**Department of Infrastructure Engineering/Department of Chemical and Biomolecular Engineering**

# ENGR30002 Fluid Mechanics

**Semester 1, 2018**

**Reading Time: 15 minutes – no writing or annotating allowed anywhere**

**Writing time: 180 minutes**

**This paper has 12 pages including this page.**

**Authorised Materials**
**Calculators:** Casio FX82 calculators are permitted

**Instructions to Invigilators**

- The examination paper IS TO REMAIN in the examination room
- Students are to be provided with 1 script book
- Students should include detached pages from question paper in the script book for collection
- Provide extra script books on request

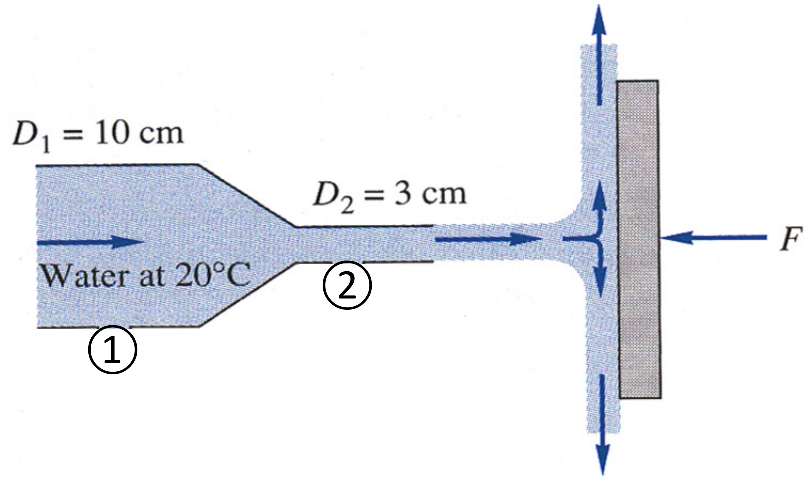
**Instructions to Students**

- This total mark for this paper is 100. All questions should be attempted.
- Ensure your student number is written on all script books during writing time. No annotating is allowed in reading time or after the end of writing time.
- Answer all questions on the right-hand lined pages of the script book
- Start the answer to each question on a new page in the script book and write the question number in the top right hand corner.
- The left-hand unlined pages of script books are for draft working and notes and <u>will not be marked</u>
- State clearly and justify any assumptions made.
- Write legibly in pencil, blue pen or black pen.
- Show all working for each problem.
- Mobile phones, tablets, laptops, and other electronic devices, wallets and purses must be placed beneath your desk.
- All electronic devices (including mobile phones and phone alarms) must be switched off and remain under your desk until you leave the examination venue.
- No items may be taken to the toilet.
- Attachments after the questions:
    - Moody diagram (Page 9)
    - Fluid properties and physical constants (Page 10)
    - Equation sheet (Pages 11-12)

***Paper to be lodged with Baillieu Library***

## Question 1

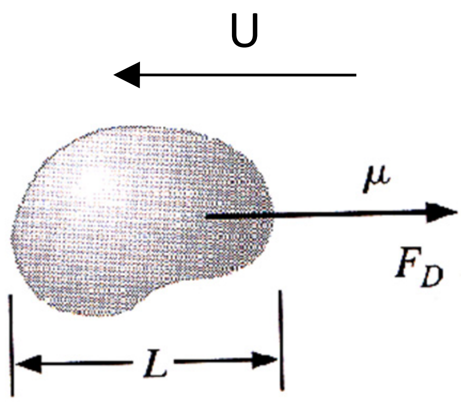
Water flows through a 10-cm-diameter pipe (which has a 3-cm-diameter circular nozzle at its end) before exiting into the air as a jet. The jet strikes a round plate (with a diameter of 20 cm). If the force required to hold the plate steady ($F$) is 70 N, determine the velocity and pressure in the pipe (i.e. at ①).




**(Total for Question 1 = 15 marks)**

## Question 2

When small particles move through a fluid, the Reynolds number is typically very small ($Re << 1$). Such flows are called creeping flows. The drag force ($F_D$) on an object in creeping flow is a function only of its speed $U$, its characteristic length scale $L$ and the dynamic fluid viscosity, $\mu$.




(a) Use dimensional analysis to derive the dependence of the drag force on the independent variables. **(6 marks)**

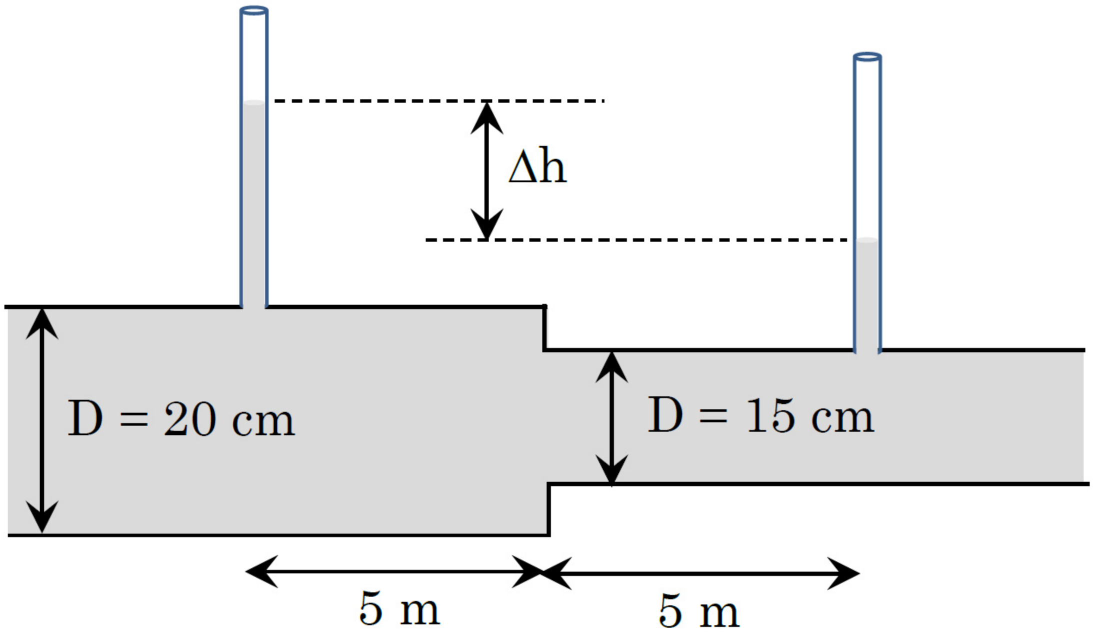
(b) Consider pollen particles falling at terminal velocity through air. At terminal velocity, the drag force on the object is equal and opposite to its weight. Assuming the density of pollen to be a constant, if the length scale of a pollen particle is doubled, what happens to its terminal velocity?

**(6 marks)**

**(Total for Question 2 = 12 marks)**

## Question 3

Water flows at 0.025 $m^3/s$ through a contraction in a smooth pipe as shown in the figure below. Estimate the difference in the two piezometer readings ($\Delta h$).




**(Total for Question 3 = 15 marks)**

## Question 4

The figure below shows channel flow over a bump, which results in a hydraulic jump downstream. The depth at Point 1 is 100 cm and the flowrate per unit width in the channel is 1.6 $m^2s^{-1}$.

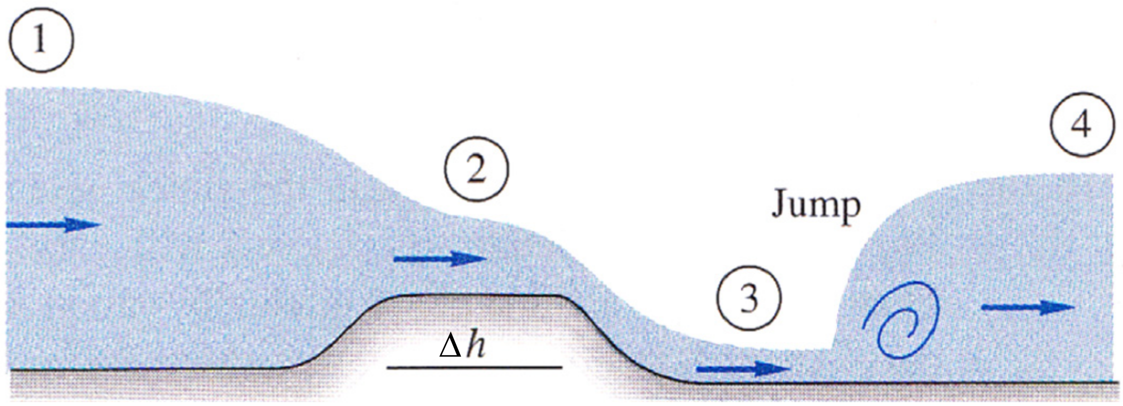
(a) Determine the depth at Point 2 **(4 marks)**

(b) Determine the bump height, $\Delta h$ **(5 marks)**

(c) Through energy considerations, show that the depth at Point 3 is 43.3 cm, and thus calculate the depth at Point 4. **(5 marks)**

(d) Without worrying about numbers, sketch the specific energy curve for this channel, and indicate (roughly) the locations 1, 2, 3 & 4 on the curve.

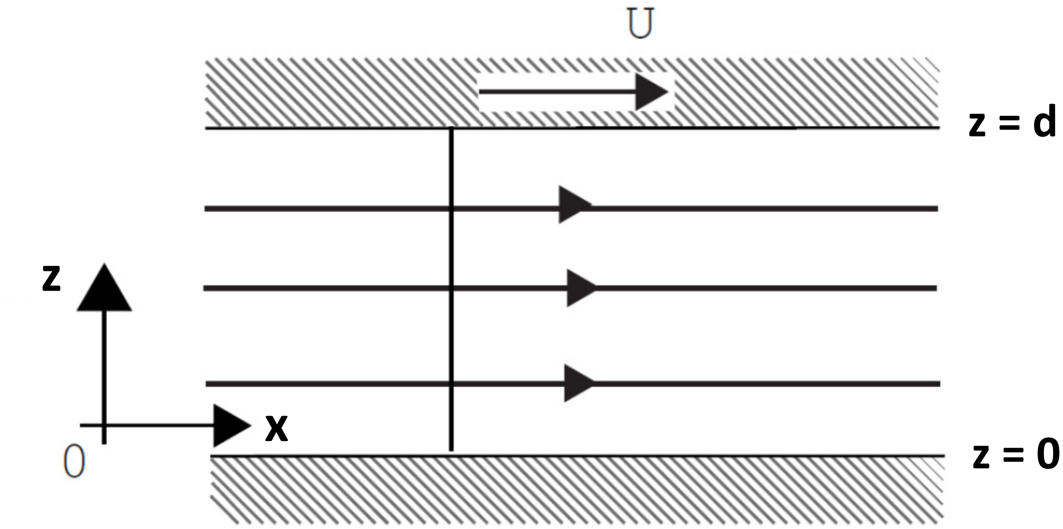
**(4 marks)**




**(Total for Question 4 = 18 marks)**

## Question 5

In class, we looked at Couette flow, where we have steady flow between two infinite, parallel plates. The top plate is moving at a speed $U$ and the lower plate is stationary.




Consider the same geometry but with a constant applied pressure gradient, $\mathrm{d}p/\mathrm{d}x = \beta$. Note that $\beta < 0$, as the pressure exerts a force in the positive $x$-direction.

(a) Derive the expression for the velocity profile, $u(z)$. **(5 marks)**

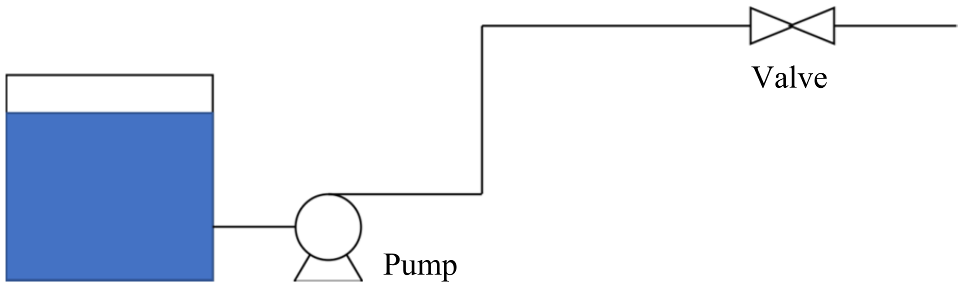
(b) What is the applied pressure gradient that will create zero shear stress on the moving plate? **(5 marks)**

(c) What is the ratio of the flowrate for the flow in (b) to that in pure Couette flow (i.e. with no pressure gradient)? **(5 marks)**

**(Total for Question 5 = 15 marks)**

## Question 6

A process liquid is pumped from a closed tank through the piping system sketched below. The space above the liquid is slightly pressurized to 0.1 kPa above atmospheric pressure. The pump draws the fluid from a point 1.5 m below the liquid surface in the tank which is maintained at a constant level. The piping leading from the pump begins with a 4 m long section having an internal diameter of 5 cm and containing two elbows. Between the second elbow and the (open) gate valve, the pipe expands to an 8 m long pipe section having an internal diameter of 10 cm. The 8 m section of pipe is 1.8 m above the centreline of the pump. The liquid discharges freely to the atmosphere.




| | |
|---|---|
| Liquid density | 1157 kg/m$^3$ |
| Liquid dynamic viscosity | 0.0019 Pa.s |
| Absolute roughness of pipes | 0.01 cm |

(a) Calculate the power required to pump the liquid at a rate of 1.5 kg/s.

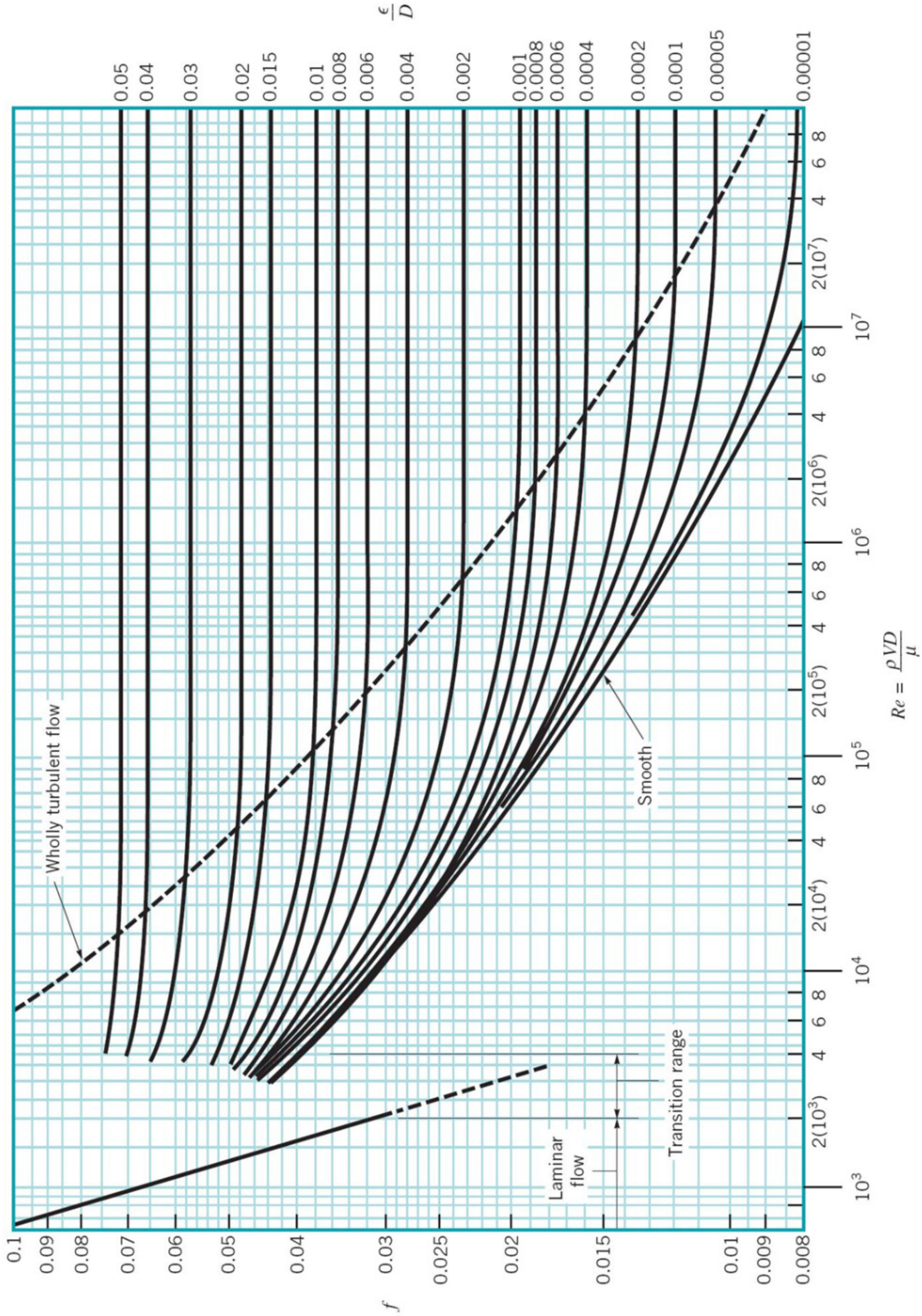
**(18 marks)**

(b) Determine the gauge pressure at the discharge of the pump. Ignore the friction that occurs in the short segment of pipe between the storage vessel and the suction side of the pump. **(7 marks)**

**(Total for Question 6 = 25 marks)**

# END OF EXAM

## MOODY DIAGRAM (FOR THE DARCY FRICTION FACTOR)



## FLUID PROPERTIES AND PHYSICAL CONSTANTS

| | **Water** | **Air** |
|---|---|---|
| $\rho$ | 998 kg/m$^3$ | 1.2 kg/m$^3$ |
| $\nu$ | $1 \times 10^{-6}$ m$^2$s$^{-1}$ | $1.5 \times 10^{-5}$ m$^2$s$^{-1}$ |
| $\sigma$ | 0.07 N/m at the air-water interface | |

Acceleration due to gravity, $g = 9.8$ ms$^{-2}$

Standard atmospheric pressure, $p_{atm} = 101.3$ kPa

Gas constant, $R = 8.314$ J mol$^{-1}$ K$^{-1}$

# EQUATION SHEET

Conservation of mass

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Navier-Stokes equations

$$(x)\quad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+g_x+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$(y)\quad \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+g_y+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$(z)\quad \frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+g_z+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

Hydrostatic force on vertical plates

$$F_R=\rho gab\left(s+\frac{a}{2}\right)$$

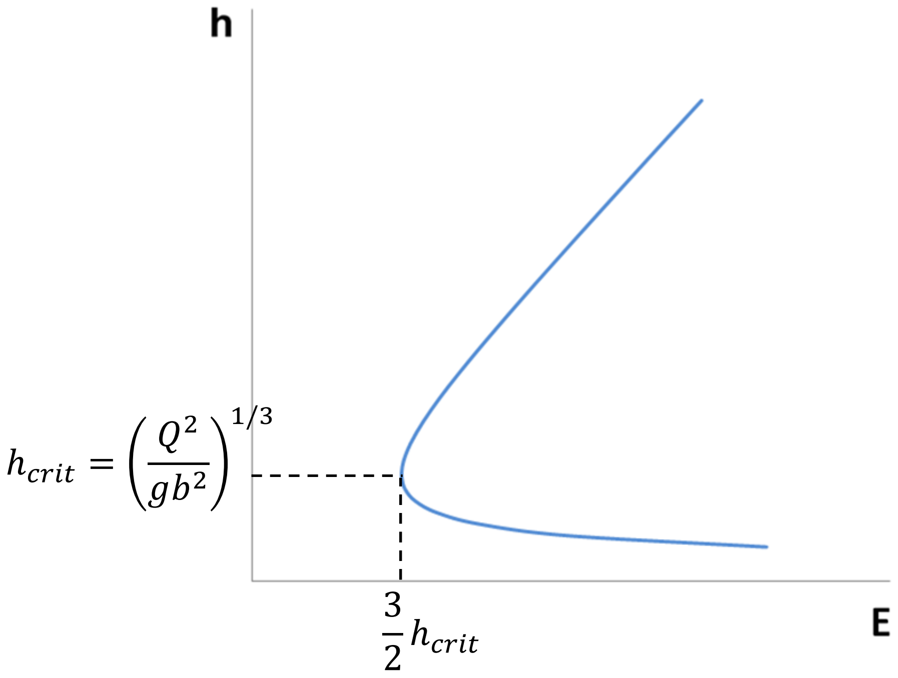
$$h_R=\frac{a}{2}+\frac{a^2/12}{s+(a/2)}$$

Manning's equation

$$U=\frac{1}{n}R_h^{2/3}S^{1/2}$$

Specific energy

$$E(h)=\frac{Q^2}{2gb^2h^2}+h$$




Hydraulic jumps

$$\frac{h_2}{h_1} = \frac{-1 + \sqrt{1 + 8Fr_1^2}}{2}$$

Pump/turbine power

$$P_P = \rho g Q h_P$$

$$P_T = \rho g Q h_T$$

Minor loss coefficients

| | |
|---|---|
| Expansion | $\left(1 - \frac{D_{small}{}^2}{D_{large}{}^2}\right)^2$ |
| Contraction | $0.5\left(1 - \frac{D_{small}{}^2}{D_{large}{}^2}\right)$ |
| Open valve | 0.15 |
| Elbow | 0.8 |

Compressible flow

$$\frac{P_2^2 - P_1^2}{2(RT/M)} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{2f_F L}{D}\left(\frac{G}{A}\right)^2 = 0$$

$$\frac{4f_F L_{crit}}{D} = \left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1$$