School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 2. Some answers

Question 1

(a) Fixed point: $(0, 0)$ - saddle node. Nullclines: $y = x$ (vertical), $x = 0$ (horizontal).

(b) Fixed point: $(0, 0)$ - saddle node, $(-1, 0)$ - stable node, $(1, 0)$ - stable node. Nullclines: $x = 0, \pm 1$ (vertical), $y = 0$ (horizontal).

(c) Fixed point: $(0, 0)$ - linear stability analysis gives zero eigenvalue. Nullclines: $y = x$ and $x = 0$ (vertical), $y = 0$ and $y = 2x$ (horizontal).

(d) Fixed point: $(1, 0)$ - saddle node. Nullclines: $y = 0$ (vertical), $y = -1 + 1/x$ (horizontal).

(e) Fixed points: $(0, 0)$ - saddle node, $(1, 1)$ - stable spiral (counter clockwise). Nullclines: $x = 0$ and $y = 2 - x$ (vertical), $y = x$ (horizontal).

(f) Fixed points: $(0, 0)$ - stable spiral (counter clockwise), $(1, 1)$ - saddle node. Nullclines: $y = x^2$ (vertical), $y = x$ (horizontal).

Question 2

(a) $\dot{x} = y$, $\dot{y} = x^3 - x$

(b) $(0, 0)$ - centre (clockwise), $(\pm 1, 0)$ - saddle node.

(c) $E = \frac{1}{2}(x^2 + y^2) - \frac{1}{4}x^4$

Question 3

(a) $\dot{x} = y$, $\dot{y} = x - x^2$

(b) $(0, 0)$ - saddle node, $(\pm 1, 0)$ - centre (clockwise).

(c) $E = \frac{1}{2}(y^2 - x^2) + \frac{1}{3}x^3$

(e) $3(y^2 - x^2) + 2x^3 = 0$

Question 4

(a) $\dot{x} = y$, $\dot{y} = -x - \epsilon x^3$

(b) Fixed point: $(0, 0)$.

(c) Fixed points: $(0,0)$ - nonlinear centre, $\pm 1/\sqrt{\epsilon}$ - saddle node. Trajectories not closed far from origin.

(d) $\frac{1}{2}(x^2+y^2)+\frac{1}{4}\epsilon x^4+\epsilon^{-1}=0$