MAT4MDS — Practice 1

Indices and logarithms

Index laws
Let a, b, m and n be real numbers with $a > 0$ and $b > 0$. **Law 1.** $a^m.a^n = a^{m+n}$ **Law 2.** $\frac{a^m}{a^n} = a^{m-n}$ **Law 3.** $(ab)^m = a^m b^m$ and $(\frac{a}{b})^m = \frac{a^m}{b^m}$ **Law 4.** $(a^m)^n = a^{mn}$ Consequences of these laws are: $a^0 = 1$ and $a^{-x} = \frac{1}{a^x}$.

Question 1. Simplify the following expression using index laws:

$$\left(\frac{(75x^3y^{-2}z)^2}{3z^9y^4x^2}\right)\left(\frac{(5xyz)}{15x^2z^{-1}}\right)^{-1}$$

Question 2. Solve the following for x:

(a) $x^2 + 4x = 21$
(b) $e^{2x} - 4e^x + 3 = 0$
(c) $6^x - 1 = 6^{1-x}$
(d) $4^x - 2^{x+3} + 12 = 0$
(e) $9^x = 3^x + 1$

Question 3. Sketch the graph of $y = 1 - e^{-x}$ by following these steps:

- Sketch the graph of e^x (see the reading on revision of exponential growth and decay).
- Reflect your graph in the y-axis to obtain the graph of e^{-x}.
- Reflect your graph in the x-axis to obtain the graph of $-e^{-x}$.
- Finally, shift your graph up (parallel to the y-axis) by one unit.

Logarithm laws

Let $x > 0$ and $y > 0$.

1. $\log_a(x) + \log_a(y) = \log_a(xy)$ **(from index law 1)**
2. $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ **(from index law 2)**
3. $\log_a(x^p) = p\log_a(x)$ **(from index law 4)**

Question 4. Consider the equation $a = b^m$. Taking the logarithm of this equation twice, first using base b and then using an unrelated base d, show that

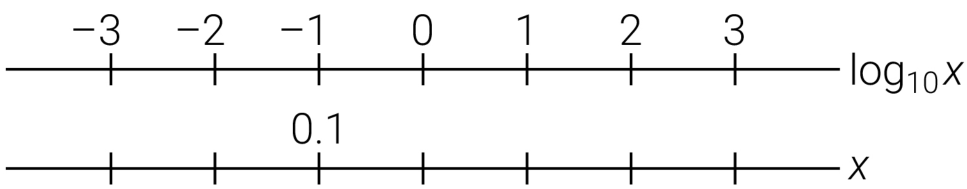
$$\log_b(a) = \frac{\log_d(a)}{\log_d(b)}$$

Explain why this is called the change of base rule for logarithms. Do calculators really need two logarithm buttons?

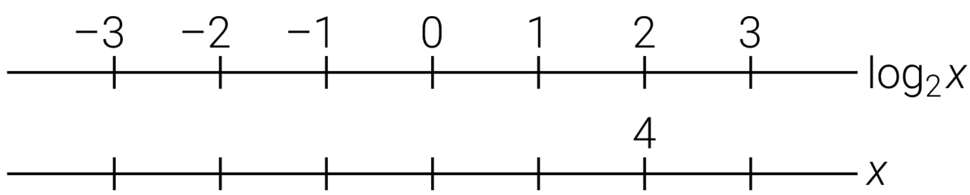
Question 5. Solve the following equations for x:

(a) $2\log_3(x) + \log_9(x) = 10$
(b) $\frac{1}{2}\log_e(x) = \log_e(2x - 1)$

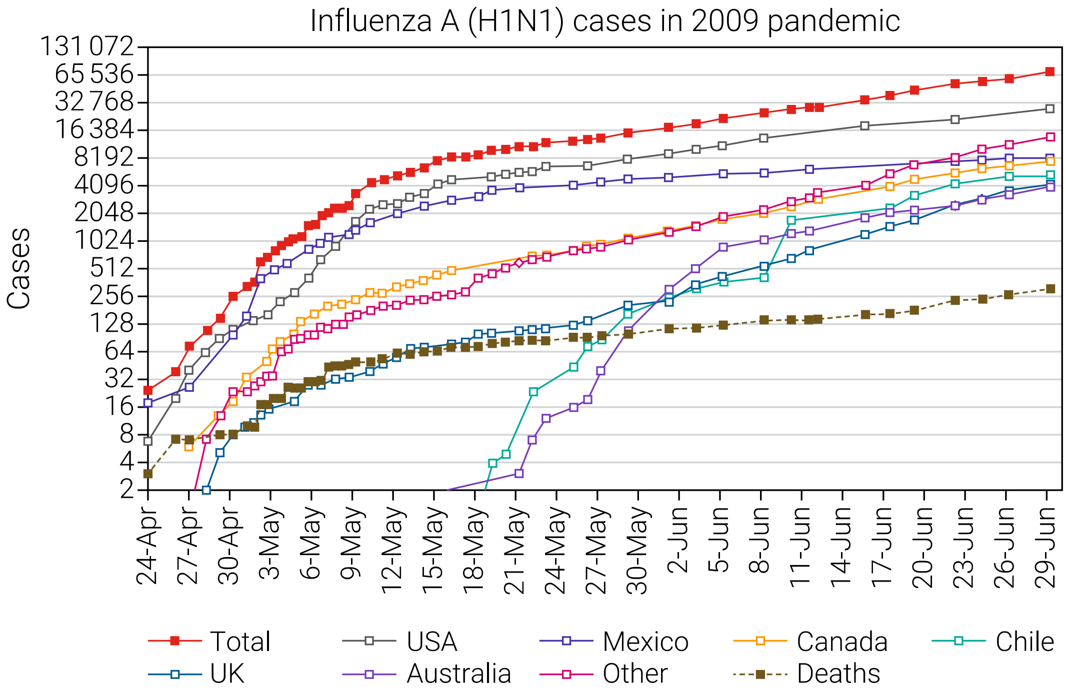
Question 6. Consider the following logarithmic scale. If this scale represents logarithms base 10, indicate on the second scale (underneath) the corresponding actual data values. (One has been done for you.)



Question 7. Consider the following logarithmic scale. If this scale represents logarithms base 2, indicate on the second scale (underneath) the corresponding actual data values. (One has been done for you.)



Question 8. Consider this graph which plots epidemiological data from the H1N1 flu epidemic of 2009:



Source: WHO (*http://www.who.int/csr/*)

(a) What kind of graph is this (e.g. linear, log–log)?
(b) The brown line indicating fatalities is about half as high as the red line indicating total cases. Does this mean half of all infected people died? Why or why not?
(c) The grey line (USA) is about one vertical scale mark below the red line (total) for all of the graph. What does this indicate?

Question 9. One page from a longer paper on analysis of rainfall data is set below. The full paper is available in the LMS (Lloyd, 2009).

(a) What kind of graph is Figure 1? What are its limitations? Have all the data available been included?
(b) What kind of graph is Figure 2? What does the negative slope indicate?
(c) What kind of graph is Figure 3? What conclusion has the author drawn from this graph?
(d) Read the sentence above Figure 5 and the scale of Figure 5. What do you notice?

Reference

Lloyd, P. (2009). On the determination of trends in rainfall. *Water SA*, *35*(3), 237–243.
http://www.scielo.org.za/pdf/wsa/v35n3/a01v35n3.pdf

The distributions of each set of data were determined by sorting into ascending order and straightforward decomposition into classes.

The distribution of rainfall

The instantaneous (5 min) data distribution is shown in Fig. 1. The records for 0.2 to 0.6 mm are omitted as the abscissa scale then becomes so large that the form of the distribution is hidden. Note that the omission is purely for illustrative purposes; the ~6000 records in the 0.2 to 0.6mm range were included in the later analysis. Note also that while points are shown, the numbers on the ordinate are in fact the upper bounds of ranges. So a point shown as 2.0mm rain in 5 min is strictly the range >1.8 ≤2.0 mm. Again, for clarity, the range is omitted. The same practice is followed in most distributions shown in the rest of this paper.

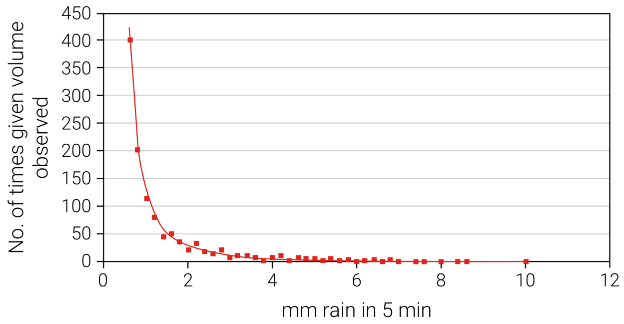


Figure 1
Rainfall measured in 5 min samples at Royal Natal National Park

When the data are re-plotted on logarithmic co-ordinates, as shown in Fig. 2, there is a very clear relationship between 5 min rainfall and frequency, as shown in Fig. 2. Figure 2 includes the data which had been omitted for clarity in Fig. 1.

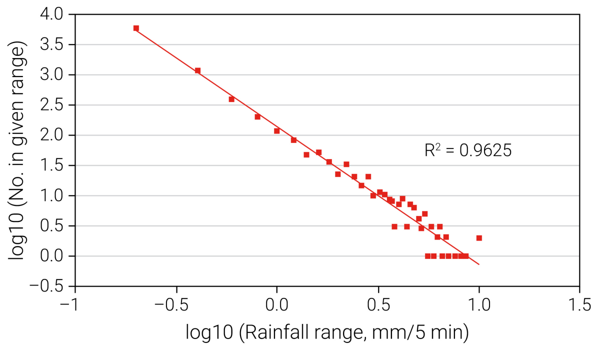


Figure 2
Log-log plot of data shown in Fig. 1

Over nearly 2 orders of magnitude of measured rainfall, and 4 orders of magnitude of frequency, a linear log-log relationship is observed with a regression coefficient of 0.9625, significant at the 0.1% level.

In Fig. 2, the data ranges are selected from the untransformed data. When the data ranges are selected from the log-transformed data, the distribution approximates that of the familiar normal distribution, as illustrated in Fig. 3.

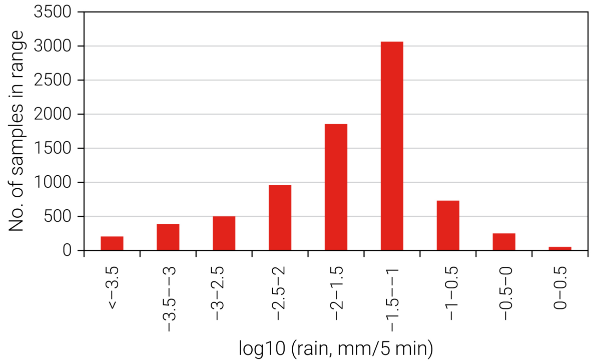


Figure 3
Distribution of log-transformed data shown in Fig. 1

The distribution of the daily rainfall in Florida is shown in Fig. 4. The distribution is clearly not a normal distribution.

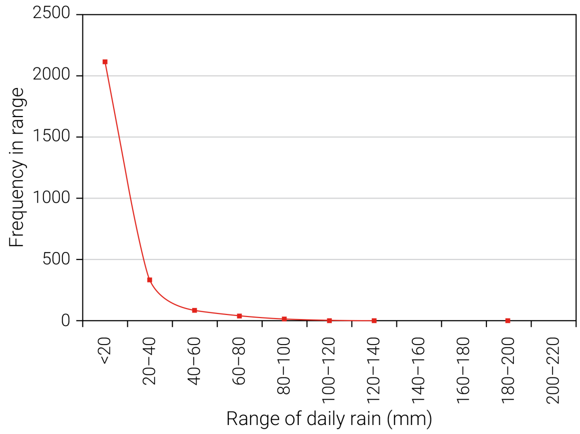


Figure 4
Frequency distribution of rain in various ranges in Florida, Site FL03

However, the frequency of the **logarithm** of the daily rainfall is again linear with a significant correlation coefficient, as shown in Fig. 5.

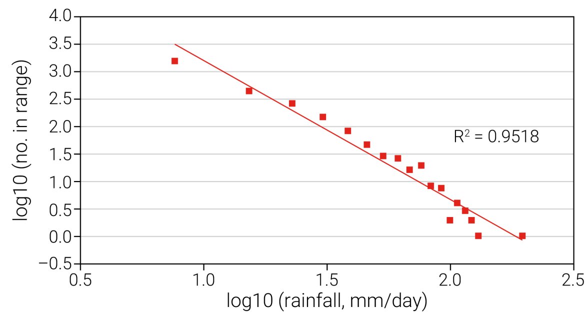


Figure 5
Frequency distribution of log_{10} (mm rain)

The 3rd data set, that from Site CA45, Hopland in California, plotted in a manner similar to that in Fig. 2, also gave a linear log-log relationship between volume and frequency, as shown in Fig. 5. In this case the correlation is poorer than in the case of the