THE UNIVERSITY OF MELBOURNE

Student number

Semester 2 Assignment 2, 2021

School of Mathematics and Statistics

MAST30022 Decision Making

Submission deadline: **4pm (Melbourne time), Friday 10 September**

This assignment consists of 12 pages (including this page)

Instructions to Students

- If you have a printer, print the assignment one-sided.

Writing

- There are 4 questions, of which 2 randomly chosen questions will be marked. Note you are expected to submit answers to all questions, otherwise **mark penalties will apply**.
- Working and reasoning **must** be given to obtain full credit. Give clear and concise explanations. Clarity, neatness, and style count.
- Write your answers in the boxes provided on the assignment that you have printed. If you need more space, you can use blank paper. Note this in the answer box, so the marker knows. The extra pages can be added to the end of the assignment to scan.
- If you have been unable to print the assignment write your answers on A4 paper. The first page should contain only your student number, the subject code and the subject name. Write on one side of each sheet only. Start each question on a new page and include the question number at the top of each page.

Scanning

- Put the pages in number order and the correct way up. Add any extra pages to the end. Use a scanning app to scan all pages to PDF. Scan directly from above. Crop pages to A4. Make sure that you upload the correct PDF file and that your PDF file is readable.

Submitting

- Go to the Gradescope window. Choose the Canvas assignment for this assignment. Submit your file. Get Gradescope confirmation on email.

Question 1

Consider the two-person, non-zero-sum, non-cooperative game with payoff bi-matrix

$$\begin{bmatrix} (9,-2) & (-1,1) \\ (7,4) & (1,10) \end{bmatrix}.$$

(a) Find the optimal security levels, the corresponding strategies, and the expected payoffs for each player if they both play their optimal security level strategies.

(b) Are the strategies found in part (a) in equilibrium?

Justify your answer.

(c) Find all equilibrium pairs using the graphical method.

(d) (i) Find $\boldsymbol{x}^*$ and $w \in \mathbb{R}$ such that

$$\boldsymbol{x}^*\boldsymbol{B} = \begin{bmatrix} w & w \end{bmatrix}.$$

((ii) Find $\boldsymbol{y}^*$ and $z \in \mathbb{R}$ such that

$$\boldsymbol{A}\boldsymbol{y}^{*T} = \begin{bmatrix} z \\ z \end{bmatrix}.$$

(iii) Does $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ found in parts (i) and (ii) give an equilibrium solution?

Justify your answer.

Question 2

Consider the two-person, non-zero-sum, cooperative game with payoff bi-matrix

$$\begin{bmatrix} (9,-2) & (-1,1) \\ (7,4) & (1,10) \end{bmatrix}.$$

(a) Find

(i) the cooperative payoff set C;

(ii) the Pareto boundary $PB(C)$;

(iii) the negotiation set $NS(C)$.

You are required to draw a graph to indicate C, $PB(C)$, and $NS(C)$ clearly. You are also required to give the expressions of $PB(C)$ and $NS(C)$ explicitly.

(b) Using the optimal security level pair (u^*, v^*) from question 1(a) as the status quo point, determine the unique Nash solution $(\underline{u}, \underline{v})$ to the game (that is, the unique point in C that satisfy Nash's Axioms).

(c) To achieve the Nash solution that you found in part (b), which pure strategy pair(s) should the players use? And with which probabilities should the players apply the pair(s)?

(d) Compare your answers to this question with your answers to question 1 where the game was treated as a non-cooperative game.

Question 3

Let $N = \{1, 2, 3\}$ and $v \in \mathrm{TU}^N$ as described in the table below with $\alpha \in \mathbb{R}$.

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	1	0	0	3	5	3	α

(a) (i) Find the minimum value of α such that v is superadditive.

(ii) Show that $C(v) = \emptyset$ for the minimum value of α.

(iii) Calculate the Shapley value by direct calculation using the minimum value of α.

(b) (i) Find the value of α so that the core consists of a single point. Find the point.

(ii) Calculate the Shapley value using the method of marginal vectors using the value of α found in part (b)(i).

(iii) Calculate v^* where (N, v^*) is the dual game of (N, v) using the value of α found in (b)(i).

(iv) Verify that $C(v) = C(v^*)$.

(v) Verify that the Shapley value for (N, v) is the same as the Shapley value for (N, v^*).

(c) In general, is the Shapley value always contained in the core?

Justify your answer.

Question 4

(a) A player i is called a *null player* in the game $(N; v)$ if for every coalition $S \subseteq N$

$$v(S) = v(S \cup \{i\}).$$

Prove that every null player is a dummy player.

(b) Consider the solution concept $\boldsymbol{\psi}$, defined for $i \in N$, by

$$\psi_i(v) = v(i).$$

Prove that $\boldsymbol{\psi}$ satisfies

(i) the dummy player property.

(ii) symmetry.

(iii) additivity.

(c) Does the solution concept $\boldsymbol{\psi}$ in part (b) give an alternative characterisation of the Shapley value?

Justify your answer.

End of Assignment