Fluid Mechanics

Topic 7

Dimensional Reasoning

The Loch Ness Monster

The Loch Ness Monster is a 20 meter monster that (supposedly) lives in Loch Ness in Scotland

The "surgeon's photograph"

Perturbations at the air-water interface

In this scenario, we're looking at a fluid-fluid interface

- Particularly, we are looking at an air-water interface

The equilibrium state of an air water interface would be a flat surface

- Think of the surface of a lake on a calm day

However, perturbations can happen that disrupt the surface of the fluid

These perturbations are called **waves**

Two main types of waves

Waves are divided into two main classes

Gravity waves

Capillary waves
(ripples)

Unique restoring forces

For each wave type, there is a **restoring force**, a force that acts to return the fluid to equilibrium

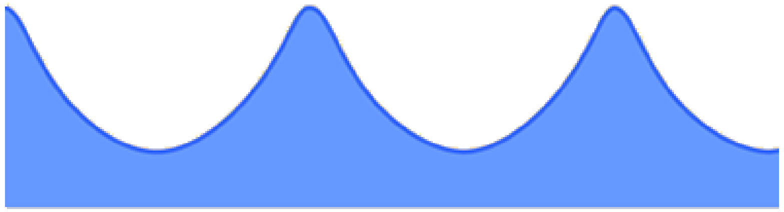
For **gravity waves**, the restoring force is **gravity**

For **capillary waves** (**ripples**), the restoring force is **surface tension**

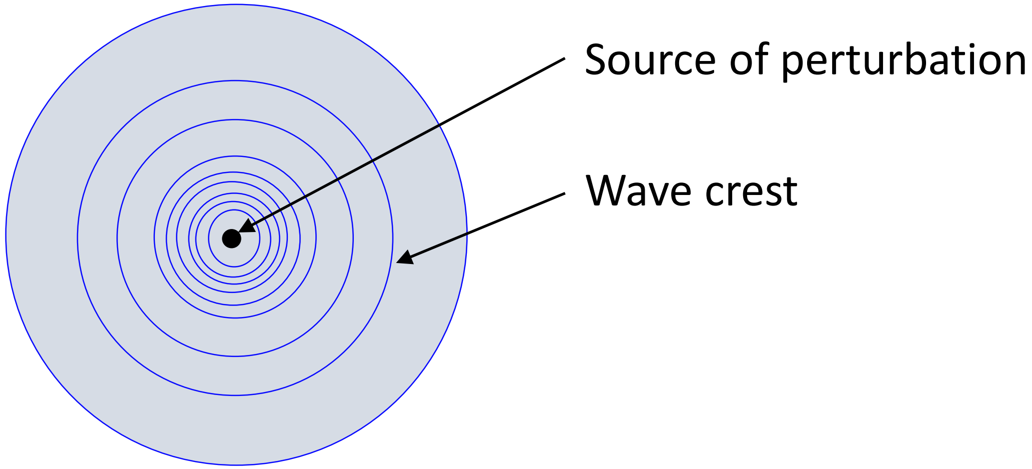
Properties of gravity waves

For gravity waves

- The restoring force is gravity
- The wavelength of gravity waves is > O(1) cm
- The waves are not perfectly sinusoidal
- The waveform (side view) of the waves have sharp peaks and long shallow troughs



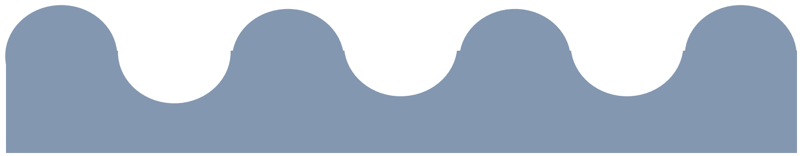
- Waves with a greater wavelength travel faster leading to a unique wave pattern



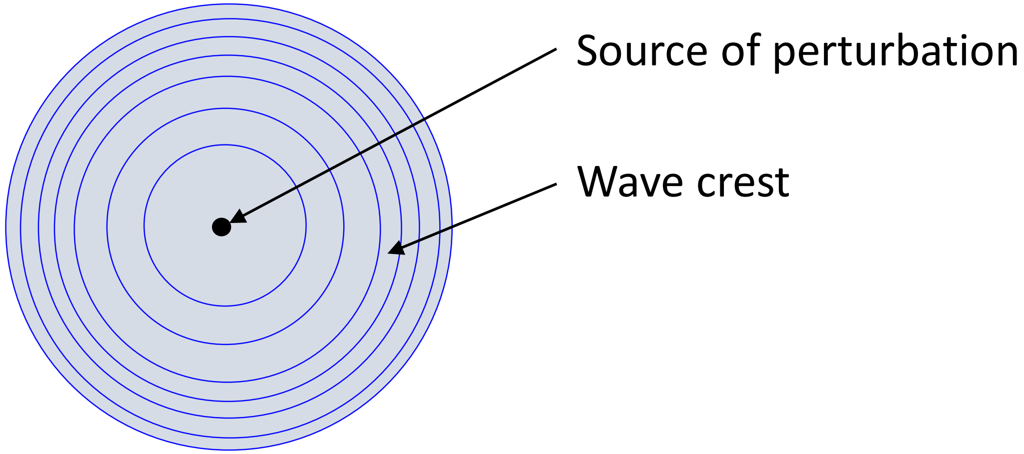
Properties of capillary waves

For capillary waves (ripples)

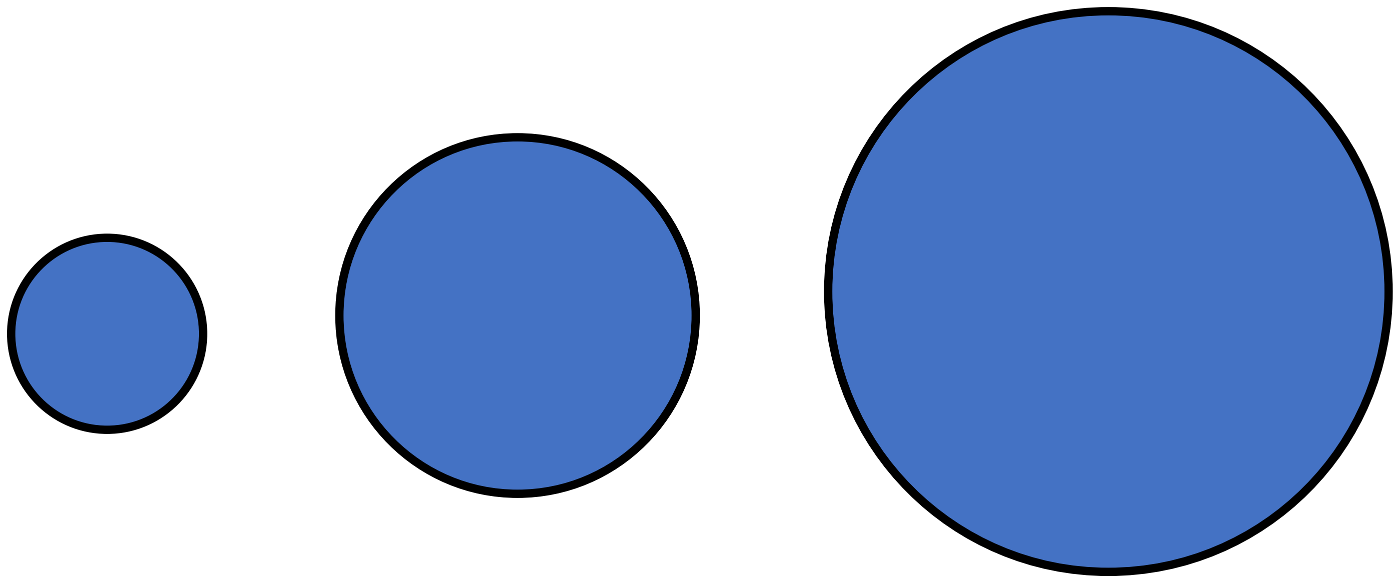
- The restoring force is surface tension
- The wavelength of capillary waves is O(1) cm or less
- Waves are not perfectly sinusoidal
- The waveform is almost hemispherical



- Waves with greater wavelengths travel slower



Which of these droplets will have the highest internal pressure?



The Loch Ness Monster

Using your knowledge of waves, how can you determine that this photograph is not of the real Loch Ness Monster?

The Loch Ness Monster – it's a hoax!!!

The story behind the hoax

- Marmaduke Weatherell wanted to prove the Loch Ness Monster existed
- He first faked "footprints" of the monster, that were determined to be faked
- Next, he worked with several others to create the "monster" in the photo from a toy submarine and wood putty
- He had a friend submit the photos to the local new paper *Daily Mail* in 1934
- The photo wasn't largely accepted as being fake until 1994, 60 years later!!!

How can be make a better hoax
(without spending a huge amount of money)?

Question: What are dimensionless parameters and how are they useful?

Forces on fluid particles – orders of magnitude

- Want to put forces in terms of characteristic length scale L and characteristic velocity U so we can construct dimensionless force balances

1) inertial forces

$$F_I = ma = m\frac{\partial u}{\partial t} = m\frac{\partial u}{\partial x/u} = mu\frac{\partial u}{\partial x}$$

$$m = \rho L^3$$

$$\therefore F_I \sim \rho L^3 U \frac{U}{L} = \rho U^2 L^2$$

Forces on fluid particles – orders of magnitude

2) Gravitational forces

$$F_g = mg$$

$$m = \rho L^3$$

$$F_g \sim \rho L^3 g$$

Forces on fluid particles – orders of magnitude

3) Viscous forces

$$F_v = \mu \frac{du}{dz} A$$

$$F_v \sim \mu \frac{U}{L} L^2 = \mu U L$$

Forces on fluid particles – orders of magnitude

4) Surface tension forces

$$F_v \sim \gamma L$$

Constructing dimensionless numbers

We can create a number of dimensionless parameters by taking the ratio of these forces

1. Reynolds Number (Re)

$$Re = \frac{inertial\ forces}{viscous\ forces} = \frac{\rho U^2 L^2}{\mu U L} = \frac{\rho U L}{\mu} = \frac{U L}{\nu}$$

$$\boxed{Re = \frac{\rho U L}{\mu}}$$

Constructing dimensionless numbers

We can create a number of dimensionless parameters by taking the ratio of these forces

2. Bond Number (Bo)

$$Bo = \frac{gravitational\ forces}{surface\ tension\ forces} = \frac{\rho L^3 g}{\gamma L}$$

$$\boxed{Bo = \frac{\rho L^3 g}{\gamma L}}$$

Constructing dimensionless numbers

We can create a number of dimensionless parameters by taking the ratio of these forces

3. Weber Number (We)

$$We = \frac{inertial\ forces}{surface\ tension\ forces} = \frac{\rho U^2 L^2}{\gamma L} = \frac{\rho U^2 L}{\gamma}$$

$$\boxed{We = \frac{\rho U^2 L}{\gamma}}$$

Constructing dimensionless numbers

We can create a number of dimensionless parameters by taking the ratio of these forces

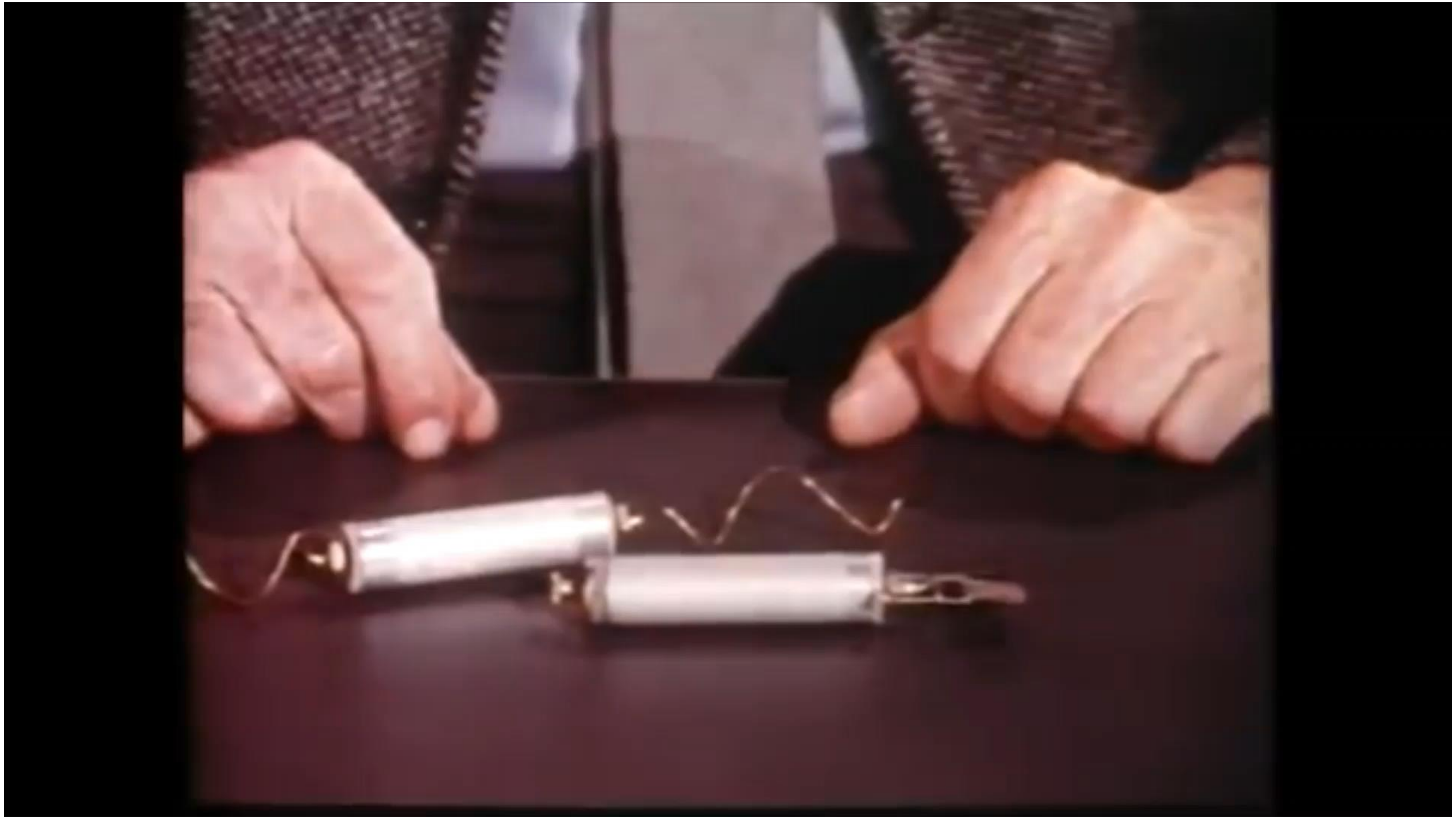
4. Froude Number (Fr)

$$Fr = \frac{inertial\ forces}{gravitational\ forces} = \frac{\rho U^2 L^2}{\rho L^3 g} = \frac{U^2}{gL} = \frac{U}{\sqrt{gL}}$$

$$\boxed{Fr = \frac{U^2}{gL}}$$

Introduction to the Reynolds Number (Re)

$$Re = \frac{\rho U L}{\mu}$$



When is Re important?

2. Turbulence is a function of Re

- The transition from laminar to turbulent flow occurs at ~ Re = 1000
  - The exact transition point will be system dependent

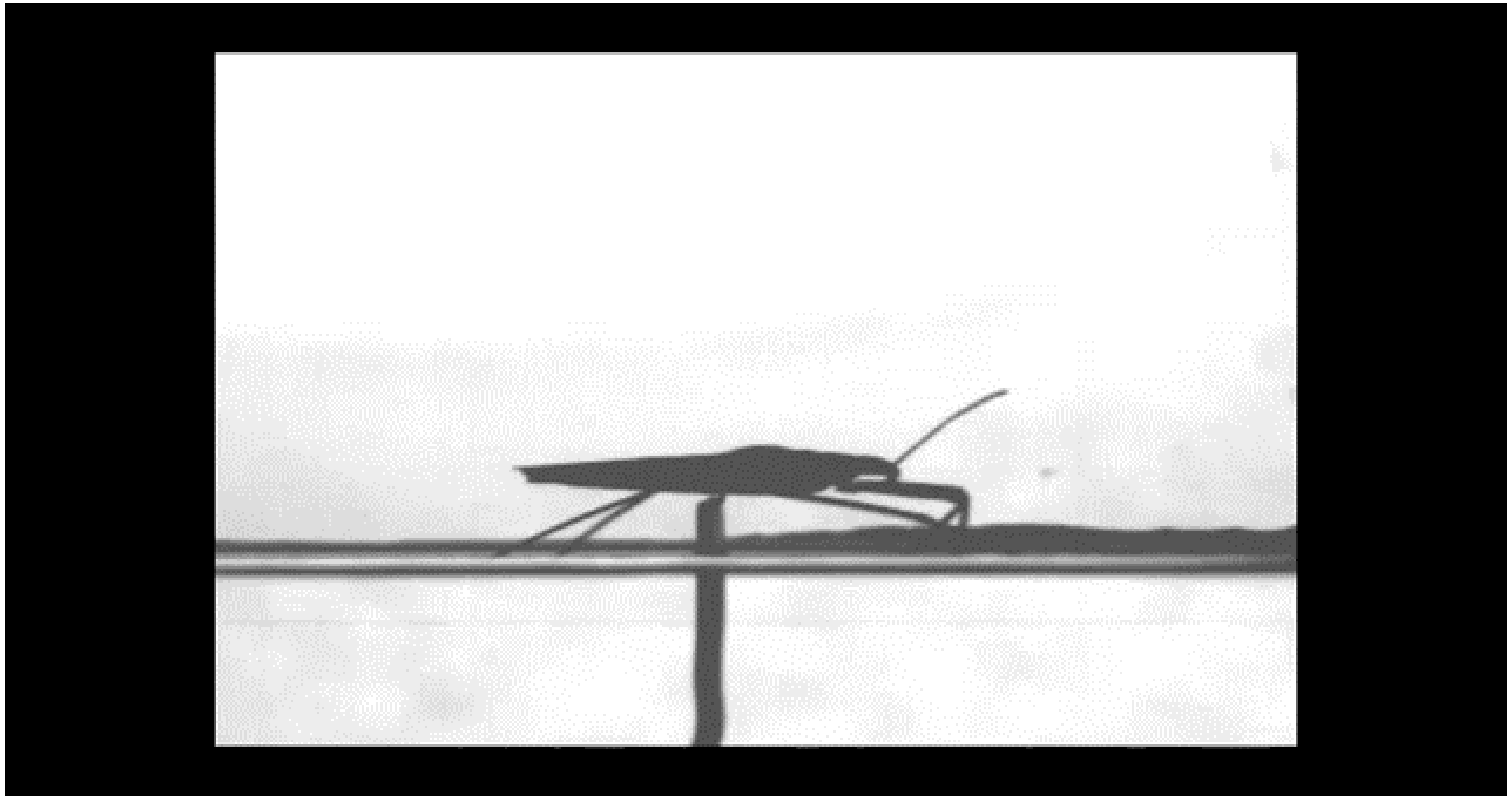
- Laminar flow occurs at lower Re – it is also referred to as smooth flow
  - Little mixing occurs in laminar flow systems as mixing occurs by Brownian motion only

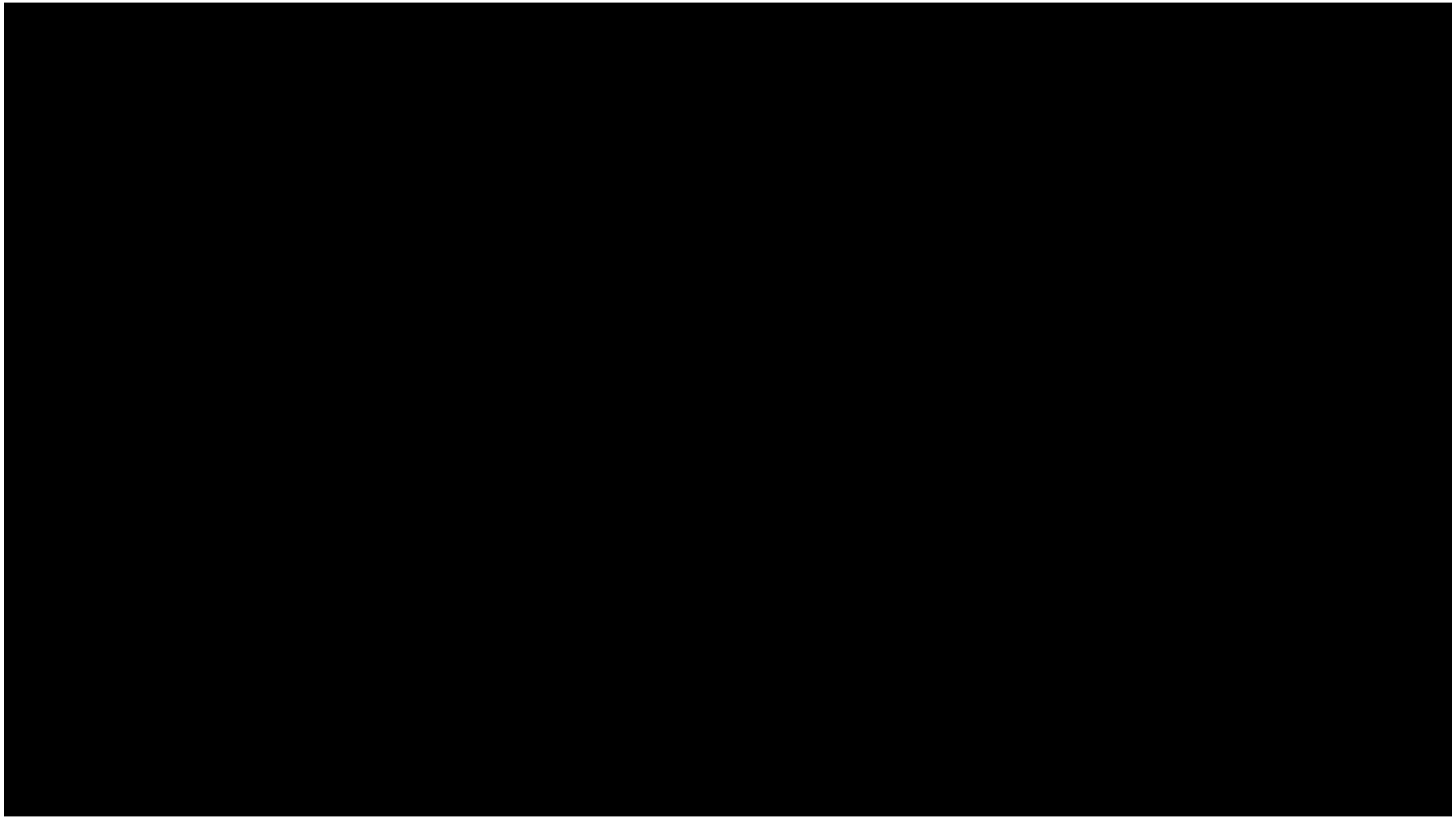
- Turbulent flow occurs at higher Re – it is also referred to as chaotic flow
  - A lot of mixing occurs in turbulent flow systems due to rotational eddies superimposed over the mean flow

Introduction to the Bond Number (Bo)

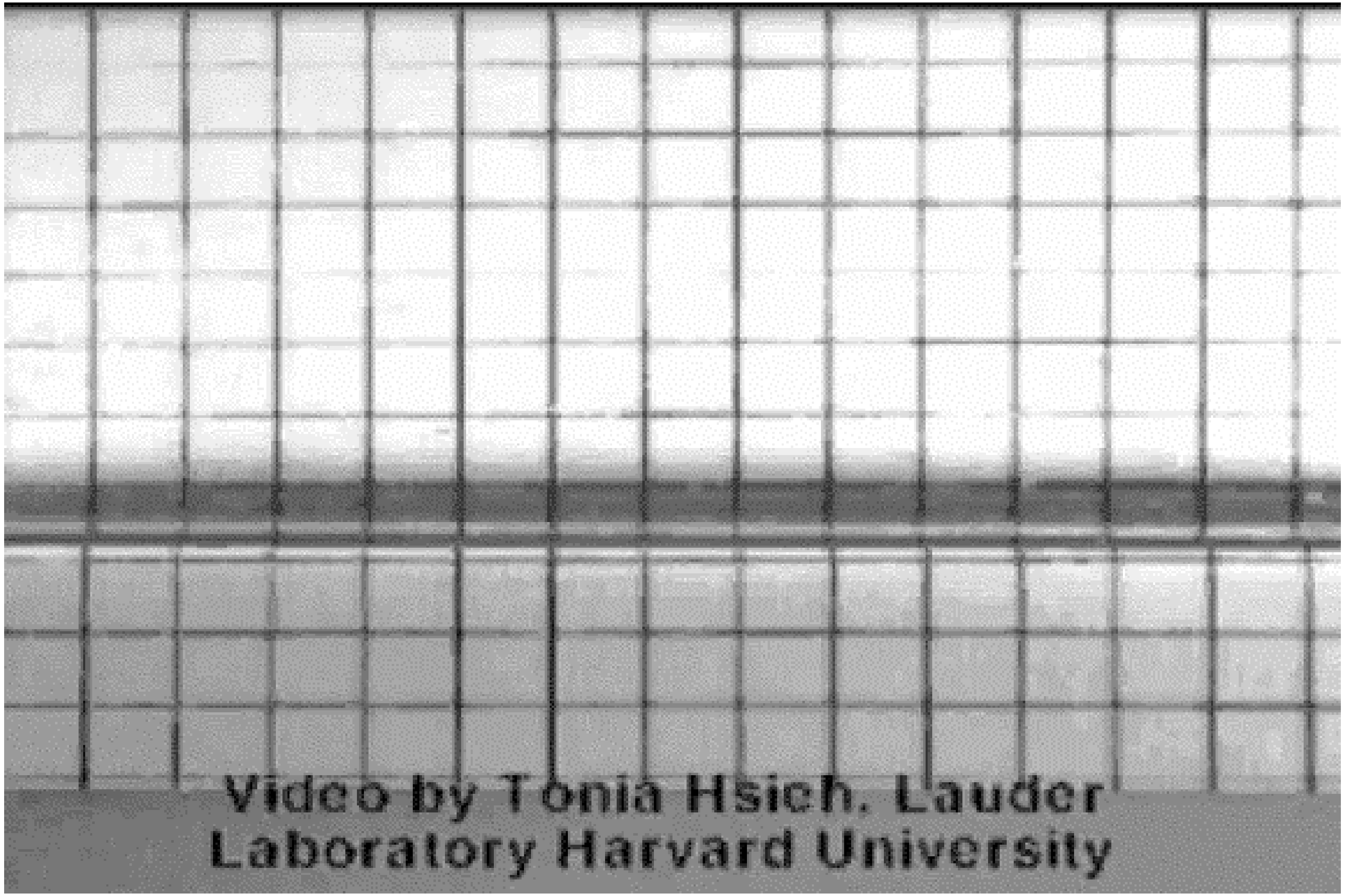
$$Bo = \frac{\rho L^3 g}{\gamma L}$$

What systems are relevant for the Bond number?





Is the basilisk supported by surface tension?



Glasheen JW, McMahon TA. A hydrodynamic model of locomotion in the basilisk lizard. Nature. 1996 Mar;380(6572):340-2.

Support at a fluid-fluid interface

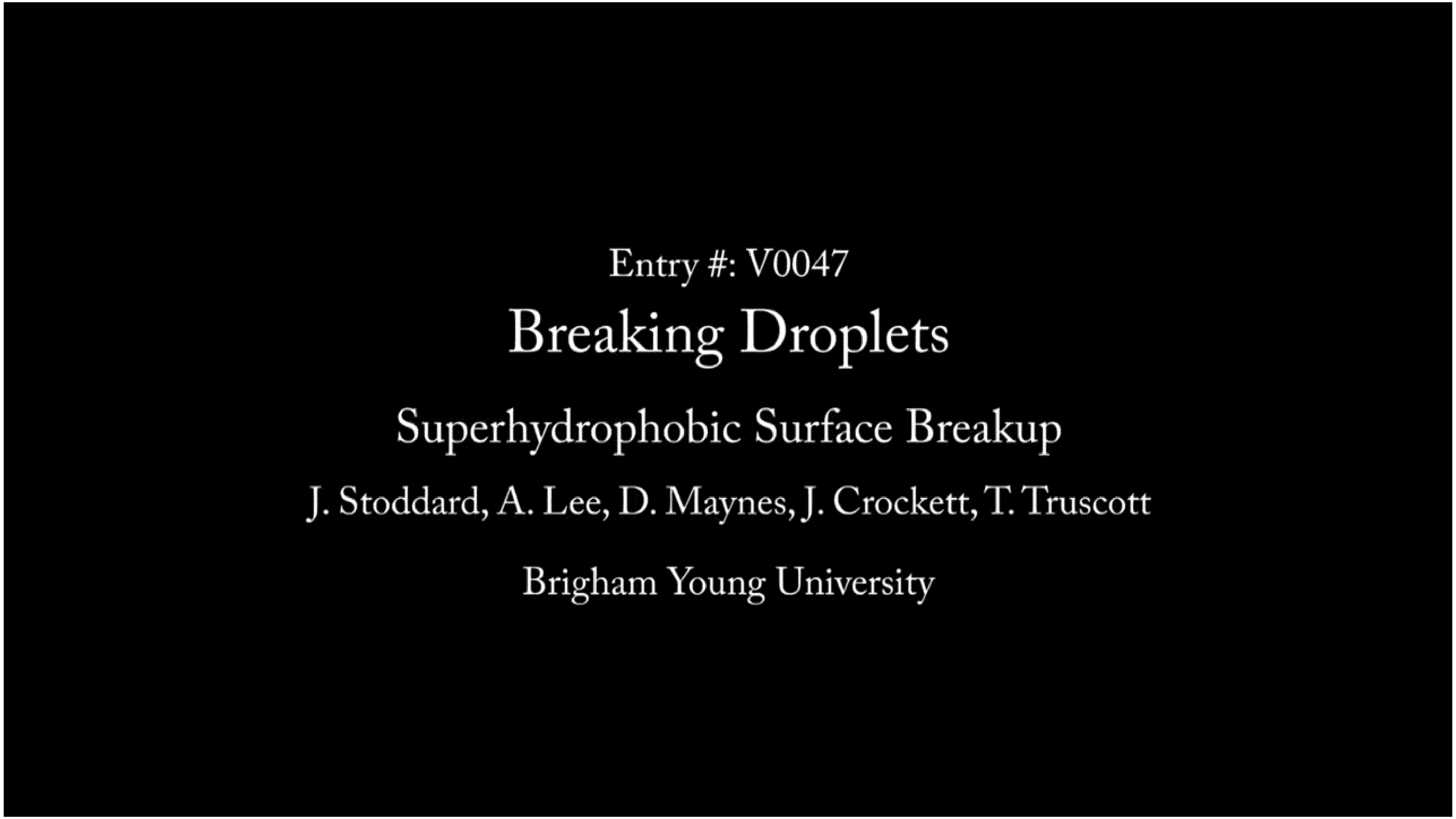
Many structures are supported at a fluid-fluid interface by buoyancy

This is due to differences in density and by how much water the structure displaces

Boats, buoys, etc. are not supported by surface tension – they have high Bond number!!!

Introduction to the Weber Number (We)

$$We = \frac{\rho U^2 L}{\gamma}$$



Stoddard JG. Jet and droplet impingement on superhydrophobic surfaces.

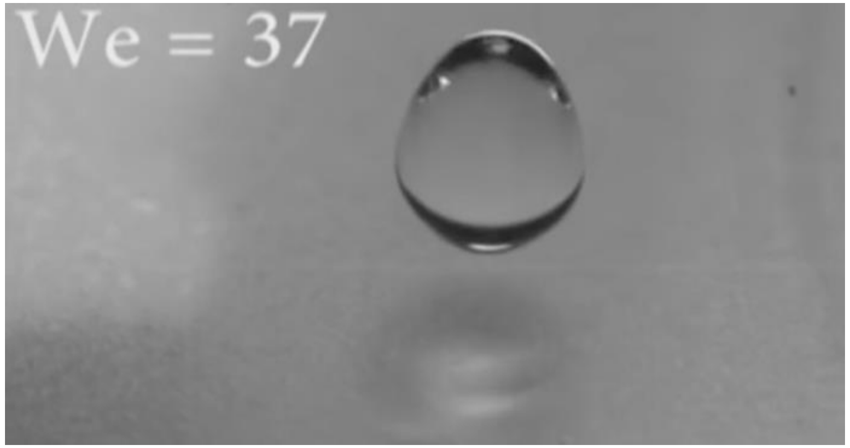
When is the Weber number important?

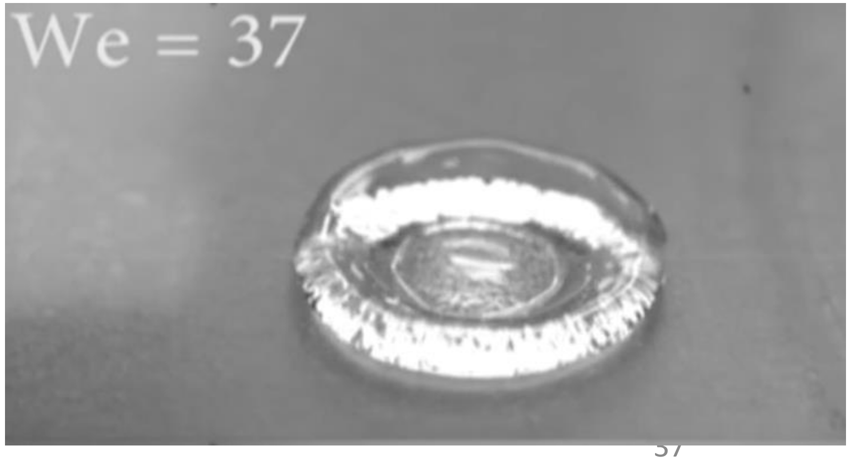
Rain is an excellent example of the Weber number

The size of rain droplets are limited by the Weber number

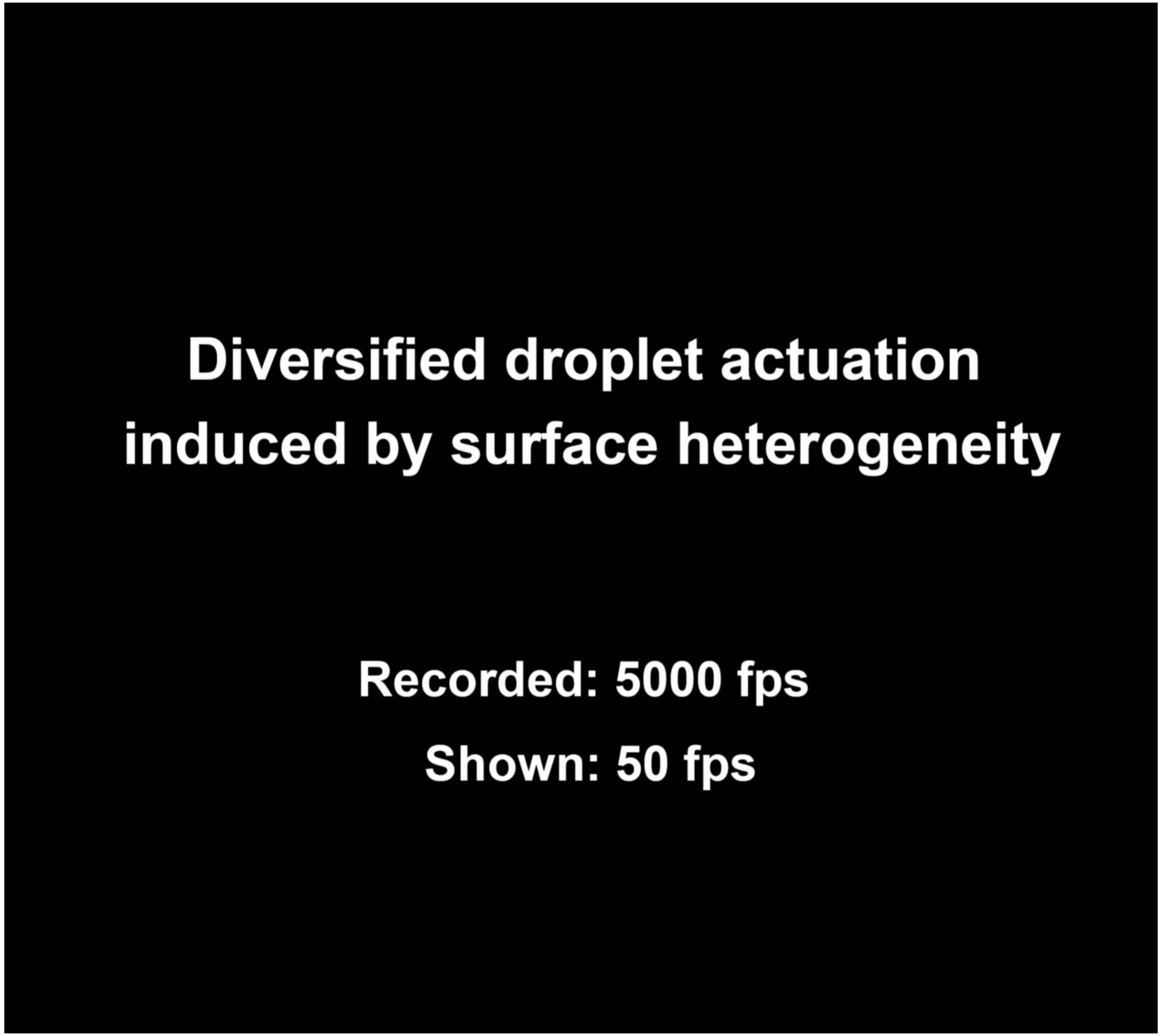
- If a big droplet falls, it will have a higher terminal velocity
- If those inertial forces are too great, the shear stresses on the surface of the droplet will cause the droplet to destabilize and fracture into smaller droplets
- This process will continue until the droplet can fall with low Weber number at its terminal velocity

The Weber number will also determine if a droplet remains cohesive upon impacting a surface



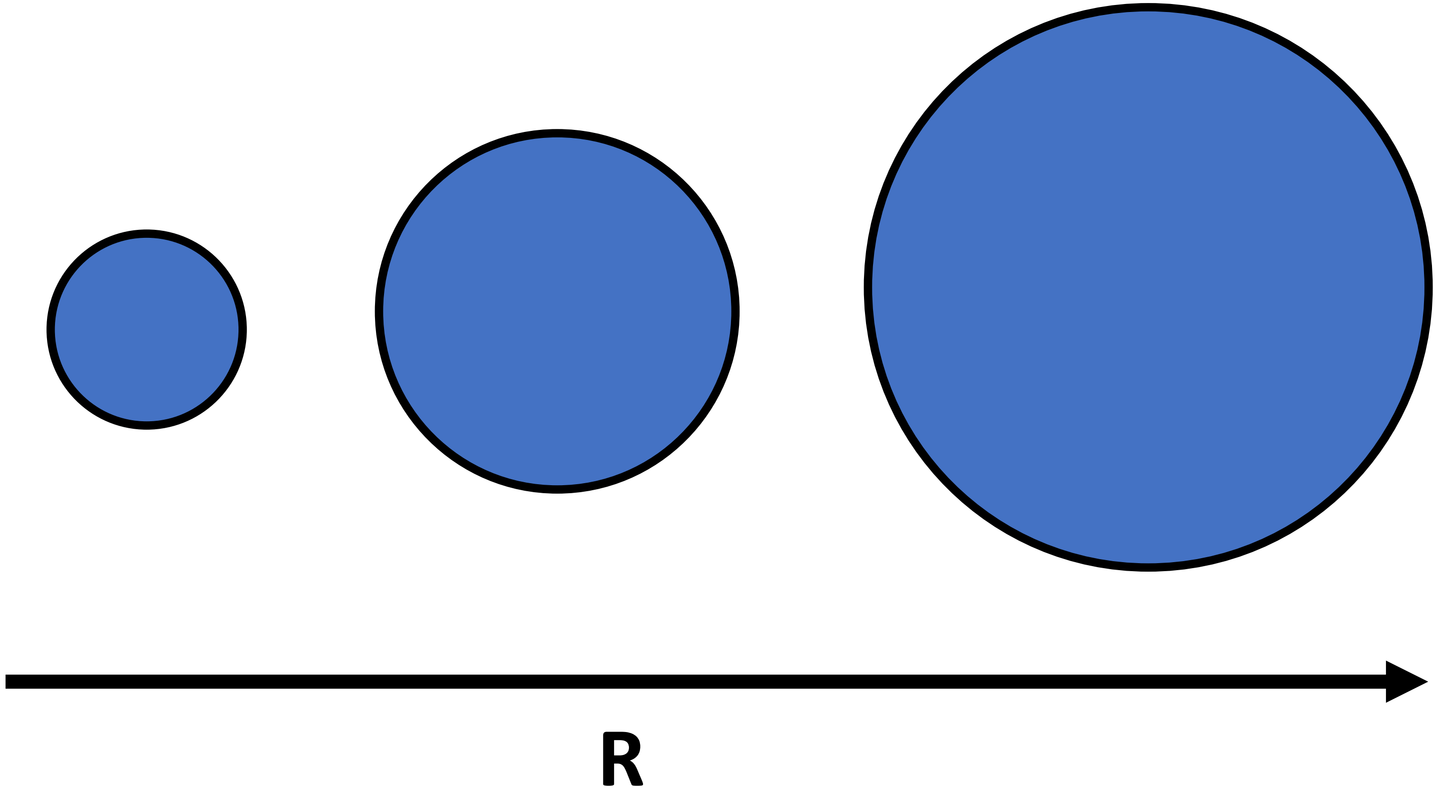


Which force is dominant in this system?




Episode 13: Astro Puffs
PHYSICSCENTRAL.COM
Learn how your world works

How will dimensions affect behavior?



Introduction to the Froude Number (Fr)

$$Fr = \frac{U^2}{gL}$$

What systems are relevant for the Froude number?

Froude number and open channel flow

Here is a spillway created in a lab

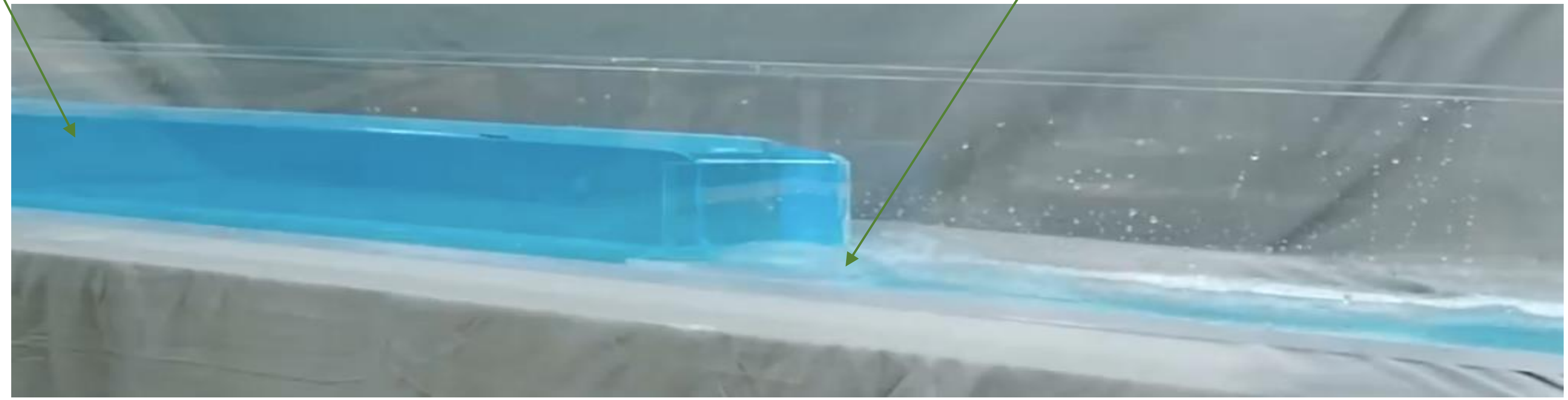
- The flow rate is constant due to conservation of mass
- The flow downstream must be much faster

Low Froude number (Fr < 1)

- Smooth flow
- Subcritical flow

High Froude number (Fr > 1)

- Rough flow
- Supercritical flow



$$Fr = \frac{V}{\sqrt{gh}}$$

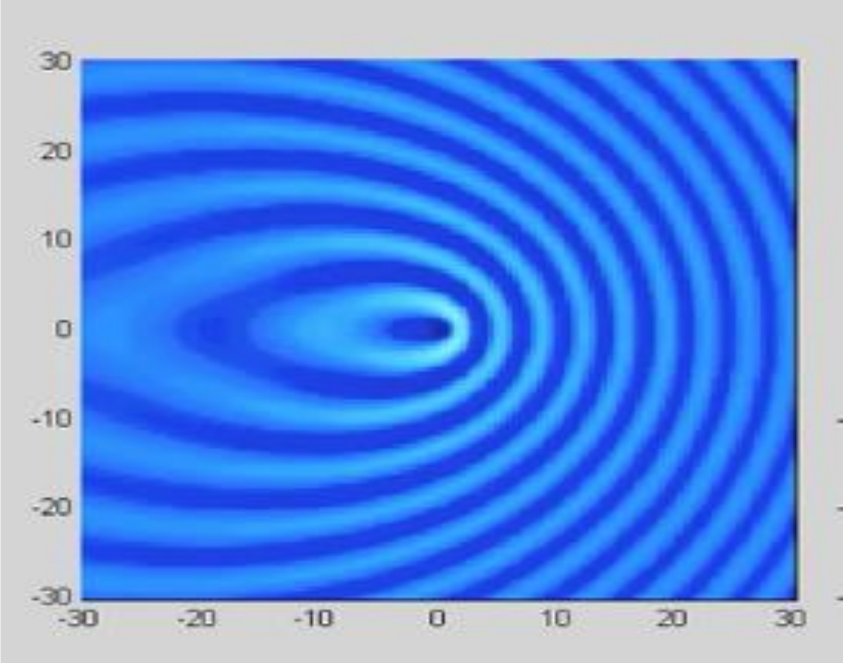
Where ‘h’ is the depth of the flow

Froude number and open channel flow

Let’s look at subcritical flow (Fr < 1)

- The speed of a wave is greater than the speed of the bulk flow
- This means that waves can travel upstream

The bulk flow in this image is from right to left



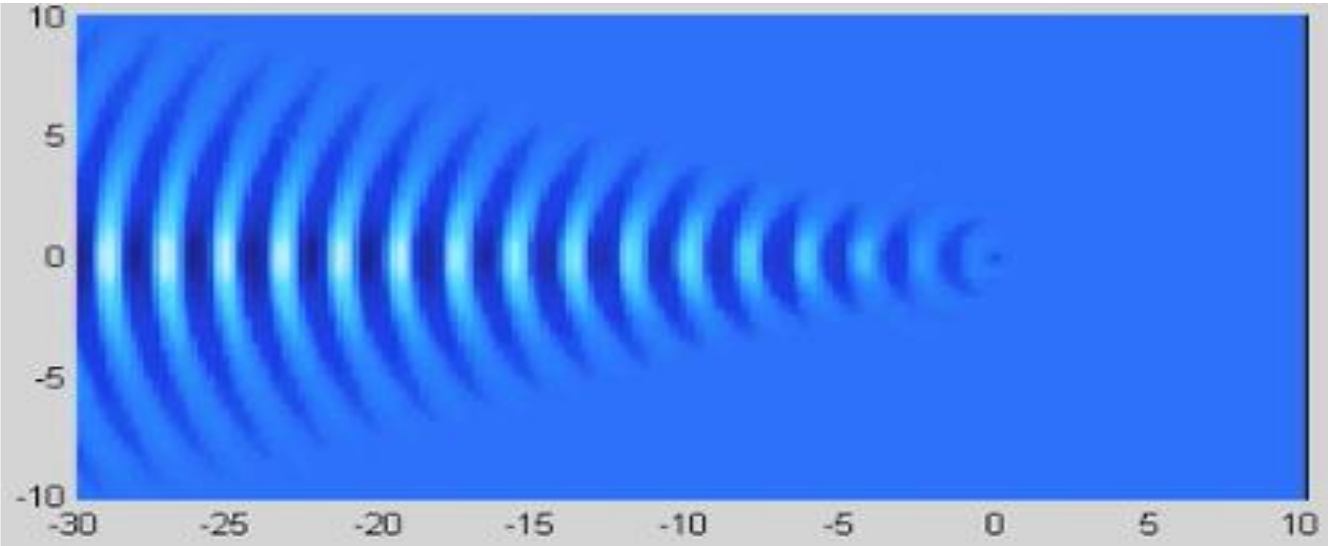
Flow direction

Froude number and open channel flow

For supercritical flow (Fr > 1)

- The speed of the flow is greater than the speed of the wave
- This means that waves cannot propagate upstream

Bulk flow is from right to left



Flow direction

In which systems do these dimensionless numbers govern the fluid mechanics?

1) Reynolds number

2) Bond number

3) Weber number

4) Froude number

Which is the most relevant dimensionless number for this system?

Humans Running in Place on Water at Simulated Reduced Gravity
https://doi.org/10.1371/journal.pone.0037300

How can we create a more realistic Lochness Monster Hoax?

Example 7.1 Modeling a submarine

The Australian military has asked you to evaluate how a new technology impacts the propulsion of their submarines as they travel underwater. However, the government will not let you study the *real* submarine. Instead, you must make a model of it in the lab.

At what speed would the model need to travel at to accurately reflect the propulsion of the actual submarine? The model is 1/200th the size of the real submarine.

Example 7.2 Port Phillip Bay

The city has asked you to model how tides impact erosion in Port Phillip Bay. To accomplish this task, you are to generate a model of the bay in the lab.

At what speed will the tide need to come in in the model? Derive an expression for the model's velocity.