

School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 6. Some answers

Question 1

The body floats if its mass equals the sum of the masses of the displaced fluids.

Question 2

The flow is not irrotational so we are not entitled to equate the Bernoulli head across different streamlines. So the assertion 'by Bernoulli' is wrong.

After integrating the Euler equations for the pressure (easy since we're given the velocity field) we get surfaces of constant pressure obeying

$$z = C + \frac{\Omega^2}{2g}(x^2 + y^2)$$

as we expect.

Question 3

Use the fact that $\nabla \times \mathbf{u} = 0$ and so $\mathbf{u} = \nabla \phi$ where ϕ is the velocity potential (*not* the azimuthal angle!!)

The choice of $F(t)$ can't matter since we could absorb it into ϕ but $\mathbf{u} = \nabla \phi$ wouldn't be affected.

Question 4

Just express the velocity components in terms of Ψ and impose the irrotationality condition.