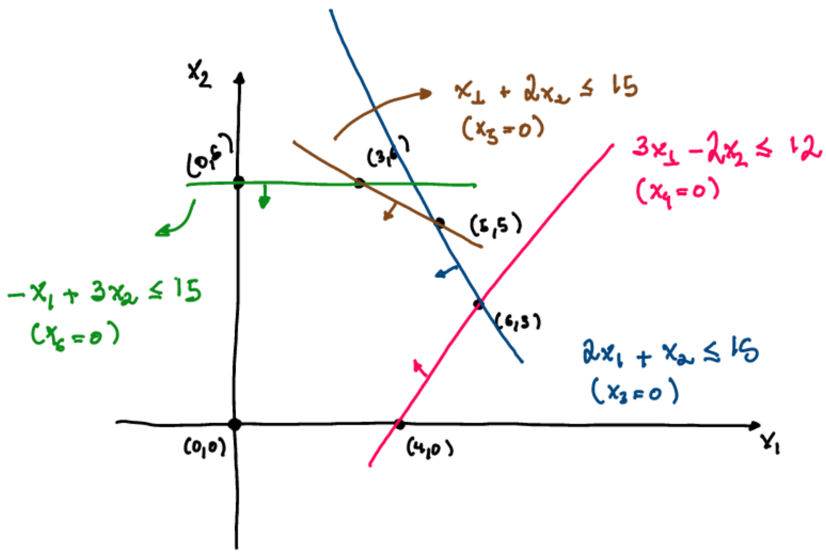
# Assignment 2 solutions

**Feasible space**:




**Canonical form**:

$$
\begin{aligned}
&\max 5x_1 + 2x_2 \\
s.t. \quad & 2x_1 + x_2 + x_3 = 15 \\
& 3x_1 - 2x_2 + x_4 = 12 \\
& x_1 + 2x_2 + x_5 = 15 \\
& -x_1 + 3x_2 + x_6 = 15 \\
& x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}
$$

**Iteration 1**:

Basic variables $x_3, x_4, x_5, x_6$
Non basic variables $x_1, x_2$

Tableau:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS | Ratio Test |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 2 | 1 | 1 | 0 | 0 | 0 | 15 | 7.5 |
| $x_4$ | $\boxed{3}$ | -2 | 0 | 1 | 0 | 0 | 12 | 4 |
| $x_5$ | 1 | 2 | 0 | 0 | 1 | 0 | 15 | 15 |
| $x_6$ | -1 | 3 | 0 | 0 | 0 | 1 | 15 | - |
| $z$ | -5 | -2 | 0 | 0 | 0 | 0 | 0 | |

$$A_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_B^{-1} \qquad y = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -1 \end{bmatrix}$$

Basic solution is $(0, 0, 15, 12, 15, 15)$, corresponding extreme point is $(0, 0)$. Entering variable is $x_1$, exiting variable is $x_4$.

**<u>Iteration 2</u>**:

Basic variables $x_3, x_1, x_5, x_6$
Non basic variables $x_4, x_2$

Tableau:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS | Ratio Test |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 0 | $\boxed{7/3}$ | 1 | -2/3 | 0 | 0 | 7 | 3 |
| $x_1$ | 1 | -2/3 | 0 | 1/3 | 0 | 0 | 4 | - |
| $x_5$ | 0 | 8/3 | 0 | -1/3 | 1 | 0 | 11 | 33/8 |
| $x_6$ | 0 | 7/3 | 0 | 1/3 | 0 | 1 | 19 | 57/7 |
| $z$ | 0 | -16/3 | 0 | 5/3 | 0 | 0 | 20 | |

$$A_B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \; A_B^{-1} = \begin{bmatrix} 1 & -2/3 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & -1/3 & 1 & 0 \\ 0 & 1/3 & 0 & 1 \end{bmatrix} \; y = \begin{bmatrix} 7/3 \\ -2/3 \\ 8/3 \\ 7/3 \end{bmatrix}$$

Basic solution is $(4, 0, 7, 0, 11, 19)$, corresponding extreme point is $(4, 0)$. Entering variable is $x_2$, exiting variable is $x_3$.

**Iteration 3**:

Basic variables $x_2, x_1, x_5, x_6$
Non basic variables $x_4, x_3$

Tableau:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS |
|---|---|---|---|---|---|---|---|
| $x_2$ | 0 | 1 | 0.429 | -0.286 | 0 | 0 | 3 |
| $x_1$ | 1 | 0 | 0.286 | 0.143 | 0 | 0 | 6 |
| $x_5$ | 0 | 0 | -1.14 | 0.429 | 1 | 0 | 3 |
| $x_6$ | 0 | 0 | -1 | 1 | 0 | 1 | 12 |
| $z$ | 0 | 0 | 2.29 | 0.143 | 0 | 0 | 36 |

$$A_B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{bmatrix} A_B^{-1} = \begin{bmatrix} 0.429 & -0.29 & 0 & 0 \\ 0.29 & 0.14 & 0 & 0 \\ -1.14 & 0.43 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

Basic solution is $(6, 3, 0, 0, 3, 12)$, corresponding extreme point is $(6, 3)$. There are no negative reduced costs, so this is the optimal solution.

**Marking**:

- (1 mark) Correct sketch of the feasible region
- (1 mark) Correct labelling of the sketch (equations and points)
- (1 mark) Correct canonical tableau at the start of each iteration
- (1 mark) Correct display of $A_B$, $A_B^{-1}$ and $y$
- (1 mark) "Logical" path to optimal (i.e. picking a variable with negative reduced costs)
- (1 mark) Correct path to optimal (as shown in solutions)
- (1 mark) Correct optimal solution