STM4PSD – Workshop 10 Solutions

1.
```
x <- c(3.06, 2.00, 3.36, 3.60, 3.72, 4.65, 3.00, 3.74,
       3.47, 4.33, 5.97, 3.11, 1.41, 3.63, 3.10, 4.76, 4.86, 7.91, 2.51)
t.test(x, mu=86)
```

This yields the output shown below.

```
        One Sample t-test

data:  x
t = -1.2047, df = 18, p-value = 0.2439
alternative hypothesis: true mean is not equal to 4.2
95 percent confidence interval:
 3.101003 4.497944
sample estimates:
mean of x
 3.799474
```

This is the same as the answers from Question 1 (with less rounding).

2. (a) Let μ denote the mean lifetime of the new lightbulb model. We are testing

$$H_0: \mu \leqslant 16000 \text{ versus } H_1: \mu > 16000$$

(b) Assuming the file is imported into the variable `lights`, we use the command below.

```
t.test(lights$lifetime, mu=16000, alternative="greater")
```

This gives the output below.

```
        One Sample t-test

data:  lights$lifetime
t = 15.309, df = 299, p-value < 2.2e-16
alternative hypothesis: true mean is greater than 16000
95 percent confidence interval:
 16390.22      Inf
sample estimates:
mean of x
 16437.36
```

(c) As the p-value is smaller than $\alpha = 0.05$, we reject the null hypothesis. The data provides strong evidence that the mean lifetime of the new model is greater than 16,000 hours.

3. (a) It does not suggest that the improved performance is meaningful. This is discussed in more detail in the following questions.

(b) The following commands will produce a 95% confidence interval for the mean running time.

```
x <- lights$lifetime
mean(x) + c(-1,1)*qt(0.975, df=length(x)-1)*sd(x)/sqrt(length(x))
```

This results in a 95% confidence interval of $(16381, 16494)$.

(c) The 95% confidence interval suggests that the mean lifetime is only about 380–500 hours more than the previous model. Relative to the 16000 hour lifetime of the old model, this is only about a 2–3% improvement. Certainly not worth the extra 10% in price!

(d) The p-value gives only an indication of the statistical significance, whereas the confidence interval gives an indication of clinical significance.

4.
```
my.t.test <- function(mean, sd, n, mu) {
        # fill in the blanks to complete the function
        df <- n-1 # compute the degrees of freedom
        se <- sd/sqrt(n) # compute the standard error
        t <- (mean-mu)/se # compute the test statistic
        p.value <- 2*(1-pt(abs(t), df=df)) #compute the p-value

        critical.value <- qt(0.975, df=df) # calculate the coefficient of 'SE'
        lower <- mean - critical.value*se
                        # find the lower half of the 95% confidence interval
        upper <- mean + critical.value*se
                        # find the upper half of the 95% confidence interval

        # You do not need to modify the lines of code below.
        writeLines("\t\tOne Sample t-test for a two-sided hypothesis")
        writeLines("\t\t\tUsing a significance level of 0.05\n")
        writeLines(sprintf("t = %.4f, df = %.2f, p-value = %.4f", t, df, p.value))
        writeLines(sprintf("alternative hypothesis: true mean not equal to %.0f", mu))
        writeLines(sprintf("95 percent confidence interval:\n %.5f %.5f", lower,upper))
        writeLines(sprintf("sample estimates:\nmean of x\n\t%.5f", mean))
}
```

5. Assuming you loaded the `milk.csv` file into a variable called `milk`, then

```
t.test(milk$Isotopic, milk$Test.weighing, paired=TRUE)
```

will give the output from Q1(c), and

```
t.test(milk$Isotopic, milk$Test.weighing)
```

will give the results for Q1(d).
Using

```
t.test(milk$Isotopic[1:4], milk$Test.weighing[1:4], paired=TRUE)
```

will perform the calculations using just the first four data points.

6. (a) Assuming you loaded the file into a data frame called `BatteryLife`, the following command will produce the results

```
t.test(BatteryLife$BatteryA, BatteryLife$BatteryB)
```

You should have the following output.

```
        Welch Two Sample t-test

data:  BatteryLife$BatteryA and BatteryLife$BatteryB
t = 2.832, df = 997.569, p-value = 0.004719
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.1108844 0.6112880
sample estimates:
mean of x mean of y
 148.0465  147.6854
```

(b) 2.832.

(c) From the output, p-value $= 0.004719 \approx 0.005$. This p-value is very small so we reject H_0.

(d) For Battery A the sample mean is 148.0465 hours. For Battery B it is 147.6854.

(e) The estimated difference is $148.0465 - 147.6854 = 0.3611$ hours which is 21.666 minutes.

(f) The 95% confidence interval for $\mu_A - \mu_B$ is $(0.1108844,\ 0.6112880)$.

(g) Yes, the value for $\mu_A - \mu_B$ under the null hypothesis is 0 which does not fall in our interval.

(h) The interval is narrow, suggesting that we have an accurate estimate of the difference in means. However, even the upper limit of our interval is small so we are confident that the difference in means is not very large. 10% extra seems to be a big price to pay for a battery that appears to be only marginally superior.

(i) If only the p-value was reported, we would not have been able to tell that the difference in lifetimes is likely to be small. That is, while the result was statistically significant, it does not appear to be clinically significant.

(j) While we found evidence the Battery A has a longer average lifetime than Battery B ($p = 0.005$), the actual difference is likely to be small. We estimate, with 95% confidence, that difference in means is in the interval $(0.111,\ 0.611)$ hours.

7. (a) $\hat{p}_1 = 27/360 = 0.075$ and $\hat{p}_2 = 24/215 \approx 0.112$.

(b) The command

```
prop.test(c(27, 24), c(360, 215))
```

produces

```
        2-sample test for equality of proportions with continuity correction

data:  c(27, 24) out of c(360, 215)
X-squared = 1.8041, df = 1, p-value = 0.1792
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.09046348  0.01720767
sample estimates:
   prop 1    prop 2
0.0750000 0.1116279
```

i. They are on the bottom line of the output.

ii. To three decimal places the interval is $(-0.090,\ 0.017)$.

iii. p-value = 0.1792.

(c) No, the p-value is greater than 0.05 so we cannot reject that p_1 and p_2 are equal at this level of significance.

(d) No, and we do not expect it to since the interval will agree with the conclusion based on the p-value. Zero is within the interval.

(e) We would need a level of significance of $\alpha = 0.1792$. This seems to big a risk for falsely rejecting H_0 and incorrectly claiming that one company's product is superior.

(f) The data collected did not provide enough evidence for us to confidently claim that the chances of the fire alarms being defective is different between the two companies (p-value $= 0.1792$).