Assignment 2 MAT5OPT Michael Le

Question 1

```
%Do not use the function addRow from the solutions to Workshop 4
%question 10,
% but write your own code,e.g.
% using a for statement which runs though the rows.

% If you don't succeed in this question, then you may use the code
% provided in Workshop 4 question 10 in subsequent questions of this
% assignment.

%pivot script.
 function M = pivot(M, i, j)
 entry = M(i,j);
 if (entry ~= 0)
     M(i,:) = M(i,:)/entry;
     [numrows, ~] = size(M);
     column =-M(:,j);
     column(i) = 0;
     %Do I have to modify the addRow function, if achievable.
     %Otherwise, I just submitted this line of code below!
     M(1:numrows,:) = M(1:numrows,:) + column.*M(i,:);

 end
 end
```

Question 3
Part a

```
% Use the MATLAB program pivot from question 1 to bring M into
 % canonical form,
 % where columns 2, 4 and 5 are basic.
 % M= [1 2 3 4 5 6;
 %     1 0 1 0 1 0;
 %     5 4 3 2 1 0];


%Is it in canonical?
%First step we perform these row-operations first!

%R3 = [5 4 3 2 1 0];
%R1 = [1 2 3 4 5 6];
%R1_5 = [5 10 15 20 25 30];
%new_R3 = R3 - R1_5;

% M= [1  2  3  4  5  6;
%     1  0  1  0  1  0;
%     0 -6 -12 -18 -24 -30];


 % MAIN PROGRESS
 % [r,c] = minimumRatio(M);
 % M = pivot(M, r, c);
```

```
%Iterations
 %M = pivot(M, 2, 5);
 %M = pivot(M, 1, 4);


% M = [-1  0.5  -0.5  1  0  1.5;
%     1     0    1    0  1   0;
%     6     3    3    0  0  -3];

%After dividing the last row by 3,
% M = [-1  0.5  -0.5  1  0  1.5;
%     1     0    1    0  1   0;
%     2     1    1    0  0  -1];

%Performing row-operations,
%R1 = 2*[-1 0.5 -0.5 1 0 1.5];
%R3 = [2 1 1 0 0 -1];
%New_R1 = (R3-R1)/-2;

%Final Matrix will be, in canonical form with columns
%2,4 and 5 are basic.
% M = [-2   0   -1    1  0   2;
%     1     0    1    0  1   0;
%     2     1    1    0  0  -1];
```

Question 3
Part b.

```
%According to the result from Q3a, this is the only unique
%solution matrix that has columns 2,4 and 5 that are basic.
```

Question 5.
Part a.

```
 %Matrix for Phase 1

 %Q5a_P1 = [1/2 -1 1 0 0 0 0 1;
 %    1/2 1 0 1 0 0 0 4;
 %    3 1 0 0 -1 0 1 5;
 %    2 -1 0 0 0 1 0 2;
 %   -3 -1 0 0 1 0 0 -5];

 % MAIN PROGRESS
 % [r,c] = minimumRatio(Q5a_P1);
 % Q5a_P1 = pivot(Q5a_P1, r, c);
 % Repeat the MAIN PROGRESS until you get w = 0.

 %Iterations for Q5a for the first phase
 %Q5_P1a = pivot(Q5_P1a, 4, 1)
 %Q5_P1a = pivot(Q5_P1a, 3, 2)




% After 2 iterations it gives us the solution,
```

```
% Q5a_P1 =

%   0         0    1.0000         0   -0.3000   -0.7000    0.3000    1.1000
%   0         0         0    1.0000    0.5000    0.5000   -0.5000    2.5000
%   0    1.0000         0         0   -0.4000   -0.6000    0.4000    0.8000
%   1.0000    0         0         0   -0.2000    0.2000    0.2000    1.4000
%   0         0         0         0         0         0    1.0000         0


% Starting Phase 2 for Q5a!
% Q5a_P2 = [0 0 1 0 -0.3 -0.7 1.1; 0 0 0 1 0.5 0.5 2.5;
% 0 1 0 0 -0.4 -0.6 0.8;1 0 0 0 -0.2 0.2 1.4; -1 -1 0 0 0 0 0]

% MAIN PROGRESS
% [r,c] = minimumRatio(Q5a_P2);
% Q5a_P2 = pivot(Q5a_P2, r, c);
% Repeat the MAIN PROGRESS until you get the optical solution.

 %Iterations for Q5a for the second phase
 %Q5a_P2 = pivot(Q5a_P2, 4, 1)
 %Q5a_P2 = pivot(Q5a_P2, 3, 2)
 %Q5a_P2 = pivot(Q5a_P2, 2, 5)

% After 3 iterations it gives us the solution,
% Q5a_P2 =

%         0         0    1.0000    0.6000         0   -0.4000    2.6000
%         0         0         0    2.0000    1.0000    1.0000    5.0000
%         0    1.0000         0    0.8000         0   -0.2000    2.8000
%    1.0000         0         0    0.4000         0    0.4000    2.4000
%         0         0         0    1.2000         0    0.2000    5.2000

%Overall, our final solution
%is z = 5.2, given x* = (2.4, 2.8, 2.6, 0, 5, 0)
```

Question 5.

Part b.

```
 %Phase 1.

 %Q5b_P1 = [1/2 -1 1 0 0 0 0 -1;
 %    1/2 1 0 1 0 0 0 4;
 %    3 1 0 0 -1 0 1 5;
 %    2 -1 0 0 0 1 0 2;
 %    -3 -1 0 0 1 0 0 -5];

  % MAIN PROGRESS
 % [r,c] = minimumRatio(Q5b_P1);
 % Q5b_P1 = pivot(Q5b_P1, r, c);
 % Repeat the MAIN PROGRESS until you get w = 0.

 %Iterations for Q5b for the first phase
 %Q5b_P1 = pivot(Q5b_P1, 1, 1)
 %Q5b_P1 = pivot(Q5b_P1, 3, 2)

% After two iterations the final table for Phase 1 is,

%Q5b_P1 =

% 1.0000    0    0.2857         0   -0.2857         0    0.2857    1.1429
% 0         0    0.7143    1.0000    0.2857         0   -0.2857    1.8571
% 0    1.0000   -0.8571         0   -0.1429         0    0.1429    1.5714
% 0         0   -1.4286         0    0.4286    1.0000   -0.4286    1.2857
% 0         0         0         0         0         0    1.0000         0

%Starting Phase 2 for Q5b
 %Q5b_P2 = [1         0    0.2857   0   -0.2857  0  1.1429;
 %         0         0    0.7143   1    0.2857  0  1.8571;
 %         0         1   -0.8571   0   -0.1429  0  1.5714;
 %         0         0   -1.4286   0    0.4286  1  1.2857;
 %         1         1         0   0         0  0       0];
%NOTE: Not in canonical form

%After performing these operations I get
%R5 = [1 1 0 0 0 0 0];
%R1 = [1 0 0.2857   0   -0.2857  0  1.1429];
%R3 = [0 1 -0.8571  0   -0.1429  0  1.5714] ;
%R5 = R1 - R5;
%R5 = R5 + R3;

 %Q5b_P2 = [1         0    0.2857   0   -0.2857  0  1.1429;
 %         0         0    0.7143   1    0.2857  0  1.8571;
 %         0         1   -0.8571   0   -0.1429  0  1.5714;
 %         0         0   -1.4286   0    0.4286  1  1.2857;
 %         0         0   -0.5714   0   -0.4286  0  2.7143];


% MAIN PROGRESS
% [r,c] = minimumRatio(Q5b_P2);
% Q5b_P2 = pivot(Q5b_P2, r, c);
% Repeat the MAIN PROGRESS until you get the minimial solution.

 %Iterations for Q5b for the second phase
 %Q5b_P2 = pivot(Q5b_P2, 2, 3)
 %Q5b_P2 = pivot(Q5b_P2, 4, 5)
```

```
%After two iterations we get,
%Q5b_P2 =
%    1.0000         0         0    0.4000         0    0.4000    2.3999
%         0         0    1.0000    0.6000         0   -0.4000    0.6001
%         0    1.0000         0    0.8001         0   -0.1999    2.8002
%         0         0         0    2.0000    1.0000    1.0000    4.9999
%         0         0         0    1.2001         0    0.2001    5.2001

%The minimial solution x* = (2.3999, 2.8002, 0.6001, 0, 4.9999,0)
%z = 5.2001, Back to the minimisation problem
```

Q2) a.)

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 2 \\ \underline{0} & 5 & 0 & \underline{0} & 1 & 3 \end{array}\right]$$

Not in Canonical form e_1 appears twice, e_3 appears once. But, e_2 does not appear.

b.)

$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 1 & 0 & 0 & 3 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & \underline{0} & \underline{0} & -1 & \underline{1} & 2 \end{array}\right]$$

It is in canonical form because e_2 appears in the third column. e_1 appears in the fourth column and e_3 is in the sixth column.

The basic variables are x_3, x_4 and x_6. We set $x_1 = 0, x_2 = 0, x_5 = 0$. This gives.

$\cancel{x_1} + 2\cancel{x_2} + x_4 = 3 \Rightarrow x_4 = 3$
$\cancel{x_1} + \cancel{x_2} + x_3 = 1 \Rightarrow x_3 = 1$
$\cancel{x_2} - \cancel{x_5} + x_6 = 2 \Rightarrow x_6 = 2$

The basic solution is $(0,0,1,3,0,2)$.

c).

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 5 & 0 & 3 \\ \underline{1} & 1 & \underline{0} & 3 & \underline{0} & 1 \end{array}\right]$$

It is in canonical form, the basic variables are x_1, x_3 and x_5.

To find the basic solution,
we need to find x_1, x_3 and x_5, set $x_2 = x_4 = 0$.

$\cancel{x_2} + x_5 = 2 \Rightarrow x_5 = 2$

$x_3 + 5\cancel{x_4} = 3 \Rightarrow x_3 = 3$

$x_1 + \cancel{x_2} + 3\cancel{x_4} = 1 \Rightarrow x_1 = 1$

The basic solution $(1, 0, 3, 0, 2)$.

Q4a).

$$\min z = 4x_2 - 2x_1$$
$$\text{s.t. } x_1 - 2x_2 \leq 2$$
$$2x_1 + 3x_2 \leq 12$$
$$x \geq 0.$$

In canonical form,

$$\max -z = -4x_2 + 2x_1$$
$$\text{s.t. } x_1 - 2x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + x_4 = 12$$
$$x \geq 0$$

where $-z + (-2x_1) + 4x_2 = 0$

$-z - 2x_1 + 4x_2 = 0$

Continuing.

Q4a).

Initial Matrix

BV	x_1	x_2	x_3	x_4		Ratio
x_3	$\boxed{1}$	-2	1	0	2	$2/1=2$
x_4	2	3	0	1	12	$12/2=6$
	-2	4	0	0	0	

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_1	1	-2	1	0	2
x_4	0	7	-2	1	8
	0	0	2	0	4

$R_2' = -2R_1 + R_2$
$R_3' = R_1 + R_3$

→ No negative entries.
The optical solution is $x^* = (2, 0, 0, 8)$.

However this is only one solution
gives us $z = 4(0) - 2(2) = -4$ (minimization problem)

For other solutions, we observe that the second column has a zero entry, but x_2 is not a basic variable. We can choose x_2 as an entering basic variable and pivot once more:

BV	x_1	x_2	x_3	x_4		Ratio	
x_1	1	-2	1	0	2	$2/-2=-1$	(invalid)
x_4	0	$\boxed{7}$	-2	1	8	$8/7$	(valid)
z	0	0	2	0	4		

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_1	1	-2	1	0	2
x_2	0	$\boxed{1}$	$-2/7$	$1/7$	$8/7$
z	0	0	2	0	4

$R_2' = R_2/7$
$R_1' = 2R_2^* + R_1$

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_1	1	0	$3/7$	0	$30/7$
x_2	0	1	$-2/7$	0	$8/7$
z	0	0	2	0	4

Now, we find another optimal solution.

$x_1 = 30/7$

$x_2 = 8/7$

$x_3 = 0$

$x_4 = 0$

Gives us $-z = -4x_2 + 2x_1$

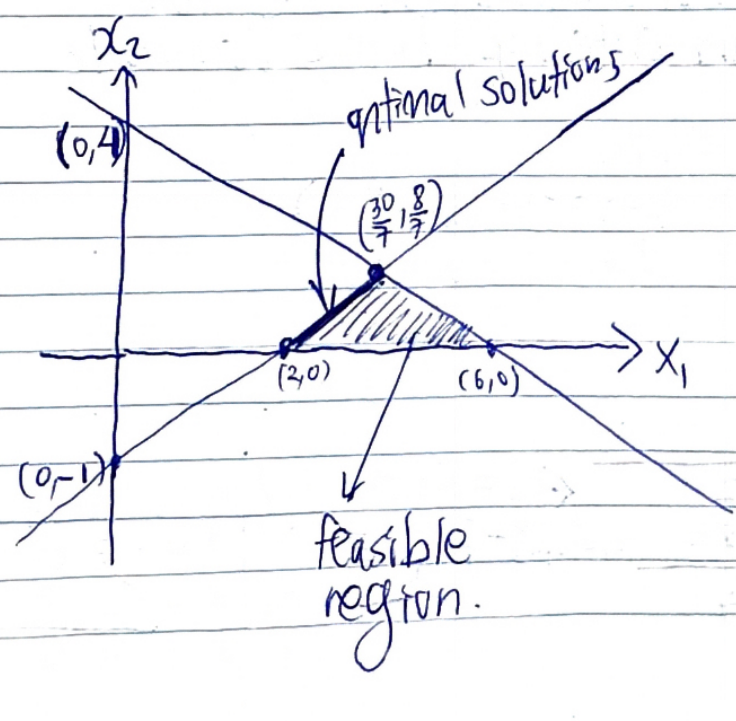
$$= -4\left(\frac{8}{7}\right) + 2\left(\frac{30}{7}\right) = -\frac{32}{7} + \frac{60}{7} = \frac{28}{7} = 4.$$

revert back to the minimization problem $z = -4$.
this problem lie on the line segment joining the points $(2,0)$ and $(30/7, 8/7)$.

$$(x_1, x_2) = \alpha(2,0) + (1-\alpha)\left(\frac{30}{7}, \frac{8}{7}\right)$$

$$= \left(2\alpha + 30/7 - 30/7\,\alpha,\ \frac{8}{7}(1-\alpha)\right)$$

$$= \left(-\frac{16}{7}\alpha + \frac{30}{7},\ \frac{8-8\alpha}{7}\right)$$, where $\alpha \in [0,1]$.



Q4b).

$\max\ z = x_1 + 2x_2$

s.t. $x_1 + 4x_2 \leq 12$

$x_1 + x_2 \geq 3$

$x \geq 0$.

Degeneracy?

But $(x_1, x_2) = (0,0)$ fails the second constraint,
Hence,

$$\max z = x_1 + 2x_2$$
$$x_1 + 4x_2 + x_3 = 12$$
$$x_1 + x_2 - x_4 + x_5 = 3$$
$$x \geq 0.$$

NOTE: Take the - sign for x_4 because we wanted to maximize the objective function. We introduce the variable x_5, where x_5 is the artifical variable. This problem requires a 2-phase method.

Must satisfy $x_1 + x_2 - x_4 = 3$, $x_5 > 0$ which violates the original constraint maximises the objective function $w = -x_5$.

The second constraint gives $x_5 = 3 + x_4 - x_1 - x_2$
Adding equation,

$w = -x_5 = x_1 + x_2 - x_4 - 3$
$w - x_1 - x_2 + x_4 = -3$ (Adding equation).

BV	x_1	x_2	x_3	x_4	x_5	
x_3	1	4	1	0	0	12
x_5	1	1	0	-1	1	3
$w = -x_5$	-1	-1	0	1	0	-3

initial feasible solution:
$X^* = (0, 0, 12, 0, 3)$

Using Bland's rule,
Picking between x_1, x_2 for the entering variable, pick the left most one x_1.

Ratio test

BV	x_1	x_2	x_3	x_4	x_5	
x_3	1	4	1	0	0	12
x_5	$\boxed{1}$	1	0	-1	1	3
$w=-x_5$	-1	-1	0	1	0	-3

$12/1 = 12$
$3/1 = 3$

$\equiv$

BV	x_1	x_2	x_3	x_4	x_5	
x_3	0	3	1	1	-1	9
x_1	$\boxed{1}$	1	0	-1	1	3
$w=-x_5$	0	0	0	0	1	0

$R_1' = R_1 - R_2$
$R_3' = R_2 + R_3$
Not interested in all solutions in Phase 1. All that matters is that we have $w=0$ at the end.

Phase 1 completed,

→ Proceed to Phase 2,
objective $z = x_1 + 2x_2$

BV	x_1	x_2	x_3	x_4		Ratio
x_3	0	3	1	1	9	$9/3=3$
x_1	1	$\boxed{1}$	0	-1	3	$3/1=3$
z	-1	-2	0	0	0	

Get rid of the artificial column x_5. Re-write the last row z, our objective function.

Bland's rule.

BV	x_1	x_2	x_3	x_4	
x_3	1	0	$-1/3$	$-4/3$	0
x_2	1	$\boxed{1}$	0	-1	3
z	$1/2$	0	0	-1	3

$R_1' = R_2 - (R_1/3)$
$R_3' = R_2 + (R_3/2)$

BV	x_1	x_2	x_3	x_4	
x_3	1	0	$-1/3$	$\boxed{-4/3}$	0
x_2	1	1	0	-1	3
z	$1/2$	0	0	-1	3

Ratio
$0/-4/3 = 0$
$3/-1 = -3$

BV	x_1	x_2	x_3	x_4	
x_4	$-3/4$	0	$1/4$	$\boxed{1}$	0
x_2	1	1	0	-1	3
z	$1/2$	0	0	-1	3

$R_1' = -\frac{3}{4} R_1$

BV	x_1	x_2	x_3	x_4	
x_4	$-3/4$	0	$1/4$	1	0
x_2	$1/4$	1	$1/4$	0	3
z	$\boxed{-1/4}$	0	$1/4$	0	3

$R_2' = R_1 + R_2$
$R_3' = R_1 + R_3$

BV	x_1	x_2	x_3	x_4	
x_4	$\boxed{-3/4}$	0	1/4	1	0
x_2	1/4	1	1/4	0	3
z	$\boxed{-1/4}$	0	1/4	0	3

Ratio test

$0/-3/4 = 0$

$3/1/4 = 12$

$\equiv$

BV	X_1	X_2	X_3	X_4	
x_1	1	0	$-1/3$	$-\frac{4}{3}$	0
x_2	1/4	1	1/4	0	3
z	$-1/4$	0	1/4	0	3

Ratio-test

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_1	1	0	$-1/3$	$\boxed{-4/3}$	0
x_2	0	-4	$-4/3$	$-4/3$	-12
z	0	0	2/3	$-4/3$	3

0

$(-12)/(-4/3) = 9$

$R_2' = R_1 - 4R_2$

$R_3' = R_1 + 4R_3$

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_4	$-3/4$	0	1/4	1	0
x_2	$-3/4$	-3	$-3/4$	0	-9
z	$-3/4$	0	3/4	0	9/4

$R_1' = -3/4\, R_1$

$R_2' = R_1 + 3/4\, R_2$

$R_3' = R_1 + 3/4\, R_3$

$\frac{1}{4} + \frac{3}{4}\left(\frac{2}{3}\right) = 3/4$

$\equiv$

BV	X_1	X_2	X_3	X_4	
X_4	$-3/4$	0	1/4	1	0
X_2	1/4	1	1/4	0	3
z	$-1/4$	0	1/4	0	3

$R_2' = -R_2/3$

$R_3' = R_3/3$

Back to the original matrix

Bland's rule,

BV	X_1	X_2	X_3	X_4	
x_4	$-3/4$	0	$1/4$	1	0
x_1	$\boxed{1/4}$	1	$1/4$	0	3
z	$-1/4$	0	$1/4$	0	3

BV	X_1	X_2	X_3	X_4	
x_4	0	4	$4/3$	$4/3$	12
x_1	$\boxed{1}$	4	1	0	12
z	0	4	2	0	24

$R_1' = R_2 + \frac{4}{3} R_1$

$R_3' = R_2 + 4R_3$

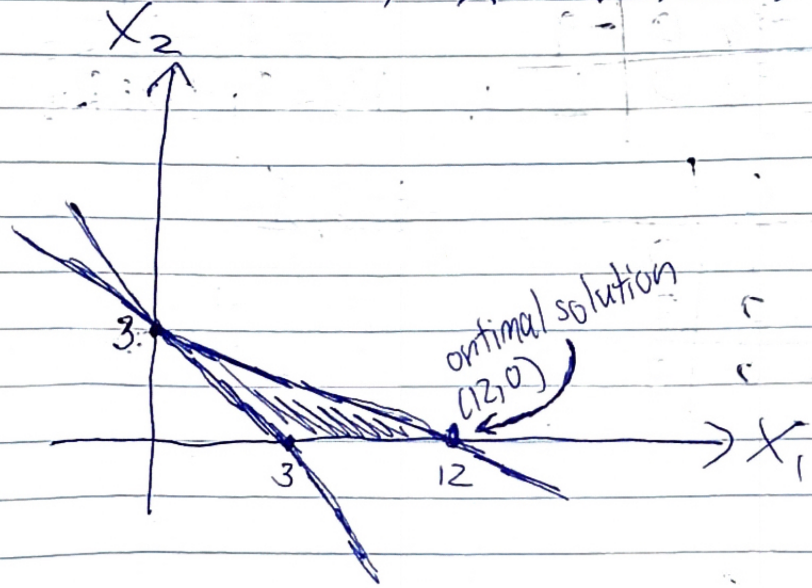
BV	X_1	X_2	X_3	X_4	
x_4	0	3	1	1	9
x_1	1	4	1	0	12
z	0	2	1	0	12

$R_1' = 3R_1/4$

$R_3' = R_3/2$

Gives us optimal solution, where

$z = 12$, $x^* = (12, 0, 0, 9)$



Q4c).

$\max z = x_1 + 3x_2 + 2x_3$

subject to $x_1 + 2x_2 + x_3 \geq 1$

$x_1 + 2x_2 + x_3 \leq 1$

$2x_1 - x_2 \geq -2$

$x \geq 0$

Convert into canonical form

$\max z = x_1 + 3x_2 + 2x_3$

Step 1: s.t. $x_1 + 2x_2 + x_3 = 1$ ← combine from the first 2 constraints.

Step 2: change the sign (inequality), multiply on both sides by minus 1.

$$-2x_1 + x_2 \leq 2.$$

Now, my canonical form,

$\max z = x_1 + 3x_2 + 2x_3$

s.t. $x_1 + 2x_2 + x_3 = 1$

$-2x_1 + x_2 + x_4 = 2$

$x \geq 0.$

BV	x_1	x_2	x_3	x_4		Ratio
x_3	1	$\boxed{2}$	1	0	1	1/2
x_4	-2	1	0	1	2	2/1 = 2
z	-1	-3	-2	0	0	

BV	x_1	x_2	x_3	x_4		
x_2	1/2	1	1/2	0	1/2	$R_1' = R_1/2$
x_4	-2	1	0	1	2	
z	-1	-3	-2	0	0	

BV	x_1	x_2	x_3	x_4	
x_2	1/2	$\boxed{1}$	1/2	0	1/2
x_4	-2	1	0	1	2
z	-1	-3	-2	0	0

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_2	1/2	1	1/2	0	1/2
x_4	5/2	0	1/2	-1	-3/2
z	1/2	0	-1/2	0	3/2

$R_2' = R_1 - R_2$

$R_3' = 3R_1 + R_3$

$\equiv$

BV	x_1	x_2	x_3	x_4		Ratio
x_3	1	2	$\boxed{1}$	0	1	1
x_4	5/2	0	1/2	-1	-3/2	-3
z	1/2	0	-1/2	0	3/2	

$R_1' = 2R_1$

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_3	1	2	$\boxed{1}$	0	1
x_4	-4	2	0	2	4
z	2	2	0	0	4

$R_2' = R_1 - 2R_2$

$R_3' = R_1 + 2R_3$

$\equiv$

BV	x_1	x_2	x_3	x_4	
x_3	1	2	1	0	1
x_4	-2	1	0	1	2
z	1	1	0	0	2

$R_2' = \frac{R_2}{2}$

$R_3' = \frac{R_3}{2}$

gives us the optimal solution,
$x^* = (0, 0; 1, 2)$ with the z-value
of $z = 0 + 3(0) + 2(1) = 0 + 0 + 2 = 2$ $(z = 2)$.

Q5)

a) max $z = x_1 + x_2$

s.t. $\frac{1}{2}x_1 - x_2 \leq -1$

$\frac{1}{2}x_1 + x_2 \leq 4$

$3x_1 + x_2 \geq 5$

$2x_1 - x_2 \leq 2$

$x \geq 0.$

Canonical form.

max. $z = x_1 + x_2$

s.t. $\frac{1}{2}x_1 - x_2 + x_3 = 1$

$\frac{1}{2}x_1 + x_2 + x_4 = 4$

$3x_1 + x_2 - x_5 + x_7 = 5$

$2x_1 - x_2 + x_6$

Martix for the first phase,

$w = -x_7 = -5 + 3x_1 + x_2 - x_5$

$\Rightarrow w = -3x_1 - x_2 + x_5 = -5.$

$$\left(\begin{array}{cccccccc|c} 1/2 & -1 & 1 & 0 & 0 & 0 & 0 & & 1 \\ 1/2 & 1 & 0 & 1 & 0 & 0 & 0 & & 4 \\ 3 & 1 & 0 & 0 & -1 & 0 & 1 & & 5 \\ 2 & -1 & 0 & 0 & 0 & 1 & 0 & & 2 \\ \hline -3 & -1 & 0 & 0 & 1 & 0 & 0 & & -5 \end{array}\right)$$

Rest of the code is computed in pivot file in MATLAB! (from above)

Q5b).

min $z = x_1 + x_2$

s.t.

$$\frac{1}{2}x_1 - x_2 \leq -1$$

$$\frac{1}{2}x_1 + x_2 \leq 4$$

$$3x_1 + x_5 \geq 5$$

$$2x_1 - x_2 \leq 2$$

$$x \geq 0$$

Change from min. to max.

max $-z = -x_1 - x_2$

s.t.

$$\frac{1}{2}x_1 - x_2 \leq -1$$

$$\frac{1}{2}x_1 + x_2 \leq 4$$

$$3x_1 + x_2 \geq 5$$

$$2x_1 - x_2 \leq 2$$

$$x \geq 0.$$

Change to canonical form,

max $-z = -x_1 - x_2$

s.t.

$$\begin{aligned} 1/2\,x_1 - x_2 + x_3 \quad &= -1 \\ 1/2\,x_1 + x_2 \quad + x_4 \quad &= 4 \\ 3x_1 + x_2 \quad - x_5 + x_7 &= 5 \\ 2x_1 - x_2 \quad + x_6 \quad &= 2 \end{aligned}$$

where

$$w = -x_7 = -(5 - 3x_1 - x_2 + x_5)$$
$$= -5 + 3x_1 + x_2 - x_5$$

Initial Table for Phase 1,

$$\begin{bmatrix} 1/2 & -1 & 1 & 0 & 0 & 0 & 0 & -1 \\ 1/2 & 1 & 0 & 1 & 0 & 0 & 0 & 4 \\ 3 & 1 & 0 & 0 & -1 & 0 & 1 & 5 \\ 2 & -1 & 0 & 0 & 0 & 1 & 0 & 2 \\ -3 & -1 & 0 & 0 & 1 & 0 & 0 & -5 \end{bmatrix}$$

→ see the rest of the steps in MATLAB pivot file. (from above)

Phase 2,

$$\begin{bmatrix} 1 & 0 & 0.2857 & 0 & -0.2857 & 0 & 1.1429 \\ 0 & 0 & 0.7143 & 1 & 0.2857 & 0 & 1.8571 \\ 0 & 1 & -0.8571 & 0 & -0.1429 & 0 & 1.5714 \\ 0 & 0 & -1.4286 & 0 & 0.4286 & 1 & 1.2857 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow$$

Not in canonical

strictly in that order,

$\equiv$ • $R_5' = R_1 - R_5$

• $R_5' = R_5 + R_3$

$$\begin{pmatrix} 1 & 0 & 0.2857 & 0 & -0.2857 & 0 & 1.1429 \\ 0 & 0 & 0.7143 & 1 & 0.2857 & 0 & 1.8571 \\ 0 & 1 & -0.8571 & 0 & -0.1429 & 0 & 1.5714 \\ 0 & 0 & -1.4286 & 0 & 0.4286 & 1 & 1.2857 \\ 0 & 0 & -0.5714 & 0 & -0.4286 & 0 & 2.7143 \end{pmatrix}$$

Now in canonical form,

→ see the rest of the steps in MATLAB pivot file. (from above)