School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 2.
Phase Portraits of Nonlinear Dynamical Systems

Question 1

For the following dynamical systems, calculate the fixed points and perform a linear stability analysis to determine the behaviour of local trajectories. Also locate the nullclines. Using this information, draw a sketch of the phase portrait.

(a) $\dot{x} = x - y$, $\dot{y} = 1 - e^x$
(b) $\dot{x} = x - x^3$, $\dot{y} = -y$
(c) $\dot{x} = x(x - y)$, $\dot{y} = y(2x - y)$
(d) $\dot{x} = y$, $\dot{y} = x(1 + y) - 1$
(e) $\dot{x} = x(2 - x - y)$, $\dot{y} = x - y$
(f) $\dot{x} = x^2 - y$, $\dot{y} = x - y$

Question 2

Consider the following dynamical system

$$\ddot{x} = x^3 - x$$

(a) Rewrite this second order system as a two-dimensional system.
(b) Find all the equilibrium points and classify their stability.
(c) Show that this system is conservative, and find a conserved quantity.
(d) Using the above information, sketch the phase portrait.

Question 3

Consider the following dynamical system

$$\ddot{x} = x - x^2$$

(a) Rewrite this second order system as a two-dimensional system.
(b) Find all the equilibrium points and classify their stability.
(c) Show that this system is conservative, and find a conserved quantity.
(d) Using the above information, sketch the phase portrait.
(e) Find an equation for the homoclinic orbit that separates closed and nonclosed trajectories.

Question 4

Consider the following dynamical system (Duffing equation)

$$\ddot{x} + x + \epsilon x^3 = 0$$

(a) Rewrite this second order system as a two dimensional system.
(b) For $\epsilon > 0$, show that the system has a single fixed point and it is a nonlinear centre. Sketch its phase portrait.
(c) For $\epsilon < 0$, show that trajectories near the origin are closed. Are trajectories far from the origin closed? Sketch the phase portrait.