MAST30013 Assignment 1 2021

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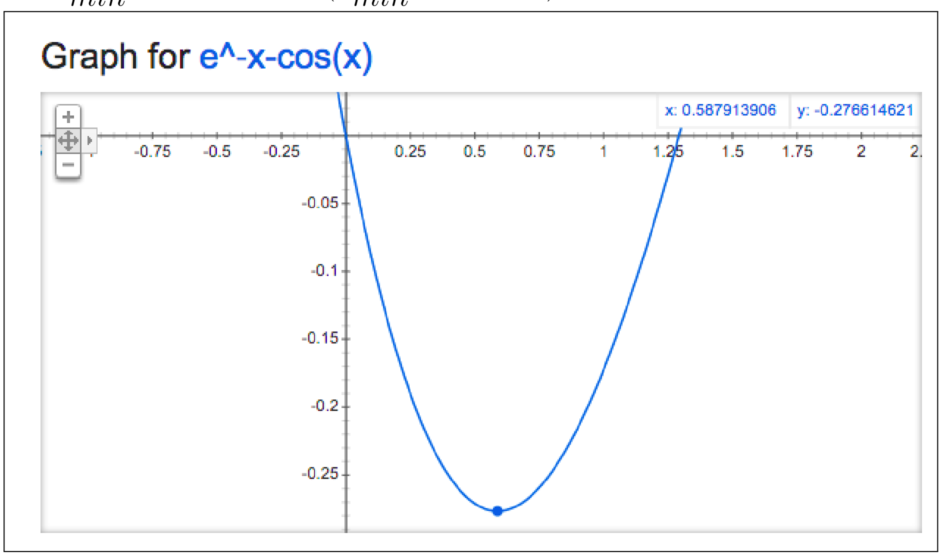
Question 1a Solution:

$$minf(x) := exp(-x) - cos(x)$$

$\boldsymbol{With\,respect\,to x}$:

$$f'(x) := -exp(-x) + sin(x) = 0$$

Since we expect to solve x from the domain, $x \in [0, 1]$.
$x_{min} \approx 0.58853$, this value was taken from WolframAlpha.
sub $x_{min} \approx 0.58853$ into f($x_{min} \approx 0.58853$) ≈ -0.2766146898253521



We proof that f is a uni-modal function and there is a unique global minimum in the interior of [0,1].
NOTE: Using radians instead of degrees!

Question 1b Solution:
We want one f-calculation in this iteration, and

$$\tau_2(1-\tau_1) = 1 - 2\tau_1$$

Assume that the ratio τ_k is independent of iteration k.

$$\tau_1 = \tau_2 = \tau$$

$$\tau^2 - 3\tau + 1 = 0$$

gives us,
$\tau_1 = \frac{3-\sqrt{5}}{2}$
$\tau_2 = \frac{3+\sqrt{5}}{2}$ disregarded
1 - τ = $\frac{\sqrt{5}-1}{2} = \gamma$

$$\gamma \approx 0.618$$

is the golden ratio!!

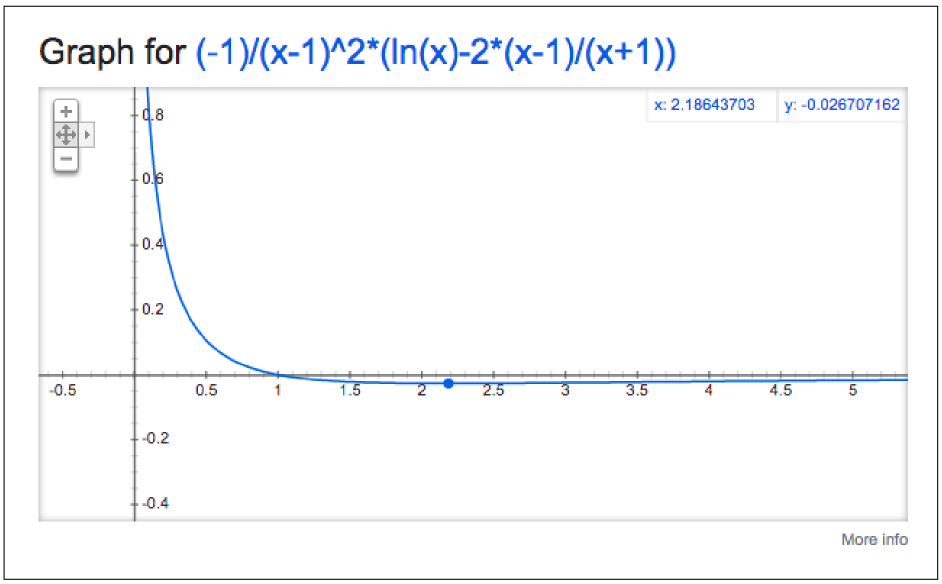
The Golden Section Search:
Step 1:
Set k = 1:
p = b - γ(b-a) = 1 - 0.618(1-0) = 0.382
q = a + γ(b-a) = 0 + 0.618(1-0) = 0.618
f(0.382) = f(p) = -0.245426
f(0.618) = f(q) = -0.276017
Step 2:
f(p)> f(q)
Set k = 2:
a = p = 0.382
p = q = 0.618
q = p + γ(b-a) = 0.382 + 0.618(1-0.382) = 0.763924
f(p) = f(0.618) = -0.2760174953761674
f(q) = f(0.763924) = -0.256292209746808
f(p) $\leq$ f(q)
Step 3:
b = q = 0.763924
q = p = 0.618
p = b - γ(b - a) = 0.763924 - 0.618(0.763924-0.382) = 0.527894968

f(q) = f(0.618) = -0.2760174953761674
f(p) = f(0.527894968) = -0.274024015
as, b - a = 0.381924 $\leq$ 0.5 = 2ϵ
Thus, $x_{min} \in$ [0.382,0.763924] that is
is x_{min} = 0.572962 $\pm$ 0.190962

Question 2 solution:

Given the function

f(x) = $\frac{-1}{(x-1)^2}$ (log(x) - $\frac{2(x-1)}{x+1}$)



f'(x) = $\frac{-5x^3+3x^2+x+2(x+1)^2 x log(x)+1}{(x-1)^3 x(x+1)^2}$

f'(x) = 0 = -5x^3+3x^2+x+2$(x+1)^2$xlog(x)+1
Solving x gives us $x_{min} \approx$ 2.1887 from WolframAlpha.
f($x_{min} \approx$ 2.1887) = -0.02670719022

Step 1:

Solve for n, $\frac{4.5-1.5}{F_n} = \frac{3}{F_n} < 2\epsilon$.

$\frac{3}{F_n} < 2\epsilon = \frac{1}{7}$ = 0.1428571429
F$_n$>21

$\Rightarrow$:Fn = 34, n = 8
We need 8 calculations!
$F_0 = 1$
$F_1 = 1$
$F_2 = 2$
$F_3 = 3$
$F_4 = 5$
$F_5 = 8$
$F_6 = 13$
$F_7 = 21$
$F_8 = 34$

Step 2: k = 8

$p = 4.5 - \frac{F_7}{F_8}(4.5\text{-}1.5) = 4.5 - \frac{21}{34}(4.5\text{-}1.5) = 2.647058824$
$q = 1.5 + \frac{F_7}{F_8}(4.5\text{-}1.5) = 1.5 + \frac{21}{34}(4.5\text{-}1.5) = 3.352941176$
f(p) = f(2.647058824) = -0.025885899
f(q) = f(3.352941176) = -0.0232567047

Step 3: k = 7

f(p) $\leq$ f(q)
b = q = 3.352941176
q = p = 2.647058824
$p = b - \frac{F_6}{F_7}(3.352941176\text{-}1.5) = 3.352941176 - \frac{13}{21}(3.352941176\text{-}1.5) = 2.205882353$
f(p) = f(2.205882353) = -0.026705602
f(q) = f(2.647058824) = -0.025885899

Step 4: k = 6

f(p) $\leq$ f(q)
b = q = 2.647058824
q = p = 2.205882353
$p = b - \frac{F_5}{F_6}(b\text{-}a) = 2.647058824 - \frac{8}{13}(2.647058824\text{-}1.5) = 1.941176471$
f(p) = f(1.941176471) = -0.026296988
f(q) = f(2.205882353) = -0.026705602

Step 5: k = 5

f(p) > f(q)
a = p = 1.941176471
p = q = 2.205882353
$q = a + \frac{F_4}{F_5}(b\text{-}a) = 1.941176471 + \frac{5}{8}(2.647058824 - 1.941176471) = 2.382352942$

f(p) = f(2.205882353) = -0.026705602
f(q) = f(2.382352942) = -0.026530606

Step 6: k = 4

f(p) $\leq$ f(q)
b = q = 2.382352942
q = p = 2.205882353
p = b - $\frac{F_3}{F_4}$(b-a) = 2.382352942 - $\frac{3}{5}$(2.382352942-1.941176471) = 2.117647059
f(p) = f(2.117647059) = -0.026678032
f(q) = f(2.205882353) = -0.026705602

Step 7: k = 3

f(p) > f(q)
a = p = 2.117647059
p = q = 2.205882353
q = a + $\frac{F_2}{F_3}$(b-a) = 2.117647059 + $\frac{2}{3}$(2.382352942-2.117647059) = 2.294117648
f(p) = f(2.205882353) = -0.026705602
f(q) = f(2.294117648) = -0.026651303

Step 8: k = 2

f(p) $\leq$ f(q)
b = q = 2.294117648
q = p = 2.205882353
p = b - 2ϵ = 2.294117648 - $\frac{1}{7}$ = 2.151260505
f(p) = f(2.151260505) = -0.02669930942844322
f(q) = f(2.205882353) =.-0.026705602

Step 9: k = 1

f(p) > f(q)
a = p = 2.151260505
p = q = 2.205882353
q = a + 2ϵ = 2.151260505 + $\frac{1}{7}$ = 2.294117648
as b - a = 2.205882353 - 2.151260505 = 0.054621848 $\leq$ 2ϵ = 0.1428571429
f(p) = f(2.205882353) = -0.026705602
f(q) = f(2.294117648) = -0.026651303
Thus, $x_{min} \in$ [2.151260505,2.205882353]
that is x_{min} = 2.178571429 $\pm$ 0.02731092403

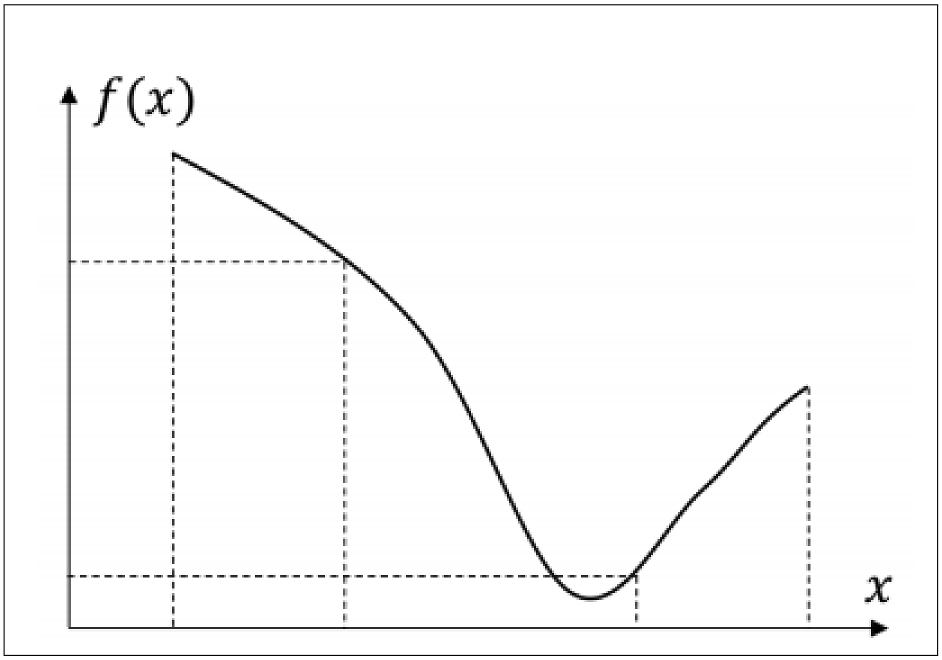
Question 3 Solution:

Proposition 1:

Let 0< $\sigma \leq \mu < 1$. The above line search procedure either finds a point such that f(t) $\leq$ f(0) + tσf'(0) and f'(t) $\geq$ μf'(0) in finitely many steps or it produces many steps or it produces a sequence f(t_k) $\rightarrow -\infty$ as k $\rightarrow \infty$.

This will satisfy this proposition because were given information that f has a continuous uni-modal function. (i.e. by definition the continuous function f is uni-modal on [0,∞) if it has only one local minimum. Meaning this local minimum is also the global minimum.

For example,



The Armijo-Goldstein function:

Let $\sigma \in$ (0,1). Let ***t*** *be the step size with weight* $\boldsymbol{\sigma}$,

$$f(t) \leq f(0) + t'(0)$$

Equation 1

given f'(0) < 0, where t has to be large!

The Wolff condition:

$$f'(t) \geq \mu f'(0)$$

Equation 2

where t step size is too small where $\mu \in [\sigma,1)$.

Ensure t does not get too close to 0.

Let T be the largest value that can satisfy Armijo-Goldstein.

$\Rightarrow$ f(T) $\leq$ f(0) $+T\sigma$f'(0)

As t increases for the Wolf condition does not satisfy for t $\leq$ T is not satisfied

$\Rightarrow$ f(t) $>$ f(0) $+t\sigma$f'(0)

For a different line

f'(T) $\geq$ σf'(0)

y = f(0) + tσf'(0) intersects curve y = f(t) at t=T but not for t $>$ T.

since f'(0) $<$ 0 at $\mu \in$ (0,1),

f'(0) $<$ μf'(0)

$\Rightarrow$ f'(T) $\geq$ σf'(0)

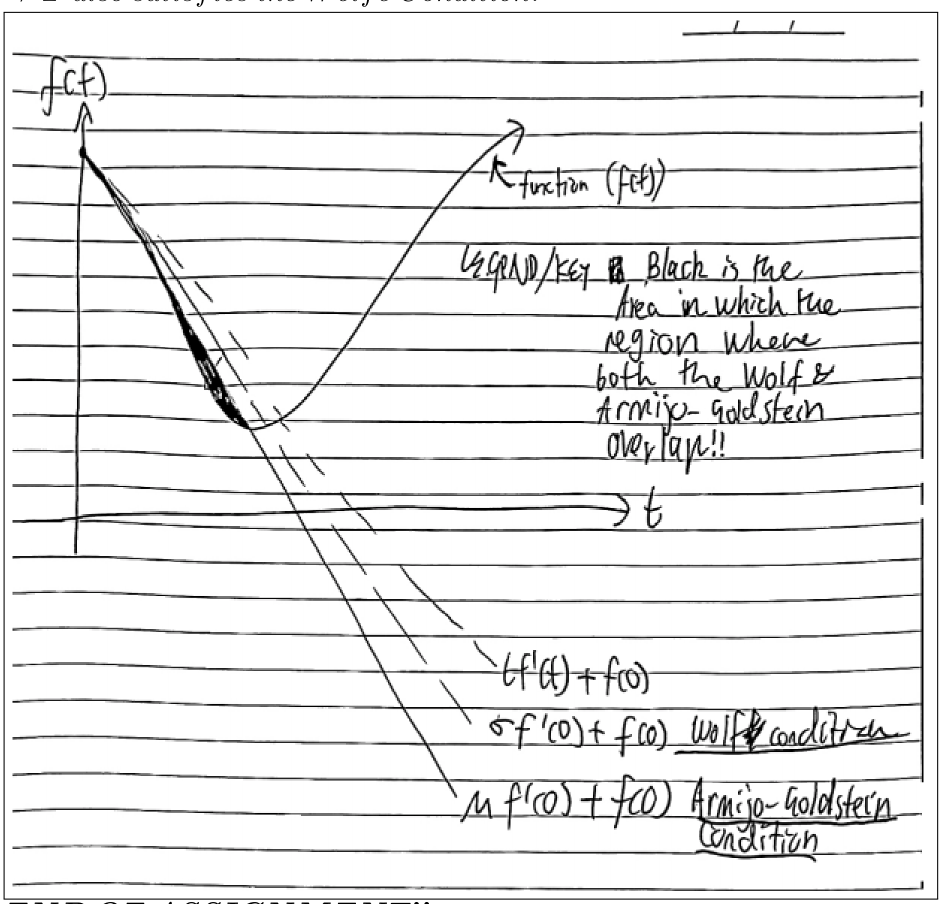
since f'(0) $<$ 0 and $\sigma \in$ (0,1),

since f'(0) $<$ 0 and $\sigma \in$ (0,1),

$\Rightarrow$ f'(T) $\geq$ μf'(0)

since $\mu \in$ (σ, 1)

$\Rightarrow$ $\boldsymbol{T}$ *also satisfies the Wolfe Condition*!



END OF ASSIGNMENT!!