We distinguish nine cases: each combination of one of the possibilities $\alpha < 0$, $\alpha = 0$, $\alpha > 0$, and one of the possibilities $\beta < 0$, $\beta = 0$, $\beta > 0$. We can put these combinations in a table (see Table S1.1), and fill the entries of the table as follows.

(1) Suppose first that $\beta < 0$. Then we may take $x_1 = 0$ and make the value of x_2 arbitrarily negative without violating the constraints. The corresponding objective value becomes arbitrarily negative and, hence, the model is unbounded. So, each entry in the first row of Table S1.1 is 'unbounded model'.

(2) Suppose next that $\beta = 0$. The model becomes $\min\{x_1 + x_2 \mid \alpha x_1 \geq 1,\ x_1 \geq 0\}$. Clearly, this model is feasible if and only if $\alpha > 0$. If $\alpha > 0$, then we may take $x_1 = \frac{1}{\alpha}$ and x_2 arbitrarily negative; hence the model is unbounded in that case. Therefore, the entries in the second row of Table S1.1 are 'infeasible model', 'infeasible model', and 'unbounded model', respectively.

(3) Finally, suppose that $\beta > 0$. If $\alpha \leq 0$, then the optimal value of x_1 is 0, because decreasing the value of x_1 decreases the objective value, while it does not decrease the left hand side value of the constraint. Since $\beta > 0$, it then follows that the optimal value

	$\alpha < 0$	$\alpha = 0$	$\alpha > 0$
$\beta < 0$	unbounded model	unbounded model	unbounded model
$\beta = 0$	infeasible model	infeasible model	unbounded model
$\beta > 0$	unique optimal solution	unique optimal solution	unique optimal solution if $\alpha < \beta$ multiple optimal solutions if $\alpha = \beta$ unbounded model if $\alpha > \beta$

Table S1.1: The possible optimal solutions of the model in Exercise 1.8.7.

is $\frac{1}{\beta}$. The last case, i.e., $\alpha > 0$ and $\beta > 0$ is easiest explained by drawing the graph of the feasible region (which is left to the reader). The following cases can be distinguished:

(a) If $\alpha < \beta$, then the unique optimal solution is $\begin{bmatrix}1 & 0\end{bmatrix}^\mathsf{T}$.

(b) If $\alpha = \beta$, then all points in the set $\left\{\begin{bmatrix}x_1 & x_2\end{bmatrix}^\mathsf{T} \,\middle|\, x_1 + x_2 = 1, x_1 \geq 0\right\}$ are optimal.

(c) If $\alpha > \beta$, then the model is unbounded.

The answers to the questions can now be determined from the entries of Table S1.1:

(a) The model is infeasible if and only if both $\alpha \leq 0$ and $\beta = 0$.

(b) The model has a bounded optimal solution if and only if both $\alpha \leq \beta$ and $\beta > 0$.

(c) The model is unbounded if and only if either $\beta < 0$ or $\alpha > \beta \geq 0$.

(d) The model has multiple optimal solutions if and only if both $\alpha = \beta$ and $\beta > 0$.

Solution to Exercise 1.8.13.

(a) We give two options: (1) remove player p from the input data set; (2) add the constraint $\sum_{j=1}^{M} x_{pj} = 0$.

(b) We give two options: (1) remove player p and position 1 from the input data; (2) add the constraint $x_{p1} = 1$.

(c) Add the constraint $\sum_{j=1}^{M} x_{pj} = 1$.

(d) Add the constraint $\sum_{j \notin B} x_{ij} = 0$ for $i \in A$.

Filename: Document1
Directory:
Template: /Users/acosta/Library/Group Containers/UBF8T346G9.Office/User Content.localized/Templates.localized/Normal.dotm
Title:
Subject:
Author: Alysson Machado Costa
Keywords:
Comments:
Creation Date: 9/23/20 5:02:00 PM
Change Number: 1
Last Saved On:
Last Saved By:
Total Editing Time: 1 Minute
Last Printed On: 9/23/20 5:04:00 PM
As of Last Complete Printing
Number of Pages: 2
Number of Words: 0
Number of Characters: 14 (approx.)