MAST30001 2013, Recommended Problems, Chapter 2
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Instructions: Answer the following questions. Clearly show all work and give clear and concise explanations, using prose when appropriate. Clarity, neatness and style count.

1. Let $X_1, X_2, \ldots, X_n$ be independent and have exponential distributions with common rate λ (recall this means they each have density $\lambda e^{-\lambda x}$ for $x > 0$). What is the density of $X = \min\{X_1, \ldots, X_n\}$?

2. If Y has an exponential distribution with rate λ, show that Y has the *memoryless property*:
$$\mathbb{P}(Y > t + s | Y > t) = \mathbb{P}(Y > s)$$
and describe in words what this property means.

3. An electrical circuit requires 2 working diodes to function. Suppose that the lifetime of diodes in the system are independent and each have an exponential distribution with rate λ. If you have 4 diodes how long on average will the system function?

4. Bob and Doug are playing the following game. Bob starts by rolling two fair dice; if the sum of his dice is six, then he wins the game. If not, then Doug rolls the dice, and if the sum of his rolls is seven, then he wins the game. If neither player wins the game during the first round, then they repeat the process (with Bob going first) until someone wins a round. What is the probability that Bob wins this game? Is he more or less likely than Doug to win?

5. An urn initially contains one red and one blue ball. A ball is chosen uniformly at random from the urn and replaced along with another ball of the same color so that the urn contains three balls. This procedure is performed again on the urn (which now has three balls), so that in the end, the urn has four balls total in it, some (random) number of which are blue.

   (a) If X is the number of blue balls in the urn after this procedure, then what is the distribution of X? What is its mean and variance?

   (b) Answer the same questions for Y equal to the number of red balls in the urn after the procedure.