TUTORIAL 2

1. Consider the following LP problem in standard form:

$$\text{Maximise} \quad z = 8x_1 + 6x_2$$

$$\begin{aligned} x_1 + x_2 &\leq 10 \\ 2x_1 + x_2 &\leq 15 \\ x_1, x_2 &\geq 0. \end{aligned}$$

   (a) Convert this problem into canonical form.

   (b) Find all basic solutions to this problem in canonical form by using the definition of a basic solution.

   (c) Which basic solutions that you find in (b) are basic feasible solutions?

   (d) Compare the basic feasible solutions obtained in (c) with the extreme points of the feasible region of the above problem in standard form. What conclusion can you reach from this comparison?

2. To prove the Fundamental Theorem of Linear Programming in lectures, we used the following algorithm:

Input: Coefficient matrix A, vector $\mathbf{b}$, feasible solution $\mathbf{x}$ to $A\mathbf{x} = \mathbf{b}$
Output: Basic feasible solution

Relabel the variables when necessary so that $\mathbf{x} = (\mathbf{x}^+, \mathbf{0})$ and $\mathbf{x}^+$ consists of the strictly positive components of $\mathbf{x}$. Partition $A = (A^+, A^0)$ such that the number of columns of A^+ is equal to that of $\mathbf{x}^+$.
while the columns of A^+ are linearly dependent **do**
    Choose a non-zero vector $\mathbf{w}^+$ with the same dimension as $\mathbf{x}^+$ such that $A^+\mathbf{w}^+ = \mathbf{0}$ (solve these equations to find $\mathbf{w}^+$). We may assume at least one coordinate of $\mathbf{w}^+$ is strictly positive (otherwise replace $\mathbf{w}^+$ by $-\mathbf{w}^+$).
    Let $\epsilon = \min\{\frac{x_j}{w_j} : w_j > 0\}$.
    Let $\mathbf{x} := (\mathbf{x}^+ - \epsilon\mathbf{w}^+, \mathbf{0})$.
    Relabel the variables when necessary so that $\mathbf{x} = (\mathbf{x}^+, \mathbf{0})$ and $\mathbf{x}^+$ consists of the strictly positive components of $\mathbf{x}$. Partition $A = (A^+, A^0)$ such that the number of columns of A^+ is equal to that of $\mathbf{x}^+$.
end while
Output $\mathbf{x}$ (as a basic feasible solution).

Consider the linear program in standard form:

$$\text{Maximise} \quad z = 4x_1 + 6x_2$$

$$\begin{aligned} x_1 + x_2 &\leq 40 \\ x_1 + 3x_2 &\leq 90 \\ 2x_1 + x_2 &\leq 70 \\ x_1, x_2 &\geq 0. \end{aligned}$$

(a) Transform this problem into canonical form.

(b) Now use the algorithm described above to find a basic feasible solution, starting with the feasible solution $(10, 20, 10, 20, 30)$.