Course Outline

- Probability Review
- Discrete time Markov chains
  Exam probs: 10.1,2; 12.2; 13.1,2,7; 14.1,2,3,4; 15.1,3; 16.1,6; 18.1,6
  1. Definition and Markov property
  2. Transition matrix and n-step transition
  3. State space classification: Reducibility, periodicity, null and positive recurrence
  4. Long run behaviour: Ergodicity/Stationarity, description in all cases above
  5. First step analysis: "hitting times"

Course Outline

- Poisson process
  Exam probs: 10.3; 12.4; 13.4; 14.7; 15.4; 16.2,7; 18.4
  1. Definition
  2. Descriptions (N_t, T_k, τ_i) and their relationships
  3. Superposition/Thinning
  4. Compound Poisson process
  5. PASTA

Course Outline

- Continuous time Markov chains
  Exam probs: 10.4; 12.5; 13.4,5; 14.5; 15.2,6; 16.3,5; 18.2,5
  1. Definition and Markov property
  2. Descriptions (Exponential waiting times + jumps, transition matrix, generator) and their relationships
  3. Forward and backward equations
  4. State space classification: Reducibility, positive and null recurrence
  5. Long run behaviour: ergodicity/stationarity
  6. Birth-death chains

Course Outline

- Queuing Theory
  Exam probs: 10.4; 12.6; 13.5; 14.5; 15.6; 16.3; 18.5
  1. Basic framework
  2. $M/M/a$ relationship to birth death chains
  3. Stationarity
  4. Quantities of interest: L, L_q, D, W, etc
  5. PASTA
  6. Little's law

- Renewal Theory
  Exam probs: 10.6; 12.7; 13.6; 14.8; 15.5; 16.4,7; 18.3
  1. Basic framework (N_t, T_k, τ_i) and relationships
  2. Renewal limit theorems (LLN and CLT)
  3. Overshoot Y_t and time to previous arrival A_t

Course Outline

- Brownian motion
  Exam probs: 14.6; 15.7
  1. Definition
  2. Relationship to multivariate normal/computing probabilities
  3. Hitting times and maximum
  4. Properties read as limits from random walk