Semester 2 Assignment 3, 2021

School of Mathematics and Statistics

MAST30022 Decision Making

Submission deadline: **4pm (Melbourne time), Friday 1 October**

This assignment consists of 13 pages (including this page)

Instructions to Students

- If you have a printer, print the assignment one-sided.

Writing

- There are 4 questions, of which 2 randomly chosen questions will be marked. Note you are expected to submit answers to all questions, otherwise **mark penalties will apply**.
- Working and reasoning **must** be given to obtain full credit. Give clear and concise explanations. Clarity, neatness, and style count.
- Write your answers in the boxes provided on the assignment that you have printed. If you need more space, you can use blank paper. Note this in the answer box, so the marker knows. The extra pages can be added to the end of the assignment to scan.
- If you have been unable to print the assignment write your answers on A4 paper. The first page should contain only your student number, the subject code and the subject name. Write on one side of each sheet only. Start each question on a new page and include the question number at the top of each page.

Scanning

- Put the pages in number order and the correct way up. Add any extra pages to the end. Use a scanning app to scan all pages to PDF. Scan directly from above. Crop pages to A4. Make sure that you upload the correct PDF file and that your PDF file is readable.

Submitting

- Go to the Gradescope window. Choose the Canvas assignment for this assignment. Submit your file. Get Gradescope confirmation on email.

Question 1

Define the two binary relations θ and $\bar{\theta}$ on $\mathbb{Z} \times \mathbb{Z}$ by

(a) $\theta = \{(\boldsymbol{a}, \boldsymbol{b}) : a_1 a_2 - b_1 b_2 \text{ is odd}\}$;

(b) $\bar{\theta} = \{(\boldsymbol{a}, \boldsymbol{b}) : a_1 \geq b_1 \text{ or } a_2 \geq b_2\}$.

(i) Verify for each of the two relations which of the following properties are satisfied: transitivity, reflexivity, comparability, symmetry, asymmetry, antisymmetry.

(ii) Which properties are gained, lost, or kept if "odd" is replaced by "even" in θ; and if "$a_1 \geq b_1$ or $a_2 \geq b_2$" is replaced by "$a_1 > b_1$ or $a_2 > b_2$" in $\bar{\theta}$?

Carefully explain your answers by providing proofs or counterexamples.

Answer (a)(i) in the box below.

Continue your answer to (a)(i) in the box below.

Answer (a)(ii) in the box below.

Answer (b)(i) in the box below.

Answer (b)(ii) in the box below.

Question 2

(a) Let θ be a binary relation on a set A which is not necessarily transitive.

Define a binary relation θ^* on A as follows: for any $a, b \in A$, $a\theta^* b$ if and only if there exists a sequence $a_1, a_2, \ldots, a_k \in A$, where $k \geq 2$ is an integer (which relies on a and b), such that $a = a_1, b = a_k$ and $a_i \theta a_{i+1}$ for all $i = 1, 2, \ldots, k-1$.

θ^* is called the *transitive closure* of θ.

Prove that θ^* is a transitive relation on A.

(b) Let (T, r) be a rooted tree. Denote the set of vertices of (T, r) by V. Let θ be the binary relation defined by

$$\theta \;=\; \{(a, b) \mid a, b \in V,\; a \text{ is a child of } b\}.$$

(i) Show that θ is not transitive.

(ii) Let θ^* be the transitive closure of θ and $a, b \in V$.

Describe what relation holds between a and b if $a\theta^* b$.

Question 3

Let $A = \{(1, 2, -1), (2, 1, -1), (-1, 2, 1), (2, -1, 1), (2, -1, -1), (2, -2, 1), (-1, 1, 1)\}$.

(a) For the lexicographic order on A, use the Boolean matrix representation to find the lexicographic-maximal and lexicographic-minimal element sets $L_{\max}(A)$ and $L_{\min}(A)$, and the lexicographic greatest and least elements (if any) of A.

(b) Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by

$$f(\boldsymbol{x}) = (x_1 + x_2, x_3)$$

for all $\boldsymbol{x} = (x_1, x_2, x_3) \in \mathbb{R}^3$. Denote the Pareto order on $\mathbb{R}^2$ by P. You may freely use the results from the lectures that P is reflexive, transitive, and antisymmetric. Define θ^P by

$$\theta^P = \{(\boldsymbol{a}, \boldsymbol{b}) | \boldsymbol{a}, \boldsymbol{b} \in A, f(\boldsymbol{a}) P f(\boldsymbol{b})\}.$$

(i) Determine whether θ^P satisfies the properties of reflexivity, transitivity, and anti-symmetry.

(ii) Determine all maximal/minimal elements and greatest/least elements in A with respect to θ^P.

Question 4

Consider the decision table (where x is a real number):

		States			
		θ_1	θ_2	θ_3	θ_4
Actions	a_1	16	1	7	16
	a_2	4	1	25	7
	a_3	4	4	10	4
	a_4	7	10	16	x

(a) Find which decision will be taken, as a function of x, according to

(i) Wald's maximin criterion;

(ii) Hurwicz's α-criterion (take $\alpha = \frac{1}{2}$);

(iii) Laplace's criterion;

(iv) Savage's minimax regret criterion.

(b) Find the range(s) of x for which all four criteria lead to the same choice.

(c) Use the decision table above to show that Hurwicz's α-criterion (with $\alpha = \frac{1}{2}$) does not satisfy the axiom of *independence of addition of a constant to a column.*

End of Assignment