F cdf τ_i. $\quad 1-F(y) = P(\tau_i > y)$ $\quad Y_t \xrightarrow[t\to\infty]{}$ distribution of density $\dfrac{P(\tau_i > y)}{\mu}$.

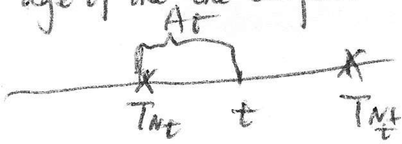
of arrival time t

ystem ed

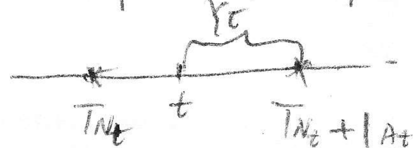
$z \geq 0$ $\quad E(Z) = \int_0^\infty 1 \cdot P(Z > z)\, dt$.

$E(Z) = \sum_k k \cdot P(Z = k) = \sum_{k=1}^{\infty} P(Z \geq k)$

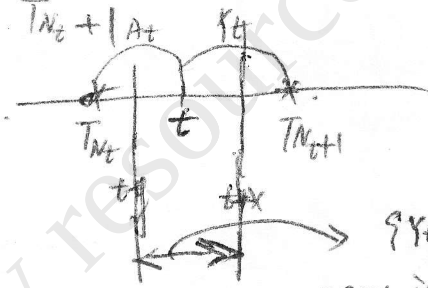
A_t: age of the the component at time t := $A_t = t - T_{N_t}$.



Y_t: The residual lifetime of component at time t : $Y_t := T_{N_t+1} - t$



Event $\{Y_t > x, A_t > y\}$



$\{Y_t > x, A_t > y\}$ means in this interval no renews.

which is same as $Y_{t-y} > x+y$

$\Rightarrow \lim_{t\to\infty} P(Y_t > x, A_t > y)$

$= \lim_{t\to\infty} P(Y_{t-y} > x+y)$

$N_t \sim$ Poisson process $\quad \tau_i \sim Exp(\lambda)$

(Y_t, A_t). joint dist

$P(Y_t > x, A_t > y)$

$= \lambda \int_{x+y}^{\infty} e^{-\lambda t}\, dz = e^{-(\lambda x + \lambda y)}$

$= e^{-\lambda x} \cdot e^{-\lambda y}$.

Recall central limit Therom .

WLLN 1^{st} order X_i iid $\frac{\sum X_i}{n} \to \mu$.

CLT (Central limit Therom) Fluctuation $\frac{\frac{\sum X_i}{n} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \to N(0,1)$.

if $E[\tau_j] = \mu$. $Var(\tau_j) = \sigma^2 < \infty$.

$N_t \approx N_0\left(\frac{t}{\mu}, \frac{t\sigma^2}{\mu^3}\right)$ large t

$\frac{N_t}{t} \approx N_0\left(\frac{1}{\mu}, \frac{\sigma^2}{t\mu^3}\right)$

$\Rightarrow \frac{N_t - \frac{t}{\mu}}{\sqrt{\frac{t\sigma^2}{\mu^3}}} \xrightarrow{d} N(0,1)$.

$\forall x$. $\lim_{t\to\infty} P\left(\frac{N_t - \frac{t}{\mu}}{\sqrt{t\sigma^2/\mu^3}} \le x\right) = \Phi(x)$ normal distribution function

let $Z = \frac{T_i - i\mu}{\sqrt{i\sigma^2}} \overset{d}{\approx} N(0,1)$ $i \to \infty$.

$P(N_t \ge i) = P(T_i \le t)$

$\approx P\left(Z \le \frac{t - i\mu}{\sqrt{i\sigma^2}}\right)$

$= P\left(Z \ge \frac{i\mu - t}{\sqrt{i\sigma^2}}\right)$.

e.g. $\tau_i \sim U(30, 60)$. $N_t \sim \frac{t}{45}$ $\mu = 45$. $\sigma^2 = \frac{(60-30)^2}{12}$.

For large t: $N_t \simeq N\left(\frac{t}{45}, \frac{t(30)^2}{12(45)^3}\right)$

After 1000 hours of 95% chance. Jenny will use battery

$\frac{1000}{45} \pm \sqrt{\frac{(1000 \times (30)^2}{12 \cdot (45)^3}} \times 1.96$