MAST30013 – Techniques in Operations Research

Semester 1, 2021

Tutorial 9

1. Consider the constrained nonlinear program

$$\begin{array}{rl} \min & \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 - x_1 + x_2 \\ \text{subject to} & x_1, x_2 \leq 0. \end{array}$$

   (a) Write down the l_2-penalty function $P_k(\boldsymbol{x})$ with penalty parameter k.

   (b) Write down $\boldsymbol{\nabla} P_k(\boldsymbol{x})$ and show that the stationary points for $P_k(\boldsymbol{x})$ only occur when $x_1 > 0$ and $x_2 < 0$.

   (c) Find all stationary points $\boldsymbol{x}^k = (x_1^k, x_2^k)$ for $P_k(\boldsymbol{x})$ such that $x_1^k > 0$ and $x_2^k < 0$. Write down the limit $\boldsymbol{x}^* = \lim_{k\to\infty} \boldsymbol{x}^k$.

   (d) For each stationary point, write down an estimate $\boldsymbol{\lambda}^k$ of the optimal Lagrange multiplier vector, and find the limit $\boldsymbol{\lambda}^* = \lim_{k\to\infty} \boldsymbol{\lambda}^k$.

2. Consider the non-linear program

$$\begin{array}{rl} \min & \frac{1}{4}x_1^4 - \frac{1}{2}x_1^2 + x_2^2 \\ \text{subject to} & x_1 \geq 0 \\ & x_2 \geq 2. \end{array}$$

   (a) Write down the l_2-penalty function $P_k(\boldsymbol{x})$ with penalty parameter k.

   (b) Write down $\boldsymbol{\nabla} P_k(\boldsymbol{x})$ and show that the stationary points for $P_k(\boldsymbol{x})$ only occur when $x_1 \geq 0$ and $x_2 < 2$.

   (c) Find all stationary points $\boldsymbol{x}^k = (x_1^k, x_2^k)$ for $P_k(\boldsymbol{x})$ such that $x_1^k \geq 0$ and $x_2^k < 2$. Write down the limit $\boldsymbol{x}^* = \lim_{k\to\infty} \boldsymbol{x}^k$.

   (d) For each stationary point, write down an estimate $\boldsymbol{\lambda}^k$ of the optimal Lagrange multiplier vector, and find the limit $\boldsymbol{\lambda}^* = \lim_{k\to\infty} \boldsymbol{\lambda}^k$.