Practice Exam for MAT4MDS

Question 1. *Total: 8 marks*

(a) Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ *2 marks*

(b) Hence find the inverse of the matrix $K = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. *2 marks*

(c) Find the three eigenvalues of K. *3 marks*

(d) Is K of full rank? Why or why not? *1 mark*

Question 2. *Total: 10 marks*

(a) In the following, you may not use the identity matrix, the matrix of ones or the zero matrix as your examples.

(i) Write down an example of a 2×2 matrix which does not have an inverse. *1 mark*

(ii) Write down an example of a 3×3 matrix N such that $N = N^T$. *1 mark*

(iii) Write down an example of a 2×2 matrix M such that $\text{trace}(M) = 0$. *1 mark*

(b) Find the least squares line $y = \alpha x + \beta$ for the following data:

y	4	5	7	10
x	-2	-1	2	3

5 marks

(c) Calculate the average (mean) value of the independent variable in part (b), and use the least squares line to calculate the average value of the dependent variable. *2 marks*

Question 3. *Total: 12 marks*

Consider the function

$$h : \mathbb{R} \to \mathbb{R},\ h(x) = x^2 - e^{-x}$$

(a) Find the x coordinate of the point of inflection of $h(x)$. *3 marks*

(b) What is the slope of the graph at $x = 0$?
What is the value of the function at $x = 0$? *2 marks*

(c) You are given that $h(x)$ has no local maximum or minimum points. In principle, does this function possess an inverse? (Note: Do not attempt to find such an inverse, if it exists.) *1 mark*

(d) Sketch the graph of h, showing clearly the features found in (a) and (b). *3 marks*

(e) Find the area under part of the graph of h given by *3 marks*

$$\int_1^4 h(x)dx$$

Use this page if you need more space. Label your answers with the question number.

Question 4. *Total: 7 marks*

(a) Evaluate *3 marks*

$$\int_1^9 \sqrt{x}\log_e(x)dx$$

(b) Using substitution, find the following

(i) $\int_0^\infty x^2 e^{-x^3}dx$ *2 marks*

(ii) $\int_0^\infty x^4 e^{-x^3}dx$ *2 marks*

Question 5. *Total: 12 marks*

Consider the function of two variables

$$f(x, y) = x^3 y - 2e^{y^2}$$

(a) Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ *2 marks*

(b) Find $\dfrac{\partial^2 f}{\partial x^2}$ and $\dfrac{\partial^2 f}{\partial y^2}$ and $\dfrac{\partial^2 f}{\partial y \partial x}$ *3 marks*

(c) Hence find the second order Taylor polynomial for $f(x, y)$ near $(1, 0)$. *3 marks*

(d) Let $g(x, y) = x^2y - x\log_e(y)$.

Calculate:

$$\int_1^e \int_0^3 g(x, y)\ dx\ dy$$

4 marks

Question 6. *Total: 6 marks*

(a) Show that

$$xB(1, x) = 1$$

2 marks

(b) A student plots data using a log-log plot, and observes that it falls on a line with slope 3. If the variables plotted are x on the horizontal axis and T on the vertical axis, what is the form of the relationship between T and x? *2 marks*

(c) Another student plots different data using a linear-log plot, and observes that it falls on a line with slope -3. If the variables plotted are x on the horizontal axis and z on the vertical axis, what is the form of the relationship between z and x? *2 marks*

$* * * *$ **End of Questions** $* * * *$