Formula Sheet : Fluid Mechanics

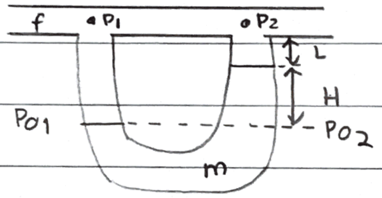
Hydrostatics

$\Delta P = \rho g h$

$P_2 - P_1 = \rho g h$ where point 2 is deeper

$P_{abs} = P_{atm} + P_{gauge}$

Manometer



$P_{o1} = P_1 + \rho_f g (L+H)$

$P_{o2} = P_2 + \rho_f g L + \rho_m g H$

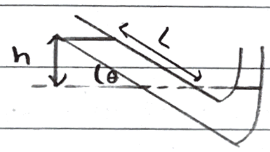
$\Big] \; P_{o1} = P_{o2}$

$P_1 - P_2 = (\rho_m - \rho_f) g H$

Inclined manometer

$P_{1a} = P_1 + \frac{\rho_w V^2}{2}$

$h = L \sin \theta$



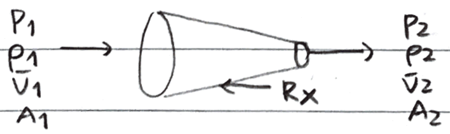
Conservation Laws for Fluid Flow

$G = \rho V A$ mass flowrate

$Q = VA$ volumetric flowrate

momentum $p = m \times V$

conservation



Rx : force exerted by duct on fluid

rate of change out in

$P_1 A_1 - P_2 A_2 - R_x = \rho_2 A_2 \bar{V}_2^2 - \rho_1 A_1 \bar{V}_1^2$

energy conservation

$\frac{\Delta P}{\rho} + \Delta\left(\frac{1}{2}\bar{V}^2\right) + g\Delta z + \Delta W_s + F = 0$ (J/kg)

Real Flow in Pipes

Friction factors

$$\phi = \frac{\tau_w}{\rho \bar{v}^2}$$

$$F = \frac{4\phi L \bar{v}^2}{D}$$

Fanning friction factor $f_F = 2\phi$

Darcy - Weisbach friction factor $f_D = 8\phi$

$$Re = \frac{\rho \bar{v} d}{\mu}$$

In laminar zone ($Re < 2000$) $f_F = \frac{16}{Re}$

friction factor plot [Re vs ϕRe^2 plot]

$$\phi Re^2 = \frac{F D^3 \rho^2}{4 L \mu^2}$$

→ use with $\frac{e}{D}$ to find Re and $\bar{v}$

Hydraulic mean diameter

$$D_e = \frac{4 \times \text{cross section}}{\text{wetted perimeter}}$$

1) annulus

$$D_e = D_{large} - D_{small}$$

2) rectangular duct

$$D_e = \frac{2 \times W \times H}{(W + H)}$$

Minor losses

$$F = \frac{2 f_F L \bar{v}^2}{D} + \frac{1}{2} \sum K \bar{v}^2$$

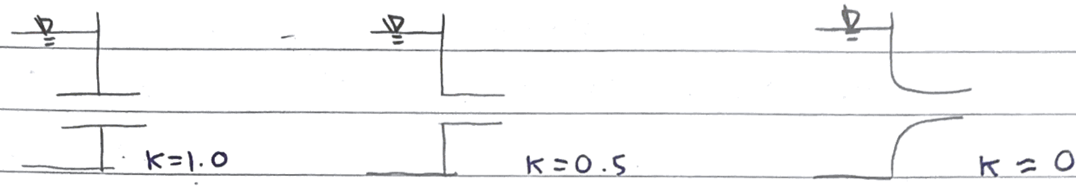
note for expansion and contraction: always use small pipe velocity

n different pipe diameter => n different F terms

$$K_{expansion} = \left[1 - \frac{A_s}{A_L}\right]^2 = \left[1 - \left(\frac{D_s}{D_L}\right)^2\right]^2$$

$$K_{contraction} = 0.5\left(1 - \frac{A_S}{A_L}\right) = 0.5\left(1 - \left(\frac{D_S}{D_L}\right)^2\right)$$

Fluid entering pipe from a tank



Branched Network

$$\frac{P_1}{\rho g} + \cancel{\frac{\bar{V}_1^2}{2\alpha g}}^{\,0} \text{ free surface} + z_1 = \frac{P_{1J}}{\rho g} + \frac{\bar{V}_{1J}^2}{2\alpha g} + z_J + \frac{F_1}{g}$$

$$h_1 \qquad\qquad h_J$$

$$h_1 - h_J = \frac{F_1}{g}$$

$$h_1 = h_J + \frac{F_1}{g} = h_J + \frac{2 f_{F,1} L_1 \bar{V}_1^2}{D_1 g}$$

rearranged

$$V_1 = \sqrt{D_1 g\left(\frac{h_1 - h_J}{2 f_1 L_1}\right)} \quad \ldots \text{ assume } h_J$$

Pumps

Efficiency

Mechanical

$$\frac{P_F}{P_B} = \frac{-W_s G}{P_B} = \frac{h_p g \rho Q}{P_B} \leftarrow \text{usually known}$$

Hydraulic

$$\eta = \frac{h_{actual}}{h_{theoretical}}$$

Pump head

$$h_p = h_{discharge} - h_{suction}$$

Theoretical pump head

$$h_p = \frac{\omega^2 r_2^2}{g} - \frac{Q\omega}{2\pi b g \tan\beta}$$

ω angular velocity in rad/s

r_2 outer radius of vanes

b width of vanes

β angle with outside radius

Net +ve suction head

$$NPSH_A = h_s - \frac{P_{vap}}{\rho g}$$

$$= \frac{P_1 - P_{vap}}{\rho g} + z_1 - h_{fs}$$

$$NPSH_A \geq NPSH_R$$

if used for fluid other than water

$$NPSH_{A,\,fluid} = NPSH_{A\,water} \times \frac{\rho_{water}}{\rho_{fluid}}$$

Compressible Flow

Ideal gas flow

$$P = \frac{\rho RT}{M} \Rightarrow Pv = \frac{RT}{M}$$

Pressure and density

Isothermal

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2}$$

Isoentropic

$$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\varphi} \quad \text{where } \varphi = \frac{C_p}{C_v} \qquad P_1 v_1^{\varphi} = P_2 v_2^{\varphi}$$

Velocity of wavefront

Isothermal

$$c = \sqrt{\frac{RT}{M}}$$

Isoentropic

$$c = \sqrt{\frac{\varphi RT}{M}}$$

Mechanical energy balance for compressible fluids

Isothermal

$$\frac{P_2^2 - P_1^2}{2 \times \frac{RT}{M}} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{4\phi}{D}\left(\frac{G}{A}\right)^2 L = 0$$

$$\left(\frac{G_{max}}{A}\right)^2 = \frac{P_w^2}{RT/M}$$

Choked flow

$$\frac{8\phi L}{D} = \left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1$$

Isoentropic

$$\frac{8\phi L}{D} = \left[\frac{\varphi-1}{2\varphi} + \frac{P_1}{v_1}\left(\frac{A}{G}\right)^2\right]\left(1-\left(\frac{v_1}{v_2}\right)^2\right) - \frac{\varphi+1}{\varphi}\ln\left(\frac{v_2}{v_1}\right)$$

Extra formulas

$$Re = \frac{GD}{A\mu}$$

$$G = \rho_1 V_1 A$$

$$\rho_1 = \frac{P_1}{RT/M}$$

Dimensional Reasoning

Forces

inertial

$F_I = m.a$

$m = \rho L^3$

$F_I \sim \rho L^3 u \frac{u}{L} = \rho L^2 u^2$

gravitational

$F_g = mg$

$F_g \sim \rho L^3 g$

viscous

$F_v = \mu \frac{du}{dz} \cdot A$

$F_v \sim \mu \frac{u}{L} L^2 = \mu u L$

surface tension

$F_s \sim \varphi L$

Dimensionless numbers

Reynolds

$Re = \frac{\text{inertial}}{\text{viscous}} = \frac{\rho u^2 L^2}{\mu u L} = \frac{\rho u L}{\mu}$

Bond

$Bo = \frac{\text{gravitational}}{\text{surface tension}} = \frac{\rho L^3 g}{\varphi L}$

Weber

$We = \frac{\text{inertial}}{\text{surface tension}} = \frac{\rho u^2 L^2}{\varphi L} = \frac{\rho u^2 L}{\varphi}$

Froude

$$Fr = \frac{\text{inertia}}{\text{gravitational}} = \frac{\rho u^2 L^2}{\rho L^3 g} = \frac{u^2}{gL} = \frac{u}{\sqrt{gL}}$$

OPEN CHANNEL FLOW

Uniform flow

$$\tau_b = \rho g \sin\theta \left(\frac{A}{\text{Perimeter}}\right)$$

Manning's Equation

$$\bar{V} = \frac{1}{n} R_h^{2/3} S^{1/2}$$

$$R_h = \frac{A}{P}$$

n is Manning's n

R_h is hydraulic radius

S is slope of flow

Hydraulic radius

rectangular

$$R_h = \frac{bh}{2h+b}$$

, wide channel where $b \gg h$, $R_h \rightarrow h$

unfilled circular channel

$$R_h = \frac{R}{2} / \frac{D}{4}$$

Specific energy

$$E = \frac{V^2}{2g} + h$$

$$= \frac{Q^2}{2gb^2h^2} + h$$

E_1 − bump height = E_2

Δh free surface = $h_1 - (h_2 + \Delta h)$

$\llcorner$ bump height

critical height

$$h = \left(\frac{Q^2}{gb^2}\right)^{1/3}$$

critical specific energy

$$E_{crit} = \frac{3}{2} h_{crit}$$

Hydraulic jump

$$Fr_1 = \frac{V_1}{\sqrt{gh_1}}$$ should be bigger than 1

$$\frac{h_2}{h_1} = \frac{-1 + \sqrt{8Fr_1 + 1}}{2}$$

$$h_L = \frac{(h_2 - h_1)^3}{4h_1h_2}$$ (difference in specific energy before and after hydraulic jump)

$E_{after} + h_L = E_{before}$

Rheology

Newton's Law of Viscosity

$$\tau_{yx} = -\mu \frac{dv_x}{dy}$$

Power law model

$$|\tau_{yx}| = k \left| \frac{dv_x}{dy} \right|^n$$

$$= \underbrace{k \left| \frac{dv_x}{dy} \right|^{n-1}}_{\mu_a \text{ - apparent viscosity}} \left| \frac{dv_x}{dy} \right|$$

Bingham plastic model

$$|\tau_{yx}| = \tau_y + \mu_p \left| \frac{dv_x}{dy} \right| \quad \text{when } |\tau_{yx}| > \tau_y$$

$$\left| \frac{dv_x}{dy} \right| = 0 \quad \text{when } |\tau_{yx}| < \tau_y$$

Generalised Bingham equation

$$|\tau_{yx}| = \tau_y + k_p \left| \frac{dv_x}{dy} \right|^n$$

Conservation Laws & Navier Stokes equation

conservation of mass

$$\iiint_V \partial\rho/\partial t \, dV + \iint_S \rho v \cdot \hat{n} \, dS = 0$$

conservation of momentum

$$\iiint_V \partial/\partial t (\rho v) \, dV + \iint \rho v v \cdot \hat{n} \, dS = \iiint_V \rho g \, dV - \iint_S P \cdot \hat{n} \, dS + \iint \tau \, dS + F_{ext}$$

NSE mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

laminar relationship between pipeflow and pressuredrop

$$\Delta P = \frac{8 \mu L Q}{\pi R^4}$$

Application of hydrostatics

Pivot

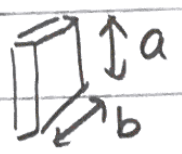
Σ moments $= 0$

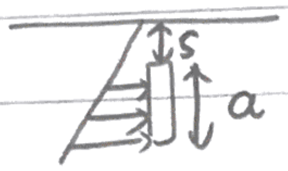
$= -F_{cw} L_{cw} + F_{ccw} L_{ccw}$

length taken from pivot

Hydrostatic forces of vertical surfaces

$|F| = a \cdot b \, \rho g (s + a/2)$





$$h_r = \frac{a}{2} + \frac{a^2/12}{s + a/2}$$

for protruding objects

$|F| = \rho g b h^2 / 2$

$$h_R = \frac{2h}{3}$$

$F = \rho_w A V^2$