Student ID ___________

THE UNIVERSITY OF MELBOURNE

Department of Chemical and Biomolecular Engineering

ENGR30002 Fluid Mechanics

Semester 2 2018

Reading Time: 15 minutes – no writing or annotating allowed anywhere

Writing time: 180 minutes

This paper has 15 pages including this page and any Appendices.

Authorised Materials
Calculator: Casio fx82 or fx100 calculators are permitted
Notes: One A4 sheet of handwritten notes (front and back) is permitted

Instructions to Invigilators

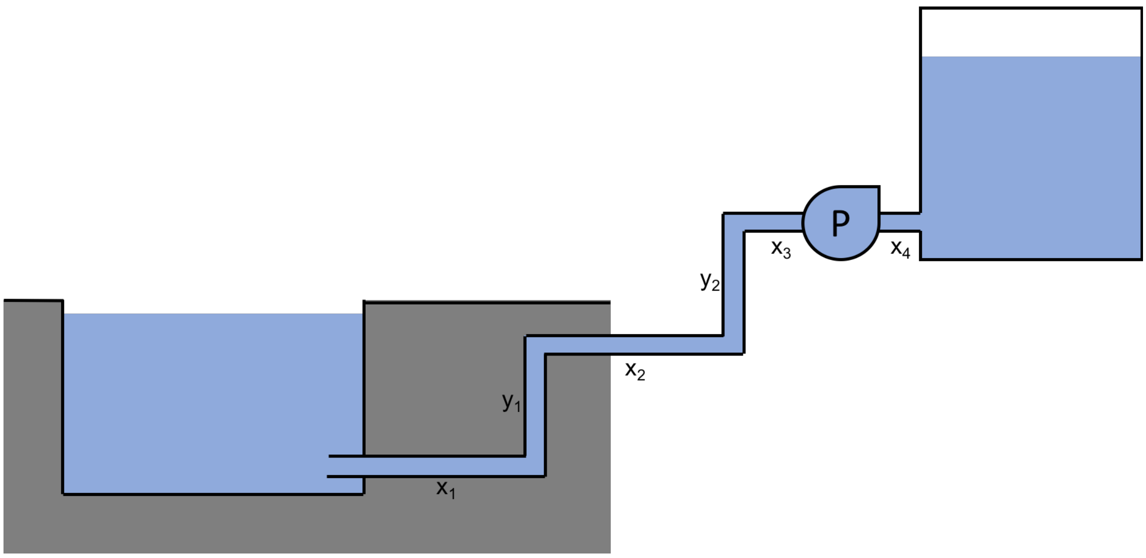
- The examination paper IS TO REMAIN in the examination room
- Students should include detached pages from question paper in the script book for collection
- Students are to be provide with one script book
- Provide extra script books on request

Instructions to Students

- Total marks for this paper is 180
- Ensure your student number is written on all script books and answer sheets during writing time. No annotating is allowed in reading time or after the end of writing time
- Attempt all 7 questions, which are of unequal marks value
- Answer all questions on the right-hand lined pages of the script book
- Start the answer to each question on a new page in the script book and write the question number in the top right hand corner
- The left-hand unlined pages of script books are for draft working and notes and will not be marked
- State clearly and justify any assumptions made
- Write legibly in blue or black pen
- Show all working for each problem
- Pages 11 to 15 contain information and formulae that may be useful in answering the questions
- Mobile phones, tablets, laptops, and other electronic devices, wallets and purses must be placed beneath your desk
- All electronic devices (including mobile phones and phone alarms) must be switched off and remain under your desk until you leave the examination venue. No items may be taken to the toilet

Question 1 (30 marks)

You work for the consulting company *Intergalactic Engineering*. The company provides engineering solutions for space colonies on different worlds. As part of your job, you regularly travel between three different planets: *Planet X*, *Planet Y*, and *Planet Z*. The process illustrated in the schematic below is operating on each of the three worlds in order to pump water (a precious commodity for the colonies!) into a pressurized storage vessel. The lake from which the water is being pumped is open to the planet's atmosphere. The pressure above the fluid in the tank is 200 kPa. The free surface of the lake is 4 m below the centreline of the pump, and the free surface in the storage vessel is 5 m above the centreline of the pump. The lake and the tank are connected by six segments of pipes and four right-angle square elbows. The two segments of vertical pipe have lengths of 2 m and 1.5 m, labelled y_1, and y_2 respectively. The four segments of horizontal pipe have lengths of 4 m, 5 m, 2 m, and 1 m, labelled x_1, x_2, x_3, and x_4 respectively. The diameter of all pipes is 50 cm. Additionally, these pipes are made from a futuristic material. As such, the surface of the pipes is perfectly smooth. The pump in the schematic is labelled as P. Take the resistance coefficients of the entrance and exit to be 1. Also assume that the Moody Diagram is valid on all planets.



A. You've been traveling for work so much recently that you've forgotten which planet you are on. However, the pump in the process you are currently working on is providing 92 J/kg of energy to the fluid and the volumetric flow rate through the system is 2 L/s. Using the information in the table below, determine which planet you are on.

Planet	**Atmospheric pressure (kPa)**	**Acceleration due to gravity (m/s^2)**
Planet X	70.5	12.2
Planet Y	101.3	9.8
Planet Z	176	7.5

(20 marks)

Question 1 continued on next page

Question 1 (continued)

B. Is the pump cavitating? Take the vapour pressure of water to be 2.5 kPa. The $NPSH_R = 9\ m$ for this pump on this planet and at this flow rate.

(5 marks)

C. Explain why more shaft work is needed to pump the same volumetric flow rate of water on Planet X than on Planet Z.

(3 marks)

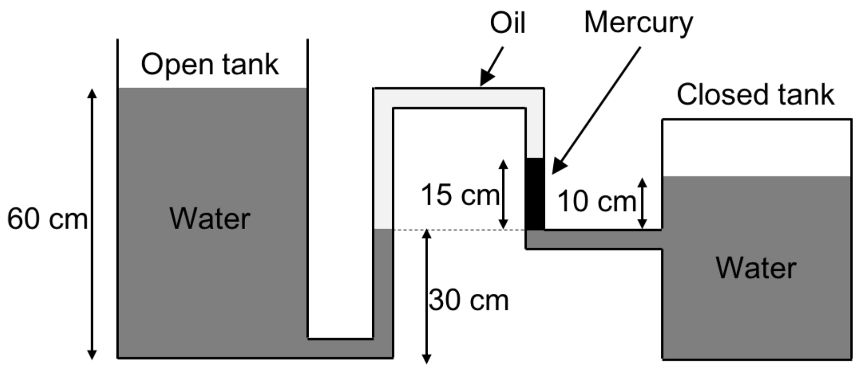
D. Why is there still friction from flow that happens in a perfectly smooth pipe?

(2 marks)

(Total for Question 1 = 30 marks)

Question 2 (15 marks)

You are currently on *Planet Fudge* (named after the intergalactic deity Fudge the Cat). The planet has the same atmospheric pressure as Earth, but it's acceleration due to gravity is only 8.2 m/s^2. You are working with a system of tanks, one of which is open to the atmosphere and the other of which is closed. The two tanks are connected by tubing that contains two manometer fluids, mercury and oil, as shown in the schematic below. The locations of water, mercury, and oil sections are specified by the vertical distances marked on the diagram, and the fluids are stationary.



A. Calculate the absolute pressure above the surface of the water in the closed tank.

(9 marks)

B. You are curious as to what the absolute pressure in the closed tank would need to be on Earth in order to achieve the same position of the manometer fluids. Calculate this absolute pressure.

(3 marks)

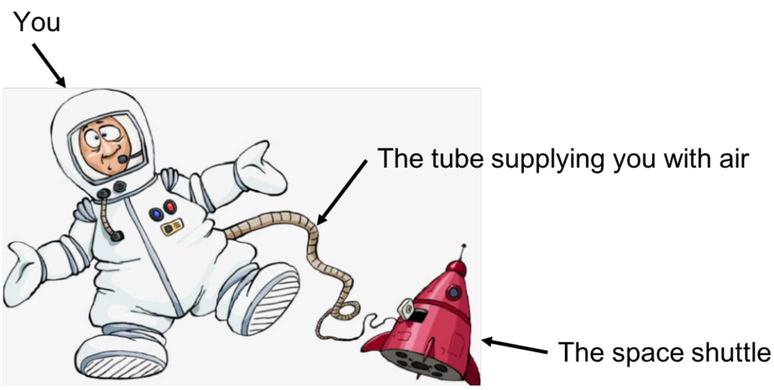
C. If the atmospheric pressure were decreased, would you need more, less, or the same pressure in the tank to maintain the position of the manometer fluids? Explain your answer in one to two sentences.

(3 marks)

(Total for Question 2 = 15 marks)

Question 3 (48 marks)

On another consulting job with *Intergalactic Engineering* you are performing a spacewalk in order to fix a satellite. This means that you are in a specially designed suit to protect you from the harsh environment of space while you repair the satellite. Your space suit is connected to the space shuttle by a tube that is supplying you with fresh air. This tube has a length of 200 m, a diameter of 5 cm, and an absolute roughness of 0.1 cm. The air within your suit and the tube is at 23 ^oC and 1 atm of pressure. The gas flow is isothermal.



Unexpectedly, the tube detaches from the space shuttle. This causes air to leave your suit, travel down the 200 m of the tube, and exit into the vacuum of space. You know that a person loses consciousness when the ambient pressure drops below 0.57 atm. You must reattach your tube to the air supply before the pressure in your suit drops to this point.

A. What is the initial velocity at which the gas exits the end of the tube into space? The pressure of space is 1.3x10^{-14} kPa.

(16 marks)

B. What is the velocity of the gas exiting the end of the tube into space when the pressure inside the suit is 0.57 atm?

(16 marks)

C. What is the initial mass flow rate at which gas leaves your suit? Neglect the contribution of kinetic energy to the flow in this calculation.

(7 marks)

Question 3 continued on next page

Question 3 (continued)

D. There is a fixed volume of 50 L of air in your suit. Using the initial flow rate you solved for in part C, estimate how long you have to reattach your tube before you lose consciousness.

(5 marks)

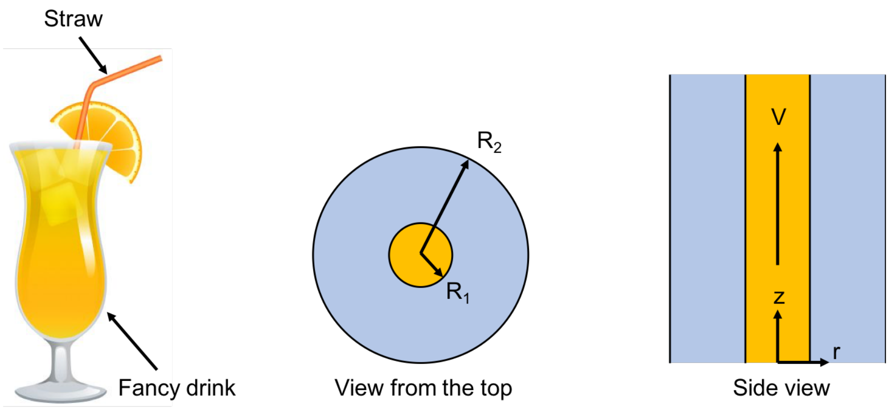
E. In the real life scenario, would you have more or less time that was calculated in part D? Explain your answer in one to two sentences.

(4 marks)

(Total for Question 3 = 48 marks)

Question 4 (42 marks)

Thankfully, you were able to reconnect your air tube before you lost consciousness. After such a stressful situation, you decided to take a vacation. On your vacation, you are sitting on a beach on Earth drinking a cocktail (a non-alcoholic one, of course). It's one of those fancy drinks with a straw, as shown below on the left. As an engineer who is fascinated by fluid flow, you start wondering about the velocity profiles within the fluid that you create by moving the straw. To simplify your analysis, you assume that the glass and the straw can be modelled as two infinitely long concentric cylinders, as shown in the middle picture of the schematic. The inner cylinder has a radius of R_1 and the outer cylinder has a radius of R_2. The space between the inner and outer cylinder contains only a Newtonian fluid (no ice). Additionally, assume that the cylinders are vertical so that gravity is acting only in the axial direction. Now imagine that the outer cylinder is stationary and the inner cylinder is moving upwards in the positive z-direction with a velocity of V, as shown in the image on the right.



A. Sketch the steady state velocity profile of the fluid. In which directions are the fluid flowing? In which directions are there gradients in the fluid velocity?

(6 marks)

B. Derive an expression that can be used to solve for the velocity profile within the fluid. Evaluate the expression at R_1 and R_2 to illustrate that it fulfils the boundary conditions.

(30 marks)

C. On one schematic, sketch the velocity profiles if the fluid were (i) a shear thinning fluid, (ii) a shear thickening fluid, and (iii) a Bingham plastic. Label each velocity profile. In one to two sentence each, explain how the flow of each of the fluids would differ from that of a Newtonian fluid.

(6 marks)

(Total for Question 4 = 42 marks)

Question 5 (21 marks)

After your holiday, you decide that you do not like working in outer space, so you apply for a new job with *Earthbound Solutions*, an engineering firm that works only on planet Earth. Part of your interview is to answer several fundamental questions on engineering fluid mechanics.

State whether the following statements are true or false. If you believe the statement is false, write the correct statement.

A. High Reynolds number flows are characterised by their slow rates of mixing and 'reversibility'.

(3 marks)

B. Gravity waves have a lower Bond number than capillary waves.

(3 marks)

C. If the Froude number is greater than 1 in an open-channel flow, waves can only propagate downstream.

(3 marks)

D. A hydraulic jump is a mechanism for a flow to transition from a low Froude number to a high Froude number.

(3 marks)

E. Droplets with low Weber numbers are unstable.

(3 marks)

F. When considering the dominant forces in a flow, surface tension is more likely to be important when the scale of the flow is small.

(3 marks)

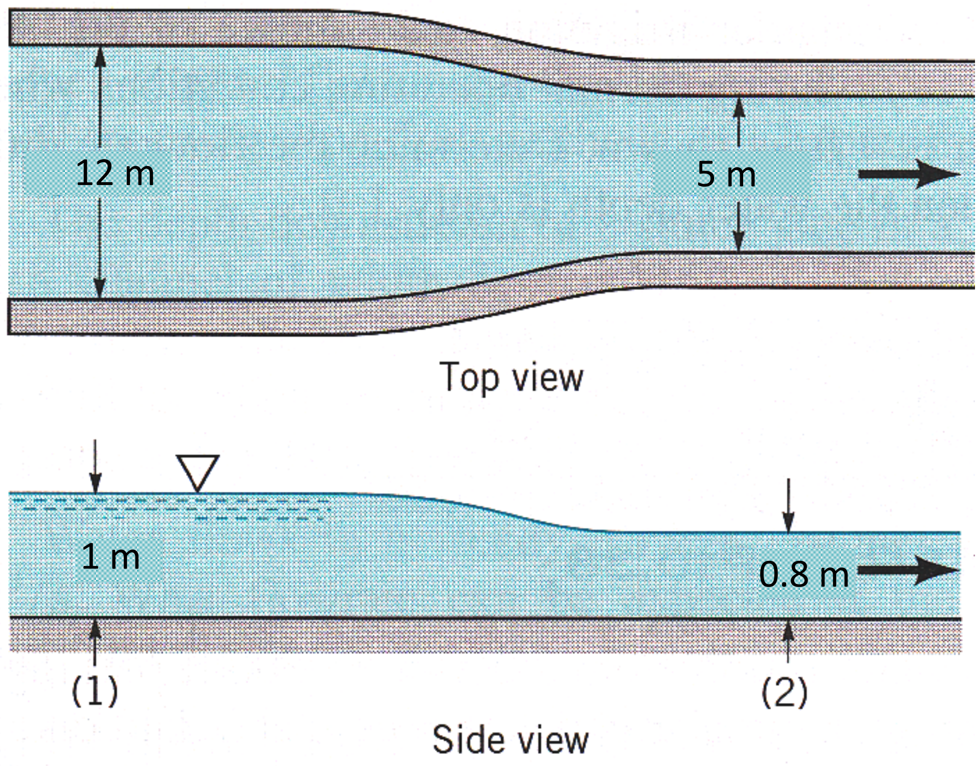
G. When creating a small-scale model of a large prototype, you can only match both the Reynolds number and Froude number if the fluid in the model is different from that in the prototype.

(3 marks)

(Total for Question 5 = 21 marks)

Question 6 (10 marks)

You received a job offer from *Earthbound Solutions*! On your first assignment, you are working with the frictionless flow of water in a wide rectangular channel (as shown in the figure below). The flow goes through a contraction, causing a decrease in the flow depth. Compute the flowrate in the channel.



(Total for Question 6 = 10 marks)

Question 7 (14 marks)

On your section assignment, you are to design a channel lined with wood (n = 0.012) to carry up to 2.0 m^3/s of water down a 1:1000 slope. The channel cross-section can be either a square OR a right-angled triangle. Which channel would require less wood and by what percentage?

(Total for Question 7 = 14 marks)

(Total for Exam = 180 marks)

Supplemental material only beyond this point

Physical properties of various fluids

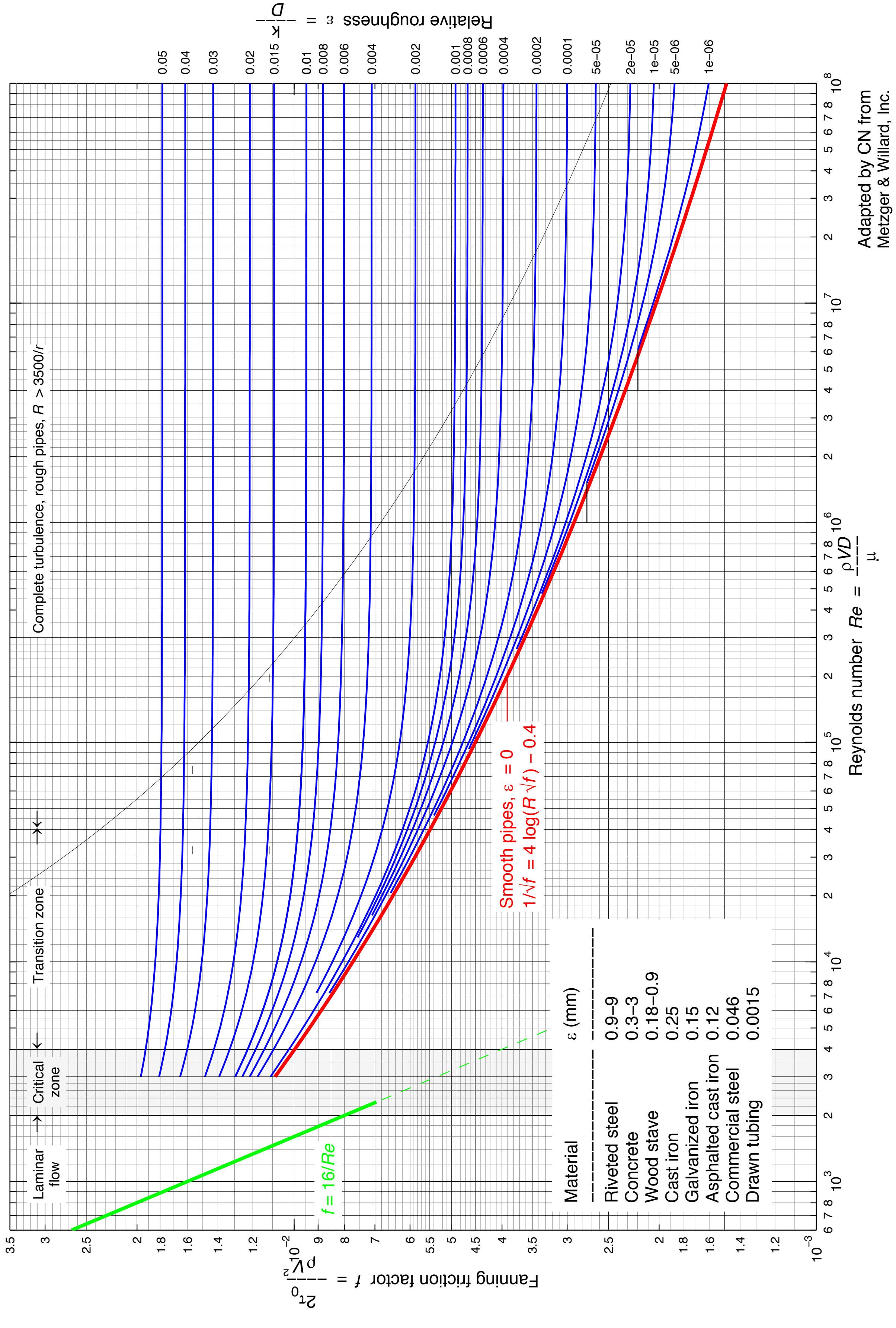
Parameter	Value	Units
Density of air	1.18	kg/m^3
Density of oil	800	kg/m^3
Density of water	1000	kg/m^3
Density of mercury	13600	kg/m^3
Viscosity of air	0.018	cP
Viscosity of water	0.89	cP
Ideal gas constant	8.314	J/mol K
Molar mass of air	28.9	g/mol
Atmospheric pressure on Earth	101.3	kPa

Equivalent lengths

Fitting	L_{eq}
45^o elbow	15D
90^o elbow	30 – 40D
90^o elbow square	60D
Entry from leg of T-piece	60D
Entry into leg of T-piece	90D
Unions and couplings	Very small
Gate valve – full open – half open – quarter open	7D 200D 500D

Absolute roughnesses

Pipe material	Roughness, e (mm)
Riveted steel	0.9 - 9
Concrete	0.3 - 3
Wood stave	0.2 – 0.9
Cast iron	0.26
Galvanized iron	0.15
Asphated cast iron	0.12
Commercial steal/wrought iron	0.046
Drawn tubing	0.0015


Relative roughness ε = k/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
5e−05
2e−05
1e−05
5e−06
1e−06
Complete turbulence, rough pipes, R > 3500/r
Transition zone
Critical zone
Laminar flow
f = 16/Re
Smooth pipes, ε = 0
1/√f = 4 log(R √f) − 0.4
Material
ε (mm)
Riveted steel 0.9–9
Concrete 0.3–3
Wood stave 0.18–0.9
Cast iron 0.25
Galvanized iron 0.15
Asphalted cast iron 0.12
Commercial steel 0.046
Drawn tubing 0.0015
Reynolds number Re = ρVD/μ
Fanning friction factor f = 2τ0/ρV²
Adapted by CN from Metzger & Willard, Inc.

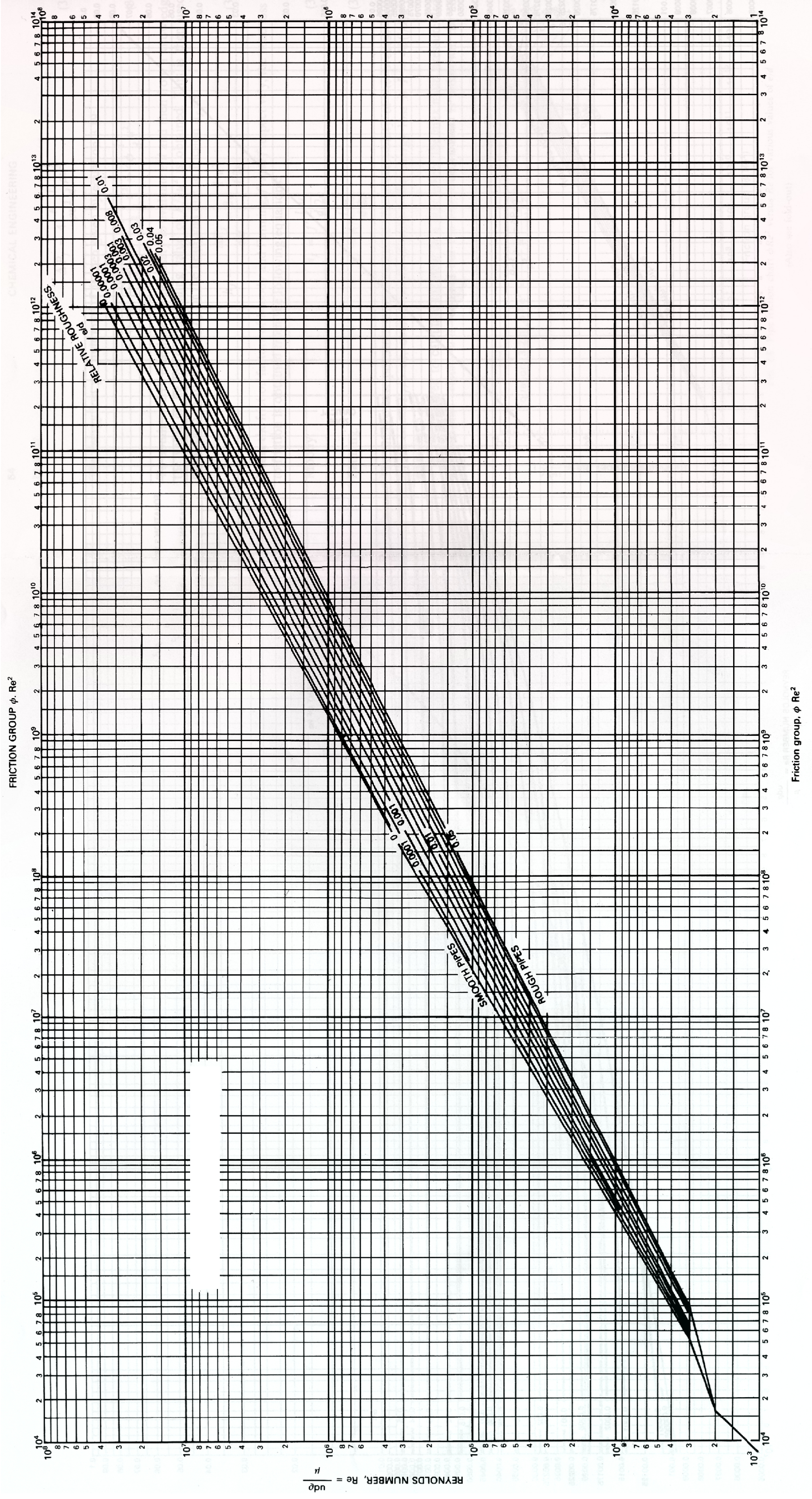


FIG. 3.8. Pipe friction chart ϕRe^2 versus Re for various values of e/d.

§B.4 THE EQUATION OF CONTINUITY[a]

$$[\partial\rho/\partial t + (\nabla \cdot \rho\mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Spherical coordinates (r, θ, ϕ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

§B.6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

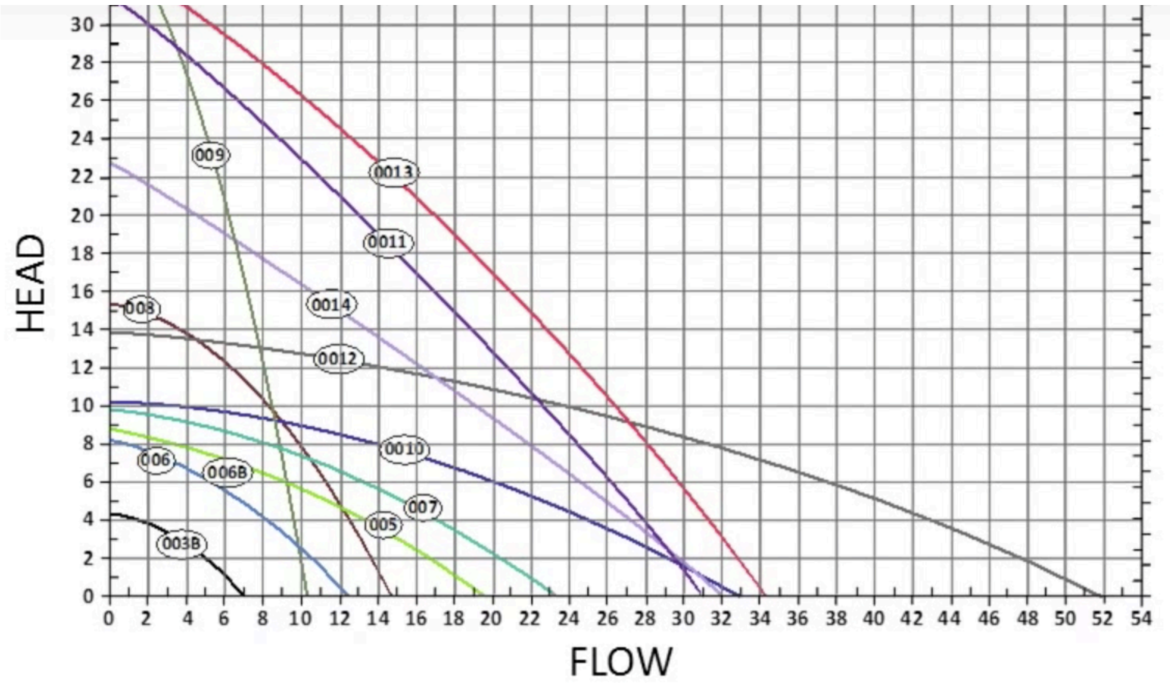
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi$$



In the chart above, head is in feet and flow is in gal/min

End of exam