The University of Melbourne

School of Engineering

Semester 2 Assessment 2017

ENGR30002 – Fluid Mechanics

Exam Duration: 3 hours

Reading Time: 15 minutes

This paper has FIFTEEN (15) pages consisting of EIGHT (8) questions.

Authorized material:
Only Casio fx82 or fx100 calculators are permitted
SIX (6) pages of supplemental material are attached.

Instructions to Invigilators:
Script books to be provided.
Charts and equation sheets can be detached.

Instructions to Students:
All eight questions are to be attempted.
Total marks for the exam = 100.
Charts and equation sheets can be detached.

This paper is to be held by the Baillieu Library

Question 1

Provide short answers to the following questions:

(a) Three constitutive equations that can be used to describe the rheology of liquids are provided below. Identify what types of fluids can be modeled by each of the equations. Write one to two sentences to describe the unique flow characteristics of each of these types of fluids.

$$\left|\tau_{yx}\right| = \tau_y + \mu_p \left|\frac{\delta V_x}{\delta y}\right|$$

$$\left|\tau_{yx}\right| = k_p \left|\frac{\delta V_x}{\delta y}\right|^n$$

$$\tau_{yx} = -\mu \frac{\delta V_x}{\delta y}$$

(4 marks)

(b) Describe in one to two sentences why laminar pressure driven flow through a horizontal and cylindrical pipe has a parabolic velocity profile?

(2 marks)

(c) Describe in one to two sentences why low viscosity fluids can be effectively mixed with small impellers but high viscosity liquids cannot.

(2 marks)

(d) Describe in one to two sentences why swirling is undesirable during mixing processes.

(2 marks)

(Total for Question 1 = 10 marks)

Question 2

Below you will find five statements. Analyze these statements using your knowledge of fluid mechanics. In your exam script book, write if the statements are always true, sometimes true, or never true. Write one to two sentences explaining your answer.

(a) Fluids flow from regions of high pressure to low pressure.

(2 marks)

(b) Liquids behave as incompressible fluids.

(2 marks)

(c) Under steady state flow conditions, centrifugal pumps provide kinetic energy to the fluid.

(2 marks)

(d) Under steady state flow conditions, the system head is equal to the pump head of a pumping system.

(2 marks)

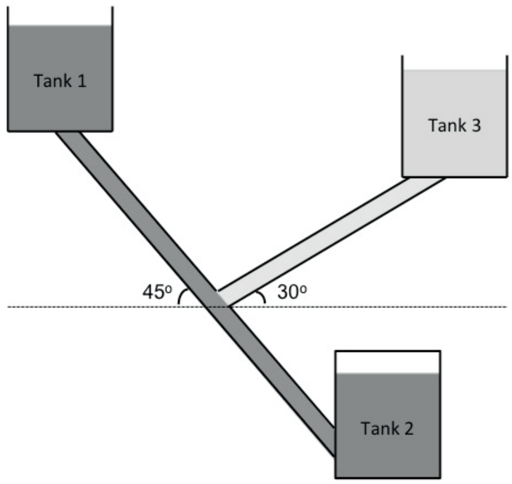
(e) The speed of sound is a constant value.

(2 marks)

(Total for Question 2 = 10 marks)

Question 3

Two water storage tanks (tank 1 and tank 2) are connected by a pipe, as shown in the schematic diagram below. These two tanks and the connecting pipe are coloured dark gray. Tank 1 is opened to the atmosphere. Tank 2 is enclosed and the pressure in the tank can be controlled. Your boss asks you to incorporate a third tank (tank 3) into the system and to connect this tank to the network via a new pipe, as shown in the schematic below. This new tank and pipe are coloured light gray. Tank 3 is also open to the atmosphere. The pipe connecting tank 1 to the joint is at a 45^o angle to the horizontal. The fluid at the free surface in tank 1 is 10 m above the joint, and the height of fluid in the tank is 2 m. The pipe connecting tank 3 to the joint will be at a 30^o angle to the horizontal, and the liquid level in tank 3 is desired to be 1.5 m. The bottom of tank 2 is 10 m below the joint, and the height of fluid in tank 2 is 3 m. Assume the tanks are large. Under normal operating conditions, it is desired that there is no flow of fluid between the tanks. Note, the schematic is not drawn to scale.



(a) What length of pipe should be used to connect tank 3 to the joint in metres?

(5 marks)

(b) If the liquid in the system is water ($\rho_{water} = 1000 kg / m^3$), what should the gauge pressure be in tank 2 in kilopascals?

(5 marks)

(Total for Question 3 = 10 marks)

Question 4

A ski resort needs to pump water uphill from a pond to a snow machine. The pond is at 2200 m in elevation, and the snow machine is at 2450 m in elevation. Assume that the water is at a constant temperature of $10^{o}C$ and that it is being pumped through 1,000 m of riveted steel pipe with a diameter of 10 cm. The pipe system contains centrifugal pump with 70% efficiency, a gate valve in the fully open position, and two 90^{o} square elbows. 7 L/s of water must be delivered to the snow machine with a gauge pressure of 7 kPa. Using the following physical properties for water.

Density: 1000 kg/m^3
Viscosity: $1.307x10^{-3}$ $N*s/m^2$

(a) Calculate the break power.

(10 marks)

(b) Several pump curves are provided in the reference material. Are any of these pumps suitable for this application? Explain your answer. In 1 m there are 3.28 ft. In 1 L there are 0.26 gallons.

(3 marks)

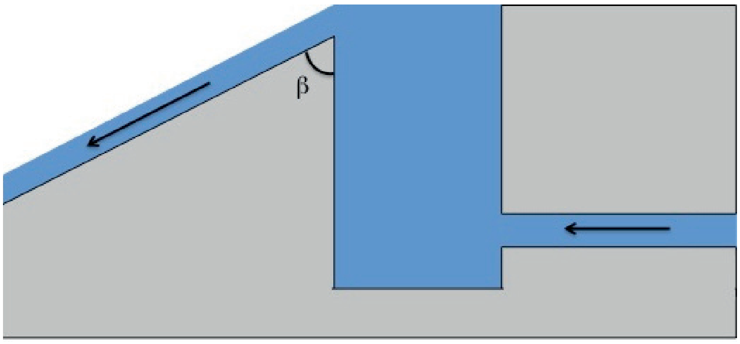
(c) If there are pumps presented on the pump curve diagram that can achieve the flow, select an appropriate pump. If there are no appropriate pumps, explain how you would engineer the system in order to successfully achieve the desired flow rate using pumps present on the pump curve diagram.

(2 marks)

(Total for Question 4 = 15 marks)

Question 5

Water flows from a pipe on the right-hand side into a storage reservoir that is open to the atmosphere. When the reservoir is full, the water will flow over a spillway on the left-hand side. The layer of fluid that flows over the spillway has a thickness of d. The length of the spillway is L and the width (going into the plane of the board) is b, and it is at an angle β to the vertical. Assume that the flow is steady and fully developed and that length and width of the spillway are much larger than the depth of the fluid.



(a) Find an analytical expression that describes the velocity of the fluid flowing down the spillway.

(12 marks)

(b) Qualitatively sketch the velocity profiles of a Newtonian, shear thinning, shear thickening, and Bingham fluid flowing over the spillway. In one to two sentences, describe how the flow profiles are different from each other.

(4 marks)

(Total marks for Question 5 = 16 marks)

Question 6

Nitrogen gas flows isothermally at 50 ^{o}C through a straight pipe of length 20 m and a uniform diameter of 100 mm. The inlet pressure is 700 kPa and the pressure outside the pipe exit is 300 kPa. The flow is assumed to be horizontal, isothermal, ideal, and compressible. The Fanning friction factor is f_F=0.005.

Molecular weight of nitrogen: 28 g/mol
Ideal gas constant: 8.314 J mol^{-1} K^{-1}

(a) Determine if the flow is choked or not. Give a reason for your conclusion.

(4 marks)

(b) Calculate the mass flow rate of nitrogen through the pipe

(7 marks)

(c) If the value of the inlet pressure is doubled, would the flow then be choked or not? Give a reason for your conclusion.

(3 marks)

(Total marks for Question 6 = 14 marks)

Question 7

Consider flow in an open channel with slope 1:500 and a Manning's *n* of 0.030. The channel has a rectangular cross-section with a width of 5 m.

(a) Uniform flow in this channel has a depth of 1.6 m. What is the flowrate in the channel?

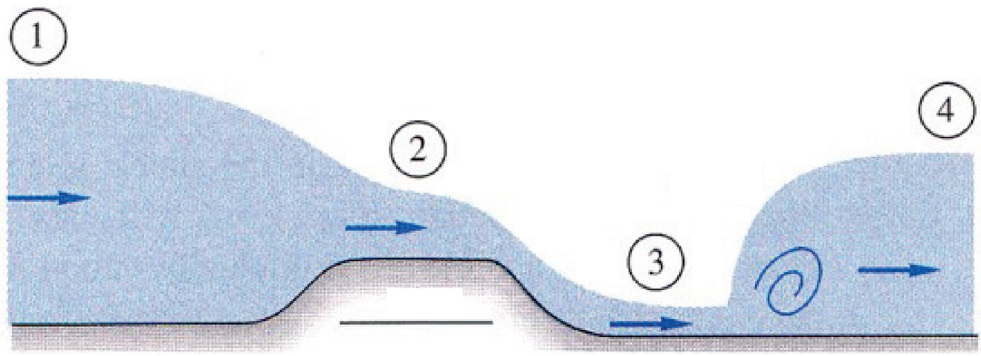
(4 marks)

(b) Suppose that this uniform flow (at point 1 in the figure below) encounters an obstacle on the channel bed (at point 2). How big would the obstacle need to be to cause a transition to supercritical flow downstream?

(5 marks)

(c) If a transition to supercritical flow does occur, show that the depth after the transition (at point 3) is approximately 48 cm, and thus determine the flow velocity after the hydraulic jump that forms (at point 4).

(6 marks)



(Total marks for Question 7 = 15 marks)

Question 8

(a) Consider the Loch Ness Monster hoax that has been discussed in class. To create an effective hoax, what is the minimum size of rock that *should* have been thrown into the pond? Assume that the rock generates waves with a wavelength roughly equal to the rock diameter.

(5 marks)

(b) To study the flow in Question 7, you create a 1:10 scale model of the channel, with water as the working fluid. What should the flowrate in the model channel be?

(5 marks)

	Water
ρ	998 kg/m^3
ν	1×10^{-6} m^{2}s^{-1}
σ	0.07 N/m at the air-water interface

(Total marks for Question 8 = 10 marks)

Reference material only beyond this point

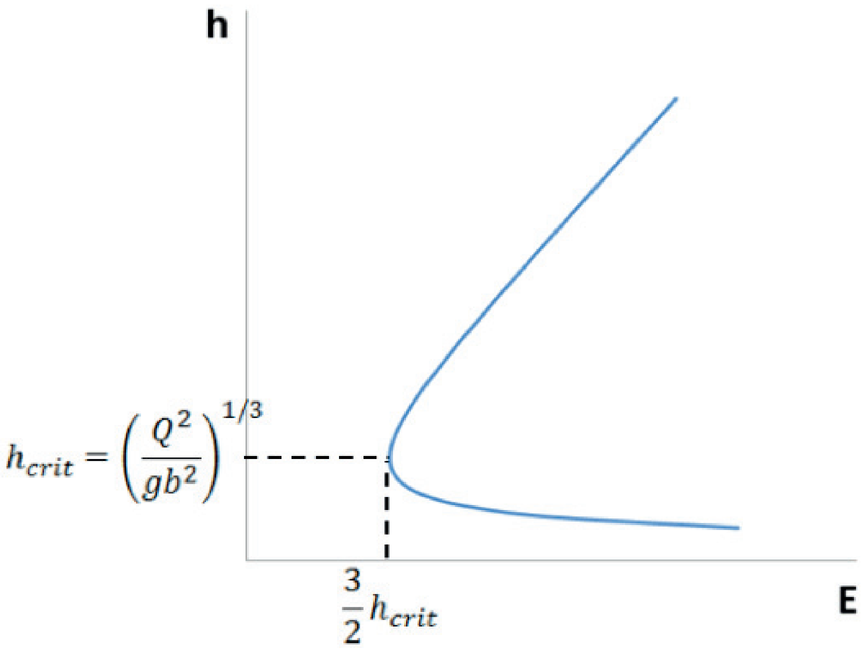
EQUATION SHEET

$$\frac{P_2^2 - P_1^2}{2(RT/M)} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{2 f_F L}{D}\left(\frac{G}{A}\right)^2 = 0$$

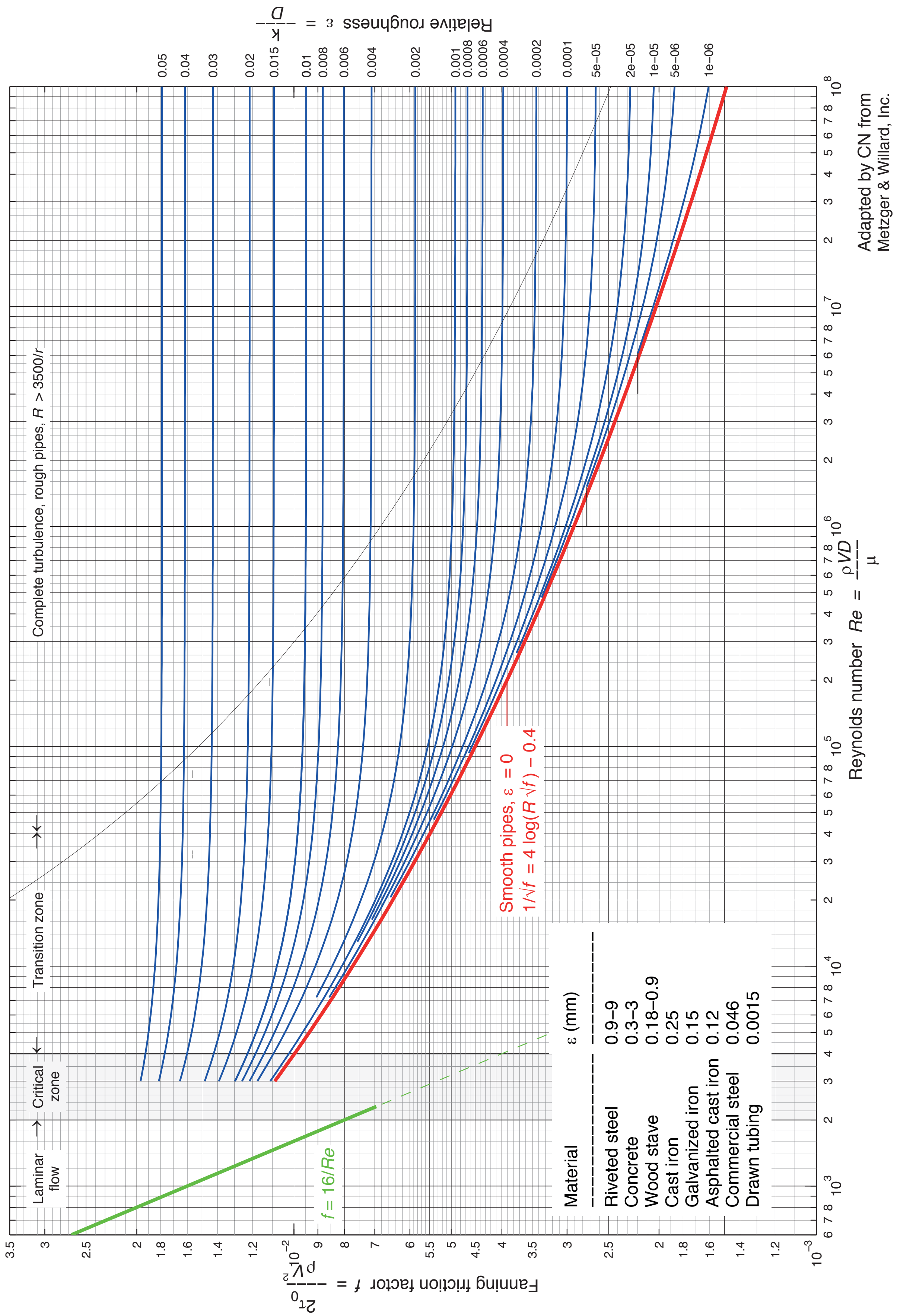
$$\frac{4 f_F L_{crit}}{D} = \left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1$$

$$U = \frac{1}{n} R_h^{2/3} S^{1/2}$$

$$E(h) = \frac{Q^2}{2gb^2h^2} + h$$



$$\frac{h_2}{h_1} = \frac{-1 + \sqrt{1 + 8Fr_1^2}}{2}$$


Relative roughness $\varepsilon = \frac{k}{D}$
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
5e−05
2e−05
1e−05
5e−06
1e−06
Fanning friction factor $f = \frac{2\tau_0}{\rho V^2}$
Reynolds number $Re = \frac{\rho V D}{\mu}$
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes, $R > 3500/r$
$f = 16/Re$
Smooth pipes, $\varepsilon = 0$
$1/\sqrt{f} = 4 \log(R\sqrt{f}) - 0.4$
Material | ε (mm)
Riveted steel | 0.9–9
Concrete | 0.3–3
Wood stave | 0.18–0.9
Cast iron | 0.25
Galvanized iron | 0.15
Asphalted cast iron | 0.12
Commercial steel | 0.046
Drawn tubing | 0.0015
Adapted by CN from
Metzger & Willard, Inc.

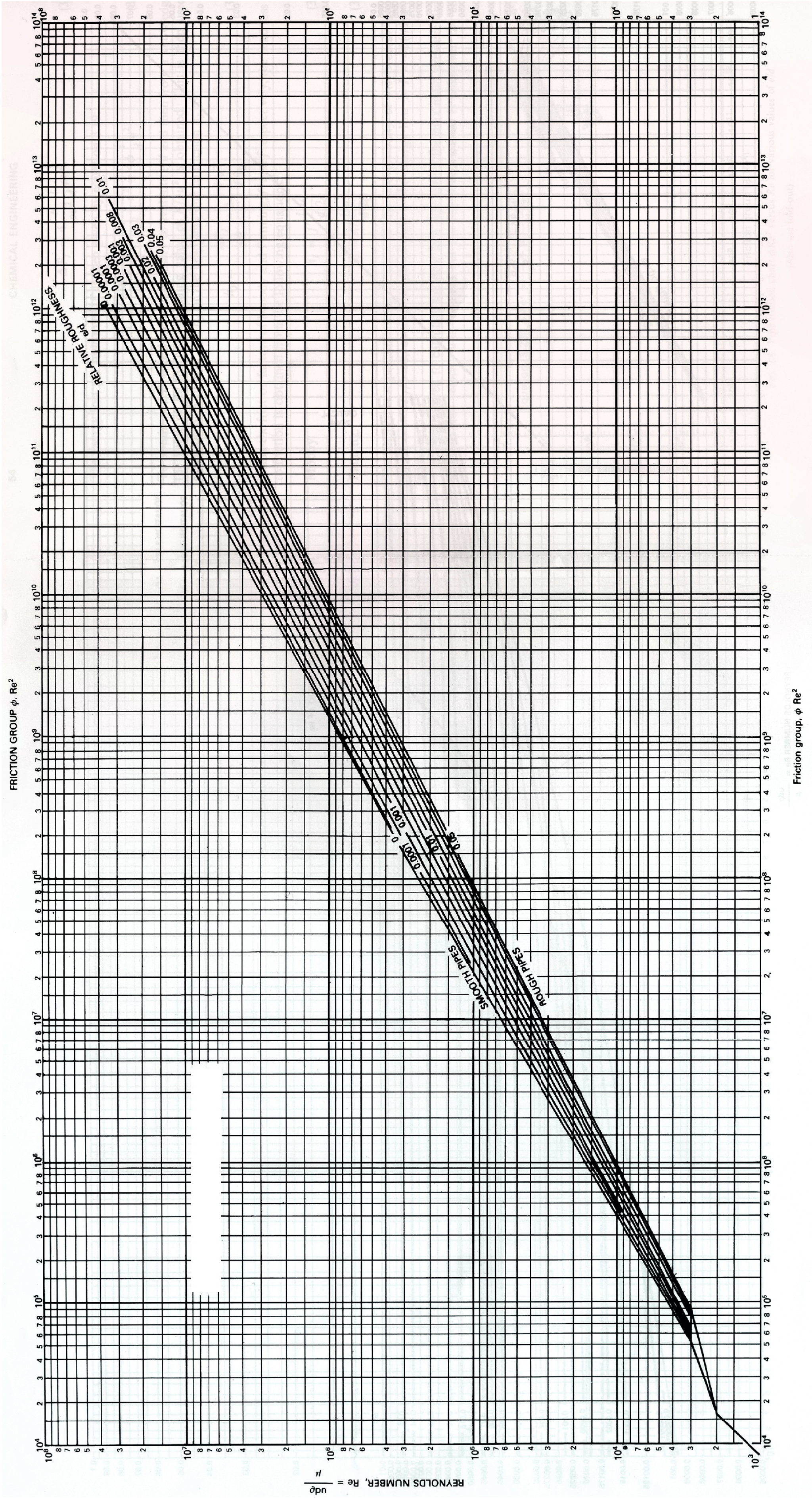


Fig. 3.8. Pipe friction chart ϕRe^2 versus Re for various values of e/d.

§B.4 THE EQUATION OF CONTINUITY[a]

$$[\partial\rho/\partial t + (\nabla\cdot\rho\mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Spherical coordinates (r, θ, ϕ):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho v_\phi) = 0$$

§B.6 EQUATION OF MOTION FOR A NEWTONIAN FLUID WITH CONSTANT ρ AND μ

$$[\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}]$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] + \rho g_\theta$$

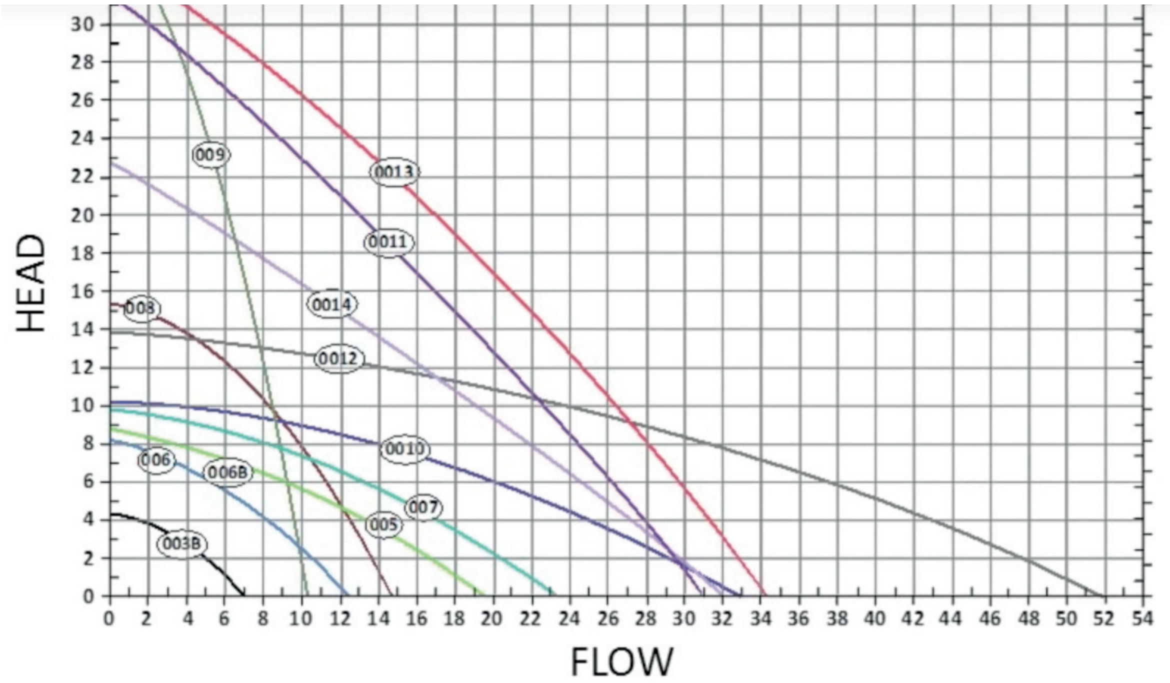
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial\theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] + \rho g_\phi$$



In the chart above, head is in feet and flow is in gal/min

Equivalent lengths and absolute roughness

Fitting	**L_{eq}**
45^o elbow	15D
90^o elbow	30 – 40D
90^o elbow square	60D
Entry from leg of T-piece	60D
Entry into leg of T-piece	90D
Unions and couplings	Very small
Gate valve – full open – half open – quarter open	7D 200D 500D

Pipe material	**Roughness, e (mm)**
Riveted steel	0.9 - 9
Concrete	0.3 - 3
Wood stave	0.2 – 0.9
Cast iron	0.26
Galvanized iron	0.15
Asphated cast iron	0.12
Commercial steal/wrought iron	0.046
Drawn tubing	0.0015

END OF EXAM