1. Please submit via the LMS portal by 23:59pm on March 21.
2. In submitting your work, you are consenting that it may be copied and transmitted by the University for the detection of plagiarism. Please start with the following statement of originality, which must be signed and dated by you:
   `"This is my own work.  I have not copied any of it from anyone else."`
3. This assignment is worth 10% of your final mark.

1. Wordy scenario. A wine-maker can't keep up with demand, their wine has a low sulfite content and has become very popular because of that. They can only produce 1250 bottles (1 bottle = 700 ml) per year. Because the wine-maker expects to sell at least 2750 bottles next year, they need to get wine supplied. They can get cheap wine with more sulfites or expensive wine with less sulfites. All wines will be blended and the final sulfite level should stay below 100 mg/L. The following table provides valuable info on costs, supply limits and sulfite levels.

Wine	Own production	Cheap wine	Expensive wine
Cost ($/100 liter)	500	400	900
Supply limit (bottles)	1250	2000	800
Sulfite level (mg/liter)	80	200	60

Please help the wine-maker and formulate his optimisation problem in mathematical terms. (No need to solve it.)

2. Visual method. Solve the linear optimisation problem

$$\begin{array}{rl} \text{Maximise} & z = x_1 + x_2 \\ \text{Subject to} & \frac{1}{2}x_1 - x_2 \leq -1 \\ & \frac{1}{2}x_1 + x_2 \leq 4 \\ & 3x_1 + x_2 \geq 5 \\ & 2x_1 - x_2 \leq 2 \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

by sketching the feasible region and drawing some level sets of the objective function. State the maximum and the point at which the maximum occurs. (Hint: this happens at the intersection of two lines.)

3. MATLAB functions manipulating arrays.

(a) Observe and explain the difference between `[1 2 3]^2` and `[1 2 3].^2`, and find out what happens if you add 1 to `[1 2 3]`.

(b) Create an array `X` with 9 elements, whose jth element is 5+3j, using each symbol 1 2 3 9 X + * [ ] = : exactly once.

(c) Write a function, called `F`, which takes an array and outputs the same array with every second element replaced by its square plus 1, and does not use a loop. E.g. the output of `F([1 2 3 4])` should be `[1 5 3 17]`.

(d) Write a function, called `D`, which takes a array, say x, and uses a loop to output its first difference, $y = \Delta x$, that is, $y_i = x_{i+1} - x_i$. (Hint: use the function `length` and find out how to use `zeros` to create an array of a certain length.)

(e) Write a anonymous function `A` that takes an array as input and outputs the sum of its elements that are divisible by 12, and apply it to `D(F(X))`.

4. Definiteness. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

(a) Write out the quadratic form corresponding to $\mathbf{A}$.

(b) Determine the definiteness of $\mathbf{A}$.

(c) Determine the definiteness of $\mathbf{A}$ on the subspace $\{\mathbf{x} : x_3 - x_1 + 2x_2 = 0\}$.

5. Conditions for minimisers, an unconstrained problem. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$, be given by

$$f(\mathbf{x}) = x_1^3 + (2x_1 + x_2 + 1)x_2.$$

(a) Determine ∇f and $D^2 f$.

(b) Find all points $\mathbf{p} \in \mathbb{R}^2$ that satisfy the FONC for f.

(c) For each point $\mathbf{p}$ found above determine the definiteness of $D^2 f(\mathbf{p})$, and make suitable deductions about these points using the SONC and the SOSC.

(d) Decide if any of the points are global extremisers. Then prove your statement, referring to the definitions given in section 1.2 of the text.

6. Conditions for minimisers, a constrained problem.

(a) Sketch the set $\Omega = \{\mathbf{x} \geq \mathbf{0}, x_1^2 - x_1x_2 + x_2^2 \leq 1\}$.

(b) Determine the feasible directions at the point $\mathbf{x}^* = \begin{pmatrix} 0 & 1 \end{pmatrix}^T$.

(c) Determine whether $\mathbf{x}^*$ is

  (i) a local minimiser [the FOSC, Theorem 5, is satisfied]

  (ii) not a local minimiser [the FONC is not satisfied]

  (iii) possibly a local minimiser [the FONC is satisfied]

of a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ with the property $\nabla^T f(\mathbf{x}^*) = \begin{pmatrix} 1 & -1 \end{pmatrix}$, subject to $\mathbf{x} \in \Omega$.

(d) Are the conditions of the FOSC, Conjecture 1, satisfied?

Hints: you may use `https://www.wolframalpha.com/` to help you plot Ω. To find the feasible directions, you may want to determine the slopes of the tangent lines to the boundary at $\mathbf{x}^*$ (which you can obtain by implicit differentiation).