School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 6.
Hydrostatics/Bernoulli equation/potential flow

Question 1

Consider a solid body partially immersed at the horizontal surface between two fluids of ρ_1 and ρ_2. Derive the condition that the body experiences no net force, assuming constant gravity. Assume the pressure is continuous at the interface.

Question 2

An inviscid fluid is rotating under gravity with constant angular velocity Ω so that relative to fixed Cartesian axes $\mathbf{u} = -\Omega y\mathbf{i} + \Omega x\mathbf{j}$. We wish to find the surfaces of constant pressure and hence the surface of a uniformly rotating bucket of water (which will be at atmospheric pressure).

By BernoulliÕ, $\frac{1}{2}q^2 + \frac{p}{\rho} + gz$ is constant, so the constant pressure surfaces are

$$z = C - \frac{\Omega^2}{2g}(x^2 + y^2)$$

But this means the surface of a rotating bucket of water has its highest point in the middle! What is wrong?

Write down the Euler equations in component form, integrate them directly to find the pressure p and hence obtain the correct shape of the free surface.

Question 3

Show that for unsteady *irrotational* flow we have a Bernoulli equation

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}q^2 + \frac{p}{\rho} + \chi = F(t)$$

where ϕ is the velocity potential and $\mathbf{b} = -\nabla\chi$.

Show that the choice of the function $F(t)$ does not affect the velocity field.

Question 4

Show that the Stokes streamfunction of an axisymmetric potential flow satisfies the equation

$$\frac{1}{\sin\theta}\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial \Psi}{\partial \theta}\right) = 0$$

which is *not* Laplace's equation.