School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 7. Some answers

Question 1

Although there are no viscous *forces* acting on the fluid there are still viscous *stresses* that act on the cylinder to produce the drag.

Question 2

The viscous normal stress is $\mu\frac{\partial w}{\partial z}$ if the normal is $\mathbf{k}$. By continuity, this is -$\mu(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})$.

Then use the no-slip boundary condition.

Question 3

This is a straightforward generalization of steady Poiseuille flow through a circular pipe.

$$w = \frac{G}{4\mu}[(a^2-\sigma^2)+3a^2\log(\sigma/a)/\log 2]$$

Volume flux is just $\int_a^{2a} 2\pi\sigma w\, d\sigma$

The shear stresses on the boundaries are just $\pm\mu\frac{dw}{d\sigma}$

Question 4

Separating variables produces exponentially decaying functions of time and Bessel functions of order 1 for the radial dependence.

Finding the coefficients to satisfy the initial condition gives a ratio of integrals of Bessel functions. To get the final results, use various orthogonality conditions and recurrence relations for Bessel functions. See Eqs. 11.4.5, 11.3.20 and 9.1.27 in Abramowitz and Stegun, *Handbook of Mathematical Functions*

Question 5

Include gravity in the equation of motion – it's a gravity driven flow. You can neglect the pressure gradient due to the pressure continuity condition at the free surface. Then use no-slip at the rod and zero stress BC's at the free surface.

Question 6

The outer/inner cylinder are fixed and use the fact that a free surface is a surface of constant pressure.

Question 7

Look at the Navier-Stokes equations in spherical coordinates assuming the existence of a flow

$$\mathbf{u} = w(r,\theta)\mathbf{e}_\phi$$

You should get after some work

$$\nabla^2 w = \frac{w}{r^2 \sin^2\theta}$$

Note: a scalar Laplacian.

Both boundary conditions at inner and outer surfaces take the form

$$w = K\sin\theta$$

which suggests we look for a flow of the form

$$w = f(r)\sin\theta$$

Substituting this form produces an equidimensional equation which gives the general solution

$$f = c_1 r + c_2 r^{-2}$$

The answer follows from applying the boundary conditions.

Question 8

i. Just use $\mathbf{U} = \Omega\hat{\mathbf{k}} \times a\mathbf{e}_r$

ii.

iii.