MAST30013 – Techniques in Operations Research

Semester 1, 2019

Group Project

Submissions: Reports (typeset in LaTex), to be submitted before the (strict) due date: 12pm Monday of Week 12.

Groups: must be composed of a minimum of 3 and a maximum of 4 members. It is your responsibility to talk to your colleagues and create/find a group, as early as possible in the semester.

Evaluation:

Reporting: You should present a coherent and self-contained report presenting your algorithms and results for the questions below.

Presentation: In the last week of class, groups will present their work in 10 minutes + 3 minutes for questions. You are not expected to explain the problem in detail during the presentation. Instead, focus on the innovative aspects that your group has found: did you find any nice application for the problem? Which solution algorithm did you implement? What conclusions did you get?

Project Description: Spanning Circles

You are tasked to solve the following two problems:

Problem 1:

Suppose you are given a set X of n points in $\mathbb{R}^2$. Find a circle of smallest radius that covers all points in X.

Details:

1. Model this problem as an unconstrained convex problem. Define all variables.
2. Demonstrate that the problem, as formulated in (1), is not smooth. Illustrate an example consisting of three points in the plane by drawing the lines consisting of all points at which the problem is not differentiable. Can you see any interesting properties of the lines and the regions bounded by the lines?
3. Remodel the problem as a constrained optimisation problem with a smooth objective. Prove that the problem is still convex. You should explain each constraint and variable.
4. Derive the KKT conditions for the model in (3). **Bonus question**: provide an intuitive (geometric) interpretation for each condition.

5. Implement an algorithm for solving your model in (3). Do a computational study (different initial solutions, parameter settings, etc.) for instances of different values of n and various random sets X of n points.
6. Can the problem be solved with a pre-existing Matlab (or other software) function? What is the package/function doing? Do a computational study (different initial solutions, parameter settings, etc.) for instances of different values of n.

Problem 2:

Suppose you are given a set X of n points in $\mathbb{R}^2$ and suppose that X is partitioned into two subsets X_1, X_2, where neither X_1 nor X_2 are empty. Let $d > 0$ be a real number. Find a pair of circles C_1 and C_2 such that C_1 covers all points in X_1, C_2 covers all points in X_2, the distance between the centre of C_1 and the centre of C_2 is at least d, and the radius of the larger circle is minimised.

Details:

1. Model the problem as a constrained optimisation problem with a smooth objective.
2. Implement an algorithm for solving your model. Do a computational study (different initial solutions, parameter settings, etc.) for instances of different values of n and various random sets X_1, X_2.

Important Notes

- You will be marked on the quality of your report. Make sure it is self contained and follows the structure of a scientific report.
- It is the group's responsibility to ensure that everyone contributes their fair share to the project. All members of the group will receive the same mark.
- It is recommended that members of the group share a Dropbox folder for their work or use Overleaf. This is so that all group members can have access to all parts of the project at any time.
- This project contains an extensive programming component. It is assumed that all students are able to program in Matlab. The lecturer will **not** assist in issues relating to the debugging of code.
- Students may research ideas on the web, but full credit to the relevant authors must be given if the students use any of these ideas. Code may NOT be copied from existing sources. Collaboration between groups is NOT allowed.
- All code must be in Matlab.