MAST30025 Assignment 1 2009

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Question 1 Solution:

$(\sum_{i=1}^{m} A_i)^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} A_i \ A_j = \sum_{i=1}^{m} A_i^2 = \sum_{i=1}^{m} A_i$

Question 2 Solutions:

Given, $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad A = \frac{1}{6} \begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 5 & 3 \\ 5 & 1 & -4 \\ 3 & -4 & 0 \end{bmatrix}$

$\mathrm{E}[\boldsymbol{y}] = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} \quad var\boldsymbol{y} = V = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Part a Solution:

$\mathrm{E}[y^T \mathrm{Ay}] = \mathrm{tr(AV)} + \mu^T \mathrm{A}\mu$

$= \mathrm{tr}(\frac{2}{3} \begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix}) + \frac{1}{6} \begin{bmatrix} 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} = 8 + \frac{83}{6} = \frac{131}{6}$

Part b Solution:

$\lambda = \frac{1}{2} \frac{1}{4} \begin{bmatrix} 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} = \frac{83}{48}$

There is a non central parameter of $\frac{83}{48}$
with r(A) = 2 degrees of freedom where A is symmetric and idempotent.

Part c Solution:

```{r}
A%*%V%*%B

```

```
          [,1] [,2]       [,3]
[1,] -5.333333    0   9.333333
[2,] 21.333333   -6 -17.333333
[3,] 10.666667   -6   1.333333
```

```
#No A and B are not independant since AVB is a non zero matrix according to Theorem 3.11!
```

Question 3 Solutions:

Part a:

Given,

$$\boldsymbol{y} = \begin{bmatrix} 8.5 \\ 8 \\ 7.5 \\ 10 \\ 11 \\ 15 \\ 13.5 \\ 14.5 \end{bmatrix} \qquad \boldsymbol{X} = \begin{bmatrix} 1 & 1.35 & 34 \\ 1 & 1.33 & 36 \\ 1 & 2 & 38 \\ 1 & 1.4 & 34 \\ 1 & 1.4 & 31 \\ 1 & 1.2 & 31 \\ 1 & 1.3 & 33 \\ 1 & 1.28 & 41 \end{bmatrix} \qquad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} \qquad \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \end{bmatrix}$$

$$y = X\beta + \epsilon$$

$$\begin{bmatrix} 8.5 \\ 8 \\ 7.5 \\ 10 \\ 11 \\ 15 \\ 13.5 \\ 14.5 \end{bmatrix} = \begin{bmatrix} 1 & 1.35 & 34 \\ 1 & 1.33 & 36 \\ 1 & 2 & 38 \\ 1 & 1.4 & 34 \\ 1 & 1.4 & 31 \\ 1 & 1.2 & 31 \\ 1 & 1.3 & 33 \\ 1 & 1.28 & 41 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \end{bmatrix}$$

Part b:

```
#Part b) Find the normal equations!
```

```{r}
t(X)%*%X #times beta parmeter equals
```

```
        [,1]     [,2]    [,3]
[1,]    8.00  11.2600  278.00
[2,]   11.26  16.2798  393.36
[3,] 278.00 393.3600 9744.00
```

```{r}
t(X)%*%y
```

```
         [,1]
[1,]    88.000
[2,]   120.625
[3,]  3048.000
```

Part c:

````
```{r}
b = solve(t(X)%*%X,t(X)%*%y)
b

```
````

```
             [,1]
 [1,] 19.4391195
 [2,] -7.8636388
 [3,]  0.0756533
```