MAST30013 – Techniques in Operations Research

Semester 1, 2021

Tutorial 4

1. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x_1, x_2) \;=\; x_1^2 - x_1 x_2 + x_2^3 + x_1 - x_2.$$

   (a) Show that $\boldsymbol{x}^* = \left(-\frac{1}{4}, \frac{1}{2}\right)^T$ is a local minimum, and that $\boldsymbol{x}^* = \left(-\frac{2}{3}, -\frac{1}{3}\right)^T$ is a saddle point.

   (b) Starting at $\boldsymbol{x}^0 = (0, 0)^T$, find the local minimum of f using the steepest descent method;
   Stop when $\| \boldsymbol{\nabla} f(x_1^k, x_2^k) \| < 0.01$.

2. In the steepest descent method, show that, for $k = 0, 1, 2, \ldots$, $\boldsymbol{d}^{k+1}$ is perpendicular to $\boldsymbol{d}^k$.

3. (a) Show that the rate of convergence of the sequence

$$x^k \;=\; \frac{2k}{4^k + k^4 + 1} \;\longrightarrow\; x^* \;=\; 0$$

   is linear.

   (b) What is the rate of convergence of the sequence

$$x^k \;=\; \frac{2k^2 - 3k + 8}{2k^2 + 7k - 2} \;\longrightarrow\; x^* \;=\; 1\ ?$$