# MAST30013 – Techniques in Operations Research

**Semester 1**

**Tutorial 1**

1. Consider the function

$$f(x) \;=\; x \log x - x + 5.$$

   It is known that there is a minimum of $f$ in $[0.5, 1.2]$. Apply both the Fibonacci search and the Golden Section search to estimate the minimum of $f$ to within a tolerance of 0.1.

   Compare your results.

2. (a) The Fibonacci numbers $F_n$ satisfy the recursion

$$F_n \;=\; F_{n-1} + F_{n-2} \tag{1}$$

   with $F_0 = F_1 = 1$. Show that

$$F_n \;=\; \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]. \tag{2}$$

   (Hint: The general solution to a recurrence relation $t_n = c_1 t_{n-1} + c_2 t_{n-2}$ has the form $t_n = A\lambda_1^n + B\lambda_2^n$. Let $F_n = \lambda^n$ and use (1) to find $\lambda_1$ and $\lambda_2$, and then the initial conditions to find $A$ and $B$.)

   (b) Now let $\gamma_n = F_{n-1}/F_n$. Use (1) to derive a quadratic equation for $\gamma \equiv \lim_{n\to\infty} \gamma_n$. (You may assume that this limit exists.)

   (c) Solve the equation in (b) and verify that there is a solution in $[0, 1]$ that coincides with $\lim_{n\to\infty} F_{n-1}/F_n$ derived from (2).

3. Prove that if you have at least 2 calculations available, the Fibonacci search will result in a strictly smaller interval than the Golden Section search for the same number of calculations.

   Hint: Start with an initial interval of length 1 and use induction.

   Note that this may not be the case if, in the Fibonacci search, the width of the final interval is $2\epsilon$.