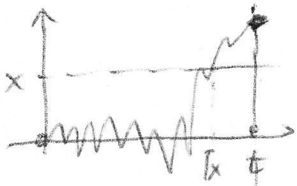
hitting time of level x. $T_x = \inf\{t : B_t = x\}$.

BM is continuous $\Rightarrow$ (If $0 < x < y \Rightarrow T_x < T_y$).

T_x is finite (since simple symmetric random walk is recurrent)



As BM is continuous. when $B_t \geq x$.

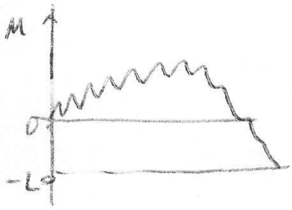
there $\exists$ a $T_x \leq t$. hitting x

$$\Rightarrow P(B_t \geq x) = P(B_t \geq x, T_x \leq t)$$

$$= P(B_t \geq x \mid T_x \leq t) \cdot P(T_x \leq t).$$

$T_x := \tau$. $\quad P(B_{\tau+s} - B_\tau > 0 \mid B_\tau = x) = P(B_s > 0 \mid B_\tau = x)$

$$= P(N(0, s) > 0) = \frac{1}{2}$$



hits $-L$ before M : $\frac{M}{L+M}$.

$S_t^{(N)}$ = sum of $\lfloor Nt \rfloor$ variables iid. each with variance $\frac{1}{N}$

$(X_1^{(N)}, \dots, X_k^{(N)} \dots$ iid. $P(X_i^{(N)} = \frac{\pm 1}{\sqrt{N}}) = \frac{1}{2})$

$\Rightarrow Var(S_t) \approx t$

(n & k grow with N.

$\Rightarrow$ as $N \to \infty$ $\frac{n}{N} \to t > 0$ $\frac{k}{\sqrt{N}} \to x$)

Slides 20.

$$Q_{\frac{n}{N}}\left(\frac{k}{\sqrt{N}}\right) = \frac{1}{2} Q_{\frac{n-1}{N}}\left(\frac{k+1}{\sqrt{N}}\right) + \frac{1}{2} Q_{\frac{n-1}{N}}\left(\frac{k-1}{\sqrt{n}}\right)$$

$\Rightarrow P_t(x)$ $\to P_{t-\frac{1}{N}}(x)$

LHS $\approx N\left[Q_t(x) - Q_{t-\frac{1}{N}}(x)\right]$. $\longrightarrow \frac{d}{dt} P_t(x)$).

RHS $\approx \frac{1}{2}\frac{\partial^2}{\partial x^2} P_t X$

$$N\left[Q_{\frac{n}{N}}\left(\frac{k}{\sqrt{N}}\right) - Q_{\frac{n-1}{N}}\left(\frac{k}{\sqrt{N}}\right)\right]$$

$$= \frac{N}{2}\left[Q_{\frac{n-1}{N}}\left(\frac{k+1}{\sqrt{N}}\right) - Q_{\frac{n-1}{N}}\left(\frac{k}{\sqrt{N}}\right)\right] - \frac{N}{2}\left[Q_{\frac{n-1}{N}}\left(\frac{k}{\sqrt{N}}\right) - Q_{\frac{n-1}{N}}\left(\frac{k-1}{\sqrt{N}}\right)\right].$$

$\frac{n}{N} \to t$. $\frac{k}{\sqrt{N}} \to x$ $(N \to \infty)$.

$\Rightarrow$ limiting stochastic process density $P_t(x)$ at time t satisfying $\frac{\partial}{\partial t} P_t(x) = \frac{1}{2}\frac{\partial^2}{\partial x^2} P_t(x)$.