

MAST30013 – Techniques in Operations Research

Semester 1, 2021

Tutorial 3

1. For each of the following matrices, determine whether it is positive definite, positive semi-definite, negative definite, negative semi-definite, or otherwise.

(a)

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 7 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. For the following functions f ,

(i) find all stationary points $\mathbf{x}^* = (x_1^*, x_2^*)^T$;

(ii) for each stationary point \mathbf{x}^* , check whether the second-order sufficient condition holds;

(iii) check whether \mathbf{x}^* is a local minimum.

(a) $f(x_1, x_2) = x_1 + x_2 + x_1^2 - 4x_1x_2 + x_2^2$.

(b) $f(x_1, x_2) = x_1^2 - 5x_1x_2 + x_2^4 - 25x_1 - 8x_2$, where you may find it helpful to note that $y = 3$ is a zero of the polynomial $8y^3 - 25y - 141$.

3. For the functions given in Question 2, determine the set of all possible descent directions when $\mathbf{x} = (1, 2)^T$, and state the steepest descent direction.