The University of Melbourne

School of Engineering

Semester 2 Assessment 2015

ENGR30002 – Fluid Mechanics

Exam Duration: 3 hours

Reading Time: 15 minutes

This paper has ELEVEN (11) pages consisting of SIX (6) questions.

Authorized material:
Only electronic calculators approved by the School of Engineering may be used.
Two Charts are attached.

Instructions to Invigilators:
Script books to be provided.

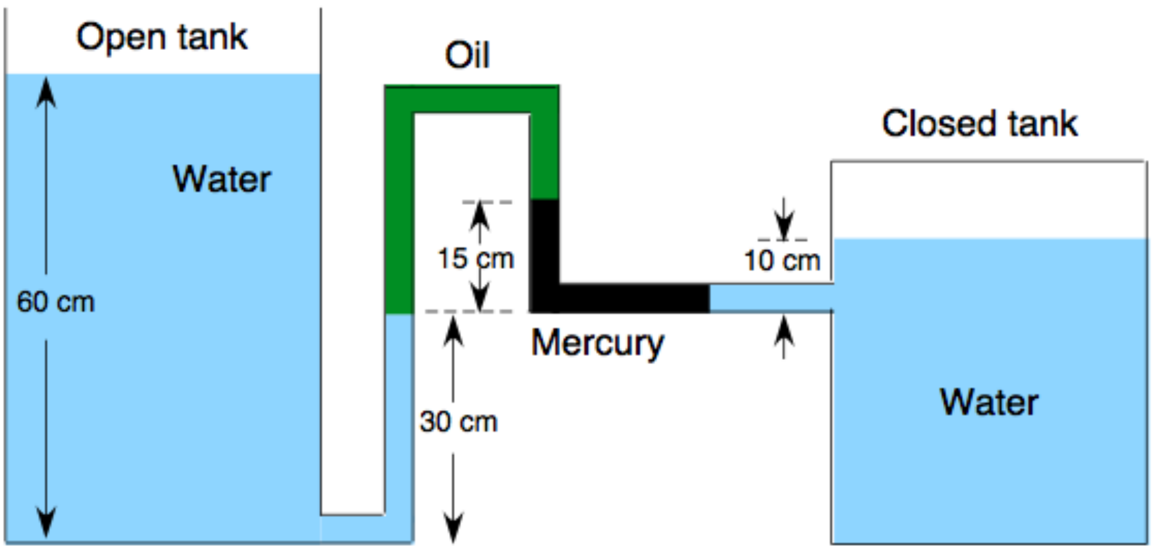
Instructions to Students:
All six questions are to be attempted.
Total marks for the exam = 100

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This paper is to be held by the Baillieu Library

Question 1

The diagram below shows an open tank and a closed tank, both containing water. The tanks are connected by piping, sections of which also contain mercury and oil. The locations of the water, mercury and oil sections are specified by the vertical distances, and by the shading, marked on the diagram. The fluids are stationary.



Density of mercury	13600 kg m^{-3}
Density of oil	800 kg m^{-3}
Density of water	1000 kg m^{-3}
Acceleration due to gravity	9.81 m s^{-2}

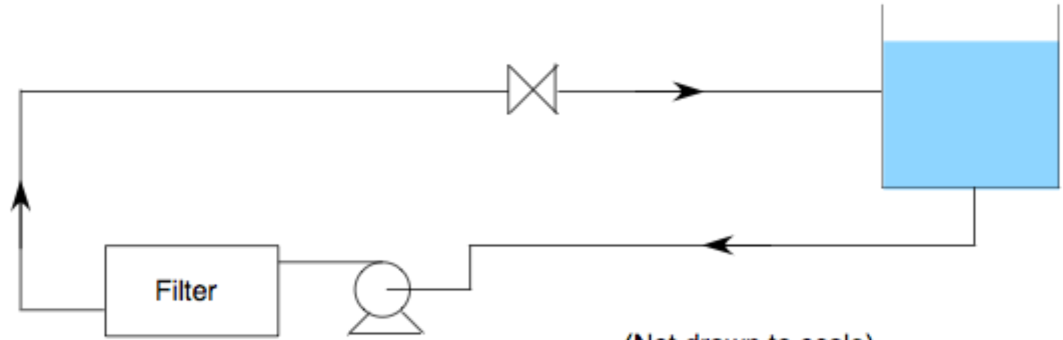
Calculate the gauge pressure above the water surface in the closed tank.

(10 marks)

(Total for Question 1 = 10 marks)

Question 2

Water is pumped from a large open tank, through a filter, and back to the tank as shown in the diagram below. The power added to the water by the pump is 30 watts. The piping has a length of 30 m on the suction side of the pump and 30 m on the discharge side, a diameter of 3 cm, and contains one valve and five elbows. The Fanning friction factor is 0.01. Include the resistance of pipe entry and exit in the calculations as appropriate.



(Not drawn to scale)

Density of water	1000 kg m^{-3}
Acceleration due to gravity	9.81 m s^{-2}
Atmospheric pressure	101.3 kPa
Water vapour pressure	2.3 kPa
Equivalent length of one elbow	25 pipe diameters
Resistance coefficient of the filter	12
Resistance coefficient of the valve	6
Resistance coefficient of pipe entry	0.8
Resistance coefficient of pipe exit	1.0

(a) Determine the volumetric flow rate through the filter in litres per minute

(10 marks)

(b) What is cavitation, when does it occur, and why should it be avoided in pumps?

(4 marks)

(c) Define NPSH and hence derive the usual expression for available NPSH when pumping liquid from a tank

(2 marks)

(Question 2 continues on next page)

Question 2 continued

(d) Calculate the available NPSH for the flow described above when the pump is located 3m below the surface of the water in the tank

(4 marks)

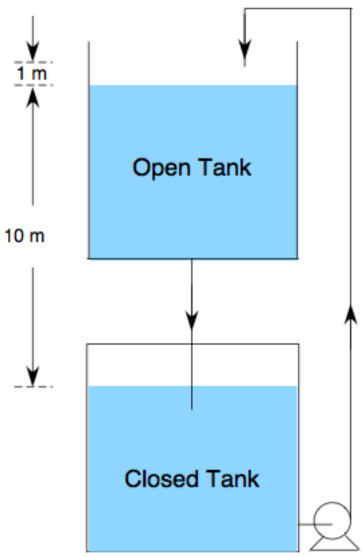
(e) If the required NPSH is 2 m, is there a cavitation problem? Explain your answer.

(2 marks)

(Total for Question 2 = 22 marks)

Question 3

Liquid in an open tank flows through a 5m long pipe into a closed tank below it, as shown in the diagram. The gauge pressure above the liquid surface in the closed tank is 100 kPa. The liquid in the closed bottom tank is pumped back into the open upper tank through a 20m long pipe (containing two elbows) which discharges freely at a height 1m above the liquid surface in the upper tank. The volumetric flow rate is such that the height difference between the liquid surfaces is maintained at 10m. All piping has an internal diameter of 25mm with an absolute roughness of 0.025mm. Ignore pipe entry and exit losses. Also ignore energy losses over the short pipe section connecting the pump to the side of the lower tank.



(Question 3 continues on next page)

Question 3 continued

Density of the liquid	1100 kg m^{-3}
Viscosity of the liquid	0.0015 Pa s
Acceleration due to gravity	9.81 m s^{-2}
Equivalent length of one elbow	25 pipe diameters

(a) Calculate the volumetric flow rate in litres per minute

(10 Marks)

(b) Calculate the brake power of the pump when its mechanical efficiency is 70%

(8 Marks)

(Total for Question 3 = 18 marks)

Question 4

(a) When discussing compressible flow, we specified the parameter γ. What is the mathematical definition of γ? Define all terms in the definition. During what flow condition is γ used when mathematically modelling the velocity of gas flow?

(2 marks)

(b) What is the water hammer effect and when does it occur? Why is it a potentially undesirable phenomenon?

(2 marks)

(c) The Froude number can be thought of as comparing two forces that are acting on the fluid. Which two forces are these? In open channel flow, what occurs when the Froude number goes from a value greater than one to a value less than one?

(2 marks)

(d) Describe two methods for preventing swirling when mixing liquid in a stirred tank? Why is one of the techniques more commonly used industrially?

(2 marks)

(Question 4 continues on next page)

Question 4 continued

(e) You are scaling up a cylindrical mixing tank from a capacity of 100 L to 1000 L. What is the value of the scaling factor (R)? Show any calculations.

(1 mark)

(f) What is the rheological property that distinguishes a Bingham plastic from other types of fluids?

(1 mark)

(Total for Question 4 = 10 marks)

Question 5

You work for the company, *Green Buildings*, that produces materials used in the construction industry. The company attempts to limit its negative impact on the environment by pumping the carbon dioxide it produces through a piping system to an underground storage site, instead of venting it into the atmosphere. Each pipe is horizontal, 2.5km long, and has an inner diameter of 15cm. The Fanning friction factor $f = 0.0005$. The inlet pressure in each pipe is 500kPa and the pressure in the underground reservoir is 150kPa. The flow is isothermal at 25°C. Consider a single pipe and neglect the kinetic energy term in the mechanical energy equation.

The mechanical energy equation for horizontal, isothermal, ideal gas flow in a pipe of uniform cross-section is

$$\frac{P_2^2 - P_1^2}{2(RT/M)} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{2fL}{D}\left(\frac{G}{A}\right)^2 = 0$$

The critical pressure P_w, associated with choked flow, satisfies the following equation:

$$\frac{4fL}{D} = \left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1$$

where all symbols have their usual meaning.

(Question 5 continues on next page)

Question 5 continued

Molecular weight of carbon dioxide	44 gm/mol
Gas constant R	8.314 J mol^{-1} K^{-1}

(a) Calculate the mass flow rate of carbon dioxide.

(5 marks)

(b) What is the pressure 1/4 of the way along the pipe from the pipe entry? What would the pressure be, at the same location, if you were pumping an incompressible liquid? Are the pressure values the same? Explain your answer.

(4 marks)

(c) An operator at your facility proudly tells you that he tripled the inlet pressure from 500kPa to 1500kPa in order to triple the flow rate of the carbon dioxide into the reservoir. Did the operator achieve his goal? Explain your answer.

(2 marks)

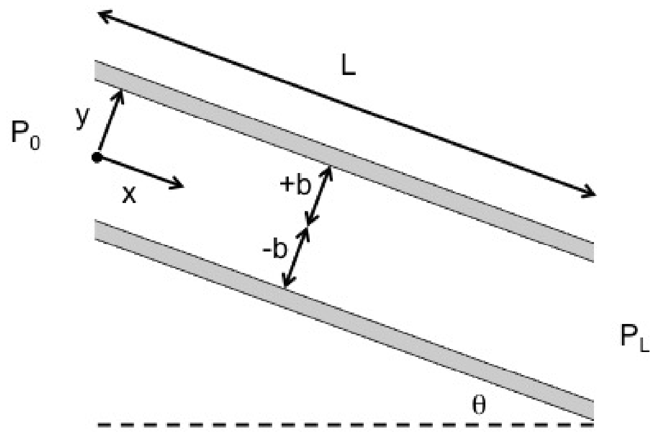
(d) What is the maximum inlet pressure you would recommend in order to maximize the flow rate of the carbon dioxide? (This calculation may require iteration). What is the velocity of the gas at the pipe exit for this condition?

(4 marks)

(Total marks for Question 5 = 15 marks)

Question 6

We have steady, fully developed, and laminar flow of a Newtonian liquid between flat, parallel plates that are at angle θ to the horizontal. The fluid is driven by both applied pressure and gravity. Consider plates of length L and lateral spacing $2b$, with pressures P_0 and P_L at the ends, as shown in the diagram below. The fluid flows only in the positive x-direction. The z-dimension is perpendicular to the plane of the paper, and velocity and pressure are independent of the z-coordinate.



The continuity and momentum equations in rectangular coordinates are:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Question 6 continues on next page)

Question 6 continued

(a) Consider the equation for the x-component of momentum and label each term with the forces they describe.

(2 marks)

(b) Show that the x-component of velocity (v_x) depends only on y.

(2 marks)

(c) Show that $\frac{\partial p}{\partial x}$ is constant

(4 marks)

(d) Derive an expression for $v_x(y)$

(9 marks)

(e) On a single sketch, qualitatively draw the velocity profile of the fluid when the applied pressure gradient, (i) is zero, (ii) assists the gravity driven flow, and (iii) opposes the gravity driven flow.

(3 marks)

(f) Now consider flow that is only due to gravity. The top plate is removed to expose the upper surface of the liquid which becomes a liquid-air interface.

- Holding all other parameters fixed, qualitatively sketch the velocity profile of the liquid under the new flow condition. Will the average velocity of the liquid be greater than, less than, or equal to the average velocity of the flow between the parallel plates? Give brief reasons your answer.

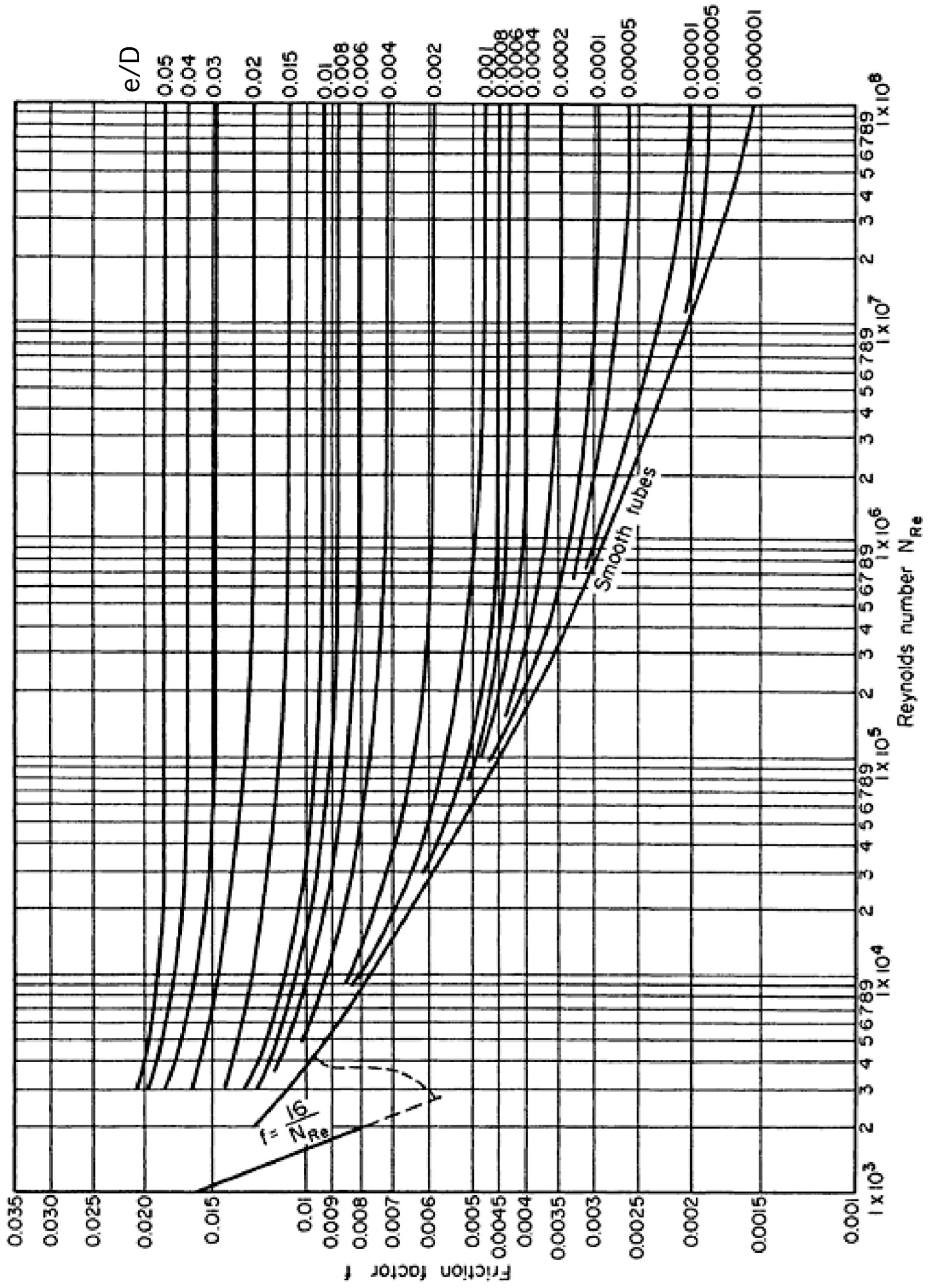
(3 marks)

- If you were to solve the Navier-Stokes equations for the new flow condition, what velocity boundary condition(s) would you use?

(2 marks)

(Total for Question 6 = 25 marks)

(Total for paper = 100 marks)


e/D
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
0.000005
0.000001
Friction factor f
0.035
0.030
0.025
0.020
0.015
0.01
0.009
0.008
0.007
0.006
0.005
0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
Reynolds number N_{Re}
1×10^3
1×10^4
1×10^5
1×10^6
1×10^7
1×10^8
Smooth tubes
$f = \frac{16}{N_{Re}}$

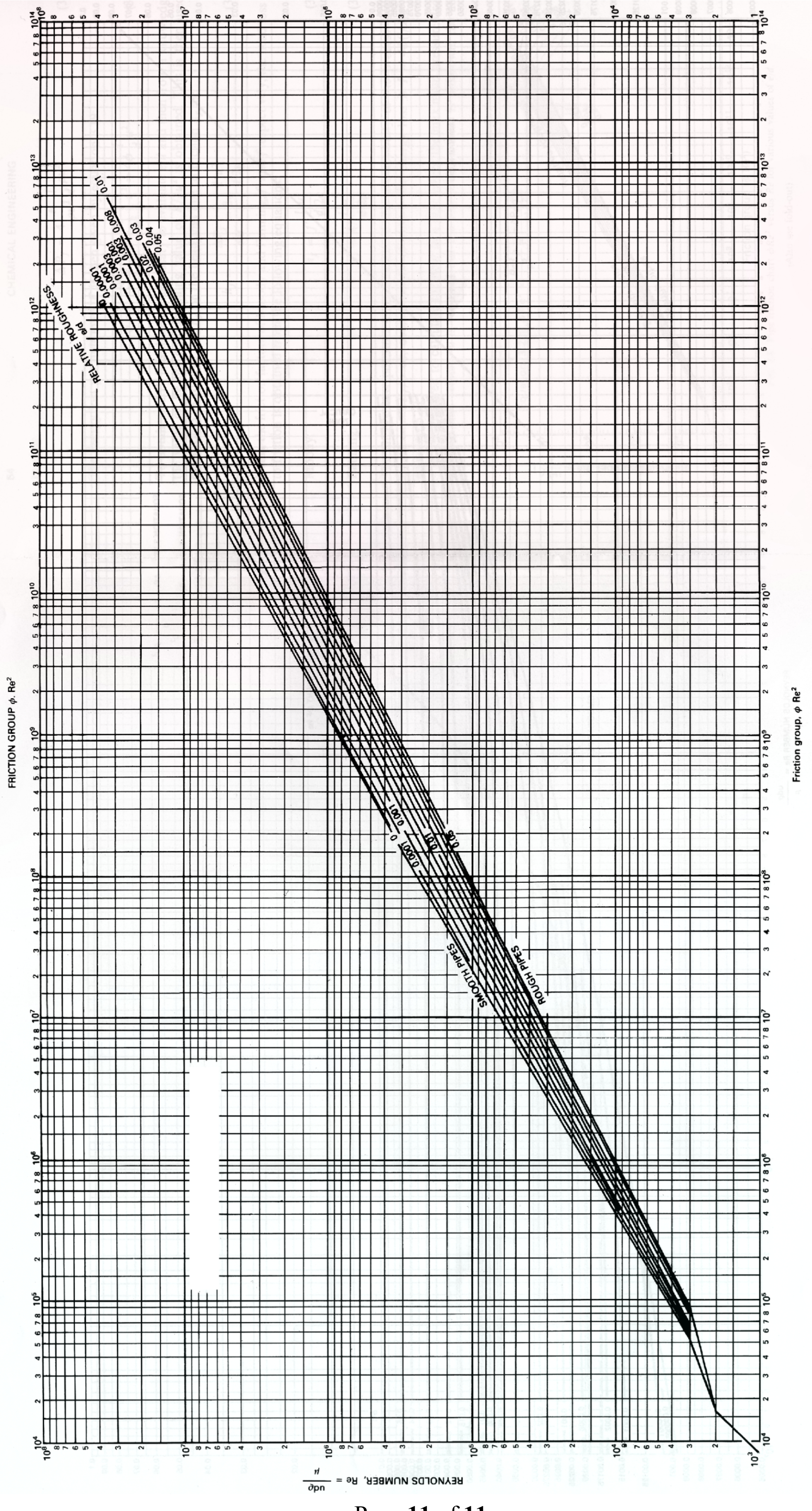


Fig. 3.8. Pipe friction chart ϕRe^2 versus Re for various values of e/d.