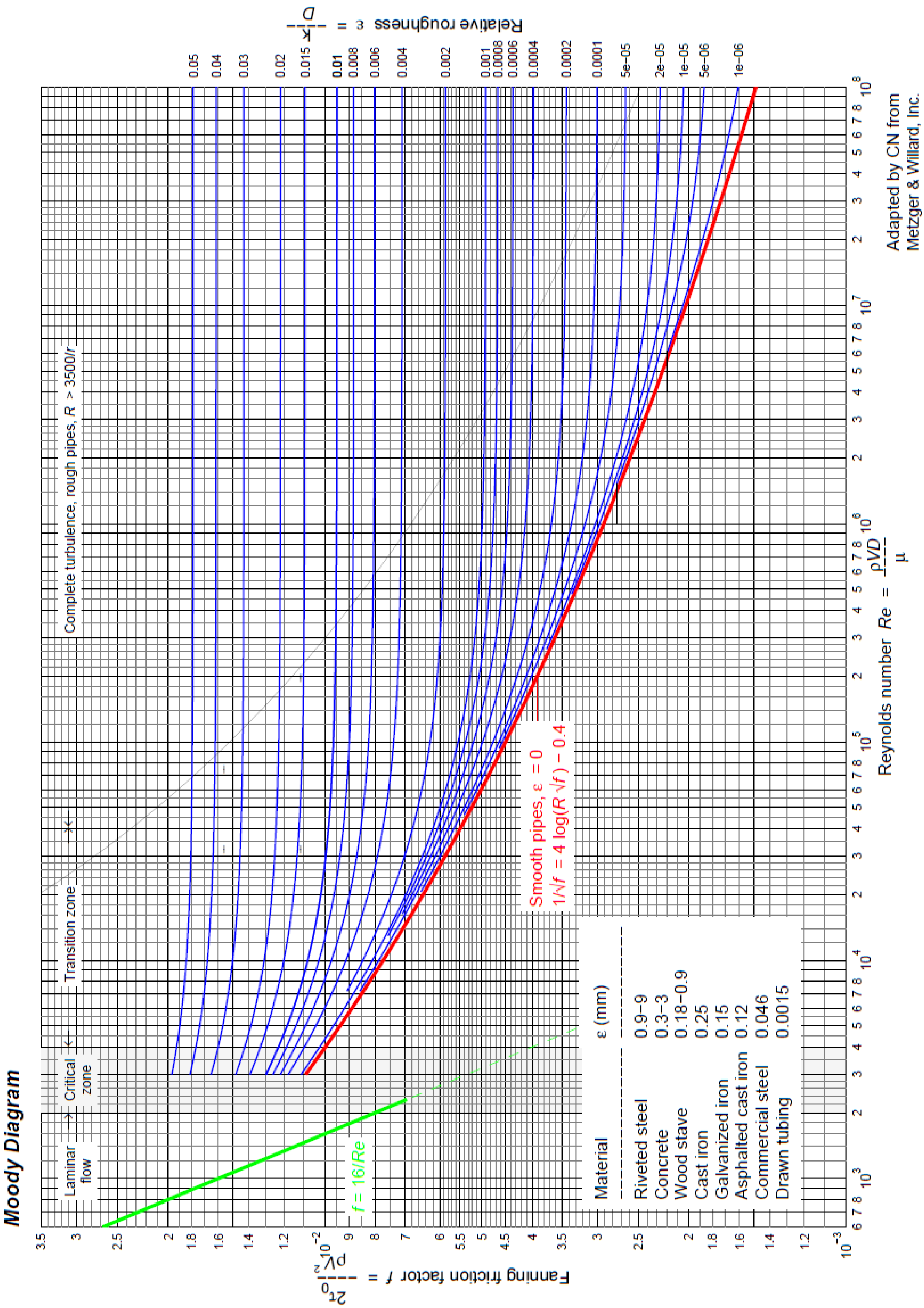
**Various constants**

| Parameter | Value | Units |
|---|---|---|
| Density of water | 1000 | kg/m$^3$ |
| Density of oil | 800 | kg/m$^3$ |
| Density of mercury | 13600 | kg/m$^3$ |
| Ideal gas constant | 8.314 | J/mol K |
| Atmospheric pressure on earth (sea level) | 101.3 | kPa |
| Molar mass of air | 29 | g/mol |
| Molar mass of carbon | 12 | g/mol |
| Molar mass of oxygen | 16 | g/mol |
| Molar mass of hydrogen | 1 | g/mol |
| gravity on earth | 9.8 | m/s$^2$ |
| gravity on mars | 3.7 | m/s$^2$ |

**Given equations**

$$\frac{h_1}{h_2} = \frac{-1 + \sqrt{8\mathrm{Fr}^2 + 1}}{2}$$

$$h_L = \frac{(h_2 - h_1)^3}{4h_1h_2}$$


Moody Diagram
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes, R > 3500/r
f = 16/Re
Smooth pipes, ε = 0
1/√f = 4 log(R √f) − 0.4
Relative roughness ε = k/D
0.05 0.04 0.03 0.02 0.015 0.01 0.008 0.006 0.004 0.002 0.001 0.0008 0.0006 0.0004 0.0002 0.0001 5e−05 2e−05 1e−05 5e−06 1e−06
Fanning friction factor f = 2τ0/ρV²
Reynolds number Re = ρVD/μ
Material
ε (mm)
Riveted steel 0.9−9
Concrete 0.3−3
Wood stave 0.18−0.9
Cast iron 0.25
Galvanized iron 0.15
Asphalted cast iron 0.12
Commercial steel 0.046
Drawn tubing 0.0015
Adapted by CN from Metzger & Willard, Inc.

## §B.4 THE EQUATION OF CONTINUITY[a]

$$[\partial\rho/\partial t + (\nabla \cdot \rho\mathbf{v}) = 0]$$

*Cartesian coordinates (x, y, z):*

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

*Cylindrical coordinates (r, θ, z):*

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

*Spherical coordinates (r, θ, φ):*

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

## §B.6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT $\rho$ AND $\mu$

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

*Cartesian coordinates (x, y, z):*

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

*Cylindrical coordinates (r, θ, z):*

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

*Spherical coordinates (r, θ, φ):*

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2 \cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta)\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2 \sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi \cot\theta}{r}\right) = -\frac{1}{r \sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi \sin\theta)\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2 \sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2 \sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi$$

**END OF EXAM**