MAST30001 Stochastic Modelling – 2015

Assignment 2

If you didn't already hand in a completed and signed Plagiarism Declaration Form (available from the LMS or the department's webpage), please do so and attach it to the front of this assignment.

Don't forget to staple your solutions and to print your name, student ID, and the subject name and code on the first page (not doing so will forfeit marks). The submission deadline is **Friday, 23 October by 4pm** in the appropriate assignment box at the north end of Richard Berry Building.

There are 2 questions, both of which will be marked. No marks will be given for answers without clear and concise explanations. Clarity, neatness and style count.

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1. Customers arrive at a shop according to a Poisson process with rate 20 per hour. Arriving customers make a purchase with probability 1/3 and among customers making a purchase, there is 6/7 chance they make a "small" purchase and 1/7 chance they make a "large" purchase. Assume customers make their purchasing decisions independently.

   (a) What is the average amount of time between now and the next time a customer makes a large purchase?

   (b) What is the chance that during a half hour period exactly three customers total enter the store and exactly one of these three makes a large purchase?

   (c) What is the chance that during a one hour period exactly five customers total enter the store and during the first thirty minutes of that period exactly four customers make a small purchase?

   (d) What is the chance that there are at least two customers who make a small purchase before the arrival of the first customer who doesn't make a purchase?

2. A car repair shop has two oil change service bays. Cars take an exponential amount of time (independent between different cars) to have their oil changed. Cars in the first service bay take about 20 minutes to have their oil changed and in the second service bay an oil change takes about 1 hour. Cars arrive for an oil change according to a Poisson process (independent of service times) with rate 2 per hour. If both service bays are empty, then an arriving car begins service in one of the two bays, chosen uniformly at random. If there is only one bay free, then an arriving car starts service in that bay. When both bays are full, cars can form a queue. Assume that if the length of the queue is two, then cars from the arrival process don't want to wait and so don't join (so the total number of cars in the system is capped at four).

   (a) What is the long run proportion of time both service bays are in use?

   (b) What is the average number of cars in the system?

   (c) What is the average amount of time that cars that enter the system have to wait for service?

   (d) What is the long run proportion of cars that enter the system that receive service from the first bay?