MAST30001 Stochastic Modelling – 2014

Assignment 2

If you didn't already hand in a completed and signed Plagiarism Declaration Form (available from the LMS or the department's webpage), please do so and attach it to the front of this assignment.

Please hand in your assignment directly to me. **Don't forget** to staple your solutions and to print your name, student ID, and the subject name and code on the first page (not doing so will forfeit marks). The submission deadline is **Friday, 24 October, 2014 at 5:10pm (end of lecture).**

There are 2 questions, both of which will be marked. No marks will be given for answers without clear and concise explanations. Clarity, neatness and style count.

---

1. Earthquakes in a certain town occur according to a Poisson process with rate 100 per year. There is a 1/1000 chance an earthquake will produce significant damage.

   (a) What is the chance that there will be 40 earthquakes in the next six months, 20 of which occur in the next three months?

   (b) What is the chance that there will be 40 earthquakes in the next six months, exactly one of which will produce significant damage?

   (c) Given there are two earthquakes in one day, what is the chance that both of them occurred in the first half of the day?

   (d) Given there are two earthquakes in one day, what is the chance exactly one of them produced significant damage and occurred in the first half of the day?

   In addition to the earthquakes, assume that typhoons strike the town according to a Poisson process with rate 1/2 per year and that there is 3/4 chance a typhoon will produce significant damage; typhoons occur independently of earthquakes.

   (e) What is the chance there are ten typhoon/earthquake events in a single year?

   (f) What is the chance that there are five typhoon/earthquake events that produce significant damage in a single year?

2. A system has two servers. Customers arrive according to a Poisson process with rate $\lambda = 2$ (per hour) and if Server A is free then they begin service with that server. If an arriving customer finds Server A busy but Server B free, then the customer will begin service with Server B. If both servers are busy, then arriving customers are turned away. The service times for Servers A and B are independent and exponential with rates $\mu_A = 3$ and $\mu_B = 2$ (per hour).

   (a) What is the long run proportion of time Server B busy?

   (b) What is the expected number of customers an **arriving** customer finds in the system after its been running for a long time?

   (c) What is the long run proportion of **entering** customers that receive service from Server B?

   (d) What is the average time **entering** customers spend in the system?

   (e) Let $M(t)$ be the mean number of customers turned away after the system is running for t hours. Find $\lim_{t\to\infty} M(t)/t$.