1. Please submit via the LMS portal by 23:59pm on May 9.
2. In submitting your work, you are consenting that it may be copied and transmitted by the University for the detection of plagiarism. Please start with the following statement of originality, which must be signed and dated by you:
   `"This is my own work.  I have not copied any of it from anyone else."`
3. This assignment is worth 10% of your final mark.

MATLAB, row operations

1. Write a MATLAB function called `pivot`, which takes a matrix M and two integers i, j, where $i \leq r$, $j \leq k$ and r, k are respectively the number of rows and the number of columns of the matrix M, and pivots the matrix M on the element (i, j), i.e. it uses row operations to turn the $(i, j)^{\text{th}}$ element into a 1, and then use that 1 to clear out the column it is in.

   Do not use the function `addRow` from the solutions to Workshop 4 question 10, but write your own code, e.g. using a for statement which runs thought the rows.

   If you don't succeed in this question, then you may use the code provided in Workshop 4 question 10 in subsequent questions of this assignment.

Canonical form, basic solutions.

2. Decide which of the following systems are in canonical form. For those that are, say what the basic variables are and write down the basic solution.

   (a) $$\begin{aligned} x_1 + 2x_3 + x_4 &= 1 \\ x_2 + x_3 &= 2 \\ 5x_2 + x_5 &= 3 \end{aligned}$$

   (b) $$\begin{aligned} x_1 + 2x_2 + x_4 &= 3 \\ x_1 + x_2 + x_3 &= 1 \\ x_2 - x_5 + x_6 &= 2 \end{aligned}$$

   (c) $$\begin{aligned} x_2 + x_5 &= 2 \\ x_3 + 5x_4 &= 3 \\ x_1 + x_2 + 3x_4 &= 1 \end{aligned}$$

3. Consider the augmented matrix

$$M = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{array}\right].$$

   (a) Use the MATLAB program `pivot` from question 1 to bring M into canonical form, where columns 2, 4 and 5 are basic.

   (b) Is the matrix you found in (a) unique? If not, write down another matrix, which is also canonical with basic columns 2, 4 and 5.

Simplex algorithm, 2-phase method.

4. Solve the following LP problems using the simplex algorithm, or 2-phase method if needed.

   (a)
$$\begin{aligned} \text{minimize} \quad & z = 4x_2 - 2x_1 \\ \text{subject to} \quad & x_1 - 2x_2 \leqslant 2 \\ & 2x_1 + 3x_2 \leqslant 12 \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned}$$

   Notice this is a minimisation problem and the solution is not unique. Give all solutions.

(b) maximize $z = x_1 + 2x_2$

subject to $x_1 + 4x_2 \leqslant 12$

$x_1 + x_2 \geqslant 3$

$\mathbf{x} \geq \mathbf{0}$.

This problem has a degeneracy.

(c) maximize $z = x_1 + 3x_2 + 2x_3$

subject to $x_1 + 2x_2 + x_3 \geqslant 1$

$x_1 + 2x_2 + x_3 \leqslant 1$

$2x_1 - x_2 \geqslant -2$

$\mathbf{x} \geq \mathbf{0}$.

Here, you can combine two constraint equations and you need to deal with a negative resource value.

5. (a) Use MATLAB and the function `pivot` from question 1 to solve the problem from assignment 1 question 2, i.e.

$$\begin{aligned} \text{maximize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & \frac{1}{2}x_1 - x_2 \leq -1 \\ & \frac{1}{2}x_1 + x_2 \leq 4 \\ & 3x_1 + x_2 \geq 5 \\ & 2x_1 - x_2 \leq 2 \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

by using the 2-phase method.

(b) Next solve the related problem

$$\begin{aligned} \text{minimize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & \frac{1}{2}x_1 - x_2 \leq -1 \\ & \frac{1}{2}x_1 + x_2 \leq 4 \\ & 3x_1 + x_2 \geq 5 \\ & 2x_1 - x_2 \leq 2 \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

by using the 2-phase method.