X_1, X_2, X_3 ~~iid taking~~ are independent random variables. uniformly distributed on $[0,t]$. show that for $i \le k$ & $x \le t$. the distribution function for the order statistic $Y_{(i)}$ is given by

$$F_{Y_{(i)}}(x) = \sum_{l=i}^{k} C_k^l \left(\frac{x}{t}\right)^l \left(1-\frac{x}{t}\right)^{k-l}$$

$F_{Y_i}(x) = P(Y_{(i)} \le x)$.

mean # $(Y_j) \ge i$ in interval $(0, x)$.

$$\Rightarrow P(Y_{(i)} \le x) = \sum_{l=i}^{k} P(l \text{ of } Y_j\text{'s} \le x, \ k-l \text{ of } Y_j\text{'s} \ge x).$$

$$= \sum_{l=i}^{k} C_k^l (F_Y(x))^l (1-F_Y(x))^{k-l}.$$

$$F'_{Y_{(i)}}(x) = \frac{k!}{(i-1)!\,(k-i)!} F(x)^{i-1} (1-F(x))^{k-i} f(x).$$

probability $i-1$ of $Y_j \le x$

pro of $k-i$ of $Y_j > x$

Poisson Process

A discrete random variable N. $\sim Po(\lambda)$.

$$p_n = \begin{cases} \frac{e^{-\lambda}\lambda^n}{n!} & n = 0, 1 \dots \\ 0 & n < 0. \end{cases}$$

$E(Po(\lambda)) = \lambda$.
$Var(\dots) = \lambda$

(exportion $f_T(t) = \lambda e^{-\lambda t}$ $E = \frac{1}{\lambda}$
$Var = \frac{1}{\lambda^2}$

- If $Y_n \stackrel{d}{=} Geo(\frac{\lambda}{n})$. $T\lambda \stackrel{d}{=} Exp(\lambda)$
  then for $t > 0$ $\lim_{n \to \infty} P(\frac{Y_n}{n} \le t) = P(T\lambda \le t)$
- If $X_n \stackrel{d}{=} Bio(n, \frac{\lambda}{n})$. $N_\lambda \stackrel{d}{=} Po(\lambda)$. Then for $k = 0, 1, \dots$
  $\lim_{n \to \infty} P(X_n = k) = P(N_\lambda = k)$ Exponential Distribution arsies as the limit of the geometric distributin

Properties of Poisson Process

- $N_{t+s} - N_t \sim Po(\lambda s)$
- for fixed h. $N_t^* := N_{t+h} - N_h$. $N^* \to$ poisson process with rate λ.
  (even h is r.v. independent of $(N_s, s > t)$.

✓ $N_{t+s} = Po(\lambda|t+s)$. $N_t \sim Po(\lambda t)$.

$N_{t+s} = \underbrace{N_{t+s} - N_t}_{} + \underbrace{N_t}_{} \to Po(\lambda t)$ $\Rightarrow E[e^{\theta(N_{t+s} - N_t)}] E[e^{\theta N_t}] = E[e^{\theta N_{t+s}}]$

$\downarrow$ $Po(\lambda(t+s))$ independent

$E[e^{\theta(N_{t+s} - N_t)}] = \frac{E[e^{\theta N_{t+s}}]}{E[e^{\theta N_t}]}$ $\to M_{Po(\lambda(t+s))}(\theta)$
$\to M(Po(\lambda t))(\theta)$