MAST30001 Stochastic Modelling

Tutorial Sheet 7

1. Yeast microbes from the air outside of a culture float by according to a Poisson process with rate 2 per minute. Each microbe that floats by joins the population of the culture with probability p and with probability $1-p$ the microbe doesn't join the culture, and this choice is made independent from the times of arrival and choice to join of all other microbes.

   (a) What is the chance that exactly four microbes float by in the first 3 minutes?

   (b) What is the chance that exactly four microbes join the culture in the first 3 minutes?

   (c) Given that 7 microbes have floated by the culture in first 3 minutes, what is the chance that at least two of the seven join the culture?

   (d) Given that 7 microbes have floated by the culture in first 3 minutes, what is the chance that exactly 3 floated by in the first 1 minute?

   (e) What is the chance that in the first 3 minutes, exactly four microbes join the culture and 3 float by that don't join the culture?

   Assume now that a second strain of yeast microbes independently float by the culture according to a Poisson process with rate 1, and each microbe joins the culture with probability q, analogous to the previous process.

   (f) What is the chance that exactly four yeast microbes from either strain float by in the first 3 minutes?

   (g) What is the chance that exactly four yeast microbes from either strain join the culture in the first 3 minutes?

2. In a Poisson process with rate 1, what is the joint density of the times of the first and second jumps? What is the joint density of the times of the ith and jth jump for $i < j$? Can you interpret these formulas similar to our discussion in lecture deriving the joint densities of order statistics?

3. Let $U_{(1)}, \ldots, U_{(n)}$ be order statistics of independent variables, uniform on the interval $(0,1)$. For $0 < x < y < 1$ what is

   (a) $\mathbb{P}(U_{(1)} > x, U_{(n)} < y)$,

   (b) $\mathbb{P}(U_{(1)} < x, U_{(n)} < y)$,

   (c) $\mathbb{P}(U_{(k)} < x, U_{(k+1)} > y)$?

4. From Tutorial 1: If N is geometric with parameter p ($\mathbb{P}(N = j) = p(1-p)^j$, $j = 0, 1, 2, \ldots$) and given $N = n$, X is gamma with parameter $n+1$, what is the density of X? Another question: If S is exponential with rate λ and given $S = s$, M is Poisson with mean s, then what is the distribution of M? A third question: If K is Poisson with mean μ and given $K = k$, J is binomial with parameters k and p, then what is the distribution of J? Can you explain (or even derive) the answers to these three questions through superposition and thinning of Poisson processes?