Analyzing spatio-temporal random fields with R

We use R packages **gstat** and **spacetime** to demonstrate spatio-temporal geostatistical modelling and interpolation.

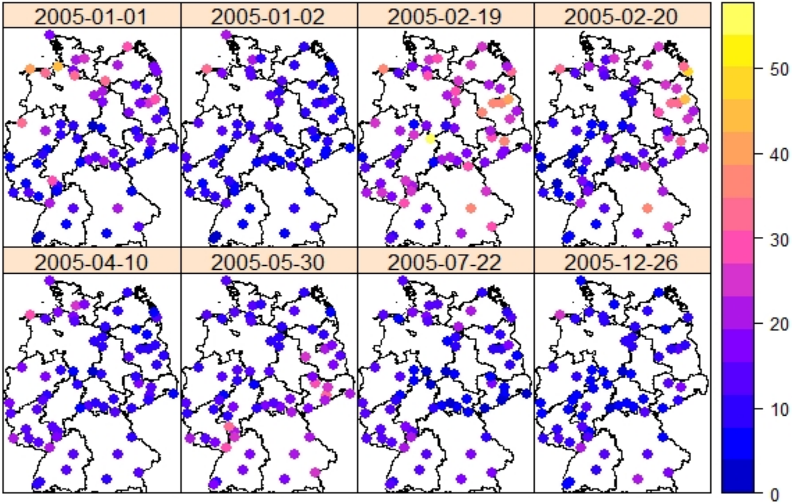
It will be only shown how to use the separable spatio-temporal covariance model, but the packages also have options for product-sum, metric and sum-metric models.

The datasets **air** and **DE_RB_2005** contain the daily mean PM10 concentration measured at rural air quality monitoring stations across Germany in 1998-2009 and 2005 respectively.

Boundaries of Germany and its regions are obtained from **DE_NUTS1**.

First we plot maps showing the monitoring stations and daily mean PM10 observed during smplDays in 2005:

```
> library(sp)
> library(spacetime)
> library(gstat)
> library(rgdal)
> data(air)
> data(DE_RB_2005)
> str(DE_RB_2005)
Formal class 'STSDF' [package "spacetime"] with 5 slots
..@ data   :'data.frame': 23230 obs. of  1 variable:
.. ..$ PM10: num [1:23230] 16.7 31.7 5 22.4 26.8 ...
> smplDays <- as.integer(c(1, 2, 50, 51, 100, 150, 203, 360))
> DE_NUTS1 <- spTransform(DE_NUTS1, CRS("+init=epsg:32632"))
> stplot( as(DE_RB_2005[, smplDays], "STFDF"),
+   col.regions = bpy.colors(120)[-(1:20)],
+   sp.layout = list("sp.polygons", DE_NUTS1),
+   scales = list(draw = F),
+   key.space = "right", colorkey = TRUE,  main = NULL)
```

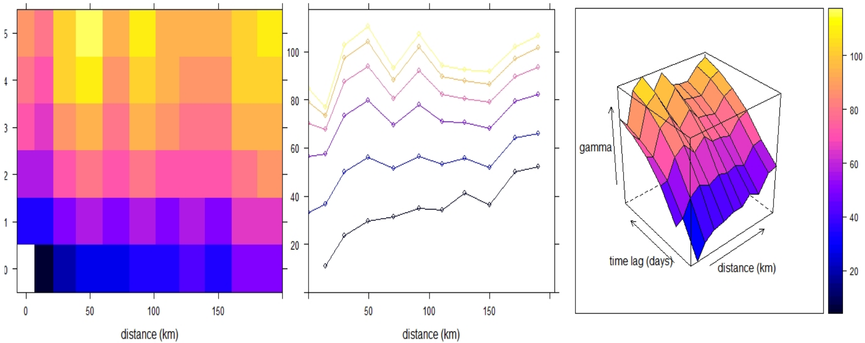

2005-01-01
2005-01-02
2005-02-19
2005-02-20
2005-04-10
2005-05-30
2005-07-22
2005-12-26
50
40
30
20
10
0

Then we consider data for the period from 2005 to 2010 and remove days with missing values. We assume that the trend is constant and use the remaining data to build an empirical spatio-temporal variogram:

```
> rural <- STFDF(stations,dates,data.frame(PM10 =
+ as.vector(air)))
> rr <- rural[,"2005::2010"]
> unsel <- which(apply(as(rr, "xts"),2, function(x)
+ all(is.na(x))))
> r5to10 <- rr[-unsel,]
> vv <- variogram(PM10~1,r5to10,width=20,cutoff=200,
+tlags=0:5)
```

Various plots of the variogram can be obtained as

```
> plot(vv)
> plot(vv, map = F)
> plot(vv, wireframe = T)
```



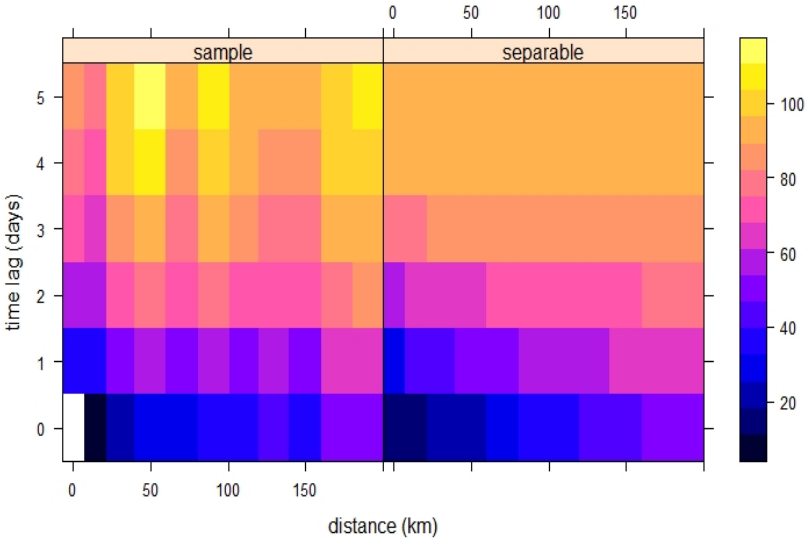
5
4
3
2
1
0
0
50
100
150
distance (km)
100
80
60
40
20
50
100
150
distance (km)
gamma
time lag (days)
distance (km)
100
80
60
40
20

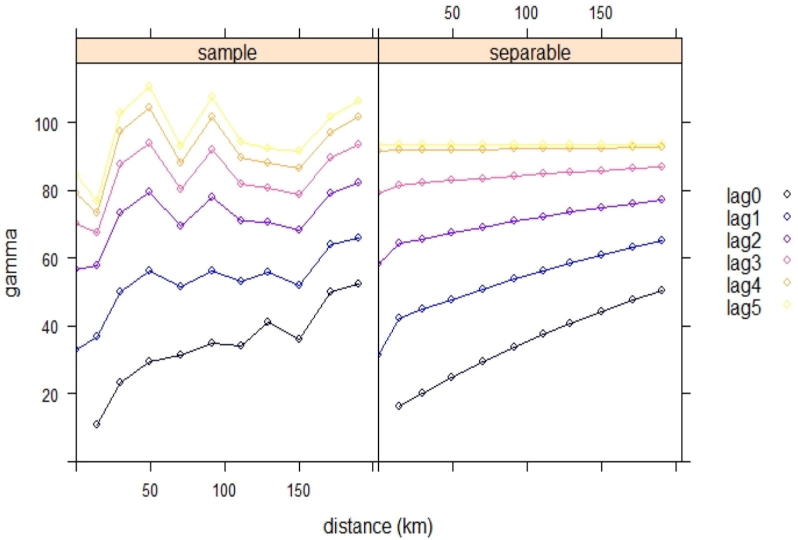
Then, using the separable model with the exponential spatial and spherical temporal variograms we fit it to the empirical spatio-temporal variogram:

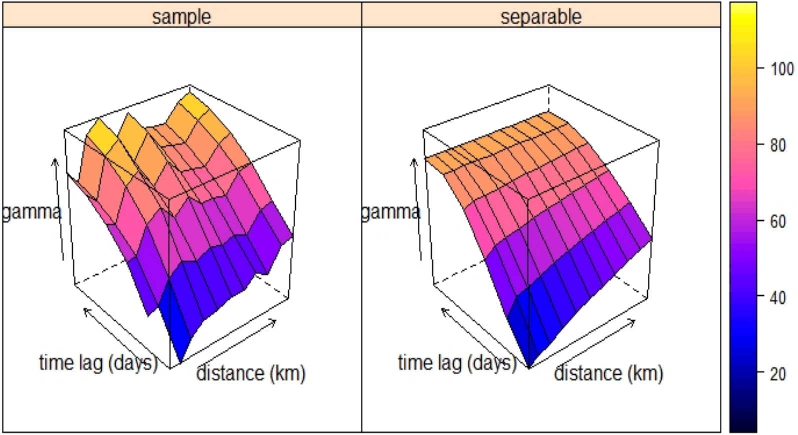
```
> separableModel0 <- vgmST("separable",
+   space = vgm(0.9, "Exp", 150, 0.1),
+   time = vgm(0.9, "Sph", 3, 0.1),  sill = 40,
+   temporalUnit = "days")

> separableModel01 <-fit.StVariogram( vv, separableModel0,
+     fit.method = 7,  stAni = 200, method = "L-BFGS-B",
+     control = list(parscale = c(100, 1, 10, 1, 100)),
+     lower = c(10, 0, 0.1, 0, 0.1),
+     upper = c(2000, 1, 12, 1, 200) )

> plot(vv, separableModel01, all = T)
> plot(vv, separableModel01, map = F, all = T)
> plot(vv, separableModel01, wireframe = T, all = T)
```


sample
separable
time lag (days)
distance (km)
0
50
100
150
0
1
2
3
4
5
20
40
60
80
100


sample
separable
gamma
distance (km)
20
40
60
80
100
50
100
150
lag0
lag1
lag2
lag3
lag4
lag5

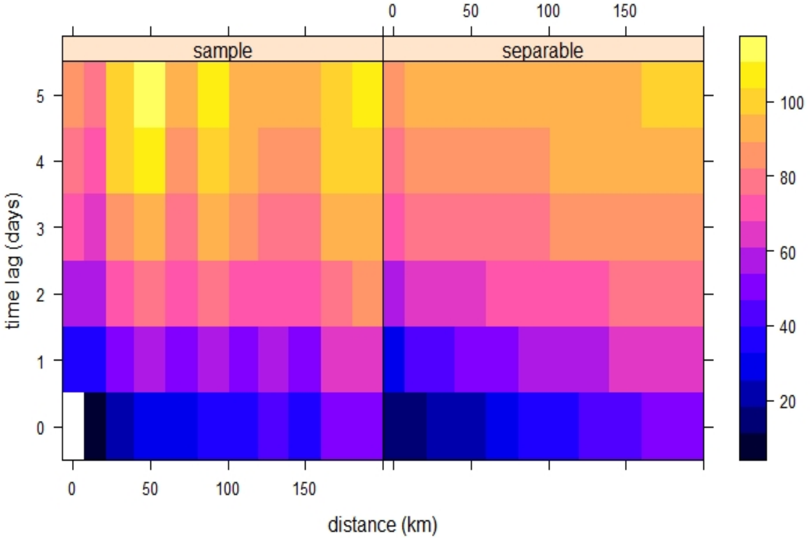

sample
separable
gamma
time lag (days)
distance (km)
gamma
time lag (days)
distance (km)
100
80
60
40
20

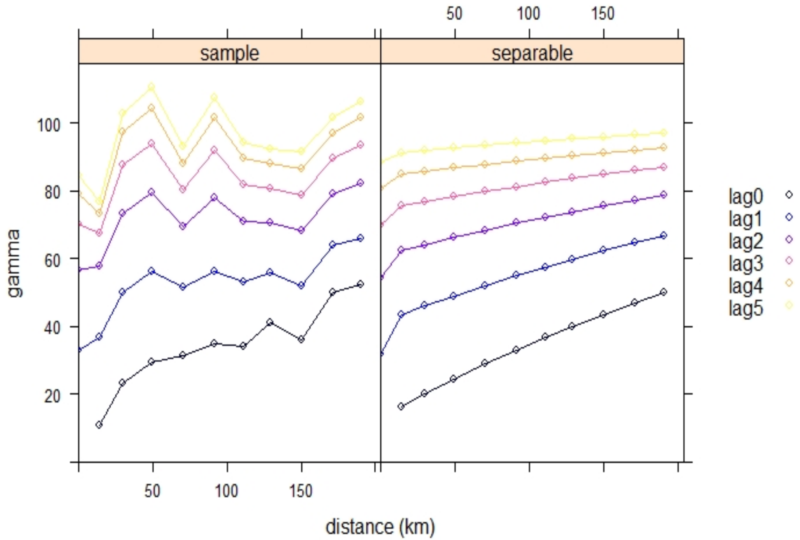
Now, we repeat the previous step but using the separable model with the exponential variogram for the both spatial and temporal dependencies.

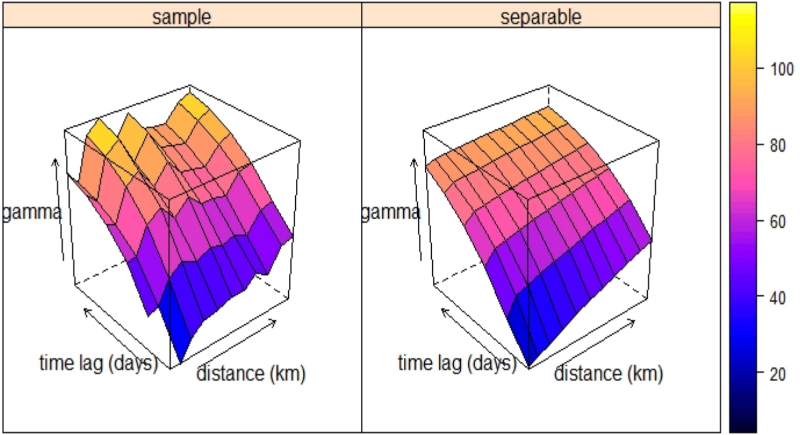
```
> separableModel <- vgmST("separable",
+   space = vgm(0.9, "Exp", 150, 0.1),
+   time = vgm(0.9, "Exp", 3, 0.1), sill = 40,
+   temporalUnit = "days")

> separableModel1 <- fit.StVariogram( vv,
+   separableModel, fit.method = 7,
+   stAni = 200, method = "L-BFGS-B",
+   control = list(parscale = c(100, 1, 10, 1, 100)),
+   lower = c(10, 0, 0.1, 0, 0.1),
+   upper = c(2000, 1, 12, 1, 200))

> plot(vv, separableModel1, all = T)
> plot(vv, separableModel1, map = F, all = T)
> plot(vv, separableModel1, wireframe = T, all = T)
```


sample
separable
time lag (days)
distance (km)
0
1
2
3
4
5
0
50
100
150
20
40
60
80
100


sample
separable
gamma
distance (km)
20
40
60
80
100
50
100
150
lag0
lag1
lag2
lag3
lag4
lag5


sample
separable
gamma
time lag (days)
distance (km)
gamma
time lag (days)
distance (km)
100
80
60
40
20

To perform spatio-temporal kriging we create spatial and temporal grids covering the area and time period respectively

```
> gridDE <- SpatialGrid(GridTopology(DE_RB_2005@sp@bbox[,1]
+ %/%10000*10000,  c(10000,10000),
+ cells.dim=ceiling(apply(DE_RB_2005@sp@bbox,1,diff)/10000)))
> proj4string(gridDE) <- CRS("+init=epsg:32632")
> fullgrid(gridDE) <- F

> DE_pred <- STF(gridDE, DE_RB_2005@time[smplDays])
> proj4string(DE_pred) <- CRS("+init=epsg:32632")
> proj4string(DE_RB_2005) <- CRS("+init=epsg:32632")
> tIDS <- unique(pmax(1,pmin(as.numeric(outer(-5:5,
+ smplDays, "+")), 365)))
```

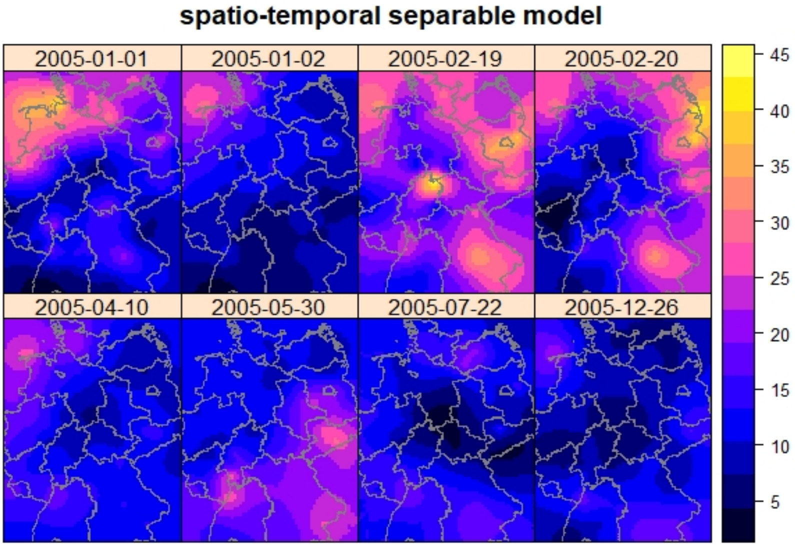
We convert the range of variogam to meters to make it consistent with $\mathrm{DE\_RB\_2005}$:

```
> separableModel0$space$range <- separableModel0$space$range*1000
> separableModel$space$range <- separableModel$space$range*1000
```

Then kriging maps are generated and plotted for a set of time moments and a spatial grid over the region covering Germany using the first separable model:

```
> sepPred0 <- krigeST(PM10~1, data=DE_RB_2005[,tIDS],
+  newdata=DE_pred, separableModel0, nmax=50,
+  stAni=200)

> stplot(sepPred0, col.regions=bpy.colors(),
+  scales=list(draw=F), sp.layout = list("sp.polygons",
+  DE_NUTS1,first=FALSE, col=gray(0.5)),
+  main="spatio-temporal separable model")
```

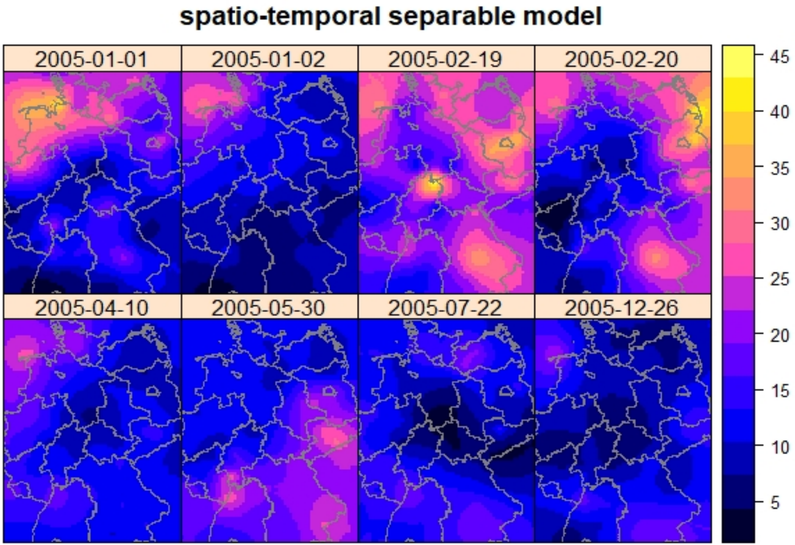

spatio-temporal separable model
2005-01-01
2005-01-02
2005-02-19
2005-02-20
2005-04-10
2005-05-30
2005-07-22
2005-12-26
45
40
35
30
25
20
15
10
5

Then we repeat it by using the second separable model:

```
> sepPred <- krigeST(PM10~1, data=DE_RB_2005[,tIDS],
+  newdata=DE_pred, separableModel, nmax=50,
+  stAni=200)

> stplot(sepPred, col.regions=bpy.colors(),
+  scales=list(draw=F),
+   sp.layout = list("sp.polygons", DE_NUTS1,first=FALSE,
+   col=gray(0.5)),
+   main="spatio-temporal separable model")
```

Comparing the kriging maps we see that they are similar in general, but different covariance models give different details on smaller scales. Also, the second model seems better predict smaller averages than the first one.


spatio-temporal separable model
2005-01-01
2005-01-02
2005-02-19
2005-02-20
2005-04-10
2005-05-30
2005-07-22
2005-12-26
45
40
35
30
25
20
15
10
5