17 Total marks

SCHOOL OF MATHEMATICS AND STATISTICS

MAST30013 Techniques in Operations Research **Semester 1, 2021**

Assignment 2 Solutions

a. function val = f(x)
val = x(1)^4+x(2)^4+x(3)^4+x(1)^2*x(2)^2-4*x(1)^3-12*x(1)*x(2)^2 ...
-x(1)*x(2)*x(3)+12*x(1)*x(3)-4*x(3)+1;
end

function grad = gradf(x)
grad(1)= 4*x(1)^3-12*x(1)^2+2*x(1)*x(2)^2-12*x(2)^2 ...
-x(3)*x(2)+12*x(3);
grad(2) = 2*x(1)^2*x(2)-24*x(1)*x(2) - x(3)*x(1)+4*x(2)^3;
grad(3) = 4*x(3)^3 + 12*x(1) - x(1)*x(2) - 4;
end

1mark

function hess = hessf(x)
hess(1,1) = 12*x(1)^2-24*x(1) +2*x(2)^2;
hess(1,2) = 4*x(1)*x(2)-x(3)-24*x(2);
hess(1,3) = 12 - x(2);

1mark

hess(2,1) = 4*x(1)*x(2)-x(3)-24*x(2);
hess(2,2) = 2*x(1)^2-24*x(1) + 12*x(2)^2;
hess(2,3) = -x(1);

No need for Matlab notation here

hess(3,1) = 12 - x(2);
hess(3,2) = -x(1);
hess(3,3) = 12*x(3)^2;
end

b.

i. The aggregated results for the steepest descent, Newton's, and the BFGS methods are given in Tables 1, 2, and 3, respectively.

f-value	Minimiser	No. of times	Av. iter. per search	Ave. time per search (sec)
-373.42	(4.32, -4.15, -2.54)	490.00	13.90	-
-296.15	(4.11, 3.96, -1.94)	430.00	19.44	-
-13.25	(-1.09, -0.0620, 1.63)	80.00	8.00	-

Table 1: Steepest Descent

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f-value	Minimiser	No. of times	Av. iter. per search	Ave. time per search (sec)
(4.32, -4.15, -2.54)	-373.42	450.00	5.33	-
(4.11, 3.96, -1.94)	-296.15	540.00	5.19	-
(-1.09, -0.0620, 1.63)	-13.25	10.00	3.00	-

Table 2: Newton's method

3 marks

f-value	Minimiser	No. of times	Av. iter. per search	Ave. time per search (sec)
(4.32, -4.15, -2.54)	-373.42	530.00	10.92	-
(4.11, 3.96, -1.94)	-296.15	430.00	11.72	-
(-1.09, -0.0620, 1.63)	-13.25	40.00	7.50	-

Table 3: BFGS

3 marks

ii. There are three local minima which the algorithms found, with objective values -13.2455, -296.1545, and -373.4173, the last being the best attained when $(x_1, x_2, x_3) = (4.3234 - 4.1541 - 2.5437)$. Most of the iterations converged to one of the two best minima in all methods, but the Steepest descent algorithm, for example, found -13.2455 about eighty times over the 1000 runs.

6 marks

Newton's method took the smallest number of iterations for any particular run compared with the BFGS method which was next best and steepest descent which was the worst, but this came at the cost of larger iteration costs (i.e., more calculations per iteration).