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2 (a) dividing equation by $(G/A)^2$ gives

$$\left(\frac{A}{G}\right)^2 \frac{P_2^2-P_1^2}{2RT/M} + \ln\left(\frac{P_1}{P_2}\right) + \frac{2fL}{D} = 0$$

differentiate G with respect to P_2 | $G = f(P_2)$

$$A^2\left(\frac{-2}{G^3}\frac{dG}{dP_2}\right)\frac{P_2^2-P_1}{2RT/M} + \left(\frac{A}{G}\right)^2 \frac{2P_2}{2(RT/M)} - \frac{1}{P_2} = 0$$

now at $G = G_{max}$, $dG/dP_2 = 0$.

Let $P_2 = P_w$

$$\left(\frac{A}{G_m}\right)^2 \frac{2P_w}{2(RT/M)} - \frac{1}{P_w} = 0.$$

$$\left(\frac{A}{G_{max}}\right)^2 = \frac{1}{P_w} \times \frac{RT/M}{P_w}$$

$$\left(\frac{G_{max}}{A}\right)^2 = \frac{P_w^2}{RT/M}.$$

$\frac{G_{max}}{A} = \rho_w V_w$ and $P = \rho\frac{RT}{M}$

$$(\rho_w V_w)^2 = \frac{(\rho_w RT/M)^2}{RT/M}$$

$$V_w = \sqrt{RT/M}.$$

It will occur at the outlet of the pipe / front of pressure wave

(b) $P_w^2 = \left(\frac{G_{max}}{A}\right)^2 \cdot \frac{RT}{M}$, substitute $\left(\frac{G_{max}}{A}\right)^2 = \frac{P_w^2}{RT/M}$ to initial equation

$$\frac{P_w^2 - P_1^2}{\frac{2RT}{M}} + \frac{P_w^2}{\frac{RT}{M}}\ln\left(\frac{P_1}{P_w}\right) + \frac{2fL}{D}\left(\frac{P_w^2}{RT/M}\right) = 0 \qquad \div \frac{P_w^2}{2RT/M}$$

$$1 - \left(\frac{P_1}{P_w}\right)^2 + 2\ln\left(\frac{P_1}{P_w}\right) + \frac{4fL}{D} = 0.$$

rearrange

$$\left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1 = \frac{4fL}{D} \;//$$ shown

(c) trial and error

$$\left(\frac{800}{P_w}\right)^2 - \ln\left(\frac{800}{P_w}\right)^2 - 1 = \frac{4\times 0.006\times 20}{50\times 10^{-3}}$$

$$\left(\frac{800}{P_w}\right)^2 = 11.9862944$$

$$P_w = 231.07\,kPa$$

Flow is choked because $P_w > P_2$ //

(d) $\left(\frac{Gmax}{A}\right)^2 = \frac{P_w^2}{RT/M}$

$$= \frac{(231072\,Pa)^2}{\frac{8.314\,J/kmol \times (100+273)\,K}{28\times10^{-3}\,kg/mol}}$$

$$= 482096\ kg^2/m^4s^2$$

$$\frac{Gmax}{A} = 694\ kg/m^2s$$

$$Gmax = 694\ kg/m^2s \times \pi \times \frac{(50\times10^{-3})^2}{4}\,m^2$$

$$= 1.36\,kg/s$$

(e) entry

$$\rho_1 = \frac{P_1}{RT/M}$$

$$= \frac{800\times10^3}{\frac{8314\times373}{28\times10^{-3}}}$$

$$= 7.22\ kg/m^3$$

$$\rho_w = \frac{231\times10^3}{\frac{8.314\times373}{28\times10^{-3}}}$$

$$= 2.09\,kg/m^3$$

$$\frac{G}{A\rho_1} = V_1$$

$$V_1 = \frac{694\ kg/m^2s}{7.22\ kg/m^3}$$

$$= 96.1\,m/s$$

$$\frac{G}{A\rho_2} = V_2$$

$$V_2 = \frac{694\ kg/m^2s}{2.09\ kg/m^3}$$

$$= 332.8\ m/s$$

sonic velocity $= \sqrt{\frac{RT}{M}}$

$= 332.8$

$V_1 = \frac{96.1}{332.8} \times 100\% = 28.9\%$ while $V_2 = 100\%$

3 (a) (i) $P_F = -W_s G$

$= h\rho g \rho Q$

$= 11.75\,m \times 1000\,kg/m^3 \times 9.81\,m/s^2 \times 40\times10^{-3}\,m^3/s$

$= 4606\,J$

(ii) $h_{poil} = h_{pwater} \times \frac{\rho_{water}}{\rho_{oil}}$

$= 11.75\,m \times \frac{1000\,kg/m^3}{886\,kg/m^3}$

$= 13.26\,m$

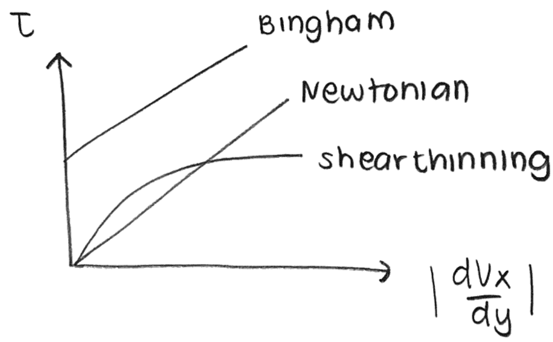
(iii) $Q = \frac{4606}{13.26 \times 9.8 \times 886}$

$= 0.04\,m^3/s$

$= 40\,L/s$

(iv) 850 rev per minute

(b) (i)



Newtonian: shear stress proporaional to viscosity regardless of shear rate/ viscosity is constant

Shear-thinning: viscosity decreases with increasing shear rate

Bingham: fluid only experiences shear stress when threshold stress is overcomed

(ii) $|\tau_{yx}| = \underbrace{K\left|\frac{dv_x}{dy}\right|^{n-1}}\left|\frac{dv_x}{dy}\right|$ → shear rate

shear stress

μ_a = apparent viscosity

n = power law index

k = term equivalent for visc

(iii) shear thinning because apparent viscosity will decrease with increasing shear rate ⇒ in this case raised to power -0.8

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4 (a) $NPSH_A = \frac{P_1 - P_{vap}}{\rho g} + z_1 - h_{fs}$

$G = 1 kg/s$, $V = G/\rho A$

$V = \frac{1 kg/s}{1000 kg/m^3} \times \frac{1}{\frac{\pi \times (0.025)^2}{4} m^2}$

$= 2.04 m/s$

$Re = \frac{\rho v d}{\mu}$

$= \frac{1000 \times 2.04 \times 25 \times 10^{-3}}{0.001}$

$= 50929$

smooth pipe with $Re = 5.1 \times 10^4$, $f_F = 0.00525$.

$NPSH_A = \frac{(101.3 - 4.2) \times 10^3}{10^3 \times 9.8} + (-3) - \frac{2 \times 0.00525 \times 15 \times 2.04^2}{25 \times 10^{-3} \times 9.8}$

$= 4.23 m$

(b) No cavitation because $NPSH_A > NPSH_R$

(c) Decrease the height of the pump so that "z_1" becomes more positive and $NPSH_A$ increases therefore greater range of safe operation w/o risk of cavitation.