Fluid Mechanics

Topic 2

Conservation Laws for Fluid Flow

What we did during the last topic

In the previous topic, we discussed fluid statics

- Stationary fluid
- No pressure gradients laterally (x- or y-directions)
- Pressure gradient in z-direction due to weight of fluid
- The magnitude of the pressure gradient depends on the density of the fluid

Manometry

- Technique using static fluids to measure pressure
- Can measure absolute pressure, gauge pressure, or pressure differences depending on the setup

Learning objectives

By the end of this lesson, students should be able to

- Define vocabulary to describe flow (ex. stress, viscosity, stream line)
- Qualitatively describe the similarities and difference between solid and fluid deformation
- Describe the forces that cause/resist flow
- Understand the derivations of the conservation laws for fluid flow
  - Mass
  - Momentum
  - Energy
- Be able to describe the assumptions that are built into each of these equations, and know when they can and cannot be used
- Be able to use the conservation laws to solve fluid flow problems

Basic terminology and principles

When a force is applied to a body, the body can respond by translating, rotating, or deforming

- In this course we are focused only on forces that deform
- When a body is deformed (or **strained**) the force that causes the deformation is referred to as a **stress**

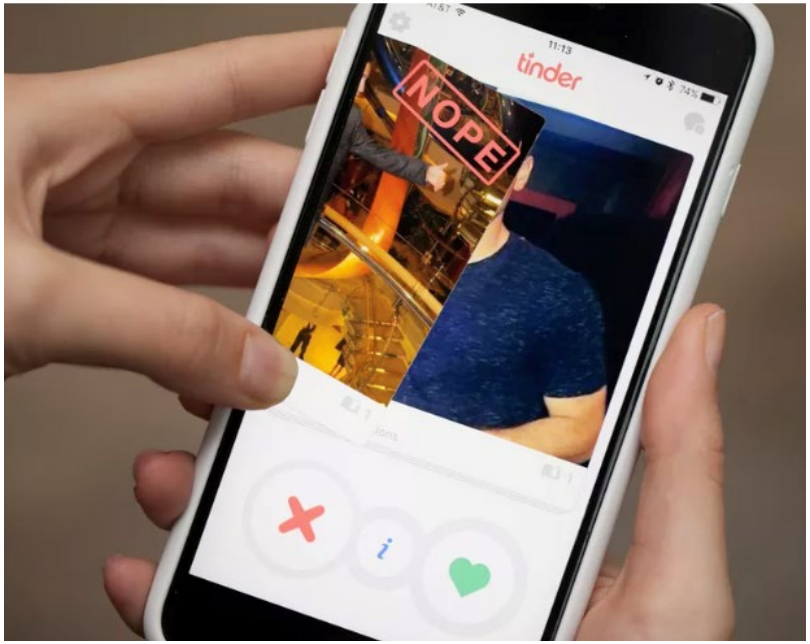
The force can act perpendicular to a surface or tangential to a surface (or both)

- Perpendicular forces are called **normal stress**
- Tangential forces are called **shear stress**

Daily examples of shear stress?

What are some daily examples of shear stresses?

- Opening a screw cap bottle
- Washing your face
- Applying moisturizer
- Counting money
- Others…?



Materials deform when stress is applied

A body will deform when a stress is applied

- "Bodies" in this context can be either solids, liquids, or gases
- The deformation depends on the **magnitude of the force** and the **properties of the material**

Solids and fluids behave differently when stressed

- Solids = finite deformation
- Liquids = continuous deformation

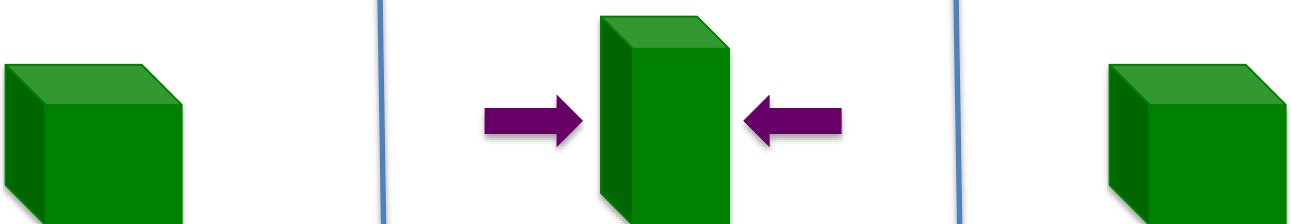
In this course we are mainly focused on the deformation of fluids

Straining an ideal elastic solid

How does an ideal solid behave under different types of stress?

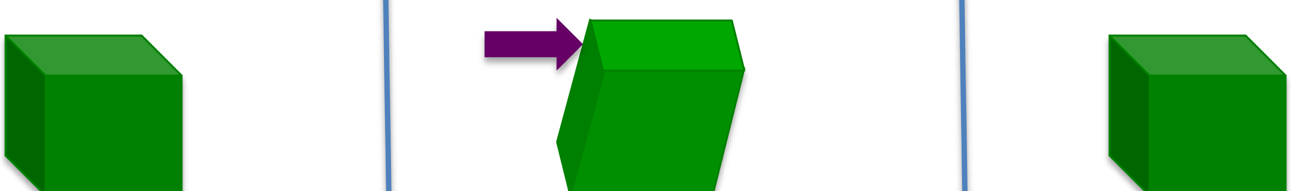
Shape of body before stress is applied	Shape of body when stress is applied	Shape of body after stress is removed

Tensile stress, a normal stress



Compressive stress, a normal stress

Ideal solids regain original shape after stress is removed



Shear stress, a tangential stress

A solid's behavior when stressed

How does an ideal solid behave under different types of stress?

- When an ideal solid is strained by a fixed amount of force it will deform a **finite** amount
- The solid will remain deformed until the force is removed
- Upon removal of the force, the solid will regain its original shape

Examples: Rope, Jelly?!

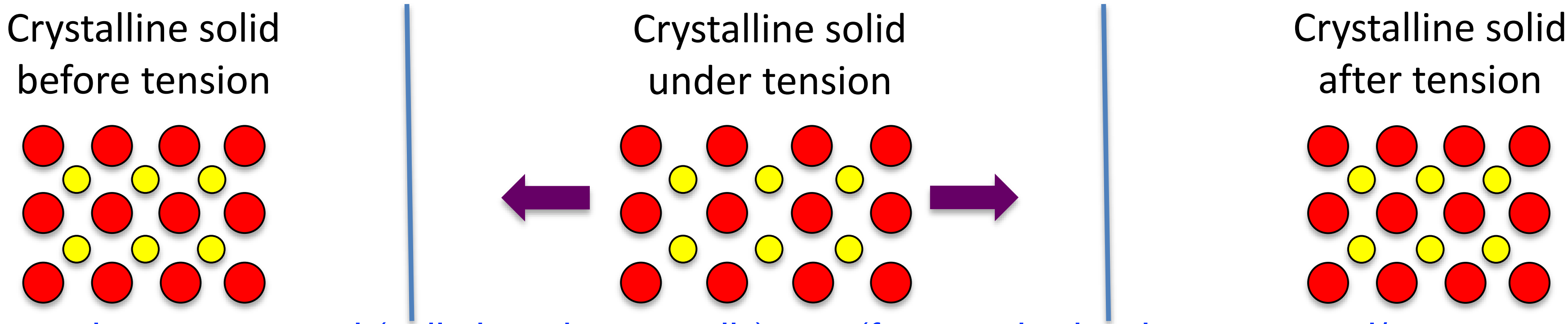
A fundamental property of a solid is **elasticity** meaning the material will recoil or regain its original shape after being deformed

The amount of deformation varies between materials

Stiffer materials will deform less under a given stress

Why do solids behave this way?

Microscopically, pieces of a solid are fixed in location to one another



original—>compressed (pulled out horizontally) —> (force on both sides is removed/maintain original shape)

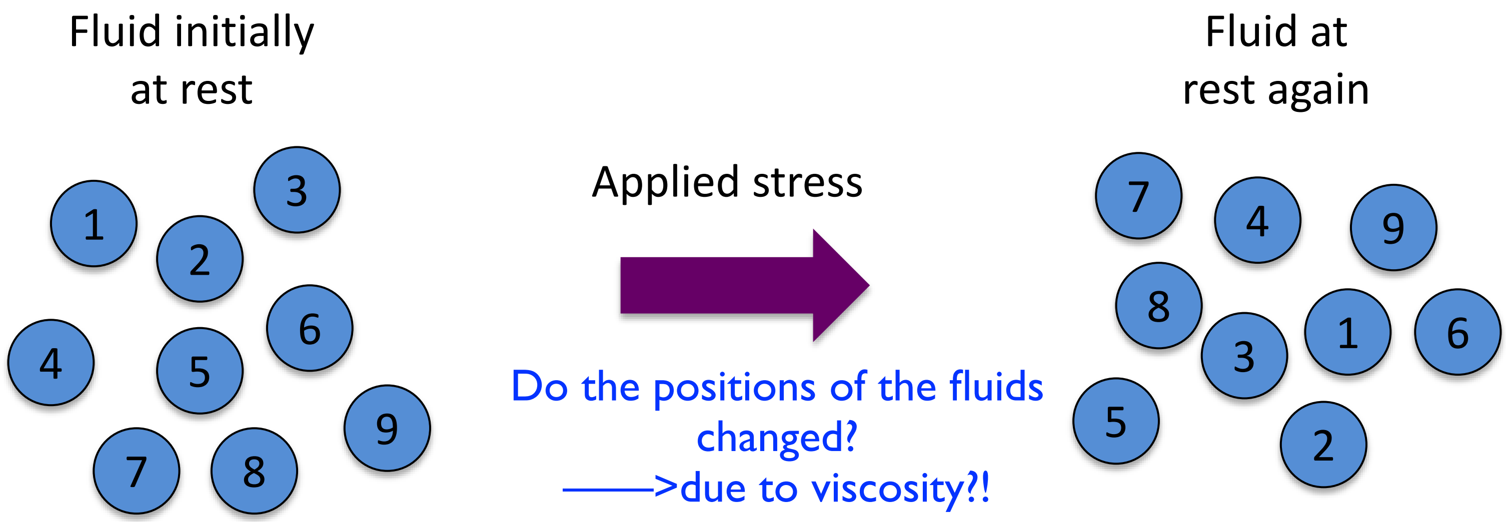
The units of the solid material can be held in place by several means

- Crystalline structure,
- Covalent crosslinks,
- or simply, the lack of enough thermal energy to move past one another

Solid materials only deform a finite amount because they develop internal stresses that balance the applied stress

Fluids versus solids?

Unlike solids, the units of a fluid can move in relation to one another

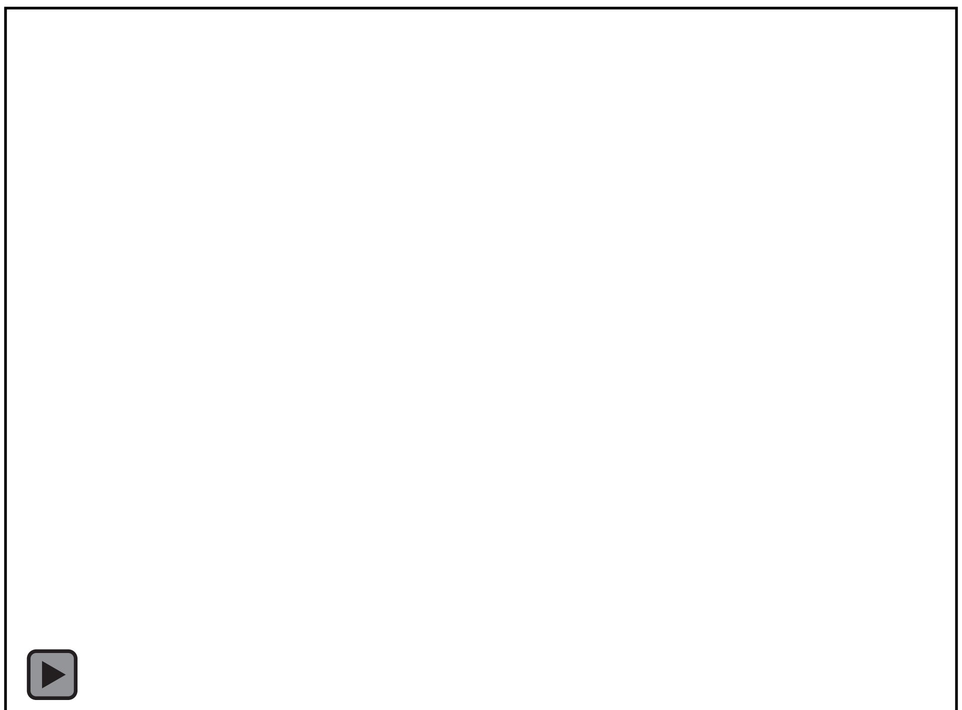


Macroscopically, this movement of fluid particles in relation to one another is what results in **fluid flow**

How do we know this occurs?

Experimental techniques have been developed to visualize

- One techniques are referred to as particle tracking velocimetry
- Many (but not all) of these techniques suspend polymer beads, glass beads, or bubbles in the flow.
- By observing the movements of the beads, we can visualize the flow of the fluid



Green glass particles are flowing in water. From movies of the flow, the velocity of the particles can be tracked, thus the flow pattern of the fluid can be visualized

Stresses that cause/resist flow

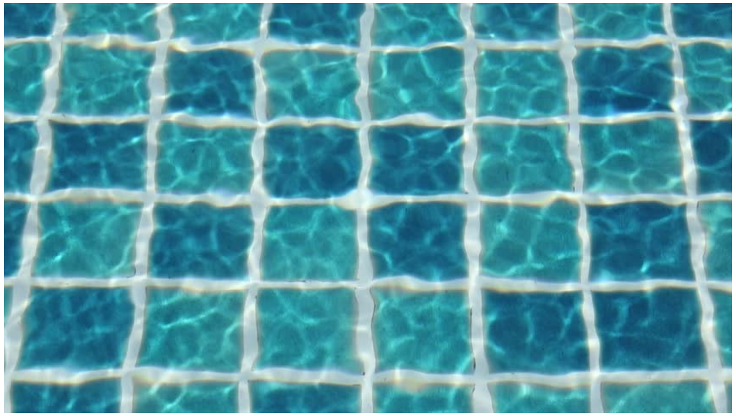
Several types of forces that either cause/resist fluid flow

Pressure differences

For humour purposes

Gravity

Shear



Stresses that cause/resist flow

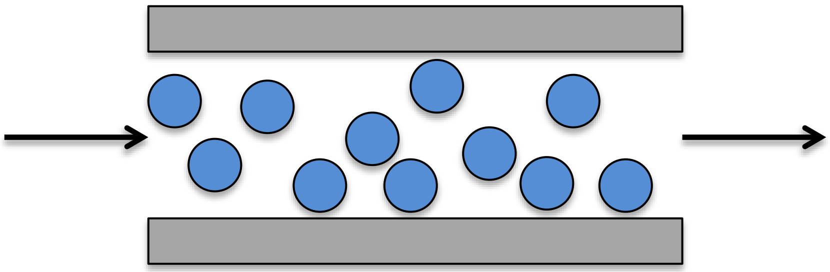
Once a stress is removed from a fluid, it will eventually cease flowing

- This is due to forces that resist flow
- What are these forces?

There are two main forces that resist flow

They are both frictional forces

1. **Viscosity** (μ) the friction of fluid particles moving past one another during flow
2. The friction associated with fluid particle moving past a solid surface



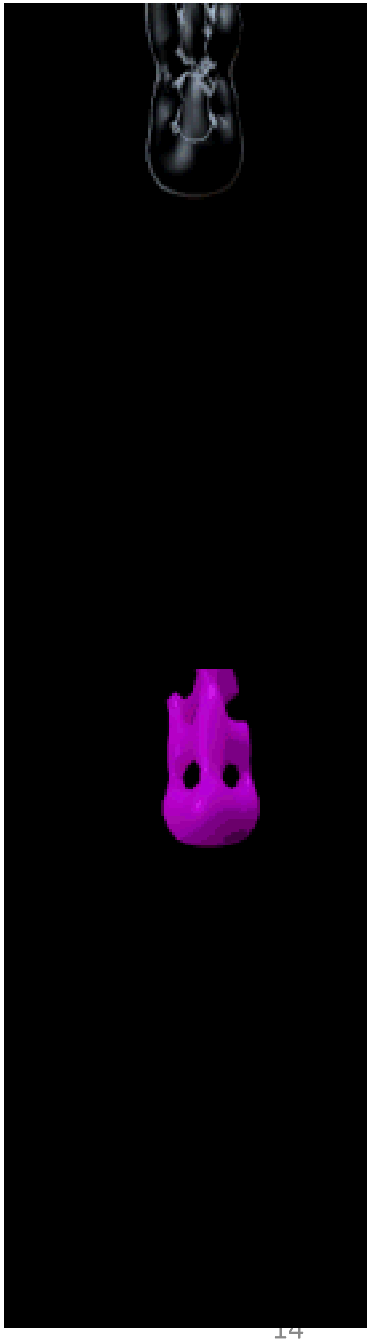
How much will a fluid deform?

A fluid's viscosity is very similar to a solid's stiffness

- These parameters determine how much a fluid will deform when a given stress is applied

Viscosity (μ) determines how easily a fluid deforms when a stress is applied

- Viscosity can be thought of as resistance to flow
  - Low viscosity fluids, or thin fluids, are easy to deform
  - High viscosity fluids, or thick fluids, are harder to deform



Viscosity (μ)

Physically, what is viscosity?

Viscosity is a resistance of the fluid that deform its shape or movement that passes through one after during flow.

Is viscosity constant?

For liquids/fluids yes depends of the resistance of the flow of that liquid, assuming it is negligible!

What factors affect viscosity

mew = density * velocity

The density of the fluid/solid and how fast the fluid flows or travels through

Quantifying viscosity

The viscosity of a fluid can be measured in order to quantify a fluid's resistance to flow

Common units for viscosity are centipoise (cP)

- μ [=] 1 cP = 1 mPa·s [=] 0.001 Pa·s [=] 0.001 N·s·m^{-2} [=] 0.001 kg·m^{-1}·s^{-1}

Fluid	Viscosity (cP)
Air	0.0186
Benzene	0.6076
Water	0.89
Olive oil	81
Glycerol	1,200
Honey	5,000

Laminar vs. Turbulent flow

Fluid flow occurs in two regimes

- **Laminar flow** (or **stream line flow**) is characterized by fluid flowing in distinct layers, with no disruption between the layers
- **Turbulent flow** is characterized by chaotic flow (eddies and vortices) and rapid changes in flow velocity in both space and time

Laminar flow

Transition flow

Turbulent flow

Laminar vs. Turbulent flow

The flow regime of a fluid (whether the flow is laminar or turbulent) is governed by three main criteria

*Is there a formula to apply all of these three main criteria?

- Fluid velocity (inertial forces)

- Fluid viscosity (viscous forces)

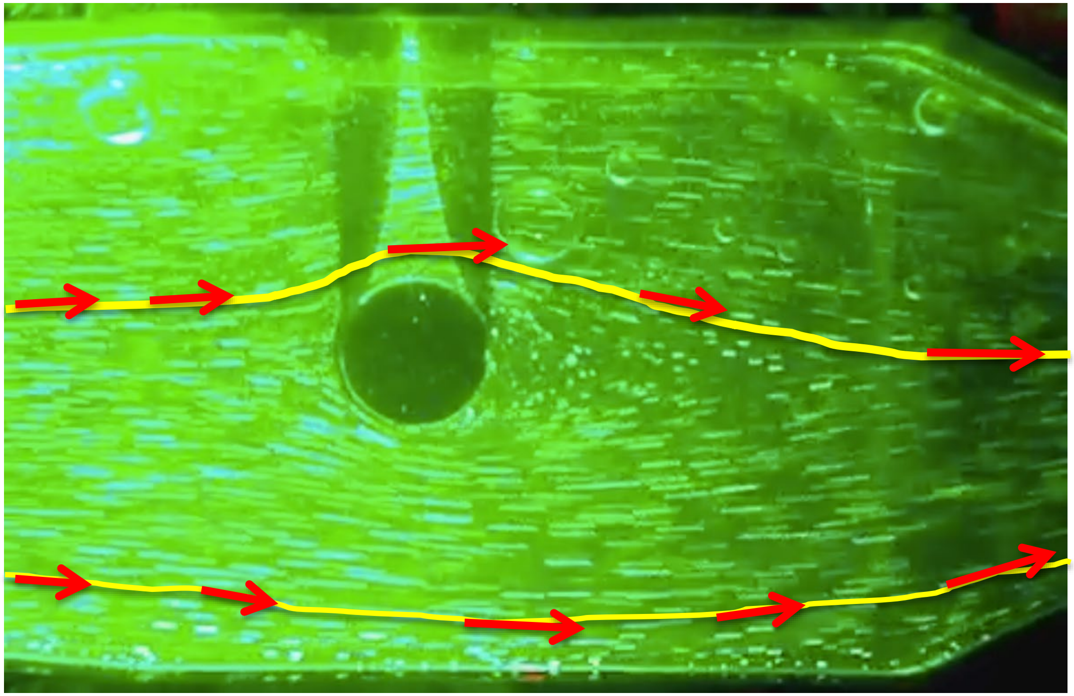
- Flow geometry

Laminar flow (stream line flow)

The video we watched earlier illustrates laminar flow

It is also called stream line flow because you can draw stream lines, or paths that the individual fluid elements travel along

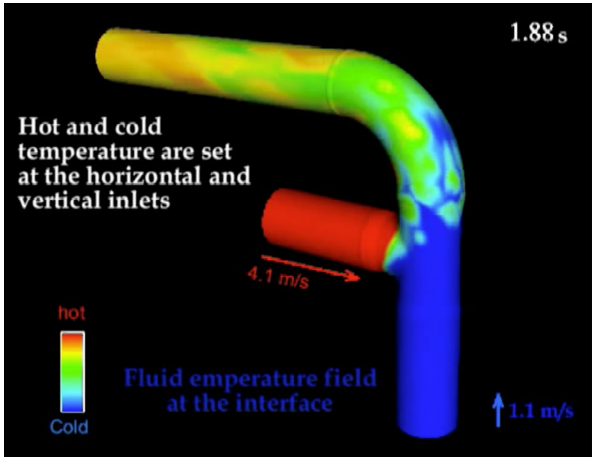
The velocity of a fluid particle is tangential to the stream line

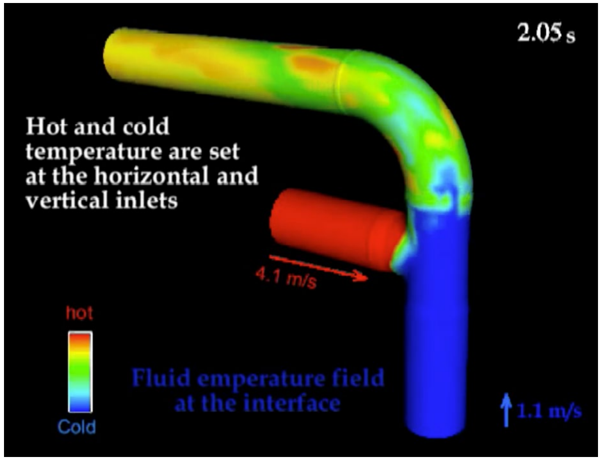


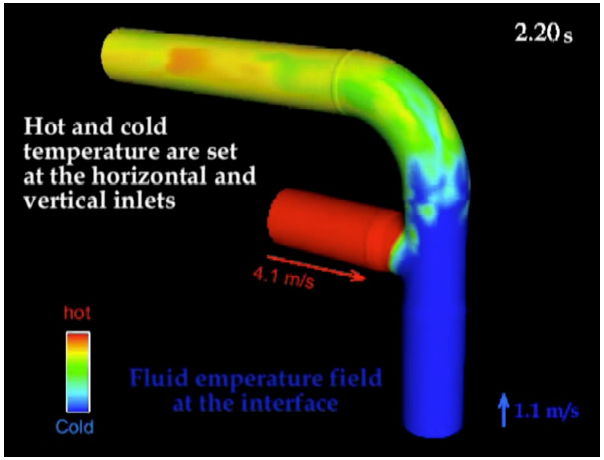
Turbulent flow

You cannot draw stream lines for turbulent flow because

- The flow field is irregular
- The flow field changes with time

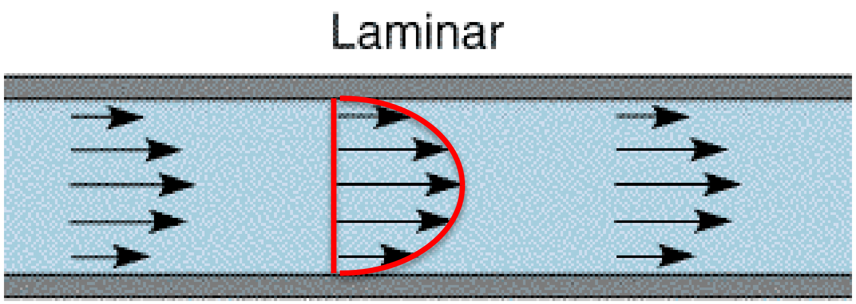






https://www.youtube.com/watch?v=GDgTwadkJ50 Watch from 1:20 – 3:10

Laminar vs. Turbulent flow

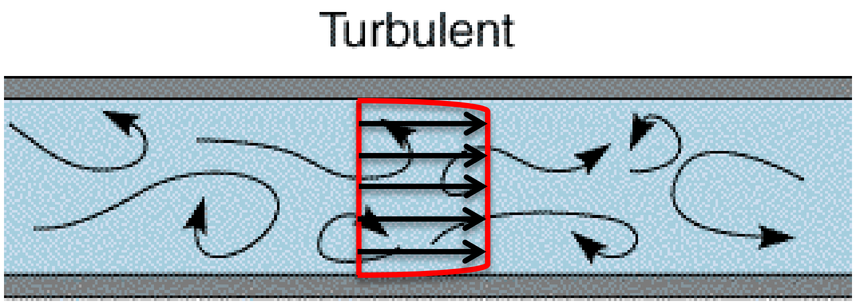


More likely to occur for:

- Fluids with low velocity
- Fluids with high viscosity
- Fluids in a regular flow geometry

Characterized by:

- Smooth stream lines
- No mixing between stream lines
- Parabolic velocity profile



More likely to occur for:

- Fluids with high velocity
- Fluids with low viscosity
- Fluids in an irregular flow geometry

Characterized by:

- Eddies and vortices that change in both space and time
- Mixing
- The velocity in the flow direction has a very blunt profile

Mini-summary

- Unlike solids, particles within a liquid have the ability to move in relation to one another – this is flow
- Multiple forces aid/resist flow
  - Gravity
  - Pressure
  - Shear
  - Viscosity
  - Friction with a surface
- Fluid flow occurs in two “regimes”
  - Laminar
  - Turbulent
- Laminar flow is characterized by well defined stream lines
- Turbulent flow is characterized by the presence of eddies, vortices, and time dependent behaviour

Mathematics to describe fluid flow

A large portion of this course is dedicated to **discovering mathematic relationships** that will allow you to describe important parameters of a flowing fluid

- Pressure
- Velocity
- Flow rate (mass flow rate and volumetric flow rate)

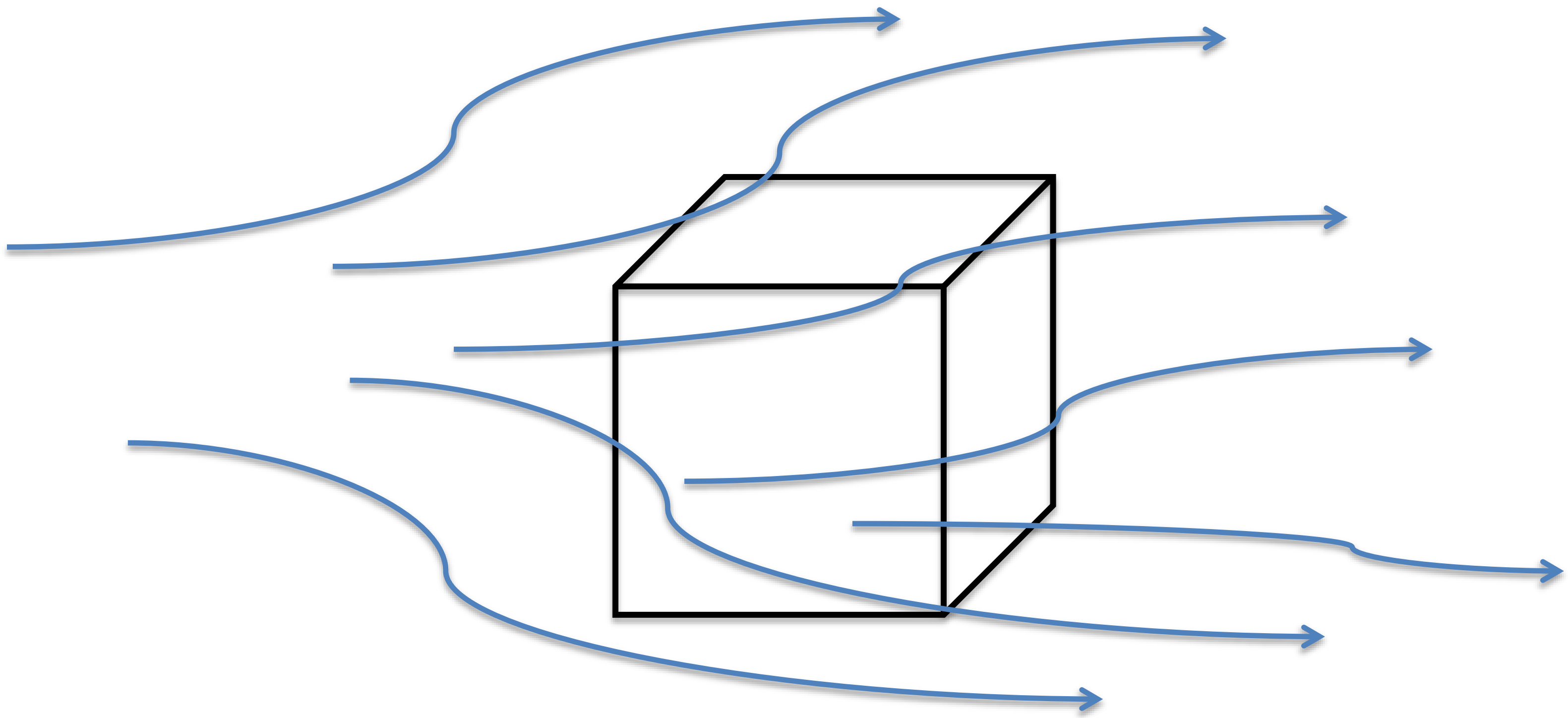
Most of these mathematic relationships are based on **laws of conservation**

- Conservation of mass
- Conservation of momentum
- Conservation of energy

More specifically, we will explore conservation of mass, momentum, and energy for **steady state** fluid flow in **closed channels**

Control volumes

In order to **generate these mathematical relationships**, we will define **control volumes** and analyze the movement of mass, energy, and momentum into and out of these volumes



Flowing fluid

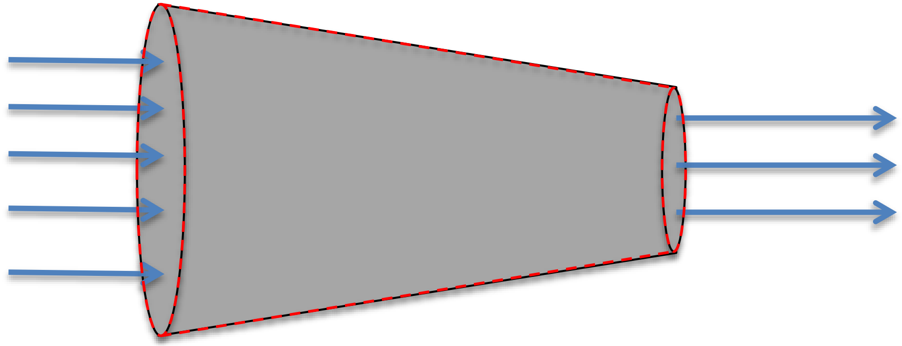
Conservation of mass

The **conservation of mass** is the easiest of the three relationships

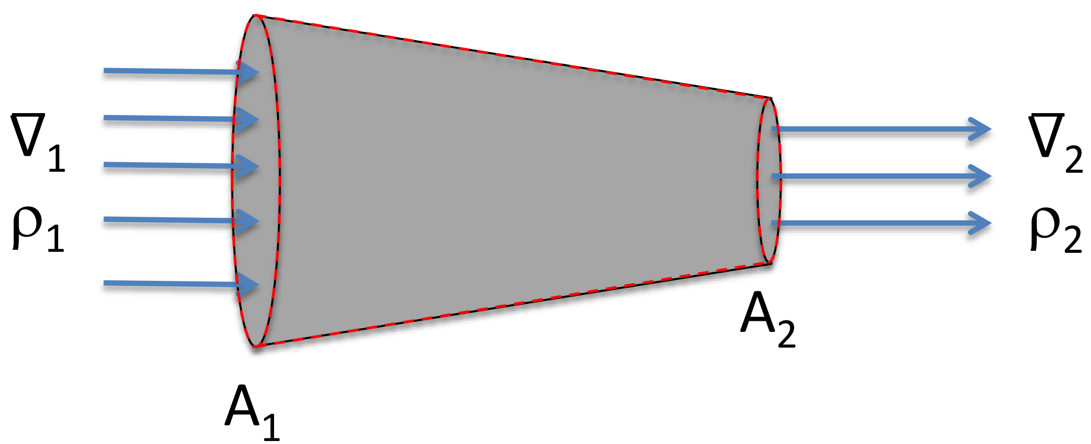
- We know that mass isn't created or destroyed, so all of the mass that flows into our control volume will have to flow out
- Applying a mass balance over a control volume results in the **equation of continuity**

We have fluid flowing through a constricting pipe.

- There is no flow of fluid through the walls of the pipe
- There is no accumulation of mass within the pipe (**steady state**)
- The entire pipe is the control volume



Equation of continuity



Where

$\bar{V}$ = average velocity

ρ = density

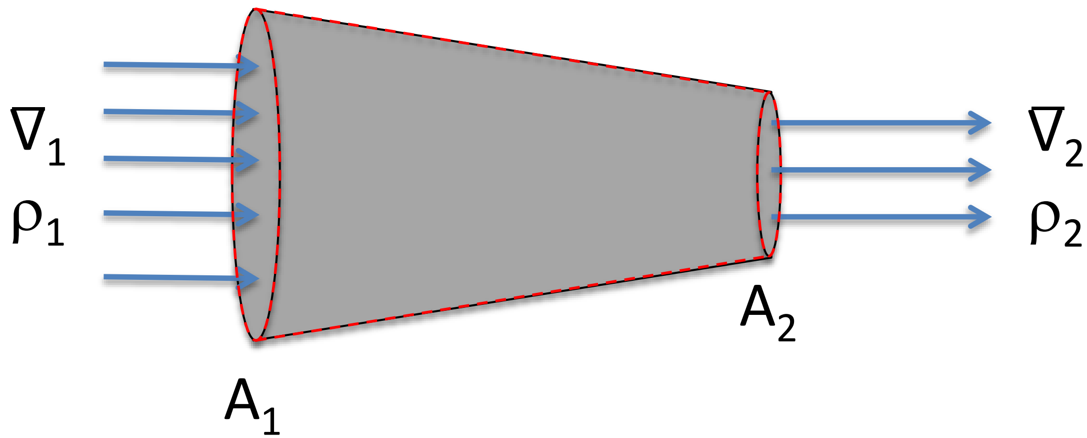
A = cross sectional area

At steady state, the mass of fluid flowing into the pipe through A_1 must be equal to the mass of fluid flowing out of the pipe at A_2

$\mathbf{G_1 = G_2}$ G is mass flow rate (mass/time)

Note: We're working with **average velocity**. This is because the fluid flow across the cross section is not constant, for either laminar or turbulent flow

Equation of continuity



Where

$\bar{V}$ = average velocity

ρ = density

A = cross sectional area

$G_1 = G_2$ G is mass flow rate (mass/time)

However, it's easier to measure the velocity of a fluid at a given point in flow than the mass, so let's write this in terms of velocity

*Assume in this case we do not consider the direction that goes pipe! But given where the velocity occurs at that approximate point/position!! V instead of V.bar

$$G = \rho V A$$

therefore

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

which still has units of mass/time

Equation of continuity for incompressible liquids

If our fluid is a liquid, it is **incompressible**

- What do we mean when we say a liquid is incompressible?

For an incompressible liquid, density does not change with pressure

$$\boxed{\frac{d\rho}{dP} = 0}$$

- Using this knowledge, we can simplify the equation of continuity

Different densities from either side

Same density on both sides

$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \rightarrow \rho V_1 A_1 = \rho V_2 A_2 \rightarrow$

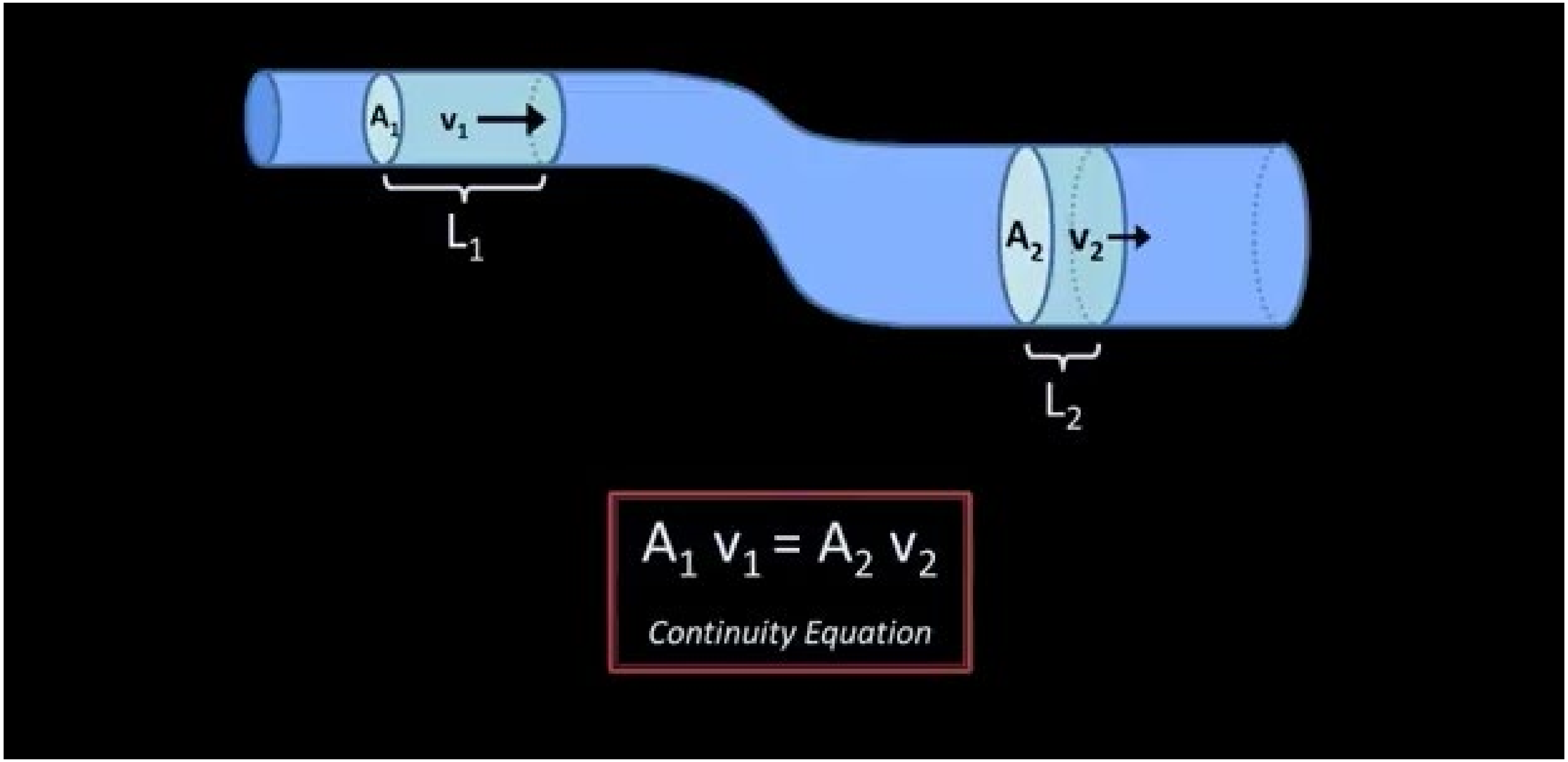
*Where are the V1 and V2 bars?!

$V_1 A_1 = V_2 A_2 \rightarrow Q_1 = Q_2$ (Q = volumetric flow rate, vol/time)

- For liquids at constant temperature, both the mass flow rate and the volumetric flow rate are constant

Note: This does not mean that the density is always constant. It will still change with other variables such as temperature. However, it is constant with pressure

Equation of continuity for incompressible liquids



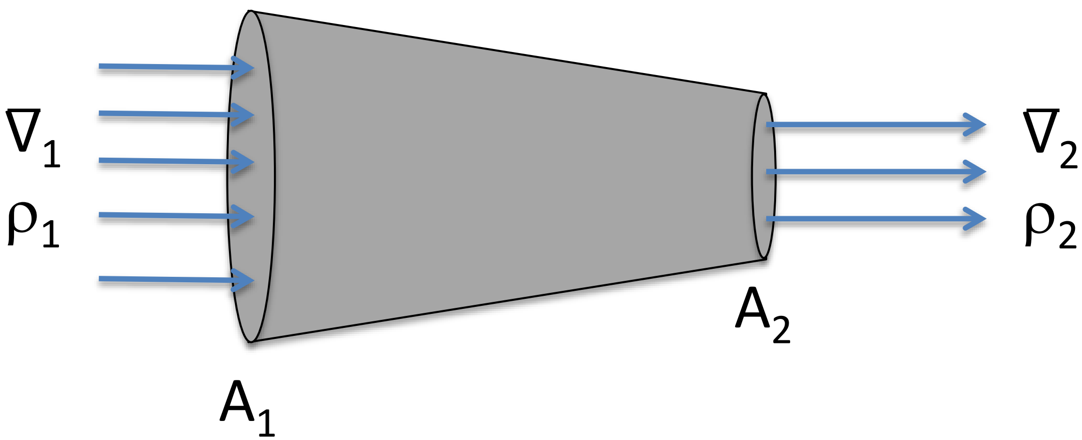
What are the units for both sides of this equation?

$$\boxed{Q = VA}$$

This is a VERY important equation in fluid mechanics

Applying the equation of continuity

An incompressible liquid is flowing through a pipe of variable cross sectional area. If $\bar{V}_1$ = 1 m/s, A_1 = 25 cm^2, and A_2 = 10 cm^2, what is $\bar{V}_2$?



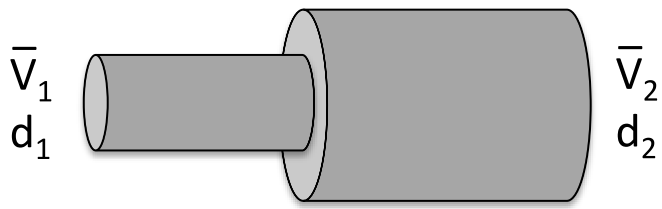
$$\rho_1 \bar{V}_1 A_1 = \rho_2 \bar{V}_2 A_2 \Rightarrow \bar{V}_1 A_1 = \bar{V}_2 A_2 \Rightarrow \bar{V}_2 = \frac{\bar{V}_1 A_1}{A_2}$$

$$\Rightarrow \bar{V}_2 = \frac{1*25}{10} \Rightarrow \bar{V}_2 = 2.5 m/s$$

Challenge problem 2.1

An incompressible liquid flows through a rapid expansion in a pipe. The pipe diameter suddenly increases from d_1 = 10 cm to d_2 = 20 cm. The volumetric flow rate is 10 liters/min. What are the average velocities in and out?

Formulas given:
Q1 = V1A1
Q2 = V2A2

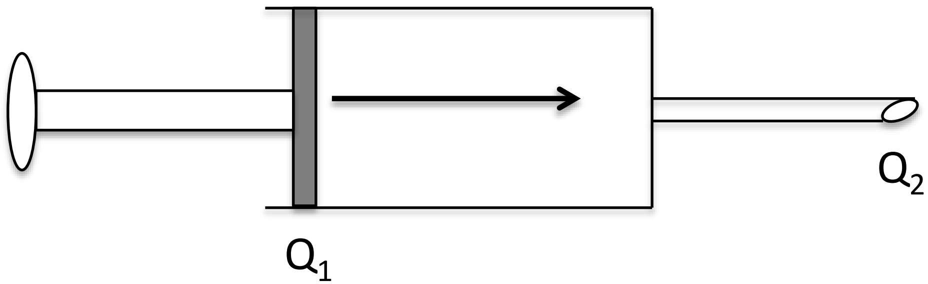


Q1=Q2=10 (dealing an incompressible liquid flow)
10liters/min = V1*A1.
A1 = pi*((0.10/2))^2 m^2
Solve for V1 for the average velocity (input)

Same idea with V2!! with d2 = 20!!
10liters/min = V2*A2.
A2 = pi*((0.20/2))^2 m^2
Solve for V2 for the average velocity (output)

Example problem 2.1.

A syringe is used to inoculate a cow. The plunger has a face area of 500 mm^2. The liquid in the syringe must be injected steadily at a rate of 0.3 liters/min. At what speed should the plunger advance?



What we know?
A = 500mm^2
Q = 0.3liters/min

Formula given:
Q = VA
0.3 = V * 500*(10^-3)^2
Solve V! in terms of m/s

Conservation of linear momentum

The next relationship we're going to develop is the **conservation of momentum**

$\Delta P = m \Delta V$

Uh, Dr. Sui, what is momentum again???

Momentum is the product of mass and velocity

$$p = m \times V$$ [=] kg*m/s [=] N*s

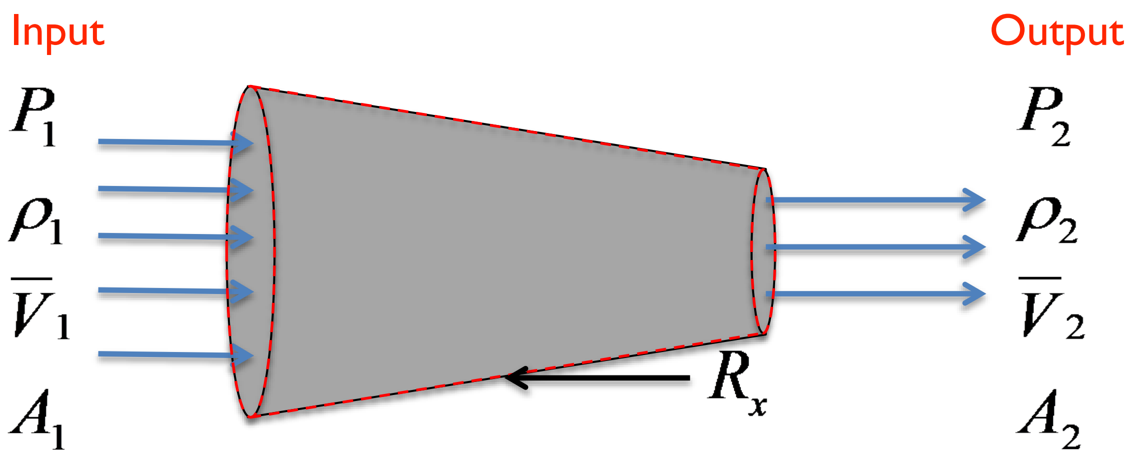
Regardless before and after?!

From Newton's 2nd Law we know that force equals the rate of change of momentum

Conservation of linear momentum

The **rate of change in momentum** as the fluid moves from point 1 to point 2 is equal to **momentum out per unit time** minus **momentum in per unit time**

Rate of change in momentum = Momentum out per time – Momentum in per time



R_x is the force exerted by the duct on the fluid

Now we need to put this relationship in terms of our system variables

Conservation of linear momentum

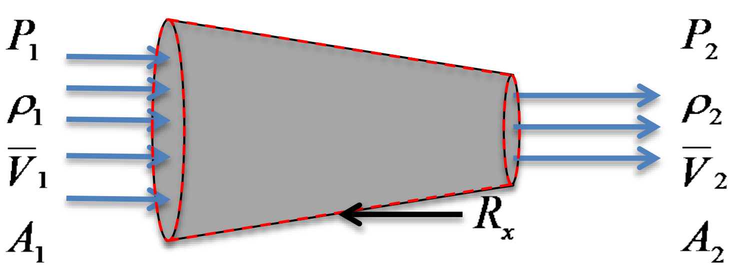
Let's determine the right hand side (RHS) of the equation first

RHS = Momentum out per time – Momentum in per time

Momentum out per time $= G_2 \overline{V}_2 \Rightarrow = \rho_2 A_2 \overline{V}_2^2$

Momentum in per time $\Rightarrow = \rho_1 A_1 \overline{V}_1^2$

RHS = $\rho_2 A_2 \overline{V}_2^2 - \rho_1 A_1 \overline{V}_1^2$



Conservation of linear momentum

Newton's 2nd Law: Force = Rate of change of momentum

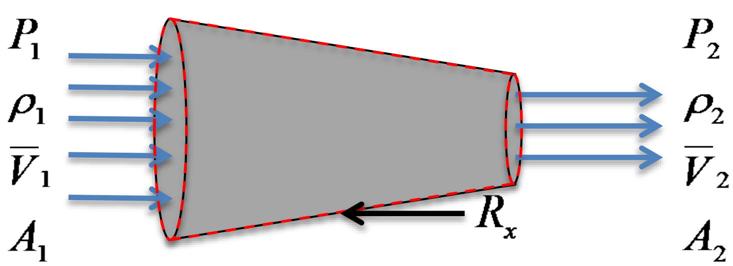
LHS = Rate of change of momentum

Therefore, we need to perform a force balance on the fluid to evaluate the left hand side (LHS) of the equation

Forces acting in direction of flow = $P_1 A_1$

Forces acting opposite direction of flow = $P_2 A_2 + R_x$

Rate of change of momentum = LHS = $P_1 A_1 - P_2 A_2 - R_x$ *Net change



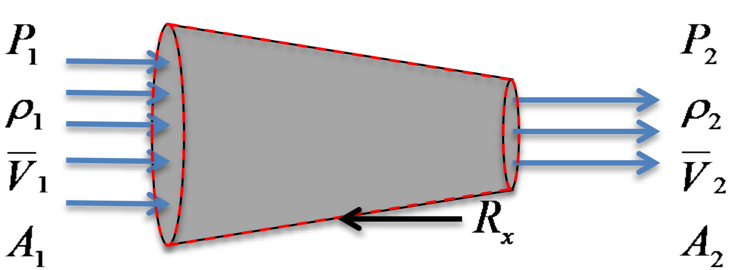
Conservation of linear momentum

Our momentum balance becomes

$$\boxed{P_1A_1 - P_2A_2 - R_x = \rho_2 A_2 \overline{V}_2^2 - \rho_1 A_1 \overline{V}_1^2}$$

But from conservation of mass we know $\boxed{\rho_2 A_2 \overline{V}_2 = \rho_1 A_1 \overline{V}_1}$

$$\boxed{R_x = \rho_1 A_1 \overline{V}_1^2 \left(1 - \frac{\rho_1 A_1}{\rho_2 A_2}\right) + P_1A_1 - P_2A_2}$$



R_x is the force exerted by the duct wall on the fluid

It is also the force required to hold the duct stationary

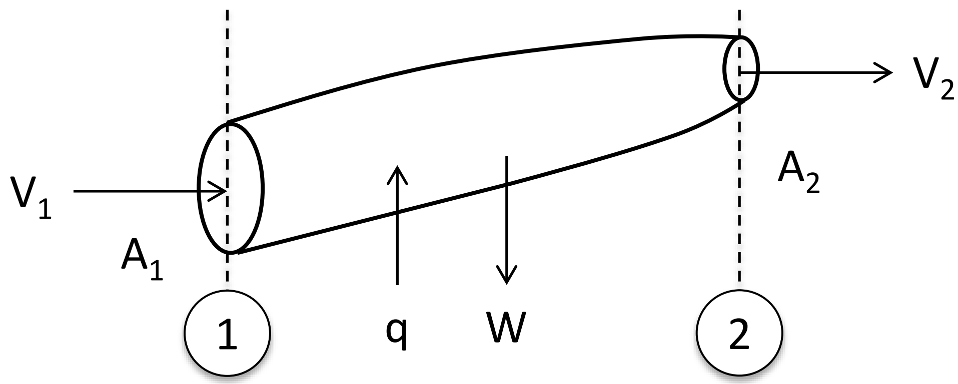
Mini-summary

- Several conservation laws can be used to quantify fluid flow
  - Conservation of mass
  - Conservation of momentum
  - Conservation of energy
- These relationships are derived by analyzing fluid flow through a control volume
- Conservation of mass
  - For steady state flow, the mass flow in is equal to the mass flow out
  - For incompressible flow, the volumetric flow in is equal to the volumetric flow out
  - When a fluid is flowing through an enclosed control volume, the velocity will increase as the cross sectional area decreases
- Conservation of momentum
  - We can determine the amount of force the pipe is experiencing from the flow

Conservation of energy

Conservation of energy will give us the most information about the flow

Consider flow through an arbitrarily shaped volume



The most general form of the energy balance has the following form

$$E_2 - E_1 = q - W$$

Increase in total stored energy per unit mass in the fluid passing from point 1 to point 2

Net heat per unit mass added to the fluid passing from point 1 to point 2

Net work done per unit mass by the fluid on the surroundings in passing from point 1 to point 2

Conservation of energy

Energy in the fluid can be stored in **three** forms

$$\underbrace{E_2 - E_1}_{\text{Increase in total stored energy per unit mass in the fluid passing from point 1 to point 2}} = q - W$$

1. **Internal Energy** (**U**), molecular and atomic energy, depends on $\nu = \frac{1}{\rho}$ and T

2. **Kinetic energy**, the energy of motion, depends on $\frac{1}{2}\overline{V}^2$

3. **Potential energy**, the energy due to position in gravitational field,

   depends on gz where z is the height above a reference height

Conservation of energy

Substitute these terms into energy balance

$$\underline{\left(U_2+\frac{1}{2}\overline{V}_2^2+gz_2\right)-\left(U_1+\frac{1}{2}\overline{V}_1^2+gz_1\right)}=q-W$$

Now let's look at the work done by the fluid

$$E_2-E_1=q-\underline{W}$$

Net work done per unit mass by the fluid on the surroundings in passing from point 1 to point 2

Conservation of energy

Work done by the fluid can occur in **two** forms

$$E_2 - E_1 = q - \underline{W}$$

Net work done per unit mass by the fluid on the surroundings in passing from point 1 to point 2

1. **Flow work**, work done on the surroundings as a result of the fluid crossing the control surfaces.

   In this instance, it is the **work done by pressure** as a fluid enters and leaves the control volume

2. **Shaft work (W_s)**, "useful work" being done as the fluid flows between points 1 & 2

   ex. driving a mechanical device

Conservation of energy

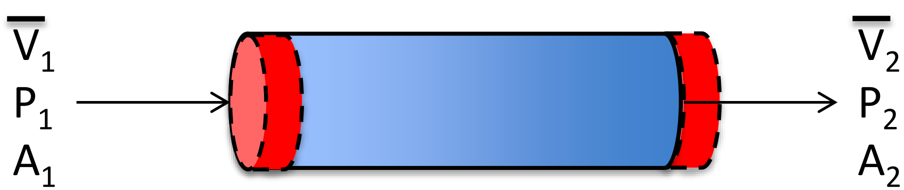
We will not elaborate more on **shaft work (W_s)** at this time

We can put **flow work** in terms of more useful variables

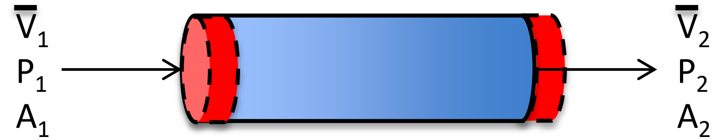
- Flow work, also called **pressure work**
- It is work done by the fluid against external pressure at the exit and
- Work done on the fluid by pressure at the entry

Consider a fluid flowing through the following volume over the time Δt

- During that time a plug of fluid will move into the control volume
- Another plug of fluid will move out of the control volume
- We can quantify the pressure work needed to achieve this fluid motion



Conservation of energy



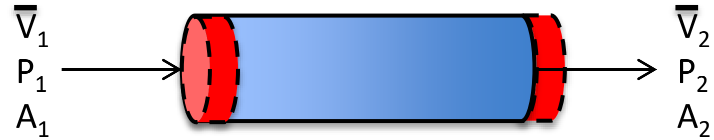
Work = Force x Distance

- The force required to move the plug of fluid into the control volume is $P_1 A_1$
- The distance the inlet fluid travels during this time is $\overline{V}_1 \Delta t$
- The work done on the fluid at point 1 $= P_1 A_1 \overline{V}_1 \Delta t$
- Similarly the work done by the fluid at point 2 $= P_2 A_2 \overline{V}_2 \Delta t$

Net flow work done by the fluid $= P_2 A_2 \overline{V}_2 \Delta t - P_1 A_1 \overline{V}_1 \Delta t$

However, we're not quite done yet, our energy balance is in a per mass basis, so this term needs to be put in a per mass basis

Conservation of energy



From conservation of mass, the mass flowing into the control volume is equal to the mass flowing out

- The mass flowing into the control volume $= \rho_1 A_1 \overline{V}_1 \Delta t$
- The mass flowing out of the control volume $= \rho_2 A_2 \overline{V}_2 \Delta t$

So the net flow work per unit mass

$$= \frac{P_2 A_2 \overline{V}_2 \Delta t}{\rho_2 A_2 \overline{V}_2 \Delta t} - \frac{P_1 A_1 \overline{V}_1 \Delta t}{\rho_1 A_1 \overline{V}_1 \Delta t}$$

$$= \frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} = P_2 v_2 - P_1 v_1$$

v is the specific volume (vol/mass)

Conservation of energy

Previously our energy balance had this form

$$\left(U_2 + \frac{1}{2}\overline{V}_2^2 + gz_2\right) - \left(U_1 + \frac{1}{2}\overline{V}_1^2 + gz_1\right) = q - W$$

We just found that net work done on surroundings is

$$W = \frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} - W_s$$

Combining yields

$$\left(U_2 + \frac{P_2}{\rho_2} + \frac{1}{2}\overline{V}_2^2 + gz_2\right) - \left(U_1 + \frac{P_1}{\rho_1} + \frac{1}{2}\overline{V}_1^2 + gz_1\right) = q - W_s$$

Conservation of energy

We can simplify our energy balance

$$\left(U_2 + \frac{P_2}{\rho_2} + \frac{1}{2}\overline{V}_2^2 + gz_2\right) - \left(U_1 + \frac{P_1}{\rho_1} + \frac{1}{2}\overline{V}_1^2 + gz_1\right) = q - W_s$$

$$\Delta U = U_2 - U_1$$

$$\Delta P = P_2 - P_1$$

$$\Delta z = z_2 - z_1$$

Δq is heat added per unit mass over an infinitely short piece of pipe

ΔW_s is the shaft work per unit mass over an infinitely short piece of pipe

$$\Delta U + \Delta\left(\frac{P}{\rho}\right) + \Delta\left(\frac{1}{2}\overline{V}^2\right) + \Delta gz = \Delta q - \Delta W_s$$

Conservation of energy

Removing internal energy (U) from the equation

$$\Delta U + \Delta\left(\frac{P}{\rho}\right) + \Delta\left(\frac{1}{2}\overline{V}^2\right) + \Delta gz = \Delta q - \Delta W_s$$

From thermodynamics we know

$$\Delta U = \Delta q - P\Delta v + \Delta F$$

Δq	heat added per unit mass
$-P\Delta v$	reversible work done on the fluid by compression
ΔF	friction, mechanical energy is converted to heat

Substitution

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{1}{2}\overline{V}^2\right) + g\Delta z + \Delta W_s + \Delta F = 0$$

After 11 slides, we've finally arrived at the **Mechanical Energy Balance**

Conservation of energy

Mechanical energy balance tells us how energy within the flow is partitioned as the fluid flows between two points

$$\underbrace{\frac{\Delta P}{\rho}}_{\substack{\text{Pressure}\\ \text{energy per}\\ \text{unit mass}}}+\underbrace{\Delta\left(\frac{1}{2}\overline{V}^{2}\right)}_{\substack{\text{Kinetic}\\ \text{energy per}\\ \text{unit mass}}}+\underbrace{g\Delta z}_{\substack{\text{Potential}\\ \text{energy per}\\ \text{unit mass}}}+\underbrace{\Delta W_{s}}_{\substack{\text{Shaft work}\\ \text{per unit}\\ \text{mass}}}+\underbrace{\Delta F}_{\substack{\text{Friction per}\\ \text{unit mass}}}=0$$

The Bernoulli Equation

The mechanical energy balance can be written in several forms

$$\frac{\Delta P}{\rho} + \Delta\left(\frac{1}{2}\overline{V}^2\right) + g\Delta z + \Delta W_s + \Delta F = 0$$

One of the most common alternative forms of the mechanical energy balance is the **Bernoulli Equation**

- No shaft work
- No friction

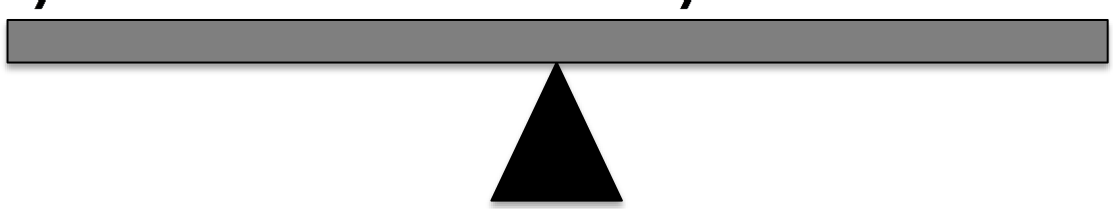
$$\frac{P_2}{\rho} + \frac{1}{2}\overline{V}_2^2 + gz_2 = \frac{P_1}{\rho} + \frac{1}{2}\overline{V}_1^2 + gz_1$$

The Bernoulli Equation

The Bernoulli Equation provides us with three key pieces of information

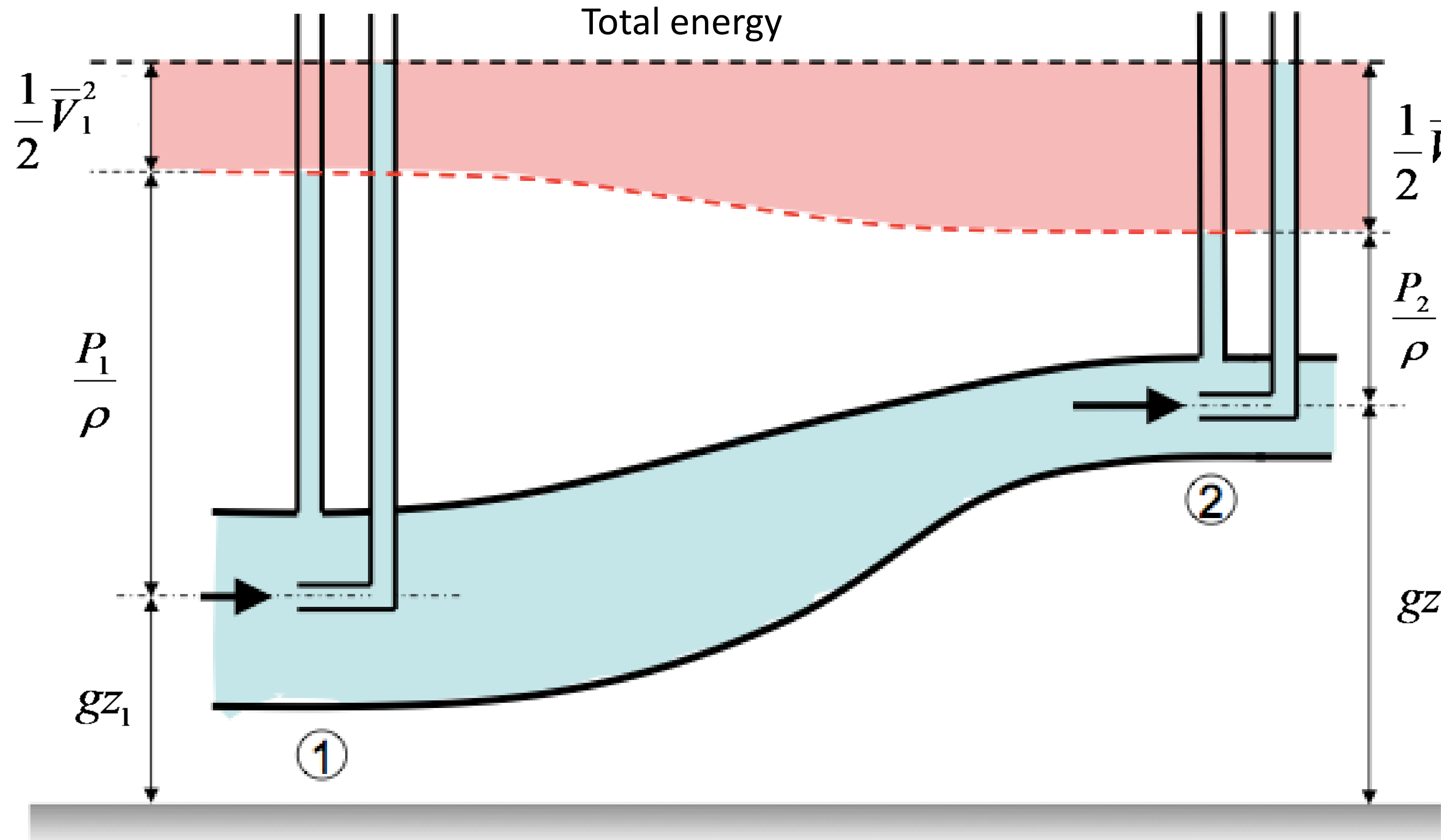
1. A stationary fluid only has potential energy (ρgh), while a flowing fluid has potential energy, kinetic energy, and pressure energy
2. The mechanical energy at poin1 equals the mechanical energy at point 2. Therefore, the total mechanical energy of the fluid is constant
3. However, the partitioning of the fluid's energy can change as a fluid flows

$$\frac{P_2}{\rho} + \frac{1}{2}\overline{V}_2^2 + gz_2 \quad = \quad \frac{P_1}{\rho} + \frac{1}{2}\overline{V}_1^2 + gz_1$$



The Bernoulli Equation

When friction is neglected and there is no shaft work, the energy at any point in the flow is constant; however, the partitioning of the energy changes



The Bernoulli Equation

The Bernoulli Equation can also be written in terms of **head** by dividing through by gravity

$$\underline{\frac{P_2}{\rho}} + \underline{\frac{1}{2}\overline{V}_2^2} + \underline{gz_2} = \underline{\frac{P_1}{\rho}} + \underline{\frac{1}{2}\overline{V}_1^2} + \underline{gz_1}$$

Bernoulli Equation written in terms of energy

- Pressure energy
- Kinetic energy
- Potential energy

$$\Downarrow$$

$$\underline{\frac{P_2}{\rho g}} + \underline{\frac{1}{2g}\overline{V}_2^2} + \underline{z_2} = \underline{\frac{P_1}{\rho g}} + \underline{\frac{1}{2g}\overline{V}_1^2} + \underline{z_1}$$

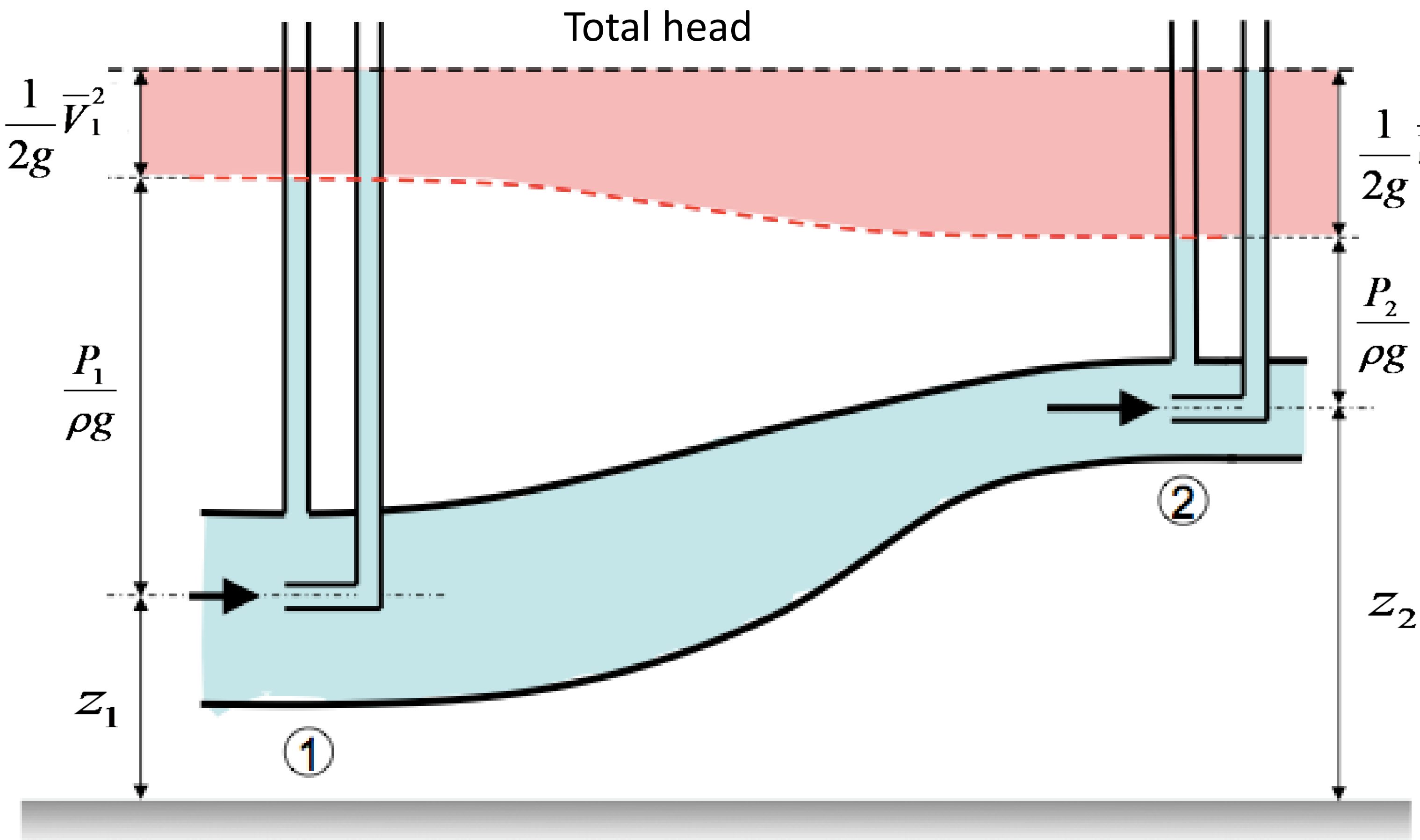
Bernoulli Equation written in terms of head

- Pressure head
- Velocity head
- Gravitational head

These equations represent the same thing, they are just different notations. Notice the units of head is meters

The Bernoulli Equation

The Bernoulli Equation can also be written in terms of head



Who is this Bernoulli?

Daniel Bernoulli was a Swiss mathematician and physicist. He lived from the 1700 – 1782. A large portion of his work focused on applying mathematics to mechanics, particularly fluid mechanics.

His name is commemorated in the **Bernoulli's principle**, an example of the conservation of energy in fluid mechanics which states that an increase in speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in potential energy.

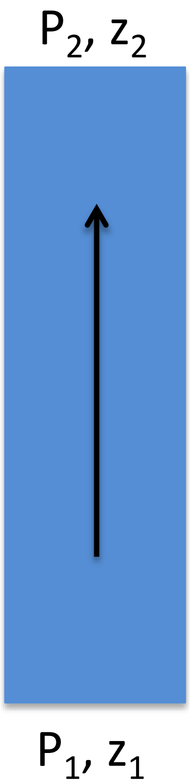
Example problem 2.2 – a thought experiment

Although the total energy is constant, it can be transformed from one type to another during flow

Example, you are pumping a fluid vertically against gravity

- In this scenario, the fluid will flow from high to low pressure, so as the fluid flow from point 1 to point 2, its pressure energy decreases

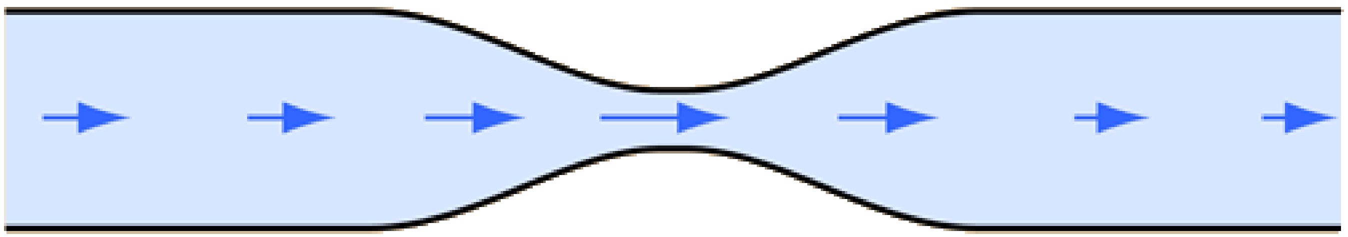
- However, due to its increase in height in the gravitational field, that pressure energy is converted into potential energy



$$\frac{P_2}{\rho}+\frac{1}{2}\overline{V}_2^2+gz_2=\frac{P_1}{\rho}+\frac{1}{2}\overline{V}_1^2+gz_1$$

Example problem 2.3 – a thought experiment

A liquid flows horizontally through a pipe with a constriction



$$\frac{P_2}{\rho}+\frac{1}{2}\overline{V}_2^2+gz_2=\frac{P_1}{\rho}+\frac{1}{2}\overline{V}_1^2+gz_1$$

- Does the potential energy of the fluid change in this system?

No, the energy due to position in gravitational field, depends on gz where z is the height above a reference height!! z1=z2=0 as the fluid goes through the pipe (horizontally).

- What will happen to kinetic energy as the fluid passes through the constriction?

Kinetic energy increases which is proportional to its velocity squared as increases from left to right!

- How will that affect the pressure energy of the system?

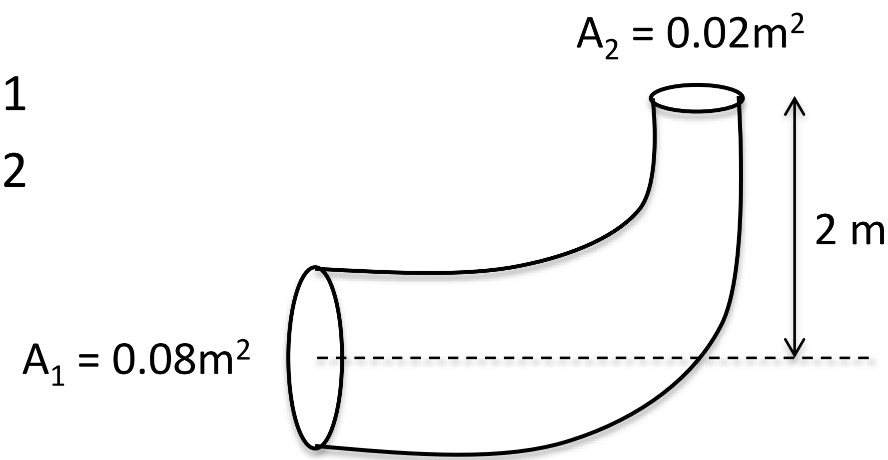
Pressure energy becomes Kinetic Energy (i.e. Pressure energy is transferred into Kinetic Energy)!

Both Pressures at each end is constant?! ahh what

Applying the Bernoulli equation

Student problem 2.2. Water is flowing through a pipe at the rate of 0.08 m^3/s. The pressure at Point 1 is 180 kPa. Find the following.

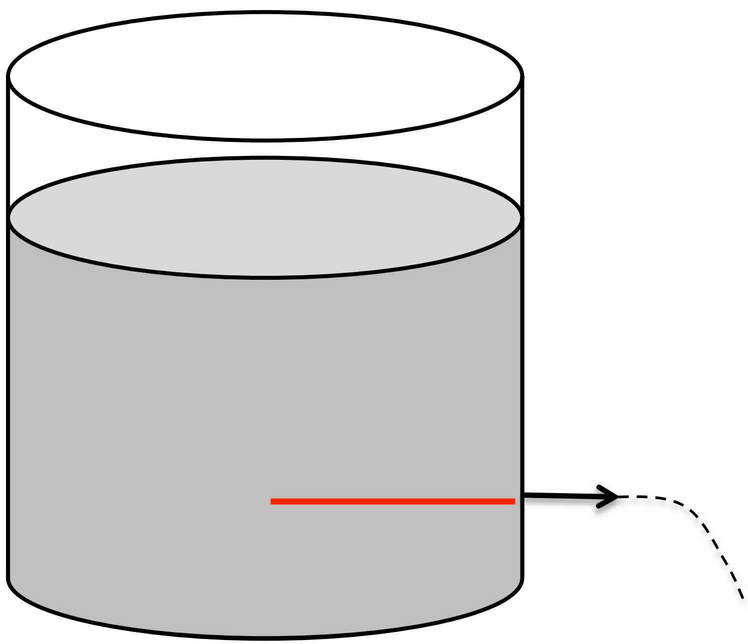
a) The velocity of the fluid at point 1
b) The velocity of the fluid at point 2
c) The pressure at point 2



See my working out through my email

Example problem 2.4

A nearsighted sheriff fires his gun at a cattle thief. Fortunately for the thief, the bullet misses him and penetrates the town's water tank instead and causes a leak. The top of the tank is open to the atmosphere. Determine the speed at which the water leave the hole when the water level is 0.5 m above the hole.



See my working out through my email

Conservation of energy

What assumptions are built into these equations?

The mechanical energy balance

$$\frac{\Delta P}{\rho}+\Delta\left(\frac{1}{2}\overline{V}^2\right)+g\Delta z+\Delta W_s+\Delta F=0$$

1. Steady state flow, the flow is not changing with time
2. Incompressible flow, density is constant

The Bernoulli equation

$$\frac{P_2}{\rho}+\frac{1}{2}\overline{V}_2^2+gz_2=\frac{P_1}{\rho}+\frac{1}{2}\overline{V}_1^2+gz_1$$

1. Steady state flow, the flow is not changing with time
2. Incompressible flow, density is constant
3. Friction is negligible
4. No shaft work

Mini-summary

- Energy can leave/enter the system through two methods
  - Heat exchange with the surroundings
  - Shaft work
- The energy within the flow can be multiple forms
  - Pressure energy
  - Kinetic energy
  - Potential energy
- The Bernoulli equation is an idealized form of the energy balance where two parameters are assumed to be zero
  - Shaft work
  - Friction
- When using the Bernoulli equation, total energy in the system is constant; however, it changes from one form to another

Learning objectives revisited

By the end of this lesson, students should be able to

- Define vocabulary to describe flow (ex. stress, viscosity, stream line)
- Qualitatively describe the similarities and difference between solid and fluid deformation
- Describe the forces that cause/resist flow
- Understand the derivations of the conservation laws for fluid flow
  - Mass
  - Momentum
  - Energy
- Be able to describe the assumptions that are built into each of these equations, and know when they can and cannot be used
- Be able to use the conservation laws to solve fluid flow problems