Programming Assignment

Q1). See - Code.

Q3).

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$P(\text{sum } 2) = \frac{1}{16} = P(\text{sum } 8)$

$P(\text{sum } 3) = \frac{2}{16} = P(\text{sum } 7)$

$P(\text{sum } 4) = \frac{3}{16} = P(\text{sum } 6)$

$P(\text{sum } 5) = 4/16.$

Q2). Symmetric (fair six-sided). distribution.

Q1).

X	1	2	3	4	5	6
$P(X)$	1/6	1/6	1/6	1/6	1/6	1/6

mean $= 1\times\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = 3.5$

Var. $= 1^2\times\frac{1}{6} + \frac{2^2}{6} + \frac{3^2}{6} + \frac{4^2}{6} + \frac{5^2}{6} + \frac{6^2}{6} - 3.5^2$

$\approx 2.91\dot{6}$ (By hand).

2ND throw

		1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
1st	3	4	5	6	7	8	9
throw	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$P(\text{sum } 2) = 1/36$
$P(\text{sum } 3) = 2/36$
$P(\text{sum } 4) = 3/36$
$P(\text{sum } 5) = 4/36$
$P(\text{sum } 6) = 5/36$
$P(\text{sum } 7) = 6/36$
$P(\text{sum } 8) = 5/36$
$P(\text{sum } 9) = 4/36$
$P(\text{sum } 10) = 3/36$
$P(\text{sum } 11) = 2/36$
$P(\text{sum } 12) = 1/36$

Q4)

mean $= \frac{1}{16}(2 + 8 + \overset{\times 2}{3} + \overset{\times 2}{7} + \overset{\times 3}{4} + \overset{\times 3}{6} + (5\times 4))$

$= \frac{80}{16} = 5.$

Var $= \frac{2^2}{16} + \frac{8^2}{16} + \frac{2}{16}3^2 + \frac{2}{16}7^2 + \frac{3}{16}4^2 + \frac{3}{16}6^2 + \frac{4}{16}5^2 - \underbrace{(5)}_{\text{mean}}^2$

$= 2.5$

Cov. $= 0$ (Since all X_i's are independent)
(sides)

Q5).

$P(1) = 1/5$
$P(2) = 2/5$
$P(3) = 1/5$
$P(4) = 1/5$.

2nd Throw

1st throw	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8.

$P(\text{sum } 2) = \frac{1}{25}$

$P(\text{sum } 8) = 1/25$

$P(\text{sum } 3) = 4/25$

$P(\text{sum } 4) = 6/25$

$P(\text{sum } 5) = 6/25$

$P(\text{sum } 6) = 5/25$

$P(\text{sum } 7) = 2/25$.

Q6)

X	1	2	3	4	5	6
P(X)	1/7	1/7	2/7	1/7	1/7	1/7

(≤ 0.5).

1st throw \ 2ND throw	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Trick.

$P(\text{Sum }2) = 1/49$

$P(\text{Sum }3) = 2/49$

$P(\text{Sum }4) = 5/49$

$P(\text{Sum }5) = 6/49$

$P(\text{Sum }\boxed{6}) = 8/49$

$P(\text{Sum }7) = 8/49$

$P(\text{Sum }8) = 7/49$

$P(\text{Sum }9) = 6/49$

$P(\text{Sum }10) = 3/49$

$P(\text{Sum }11) = 2/49$

$P(\text{Sum }12) = 1/49$

3/49

8/49

14/49

22/49

⋮

$\frac{49}{2} = 24.5$

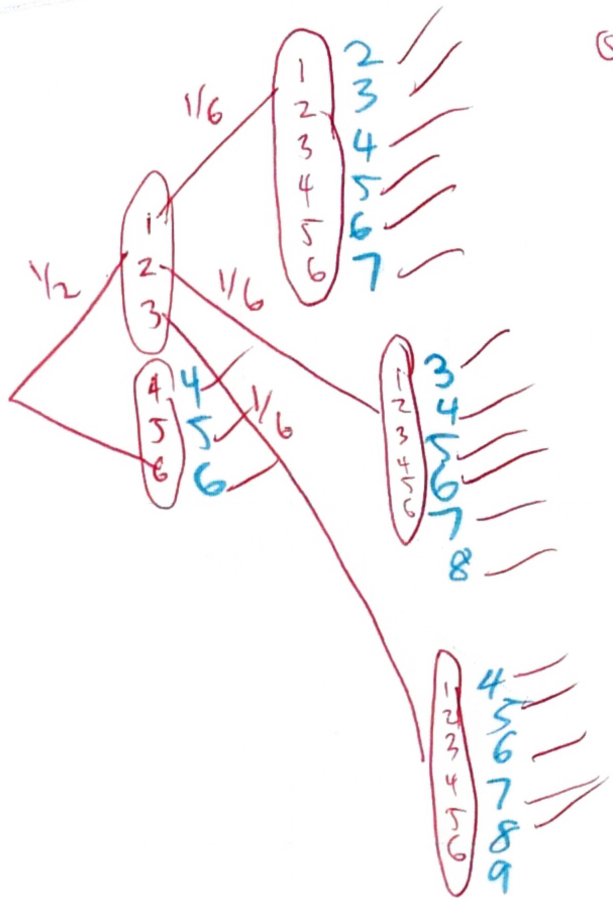
if $P(\text{Sum }1 + \ldots + \text{Sum}(?)) \leq 0.5$

$= \frac{24.5}{49}$

$\boxed{?} = 6$

→ Move Stroll/Circle to 6.

Q7)



$P(2) = 1/36$

$P(3) = 1/36 + 1/36 = 2/36$

$P(4) = 3/36 + \frac{1}{6} = 9/36$

$P(5) = 3/36 + 1/6 = 9/36$

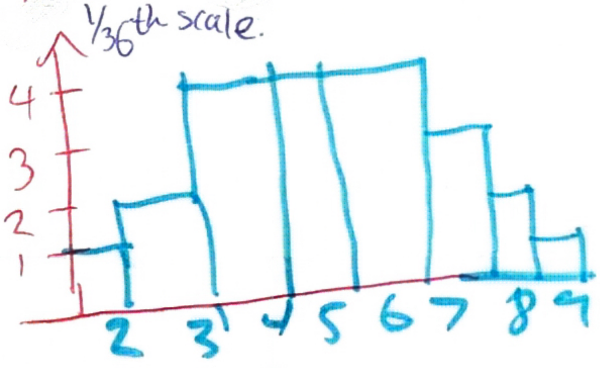
$P(6) = 3/36 + \frac{1}{6} = \frac{9}{36}$

$P(7) = 3/36$

$P(8) = 2/36$

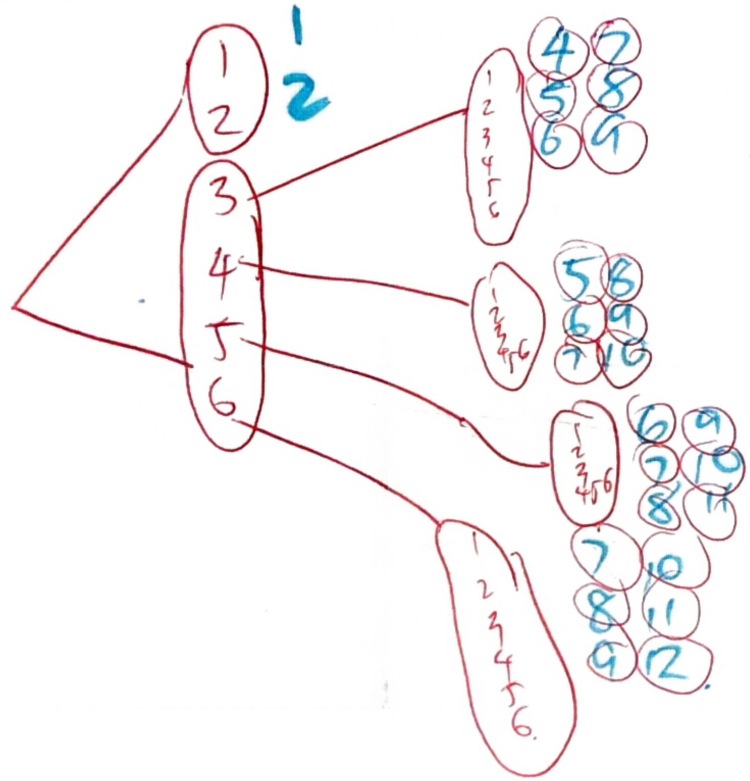
$P(9) = 1/36$

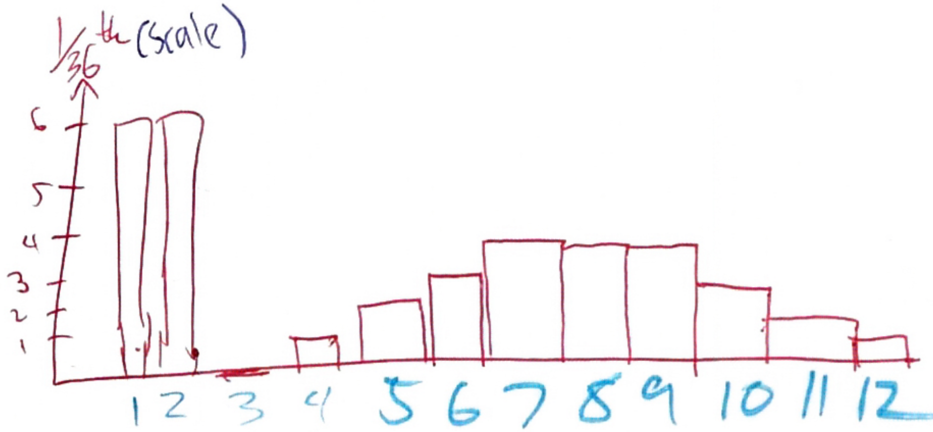
Result.



find this shape.

Q8)





find this shape.

Q9).

Case 1

n=2.

X	1	2
P(X)	1/2	1/2

Y	1	2
P(Y)	1/2	1/2

$E[X] = \frac{1}{2}(1) + \frac{1}{2}(2) = 3/2 = 1.5$

$E[Y] = \frac{1}{2}(1) + \frac{1}{2}(2) = 3/2 = 1.5$

Mean

	1	2
1	2	3
2	3	4

P(sum 2) = 1/4

P(sum 3) = 1/2

P(sum 4) = 1/4

$\rightarrow$ Mean $= 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{2}\right) + 4\left(\frac{1}{4}\right) = 3$

Conclusion
- mean ↗
- Var ↗
- Cov (No Change)

$Var. = 2^2\left(\frac{1}{4}\right) + 3^2\left(\frac{1}{2}\right) + 4^2\left(\frac{1}{4}\right) - (3)^2$

$= 0.5$

Covariances (since all sides are INDEPENDENT)

Case 2

n=4

X	1	2	3	4
P(X)	1/4	1/4	1/4	1/4

Y	1	2	3	4
P(Y)	1/4	1/4	1/4	1/4

$E[X] = \frac{1}{4}1 + \frac{1}{4}2 + \frac{1}{4}3 + \frac{1}{4}4 = 2.5$

$E[Y] = \frac{1}{4}1 + \frac{1}{4}2 + \frac{1}{4}3 + \frac{1}{4}4 = 2.5.$

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

P(sum 2) = 1/16

P(sum 3) = 2/16

P(sum 4) = 3/16

P(sum 5) = 4/16

P(sum 6) = 3/16

P(sum 7) = 2/16

P(sum 8) = 1/16

$\rightarrow$ mean $= \frac{1}{16}\left(2 + 3(2) + 4(3) + 5(4) + 6(3) + 7(2) + 8\right)$

$= 5.$

$Var = \frac{1}{16}\left(2^2 + 2(3)^2 + 3(4)^2 + 4(5)^2 + 3(6)^2 + 2(7)^2 + 8^2\right) - (5)^2$

$= 27.5 - 25$

$= 2.5$

Multiple-Choice

Q10.

Given a 6-sided loaded dice.
You throw it twice & record the sum. Which of the following statements is true?

Answer: changing the loaded side from 1-6 will yield a higher mean but the same variance.

Q11).

Given a fair n-sided dice. You throw it twice and record the sum but the second throw depends on the result of the result of the first one such as in Q7 & Q8.
Which of the following statements is true?

Answer: Changing the direction of the inequality will change the sign of the covariance.

Q12)

Given a n-sided dice (could be fair or not). You can throw it twice & record the sum (there is no dependence between the throws). If you are only given the histogram of the sums can you use it to know which are the probabilities of the dice landing on each side?

Answer: yes, but only if the die is fair.