Michael Le 21689299 #1 $\boxed{\text{MAT4MDS 2023 EXAM}}$

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$\boxed{18/4/2023}$

Michael ce 21689299 # $\boxed{2}$

Q2). ~~$f(x) = s(\ q(p(x)))$~~

$$= s(r(p(q(x))))$$

~~$g(x) = \quad q(p(x))$~~

$$g(x) = r(s(p(q(x))))$$

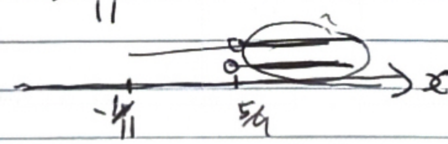
Q3) $f(x) = \dfrac{1}{\sqrt{9x-5}} + \ln(11x+1) \qquad (k, \infty)$

$x > \frac{-1}{11}$

$9x - 5 > 0$

$9x > 5$

$x > \frac{5}{9}$



$$\boxed{x > \frac{-1}{11} \cap x > 5/9}$$

$k = 5/9$

Michael Le 21689299 # $\boxed{3}$

Q4). $(BC)A = BAC$ (false) (0)

$C^TB^TA^T = C(BA)^T$ (False) (0)

$A+(B+C) = B+(A+C)$ (True) (1)

$\det(B)\det(A) = \det((AB)^T)$

$RHS = \det((AB)^T) = \det(B^TA^T)$

$= \det(B^T)\det(A^T)$

from property $= \det(B)\det(A)$ (1)

$\det(A) = \frac{1}{\det(A^{-1})}$ (Seen in Tutorial / Practice 3) (True) (1)

$B(A+C) = AB+BC$ (0) False

=

Order Matters.

Q5). ◦ $\det(M) = 1008$ $M_{4\times4}$ with eigenvalues of 6, 4, 6, 7

True (1)

◦ $Trace(M) = 23 \neq 1008$

False (0)

◦ M has full rank because all eigenvalues are non-zero (1) True.

◦ M is invertible (1) True.

if the $\det(M) \neq 0$.

Michael Le 21689299 # $\boxed{4}$

Q6)

$$A = \begin{bmatrix} 7 & x & 0 \\ 0 & 3 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned}
|A| &= 7(3(1)-1(9)) - x(0(1)-1(9)) \\
&= 7(3-9) - x(0-9) \\
&= (3-9)7 - x(-9) \\
&= (-6)7 + 9x \\
&= -42 + 9x = 0
\end{aligned}$$

$$9x = 42$$

$$x = \frac{42}{9} = \frac{14}{3}$$

Q7) $\det(BB^T C^T C^{-1})$

$$= \det(BB^T)\det(C^T) \times \frac{1}{\det(C)}$$

$$= \det(BB^T)\,\frac{\cancel{\det(C^T)}}{\cancel{\det(C^T)}}$$

$$= \det(BB^T)$$

$$= \det(B)\det(B^T)$$

$= \det(B)\det(B)$ (from property)

$$= (-6)(-6)$$

$$= 36$$

Michael Le 21689299 # $\boxed{5}$

- $Ax = \lambda x$. Find x

$$\begin{bmatrix} 7 & x \\ x & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underset{\substack{\shortparallel \\ 5}}{\lambda}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & x \\ x & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$7 + x = 5$ by ~~algebra~~ algebra fundementals.
$x + 7 = 5$ $(x = -2)$

- find the second eigenvalue.

Method 1

$$A = \begin{bmatrix} 7 & -2 \\ -2 & 7 \end{bmatrix}$$

$$\begin{vmatrix} \lambda-7 & 2 \\ 2 & \lambda-7 \end{vmatrix} = (\lambda-7)^2 - 4 = 0$$

$= \lambda^2 - 14\lambda + 49 - 4$
$= \lambda^2 - 14\lambda + 45$

$\cancel{\lambda}$ -9
$\cancel{\lambda}$ -5

$\cancel{\lambda} - 9 = 0$
$\cancel{\lambda} = 5$
$\cancel{\lambda} = 9$

$\lambda = 9$. (2ND eigenvalue).

Method 2.

$\text{Trace}(A) = \lambda_1 + \lambda_2 = 14$
$\lambda_1 = 5$

$\lambda_2 = 14 - 5 = 9$. (Still works).

Michael Le 21689299 #6

Basics

Q4) On the left graph you see it uses a linear
Part a scale on both axes. On the right you see
it is a log-log graph on both axes.

part b The level of radiation exhibits ~~how~~
over the number of days (i.e. the graph
on the right represents a striaght lines).

Q4c).

d (days)	R (radiation level)	$\log_{10}(R)$
10	128	2.11
35	30	1.48
60	9	0.95
80	5	0.70
$\boxed{(i)}$	$\boxed{(ii)}$	$\boxed{(iii)}$

Q4d). It is best to use the logarithm graph.
For larger values it is better to
use the linear graph. Depends
~~on~~ since they both show
identical data.

Michael le 21689299 # $\boxed{7}$

Q9e). We are modelling exponential decay of the radiation level.

days	$days^2$	$\log_{10}(R)$	$day \times \log_{10}(R)$
10	100	2.11	21.1
35	1225	1.48	51.8
60	3600	0.95	57
80	6400	0.70	56
185	11325	5.24.	185.9

$$\begin{bmatrix} 11325 & 185 \\ 185 & 4 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 185.9 \\ 5.24 \end{bmatrix}$$

The inverse of $\begin{bmatrix} 11325 & 185 \\ 185 & 4 \end{bmatrix}$ is $\frac{1}{11075}\begin{bmatrix} 4 & -185 \\ -185 & 11325 \end{bmatrix}$

so,

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{11075}\begin{bmatrix} 4 & -185 \\ -185 & 11325 \end{bmatrix}\begin{bmatrix} 185.9 \\ 5.24 \end{bmatrix}$$

$$= \frac{1}{11075}\begin{bmatrix} -225.8 \\ 24951.5 \end{bmatrix}$$

$$= \begin{bmatrix} -0.020388 \\ 2.25296 \end{bmatrix}$$

Michael le 21689299 #8

Then, $\log_{10}(R) = +\alpha m + \beta$

↳ exponential model.

$R = 10^{\alpha m + \beta} = (10^{\alpha})^{m} 10^{\beta}$

$= (0.95)^{m} \times 179.04$

Q10) $\boxed{\text{Student ID } 21689299}$

(a). $a = 9$

$f(x) = \sqrt{4 - 9x}$

$f'(x) = \frac{d}{dx} f(x)$ (fundamental theorem of calculus)

$= \frac{d}{dx} (4 - 9x)^{1/2}$

let $u = 4 - 9x$ (chain-rule)

$\frac{du}{dx} = -9$

$y = u^{1/2}$

$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$

$\frac{dy}{dx} = \frac{1}{2(4-9x)} \frac{-9}{1} = \frac{-9}{2(4-9x)}$

Michael Le 21689299 # $\boxed{9}$

Q(0a) $a=9.$

$f(x)=(4-9x)^{1/2}.$

$f'(x)=\frac{1}{2}(-9)(4-9x)^{-1/2}$ chain-rule.

$=1/2(-9(4-9x)^{-1/2})$

$f''(x)=\frac{1}{2}(-9)\left(-\frac{1}{2}\right)(-9)(4-9x)^{-3/2}$

$=\frac{81}{-4}(4-9x)^{-3/2}$

$f'''(x)=\frac{81}{-4}\left(\frac{-3}{2}\right)(-9)(4-9x)^{-5/2}$

$=-\frac{2187}{8}(4\ 9x)^{-5/2}$

Q(0b).

$f(0)=4^{1/2}=2^{2 \cdot 1/2}=2$

$f'(0)=-\frac{9}{2}(4)^{-1/2}=-\frac{9}{2}(2)^{2 \cdot 1/2}=-\frac{9}{2}(2)^{-1}=\frac{-9}{4}$

$f''(0)=\frac{81}{-4}(4)^{-3/2}=\frac{81}{-4}2^{2x-3/2}=\frac{81}{-4\times 8}=\frac{-81}{32}$

$f'''(0)=-\frac{2187}{8}(4)^{-5/2}=\frac{-2187}{8}2^{2x-5/2}$

$=\frac{-2187}{8\times 32}=\frac{-2187}{256}.$

Michael Le ~~#~~21689299 #1 $\boxed{10}$

$$(T_3)(x) = 2 - \frac{9}{4}x - \frac{81}{32 \times 2}x^2 - \frac{2187}{256}\frac{1}{3!}x^3$$

$$= 2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3$$

$$= 2 - \frac{a}{4}x - \frac{a^2}{64}x^2 - \cancel{3\times9\times3\times9}\ \frac{a^3 x^3}{\cancel{64}\,512}$$

Q10c) The 3rd Taylor for $\sqrt{1+x}$ around zero is given by,

$$(T_3)(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$$

and applying to $f(x)$

We see that, ~~$(1+3x$~~ $(1+x)^{1/2} = (4-ax)^{1/2}$

$$(T_n f)(x) = 2 - \frac{a}{4}(-4x) - \frac{a^2}{64}(-4x)^2 - \frac{(-4x)^3 a^3}{512}$$

$\boxed{a=9}$

$\boxed{a=a.}$ $(1+x)^{1/2} \Rightarrow \left(4 - 9\left(-\frac{4}{9}x\right)\right)^{1/2}$

$2(1+x)^{1/2}$

$$(T_n f)(x) = \cancel{2}\frac{1}{2}\left[2 - \frac{a}{4}\left(\frac{-4x}{9}\right) - \frac{9^2}{64}\left(\frac{-4x}{9}\right)^2 - \frac{a^3}{512}\left(\frac{-4x}{9}\right)^3\right]$$

Michael G 21689299 # $\boxed{11}$

$(T_3 f)(x) =$

$f(x) = \sqrt{4-ax}$

$= 2\sqrt{1-}$

$= \sqrt{4\left(1-\frac{ax}{4}\right)}$

$= 2\sqrt{1-\frac{ax}{4}}$ $\boxed{a=9}$

Re-write

$= 2\sqrt{1-\frac{9}{4}\left(-\frac{4x}{9}\right)}$

new x

$(T_n f)(x) = 2\left(1 + \left(-\frac{4}{9}x\right)/2 - \left(-\frac{4x}{9}\right)^2/8\right.$

$\left.+ \frac{1}{16}\left(\left(-\frac{4x}{9}\right)^3\right)\right)$

Michael Le 21689299 # $\boxed{12}$

Q11).

(a) $\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(e^{2ax-x^2+by} + b\ln(2-y)\right)$

$= (2a-2x)\, e^{2ax-x^2+by}$

chain-rule.

$\boxed{\begin{matrix} a=9 \\ b=9 \end{matrix}}$

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(e^{2ax-x^2+by} + b\ln(2-y)\right)$

chain rule + product rule.

$= b e^{2ax-x^2+by} + b\frac{1}{(2-y)}(-1)$

$= b e^{2ax-x^2+by} - \frac{b}{2-y}$

(b). Let $g(x) = f(x,1)$

i). $g(x) = f(x,1) = e^{2ax-x^2+b}$ ~~$+ b\ln(2-1)$~~ ~~$+ b\log(2-1)$~~

$= e^{2ax-x^2+b}$

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Q11)
b ii).

$$g'(x) = \frac{d}{dx} g(x) \quad \text{(Chain-Rule)}$$

$$= (2a-2x)e^{2ax-x^2+b}$$

$$g'(x)=0 = (2a-2x)\cancel{e^{2ax-x^2+b}}$$ term gone

$$2a-2x=0$$
$$2x=2a$$
$$x=a$$
$$a=9$$

$x=9$
$$g(a) = e^{2a^2-a^2+b}$$
$$= e^{a^2+b}$$

$\underline{(9, e^{a^2+b})}$ Stationary point.

$(9, e^{9^2+9})$, $(9, e^{90})$ using ID last 2-digits.

Q11b)
iii. since $g(x)>0$, it is a local maximum (~~...~~ Global maximum)

Q11c). Suppose that. (Next page)

Michael Le 21689299 # 14

$\cancel{\frac{\partial f}{\partial x}(1,1) = (2a-2)e^{2a-1+b} + \ln(1)}$

$$\frac{\partial f}{\partial x}(1,1) = (2a-2)e^{2a-1+b} > 0$$

$$\frac{\partial f}{\partial y}(1,1) = e^{2a-1+b} - \frac{1}{2-y} > 0$$

$$= e^{2a+b-1} + \frac{1}{y-2} > 0$$

Suppose that,

$$\frac{\partial f}{\partial x}(1,1) = (2a-2)\, e^{2a-1+b} = 2(a-1)\, e^{2a-1+b}$$

$$\frac{\partial f}{\partial y}(1,1) = b\, e^{2a-1+b} - \frac{b}{1}$$

$$= b\,(e^{2a-1+b} - 1)$$

By compariston, we increase y to improve efficiency.

Michael Le 21689299 # $\boxed{15}$

Q12) i) $F(x) = \int_{-\infty}^{a-1} 0 \, dt = 0$

ii). $F(x) = \cancel{\int_{-\infty}^{a-1} 0 \, dt} + \int_{a-1}^{x} \frac{3}{4}\left(1-(t^2-2ta+a^2)\right)dt$

$= \frac{3}{4}\left(t - \frac{t^3}{3} + t^2 a - a^2 t\right)_{a-1}^{x}$

$= \frac{3}{4}\left(x - \frac{x^3}{3} + x^2 a - a^2 x - \left(a-1 - \frac{(a-1)^3}{3} + (a-1)^2 a - a^2(a-1)\right)\right)$

Q12 iii).

$F(x) = \int_{-\infty}^{a+1} 0 \, dt + \int_{a+1}^{\infty} f(t)\, dt$

$= F(a+1) + \cancel{\int_{a+1}^{\infty} 0 \, dt}$

$= \frac{3}{4}\left(a+1 - \frac{(a+1)^3}{3} + (a+1)^2 a - a^2(a+1)\right)$

Q12 iv).

$$F(x) = \begin{cases} 0, & x < a-1 \\ \frac{3}{4}\left(x - \frac{x^3}{3} + x^2 a - a^2 x - \left(a-1 - (a-1)^3/3 + (a-1)^2 a - a^2(a-1)\right)\right) & a-1 \le x \le a+1 \\ 3/4\left(a+1 - ((a+1)^3/3 + (a+1)^2 a - a^2(a+1)\right), & x > a+1 \end{cases}$$

Q12 v). $F(a) = \frac{3}{4}\left(a - \frac{a^3}{3} + a^3 + a^3\right)$ $\frac{6}{3}$

$= 3/4\left(a + \frac{5}{3}a^3\right)$