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1) (i) $V = \dfrac{100 m^3}{3600 s \times \pi \times \frac{0.1^2}{4} m^2}$

$= 3.54 m/s$

$Re = \dfrac{1000 \times 3.54 \times 0.1}{10^{-3}}$

$= 353678$

$\dfrac{e}{D} = 0.02.$

$f_F = 0.0122.$

$$\underbrace{\cancel{\frac{\Delta P}{\rho}}}_{\text{both Patm}}^{0} + \underbrace{\cancel{\frac{1}{2}\Delta v^2}}_{\text{both free surface}}^{0} + \overbrace{\cancel{g\Delta z}}^{\text{no elevation}} + W_s + F = 0$$

$W_s + F = 0$

$-W_s = \dfrac{2 \times 0.0122 \times 10 \times 3.54^2}{0.1} + \dfrac{1}{2} \times 0.6 \times 3.54^2$

$= 34.34 \quad J/kg$

$P_F = -W_s G$

$= 34.34 J/kg \times \dfrac{100 m^3}{3600 s} \times 1000 kg/m^3.$

$= 953.794 J$

$P_B = \dfrac{P_F}{0.7} = 1.36 kW$ //

(ii) $NPSH_A = \dfrac{(101.7 - 2.3) \times 10^3}{10^3 \times 9.8} + 0 - 0.$

$= 10.07 m.$

as $NPSH_A > NPSH_R$, it is sufficient to prevent cavitation.

(iii) hsystem = F only here

$= \dfrac{-W_s}{g} = 3.50 m$ // as hsystem < h pump, install a valve downstream that will ↑ hf to meet the pump specifications //

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1 (iv). $-W_s = \dfrac{2 \times 0.0122 \times 20 \times 3.54^2}{0.1} + \dfrac{1}{2} \times 0.6 \times 3.54^2$

$= 64.91 \, J/kg$.

$NPSH_A = \dfrac{P_1 - P_{vap}}{\rho g} + z_1 - h_{fs}$

$= \dfrac{(101 - 2.3) \times 10^3}{10^3 \times 9.8} + 0 - \dfrac{64.91}{9.8}$

$= 3.44 m$

which still meets (greater than $NPSH_R$) vv.

(v)

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2 (i) $$\frac{(560\times10^3)^2-(700\times10^3)^2}{2\times8.314\times\frac{(306)}{16\times10^{-3}}} + \left(\frac{G}{A}\right)^2\left[\ln\left(\frac{700}{560}\right)+\frac{2\times0.0065\times100}{0.1}\right] = 0.$$

$$-554699 \quad + \left(\frac{G}{A}\right)^2\left[\ 13.22\right] \quad = 0$$

$$\left(\frac{G}{A}\right)^2 = 41949$$

$$\frac{G}{A} = 2048\ \mathrm{kg/sm^2}$$

$$G = 204.8\times\frac{\pi\times0.1^2}{4}\ \mathrm{kg/s}$$

$$= 1.61\ \mathrm{kg/s}$$

(ii) $$\frac{4fL_{min}}{D} = \left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1$$

$$L_{min} = \frac{0.1}{4\times0.0065}\left[\left(\frac{700}{560}\right)^2-\ln\left(\frac{700}{560}\right)^2-1\right]$$

$$= 0.446\ \mathrm{m}$$

as $L_{min} < L_{pipe}$ then flow is not choked. Trial and error gives $P_w = \sim 127\ \mathrm{kPa}$

(iii) $1.61\ \mathrm{kg/s} \times 55.6\times10^6\ \mathrm{J/kg} \times 0.1 = 8951600\ \mathrm{J/s}$

$= 8.95\ \mathrm{MW}$

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4) (i) $\frac{\partial V_z}{\partial z} = 0$ or V_z is not a function of z

(ii) z-component of momentum equations

(a) $V_x = V_y = 0$

(b) $\frac{\partial V_z}{\partial z} = 0$ from (i)

(c) steady state means any term with time dependence $= 0$

$$\rho\left(\cancelto{0}{\frac{\partial V_z}{\partial t}} + \cancelto{0}{V_x}\frac{\partial V_z}{\partial x} + \cancelto{0}{V_y}\frac{\partial V_z}{\partial y} + V_z \cancelto{0}{\frac{\partial V_z}{\partial z}}\right)$$

(iii) $$-\cancelto{0}{\frac{\partial p}{\partial z}} + \mu\left(\frac{\partial^2 V_z}{\partial x^2} + \cancelto{0}{\frac{\partial^2 V_z}{\partial y^2}} + \cancelto{0}{\frac{\partial^2 V_z}{\partial z^2}}\right) + \rho g_z$$

$$\mu \frac{\partial^2 V_z}{\partial x^2} = \rho g_z$$

$$\frac{\partial^2 V_z}{\partial x^2} = \frac{\rho g_z}{\mu}$$

$$\frac{\partial V_z}{\partial x} = \frac{\rho g_z}{\mu} x + C_1$$

$$\left.\frac{\partial V_z}{\partial x}\right|_{z=h} = 0.$$

$$\Rightarrow -\frac{\rho g_z h}{\mu} = C_1.$$

$$\frac{\partial V_z}{\partial x} = \frac{\rho g_z}{\mu}(x-h).$$

$$V_z(x) = \frac{\rho g_z}{\mu}\left(\frac{x^2}{2} - hx\right) + C_2.$$

$$V_z(x=0) = 0, \quad C_2 = 0$$

(iv) $$Q = \bar{V}A$$
$$= \int_0^w \int_0^h V_z(x)\, dx\, dy$$
$$= \int_0^w \int_0^h \frac{\rho g_z}{\mu}\left(\frac{x^2}{2} - hx\right) dx\, dy$$

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4) (v) $$\int_0^{1.5m} \int_0^{0.001} \frac{\rho g z}{\mu} (x^2 - hx)\, dx\, dy$$

$$= \frac{850 \times 9.81}{0.1} \times 1.5 \left[\frac{x^3}{6} - \frac{hx^2}{2} \right]_0^{0.001}$$

$$= -0.00004165 \, m^3/s$$

or $4.2 \times 10^{-5} \, m^3/s$ in the negative z direction,,