MAST30001 Stochastic Modelling – 2014

Assignment 1

Please complete and sign the Plagiarism Declaration Form (available from the LMS or the department's webpage), which covers all work submitted in this subject. The declaration should be attached to the front of your first assignment.

Please hand in your assignment directly to me. **Don't forget** to staple your solutions and to print your name, student ID, and the subject name and code on the first page (not doing so will forfeit marks). The submission deadline is **Friday, 12 September, 2014 at 5:10pm (end of lecture).**

There are 2 questions, both of which will be marked. No marks will be given for answers without clear and concise explanations. Clarity, neatness and style count.

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1. Let X_n be a Markov chain with transition matrix

$$P = \begin{pmatrix} 1/6 & 1/3 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}.$$

   (a) Define

$$Y_n = \begin{cases} 1 & X_n \in \{1,2\}, \\ 2 & X_n = 3. \end{cases}$$

   Is $(Y_n)_{n\geq 0}$ a Markov chain? If so, find its transition matrix.

2. N balls are distributed among two urns, labelled A and B. At discrete time steps, an urn is chosen at random and a ball from that urn is moved to the other urn; if the chosen urn is empty, then no action is taken. The chance of choosing urn A at each step is $1-p$, where $0 < p < 1$. Let X_n be the number of balls in urn A after n time steps.

   (a) Model $(X_n)_{n\geq 0}$ as a Markov chain and compute its transition probabilities.

   (b) Analyse the state space of the Markov chain and describe its long run behaviour.

   (c) If $T(i) = \min\{n \geq 1 : X_n = i\}$, find $E[T(i)|X_0 = i]$ for $0 \leq i \leq N$.

   (d) For $T(i)$ as above and assuming $N \geq 3$, compute $E[T(0)|X_0 = j]$ for $j = 0, 1, 2, 3$.

   (e) Compute $P(T(N) > T(0)|X_0 = j)$ for each $0 \leq j \leq N$.

   (f) Now assume that N is even and the process is the same *except* if the chosen urn has at least two balls in it, then two balls are moved from it to the other urn. If the chosen urn only has one ball in it, then it is moved to the other urn. Analyse the state space of the Markov chain and describe its long run behaviour.