ENGR30002 Fluid Mechanics

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2 Dimensional Reasoning

What we did in the last module

- In the previous module, we discussed fluid statics
    - Stationary fluid
    - No pressure gradients laterally (x- or y-directions)
    - Pressure gradient in z-direction due to weight of fluid
    - The magnitude of the pressure gradient depends on the fluid density
- Manometry
    - Technique using hydrostatics to measure pressure
    - Can measure absolute pressure, gauge pressure, or pressure differences depending on the setup

Learning objectives

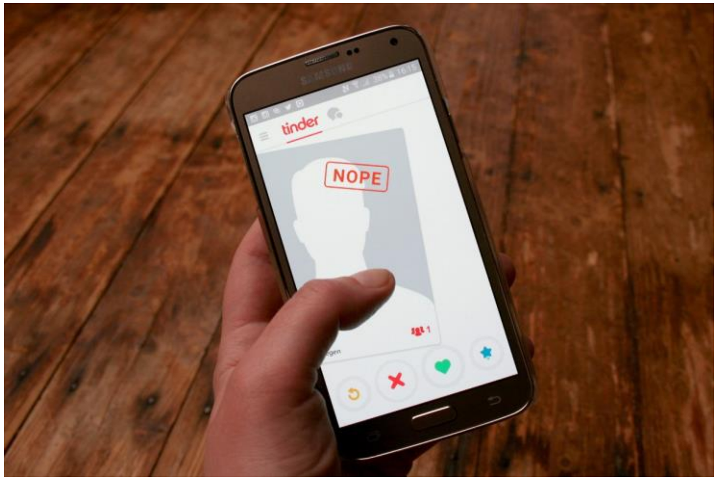
- By the end of this lesson, students should be able to:
  - Define vocabulary to describe flow (e.g. stress, viscosity, streamline)
  - Qualitatively describe the similarities and differences between solid and fluid deformation
  - Describe the forces that cause/resist flow
  - Apply dimensional analysis to derive various dimensionless parameters key to understanding fluid flow
  - Explain scenarios in which the values of the Reynolds, Weber, Bond, and Froude numbers are important

Basic terminology & principles

- When a force is applied to a body, the body can respond by translating, rotating, or deforming
  - In this course, we are focused only on forces that deform
  - When a body is deformed (or **strained**), the force (per unit area) that causes the deformation is referred to as a **stress**
- The force can act perpendicular to a surface or tangential to a surface (or both)
  - Perpendicular forces result in **normal stresses**
  - Tangential forces result in **shear stresses**

Daily examples of shear stresses?

- Opening a screw cap bottle
- Washing your face
- Applying sunscreen
- Counting money

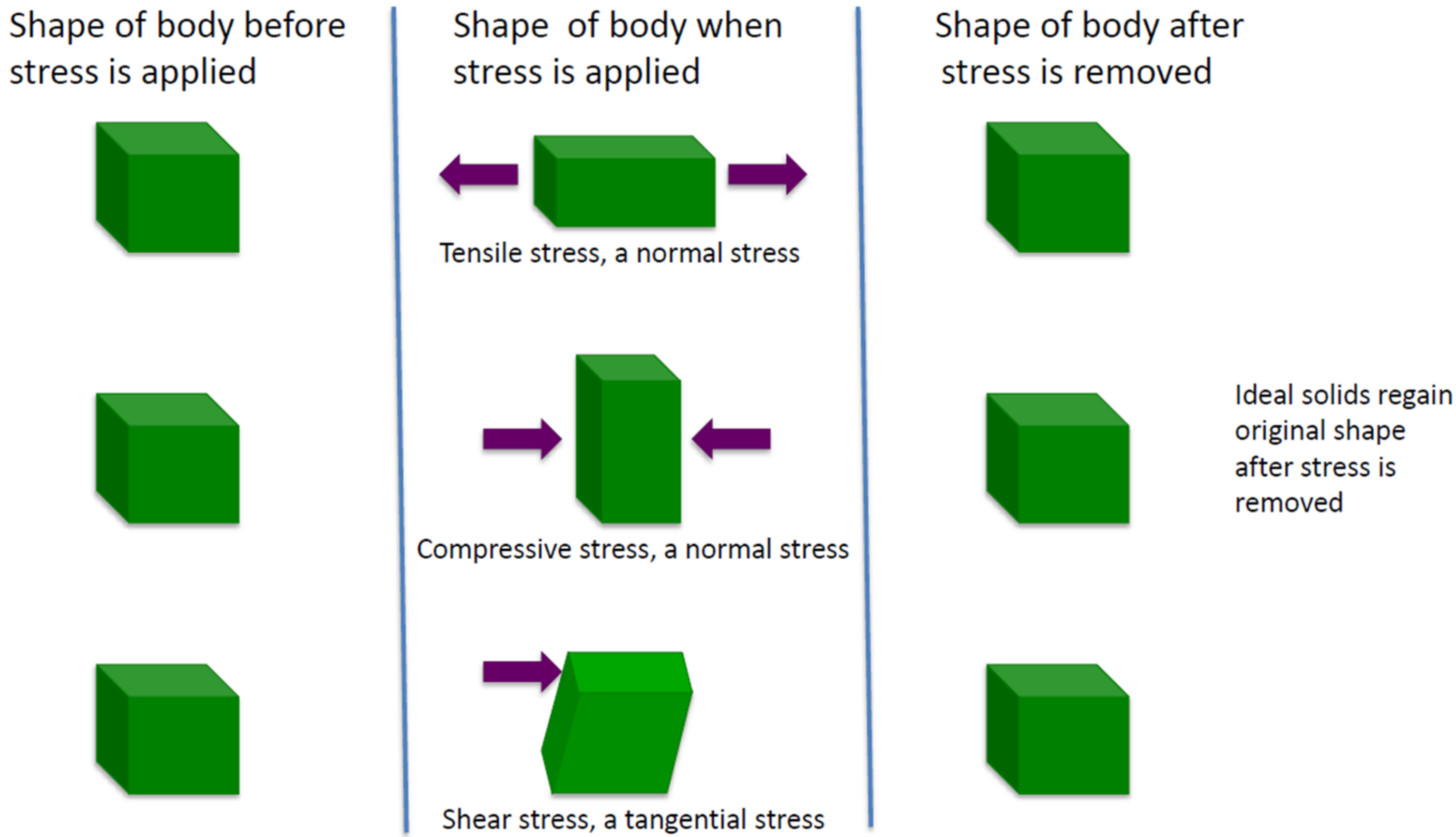


Materials deform when stress is applied

- A body will deform when stress is applied
  - "Bodies" in the context can be either solids, liquids, or gases
  - The deformation depends on the **magnitude of the force** and the **properties of the material**
- Solids and fluids behave differently when stressed
  - Solids: finite deformation
  - Liquids: continuous deformation
- In this course we are mainly focused on the deformation of fluids

Straining an ideal elastic solid

- How does an ideal solid behave under different types of stress?

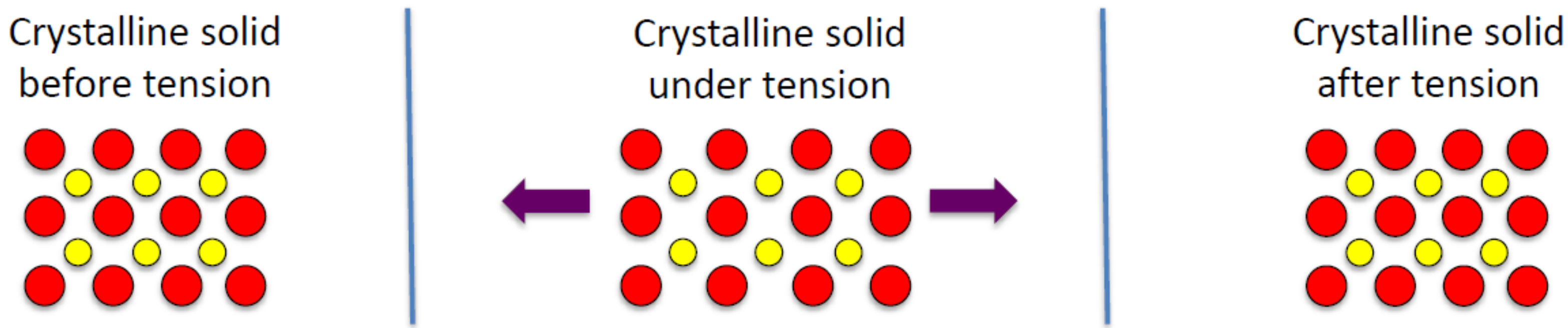


A solid's behaviour when stressed

- How does an ideal solid behave under different types of stress?
  - When an ideal solid is strained by a fixed amount of force, it will deform a **finite** amount
  - The solid will remain deformed until the force is removed
  - Upon removal of the force, the solid will regain its original shape
- A fundamental property of a solid is **elasticity**, meaning the material will recoil or regain its original shape after deformation
  - The amount of deformation varies between materials
  - **Stiffer** materials will deform less under a given stress

Why do solids behave this way?

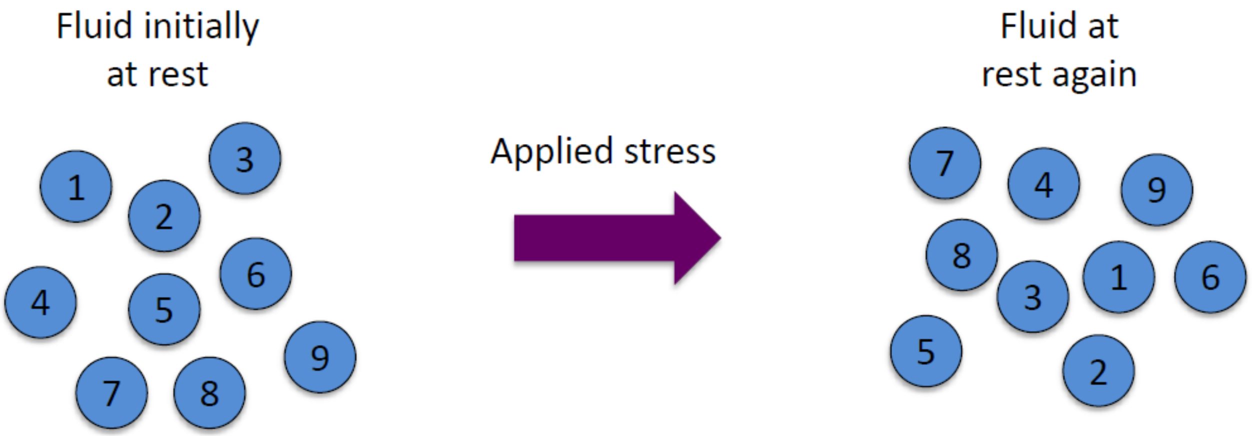
- Microscopically, pieces of a solid are fixed in location with respect to one another



- The units of the solid material can be held in place by several means:
    - Crystalline structure
    - Covalent crosslinks
    - Just the lack of enough thermal energy to move past one another
- Solid materials only deform a finite amount because they develop internal stresses that balance the applied stress

Fluids vs. solids?

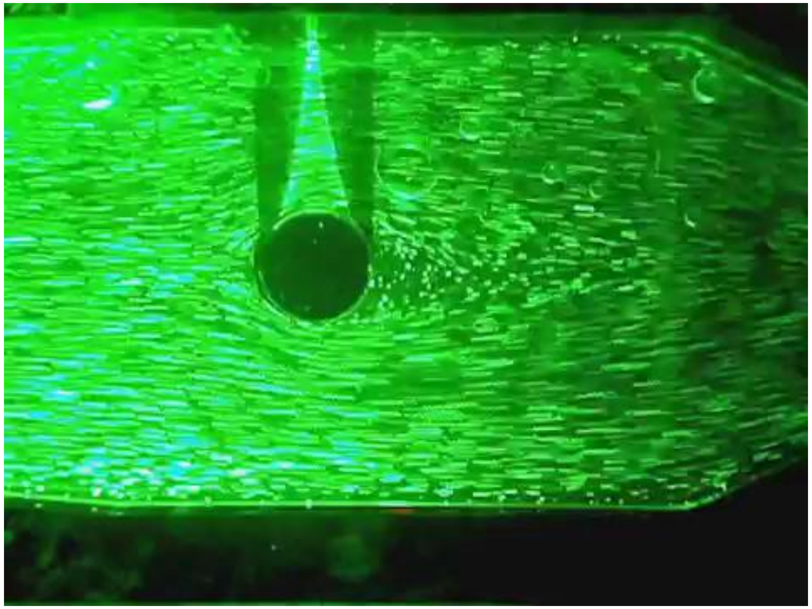
- Unlike solids, the units of a fluid can move in relation to one another



- Macroscopically, this movement of fluid particles in relation to one another is what results in **fluid flow**

How do we know this occurs?

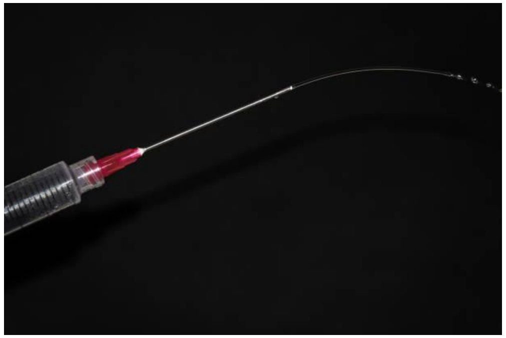
- Experimental techniques have been developed to visualise this
  - Techniques are referred to as “particle tracking velocimetry”
  - Many (but not all) of these techniques suspend polymer beads, glass bead, or bubble in the flow
  - By observing the movements of the beads, we can visualise the fluid flow



Green glass particles are flowing in water. From movies of the flow, the velocity of the particles can be tracked and thus the flow pattern of the fluid can be visualised.

Stresses that cause/resist flow

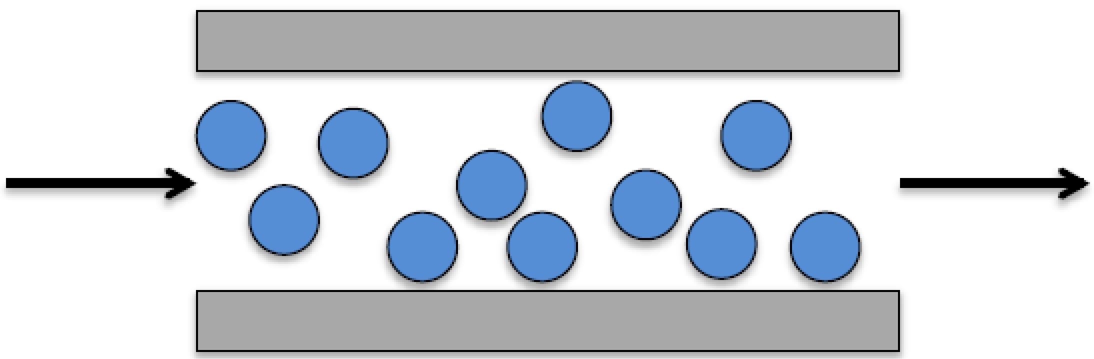
- Gravity
- Pressure differences
- Shear



MacKenzie Falls, Grampians National Park

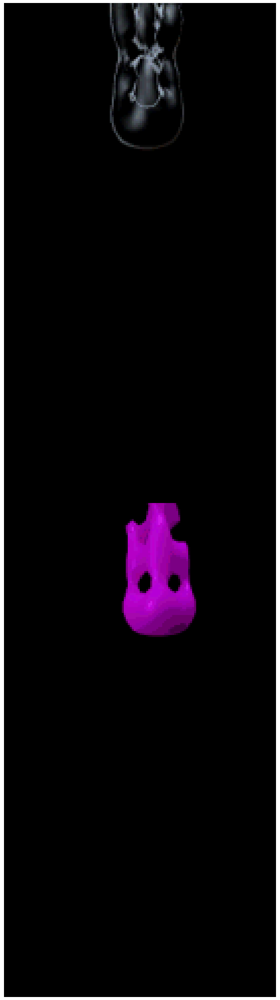
Stresses that cause/resist flow

- Once a stress is removed from a fluid, it will eventually cease flowing
  - This is due to forces that resist flow
  - What are these forces?
- There are two main forces that resist flow (both frictional)
  - **Viscosity (μ):** the friction of fluid particles moving past one another during flow
  - The friction associated with fluid particles moving past a solid surface



How much will a fluid deform?

- A fluid's viscosity is very similar to a solid's stiffness
  - These parameters determine how much a fluid will deform when a given stress is applied
- **Viscosity (μ)** determines how easily a fluid deforms when a stress is applied
  - Can be thought of as resistance to flow
    - Low viscosity fluids, or "thin" fluids, are easy to deform
    - High viscosity fluids, or "thick" fluids, are harder to deform



Quantifying viscosity

- The viscosity of a fluid can be measured in order to quantify a fluid's resistance to flow
- Common units for viscosity are centipoise [cP]

$$\mu = 1\ [cP] \equiv 1\ [mPa\ s] \equiv 0.001\ [Pa\ s] \equiv 0.001\ [N\ s\ m^{-2}] \equiv 0.001\ [kg\ m^{-1} s^{-1}]$$

Fluid	Viscosity [cP]
Air	0.0186
Benzene	0.6076
Water	0.89
Olive oil	81
Glycerol	1200
Honey	5000

- What factors affect viscosity?

Laminar vs. turbulent flow

- Fluid flow occurs in two regimes
  - Laminar flow is characterised by fluid flowing in distinct layers, with no disruption between the layers
  - Turbulent flow is characterised by chaotic flow (eddies and vortices), and rapid changes in flow velocity in both space and time

laminar flow

transitional flow

turbulent flow

https://www.youtube.com/watch?v=XOLI2KeDiOg

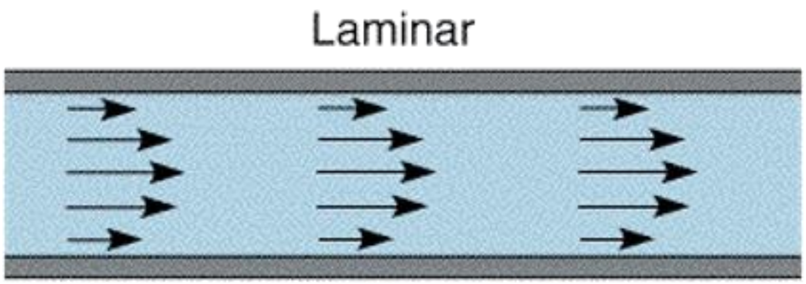
Laminar vs. turbulent flow

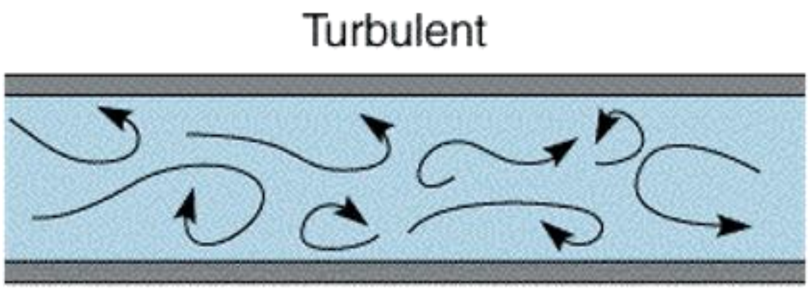
- The flow regime of a fluid (i.e. laminar or turbulent flow) is governed by three main criteria:
  - Fluid velocity (inertial forces)
  - Fluid viscosity (viscous forces)
  - Flow geometry
- These are condensed into a dimensionless parameter called the **Reynolds number, Re:**

$$\boxed{Re = \frac{\rho V L}{\mu}}$$

  - For flow in pipes:
    - **Laminar flow:** Re<2000
    - **Transitional flow:** Re=2000 to 3000
    - **Turbulent flow:** Re>3000

Laminar vs. turbulent flow





Laminar

- More likely to occur for:
  - Fluids with low velocity
  - Fluids with high viscosity
  - Fluids in a regular flow geometry
- Characterised by:
  - Smooth streamlines
  - No mixing between streamlines
  - Parabolic velocity profile (in pipes)

Turbulent

- More likely to occur for:
  - Fluids with high velocity
  - Fluids with low viscosity
  - Fluids in an irregular flow geometry
- Characterised by:
  - Eddies and vortices that change in both space and time
  - Mixing
  - “Plug” velocity profile (in pipes)

The road so far…

- Unlike solids, particles within a fluid have the ability to move in relation to one another – this is flow
- Multiple forces aid/resist flow:
    - Gravity
    - Pressure
    - Shear
    - Viscosity
    - Friction with a surface
- Fluid flow occurs in two regimes:
    - Laminar
    - Turbulent
- Flow regimes are determined by calculating the Reynolds number

Dimensionless numbers/parameters

- The Reynolds number is just one of many dimensionless numbers/parameters useful in characterising fluid behaviour – let’s explore more!
- This picture of the Loch Ness Monster is fake (unfortunately), but how can engineering concepts tell us this?

Perturbations at an air-water interface

- In this scenario, we're looking at a fluid-fluid interface
- The equilibrium state of an air-water interface would be a flat surface
  - The surface of lake on a calm day
  - The water surface in a glass of water without 🦖
- However, perturbations can happen that disrupt the surface of the fluid – these are called **waves**

Types of waves

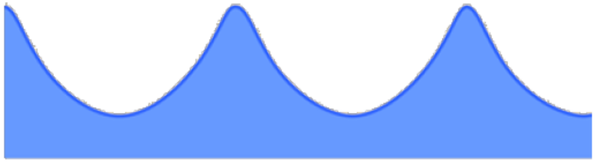
Waves are divided into two main classes:

Gravity waves

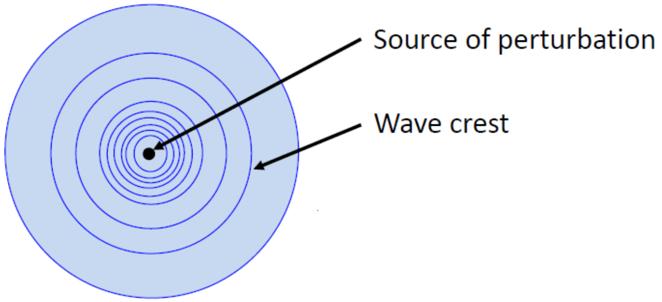
Capillary waves (ripples)

Properties of gravity waves

- For gravity waves:
  - The restoring force (back to equilibrium) is **gravity**
  - The wavelength of gravity waves is >O(cm)
    - O: “order of magnitude”
  - The waves are not perfectly sinusoidal
    - The waveform (side view) of the waves have sharp peaks and shallow troughs



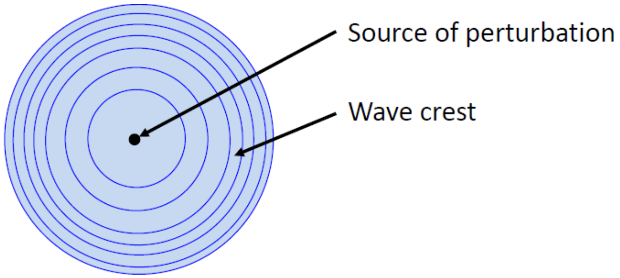
  - Waves with a greater wavelength travel faster, leading to a unique wave pattern



Properties of capillary waves

- For capillary waves (ripples):
  - The restoring force is **surface tension**
  - The wavelength of capillary waves is O(cm) or less
  - Waves are not perfectly sinusoidal
    - The waveform is almost hemispherical

  - Waves with greater wavelengths travel slower

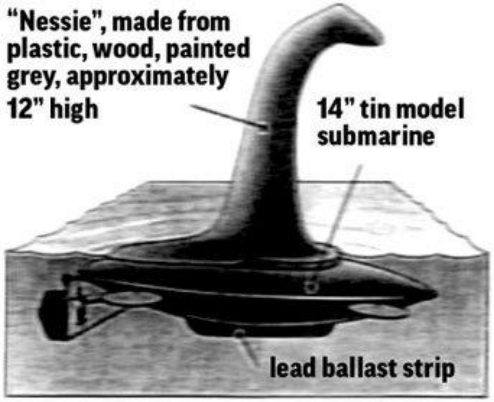


Back to Nessie

- Using your knowledge of waves, how can you determine that this photograph of the Loch Ness Monster is a hoax?

The story behind the hoax

- Marmaduke Weatherell first faked "footprints" of the Monster, that were determined to be fake
- Next, he worked with several others to create the Monster in the photo using a toy submarine and wood putty
- His friend submitted the photos to the local newspaper, the Daily Mail, in 1934 – the photo wasn't largely accepted as being fake until 1994, 60 years later!



What does this have to do with fluid mechanics?

- To study fluid systems, we often make scaled-down models of larger systems
- However, when we just linearly scale systems down, the result isn't necessarily representative of the larger system
- This is because the balance of key forces acting on the fluid is altered
  - Marmaduke tried scaling the "system" down by a factor of ~100
  - The size of the Monster relative to the waves looks about right
  - However, the waves themselves look wrong because the relative influence of surface tension and gravity in the system are no longer appropriately balanced
- To appropriately scale up/down a system, you must scale the **geometry** (geometric similarity), and also ensure that the **balance of important forces is maintained** (dynamic similarity)

Dynamic similarity – how?

- To appropriately scale a fluid system, we need to ensure the key forces are proportional to the original system
- This is done through matching **dimensionless parameters/numbers**
    - We've already seen one of these: the Reynolds number, Re
    - There are many other dimensionless numbers – we'll discuss several key ones
    - We will discover that these dimensionless numbers are just ratios of forces
- When scaling for dynamic similarity, we must make sure that the appropriate dimensionless parameter(s) are the same between the model and the original system
    - By doing so, we will ensure that the key forces are in the appropriate ratio and thus the model's results will accurately reflect the real fluid system

But I’m still not sold on these parameters…

- We’ll discover later in the subject that much of the math describing fluid flow systems is unsolvable (analytically) in its full form
  - The full form of these equations are called the Cauchy momentum equations
  - When focusing on simple fluids, these reduce to the Navier-Stokes equations
- Since we cannot solve these full equations for complex systems, we need much simple descriptors of the fluid system
  - The dimensionless parameters we will discuss fill this need
  - They provide a simpler way of describing complex systems

An order of magnitude analysis

- Dimensionless numbers are just ratios of forces
- The analysis we will do to derive them is very rough
  - We will analyse the **"order of magnitude"** of forces, not their exact values
    - Don't care if the force is 0.1 [N] or 0.2 [N]
    - Care if they are on the [N] or [kN] scale
- Also, we want to determine the parameters in the system that the ratio of forces depends on and how they impact that ratio
  - Does it depend on velocity? Or the square of velocity? Or perhaps the cube of velocity?

What forces are we going to consider?

- We will consider four main forces that impact fluid behaviour:
    1. Inertial forces – the force that must be applied to a moving fluid to stop its motion
    2. Gravitational forces
    3. Viscous/shear stress forces
    4. Surface tension forces
- Let's consider these forces in the context of a fluid system with a characteristic length scale of L and a characteristic velocity of V
    - L might be the diameter of a pipe
    - V might be the velocity of the fluid through the pipe
    - These parameters are unique to each system we consider

Inertial forces

- First we will consider inertial forces, F_I – the force that must be applied to a moving fluid to halt its motion

$$F_I = ma$$

$$F_I \sim m\frac{\partial V}{\partial t}$$

$$F_I \sim m\frac{\partial V}{\partial\left({}^{L}/_{V}\right)}$$

$$F_I \sim (\rho L^3)(V)\left(\frac{V}{L}\right)$$

$$\boxed{\therefore \boldsymbol{F_I \sim \rho V^2 L^2}}$$

- The process above is called dimensional analysis
  - It is a method of breaking down the units into its fundamental contributors to determine which and how each key parameter impacts the system

Other forces

- Using dimensional analysis, we can find scales for the other key forces:
  - Gravitational force, F_G

$$F_G = mg \sim \rho L^3 g$$

  - Viscous force, F_V

$$F_V = \mu \left(\frac{dV}{dz}\right) A \sim \mu V L$$

Newton's Law of Viscosity, will see this later!

  - Surface tension force, F_T

$$F_T = \sigma L \sim \sigma L$$

Dimensionless numbers

- The ratio of these forces lead to several key dimensionless numbers:
  - Reynolds number, Re

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho V^2 L^2}{\mu V L} = \frac{\rho V L}{\mu}$$

  - Weber number, We

$$\mathrm{We} = \frac{\text{inertial forces}}{\text{surface tension forces}} = \frac{\rho V^2 L^2}{\sigma L} = \frac{\rho V^2 L}{\sigma}$$

  - Bond number, Bo

$$\mathrm{Bo} = \frac{\text{gravitational forces}}{\text{surface tension forces}} = \frac{\rho L^3 g}{\sigma L} = \frac{\rho L^2 g}{\sigma}$$

  - Froude number, Fr

$$\mathrm{Fr} = \frac{\text{inertial forces}}{\text{gravitational forces}} = \frac{\rho V^2 L^2}{\rho L^3 g} \rightarrow \frac{V}{\sqrt{gL}}$$

(by convention, the square root is used)

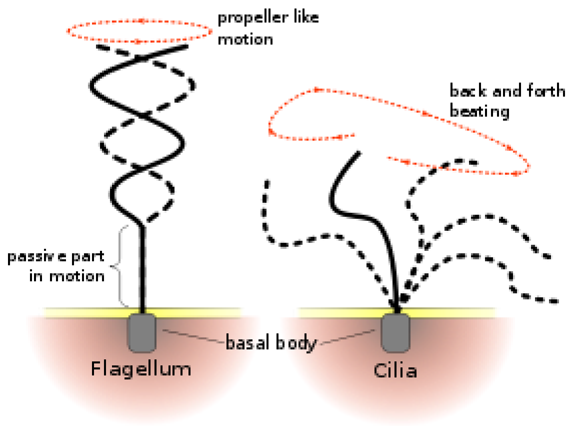
Reynolds number, Re

$$\text{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho V L}{\mu}$$

- The value of Re tells us the relative impact of each type of force:
  - When Re>>1, inertial forces dominate
  - When Re<<1, viscous forces dominate
- We (humans) generally operate at high Re
  - Air & water are relatively low viscosity fluids
  - At the length scales & speeds we typically operate at, Re>>1
- But when the value of Re is small, the world behaves very differently…

Low Re flow is strange

Flow is reversible?!
https://youtu.be/p08_KlTKP50



Locomotion considerations at the microscale

When is Re important?

- Turbulence is a function of Re
  - For pipe flow, the transition from laminar to turbulent flow occurs at around Re=2000-3000
    - The exact transition point will be system-dependent
- Laminar flow occurs at lower Re
  - Little mixing occurs in laminar flow systems as mixing occurs by Brownian motion only

- Turbulent flow occurs at higher Re
  - A lot of mixing occurs in turbulent flow systems due to rotational eddies superimposed over the mean flow

Weber number, We

$$\text{We}=\frac{\text{inertial forces}}{\text{surface tension forces}}=\frac{\rho V^2 L^2}{\sigma L}=\frac{\rho V^2 L}{\sigma}$$

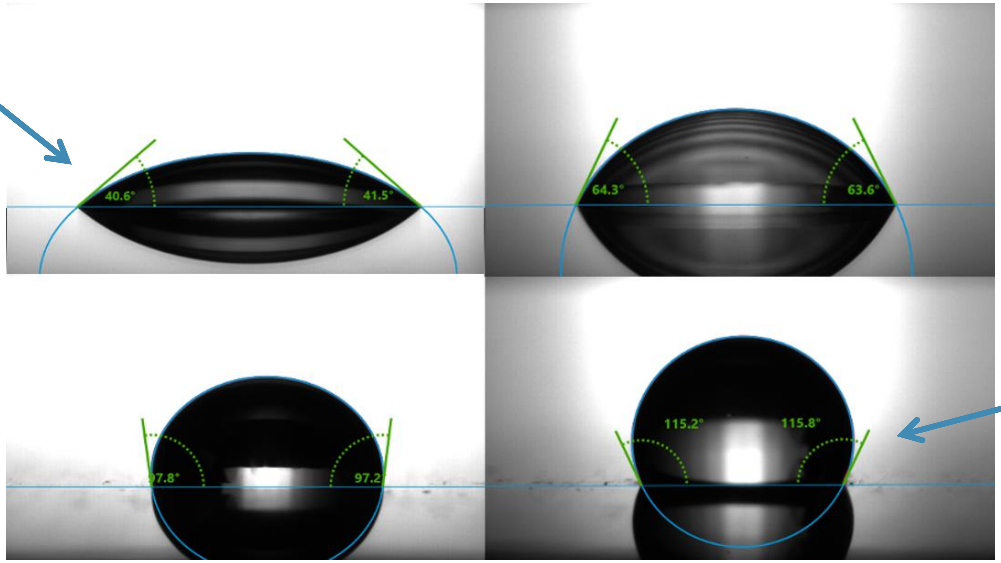
- The value of We tells us the relative impact of each type of force:
  - When We<<1, surface tension dominates
  - When We>>1, inertia dominates

Psst… You should review surface tension!

An aside: surface tension

- Some fluids are miscible, others are not:
  - If you add a drop of ethanol to water, it will dissolve
  - If you add a drop of oil to water, it will remain as a distinct phase
- This is governed by intermolecular forces and thermodynamics
  - Let's observe the shape of a droplet on a variety of different surfaces
  - The degree to which a droplet spreads is called its **contact angle**

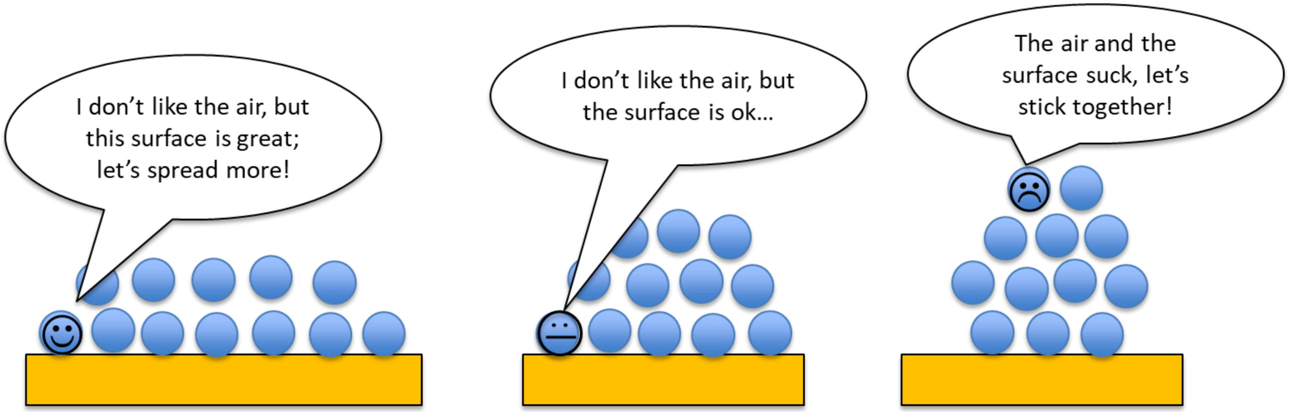
Most hydrophilic



Most hydrophobic

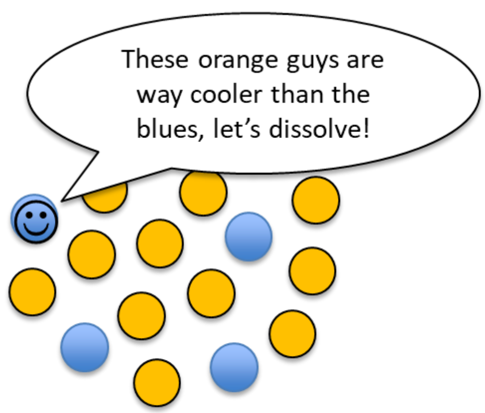
An aside: surface tension

- Surface tension can be defined as the work required to make new surface area
- The degree of fluid spreading will be determined by how much the fluid "likes" the surface and the surrounding fluid phase (air, in this case)
    - A more hydrophilic fluid will spread more on a more hydrophilic surface
    - A more hydrophobic fluid will spread more on a more hydrophobic surface

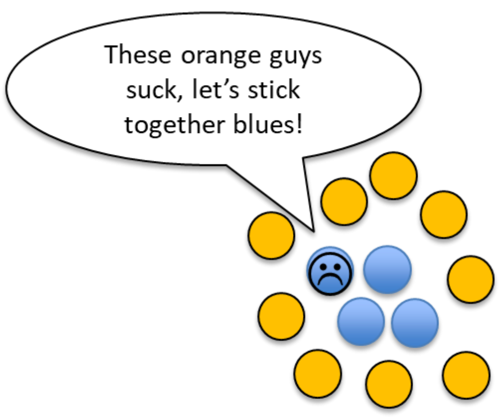


An aside: surface tension

- The degree to which a fluid spreads on a surface is determined by its interactions with the surface and the other fluid phase
- Similarly, when a droplet of liquid or gas is put into an immiscible fluid, it forms a droplet or bubble
    - It forms a spherical shape as this is the smallest surface area to volume ratio available
    - The cohesive strength of the droplet or bubble is determined by how much it dislikes the other phase, i.e. the surface tension



Miscible



Immiscible

When is We important?

- The Weber number is most important in determining the stability of moving droplets or bubbles
    - At high We, inertial forces dominate, the droplet is unstable and it will fracture into smaller droplets
    - At low Weber numbers, surface tension forces dominate, the droplet is stable and it will remain as a cohesive whole
- What are the characteristic length scale and velocities?
    - The characteristic length scale would be the diameter of the droplet or bubble
    - The characteristic velocity would be the droplet velocity

Bond number, Bo

$$\text{Bo} = \frac{\text{gravitational forces}}{\text{surface tension forces}} = \frac{\rho L^3 g}{\sigma L} = \frac{\rho L^2 g}{\sigma}$$

- The Bond number can be used under two different scenarios:
  - Sometimes we are interested in the impact of gravity and surface tension on the fluid itself (e.g. the Loch Ness Monster example)
  - Sometimes we are interested in the impact of gravity on an object that is using surface tension to support itself at a fluid-fluid interface (e.g. paper clip on water)

Bond number, Bo

The form of Bo depends on which scenario we have:

$$Bo = \frac{\rho L^2 g}{\sigma}$$

- ρ: fluid density
- L: characteristic length
- g: gravitational acceleration
- σ: surface tension of fluid-fluid interface

$$Bo = \frac{mg}{\sigma L}$$

- m: mass of object
- g: gravitational acceleration
- σ: surface tension of fluid-fluid-solid interface
- L: characteristic length

Bond number, Bo

- The value of Bo tells us the relative impact of each type of force:
  - When Bo<<1, surface tension dominates
  - When Bo>>1, gravity dominates & surface tension effects are negligible
- The Bond number is most important in the following scenarios:
  1. Determining if a perturbation at a fluid-fluid interface (a wave) will be a gravity or a capillary wave
     - We've already seen this with the Loch Ness Monster example
     - The characteristic length scale (L) in this scenario is the wavelength (λ)
     - The transition from capillary to gravity waves occurs at Bo$\approx$40
  2. When an object at an interface is supporting itself by surface tension

Support at a fluid-fluid interface
Water striders?

- Nature has used the dominance of surface tension over gravity to provide evolutionary advantage → **water striders**
  - An aquatic insect that "skates" around the surface of an air-water interface
  - They also eat other insects that fall into the fluid and get trapped!
  - In order to skate around on the interface, we would need Bo<<1.

$$Bo = \frac{mg}{\sigma L}$$

- $m \approx 0.01\ [g]$
- $\sigma = 0.07\ [N/m]$
- $L \approx 2\ [cm]$

$$\therefore Bo \approx 0.07 \ll 1$$

so surface tension does dominate!

Support at a fluid-fluid interface

Ships?

- Not everything at a fluid-fluid interface is supported by surface tension
  - Many structures are supported at a fluid-fluid interface by **buoyancy**
  - This is due to differences in density and by how much water the structure displaces
  - Boats, buoys, etc. are not supported by surface tension – they have high Bond numbers!

Support at a fluid-fluid interface

Basilisks?

- How about the basilisk? Also called the "Jesus Christ lizard", it is a lizard that can run on water
    - Do you think that the lizard is supported by surface tension?

https://youtu.be/CW0TijmAUqY

$$Bo = \frac{mg}{\sigma L}$$

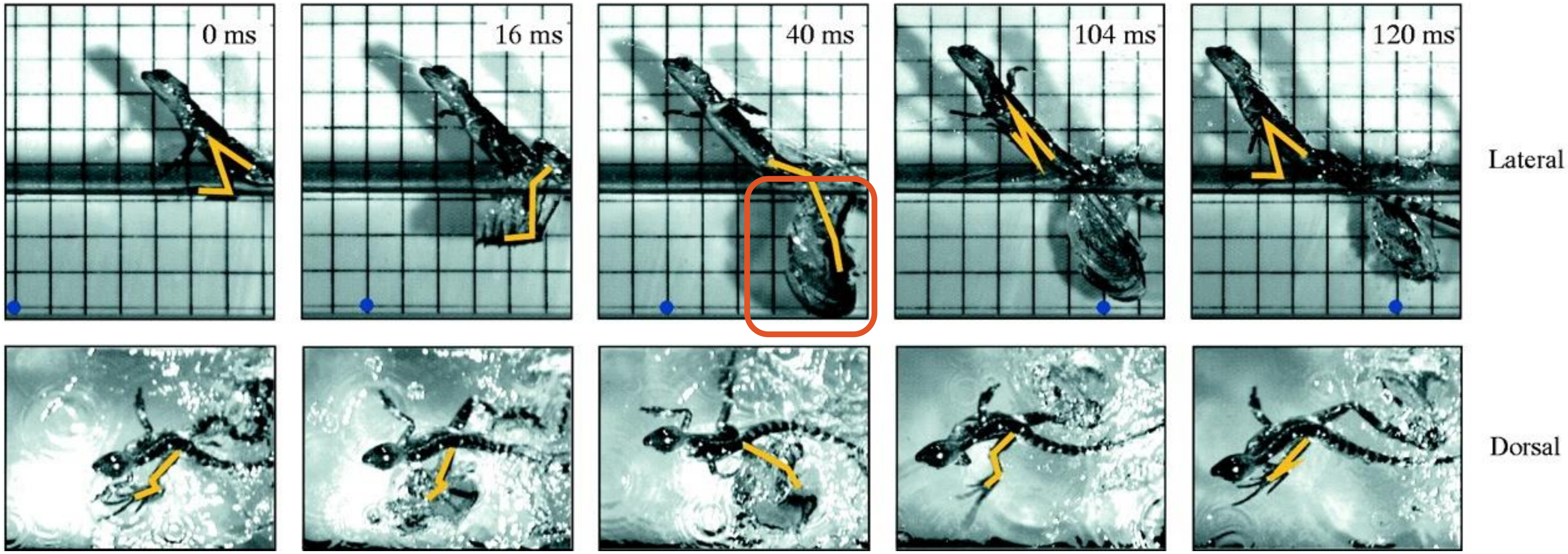
- $m \approx 100\ [g]$
- $\sigma = 0.07\ [N/m]$
- $L \approx 4\ [cm]$

$$\therefore Bo \approx 350 \gg 1$$

so it can't be surface tension at play… How does it run on water?!

Support at a fluid-fluid interface

Basilisks?



Hsieh ST, J Experimental Biology 2003.

- For us humans to achieve this, we would need to kick at ~100 [km/h]!

Froude number, Fr

$$\mathrm{Fr} = \frac{\text{inertial forces}}{\text{gravitational forces}} = \frac{\rho V^2 L^2}{\rho L^3 g} \rightarrow \frac{V}{\sqrt{gL}}$$

- The value of Fr tells us the relative impact of each type of force
  - When Fr>>1, inertial forces dominate
  - When Fr<<1, gravity forces dominate
- The Froude number is important any time you have a fluid with a free surface (liquid-gas interface) that is moving with gravity, against gravity, or has a perturbation at the surface
  - E.g. river flow, tidal flow, waves

Fr and open channel flow

- Fr is particularly important in open channel flow
- For example, if we look at flow over a spillway (or weir)…

  - The same waterway can have very different behaviour due to channel depth and this is described by the Froude number

Fr and open channel flow

- In the following lab model of a spillway:
  - The flow rate is constant due to conservation of mass (see later, but basically what goes in has to come out)
  - This means the flow downstream must be much faster

Low Froude number (Fr<1)
- Smooth flow
- Subcritical flow

High Froude number (Fr>1)
- Rough flow
- Supercritical flow

$$Fr = \frac{V}{\sqrt{gh}}$$

Fr & open channel flow

- Fr also dictates wave travel in systems with bulk flow

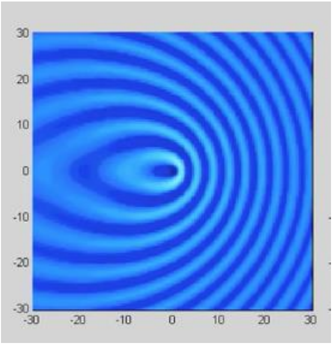
$$Fr = \frac{V}{\sqrt{gh}}$$

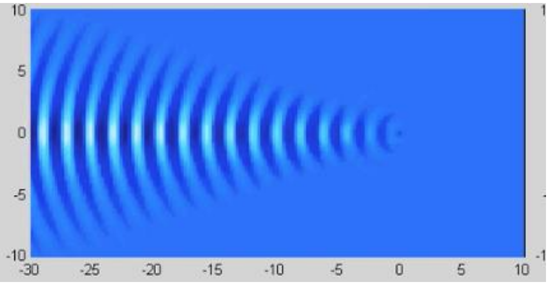
V ← bulk fluid velocity

$\sqrt{gh}$ ← speed of a wave

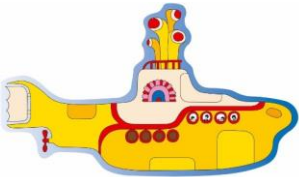
- For subcritical flow (Fr<1)
  - The speed of a wave is greater than the speed of the bulk flow
  - This means that waves can travel upstream
- For supercritical flow (Fr>1)
  - The speed of the flow is greater than the speed of the wave
  - This means that waves cannot propagate upstream

FLOW DIRECTION





Example 2.1
Modelling a submarine



- The Australian military has asked you to evaluate how a new technology impacts the propulsion of their submarines as they travel underwater. However, the government will not let you study the *real* submarine. Instead, you must make a model of it in the lab.
  - At what speed would the model need to travel at to accurately reflect the propulsion of the actual submarine?

Example 7.2
Port Phillip Bay

- The city has asked you to model how tides impact erosion in Port Phillip Bay. To accomplish this task, you need to construct a scale model of the bay in the lab.
  - At what speed will the tide need to come in in the scale model?

Summary

- Fluids differ from solids in that they flow (deform continuously) in response to stress
- Flow regimes (laminar/turbulent) are dependent on the Reynolds number – this is just one of many dimensionless parameters/numbers
- Dimensionless parameters/numbers provide insight into how key forces in systems impact fluid flow behaviour
    - Reynolds number, Re
        - Pipe flow, propulsion, flow regimes
    - Bond number, Bo
        - Support/disturbances at a fluid-fluid interface
    - Weber number, We
        - Moving droplets & bubbles
    - Froude number, Fr
        - Fluids with a shifting free surface

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho V L}{\mu}$$

$$\mathrm{Bo} = \frac{\text{gravitational forces}}{\text{surface tension forces}} = \frac{\rho L^2 g}{\sigma}$$

$$\mathrm{We} = \frac{\text{inertial forces}}{\text{surface tension forces}} = \frac{\rho V^2 L}{\sigma}$$

$$\mathrm{Fr} = \frac{\text{inertial forces}}{\text{gravitational forces}} = \frac{V}{\sqrt{gL}}$$