

on $[0, \tau]$. show ∇_{not} for $i \leq k$ & $x \leq t$, the distribution function

for the order statistic $Y_{(i)}$ is given by

$$F_{Y(i)}(x) = \sum_{l=i}^k C_k \left(\frac{x}{t}\right)^l \left(1 - \frac{x}{t}\right)^{k-l}$$

$$F_{Y_i}(x) = P\left(\frac{1}{h^i} \leq x\right)$$



mean $\neq z_i$ in interval $(0, x)$.

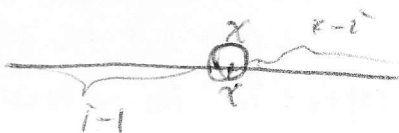
$$\Rightarrow P(Y_{(n)} \leq x) = \sum_{k=0}^n P(\text{at least } k \text{ of } Y_j\text{'s} \leq x, \text{ } n-k \text{ of } Y_j\text{'s} \geq x)$$

$$= \sum_{l=i}^K C_K^l (F_Y(x))^l (1 - F_Y(x))^{K-l}$$

$$F_{(i)}^{(k)}(x) = \frac{K!}{(i-1)!(k-i)!} F(x)^{i-1} (1-F(x))^{k-i} f(x)$$

prohibit $i-1$ of Y_j
 $X \leq$

pro of $k-i$ of $\forall j > x$



Poisson Process

A discrete random variable $N \sim \text{Po}(\lambda)$.

$$p_n = \begin{cases} \frac{e^{-\lambda} \lambda^n}{n!} & n=0, 1, \dots \\ 0 & n < 0 \end{cases} \quad \begin{aligned} E(\text{Po}(\lambda)) &= \lambda \\ \text{Var}(\dots) &= \lambda \end{aligned}$$

(exponential) $f_T(t) = \lambda e^{-\lambda t}$ $E = \frac{1}{\lambda}$
 $\text{Var} = \frac{1}{\lambda^2}$

• If $Y_n \stackrel{d}{=} \text{Geo}(\frac{\lambda}{n})$, $T_\lambda \stackrel{d}{=} \text{Exp}(\lambda)$

then for $t > 0$ $\lim_{n \rightarrow \infty} P(\frac{Y_n}{n} \leq t) = P(T_\lambda \leq t)$

• If $X_n \stackrel{d}{=} \text{Bio}(n, \frac{\lambda}{n})$, $N_\lambda \stackrel{d}{=} \text{Po}(\lambda)$. Then for $k=0, 1, \dots$

$\lim_{n \rightarrow \infty} P(X_n = k) = P(N_\lambda = k)$ Exponential Distribution arises as the limit of the geometric distribution

Properties of Poisson Process

• $N_{t+h} - N_t \sim \text{Po}(\lambda s)$

• for fixed h , $N_t^h := N_{t+h} - N_t \sim \text{Po}(\lambda h)$ $N^h \rightarrow$ poisson process with rate λ .
 (even h is rv independent of $(N_s, s \leq t)$).

✓ $N_{t+h} = \text{Po}(\lambda(t+h))$, $N_t \sim \text{Po}(\lambda t)$.

$N_{t+h} = N_{t+h} - N_t + N_t \rightarrow \text{Po}(\lambda t) \Rightarrow E[e^{\theta(N_{t+h} - N_t)}] E[e^{\theta N_t}] = E[e^{\theta N_{t+h}}]$
 \downarrow
 $\text{Po}(\lambda h) \quad \text{Po}(\lambda t) \quad \text{Po}(\lambda(t+h)) \quad \Rightarrow \frac{E[e^{\theta N_{t+h}}]}{E[e^{\theta N_t}]} \rightarrow M_{\text{Po}(\lambda(t+h))}(\theta)$
 \downarrow
 $\text{Po}(\lambda h) \quad \text{Po}(\lambda t) \quad \text{Po}(\lambda(t+h)) \quad \Rightarrow \frac{E[e^{\theta N_{t+h}}]}{E[e^{\theta N_t}]} \rightarrow M_{\text{Po}(\lambda(t+h))}(\theta)$