SCHOOL OF MATHEMATICS AND STATISTICS

MAST30013 Techniques in Operations Research **Semester 1, 2021**

Assignment 1 **Due: 5 pm, Wednesday 31 March**

- Please make sure that you have written your name and student number on the front page. Solution must be typeset in LaTex and uploaded to Canvas as a pdf.

- Upload your solution to Canvas by the due date.

- Show all necessary working.

- There are three questions, two of which will be picked for marking.

1. Consider the problem:
$$\min f(x) := e^{-x} - \cos x$$
where $x \in [0, 1]$.
   (a) Prove that f is a unimodal function and there is a unique global minimum in the interior of $[0, 1]$.
   (b) Reduce the size of the interval containing the global minimum to less or equal to 0.5 using Golden section method.

2. Use the Fibonacci algorithm to find the minimum of
$$f(x) = -\frac{1}{(x-1)^2}\left(\log x - \frac{2(x-1)}{x+1}\right),$$
which is known to lie in $[1.5, 4.5]$. Reduce the interval to $1/21$ of the original.

3. Suppose that $f : [0, \infty) \to \mathbb{R}$ is a continuous unimodal function with $f'(0) < 0$. Show that, for $\sigma \in (0, 1)$ and $\mu \in [\sigma, 1)$, there exists a stepsize $t > 0$ that satisfies both the Armiji-Goldstein condition,
$$f(t) \ \leq \ f(0) + t\sigma f'(0),$$
and the Wolfe condition,
$$f'(t) \ \geq \ \mu f'(0).$$
In other words, if $\sigma \in (0, 1)$ and $\mu \in [\sigma, 1)$, the two regions defined by the Armijo-Goldstein and Wolfe conditions overlap.