

School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 4. Some answers

Question 1

The particle paths are lines but the streamlines are not.

Question 2

For the first flow

- streamlines are hyperbolae
- $\mathbf{r}(t; \mathbf{X}) = (X \cosh t + Y \sinh t, X \sinh t + Y \cosh t)$

For the second

- streamlines are circles
- $\mathbf{r}(t; \mathbf{X}) = (X \cos t + Y \sin t, -X \sin t + Y \cos t)$
- $\nabla \times \mathbf{u} = -2\mathbf{k}$
- rigid body rotation about the z-axis

Question 3

The second flow in Question 2 is incompressible.

The streamfunction ψ is $\psi = (x^2 + y^2)/2$

Question 4

- i. $\psi = -\frac{C}{2}\sigma^2 + c$; $\nabla \times \mathbf{u} = 2C$; rigid body rotation
- ii. $\psi = -D \log_e \sigma + c$; $\nabla \times \mathbf{u} = 0$; irrotational flow (a ‘line vortex’)

Question 5

- Flow tends to uniform flow at large σ
- streamline $\psi = 0$ is the circle $\sigma = a$
- stagnation points at $\sigma = a, \varphi = 0, \pi$
- $\nabla \times \mathbf{u} = 0$
- potential flow past an infinitely long cylinder (axis perpendicular to \mathbf{U})