ENGR30002 Fluid Mechanics

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1 Hydrostatics

About me

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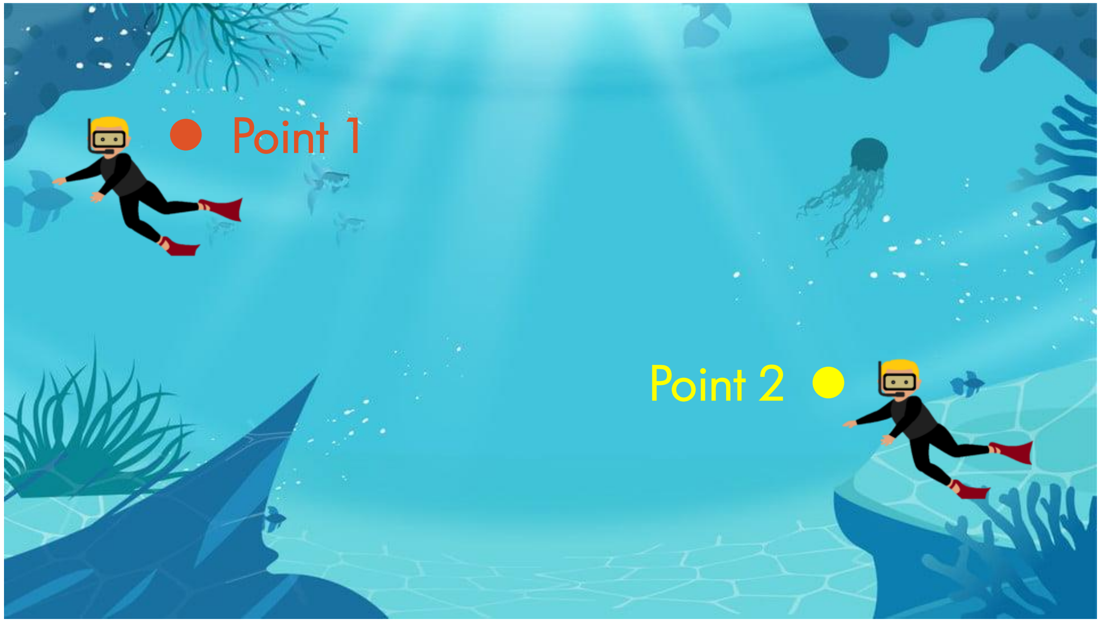
Analysis of
Transport Phenomena
Inclusive Meeting Norms

Learning objectives

- By the end of this module, students should be able to:
  - Explain the pressure gradients in a static fluid in the x-, y-, and z-directions
  - Derive the pressure gradients in a static fluid in the x-, y-, and z-directions
  - Explain the physics behind how a U-tube manometer and a differential manometer function
  - Explain the difference (verbally and mathematically) between absolute pressure and gauge pressure
  - Use manometers to calculate absolute pressure and/or the pressure difference in a system

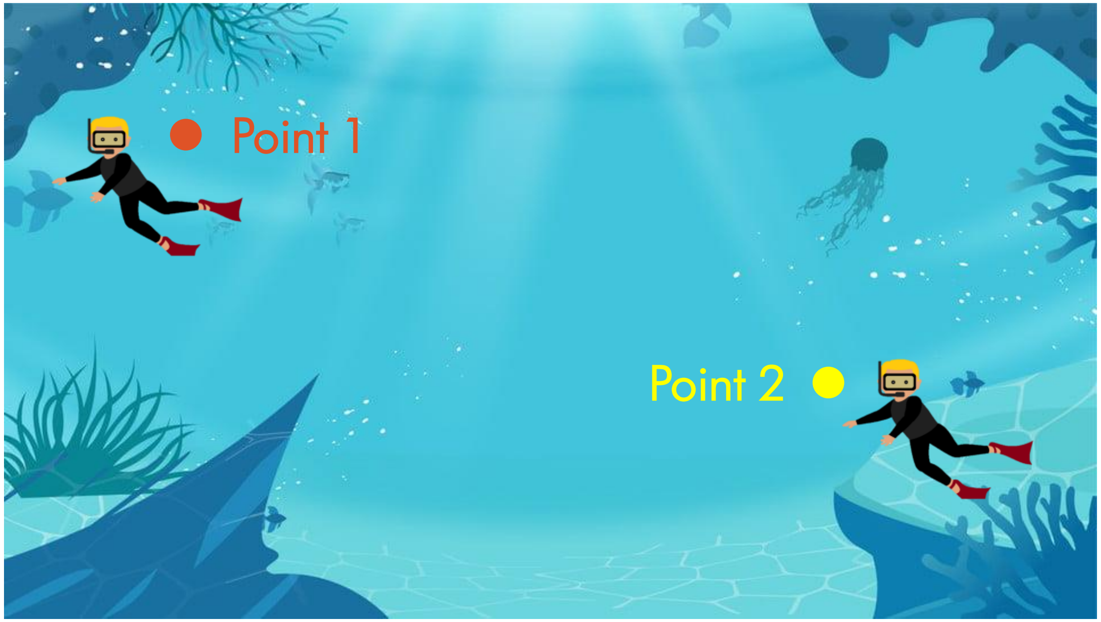
Fluids at rest

- Before we start talking about **fluid dynamics (fluid flow)**, we first need to understand **fluid statics (fluids at rest)**
- The first concept we're going to investigate is **hydrostatic pressure**, or the pressure variations that occur within a stationary body of fluid



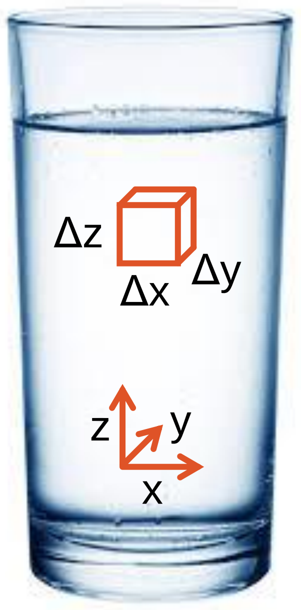
What is pressure?

- What exactly is pressure?
- What causes pressure variations within a static fluid?
  - Is the diver experiencing more, less, or the same pressure when he is at Point 2 compared to Point 1?
  - How does each component of pressure (P_x, P_y, P_z) change as our diver moves from Point 1 to Point 2?



Force balances on static fluids

- We have a glass of water
  - The fluid within the glass is stationary
  - Let's consider the fluid contained within a **control volume** with dimensions Δx, Δy, Δz
- Let's perform a **force balance** on the fluid in the volume
  - Since the fluid is stationary, the sum of forces in each direction must be equal (Newton's First Law of Motion)
  - This means the pressures acting on opposing sides of the cube must be equal



- First, the **x-direction**:

$$F_{x1} = F_{x2} \Rightarrow P_{x1}\Delta y \Delta z = P_{x2}\Delta y \Delta z \Rightarrow P_{x1} = P_{x2}$$

$$\Rightarrow P_{x2} - P_{x1} = 0 \Rightarrow \lim_{\Delta x \to 0}\left(\frac{P_{x2} - P_{x1}}{\Delta x}\right) = 0 \Rightarrow \boxed{\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{x}} = \boldsymbol{0}}$$

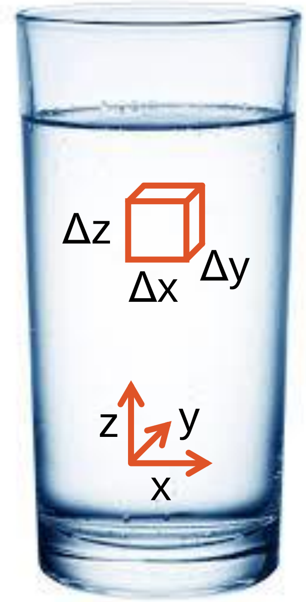
no pressure gradients in the x-direction

Force balances on static fluids

- Using the same logic in the **y-direction:**

$$F_{y1} = F_{y2} \Rightarrow \boxed{\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{y}} = \mathbf{0}}$$

no pressure gradients in the y-direction



- But the **z-direction** is more complicated due to gravity …
  - Quantify the downward force acting on the fluid in the box (includes weight!)

$$F_{\downarrow} = P_{z2}\Delta x\Delta y + \rho g\Delta x\Delta y\Delta z$$

  - Quantify the upwards force acting on the fluid in the box

$$F_{\uparrow} = P_{z1}\Delta x\Delta y$$

  - Since the fluid in the box is stationary, these forces must be equal

$$P_{z2}\Delta x\Delta y + \rho g\Delta x\Delta y\Delta z = P_{z1}\Delta x\Delta y$$

Force balances on static fluids

- Simplifying our force balance in the **z-direction**:

$$P_{z2}\Delta x\Delta y + \rho g\Delta x\Delta y\Delta z = P_{z1}\Delta x\Delta y$$

$$\Downarrow$$

$$P_{z2} + \rho g\Delta z = P_{z1}$$

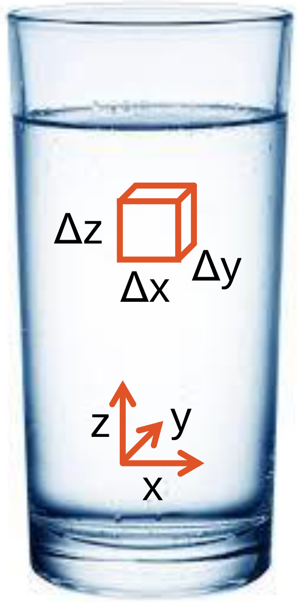
$$\Downarrow$$

$$\lim_{\Delta z\to 0}\left(\frac{P_{z2} - P_{z1}}{\Delta z}\right) = -\rho g$$

$$\Downarrow$$

$$\boxed{\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{z}} = -\boldsymbol{\rho g}}$$

there is a pressure gradient in the z-direction and it is equal to –ρg



Calculating pressure differences in static fluids

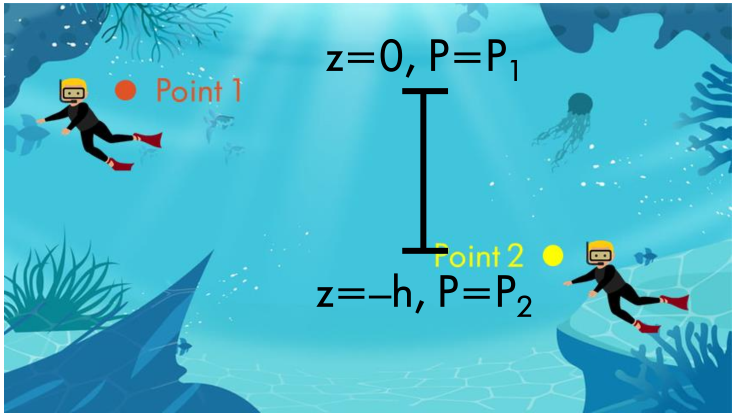
- Let's use these relationships to calculate the pressure difference experienced by our diver
  - No pressure differences occur when he moves in the x- or y-directions
- However, he does experience a pressure difference as he descends deeper into the water
  - We can quantify this by integrating the $\partial P/\partial z$ relationship we derived

$$\frac{dP}{dz} = -\rho g$$

$$\Rightarrow \int_{P=P_2}^{P_1} dP = -\rho g \int_{z=-h}^{0} dz$$

$$\Rightarrow P_1 - P_2 = -\rho g h$$

$$\boxed{\therefore \boldsymbol{\Delta P = \rho g h}}$$



pressure **increases** with depth;
this is referred to as **(hydro)static pressure**

Calculating pressure differences in static fluids

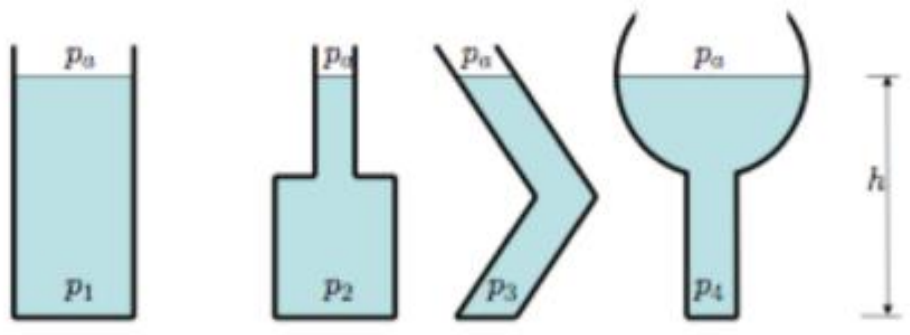
- How much does the pressure increase for each additional metre that the diver descends?

Challenge question 1.1

- What **absolute** pressure does the diver experience if he is 3 metres below the surface of the water?

The hydrostatic paradox

- We have four vessels of varying geometry that are all filled with water:



- Which pressure is highest: P_1, P_2, P_3, or $P_{4?}$

Do all fluids create the same amount of pressure?

- Three parameters are required to calculate pressure within a fluid:
  - Fluid height, h [m]
  - Gravitational acceleration, g [m/s^2]
  - Fluid density, ρ [kg/m^3]
- Therefore, if we have fluids with different densities, the amount of pressure they create will vary, for a given height of fluid

Challenge question 1.2

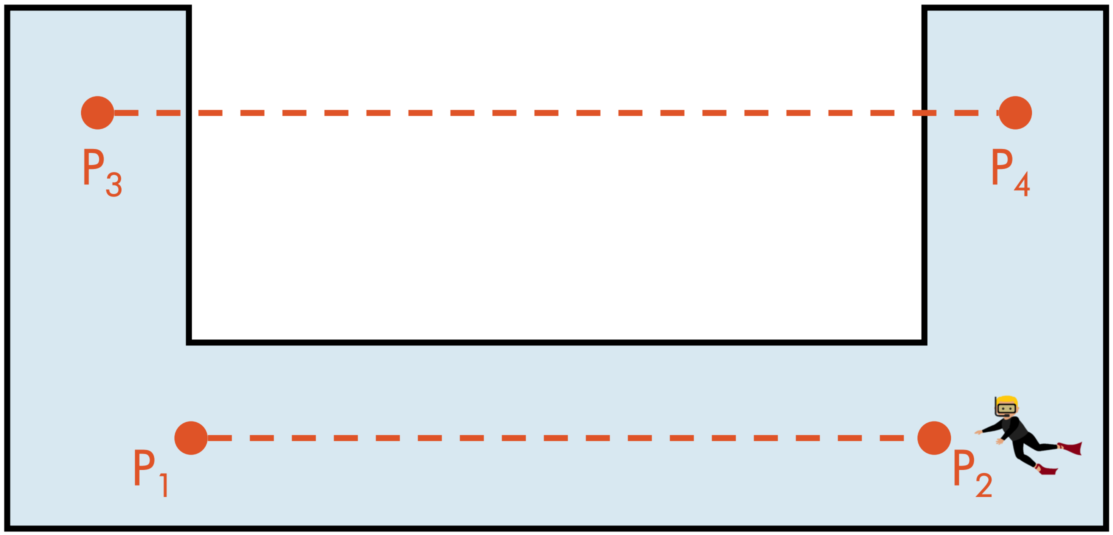
- Each metre of water produces 9.81 [kPa] of pressure (ρ_{water}=1000 [kg/m^3]). What height of mercury is needed to produce the same amount of pressure (ρ_{Hg} = 13594 [kg/m^3])?

Hydrostatic pressure head

- Often you'll hear engineers talking about pressure in terms of **head**
  - "We need three metres of **head** to pump this fluid."
- We will also use the term **head** a lot to describe changes in pressure – what this refers to is **hydrostatic pressure head**
  - "We need three metres of **hydrostatic pressure head** to pump this fluid."
- What this actually means is…
  - "We need the **equivalent amount of pressure** created by three metres of water."

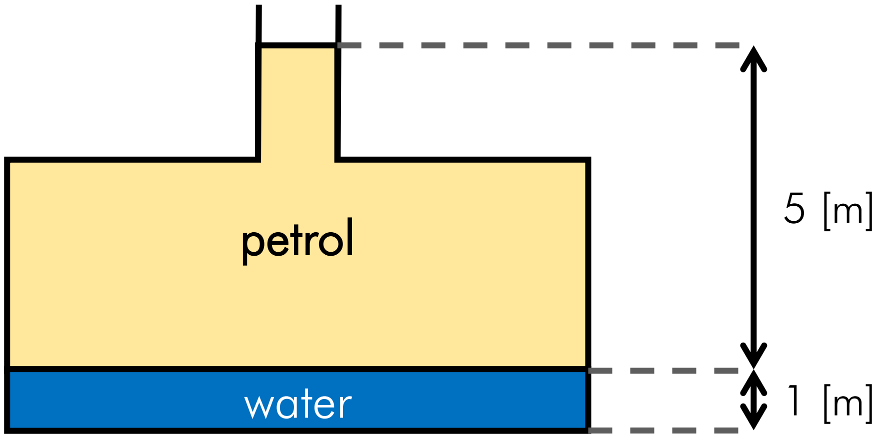
Static pressure in complex shapes

- Our diver is diving in a cave that contains static water. The cave has two exits to the atmosphere as shown in the figure.
  - What is the pressure at each cave entrance?
  - What is the relationship between P_1 and P_2?
  - What is the relationship between P_3 and P_4?



Example Problem 1.1

- Your company owns a buried petrol storage tank. Because of a leak, water has seeped into the tank as shown in the diagram below. Determine the gauge pressure at the petrol-water interface and at the bottom of the tank. The **specific gravity** of petrol is 0.68 and the density of water is 1000 [kg/m^3].



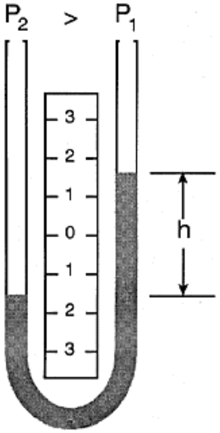
The road so far…

- Pressure gradients exist within stationary fluids
  - No gradient in the x-direction
  - No gradient in the y-direction
  - There is a gradient in the z-direction
- The pressure gradient in the z-direction is due to gravity acting on the volume of fluid above a given point
  - This is referred to as **hydrostatic pressure**
  - The pressure change from a reference point is proportional to h
  - The pressure change is also proportional to the density of the fluid
  - The change in pressure doesn’t depend on how much fluid is above you, only the height of that fluid

Applications of hydrostatic pressure?

- There are several useful applications of hydrostatics in engineering:
  - Forces on walls (dams/fluid storage vessels)
  - Buoyancy
  - Manometry
    - Manometry uses devices called **manometers** to measure pressure/pressure differences within a system
    - This is done by looking at differences in height of a column of fluid and then using the concept of hydrostatic pressure to calculate a pressure difference

Hoover Dam



U-tube manometer

Absolute and gauge pressure

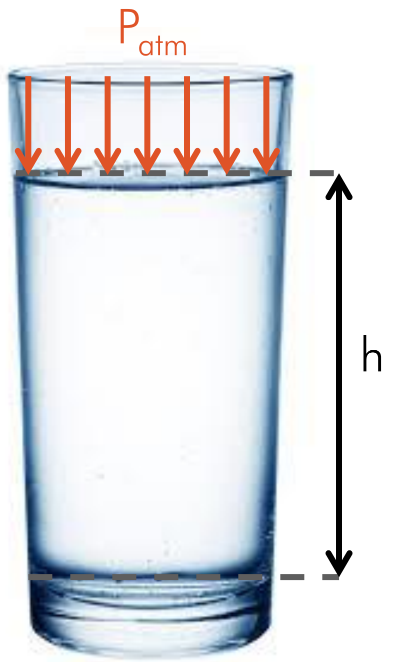
- Sometimes we will talk about pressure in terms of **absolute pressure** and at other times we will talk about pressure in terms of **gauge pressure**
  - **Gauge pressure** is the pressure **above** atmospheric pressure

$$P_g = \rho g h$$

  - **Absolute pressure** is the actual pressure, so it is the gauge pressure plus atmospheric pressure

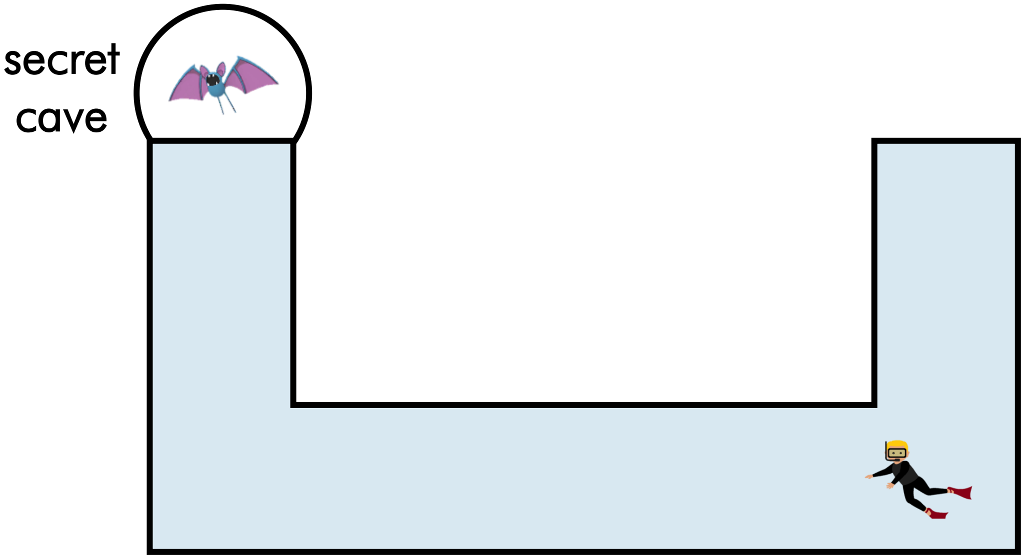
$$P_a = P_{atm} + \rho g h$$

(P_{atm}=101.3 [kPa])



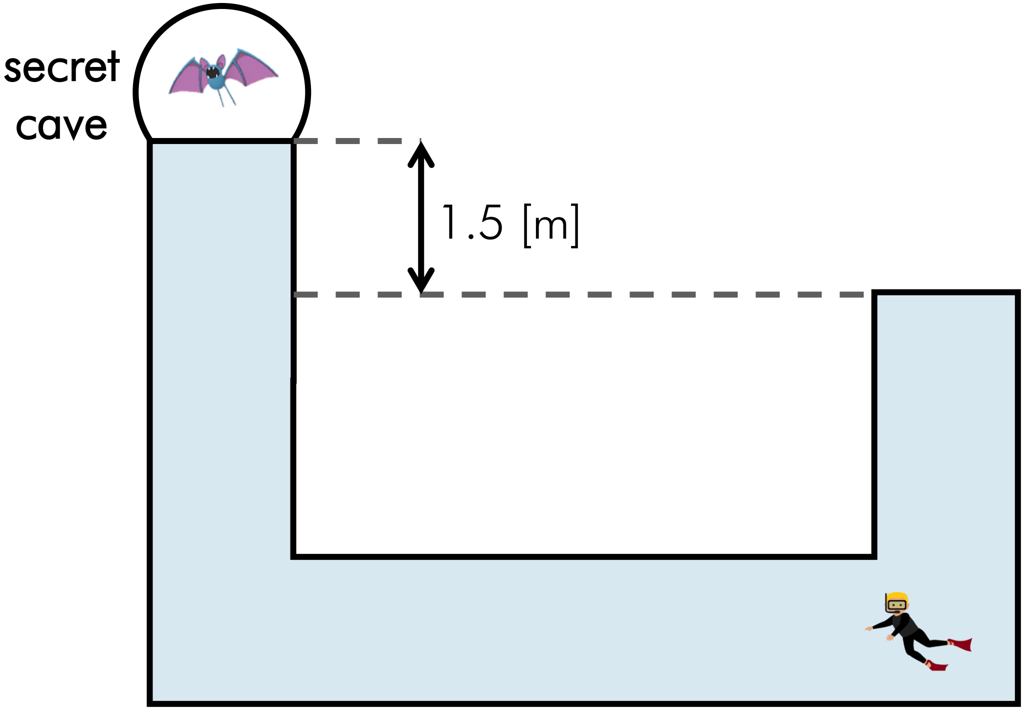
Challenge question 1.3

- Our diver is now exploring an underground passage filled with static water. One entrance to the passage is open to the atmosphere and the other leads to a fully enclosed secret cave.
  - What is the relationship between the pressure in the atmosphere and the pressure in the cave?
  - What is the pressure in the cave?



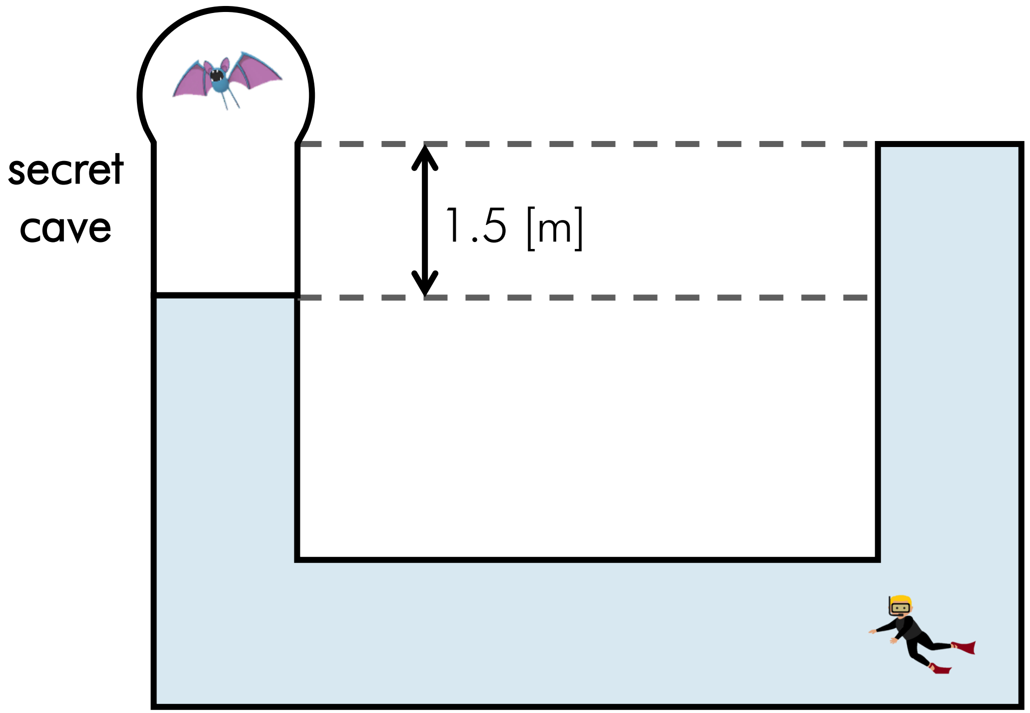
Challenge problem 1.4

- What is the pressure in the cave now?



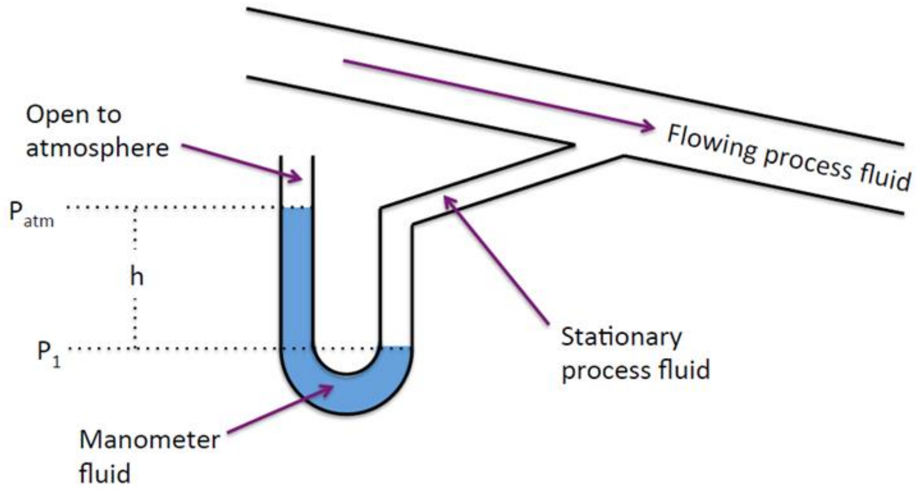
Challenge problem 1.5

- And how about now?



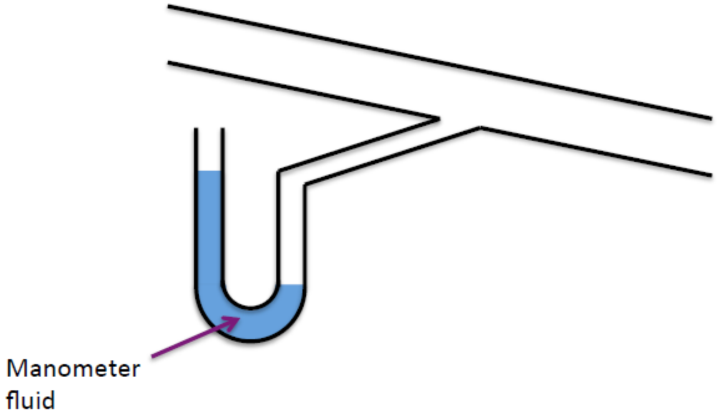
Manometry in engineering

- Most of this subject will concern itself with fluid flow through pipes
  - This flow is usually pressure-driven
  - Manometry provides a method of calculating the pressure (or pressure drop) experienced by a flowing fluid
  - In fact, manometers were one of the first pressure gauges!



Manometer fluid

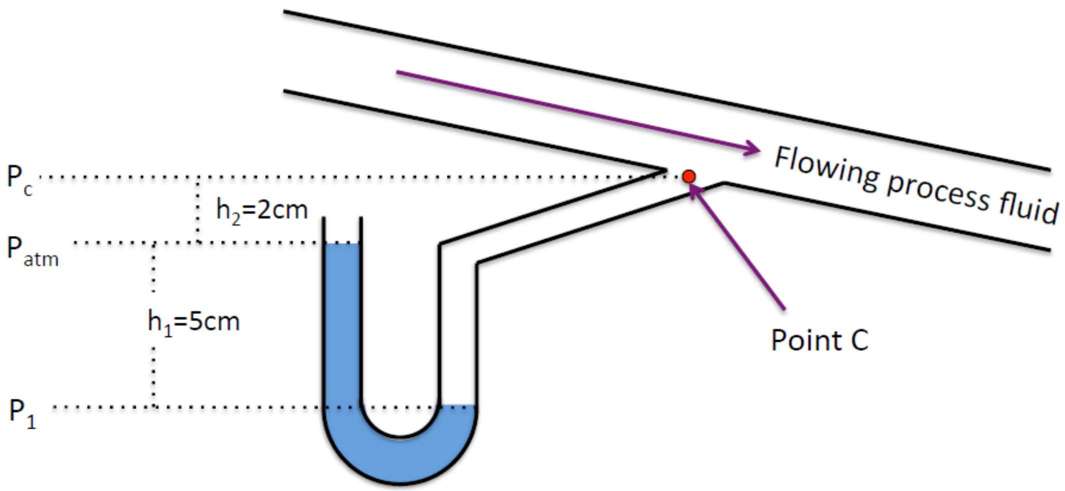
- What characteristics are necessary for a manometer fluid?
  - Immiscible with the process fluid
  - Denser than the process fluid
  - Incompressible (meaning density does not change with pressure)
- What is a common manometer fluid?
  - Mercury (Hg)
  - ρ_{Hg}=13,534 [kg/m^3]



Manometers to calculate absolute pressure

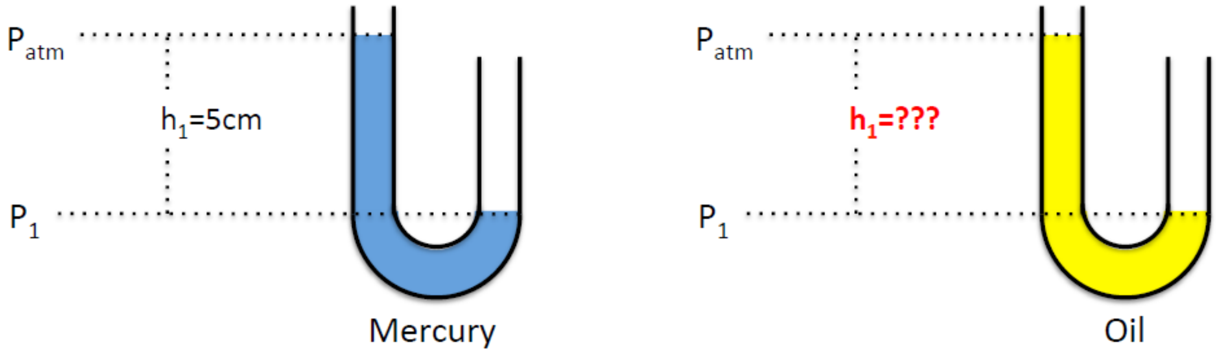
- You have water flowing through a pipe. The manometer fluid is mercury. What is the absolute pressure at **Point C**?
  - ρ_{Hg}=13,534 [kg/m^3]
  - ρ_{water}=1000 [kg/m^3]

Pa = 101.325kPa + 5*10^-2*9.81*1000kPa = P1
Pc = P1 - 7*10^-2*9.81*13534



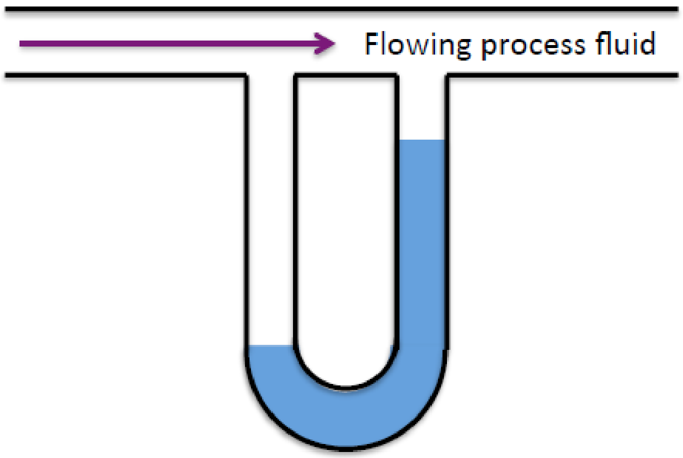
Challenge question 1.6

- What would the height of h_1 be if instead of mercury, you decided to use mineral oil as your manometer fluid (ρ_{oil}=800 [kg/m^3])?



Differential manometers

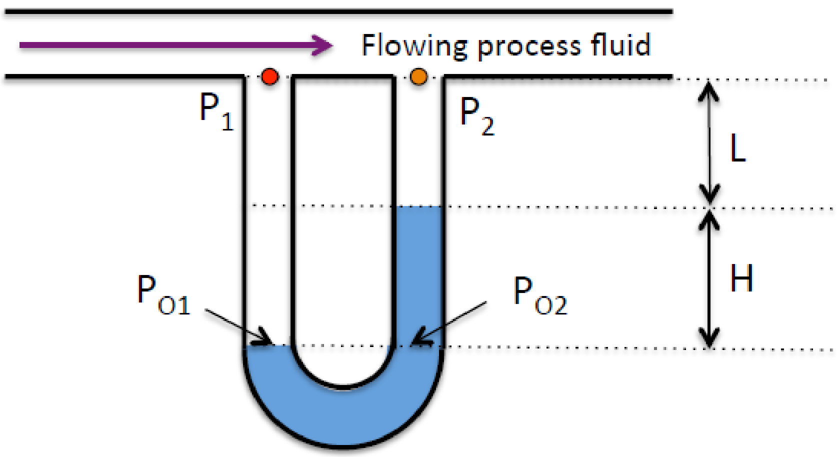
- In the previous examples, we were able to calculate an **absolute pressure** for our process fluid because the manometer was open to the atmosphere, providing us with a known reference pressure
- Sometimes we just want to measure the **pressure drop** along the length of a pipe, and for this we use a **differential manometer**



- Flow in a horizontal pipe is due to pressure
- Pressure decreases down the length of the pipe in the flow direction
- A differential manometer can be used to quantify that pressure drop down a length of pipe

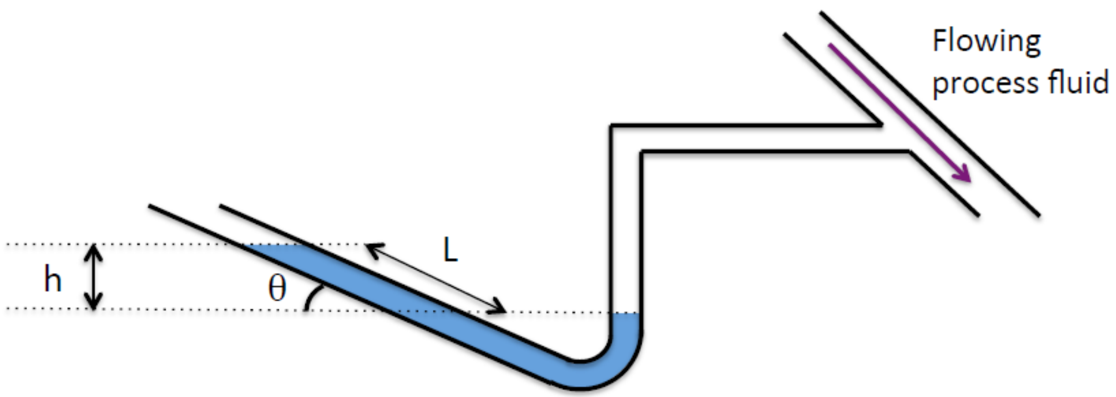
Differential manometers

- Find the pressure drop between **Point 1** and **Point 2**. The density of the process fluid is ρ_f and the density of the manometer fluid is ρ_m.



Inclined manometers

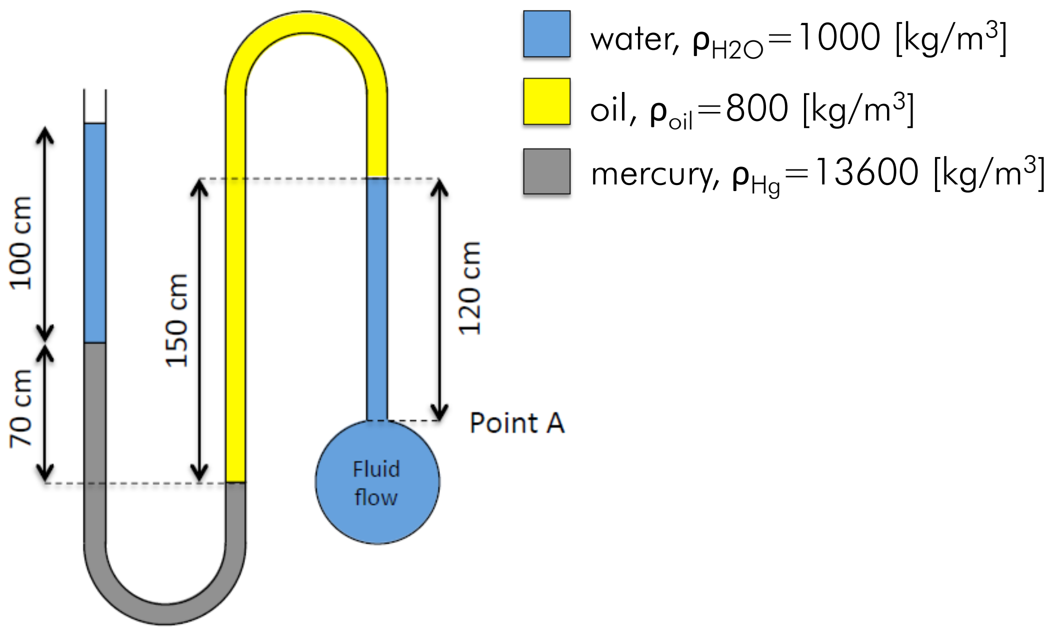
- For some systems, the pressure changes you are trying to measure are small, making it difficult to accurately measure the height of the fluid
- For small angles of θ, the change in length of the manometer fluid (L) will be large compared to the height (h), and this means getting an accurate measurement of L will be easier
  - You can then use trigonometry to solve for the change in height



$$\boxed{\begin{aligned} \frac{h}{L} &= \sin\theta \\ \therefore L &= \frac{h}{\sin\theta} \end{aligned}}$$

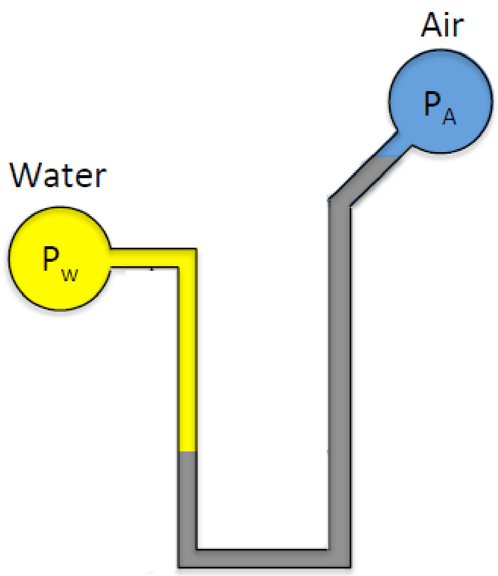
Example problem 1.2

- A fluid is flowing through a pipe in the direction perpendicular to the page. The flow is connected to a manometer that contains three fluids. The water interface is open to atmospheric pressure. Calculate the gauge pressure and absolute pressure in the pipe at **Point A**.



Example problem 1.3

- Derive a relationship between the difference in pressure between the tank that contains water and the tank that contains air ($P_W – P_A$). The manometer fluid is mercury. Assume the density of air is constant, and that the small amount of air in the manometer tubing is negligible. (This relationship will be in terms of variables, not numbers).



Summary

- The hydrostatic pressure of fluids has many engineering applications
  - Forces on walls (dams)
  - Buoyancy
  - Manometry

  to be covered later in the course
- Manometry uses the height of an immiscible fluid to determine pressure/pressure differences
  - The basic concept behind manometry is that the pressure at equal z-values in a continuous fluid is constant
  - Some manometers (those open to atmosphere or to a space with known pressure) can be used to determine the absolute pressure in a process stream
  - Differential manometers can only be used to determine pressure differences
  - Sometimes inclined manometers are used to make readings more accurate