Fluid Mechanics

Topic 3

Real flow in pipes

Fluid Mechanics

Topic 3.1

Friction factors

What we did during the last topic

In the previous topic, we applied conservation laws to liquid flow

- Conservation of mass → $G_1 = G_2$, $Q_1 = Q_2$, $A_1V_1 = A_2V_2$
- Conservation of momentum
- Conservation energy → Bernoulli equation

Although the Bernoulli equation is a nice way to think about fluid flow, it is an incomplete model

Beyond Bernoulli

Although the Bernoulli equation provides a nice way of thinking about flow, it is not correct

- We need to perform a kinetic energy correction factor
- We cannot neglect friction in the real world

When deriving Bernoulli, we averaged kinetic energy over the cross section by using the average velocity

- This is not exactly correct, nor does it accurately reflect experimental data
- We need to add a **kinetic energy correction factor**, α

$$\boxed{\frac{\Delta P}{\rho}+\Delta\left(\frac{\overline{V}^2}{2\alpha}\right)+g\Delta z=0}$$

$\alpha = 1$ for turbulent flow

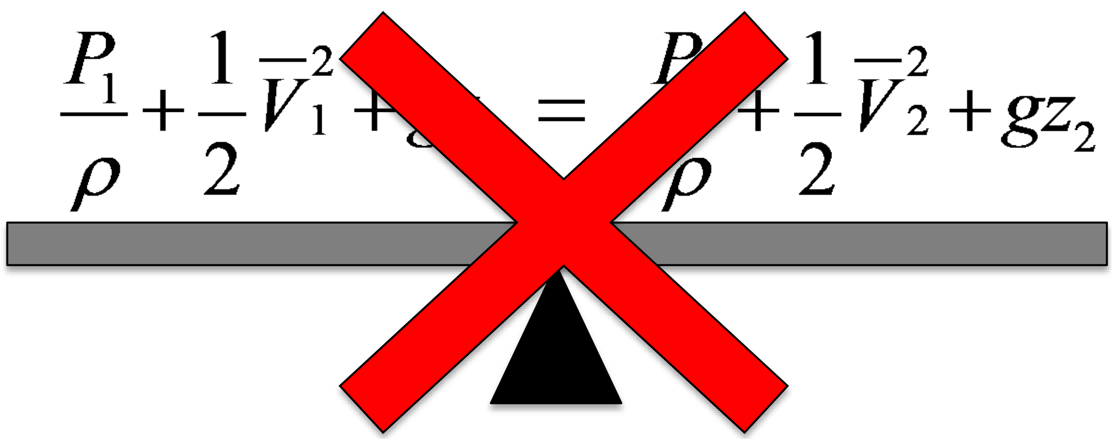
$\alpha = 0.5$ for fully developed laminar flow in circular pipes

Friction exists

According to Bernoulli, the mechanical energy of a fluid is constant down the length of your pipe —>That's one case in the real world that works!!

However, that is not true in real systems

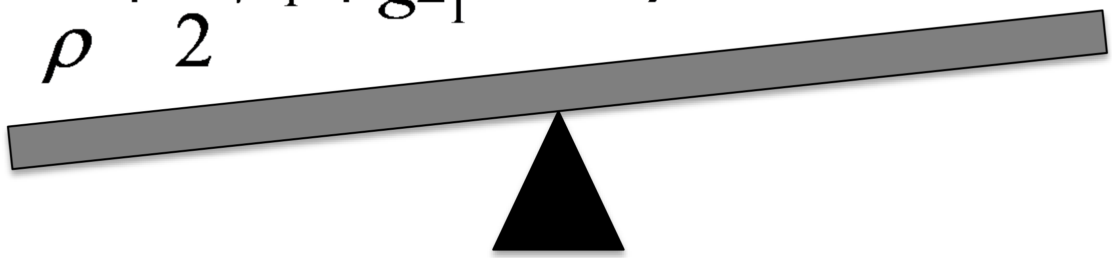
$$\frac{P_1}{\rho}+\frac{1}{2}\overline{V}_1^2+gz_1=\frac{P_2}{\rho}+\frac{1}{2}\overline{V}_2^2+gz_2$$



Friction exists

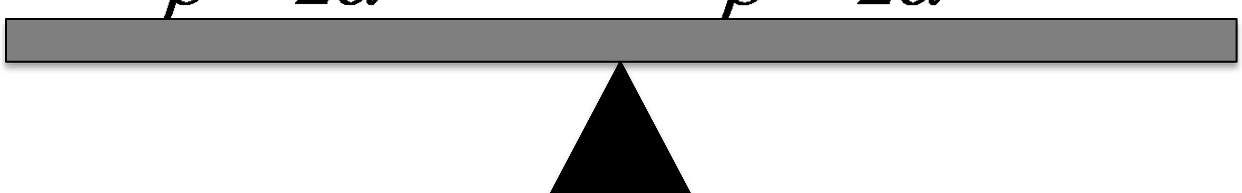
Instead, the fluid contained in the flow is greater at point 1 than point 2

$$\frac{P_1}{\rho}+\frac{1}{2}\overline{V}_1^2+gz_1 \neq \frac{P_2}{\rho}+\frac{1}{2}\overline{V}_2^2+gz_2$$



To balance energy, we must include energy that is lost due to friction

$$\frac{P_1}{\rho}+\frac{\overline{V}_1^2}{2\alpha}+gz_1 = \frac{P_2}{\rho}+\frac{\overline{V}_2^2}{2\alpha}+gz_2+F$$



The Mechanical Energy Balance

The Mechanical Energy Balance includes frictional losses

$$\frac{P_1}{\rho} + \frac{1}{2}\bar{V}_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}\bar{V}_2^2 + gz_2 + F$$

Bernoulli Equation
written in terms of energy

- Pressure energy
- Kinetic energy
- Potential energy
- Friction

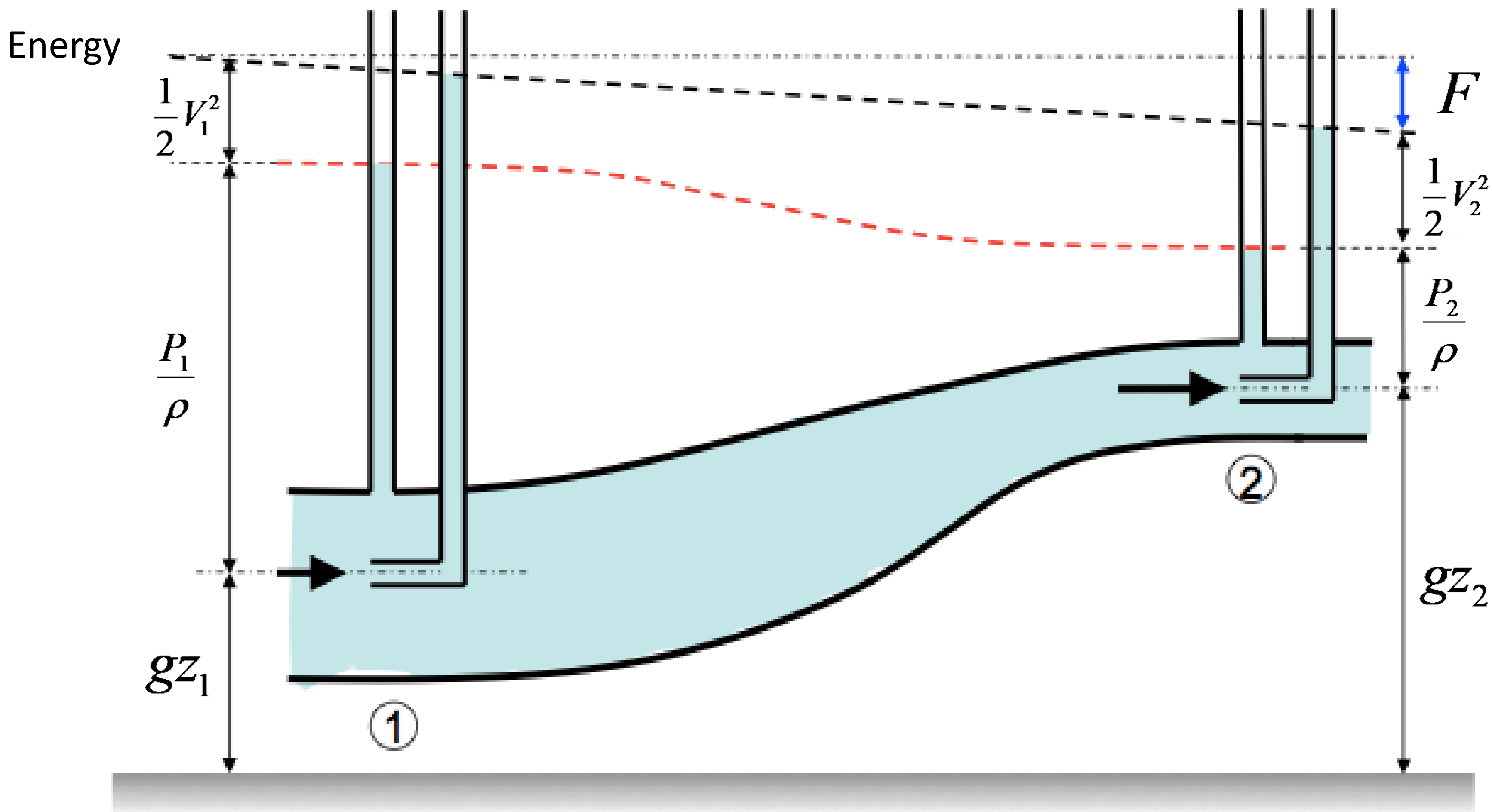
$$\frac{P_1}{\rho g} + \frac{1}{2g}\bar{V}_1^2 + z_1 = \frac{P_2}{\rho g} + \frac{1}{2g}\bar{V}_2^2 + z_2 + h_l$$

Bernoulli Equation
written in terms of head

- Pressure head
- Velocity head
- Gravitational head
- Head loss

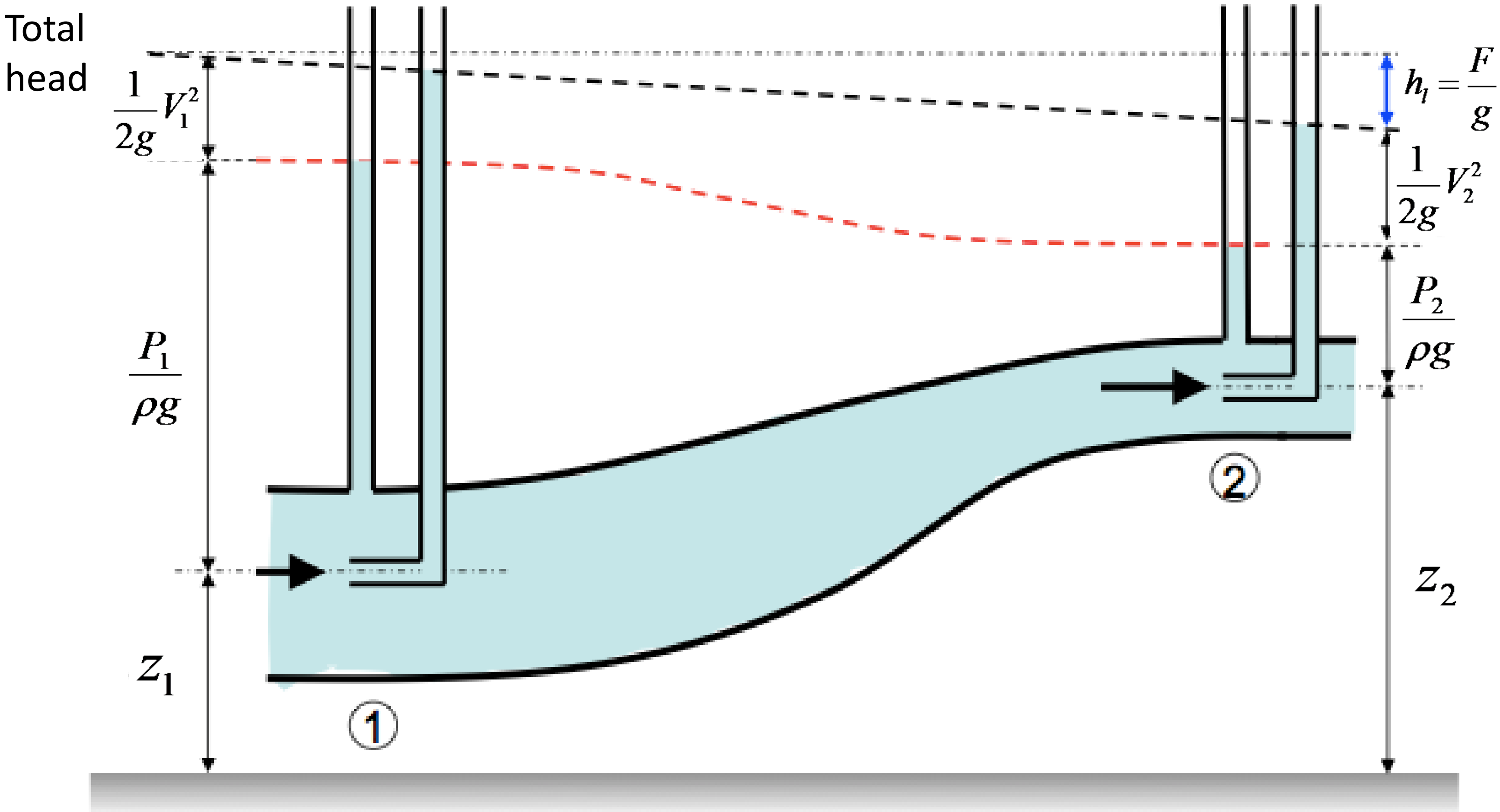
Friction exists

Schematically, we can illustrate the contribution of friction



Friction exists

We can write this in terms of head and head loss also



Calculating frictional losses

Friction occurs on the walls of the pipe as the fluid flows

This friction can be experimentally measured

$$\boxed{\left(\frac{P_2}{\rho}+\frac{\overline{V}_2^2}{2\alpha}+gz_2\right)-\left(\frac{P_1}{\rho}+\frac{\overline{V}_1^2}{2\alpha}+gz_1\right)+W_s+F=0}$$

For uniform pipe flow with constant cross section, $\overline{V}_1=\overline{V}_2$

$$\boxed{\left(\frac{P_2}{\rho}+gz_2\right)-\left(\frac{P_1}{\rho}+gz_1\right)+W_s+F=0}$$

For systems with no shaft work

$$\boxed{\left(\frac{P_2}{\rho}+gz_2\right)-\left(\frac{P_1}{\rho}+gz_1\right)+F=0}$$

Calculating frictional losses

We can rearrange this expression and solve for F

$$\boxed{F = \frac{P_1 - P_2}{\rho} + (z_1 - z_2)g}$$

Using the above expression, we could calculate the frictional losses that occur during flow by measuring the pressures and heights of the pipe

The units of F are **frictional losses per unit mass**

Although it's nice to be able to measure frictional losses within a piping system, what we really need is a way of **predicting frictional losses**

By predicting the amount of friction that will occur within a piping system, we can correctly size the pump to overcome friction and gravity before building the pipe network

Friction factors

In order to predict the value of friction that our piping system will experience, we use **friction factors**

The frictional term in the mechanical energy balance is commonly written in terms of friction factors

$$F = \frac{\text{work done against friction}}{\text{unit mass}}$$

$$F = \frac{\text{work done against friction / unit time}}{\text{mass / unit time}}$$

$$F = \frac{\text{force x distance / unit time}}{\text{mass / unit time}}$$

$$F = \frac{\tau_w 2\pi RL \times \bar{V}}{\rho \bar{V} \pi R^2}$$

τ_w *is the wall shear stress*

Friction factors

We can simplify this expression a little more

$$F = \frac{\tau_w 2\pi RL \times \overline{V}}{\rho \overline{V} \pi R^2}$$

Cancel out terms

$$F = \tau_w \frac{2L}{\rho R}$$

Let D = 2R

$$F = \tau_w \frac{4L}{\rho D}$$

Factor out a $\frac{1}{\overline{V}^2}$ *and rearrange*

$$F = \frac{\tau_w}{\rho \overline{V}^2} \times \frac{4L\overline{V}^2}{D}$$

****As long as I understand the following variables/notations!!

Friction factors

We can simplify this expression a little more

$$F = \frac{\tau_w}{\rho \overline{V}^2} \times \frac{4L\overline{V}^2}{D}$$

Define $\phi = \dfrac{\tau_w}{\rho \overline{V}^2}$

$$F = \frac{4\phi L \overline{V}^2}{D}$$

ϕ is a dimensionless parameter

F can now be plugged back into the mechanical energy balance

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\overline{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_s + \frac{4\phi L \overline{V}^2}{D} = 0$$

Includes/added work shift and Friction into the Bernoulli equation!!

So many friction factors

Depending on discipline, two different friction factors are used

- Fanning friction factor (f_F)
- Darcy-Weisbach friction factor (f_D)

$$\frac{\Delta P}{\rho}+\Delta\left(\frac{1}{2}\overline{V}^2\right)+g\Delta z+\Delta W_s+\frac{4\phi L\overline{V}^2}{D}=0$$

Fanning friction factor (f_F)

$$f_F = 2\phi$$

$$\frac{\Delta P}{\rho}+\Delta\left(\frac{\overline{V}^2}{2\alpha}\right)+g\Delta z+\Delta W_s+\frac{2f_F L\overline{V}^2}{D}=0$$

Darcy-Weisbach friction factor (f_D)

$$f_D = 8\phi$$

$$\frac{\Delta P}{\rho}+\Delta\left(\frac{\overline{V}^2}{2\alpha}\right)+g\Delta z+\Delta W_s+\frac{f_D L\overline{V}^2}{2D}=0$$

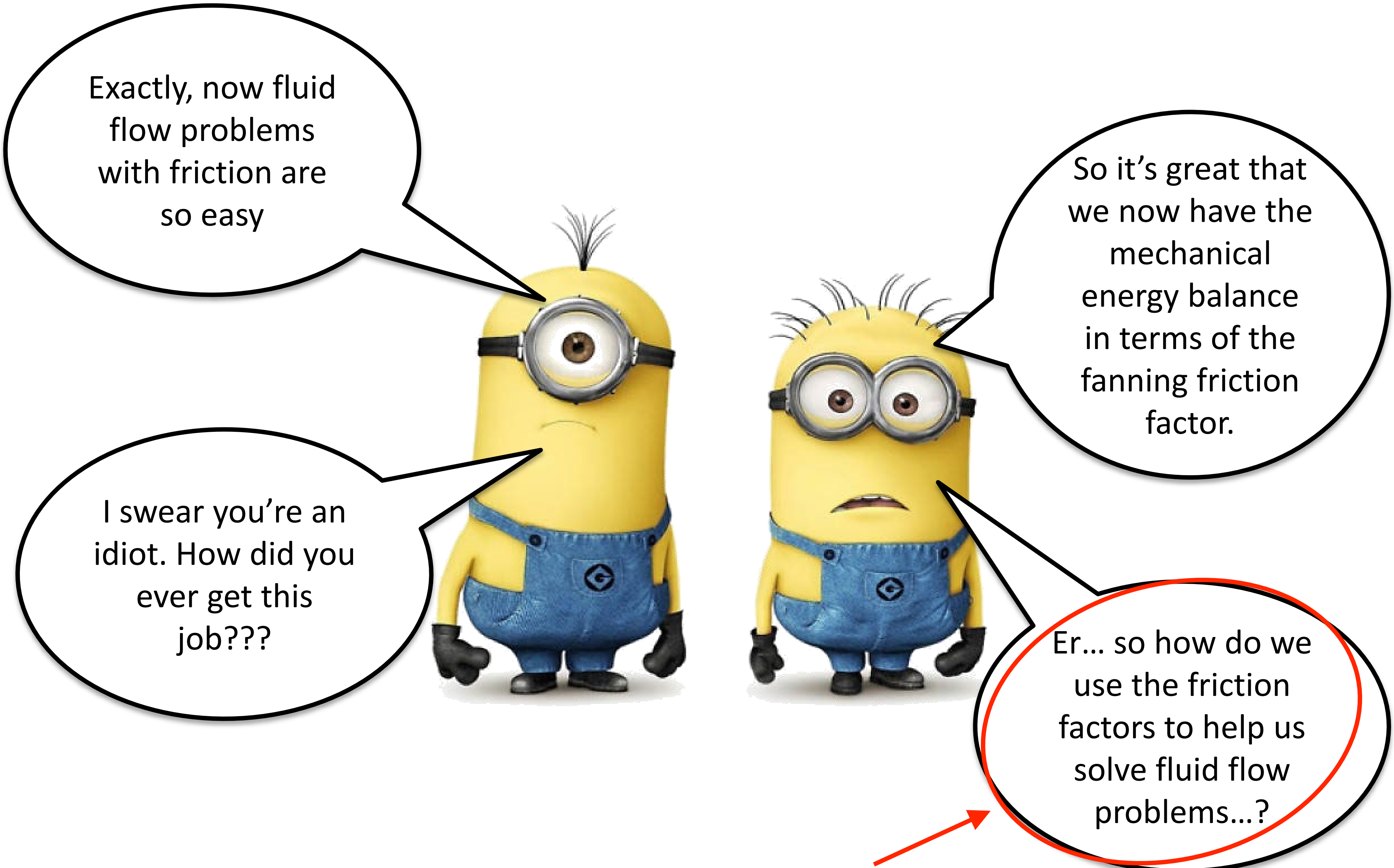
Fanning friction factor

Most chemical engineering textbooks use the Fanning friction factor, while most civil engineering textbooks use the Darcy friction factor

In this class (since I am a chemical engineer), we'll mostly use the **Fanning friction factor** (f_F)

Geez, so much for personal interest over engineers!!

$$\boxed{\frac{\Delta P}{\rho}+\Delta\left(\frac{\overline{V}^2}{2\alpha}\right)+g\Delta z+\Delta W_s+\frac{2f_F L\overline{V}^2}{D}=0}$$


Exactly, now fluid flow problems with friction are so easy
So it's great that we now have the mechanical energy balance in terms of the fanning friction factor.
I swear you're an idiot. How did you ever get this job???
Er… so how do we use the friction factors to help us solve fluid flow problems…?

Using friction factors

Remember earlier in the class, we said that the value for **friction** (**F**) could be calculated based on experimental data using the following equation

$$\boxed{F = \frac{P_1 - P_2}{\rho} + (z_1 - z_2) g}$$

Engineers have performed a huge number of experiments to see how the value of friction changes with different aspects of a flow system

1. Pipe diameter
2. Flow velocity
3. Fluid viscosity
4. Pipe roughness
5. Etc.

Using friction factors

To use the mechanical energy balances that contain friction factors, we need to know the value for friction factor

Through experimentation, engineers discovered that the **value of the friction factor** (f_F) depends on two main criteria of the fluid flow system

1. The **flow regime** of the fluid which we will quantify using the **Reynolds number**
   - Remember when we said that fluid flow could either be **laminar** or **turbulent**?
   - This is when that becomes important again because it significantly affects the value of the friction factor

2. The **relative roughness of the pipe** that the fluid is flowing through

Flow regime and Reynolds number

Previously we discussed how fluid flows in different "regimes"

- **Laminar** – defined stream lines
- **Transition** – oscillatory flow. Can behave laminar or turbulent
- **Turbulent** – no stream lines. Chaotic flow with eddies

Laminar flow

Transition flow

Turbulent flow

https://www.youtube.com/watch?v=XOLl2KeDiOg

Flow regime and Reynolds number

For pipe flow, each of these flow regimes can be quantified by a number called the **Reynolds number** (**Re**)

Laminar flow Re < 2000

Transition flow Re = 2000 – 3000

Turbulent flow Re > 3000

https://www.youtube.com/watch?v=XOLl2KeDiOg

What is this Reynolds number?

The value of the **Reynolds number** in pipe flow depends on several parameters of the flow system

- Density (ρ)
- Velocity (V)
- Pipe diameter (D)
- Viscosity (μ)

$$\boxed{\mathrm{Re} = \frac{\rho \bar{V} D}{\mu}}$$

The numerator is in terms of velocity, a parameter related to the **inertial forces** of the system

The denominator is in terms of viscosity, a parameter related to the **viscous forces** within the system

What is this Reynolds number?

The Reynolds number can be thought of as the ratio of inertial forces within the flow compared to the viscous forces within the flow

$$\boxed{\mathrm{Re} = \frac{\rho \overline{V} D}{\mu} \approx \frac{Forces_{Inertial}}{Forces_{Viscous}}}$$

At high Reynolds number

- Inertial forces are dominant
- Physically, this results in chaotic flow

At low Reynolds number

- Viscous forces are dominant
- This results in smooth/laminar flow

Relative roughness

The second quantity that controls the value of the friction factor is **relative surface roughness**

- A lot of friction arises when fluid flows over the walls of the pipe
- It turns out that the material that the pipe is made of is not a big contributor to friction
- What does greatly affect friction is the roughness of that surface

The relative surface roughness is quantified with the following expression

$$\boxed{\textit{Relative surface roughness} = \frac{e}{D}}$$

e = height of roughness element

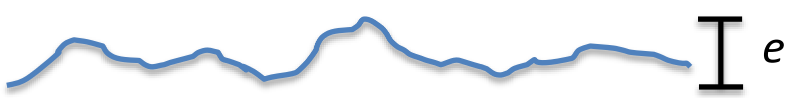
D = pipe diameter

Relative roughness

On a macroscopic level, a pipe may look quite smooth

But if you zoom in, the surface will have topography

- The increase in surface area can increase the frictional losses of energy
- The parameter *'e'* is an average height of the roughness



The values of *'e'* must be experimentally determined

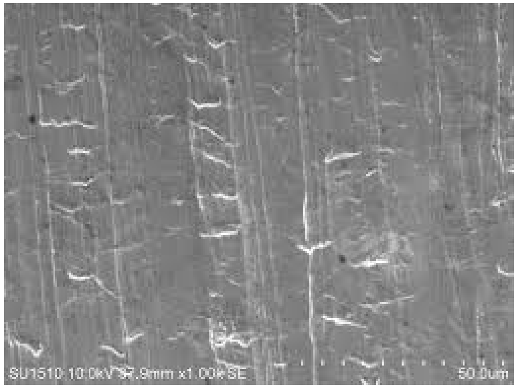
The value of *'e'* for a commercially purchased pipe will be told to you by the supplier

Relative roughness

Here are some values of *'e'* for a variety of types of pipes

Understand and come back to this table **I THINK IT'S VERY IMPORTANT TO KNOW LATER ON IN THIS MODULE!

Pipe material	Roughness, e (mm)
Riveted steel	0.9 - 9
Concrete	0.3 - 3
Wood stave	0.2 – 0.9
Cast iron	0.26
Galvanized iron	0.15
Asphated cast iron	0.12
Commercial steal/wrought iron	0.046
Drawn tubing	0.0015



Scanning electron microscope image of brass surface

Relative roughness

Bello!!
I want to know why it's e/D?

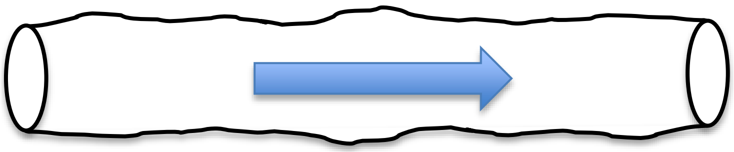
Why is pipe diameter important?

I'll tell you if you stop standing on me!

Relative roughness

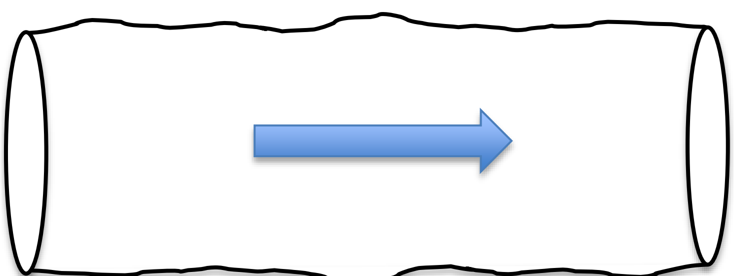
In a pipe with a smaller diameter

- A larger percentage of the fluid is in contact with the walls
- A larger portion of the fluid is experiencing wall friction



In a pipe with the same roughness but a larger diameter

- A smaller percentage of the fluid contacts the pipe walls
- A smaller portion of the fluid is experiencing wall friction



This is why the relative roughness is presented as *e/D*

The Moody diagram

We've now defined Reynolds number and relative roughness

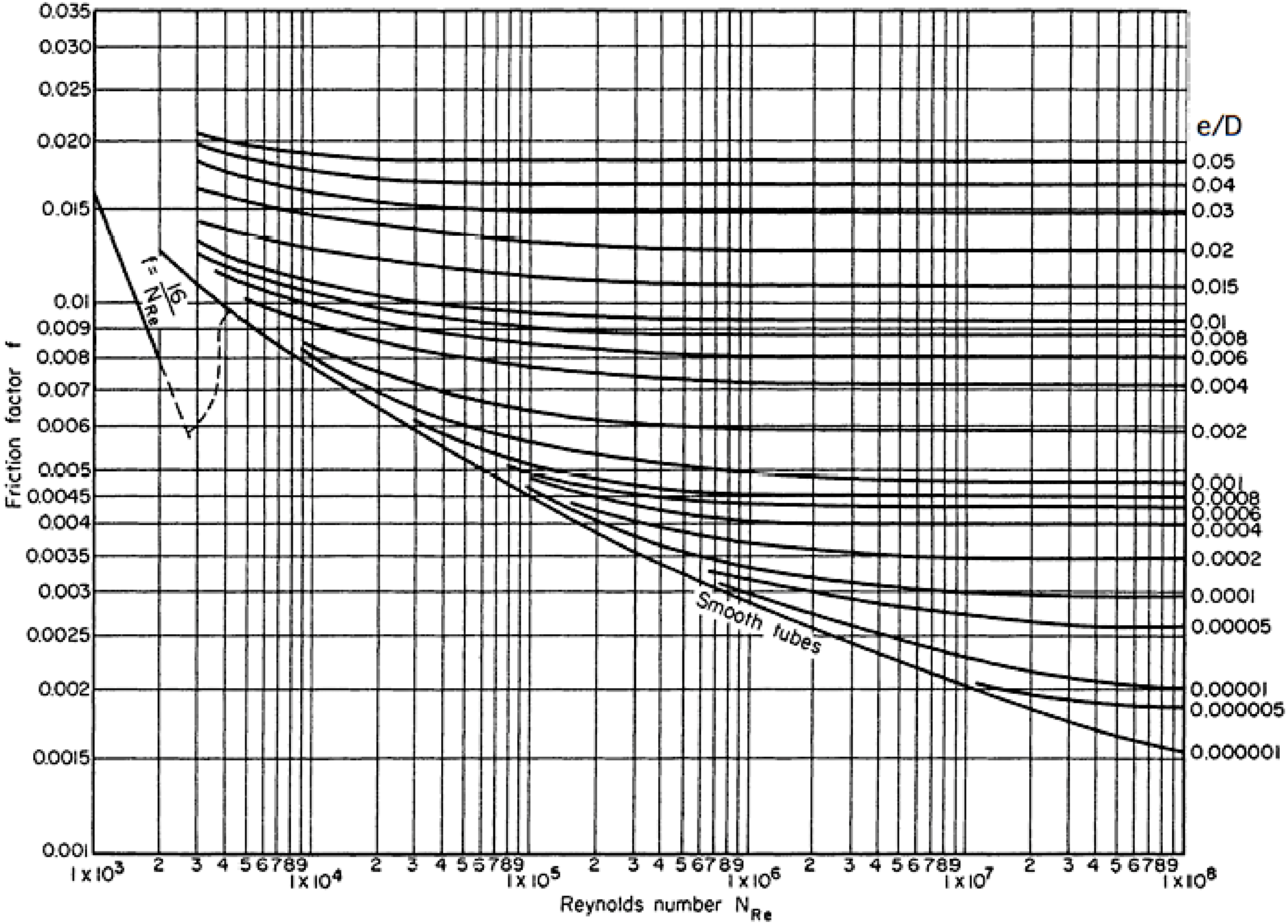
What do these parameters have to do with determining the value of the Fanning friction factor?

Engineers have completed experiments in order to generate plots that relate the **Fanning friction factor** to the **Reynolds number** of the flow and the **relative roughness** of the pipe

These plots are referred to as **Moody diagrams**

The Moody diagram

Seen this before in ENGR10004!!



The Moody diagram

The Moody diagram

- is a log-log plot
- has two y-axes and one x-axis
- the x-axis is the Reynolds number of the flow system
- the y-axis on the right is the relative roughness of the pipe '*e/D*'
- The y-axis on the left hand side of the system is Fanning friction factor

Each line on the plot corresponds to a certain relative roughness

There is a line for "smooth pipe" that assumes no surface roughness

Most of the plot is for turbulent value of flow, representing that this is the realm we often operate in

The Moody diagram

The Moody diagram allows you to determine the friction factor

In order to use the Moody diagram you must

1. Calculate the Reynolds number for your flow
2. Calculate the relative roughness of your pipe
3. Identify where those two points cross on the Moody diagram
4. Read the value on the left axis to determine your friction factor

Note: this version of the Moody diagram provides you with the **Fanning friction factor**

There are other versions of the diagram that provide you with the Darcy friction factor

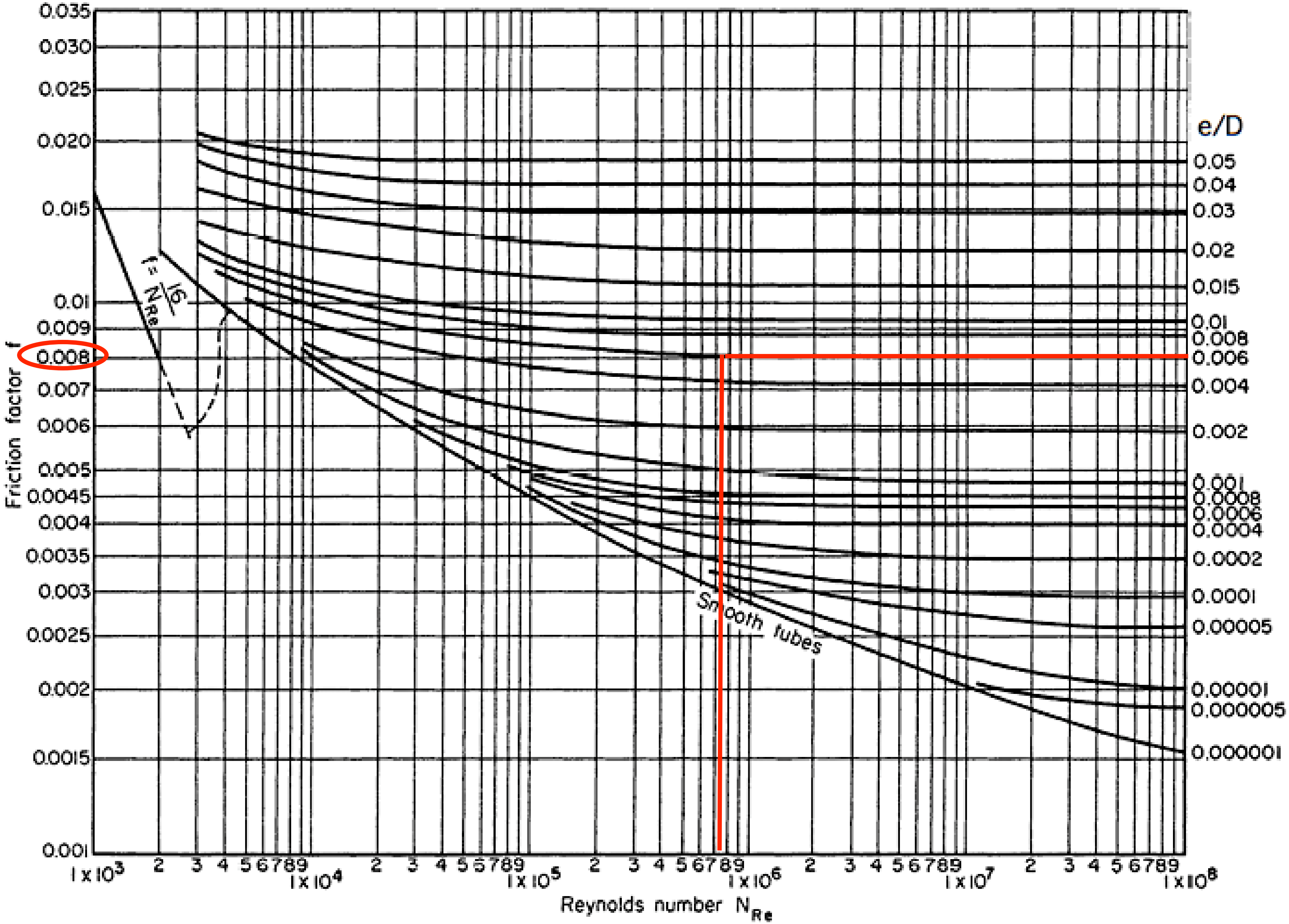
Example problem 3.1

You are pumping sea water from the ocean to a desalination plant. The fluid is flowing with an average velocity of 3 m/s through a PVC pipe with a diameter of 250 mm.

Determine the Fanning friction factor of this system.

What is the Viscosity? and relative roughness!

Example problem 3.1



Example problem 3.2

We have a 10 meter straight pipe through which 100 liters per minute of water is being pumped 1 meter uphill. The diameter of the pipe is 10cm, the roughness of the pipe is 0.001 cm, the density of the water is 1000 kilograms per cubic meter, the viscosity is 0.001 Pa s. The pressure at the exit of the pipe is P_{atm}.

What is the gauge pressure at the entrance of the pipe?

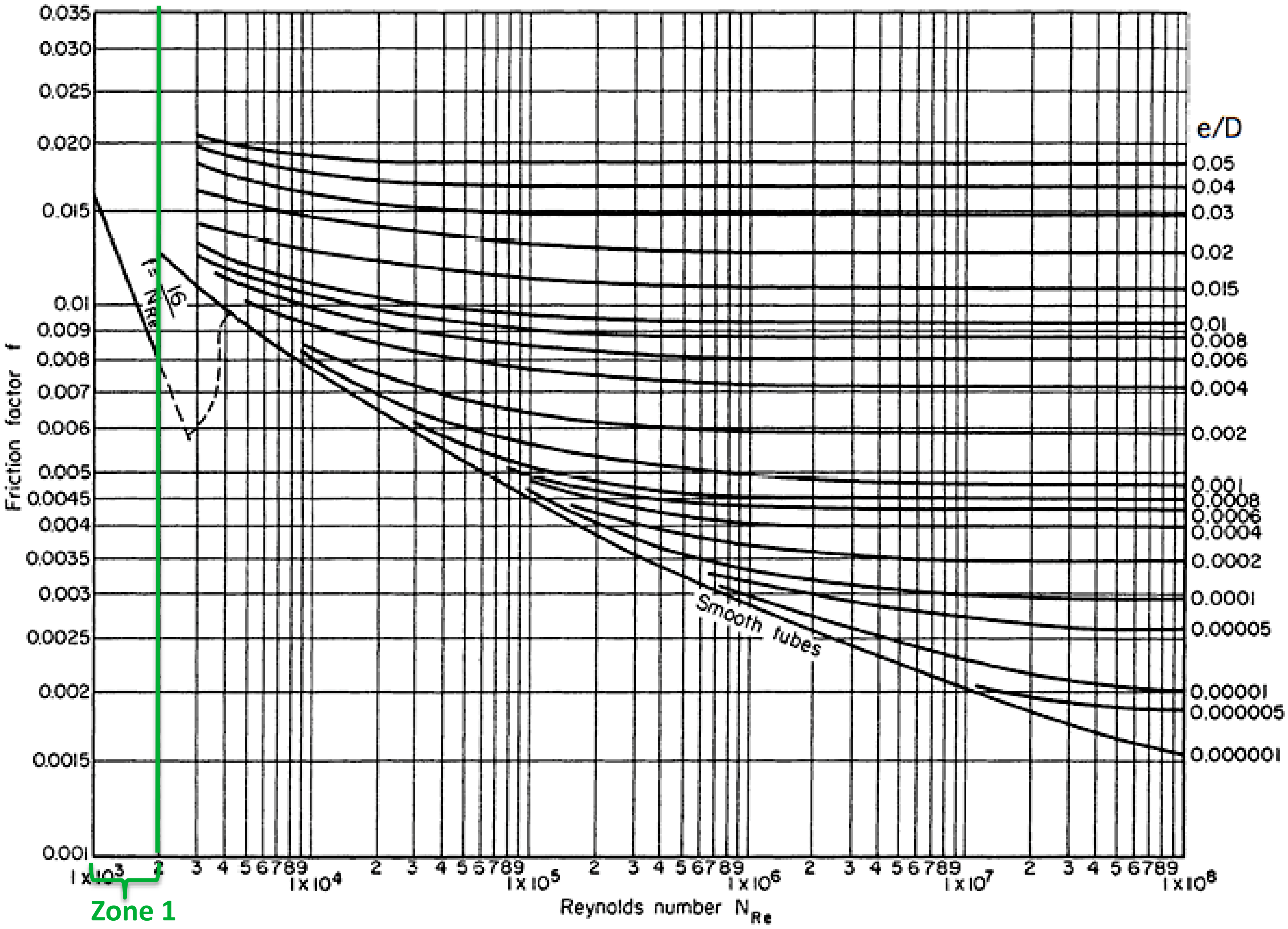
The Moody diagram

If we look more closely at the Moody diagram, we can learn some interesting things about fluid flow in pipes

Specifically, the can essentially be divided into four zones

- **Zone 1** – the laminar zone ($Re < 2000$)
- **Zone 2** – the transition zone ($2000 < Re < 3000$)
- **Zone 3** – low turbulence zone
- **Zone 4** – high Reynolds number zone

The Moody diagram – Zone 1



The Moody diagram – Zone 1

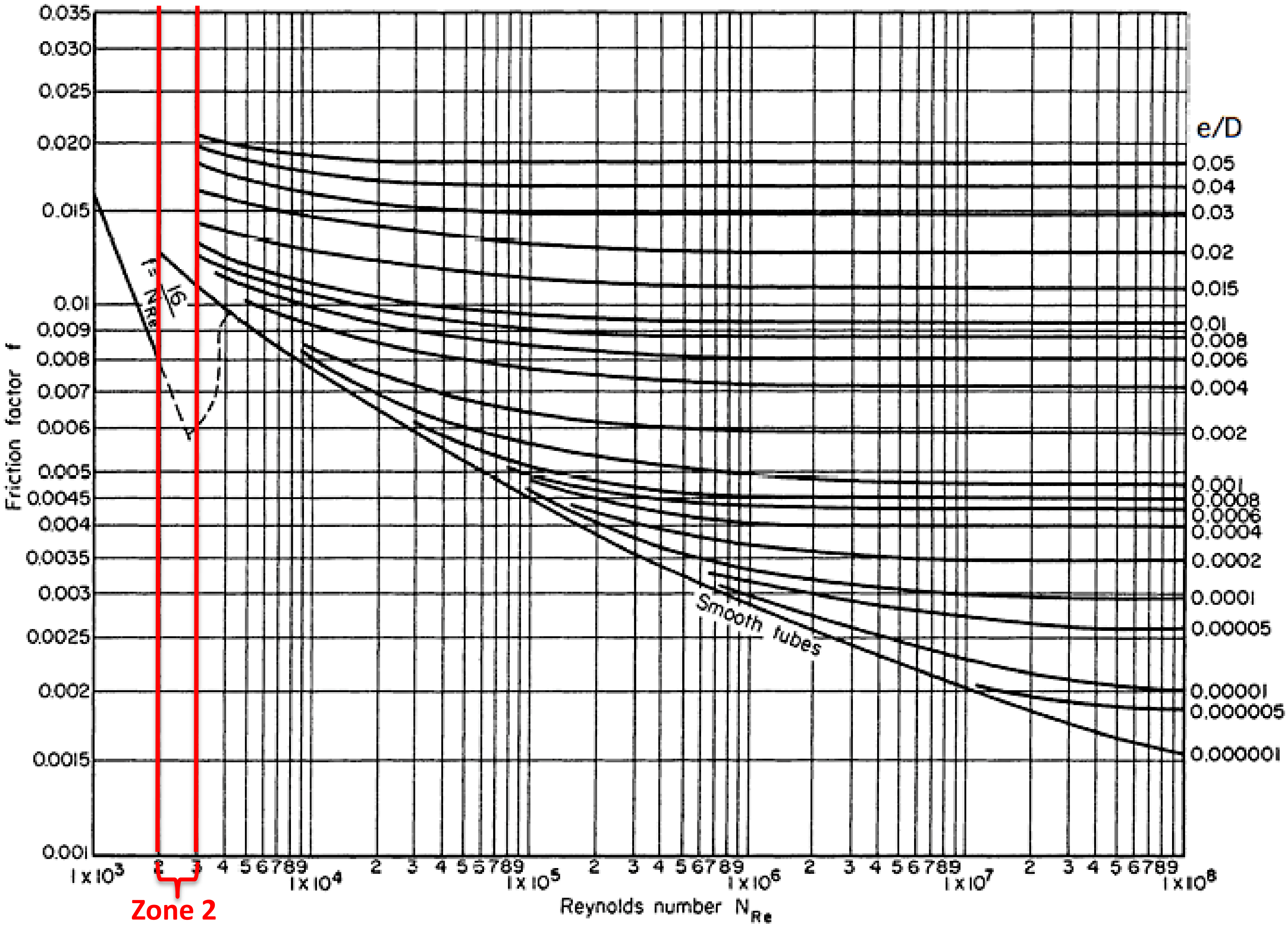
Zone 1 is laminar flow (Re < 2000)

On the Moody diagram, there is only one curve in Zone 1

- This one curve changes linearly with Reynolds number
- What this tells us is that **friction in zone 1 is independent of surface roughness**, it is only a function of the Reynolds number

We can calculate (but we won't) that in this region $\boxed{f_F = \frac{16}{\mathrm{Re}}}$

The Moody diagram – Zone 2



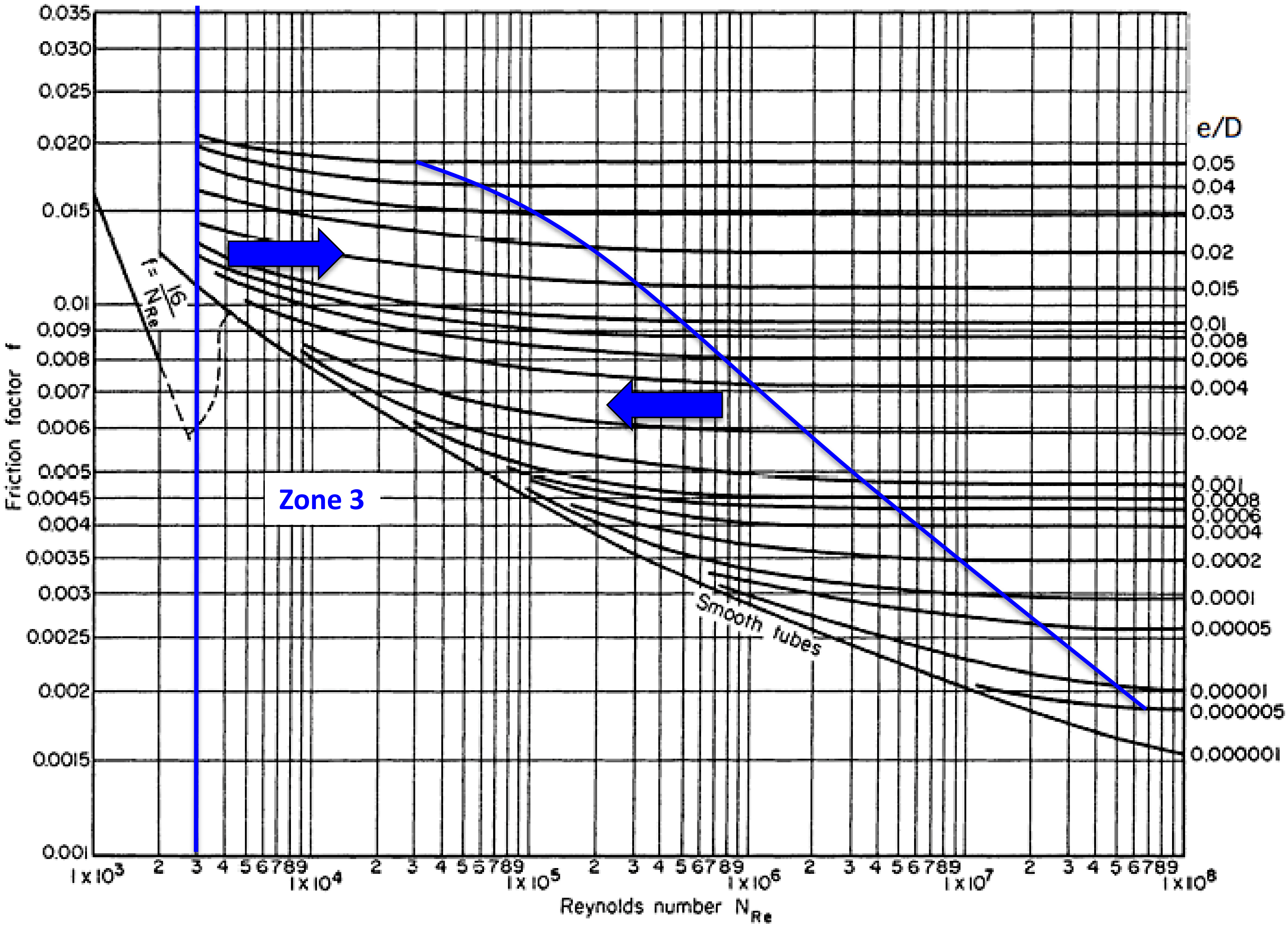
The Moody diagram – Zone 2

Zone 2 is the transition zone (2000 < Re < 3000)

On the Moody diagram, there are two curve in Zone 2

- The first curve is a continuation of the laminar flow line
- The second curve is the beginning of the turbulent flow line
- In the transition zone, flow can either behave laminar or turbulent, it will actually switch between the two
- This makes experimentally measuring reproducible values of the friction factor very difficult

The Moody diagram – Zone 3



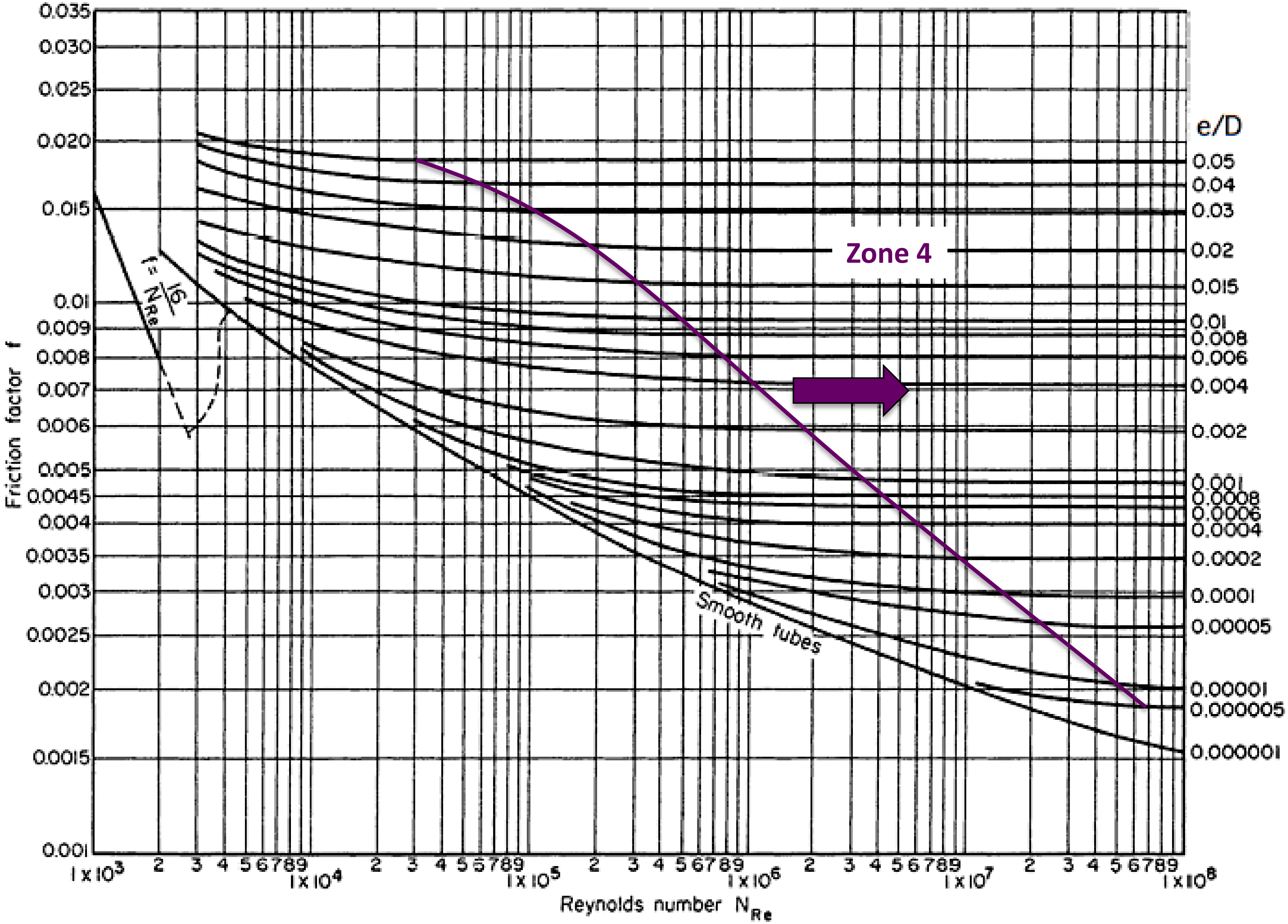
The Moody diagram – Zone 3

Zone 3 is the low turbulent flow regime

On the Moody diagram, there are many curves in zone 3

- Each curve represents a pipe with a certain relative roughness
- You will notice that in zone 3, these lines are curved
- In this region the frictional losses in the system are strong functions of both the Reynolds number and the pipe roughness

The Moody diagram – Zone 4



The Moody diagram – Zone 4

Zone 4 is high Reynolds number flow

On the Moody diagram, there are many curves in zone 4

- Each curve represents a pipe with a certain relative roughness
- You will notice that in zone 4, these lines are essentially horizontal
- This means that the friction factor (and thus the friction) **no longer change with the Reynolds number of the flow**
- **Surface roughness is the main parameter that determines the frictional losses**

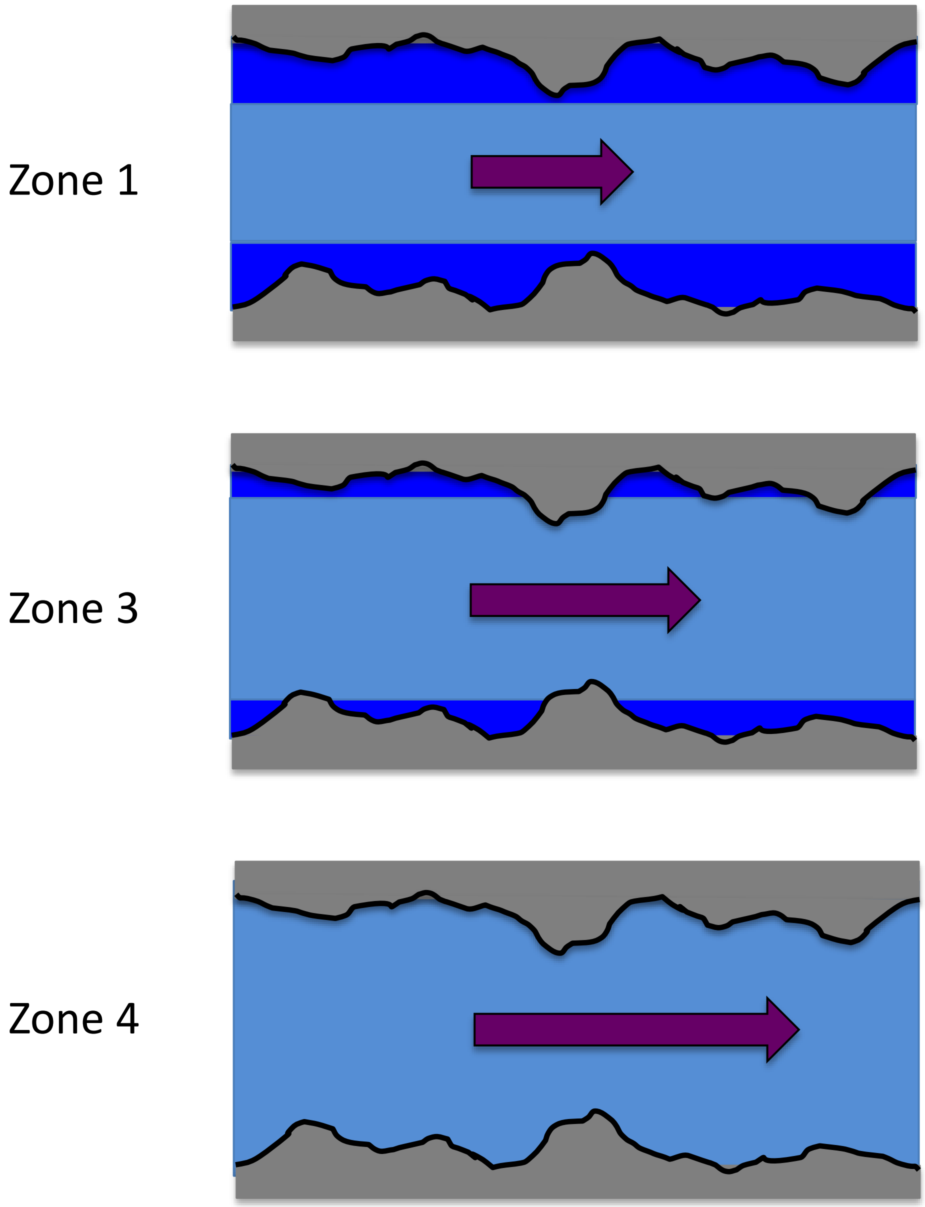
Why do these zones exist?

Why do these different regions exist?

Engineers have tried to rationalize why we observe this behaviour

- At low Reynolds numbers a layer of fluid covers the roughness elements of the pipe, shielding their impact. Therefore, roughness has no impact on laminar flow
- As the Reynolds number increases, the boundary layer thins, and the exposed roughness that increase friction
- Eventually, the whole roughness element is exposed and friction becomes independent of Reynolds number

Why do these zones exist?



The Moody diagram can only do so much

The Moody diagram has one major limitation

- In order to use the Moody diagram, you need to know the Reynolds number
- In order to calculate the Reynolds number, you need to know the average velocity
- This means you can use the **Moody diagram to calculate ΔP for a given flow rate**

However, you cannot use the Moody diagram to calculate the flow rate for a given ΔP

In order to accomplish this task, you need a different plot

- You need to use the **friction factor** plot (also known as the **Re vs ϕRe^2 plot**)

Warning: The analysis we're about to do is weird. It's even weirder than discovering the Moody diagram

Re vs ϕRe^2

To use the *Re vs* ϕRe^2 chart to find the flow rate you must:

1. Use the mechanical energy balance to solve for F

2. Use the value of F to calculate ϕRe^2

3. Use the *Re vs* ϕRe^2 chart to determine the value of the Reynolds number

4. Use the Reynolds number to solve for the velocity and flow rate

Re vs ϕRe^2

We can use the mechanical energy balance to solve for *F*. Previously we derived the following relationship for *F*

$$F = \frac{4\phi L\overline{V}^2}{D}$$

Remember, we don't know *V* or ϕ

We can rearrange this to

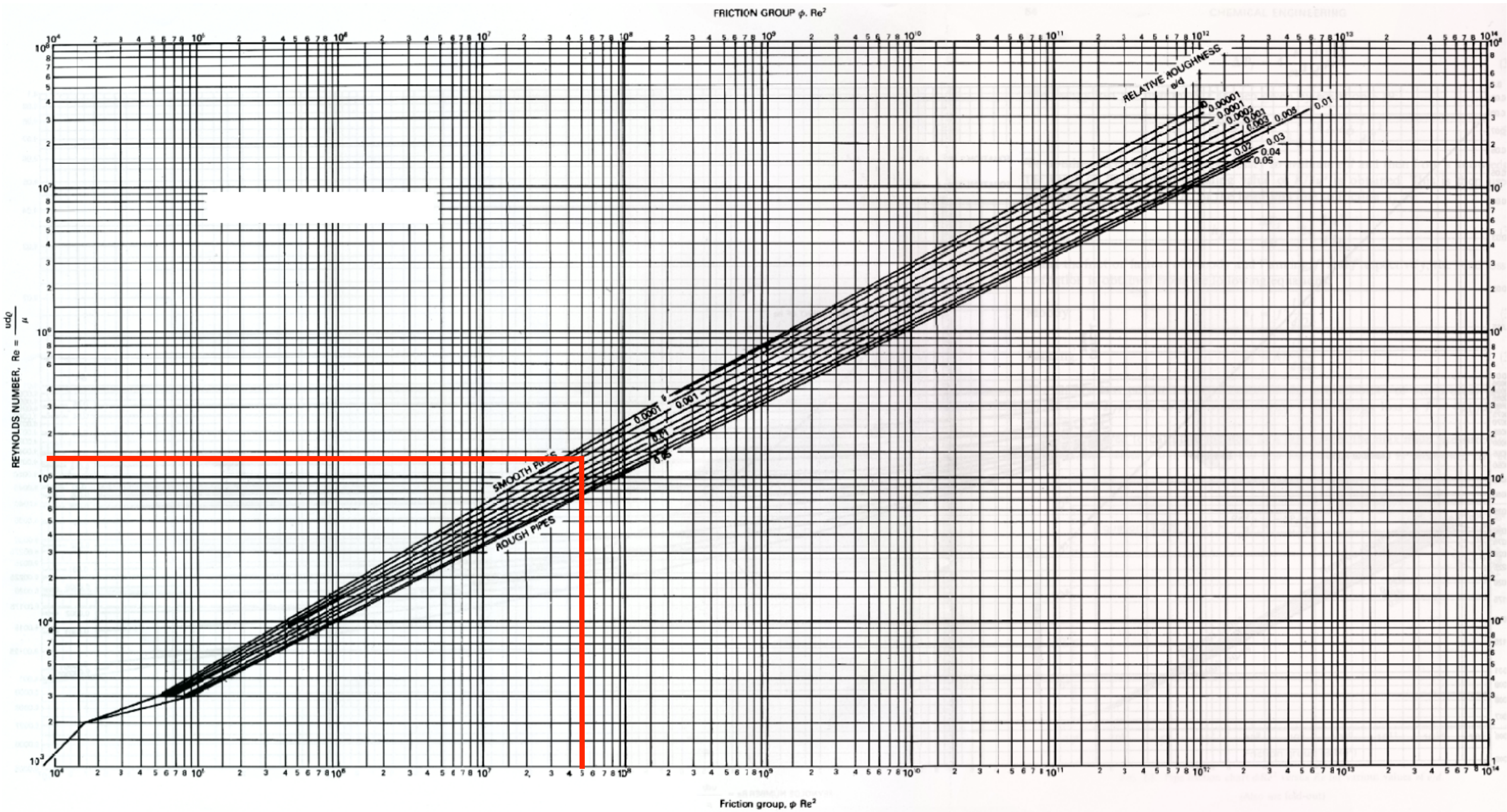
$$\frac{FD}{4L} = \phi\overline{V}^2$$

Now, let's multiply both sides by the same value

$$\left(\frac{\rho^2 D^2}{\mu^2}\right)\frac{FD}{4L} = \phi\overline{V}^2\left(\frac{\rho^2 D^2}{\mu^2}\right) \Rightarrow \frac{FD^3\rho^2}{4L\mu^2} = \phi\left(\frac{\rho\overline{V}D}{\mu}\right)^2$$

$$\Rightarrow \frac{FD^3\rho^2}{4L\mu^2} = \phi\,\mathrm{Re}^2$$

Re vs ϕRe^2



FRICTION GROUP φ. Re²
REYNOLDS NUMBER, Re = udρ/μ
RELATIVE ROUGHNESS e/d
SMOOTH PIPES
ROUGH PIPES
Friction group, φ Re²

Example problem 3.3

We have a 10 meter straight pipe through which water is being pumped 1 meter uphill. The diameter of the pipe is 10cm, the length of the pipe is 10 m, the roughness of the pipe is 0.001 cm, the density of the water is 1000 kilograms per cubic meter, the viscosity is 0.001 Pa s. The pressure at the exit of the pipe is P_{atm}, and the pressure at the inlet of the pipe is P_{atm} + 10,000 Pa

What is the flow rate of water through the pipe?

Mini-summary (1 of 2)

In this section of the class, we are focusing solely on flow through pipes, as this is what we are most interested in as engineers

Although the Bernoulli equation is an excellent tool to conceptually think of fluid flow, it is not applicable in most engineering systems as it neglects friction

To address frictional losses in our system, we manipulated the mechanical energy balance to include friction factors

To determine the value of the friction factors, we used the **Moody diagram**

Mini-summary (2 of 2)

In order to use the Moody diagram, you need to first calculate two parameters of your flow system

- They Reynolds number
- The relative roughness

The Reynolds number is a way of quantifying the flow regime of the fluid. It is a ratio of the inertial and viscous forces within your fluid

If you know the pressure drop and are asked to calculate the flow rate you will need to use the **Re vs ϕRe^2** plot

Fluid Mechanics

Topic 3.2

Flow through pipes with
non-circular cross sections

What we did during the last lesson

Last lesson we discussed how in real flow scenarios, frictional losses exist

In order to correctly predict flow behaviour, we need to include frictional losses in the mechanical energy balance

This is done through the use of friction factor, which is dependent on two quantities

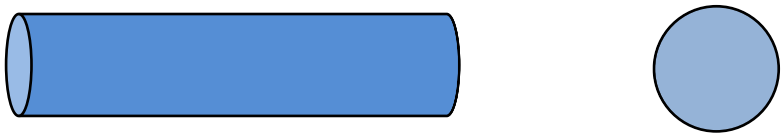
- Reynolds number
- Relative roughness

To use friction factors, we need one of two plots

- If flow rate is known, we need the Moody plot
- If pressure drop is known, we need the Re vs ϕRe^2 plot

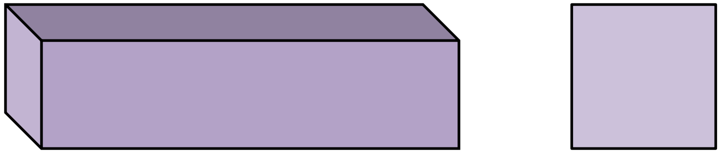
Assuming circular cross section

Until now, we have assumed the conduit in which our fluid is flowing is a cylindrical pipe with circular cross section



However, sometimes our fluid may be flowing conduits with other geometries

- Annuluses
- Ducts



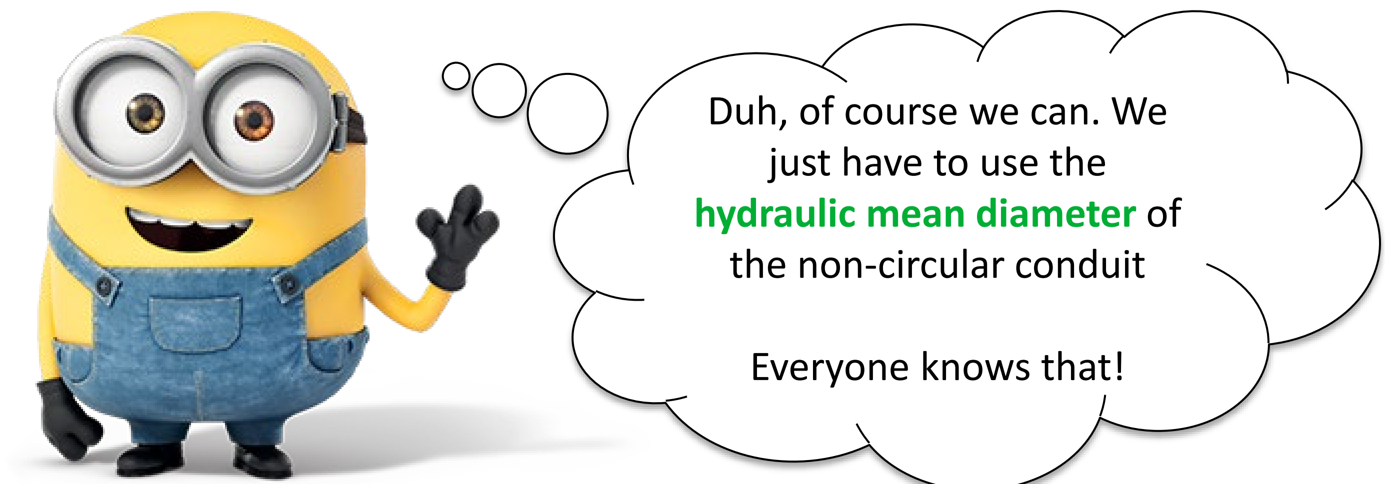
Assuming circular cross section

Currently our mechanical energy balance is in terms of pipe diameter, D

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\overline{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_s + \frac{2 f_F L \overline{V}^2}{D} = 0$$

This value for D is for the diameter of a **circular** pipe

Can we modify the mechanical energy balance so that it can be used to model flow with non-circular cross section?



Hydraulic mean diameter

Bob is correct!

In order to use the mechanical energy balance on fluid flowing in pipes of non-circular cross section, we need to replace the value of the diameter with something called the **hydraulic mean diameter**, **D_e**

The hydraulic mean diameter is an empirical relationship

$$D_e = \frac{4 \times Cross\ sectional\ area\ of\ flow}{Wetted\ perimeter}$$

Hydraulic mean diameter

When modeling flow through conduits with non-circular cross section, the mechanical energy balance becomes the following

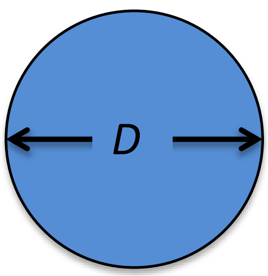
$$\frac{\Delta P}{\rho} + \Delta\left(\frac{\overline{V}^2}{2\alpha}\right) + g\Delta z + \Delta W_s + \frac{2 f_F L \overline{V}^2}{D_e} = 0$$

You just need to solve for the hydraulic mean diameter and then solve the mechanical energy balance as usual

Calculating the hydraulic mean diameter

We shall now calculate the hydraulic mean diameter for three conduits with different cross sections

1. Take a moment to determine the hydraulic mean diameter for a pipe with circular cross section



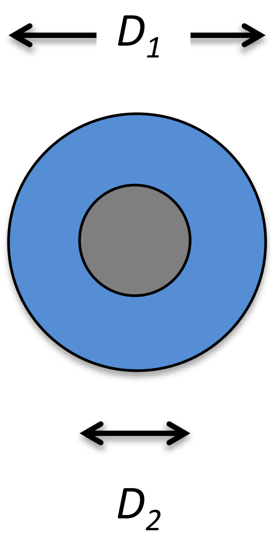
$$D_e = \frac{4 \times Cross\ sectional\ area\ of\ flow}{Wetted\ perimeter}$$

$$D_e = \frac{4 \times \pi \frac{D^2}{4}}{\pi D}$$

$$\boxed{D_e = D}$$

Calculating the hydraulic mean diameter

2. Take a moment to determine the hydraulic mean diameter for a pipe with annular cross section



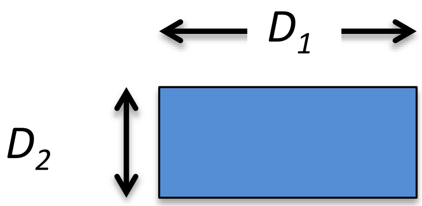
$$D_e = \frac{4 \times Cross\ sectional\ area\ of\ flow}{Wetted\ perimeter}$$

$$D_e = \frac{4 \times \pi \left(\frac{D_1^2}{4} - \frac{D_2^2}{4} \right)}{\pi (D_1 + D_2)}$$

$$D_e = D_1 - D_2$$

Calculating the hydraulic mean diameter

3. Take a moment to determine the hydraulic mean diameter for a rectangular duct



$$D_e = \frac{4 \times Cross\ sectional\ area\ of\ flow}{Wetted\ perimeter}$$

$$D_e = \frac{4D_1D_2}{2(D_1 + D_2)}$$

$$\boxed{D_e = \frac{2D_1D_2}{D_1 + D_2}}$$

Mini-summary

In this section, we discussed how to modify the mechanical energy balance in order to model flow through conduits with non-circular cross section

This is accomplished by replacing the pipe diameter with the hydraulic mean diameter of the system, D_e

$$\boxed{\frac{\Delta P}{\rho}+\Delta\left(\frac{\overline{V}^2}{2\alpha}\right)+g\Delta z+\Delta W_s+\frac{2f_F L\overline{V}^2}{D_e}=0}$$

The hydraulic mean diameter can be calculated as follows

$$\boxed{D_e=\frac{4\times Cross\ sectional\ area\ of\ flow}{Wetted\ perimeter}}$$

Fluid Mechanics

Topic 3.3

Minor losses from fittings

What we did during the last lesson

Last lesson we discussed how in real scenarios, frictional losses exist

To correctly predict flow behaviour, we need to include frictional losses in the mechanical energy balance

This is done through the friction factor, which depends on two values

- Reynolds number
- Relative roughness

To use friction factors, we need one of two plots

- If flow rate is known, we need the Moody plot
- If pressure drop is known, we need the Re vs ϕRe^2 plot

We then discussed how to tailor the energy balance to model fluid flowing in conduits of non-circular cross section using the hydraulic mean diameter

Limitations of the friction factor

Great, now we can calculate the frictional losses in any pipe system!!!

Or can we…?

Pipe network on an oil tanker

Limitations of the friction factor

Friction factors only quantify frictional losses over a straight (or gently curved) length of pipe

Significant frictional losses occur around fittings

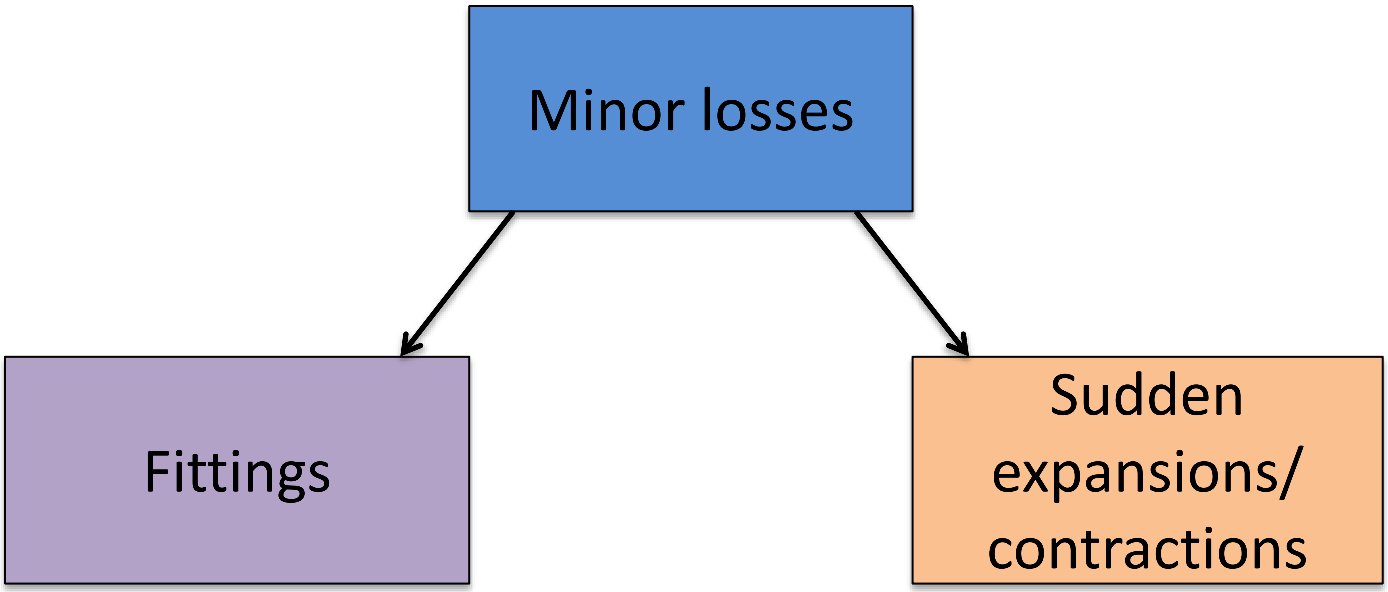
- L-bends
- Valves
- T-junctions
- Etc…

Significant frictional losses also occur around sudden expansions/contractions in the flow system

These frictional losses are often referred to as **minor losses**

These are usually referred to as minor losses as they are usually smaller than the losses due to the major losses we discussed last lesson

Types of minor frictional losses



Accounted for by:

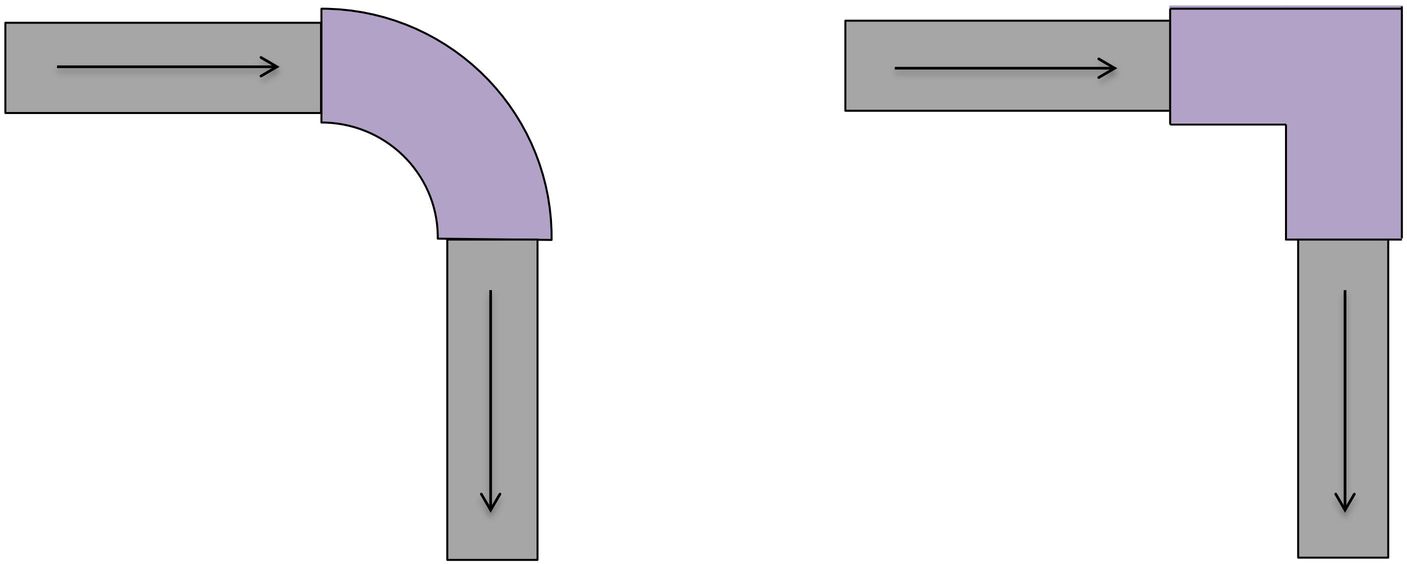
1. Equivalent length or
2. Resistance coefficient

Accounted for by:
Resistance coefficient

First we'll focus on how to account for/quantify losses due to fittings

Why do fittings result in minor losses?

Fluid must flow through an elbow. Which elbow will result in a lower frictional loss? The curve bend

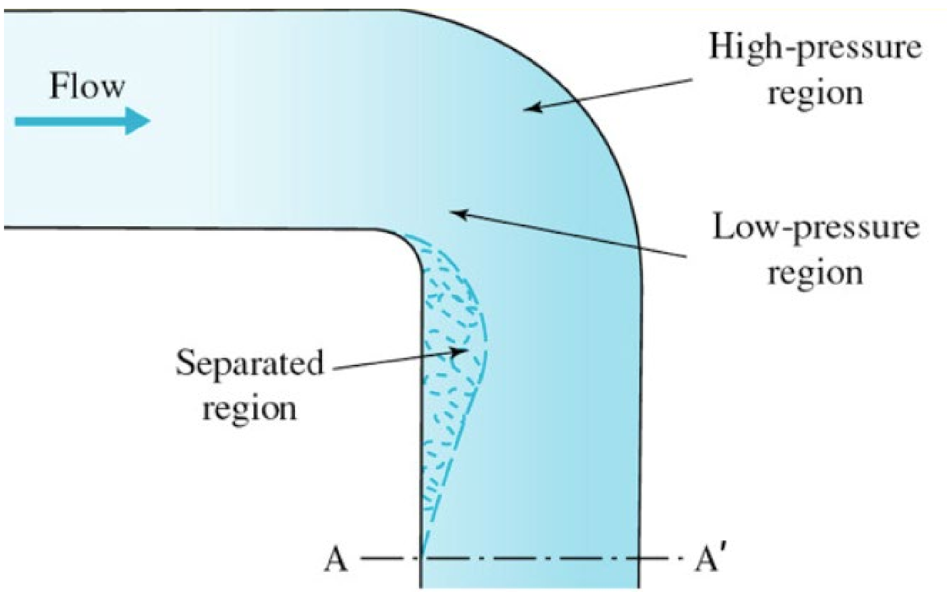


Why? See next slide!!

Why do fittings result in minor losses?

The presence of fittings causes disturbances in the flow, such as separation regions

These disturbances result in frictional losses



Quantifying minor losses due to fittings

There are two methods that can be used to account for the frictional losses that occur due to fittings

1. Equivalent length (L_{eq})
2. Resistance coefficient (K)

Engineers have performed experiments, and developed empirical methods that allow us to predict frictional losses

- This is highly analogous to how the Moody diagram was developed
- To use either of the above methods, you need to be provided with a table of quantities

Method 1: Equivalent length

Frictional losses through a fitting were experimentally measured

Engineers then calculated how long a piece of pipe the fluid would have to flow through in order to experience the same frictional loss

Essentially the engineers are saying:

"The fluid flowing through this elbow experiences the same loss in friction as the fluid would if it traveled through an extra 0 _____ *meters of pipe."*

Do I guess a certain value?

These values of equivalent pipe length are then tabulated for common types of fittings

Method 1: Equivalent length

The below table lists the equivalent length for common fittings

Very important!!

Fitting	$\mathbf{L_{eq}}$
45^o elbow	15D
90^o elbow	30 – 40D
90^o elbow square	60D
Entry from leg of T-piece	60D
Entry into leg of T-piece	90D
Unions and couplings	Very small
Gate valve – full open – half open – quarter open	7D 200D 500D

These are listed as multipliers of *D*, the diameter of the pipe

Method 1: Equivalent length

In order to use this method

1. Calculate an equivalent length, L_{eq}, for each fitting in your system
2. Add these equivalent lengths to the length of your pipe
3. Calculate F
4. Solve the mechanical energy balance as normal

$$\boxed{F = \frac{2 f_F L \bar{V}^2}{D}}$$

$$\Downarrow$$

$$\boxed{F = \frac{2 f_F (L + \sum L_{eq}) \bar{V}^2}{D}}$$

$$\Downarrow$$

$$\boxed{F = \frac{2 f_F (L + L_{eq1} + L_{eq2} + L_{eq3} + \ldots) \bar{V}^2}{D}}$$

Method 2: Resistance coefficient, *K*

To use **resistance coefficients**, you add an additional frictional loss term to the mechanical energy balance for each fitting

For each fitting, you add a $\frac{1}{2}K\overline{V}^2$ term to the energy balance, where ***K*** is found through a table

$$\boxed{F = \frac{2f_F L\overline{V}^2}{D} + \frac{1}{2}\sum K\overline{V}^2}$$

$$\Downarrow$$

$$\boxed{F = \frac{2f_F L\overline{V}^2}{D} + \frac{1}{2}K_1\overline{V}^2 + \frac{1}{2}K_2\overline{V}^2 + \ldots}$$

Method 2: Resistance coefficient, *K*

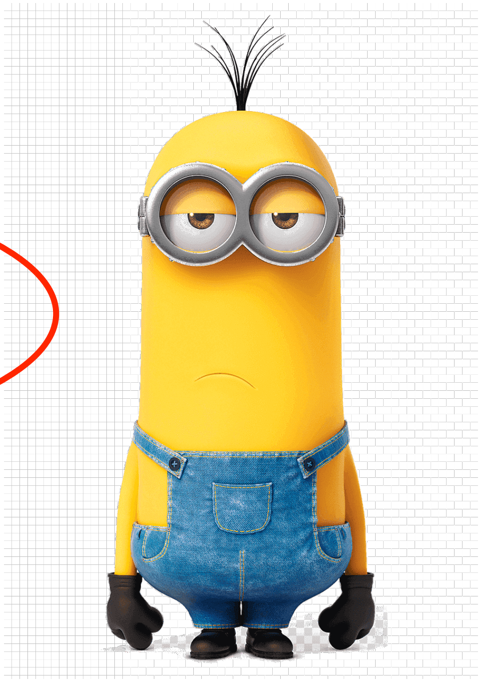
If necessary, you can use equivalent lengths and resistance coefficients together in the same energy balance

Just be sure to only account for the frictional loss of a given fitting once

$$\boxed{F = \frac{2 f_F (L + \sum L_{eq}) \overline{V}^2}{D} + \frac{1}{2} \sum K \overline{V}^2}$$

If they're minor, are they important?

So if these losses are minor, do they need to be quantified?



The frictional losses that occur in a single fitting are usually much smaller than what occurs in the pipe, especially for long pipes

However, there are often many fittings within a piping system, and the sum of their individual frictional contributions can be significant

Mini-summary

Additional frictional losses arise from fittings within a piping network

These minor losses must be accounted for in the mechanical energy balance

There are two methods to quantify the minor losses due to friction

1. Equivalent length, L_{eq}
2. Resistance coefficient, K

Either method is fine

Both methods can be used together, if necessary

However, be certain to only account for frictional losses from each fitting once

Fluid Mechanics

Topic 3.4

Minor losses from
sudden expansions/contractions

What we did during the last lesson

In the previous lesson, we explored minor frictional losses that occurred due to the presence of fittings within a pipe system

The frictional losses that occur in a single fitting are often small compared to what occurs due to friction with the pipe walls; however, the sum of the minor losses may be significant

To quantify these losses we discussed two methods

- Equivalent length
- Resistance coefficient

Other sources of minor losses

Minor losses also occur when

- A fluid flow through a sudden expansion
- A fluid flows through a sudden contraction

Each of these geometries result in disturbances in the flow field, and subsequent loss of energy from the flow due to friction

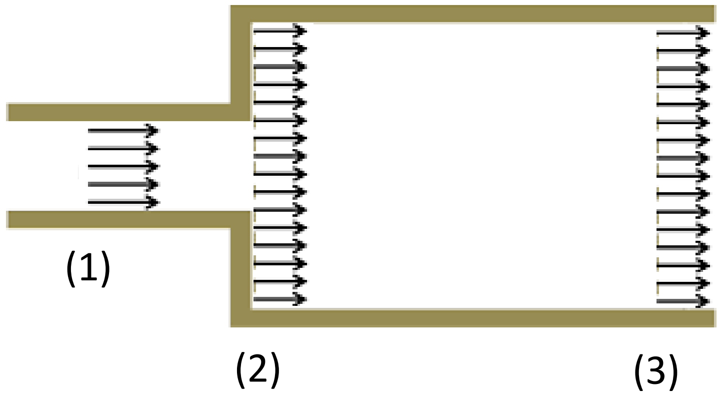
First we will discuss sudden expansion

Flow through sudden expansions

According to our relationship derived from the conservation of mass

$$V_1 A_1 = V_2 A_2$$

If we strictly apply that relationship to a sudden expansion, we would predict the following flow behavior



$V_1 > V_2$

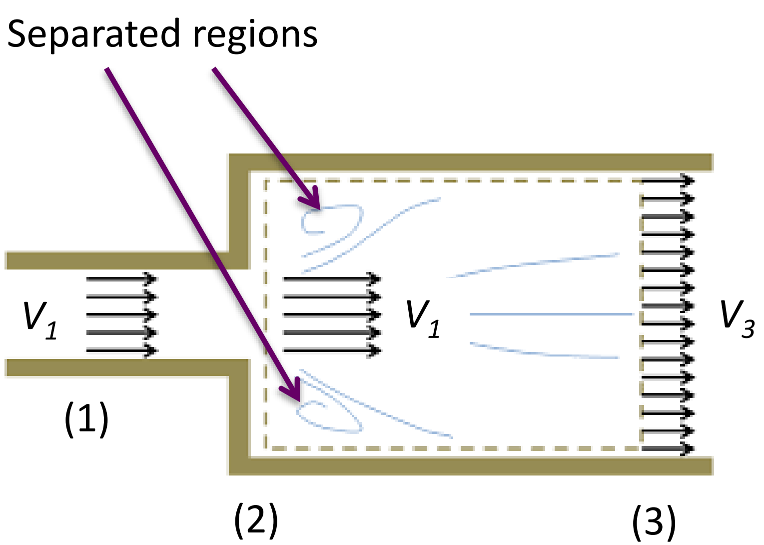
$V_2 = V_3$

However, as your intuition probably tells you, this is not the experimentally observed flow behaviour

Flow through sudden expansions

As seen with flow through a fitting, sudden expansions result in a separated region

In reality the flow profile looks more like the following

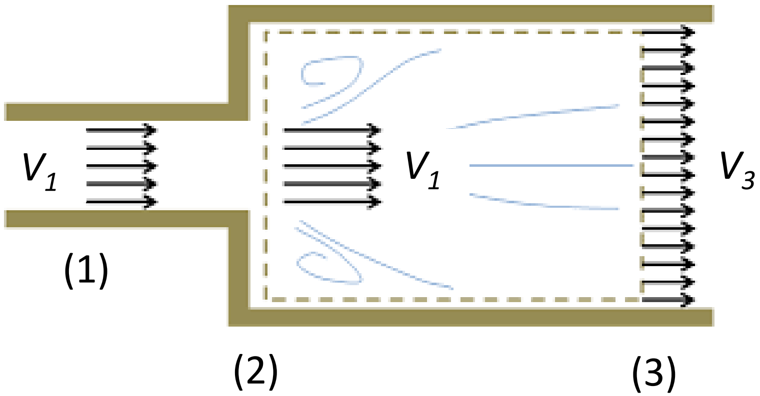


- For a certain duration into the expansion, the velocity is constant
- There are separated regions around the expansion
- Velocity eventually decreases to what is expected in order to maintain flow rate

Quantifying losses through expansions

Frictional losses through expansions are

- Quantified using a resistance coefficient, K_{ex}, as done for fittings
- The value of the coefficient is determined by the geometry of the expansion



Friction due to expansion

$$F_{ex} = \frac{1}{2} K_{ex} V_s^2$$

Resistance coefficient

$$K_{ex} = \left(1 - \frac{A_s}{A_L}\right)^2 = \left(1 - \left(\frac{D_s}{D_L}\right)^2\right)^2$$

Abbreviations

- S = small pipe
- L = Large pipe
- ex = expansion

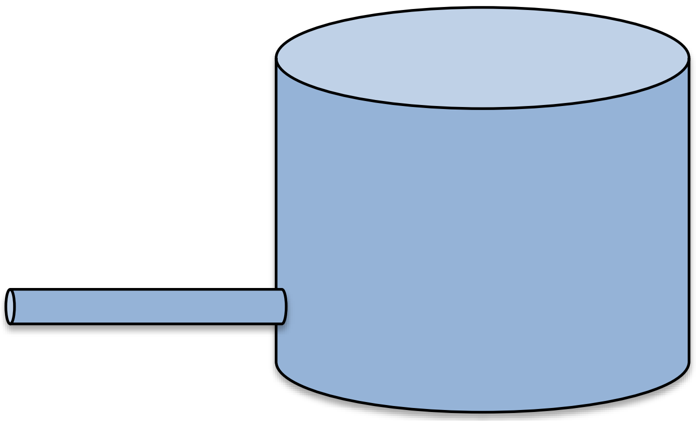
Quantifying losses through expansions

What if your pipe is flowing into a large liquid storage tank?

This is modeled the same way

However, in this case $A_L >> A_S$

This allow us to assume that $K_{ex} = 1$



Resistance coefficient

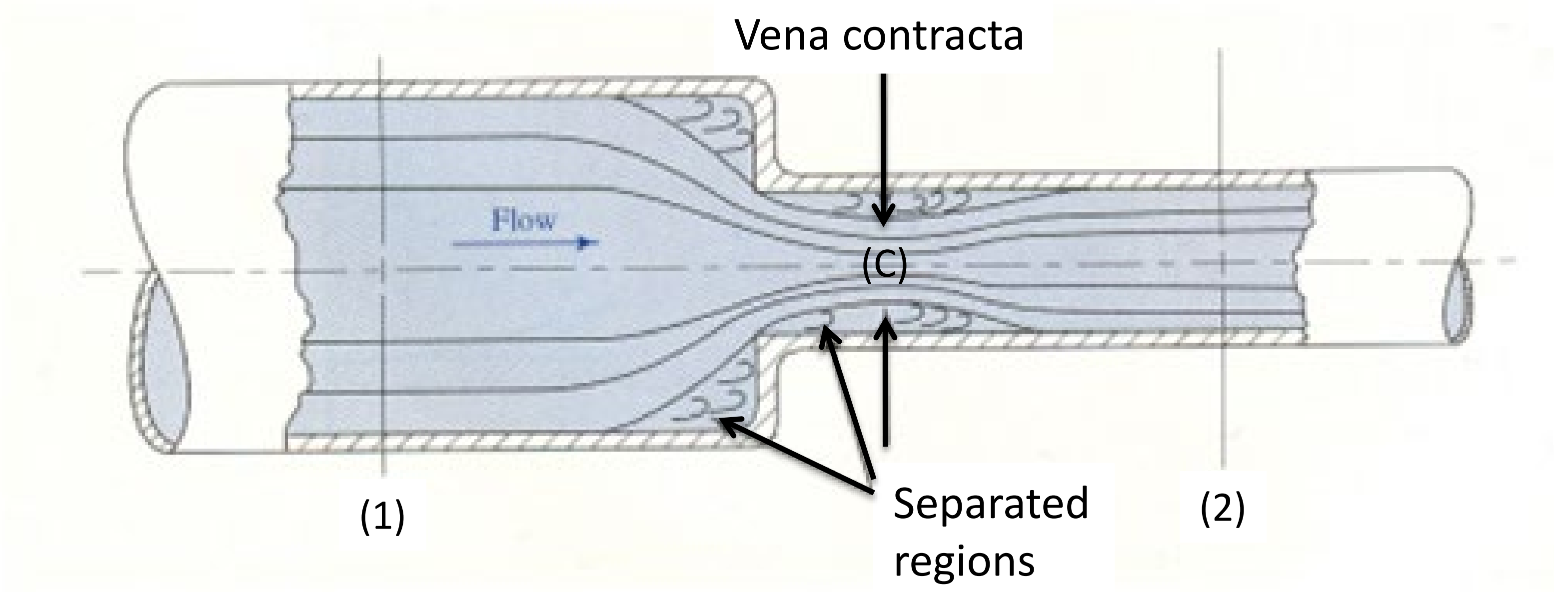
$$K_{ex} = \left(1 - \frac{A_S}{A_L}\right)^2 = \left(1 - \left(\frac{D_S}{D_L}\right)^2\right)^2$$

If pipe is discharging into a large tank

- $A_s/A_L \rightarrow 0$
- $K_{ex} = 1$

Flow through sudden contraction

Separated regions also exist at sudden contractions

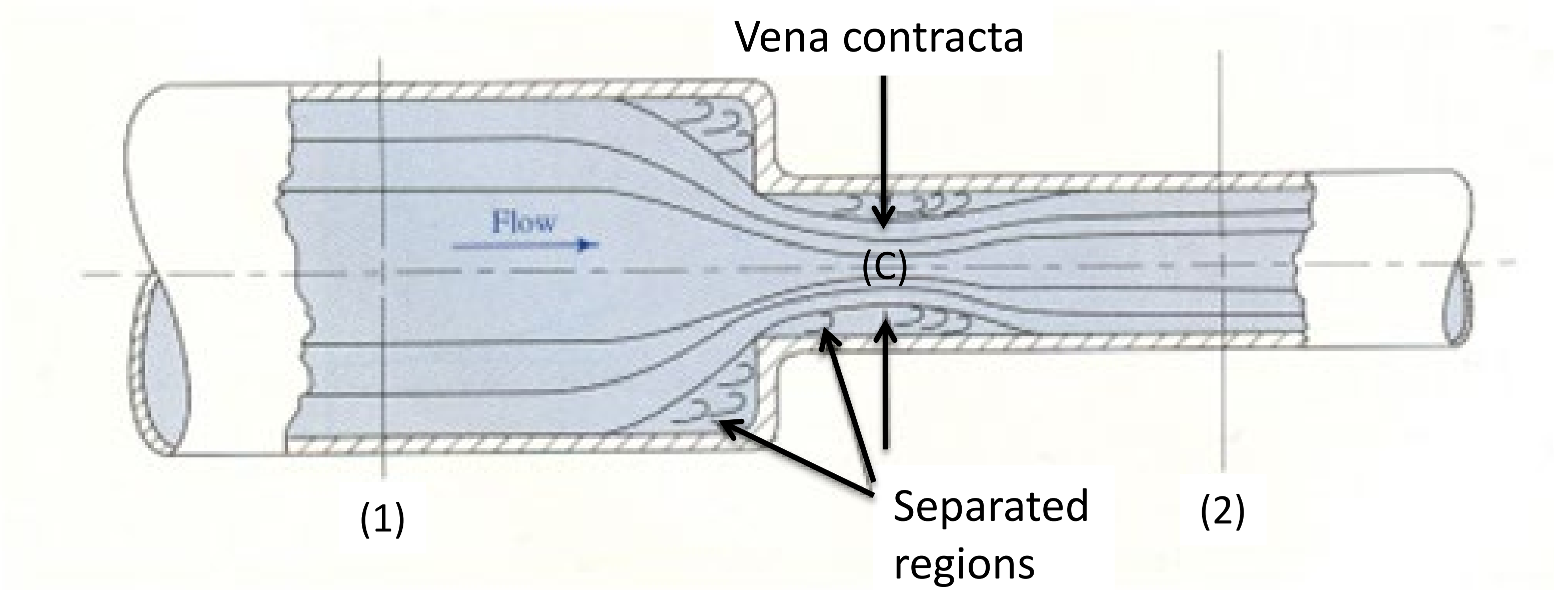


In this flow geometry, there are two separated regions

- The area immediately before the contraction
- A region along the walls within the contraction (vena contracta)
  - The point of minimum cross section within the vena contracta is point *C*

Quantifying losses through sudden contraction

Interestingly, experimental results tell us that the majority of the frictional losses occur during the re-expansion of the flow between points C and point 2

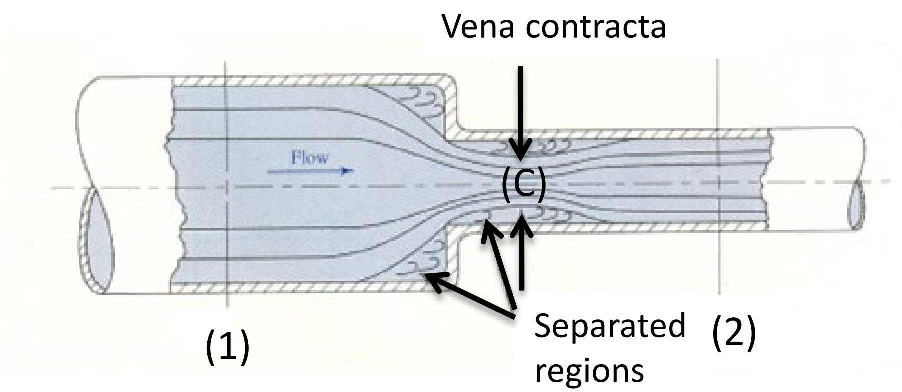


As with the expansion, the losses through contraction will be quantified using a resistance coefficient, K_c

Quantifying losses through contractions

Frictional losses through contractions are

- Quantified using a resistance coefficient, K_{con}, as done for fittings
- The value of the coefficient is determined by the geometry of the contraction



Friction due to contraction

$$F_{con} = \frac{1}{2} K_{con} V_2^2$$

Resistance coefficient

$$K_{con} = 0.5\left(1 - \frac{A_S}{A_L}\right) = 0.5\left(1 - \left(\frac{D_S}{D_L}\right)^2\right)$$

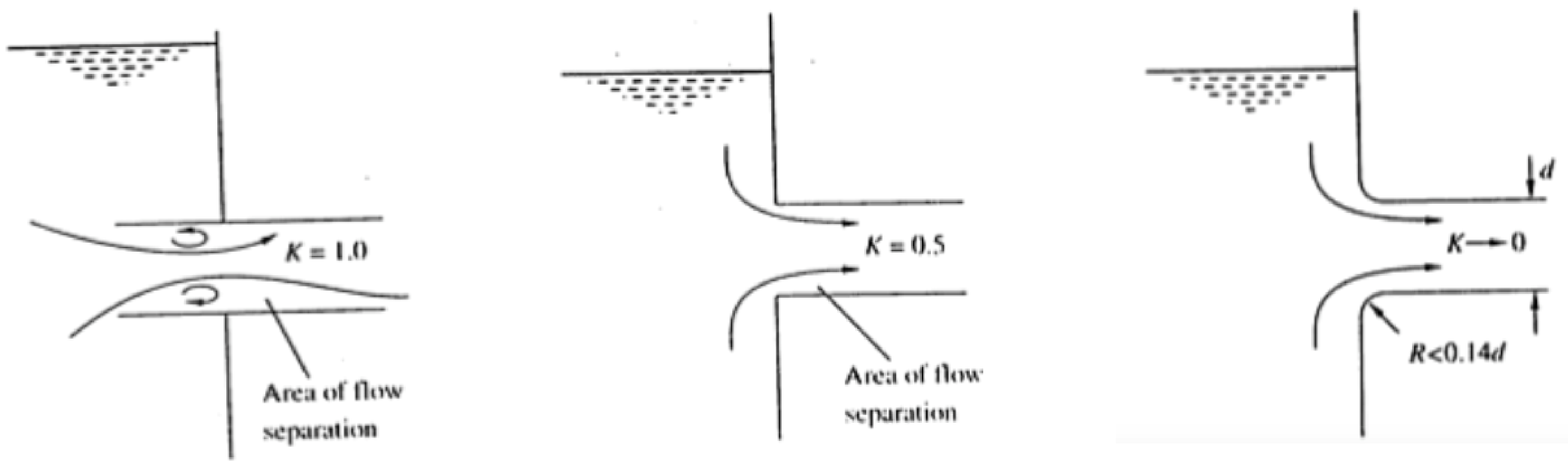
Abbreviations

- S = small pipe
- L = Large pipe
- con = contraction

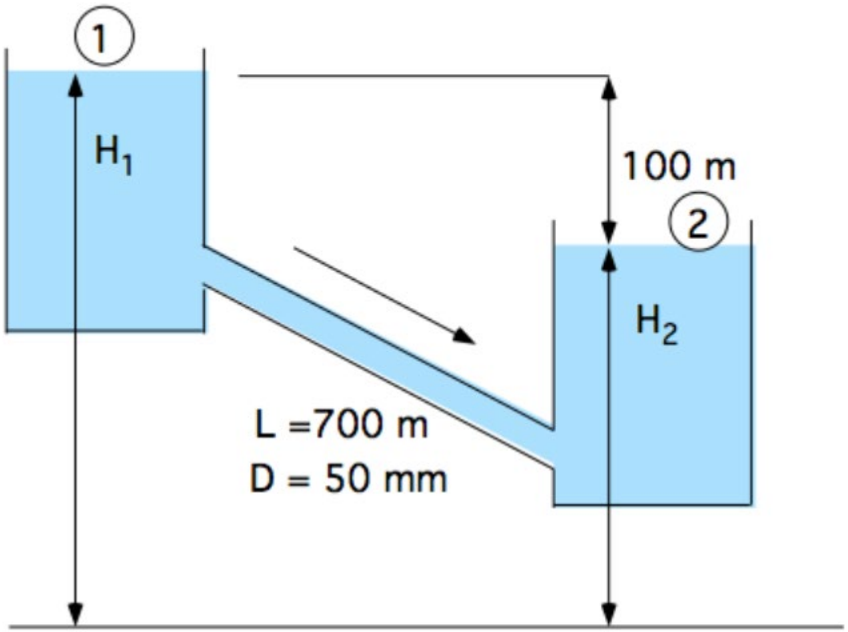
Quantifying losses through contractions

The previous conversation dealt with contractions within pipes

If you have fluid entering a pipe from a tank where the fluid can be considered as stagnant, the value for the resistance coefficient is different and depends on the geometry of the system



Example problem 3.4



Two large reservoirs are open to the atmosphere. They are connected by a smooth pipe, as shown in the figure. Calculate the average velocity within the pipe if K_{entry} = 0.5, K_{exit} = 1.0, the density of the fluid is 1000 kg/m^3 and the viscosity is 0.001 Pa*s

How do I start here?

Summary

In this segment, we looked at how to expand the applicability of the mechanical energy balance to a wider variety of flow problems

To do so, we introduced three modifications

1. We discussed how the hydraulic mean diameter can be used when modeling flow through pipes with non-circular cross section
2. We discussed minor losses due to fittings within the pipe
   - Equivalent length
   - Resistance coefficient
3. We discussed minor losses due to a sudden expansion or contraction in the pipe

Fluid Mechanics

Topic 3.5

Branched pipes and pipe networks

What we did last lesson

In the last lesson, we discussed how the mechanical energy balance that we had developed thus far could not be applied to certain flow geometries

Several assumptions were built into the equation

- The pipe in which flow was occurring has circular cross section
- The pipe was straight (no fittings)
- The pipe did not go through any sudden changes in diameter

These assumptions are not realistic in real life, so we discussed how to modify the mechanical energy balance in order to account for them

- Hydraulic mean diameter
- Equivalent length/resistance coefficients

What are we doing this lesson?

Thus far, we have assumed that our fluid is flowing through a single pipe

However, often we will have **branched pipes** or the fluid will be flowing through a **network of pipes**

The mechanical energy balance will allow us to model the fluid flow through these systems; however, it is not as straight forward as the examples we have discussed thus far

Defining "head"

Before we discuss flow in networks and branches, we're going to define a new term, **head**

Thus far, we have looked at the mechanical energy balance in this form

$$\frac{P_1}{\rho}+\frac{\overline{V}_1^2}{2\alpha}+gz_1=\frac{P_2}{\rho}+\frac{\overline{V}_2^2}{2\alpha}+gz_2+F$$

Each term in this expression has units of energy per unit mass

Sometimes it is helpful to divide through by *g*

$$\frac{P_1}{\rho g}+\frac{\overline{V}_1^2}{2\alpha g}+z_1=\frac{P_2}{\rho g}+\frac{\overline{V}_2^2}{2\alpha g}+z_2+\frac{F}{g}$$

Each term in this expression has units of length

Defining "head"

Before we discussed how the energy at point 1 equals the energy at point 2 plus the energy that was depleted from the flow due to friction

$$\underline{\frac{P_1}{\rho}}+\underline{\frac{\overline{V}_1^2}{2\alpha}}+\underline{gz_1}=\underline{\frac{P_2}{\rho}}+\underline{\frac{\overline{V}_2^2}{2\alpha}}+\underline{gz_2}+\underline{F}$$

Pressure energy
Kinetic energy
Potential energy
Frictional losses

After dividing through by *g*, these terms are now referred to as head

$$\underline{\frac{P_1}{\rho g}}+\underline{\frac{\overline{V}_1^2}{2\alpha g}}+\underline{z_1}=\underline{\frac{P_2}{\rho g}}+\underline{\frac{\overline{V}_2^2}{2\alpha g}}+\underline{z_2}+\underline{\frac{F}{g}}$$

Pressure head
Velocity head
Gravitational head
Head loss

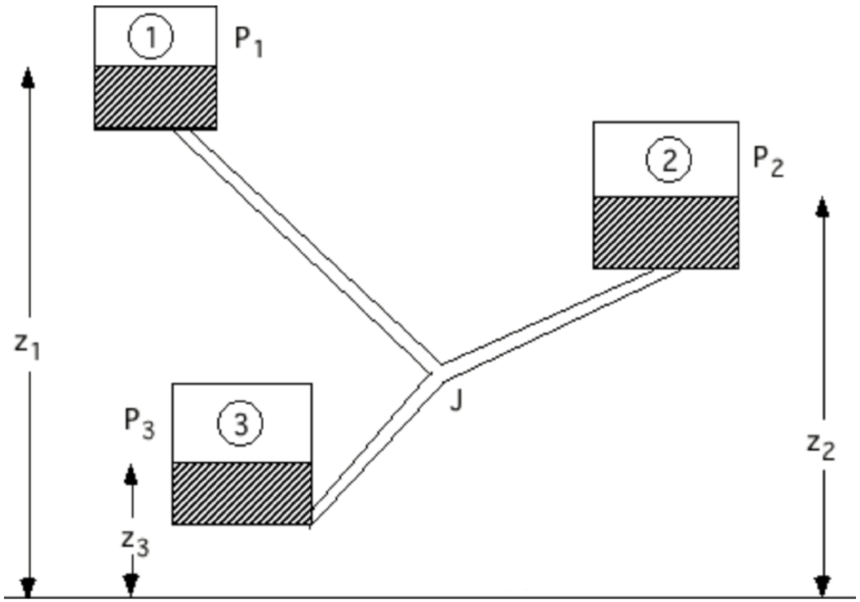
Defining "head"

The sum of each of the pressure head, velocity head, and gravitational head is referred to as the **total head** at a given point

$$\underbrace{\frac{P_1}{\rho g}+\frac{\overline{V}_1^2}{2\alpha g}+z_1}_{\substack{\text{Total head}\\ \text{at point 1}}}=\underbrace{\frac{P_2}{\rho g}+\frac{\overline{V}_2^2}{2\alpha g}+z_2}_{\substack{\text{Total head}\\ \text{at point 2}}}+\underbrace{\frac{F}{g}}_{\substack{\text{Head loss}\\ \text{Between 1 \& 2}}}$$

$$\boxed{h_1 = h_2 + h_L}$$

Thought experiment: flow in branched pipes

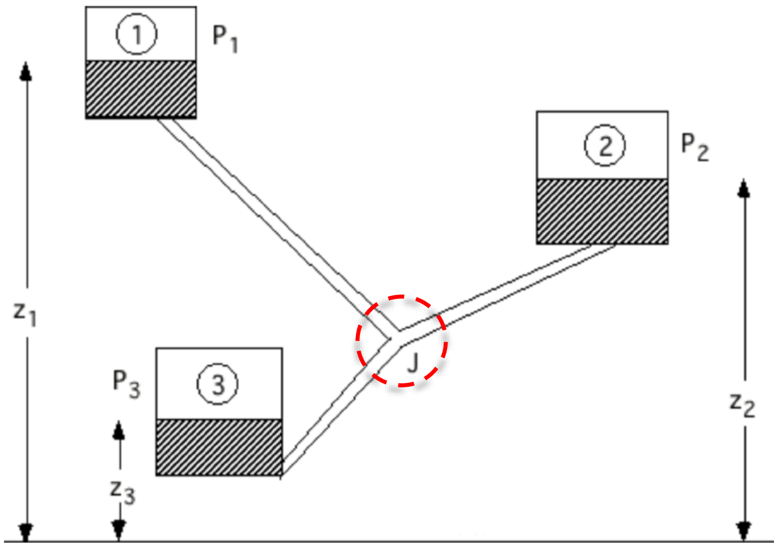


Consider three large tanks at various heights and under various pressures, interconnected at point J

Which tanks will fill and which will empty?

What are the flow rates into/out of each tank?

Thought experiment: flow in branched pipes



General considerations about branch points:

1. Mass is conserved at a branch point. The mass flow rate into a branch has to equal the mass flow rate out of a branch

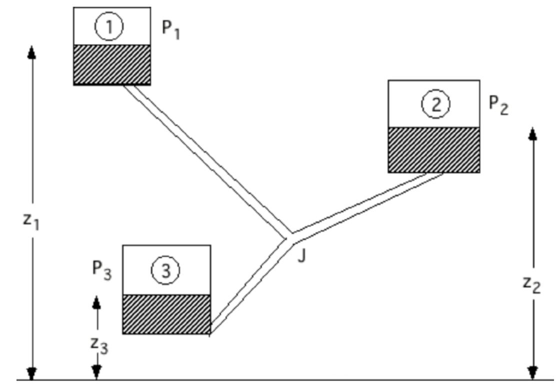
$$\boxed{Q_1 = Q_2 + Q_3}$$

2. The head is **common** to all branches at a junction. The energy may be partitioned differently, but the total heads are equal

   - Flow in Pipe 1 ends up with a total head (a total mechanical energy) of h_j
   - Flow in Pipes 2 and 3 begin with a total head (a total mechanical energy) of h_j
   - Said another way, the total head (the total mechanical energy) of the system cannot be discontinuous anywhere in the flow

Thought experiment: flow in branched pipes

If these tanks were open to atmosphere, the pressure at each fluid surface would be P_{atm}



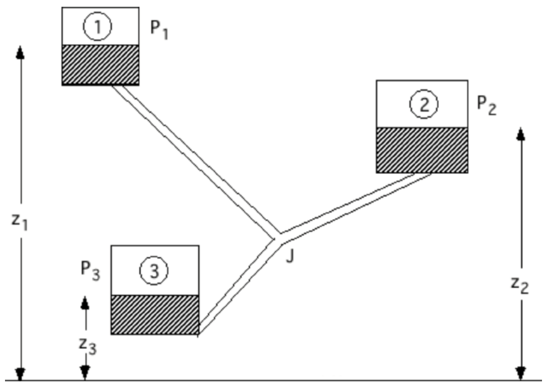
In that scenario, gravity head would be the main driver of fluid flow, and all the fluid would drain down into the third tank

However, in this scenario, the tanks are sealed providing the opportunity to adjust the pressure head in each tank

This could result in a scenario where the pressure head in tank 3 is large enough to push fluid against gravity into tanks 1 and 2

Thought experiment: flow in branched pipes

The fluid flow is governed by the mechanical energy at each of the three free surfaces



Said another way, the fluid flow is governed by the head of fluid at each of the three surfaces

Let us calculate the total head at the free surface in tank 1

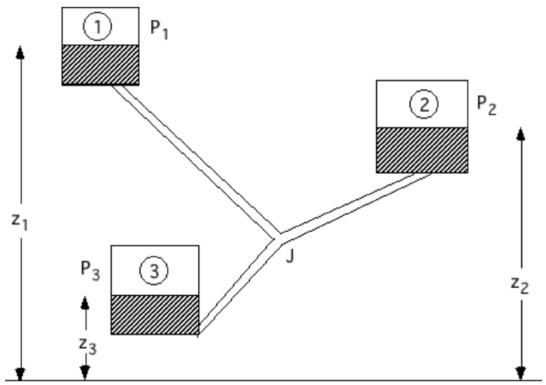
$$h_1 = \frac{P_1}{\rho g} + \frac{\overline{V}_1^2}{2\alpha g} + z_1$$

Since the tanks are large, we can assume that the kinetic energy at the surface is negligible

$$h_1 = \frac{P_1}{\rho g} + z_1$$

Thought experiment: flow in branched pipes

Using the same analysis, we can write the head at each of the free surfaces



At point 1

$$h_1 = \frac{P_1}{\rho g} + z_1$$

At point 3

$$h_3 = \frac{P_3}{\rho g} + z_3$$

At point 2

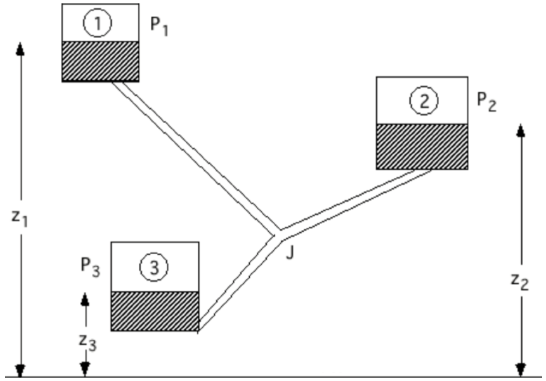
$$h_2 = \frac{P_2}{\rho g} + z_2$$

This analysis confirms our previous rationalization, that both height of the tanks and the pressure within the tanks will drive flow

Said another way, there is a contribution to flow by both the gravitational head and the pressure head

Thought experiment: flow in branched pipes

Given: $h_1 > h_j$



Using this basis, multiple scenarios can occur

Scenario 1. $h_j > h_2$ and $h_j > h_3$

Fluid will flow from tank 1 toward the joint. From the joint, a portion will go to tank 2 and a portion will go to tank 3

$Q_1 = Q_2 + Q_3$ *(conservation of mass must hold)*

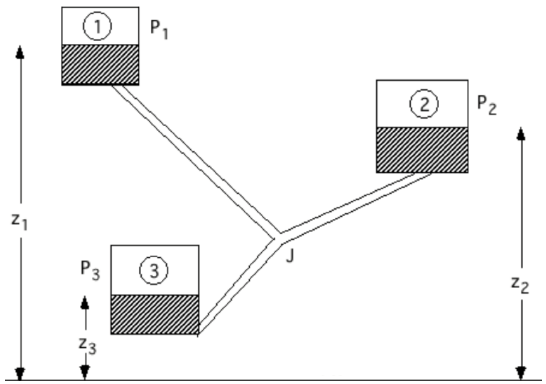
Scenario 2. $h_j < h_2$ and $h_j > h_3$

Fluid will flow from both tank 1 and tank 2 into tank 3

$Q_1 + Q_2 = Q_3$

Thought experiment: flow in branched pipes

Given: $h_1 > h_j$



Using this basis, multiple scenarios can occur

Scenario 3. $h_j > h_2$ and $h_j < h_3$

Fluid will flow from both tank 1 and tank 3 into tank 2

$Q_1 + Q_3 = Q_2$

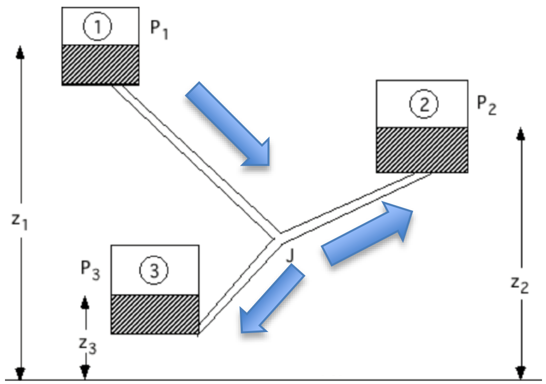
Scenario 4. $h_j < h_2$ and $h_j < h_3$

This scenario is impossible. Under these conditions, all the fluid would flow to the joint; however, the fluid is incompressible, so there's no space for the fluid to go

Flow in branched pipes

Let's assume we have scenario 1.

- $h_1 > h_j$
- $h_J > h_2$
- $h_j > h_3$



To calculate the flow, we must determine the head loss between each of the free surfaces and the joining point

Pipe 1 to J

$$\frac{P_1}{\rho g}+\frac{\overline{V}_1^2}{2\alpha g}+z_1=\frac{P_{1J}}{\rho g}+\frac{\overline{V}_{1J}^2}{2\alpha g}+z_J+\frac{F_1}{g}$$

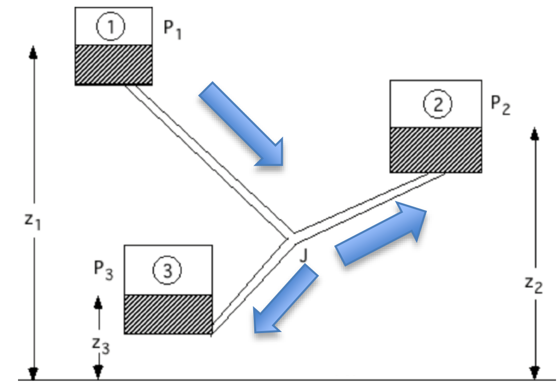
Simplify by letting $V_1 = 0$

$$\underbrace{\frac{P_1}{\rho g}+z_1}_{\text{Total head at point 1}}=\underbrace{\frac{P_{1J}}{\rho g}+\frac{\overline{V}_{1J}^2}{2\alpha g}+z_J}_{\text{Total head at point J}}+\underbrace{\frac{F_1}{g}}_{\text{Head loss Between 1 \& J}} \Rightarrow h_1-h_J=\frac{F_1}{g}$$

Flow in branched pipes

We can perform the same analysis for the other two segments of pipe

However, fluid is flowing from J to the tank, so the joint is now point 1 and the tank is now point 2



J to point 2

$$\frac{P_{2J}}{\rho g}+\frac{\overline{V}_{2J}^{2}}{2\alpha g}+z_J=\frac{P_2}{\rho g}+\frac{\overline{V}_2^{2}}{2\alpha g}+z_2+\frac{F_2}{g}$$

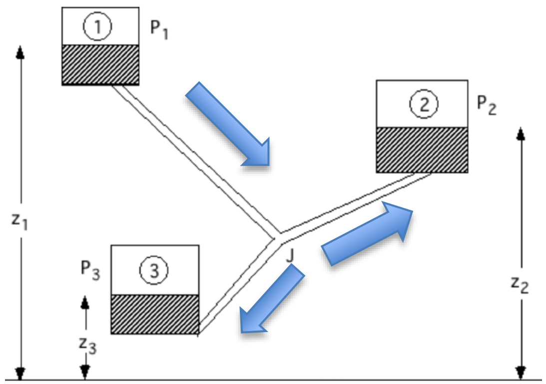
Simplify by letting $V_2 = 0$

$$\frac{P_{2J}}{\rho g}+\frac{\overline{V}_{2J}^{2}}{2\alpha g}+z_J=\frac{P_2}{\rho g}+z_2+\frac{F_2}{g} \Longrightarrow h_J-h_2=\frac{F_2}{g}$$

Flow in branched pipes

The same analysis can be performed between the joint

And tank 3



This produces the following three energy balances

$$h_1 - h_J = \frac{F_1}{g} \qquad h_J - h_2 = \frac{F_2}{g} \qquad h_J - h_3 = \frac{F_3}{g}$$

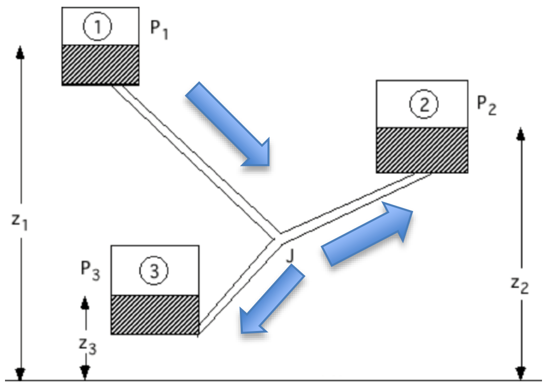
Even with the three energy balances, more information is needed before you can solve the problem

You must be supplied with additional information

- The pressure in each tank
- The height of the free surfaces
- Pipe parameters in order to calculate friction (length, diameter, and roughness)

Flow in branched pipes

If you are provided with all of the necessary information, you can solve for the flow rates and velocities of the fluid through each pipe



However, it is necessary to use an iterative approach in order to solve the problem

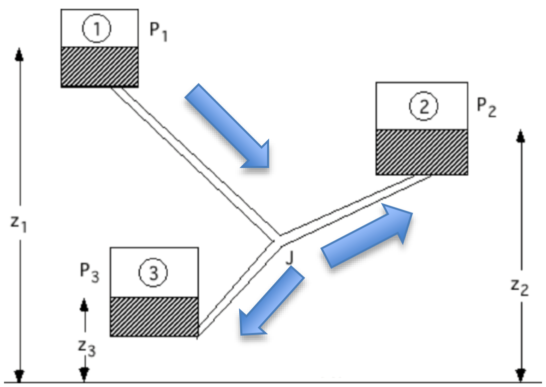
With the tank pressures, heights, and pipe parameters provided, you can calculate the head in each of the tanks

However, we still do not know the values of h_j and we cannot calculate friction without knowing the velocity of the fluid in the pipes

$$h_1 - h_J = \frac{F_1}{g} \qquad h_J - h_2 = \frac{F_2}{g} \qquad h_J - h_3 = \frac{F_3}{g}$$

Flow in branched pipes

In order to solve for the velocities and flow rates in each pipe you must use the following algorithm



1. Calculate the head at each free surface
2. Estimate a reasonable value of h_j
3. Calculate the estimated values of frictional losses: F_1, F_2, & F_3
4. Calculate the value for ϕRe^2 for each pipe to determine the estimated Reynolds number for each pipe
5. Using the estimated Reynolds number, calculate velocities and volumetric flow rates in each pipe
6. Check to see if these values of Q_1, Q_2, and Q_3 fulfill the equation of continuity
7. If continuity is violated, guess a new value of h_j and reiterate

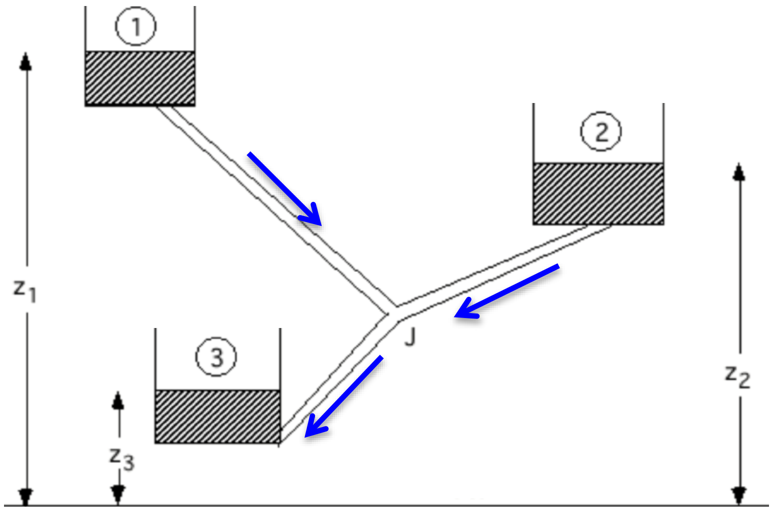
$$h_1 - h_J = \frac{F_1}{g}$$

$$h_J - h_2 = \frac{F_2}{g}$$

$$h_J - h_3 = \frac{F_3}{g}$$

Example problem 3.5

Now we will solve a (slightly easier) version of the three reservoirs problem



Three large and open tanks are interconnected as shown in the diagram

Determine the volumetric flow rate in each pipe using the information provided in the table below

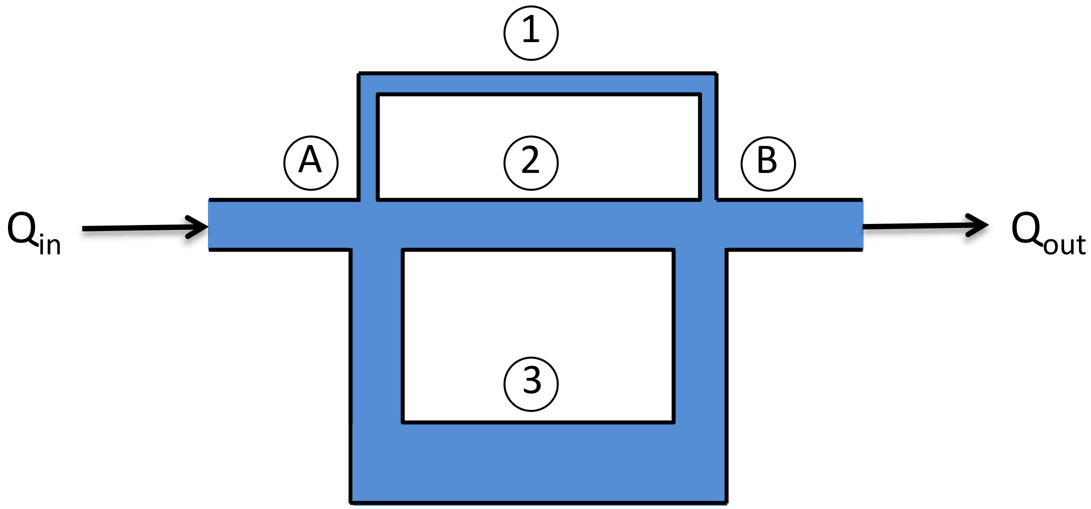
Given information

- The directions of flow in the system are provided
- The lengths, diameter, and friction factors of the pipes are given
- The value of the friction factor in each pipe is constant
- The tanks are open to the atmosphere

In reality the value of f_F changes with velocity

Flow in pipe networks

Sometimes we will also have flow through pipe networks



This particular network has two joints and three branches

Can we model the flow in this system?

- How much fluid will go through each branch?
- What is the velocity of fluid in each branch?

Thought experiment: flow in branched pipes

We had two general rules when considering flow in branched pipes

For flow in pipe networks we have three general rules

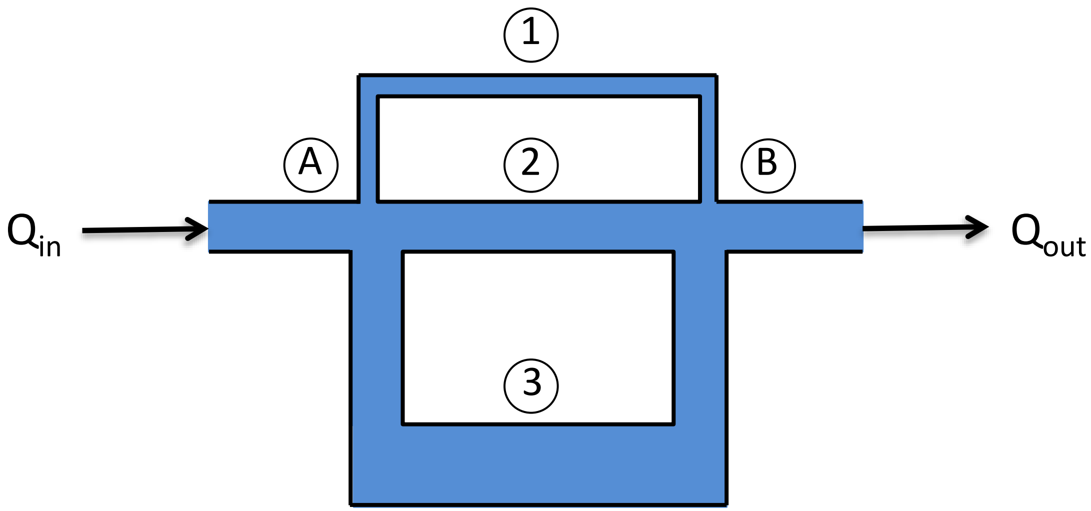
1. Mass is conserved at a branch point. The mass flow rate into a branch has to equal the mass flow rate out of a branch

2. The head is common to all streams entering or leaving the branch point

1. The frictional losses between two points must be equal regardless of the path the fluid takes

Flow in pipe networks

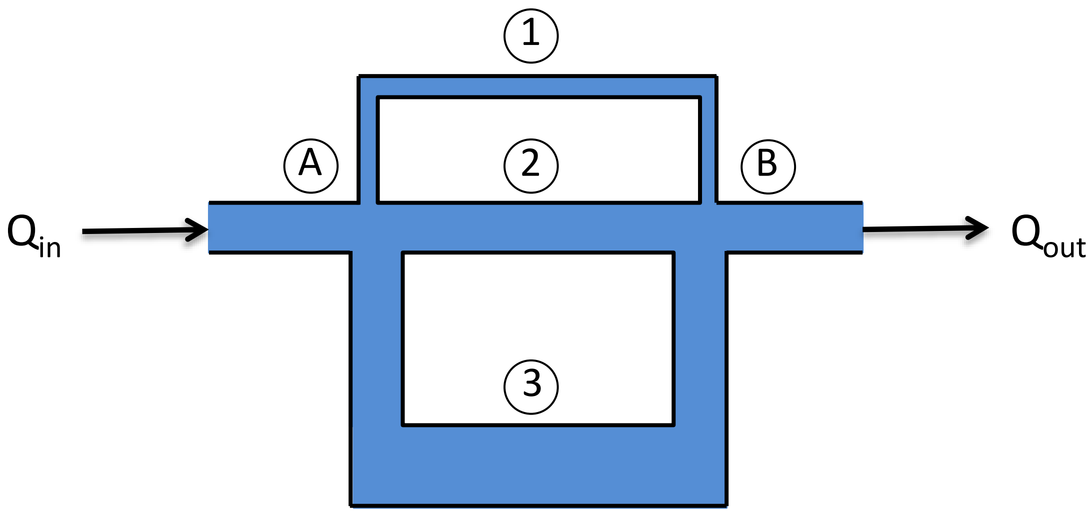
From out overall mass balance, we know that the mass into the system has to equal the mass out of the system



$$Q_{In} = Q_{Out}$$

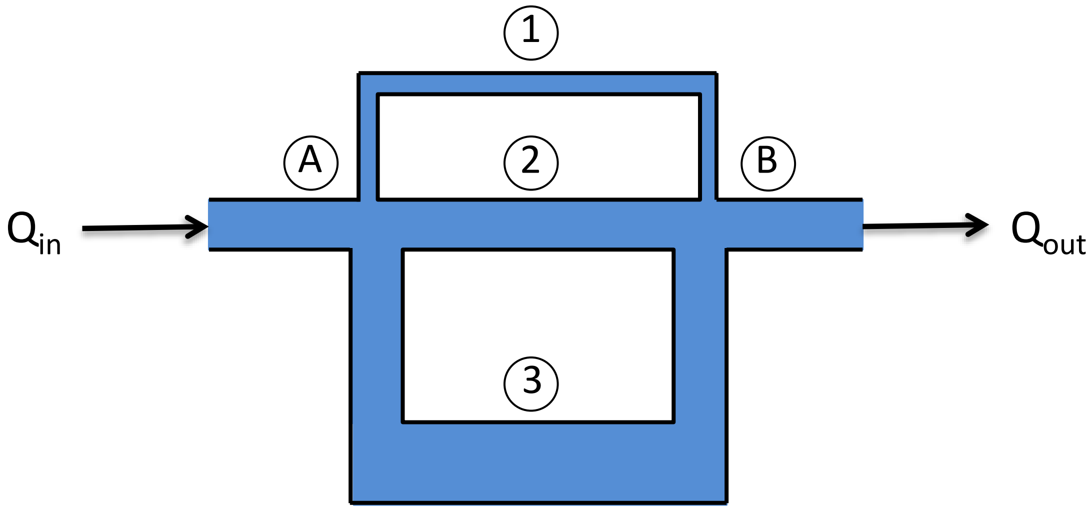
Flow in pipe networks

From **Rule 1**, the mass into a joint has to be equal to the mass out of the joint



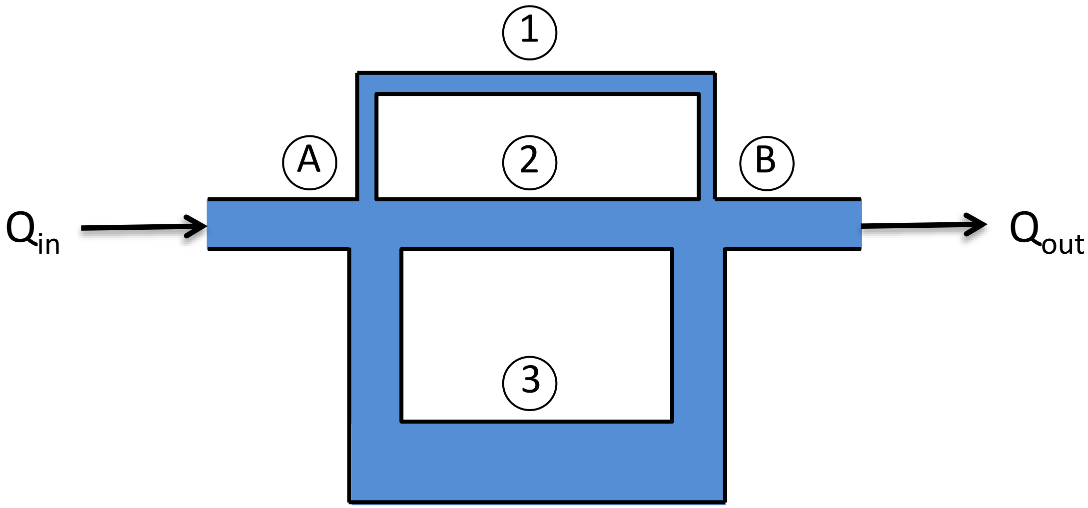
Flow in pipe networks

From **Rule 2**, the head coming into/leaving a node from each branch must be common



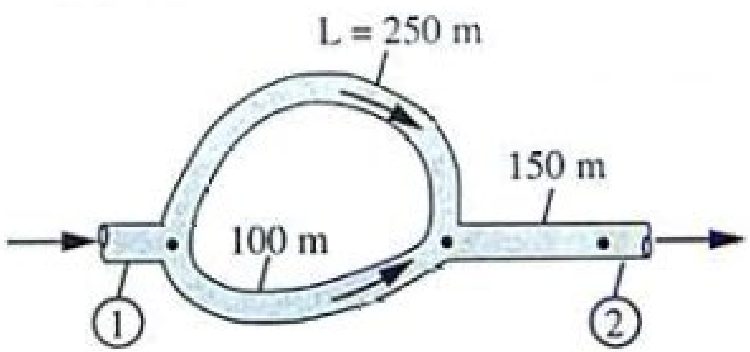
Flow in pipe networks

From **Rule 3**, the frictional losses between any two points must be the same, regardless of the path taken by the fluid



Example problem 3.6

A pipe network transports water from Point 1 to Point 2. All pipes are cast iron (e = 0.12 mm) and have a diameter of 8 cm. The total pressure drop between Points 1 and 2 is 750 kPa. Find the resulting flow rate. Neglect minor losses.



Review on pipe flow

Fluid flow in pipes is often modeled through application of the **equation of continuity** and **mechanical energy balance**

- The equation of continuity arises from performing a mass balance on the flow
- The mechanical energy balance arises from performing an energy balance

The mechanical energy balance can be simplified to the Bernoulli Equation

- Assume no frictional losses
- No shaft work

Review on pipe flow

Friction does exist, and we need to quantify it in order to predict flow behaviour

- Viscous friction and friction with pipe walls → friction factor
  - Moody Diagram
  - Re vs ϕRe^2 plot
- Friction due to fittings, valves, expansions, contractions
  - Equivalent length
  - Resistance coefficient

Use hydraulic mean diameter for conduits with non-circular conduits

For flow in pipe networks, three rules must be maintained

1. Conservation of mass through a joint
2. Head is common to each stream entering/leaving a joint
3. Frictional losses between any two points are equal, regardless of the path