Topic 1: Studying Spatial Point Processes

This topic further investigates properties of spatial point processes. In particular, we consider

- Simulation of **homogeneous Poisson spatial processes.**
- Simulation of **inhomogeneous Poisson spatial processes.**
- **Kolmogorov-Smirnov test of CSR**.
- Applications to **New Zealand and tropical rain forest trees data**.

Simulation of Poisson spatial process

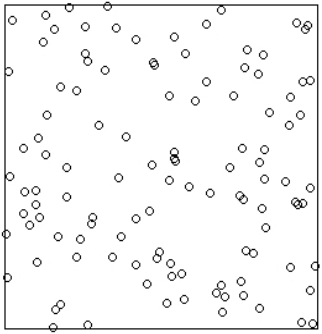
The homogeneous Poisson process of intensity $\lambda > 0$ has the properties:

- the number $X(B)$ of points falling in any region B is a Poisson random variable;
- the expected number of points falling in B is $E[X(B)] = \lambda \cdot V(B)$;
- if B_1, B_2 are disjoint sets then $X(B_1)$ and $X(B_2)$ are independent random variables;
- given that $X(B) = n$, the n points are independent and uniformly distributed in B.

To simulate the Poisson process directly one can use

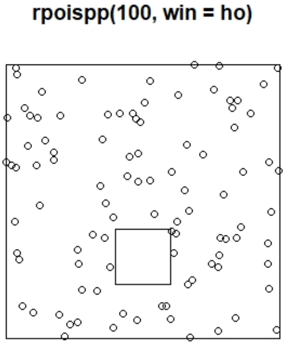
```
> a<-rpoispp(100)
> a
> plot(a)
```

a



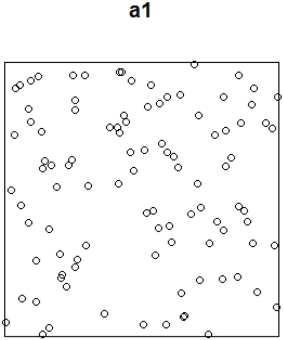
We can change the window in which to simulate. For example,

```
> ho <- owin(poly=list(list(x=c(0,1,1,0), y=c(0,0,1,1)),
+ list(x=c(0.6,0.4,0.4,0.6), y=c(0.2,0.2,0.4,0.4))))
> plot(ho)
> plot(rpoispp(100, win=ho))
```



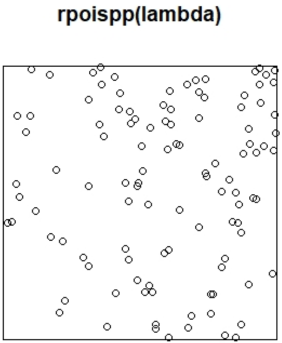
If one wants to simulate the Poisson process conditionally on a fixed number of points, use

```
> a1<-runifpoint(100)
> a1
> plot(a1)
```



To simulate the inhomogeneous Poisson process with the intensity function $\lambda(x, y)$, use

```
> lambda <- function(x, y) {100 * (x + y)}
> plot(rpoispp(lambda))
```



Kolmogorov-Smirnov test of CSR

The basic model of a point process is the homogeneous Poisson point process in the plane with intensity λ, sometimes called **Complete Spatial Randomness (CSR).** The homogeneous Poisson process is usually taken as the appropriate null model for a point pattern.

Our first task in analysing a point pattern is to find evidence against CSR.

Typically the **Kolmogorov-Smirnov test** is used in which one compares the observed and expected distributions of the values of some function T.

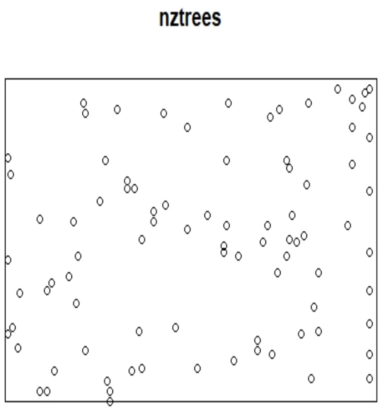
We specify a real-valued function $T(x, y)$ defined at all locations (x, y) in the window. We evaluate this function at each of the data points. Then we compare this empirical distribution of values of T with the predicted distribution of values of T under CSR, using the Kolmogorov-Smirnov test.

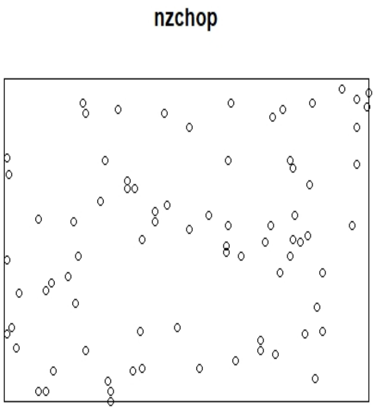
We will use the NZ trees dataset **nztrees** which represents the positions of 86 trees in a forest plot approximately 153 by 95 feet.

We discard from our analysis the eight trees at the right-hand edge of the plot (which appear to be part of a planted border) and trim the window by a 5-foot margin accordingly. The result is the NZCHOP data:

```
> library(spatstat)
> data(nztrees)
> nztrees
Planar point pattern: 86 points
window: rectangle = [0, 153] x [0, 95] feet
> plot(nztrees)

> chopped <- owin(c(0, 148), c(0, 95))
> nzchop <- nztrees[chopped]
> plot(nzchop)
```

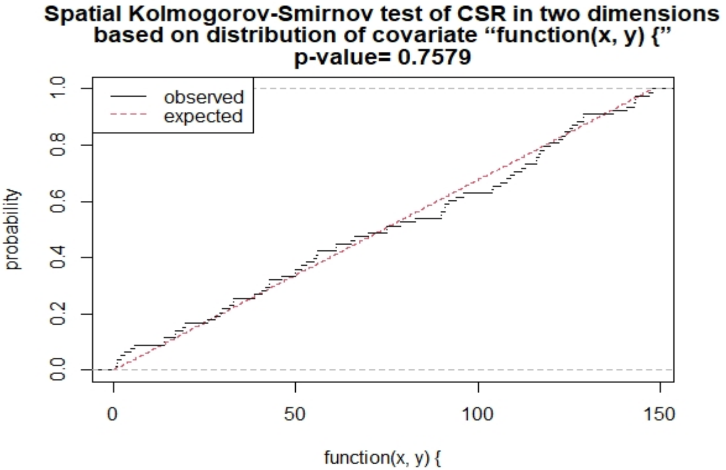

nztrees


nzchop

In SPATSTAT the spatial Kolmogorov-Smirnov test is performed by **cdf.test.** For example, let the function T be the x coordinate, $T(x, y) = x$. This means we are simply comparing the observed and expected distributions of the x coordinate:

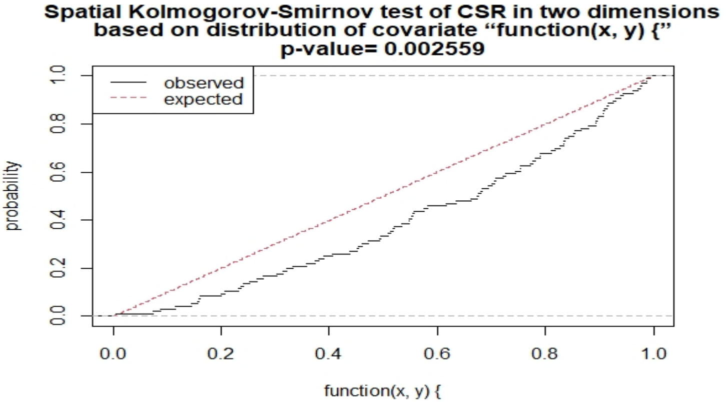
```
> cdf.test(nzchop, function(x, y) {x})
Spatial Kolmogorov-Smirnov test of CSR in two dimensions
data: covariate function(x, y){evaluated at points of nzchop
    and transformed to uniform distribution under CSR
    D = 0.075643, p-value = 0.7347
    alternative hypothesis: two-sided
> KS <- cdf.test(nzchop, function(x, y) {x})
> plot(KS)
> pval <- KS$p.value
> pval
    [1] 0.7164411
```

The result shows that the *p*-value equals 0.7579. Thus, the test does not reject the hypothesis of CSR. The plot displays the observed and expected distribution functions that also confirm it.



In the next example the Kolmogorov-Smirnov test gives us evidence against CSR for the simulated non-stationary point process:

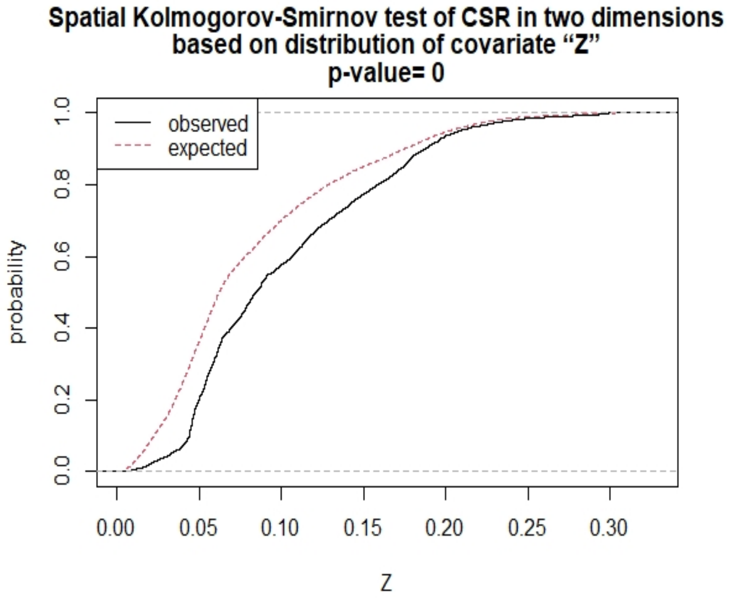
```
> lambda <- function(x, y) {100 * (x + y)}
> KS1 <- cdf.test(rpoispp(lambda), function(x, y) {x})
> plot(KS1)
```



We are often interested in testing whether the point pattern intensity depends on covariates. For example, our preliminary analysis of the tropical rainforest pattern BEI suggested that the density of trees depends on terrain slope. To test this formally we can apply the Kolmogorov-Smirnov test using the slope covariate:

```
> data(bei)
> Z <- bei.extra$grad
> KS2 <- cdf.test(bei, Z)
> KS2
Spatial Kolmogorov-Smirnov test of CSR in two  dimensions
data:  covariate Z evaluated at points of bei
and transformed to uniform distribution under CSR
D = 0.19475, p-value < 2.2e-16
alternative hypothesis: two-sided

> plot(KS2)
```


Spatial Kolmogorov-Smirnov test of CSR in two dimensions
based on distribution of covariate "Z"
p-value= 0
observed
expected
probability
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Z

Key R commands	
runifpoint(n)	*generates a random point pattern with n independent uniform random points*
rpoispp(lambda,...)	*generates Poisson point pattern*
win(xrange, yrange,...)	*creates an observation window*
cdf.test(...)	*tests goodness-of-fit of a point process model*
nztrees	*data with locations of trees in a forest plot*
bei	*data with locations of 3605 trees in a tropical rain forest*