1. Please submit via the LMS portal by 23:59pm on Thursday May 30 (week 12).
2. In submitting your work, you are consenting that it may be copied and transmitted by the University for the detection of plagiarism. Please start with the following statement of originality, which must be signed and dated by you:
`"This is my own work.  I have not copied any of it from anyone else."`
3. This assignment is worth 10% of your final mark.

1. A function $\mathbf{h} : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by
$$\mathbf{h}(\mathbf{x}) = \begin{pmatrix} y^2 + 2yz + 2z^2 - 2y - 4z \\ x^2 - 2xy + 3y^2 + 4yz - 4y \end{pmatrix}.$$
Define $\mathcal{H}$ to be the level set $\mathcal{H} = \{\mathbf{x} \in \mathbb{R}^3 : \mathbf{h}(\mathbf{x}) = (-1, 2)^T\}$.
   (a) Which of the points
   $$\mathbf{p}_1 = (2, 0, 1)^T, \quad \mathbf{p}_2 = (-1, 1, 0)^T, \quad \mathbf{p}_3 = (3, 1, 0)^T, \quad \mathbf{p}_4 = (2, 2, -1)^T$$
   are
      i. in $\mathcal{H}$?
      ii. regular points of $\mathbf{h}$?
   (b) Consider the parametrised curve, $\boldsymbol{\gamma} : \mathbb{R} \to \mathbb{R}^3$,
   $$\boldsymbol{\gamma}(t) = \big(\cos(t) + \sin(t), \sin(t) - \cos(t), \cos(t) + 1\big)^T.$$
      i. Is $\boldsymbol{\gamma}$ a curve in $\mathcal{H}$?
      ii. Which of the points $\mathbf{p}_i$ are on $\boldsymbol{\gamma}$? For those points, provide t such that $\boldsymbol{\gamma}(t) = \mathbf{p}_i$.
   (c) Find a basis for the normal space $N\mathcal{H}(\mathbf{p}_2)$.
   (d) Find a basis for the tangent space $T\mathcal{H}(\mathbf{p}_2)$.
   (e) Show that the t-derivative $D\boldsymbol{\gamma}$ at $\mathbf{p}_2$ is in the tangent space $T\mathcal{H}(\mathbf{p}_2)$, and that it is orthogonal to the normal space $N\mathcal{H}(\mathbf{p}_2)$.

2. Let P be the parabola $P = \{(x, y) : \ (y + x)^2 + y = x + \frac{3}{2}\}$. Determine, with reasons (i.e. use the SOSC), all points on P which are
   (a) closest to $\mathbf{0}$,
   (b) locally furthest to $\mathbf{0}$.

3. Let $f, g, h : \mathbb{R}^3 \to \mathbb{R}$ be given by
$$f(\mathbf{x}) = (x + y - 3)^2 + (y - z + 2)^2, \quad h(\mathbf{x}) = -x^2 + y + z - 5, \quad g(\mathbf{x}) = y - z - 1.$$
Consider the objective function f with feasible set
$$\Omega = \{\mathbf{x} \in \mathbb{R}^3 : h(\mathbf{x}) = 0, \ g(\mathbf{x}) \le 0\}.$$
Write down the KKT conditions, solve them and hence find all possible extremisers. You do **not** need to classify them using second order conditions. Hint: consider the linear combination $(-1, 1, 1) \cdot \nabla(f + \lambda h + \mu g) = 0$.

4. Find the optimal separating hyperplane, specified by a 3×1 vector $\mathbf{w}$ and a scalar b, for the 3-dimensional `Trainingdata.csv` with `Traininglabels.csv` provided to you on LMS, and use it to label the `Newdata.txt`.