1. In the file `rosenbrock.m`:

```
function f = rosenbrock(x)
f = 100*(x(2) - x(1)^2)^2 + (1-x(1))^2;
end
```

   Then run `fminsearch(@rosenbrock, [0 0])`

   The minimiser that is found is $\mathbf{x} \approx \begin{pmatrix}1 & 1\end{pmatrix}^T$.

2. (a) `M = [2:3:50; 101:-3:53];`
   (b) `M = (M+1)/3;`
   (c) `M(:, [1 2]) = [];`
   (d) `[nrow, ncol] = size(M);`
   (e) `M(:, randi(ncol)) = [];`
   (f) `M = [M(2,:); M(1,:)];`
   (g) `M(:, [7 9]) = M(:,[9 7]);`
   (h) `M = (M.^2 - M)/2;`
   (i) `M(2, :) = randi([-7 7], [1 ncol-1]);`
   (j) `M(:, 3:6) = randi([-10 10], [2 4]);`
   (k) `M(M > 0) = floor(sqrt(M(M > 0)));`
   (l) `M(M < 0) = (M(M < 0).^2 - M(M < 0))/2;`

3.

```
f = @(x) (2*x.^2 - 2*x - 4).*log(x + 1);
x = linspace(0,3,1000);
plot(x, f(x));
xlabel('x-axis');
ylabel('y-axis');
title('f(x) = 2x^2 - 2x - 4)log(x + 1)')
```

   For a fancier title using LaTeX you could do this instead:

```
f = @(x) (2*x.^2 - 2*x - 4).*log(x + 1);
x = linspace(0,3,1000);
plot(x, f(x));
xlabel('x-axis');
ylabel('y-axis');
TeXTitle = title("$f(x) = (2x^2 - 2x - 4)\cdot \log(x+1)$");
set(TeXTitle, 'Interpreter', 'Latex')
```

4. $\begin{pmatrix}3 & -1\\3 & -5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix} \implies \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3 & -1\\3 & -5\end{pmatrix}^{-1}\begin{pmatrix}4\\2\end{pmatrix} = \frac{1}{-15+3}\begin{pmatrix}-5 & 1\\-3 & 3\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix} = -\frac{1}{12}\begin{pmatrix}-18\\-6\end{pmatrix} = \begin{pmatrix}3/2\\1/2\end{pmatrix}.$

5. $$\left(\begin{array}{cc|c}3 & -1 & 4\\3 & -5 & 2\end{array}\right) \equiv \left(\begin{array}{cc|c}3 & -1 & 4\\0 & -4 & -2\end{array}\right) \quad R_2' = R_2 - R_1$$
   $$\equiv \left(\begin{array}{cc|c}1 & -1/3 & 4/3\\0 & 1 & 1/2\end{array}\right) \quad \begin{array}{l}R_1' = \frac{1}{3}R_1\\ R_2' = -\frac{1}{4}R_2\end{array}$$
   $$\equiv \left(\begin{array}{cc|c}1 & 0 & 3/2\\0 & 1 & 1/2\end{array}\right) \quad R_1' = R_1 + \frac{1}{3}R_2$$

   This gives $x = \frac{3}{2}$ and $y = \frac{1}{2}$.

6. $$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 2 & 1 & 0 & 0 & 0 \end{array}\right) \equiv \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & -2 & 0 \end{array}\right) \quad R_3' = R_3 - 2R_1$$
$$\equiv \left(\begin{array}{cccc|c} 1 & 0 & -1 & -5 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 7 & 0 \end{array}\right) \quad \begin{array}{l} R_1' = R_1 - 2R_2 \\ \\ R_3' = R_3 + 3R_2 \end{array}$$
$$\equiv \left(\begin{array}{cccc|c} 1 & 0 & -1 & -5 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \quad R_3' = \tfrac{1}{7}R_3$$
$$\equiv \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \quad \begin{array}{l} R_1' = R_1 + 5R_3 \\ R_2' = R_2 - 3R_3 \end{array}$$
The solution set is then
$$\begin{aligned} &\{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 : x_1 - x_3 = 0, \quad x_2 + 2x_3 = 0, \quad x_4 = 0\} \\ =&\{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 : x_1 = x_3, \quad x_2 = -2x_3, \quad x_4 = 0\} \\ =&\{(x_3, -2x_3, x_3, 0) : x_3 \in \mathbb{R}\}. \end{aligned}$$

7. For Question 4,

```
A = [3 -1; 3 -5];
B = [4; 2];
inv(A)*B
```

You could also write `format rational` beforehand to output the result as a rational number. To return to decimal output afterwards, write `format long` or `format short`.

For Question 5,

```
A = [3 -1 4; 3 -5 2];
A(2,:) = A(2,:) - A(1,:);
A = A./[3; -4];
A(1,:) = A(1,:) + A(2,:)/3
```

For Question 6,

```
A = [1 2 3 1 0; 0 1 2 3 0; 2 1 0 0 0];
A(3,:) = A(3,:) - 2*A(1,:);
A = A + [-2; 0; 3].*A(2,:);
A(3,:) = A(3,:)/7;
A = A + [5; -3; 0].*A(3,:)
```

8.
```
rref([3 -1 4; 3 -5 2])
rref([1 2 3 1 0; 0 1 2 3 0; 2 1 0 0 0]
```

9. (a) The objective function is
$$\begin{aligned} F(a_0,a_1) &= (-1-(-4a_1+a_0))^2 + (0-(-2a_1+a_0))^2 + (2-(2a_1+a_0))^2 + (3-(5a_1+a_0))^2 \\ &= (-1+4a_1-a_0)^2 + (2a_1-a_0)^2 + (2-2a_1-a_0)^2 + (3-5a_1-a_0)^2 \end{aligned}$$

(b) In the file `objective.m`:

```
function f = objective(a)
a0 = a(1);
a1 = a(2);
f = (-1 + 4*a1 - a0)^2 + (2*a1 - a0)^2 + (2-2*a1 - a0)^2 + (3-5*a1-a0)^2;
end
```

Then running `fminsearch(@objective, [0 0])` results in $a_0 \approx 0.8872$ and $a_1 \approx 0.4513$.

(c) The partial derivatives are

$$
\begin{aligned}
\frac{\partial F}{\partial a_0} &= -2(-1+4a_1-a_0) - 2(2a_1-a_0) - 2(2-2a_1-a_0) - 2(3-5a_1-a_0) \\
&= -2(4-a_1-4a_0), \\
\frac{\partial F}{\partial a_1} &= 8(-1+4a_1-a_0) + 4(2a_1-a_0) - 4(2-2a_1-a_0) - 10(3-5a_1-a_0) \\
&= -46+78a_1+2a_0.
\end{aligned}
$$

Setting these equal to zero results in

$$
\begin{aligned}
-2(4-a_1-4a_0) = 0 &\iff a_1+4a_0 = 4 \\
-46+78a_1+2a_0 = 0 &\iff 49a_1+a_0 = 23
\end{aligned}
$$

So we have

$$
\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 1 & 49 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 23 \end{pmatrix} = \frac{1}{4\cdot 49-1}\begin{pmatrix} 49 & -1 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 23 \end{pmatrix} = \frac{1}{195}\begin{pmatrix} 173 \\ 88 \end{pmatrix},
$$

giving $a_0 = \frac{173}{195}$ and $a_1 = \frac{88}{195}$.

(d) We have

$$
\begin{aligned}
n &= 4 \\
\sum_{i=1}^{4} x_i &= -4-2+2+5 = 1 \\
\sum_{i=1}^{4} y_i &= -1+0+2+3 = 4 \\
\sum_{i=1}^{4} x_i^2 &= (-4)^2+(-2)^2+2^2+5^2 = 49 \\
\sum_{i=1}^{4} x_i y_i &= -4\cdot -1 + -2\cdot 0 + 2\cdot 2 + 5\cdot 3 = 23.
\end{aligned}
$$

The normal equations are then

$$
\begin{aligned}
na_0 + a_1\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i &\iff 4a_0 + a_1 = 4 \\
a_0\sum_{i=1}^{n} x_i + a_1\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i &\iff a_0 + 49a_1 = 23.
\end{aligned}
$$

Using `inv([1 4; 49 1])*[4; 23]` then results in the coefficients $a_0 \approx 0.8872$ and $a_1 \approx 0.4513$. Writing `format rational` beforehand or evaluating `rats(inv([1 4; 49 1])*[4; 23])` will find the same rational answers as in (c).

(e) The matrices are

$$
\mathbf{A} = \begin{pmatrix} 1 & -4 \\ 1 & -2 \\ 1 & 2 \\ 1 & 5 \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 3 \end{pmatrix}.
$$

Then running the folllowing code will find the coefficients:

```
x = [-4 -2 2 5]';
y = [-1 0 2 3]';
A = [ones([4 1]) x];
inv(A'*A)*A'*y
```

10. (a) The residuals are

$$r_i = z_i - ax_i - by_i - c,$$

so the objective function is

$$F(a,b,c) = \sum_{i=1}^{n} (z_i - ax_i - by_i - c)^2.$$

(b) The partial derivatives are

$$\begin{aligned}
\frac{\partial F}{\partial a} &= \sum_{i=1}^{n} -2x_i(z_i - ax_i - by_i - c) = -2\sum_{i=1}^{n}(x_i z_i - ax_i^2 - bx_i y_i - cx_i) \\
\frac{\partial F}{\partial b} &= \sum_{i=1}^{n} -2y_i(z_i - ax_i - by_i - c) = -2\sum_{i=1}^{n}(y_i z_i - ax_i y_i - by_i^2 - cy_i) \\
\frac{\partial F}{\partial b} &= \sum_{i=1}^{n} -2(z_i - ax_i - by_i - c) = -2\sum_{i=1}^{n}(z_i - ax_i - by_i - c).
\end{aligned}$$

Setting $\frac{\partial F}{\partial a}$ equal to 0 results in

$$\begin{aligned}
\sum_{i=1}^{n}(x_i z_i - ax_i^2 - bx_i y_i - cx_i) = 0 &\iff \sum_{i=1}^{n} x_i z_i = \sum_{i=1}^{n} ax_i^2 + \sum_{i=1}^{n} bx_i y_i + \sum_{i=1}^{n} cx_i \\
&\iff \sum_{i=1}^{n} x_i z_i = a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i y_i + c\sum_{i=1}^{n} x_i
\end{aligned}$$

Setting $\frac{\partial F}{\partial b}$ equal to 0 results in

$$\begin{aligned}
\sum_{i=1}^{n}(x_i z_i - ax_i^2 - bx_i y_i - cx_i) = 0 &\iff \sum_{i=1}^{n} y_i z_i = \sum_{i=1}^{n} ax_i y_i + \sum_{i=1}^{n} by_i^2 + \sum_{i=1}^{n} cy_i \\
&\iff \sum_{i=1}^{n} y_i z_i = a\sum_{i=1}^{n} x_i y_i + b\sum_{i=1}^{n} y_i^2 + c\sum_{i=1}^{n} y_i
\end{aligned}$$

Setting $\frac{\partial F}{\partial c}$ equal to 0 results in

$$\begin{aligned}
\sum_{i=1}^{n}(z_i - ax_i - by_i - c) = 0 &\iff \sum_{i=1}^{n} z_i = \sum_{i=1}^{n} ax_i + \sum_{i=1}^{n} by_i + \sum_{i=1}^{n} c \\
&\iff \sum_{i=1}^{n} z_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i + nc
\end{aligned}$$

(c) In matrix form, this is $\mathbf{A}^T\mathbf{A}\mathbf{a} = \mathbf{A}^T\mathbf{z}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} c \\ b \\ a \end{pmatrix}, \quad \mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix}.$$

11. (a) –

(b) No; the first line in the file is the column headings, and for files containing mixed numeric and text data, `readmatrix` imports the data as a numeric array by default.

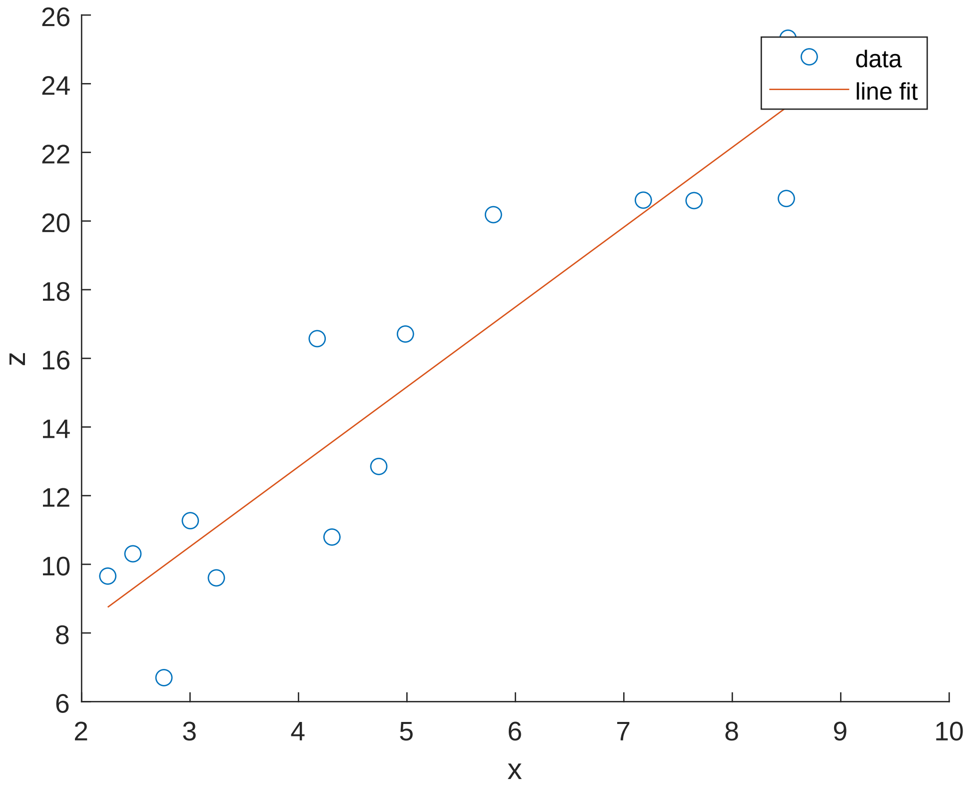
(c)
```
x = dat(:,1);
y = dat(:,2);
z = dat(:,3);
```

(d) `scatter(x,z);`

(e)
```
A = [ones(size(x)) x];
coeffs = inv(A'*A)*A'*z;
```

This gives $a_0 \approx 3.5370$ and $a_1 \approx 2.3264$.

(f)
```
hold on
scatter(x,z)
xmin = min(x);
xmax = max(x);
plot([xmin, xmax], [coeffs(1) + coeffs(2)*xmin, coeffs(1)+coeffs(2)*xmax]);
legend('data', 'line fit');
xlabel('x');
ylabel('z');
hold off
```



12.
```
A = [ones(size(x)) x y];
coeffs = inv(A'*A)*A'*z;
```

This gives $a_0 \approx 3.9680$, $a_1 \approx 2.2517$ and $a_2 \approx -0.3092$.