As $t \to \infty$. $N(t) \approx \lambda t$. $\sum_{n=1}^{N(t)} D(n) \approx N(t) \cdot D \approx \lambda t D$

$$E(\lambda(t)/t) = \frac{1}{t} E[\sum_{n=1}^{N(t)} D_n] = \frac{1}{t} E[E[\sum_{n=1}^{N(t)} D_n | N(t)]]$$

$$= \frac{1}{t}[E(N_t) D] = \frac{\lambda t D}{t} = \lambda D$$

$$\Rightarrow L = \lambda D$$

M/M/a queue:

$a \geq 1$ servers. birth rate: = Poisson arrival process → rate λ

death rate: k servers working $\exp(k\mu)$.

The expected queue length is

$$L_q = E[\max(X_t - a, 0)] = \sum_{k=a}^{\infty}(k-a)\pi_k = \frac{\pi_0}{a!}\sum_{k=a}^{\infty}(k-a)\left(\frac{\lambda}{\mu}\right)^k \frac{1}{a^{k-a}}$$

$$= \frac{\pi_0}{a!}\left(\frac{\lambda}{\mu}\right)^a \frac{\lambda}{a\mu} \cdot \frac{1}{(1-\frac{\lambda}{a\mu})^2}$$

$$P_q = \sum_{k=a}^{\infty} \pi_k = \pi_0 \frac{\lambda^a}{\mu^a a!} \frac{a\mu}{a\mu - \lambda} \qquad L_q = \frac{\lambda}{a\mu - \lambda} P_q$$

The expected number N_b of busy servers

$$E[\min(X_t, a)] = \sum k\pi_k + aP_q$$

ρ load : Expectation stationary number of custom in system

$$L = E[q_{eo}(1-\rho)] = \frac{\rho}{1-\rho}$$

$$L_q = \sum_{k\geq 1}(k-1)\pi_k = \sum_{k\geq 0}(k-1)\pi_k + \pi_0 = \frac{\rho}{1-\rho} - 1 + \frac{\pi_0}{1-\rho}$$

k-1 people waiting for service
k people in line

$X \sim Exp(\lambda)$. $E[e^{-sX}] = \frac{\lambda}{s+\lambda}$. W: waiting time.

$$E[e^{-sW}] = E[E[e^{-sW}|N]] = E\left[\left(\frac{\mu}{s+\mu}\right)^N\right]$$

$$= (1-\rho)\sum_{n=0}^{\infty}\rho^n\left(\frac{\mu}{s+\mu}\right)^n$$

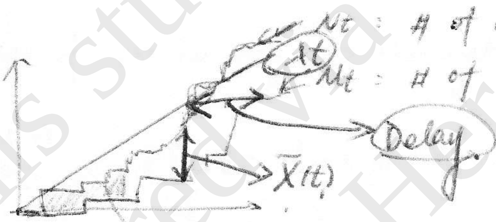
$$= \frac{(1-\rho)(s+\mu)}{s+\mu-\lambda} = 1-\rho + \rho\frac{\mu-\lambda}{s+\mu-\lambda}.$$

$$\Rightarrow W \begin{cases} 0 & \text{w.p. } 1-\rho \\ Exp(\mu-\lambda) & \text{w.p. } \rho \end{cases}$$

$E[W] = \frac{\rho}{\mu-\lambda}$ (expected waiting time)

Expected delayed time $D = E[W] + \frac{1}{\mu} = \frac{1}{\mu-\lambda}$.

Arrival process



N_t : # of arrival at time t.
M_t : # of people served on time t

$\frac{L}{D} \approx \lambda$. → the area between two curves:
$A(t)$ → area under $L(t)$

$$\frac{A(t)}{t} = \frac{1}{t}\int_0^t L(u)\,du.$$

Also you have $\frac{A(t)}{t} = \frac{1}{t}\sum_{t=1}^{N(t)} D_n$ (D_n is delay experienced by the nth customer).

⇓

look at the horizontal area approximation

$A(t) = \sum_{j=0}^{\infty} j \cdot$ (Amount of time at state j up to time t)

$\frac{A(t)}{t} = \sum j$ (proportion of time at state j up to time t)

⇓ π_j ($t \to \infty$) → L.

$\Rightarrow \frac{A_t}{t} \to E$[# people in system in stationary]