



Semester 2 Assignments 7 - 9, 2020

School of Mathematics and Statistics

## MAST 20018 Discrete Maths and Operations Research

Reading time: 30 minutes — Writing time: 3 hours — Upload time: 30 minutes

This exam consists of 7 pages (including this page)

### Permitted Materials

- This exam and/or an offline electronic PDF reader, blank loose-leaf paper and a Casio FX-82 calculator.
- One double sided A4 page of notes (handwritten or printed).

### Instructions to Students

- This assignment is worth 30 points (in the 0-100 point scale for the assignment components). The exam will follow the same format of this assignment. The instructions below are those that you will see in the exam.
- During writing time you may only interact with the device running the Zoom session with supervisor permission. The screen of any other device must be visible in Zoom from the start of the session.
- If you have a printer, print the exam one-sided. If you cannot print, download the exam to a second device, which must then be disconnected from the internet.
- Write your answers in the boxes provided on the exam that you have printed. If you are unable to answer the whole question in the answer space provided then you can append additional handwritten solutions to the end after the 7 numbered pages. If you do this you MUST make a note in the correct answer space or page for the question, warning the marker that you have appended additional remarks at the end.
- If you have been unable to print the exam write your answers on A4 paper. The first page should contain only your student number, the subject code and the subject name. Write on one side of each sheet only. Start each question on a new page and include the question number at the top of each page.
- Assemble all exam pages in correct page number order and the correct way up. Add any extra pages with additional working at the end. Use a mobile phone scanning application to scan all pages to a single PDF file. Scan from directly above to reduce keystone effects. Check that all pages are clearly readable and cropped to the A4 borders of the original page. Poorly scanned submissions may be impossible to mark.
- Submit your PDF file to the Canvas Assignment corresponding to this exam using the Gradescope window. Before leaving Zoom supervision, confirm with your Zoom supervisor that you have Gradescope confirmation of submission.

**Question 1**

Ozzy Mikks produces both interior and exterior paints from two raw materials,  $M1$  and  $M2$ . The following table provides the data of the problem:

	<b>Tonnes of raw material per tonne of</b>		<i>Availability (tonnes/day)</i>
	<i>Exterior paint</i>	<i>Interior paint</i>	
Raw material $M1$	6	4	24
Raw material $M2$	1	2	6
Profit per tonne (\$1000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than one tonne. Also, the maximum daily demand of interior paint is 2 tonnes. Ozzy Mikks wants to determine the optimum product combination of interior and exterior paints that maximises the total daily profit. Formulate, but do not solve, the problem as an LP. Clearly indicate the meaning of your variables.

**Question 2**

Consider the linear program:

$$\max z = 2x_1 + 3x_2$$

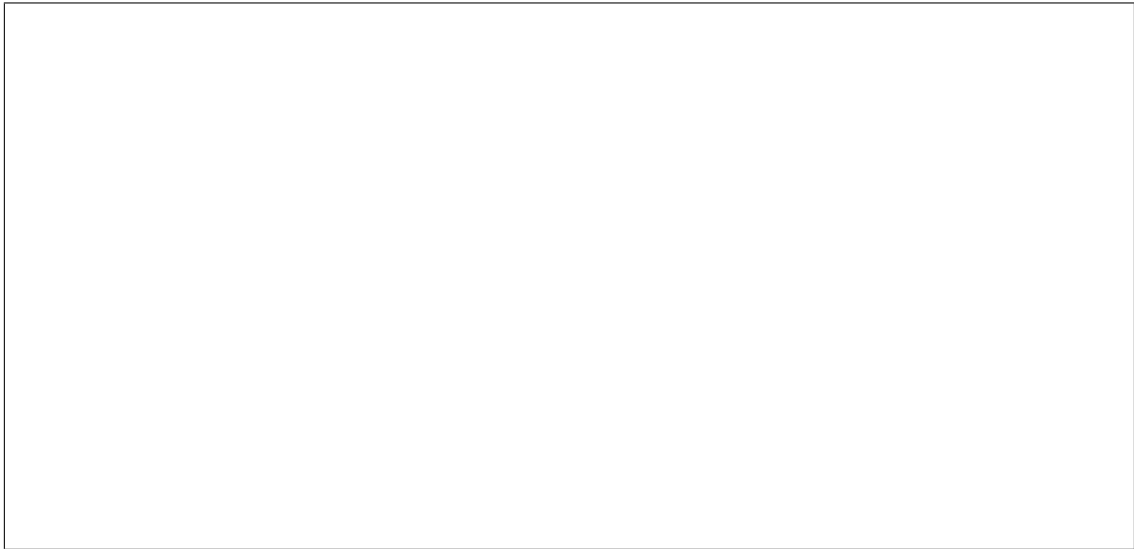
such that

$$2x_1 + x_2 \leq 4$$

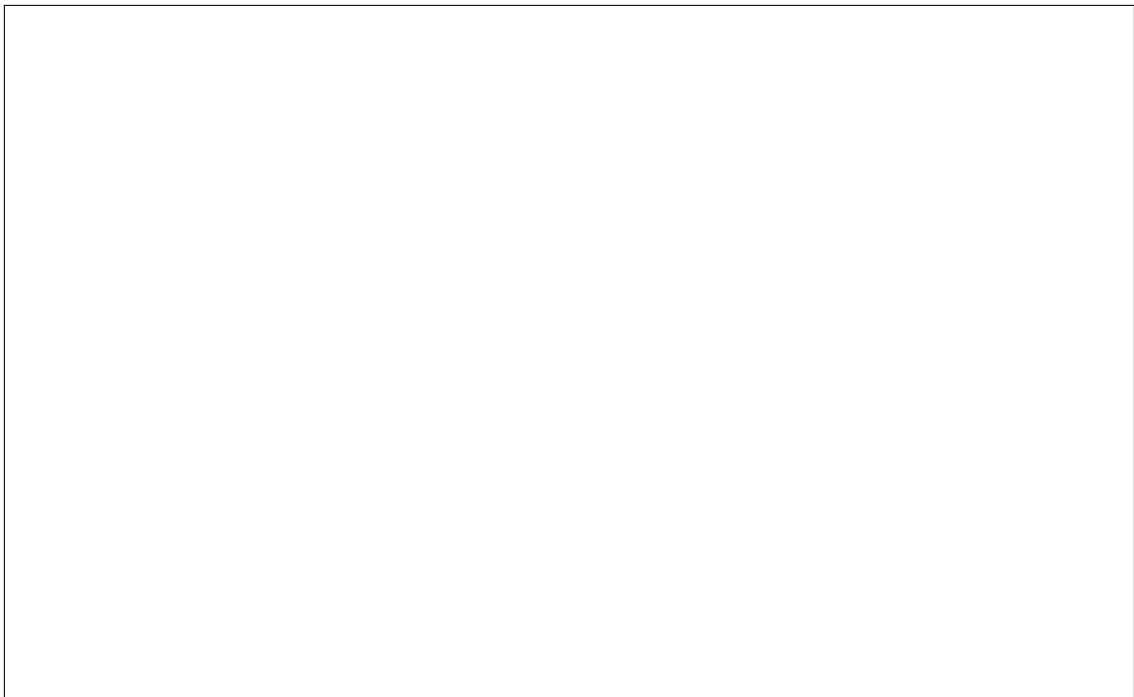
$$x_1 + 2x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

- (a) Find all basic feasible solutions of the LP using the graphical method.



- (b) Convert the LP from (a) into canonical form.



- (c) How would you use your answer in (b) to find the basic feasible solutions without using the graphical method? Note: just state the method here; do not recalculate the basic feasible solutions.

### Question 3

Consider the following LP:

$$\begin{aligned} \max z &= x_1 - x_2 + 2x_3 \\ \text{such that} \\ x_1 + x_2 + 3x_3 &\leq 15 \\ 2x_1 - x_2 + x_3 &\leq 2 \\ -x_1 + x_2 + x_3 &\leq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Let  $x_4, x_5$  and  $x_6$  denote the slack variables for the respective constraints. After applying the Simplex method, a portion of the final tableau is as follows:

BV	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	RHS
$x_4$				1	-1	-2	
$x_3$				0	1/2	1/2	
$x_2$				0	-1/2	1/2	
$z$				0	3/2	1/2	

Use the algebra of the Simplex method in order to complete the tableau. Note, you are NOT required to complete the RHS column.

#### Question 4

There are four items  $I_1, I_2, I_3, I_4$  to be distributed among three people  $A, B, C$  using the method of sealed bids. The bids are given by the following table.

bids	$A$	$B$	$C$
$I_1$	\$10,000	\$4,000	\$7,000
$I_2$	\$2,000	\$1,000	\$4,000
$I_3$	\$500	\$1,500	\$2,000
$I_4$	\$800	\$2,000	\$1,000

- (a) Extend the table to include the fair share of each player, and their contribution to the pot.

- (b) Compute the excess in the pot.

- (c) Summarize the allocation to each player.

- (d) Show, in detail and with reference to every player, that this allocation is envy free.

**Question 5**

It has been suggested that range voting should replace the preferential voting system currently used in Australian elections.

- (a) Describe the method of range voting.

- (b) What are the benefits of range voting?

- (c) What would be the drawbacks of implementing range voting in Australia?

**End of Exam**