**MAST30001 Stochastic Modelling**

**Assignment 1**

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Please complete and sign the Plagiarism Declaration Form (available through the LMS), which covers all work submitted in this subject. The declaration should be attached to the front of your first assignment.

**Don't forget** to staple your solutions and to print your name, student ID, and the subject name and code on the first page (not doing so will forfeit marks). The submission deadline is **Friday, 9 September, 2016 by 4pm** in the appropriate assignment box at the north end of Richard Berry Building (near Wilson Lab).

There are 2 questions, both of which will be marked. No marks will be given for answers without clear and concise explanations. Clarity, neatness, and style count.

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1. (a) Analyse the state space $S = \{1, 2, 3, 4\}$ for each of the three Markov chains given by the following transition matrices. That is, write down the communication classes and their periods, label each class as essential or not, and as transient or positive recurrent or null recurrent.

   i.
$$\begin{pmatrix} 0 & 1/8 & 0 & 7/8 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 3/8 & 0 & 5/8 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

   ii.
$$\begin{pmatrix} 1/4 & 0 & 0 & 3/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 0 \\ 1/6 & 0 & 0 & 5/6 \end{pmatrix}.$$

   iii.
$$\begin{pmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 0 & 3/4 & 1/4 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \end{pmatrix}.$$

   (b) For the Markov chain given by the transition matrix in part iii above, discuss the long run behaviour of the chain including deriving long run probabilities.

   (c) For the Markov chain given by the transition matrix in part iii above, find the expected number of steps taken for the chain to first reach state 3 given the chain starts at state 1.

   (d) For the Markov chain given by the transition matrix in part iii above, find the expected number of steps taken for the chain to first return to state 2 given the chain starts at state 2.

2. A Markov chain $(X_n)_{n\geq 0}$ on $\{0, 1, 2, \ldots\}$ has transition probabilities for $i = 1, 2, \ldots,$

$$p_{i,i+1} = 1 - p_{i,i-1} = \frac{1}{2}\left(\frac{i+1}{i+2}\right),$$

   and $p_{0,1} = 1 - p_{0,0} = 1/4$. Note that this chain is irreducible.

(a) Is the chain transient, null recurrent, or positive recurrent?

(b) Describe the long run behaviour of the chain (including deriving long run probabilities where appropriate).

(c) If $T(i) = \min\{n \geq 1 : X_n = i\}$, find $E[T(i)|X_0 = i]$ for $i = 0, 1, \ldots$.

(d) If $X_0 = 0$, what is the chance the chain reaches state 3 before it returns to state 0?