The University of Melbourne
School of Engineering

Semester 1 Assessment 2014

ENGR30002 – Fluid Mechanics

Exam Duration: 3 hours

This paper has THIRTEEN (13) pages consisting of FIVE (5) questions.

Authorized material:
Electronic calculators approved by the School of Engineering may be used.
One Chart is attached.

Instructions to Invigilators:
Script books to be provided.

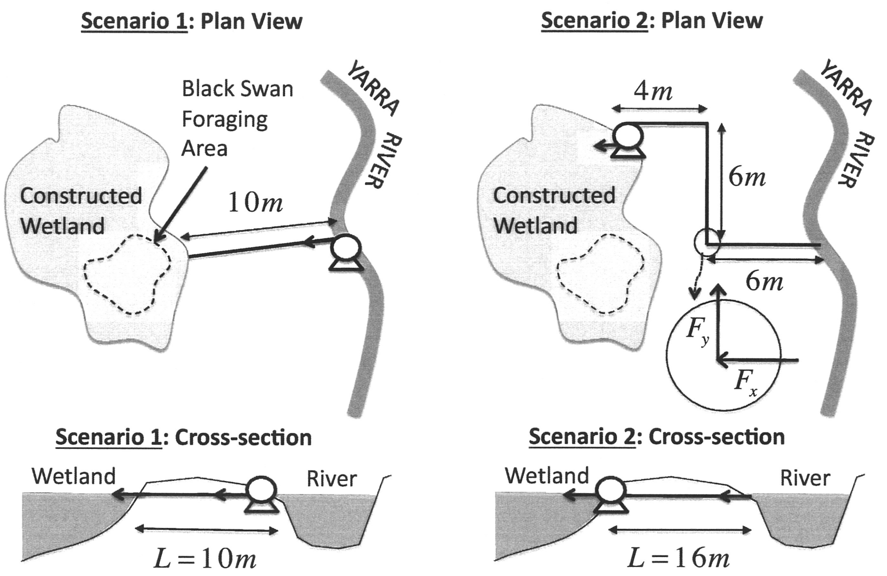
Instructions to Students:
All questions are to be attempted.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This paper is to be held by the Baillieu Library

Question 1

A pipeline is to pump water from the Yarra River to a nearby constructed wetland (see figure below). The pipe is straight and 10 m long in Scenario 1, and is 16 m long with two elbows in Scenario 2. You may assume the following: (1) the inlet and exit of the pipe are at the same elevation; (2) the pipe diameter is 10 cm; (3) the interior of the pipe has a relative roughness of 0.02; (4) the typical physical properties of water apply ($\rho = 1000 \text{ kg m}^{-3}$, $\mu = 0.001 \text{ Pa}\cdot\text{s}$); and (5) the head loss as water comes into the pipe from the river can be expressed in terms of a resistance coefficient $K = 0.6$. Gravitational acceleration is 9.81 ms^{-2}.



Answer the following questions for **Scenario 1** (see figure).

i) Your client (Darcy) has informed you that the pipe is to transfer 100 cubic meters of water per hour from the river to the wetland. Determine the brake power required to pump the water at this flow rate assuming a pump mechanical efficiency of $\eta = 0.7$

(9 marks)

(Question 1 continues on next page)

Question 1 (continued)

ii) A centrifugal pump is selected. Based on the manufacturer's performance plot, the required NPSH is 2.4 m at a volumetric flow rate of 100 cubic meters per hour. The pump is positioned at the pipe intake (in the Yarra River). If the vapor pressure of water is 2.3 kPa and atmospheric pressure is 101 kPa, calculate the available NPSH. Will the available NPSH be sufficient to prevent cavitation?

(4 marks)

iii) According to the manufacturer's data, the pump delivers 8 m of head at a flow rate of 100 cubic meters per hour. Given the information above, calculate the system head and determine if it is greater than or less than the pump head. If the pump head and system head are not equal, advise Darcy regarding whether she should install a valve downstream of the pump, or purchase another pump that can deliver the appropriate head at the design flow rate. Justify your answer with calculations.

(4 marks)

Answer the following questions for **Scenario 2** (see figure).

iv) Residents living along the Yarra River complain to the local Shire about the unsightly nature of the pump equipment along the river shore. Darcy agrees to relocate the pump from the pipe inlet in the Yarra River to the pipe outlet in the wetland. The relocation requires adding two 90-degree bends that lengthen the pipe by 6 m. Minor frictional losses are characterized by an equivalent length of $L_{eq} = 2$ m per elbow. Calculate the available NPSH. Will the required NPSH of the pump be met under this scenario?

(5 marks)

(Question 1 continues on next page)

Question 1 (continued)

v) The two 90-degree elbows were added so that noise from the pump would not disturb a pair of black swans that routinely forage in one particular part of the wetland. You inform Darcy that she will need to account for the forces generated on the pipe by the change in flow direction caused by the new elbows. Calculate for Darcy the magnitude of the x- and y- forces acting on the first elbow (see the expanded view of the elbow in the figure). For your calculations you can assume a flow rate of 100 cubic meters per hour, and the water and pipe data presented above. You can ignore frictional forces at the elbow.

(4 marks)

vi) After two weeks of running the pump in the second scenario, the pump fails. The only replacement in stock is a geometrically similar pump with an impeller that is 285 mm in diameter (the impeller diameter in the original pump was 279 mm). Assuming the mechanical efficiency of the pump does not change after the new pump is installed, use the pump affinity laws to calculate the new volumetric flow rate delivered by the pump at the same speed of rotation. Will the pump head go up or down, and by what percentage? Will the required NPSH go up or down, and by what percentage? Will the brake power go up or down, and by what percentage?

(8 marks)

vii) Water flowing through the pipe exerts frictional drag on the pipe walls. Referring to scenario 2, compute the x-component of the drag force acting on the first segment of the pipe, between the Yarra River and the first elbow. For your calculations you can assume a flow rate of 100 cubic meters per hour and the water and pipe data presented above.

(4 marks)

(Total for Question 1 = 38 marks)

Question 2

A dairy farmer plans to test out a new system (called a "manure digester") for generating renewable energy. The digester holds the manure in an airtight tank heated to 33 degrees (centigrade). Bacteria growing in the digester break down organic carbon in the manure producing a slurry of organic material (called "sludge") on the bottom of the tank and large quantities of methane gas. You are asked to design a gas pipeline to transport the methane gas from the top of the digester to a generator where the methane will be converted into electricity for the farm.

Length of the pipeline $L = 100 \text{ m}$

Diameter of the pipe $D = 10 \text{ cm}$

Fanning friction factor $f = 0.0065$

Temperature $T = 33^{o}\text{C}$ (isothermal)

Pressure at the start of the pipeline $P_1 = 700 \text{ kPa}$

Pressure at the end of the pipeline $P_2 = 560 \text{ kPa}$

Gas constant $R = 8.314 \text{ Jmol}^{-1}\text{K}^{-1}$

Dynamic viscosity of methane $\mu = 1.08 \times 10^{-5} \text{ Pa} \cdot \text{s}$

Gram molecular weight of methane: 16

i) Methane gas leaves the digester and enters a pump. After passing through the pump, the methane enters the pipeline at a pressure of $P_1 = 700 \text{ kPa}$. At the other end of the pipeline the methane enters a combustion chamber held at a constant pressure of $P_2 = 560 \text{ kPa}$. Given the set of constants listed above, calculate the mass flow rate of methane through the pipeline. You should use the mechanical energy equation for isothermal flow of an ideal gas through a horizontal pipeline where the variables have their usual meaning:

$$\frac{P_2^2 - P_1^2}{2(RT/M)} + \left(\frac{G}{A}\right)^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{2fL}{D}\left(\frac{G}{A}\right)^2 = 0$$

(8 marks)

(Question 2 continues on next page)

Question 2 (continued)

ii) The critical pressure P_w is given implicitly by the equation:

$$\left(\frac{P_1}{P_w}\right)^2 - \ln\left(\frac{P_1}{P_w}\right)^2 - 1 = \frac{4fL}{D}$$

Use this expression for the critical pressure to determine if the flow is choked or not choked. If the flow is choked, recalculate the flow rate so it satisfies the choked condition at the end of the pipeline.

(5 marks)

iii) The conversion of methane gas to electricity takes place in three steps: (a) the methane is combusted in a furnace; (b) heat from the furnace is used to make steam; and (c) the steam turns a turbine that generates electricity. Assuming a heat of combustion for methane of $55.5\text{x}10^6$ J/kg and an overall efficiency of 10% for the conversion of methane to electricity, what is the maximum electrical power that can be generated from the system at the gas flow rate calculated above?

(4 marks)

(Total for Question 2 = 17 marks)

QUESTION 3 IS ON THE NEXT PAGE

Question 3

The sludge in the digester has started foaming causing the process to shut down. The foaming stems from inadequate mixing of sludge in the digester and you must help to determine conditions that will properly mix the sludge. For the suspended solids concentration in the digester, the dynamic viscosity of the sludge is $\mu = 0.01\ \mathrm{Pa \cdot s}$ and the density is 721 kg/m^3. The agitator diameter is 6 m.

i) You decide to install baffles on the inside wall of the digester tank. The goal of adding the baffles is to minimize vortexing inside the digester. In the absence of vortexing, you can calculate the power delivered to the sludge from the agitator by assuming inertial forces are small relative to gravitational forces. Is this statement "True" or "False"?

(2 marks)

ii) You begin your analysis by assuming that the agitator should operate in the inertial range ($Re_M > 10000$). What is the smallest agitator speed (in rotations per minute) that will operate the agitator in the inertial limit?

(3 marks)

iii) In the Inertial Limit and assuming no vortexing, the power number is constant:

$$N_p = \frac{P}{\rho N^3 d^5} = C_2, \quad C_2 = 6.3$$

From this expression, calculate the power delivered to the sludge for the agitator speed determined in part ii). Assuming that the agitator is 50% efficient (i.e., 50% of the power going into the agitator from the electricity grid is converted into power delivered to the fluid), what is the power requirement of this system?

(3 marks)

(Question 3 continues on next page)

Question 3 (continued)

iv) You do not know which conditions will minimize foaming. You decide to build a scale laboratory model with which to test various agitator designs. What are the two forces that should be kept in the same ratio if the laboratory model is to represent the full-scale digester? What non-dimensional number must be equal in the laboratory model and full-scale digester in order to enforce dynamic similarity in this case? You may assume that vortexing is suppressed.

(2 marks)

v) If the laboratory model has an agitator diameter of 10 cm, what agitator rotation rate is required to enforce the dynamic similarity criterion described above? You may assume that sludge from the digester is used for the laboratory experiments.

(4 marks)

vi) You now plan to enforce dynamic similarity (assuming no vortexing) and choose the rotation rate of the laboratory agitator to be 100 times the rotation rate of the full-scale agitator. Determine the kinematic viscosity $\upsilon = \mu/\rho$ of the fluid you must use in the laboratory model to achieve this when the diameter of the laboratory agitator is 10 cm. You must give the units of the kinematic viscosity in terms of metres and seconds.

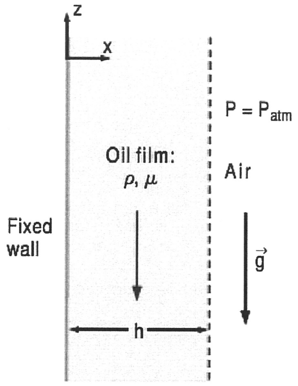
(4 marks)

(Total for Question 3 = 18 marks)

QUESTION 4 IS ON THE NEXT PAGE

Question 4

Consider a steady, incompressible, parallel, laminar flow of a film of oil falling slowly down an infinite vertical wall. The oil film thickness is h, and gravity acts in the negative z-direction. There is no applied pressure driving the flow—the oil falls by gravity alone. Because the flow is assumed to be only parallel to the wall, the only non-zero velocity component is v_z, the component in the z-direction. There is no dependence on the y-direction (not shown in the figure) perpendicular to the page. You may also assume that the pressure everywhere inside the film is equal to the atmospheric pressure.



The continuity and momentum equations in rectangular coordinates are:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

where g_x, g_y, g_z are the components of gravitational acceleration in the positive x, y, and z directions, respectively.

(Question 4 continues on next page)

Question 4 (continued)

i) Based on the continuity equation, what can you conclude about the z-component of the velocity (referred to as v_z) inside the falling film?

(2 marks)

ii) Given the set of assumptions described in the problem statement, show that the material derivative of the z-component of the velocity is zero: $Dv_z/Dt = 0$. Justify why each term in the material derivative can be neglected.

(2 marks)

iii) Simplify the z-component Navier-Stokes equation, and solve the resulting ordinary differential equation subject to the following two boundary conditions: (1) no slip boundary condition at the wall; and (2) no shear boundary condition at the free surface.

(4 marks)

iv) Write down an integral expression for the volumetric flow rate of oil flowing down the wall, that utilizes the solution developed in part iii). You may assume that the horizontal extent of the oil film in the y-direction is W.

(4 marks)

v) Evaluate the integral and calculate the rate at which oil is moving down the wall in mL per second for thickness $h = 1\ \text{mm}$, width $W = 1.5\ \text{m}$, dynamic viscosity of $\mu = 0.1\ \text{Pa}\cdot\text{s}$, and fluid density $\rho = 850\ \text{kg m}^{-3}$. Gravitational acceleration is 9.81 ms^{-2}.

(2 marks)

(Total for Question 4 = 14 marks)

Question 5

The flow rate for open channel flow based on a simple force balance and an empirical approximation of the ϕ friction factor is:

$$Q = 2AD_e^{2/3}e^{-1/6}\sqrt{g\sin\theta}$$

where all variables have their usual meaning.

Consider two rectangular channels that are identical except that one is concrete lined and the other has a natural ("earthen") bottom. The water height in the channels is denoted by y and the channel width is given by b. Assume that the flow rate and channel slope are the same for both channels.

i) If the width of the channel is much larger than its depth ($b >> y$), show that the ratio of water depths in the concrete-lined and natural-bed channels can be calculated from the ratio of respective bed roughnesses as follows:

$$\frac{(y)_{natural}}{(y)_{concrete}} = \left[\frac{(e)_{natural}}{(e)_{concrete}}\right]^{1/10}$$

You will only get points on this problem if your derivation is clear.

(7 marks)

ii) The relationship in part i) means that, for a fixed flow rate, the depth of flow in a channel will increase when the channel roughness increases. Give a physical explanation for this.

(4 marks)

(Question 5 continues on next page)

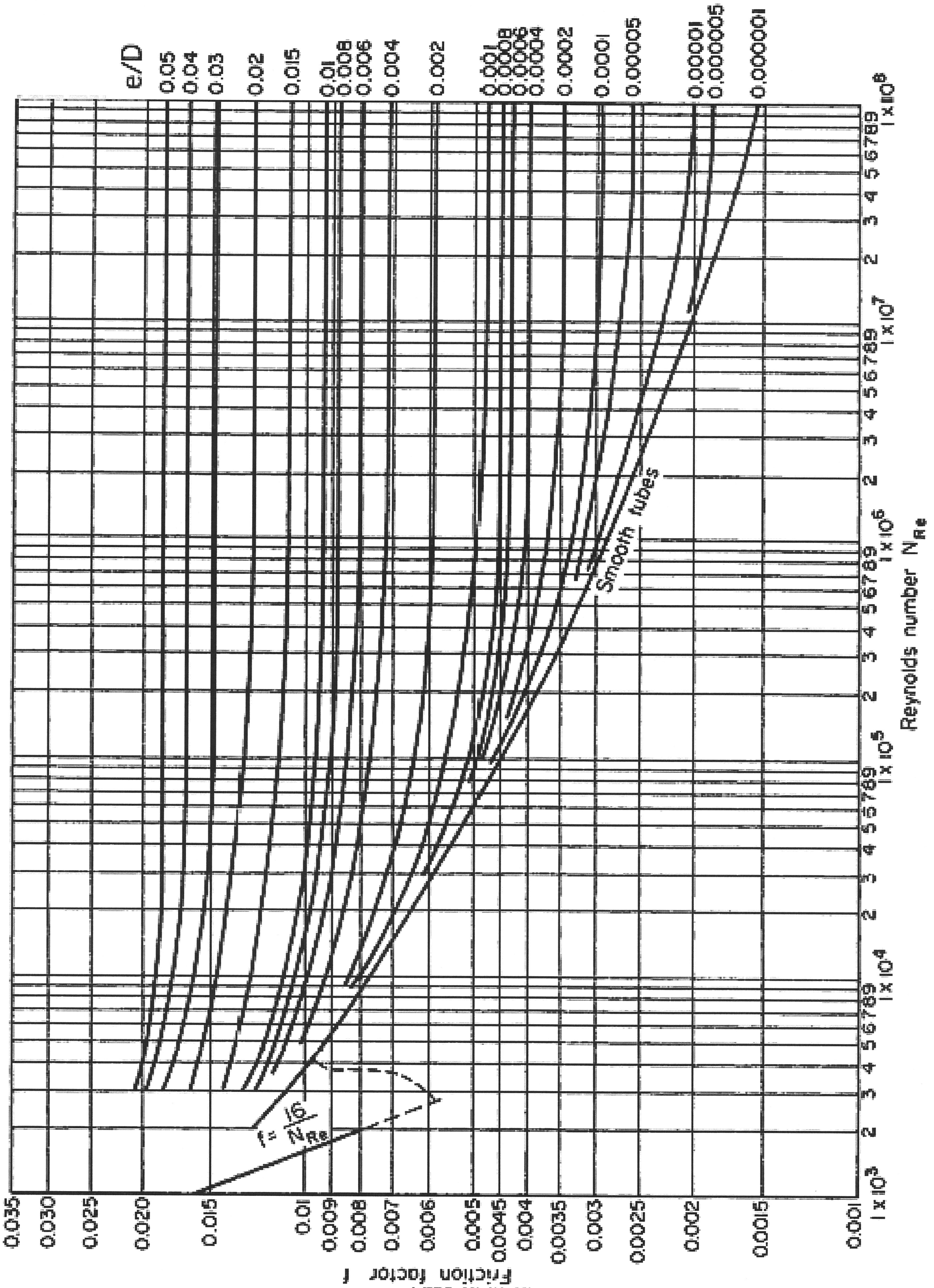
Question 5 (continued)

iii) Given the relationship derived in part i), how much deeper will the flow in the natural-bed channel be compared with the depth in the concrete-lined channel when the roughness of the natural bed is 10 times that of the concrete? Express the increase in depth as a percent of water depth in the concrete-lined channel (e.g., "water depth in the earthen channel will be X% deeper than in the concrete lined channel for the same volumetric flow rate, and bed slope").

(2 marks)

(Total for Question 5 = 13 marks)

(Total for paper = 100 marks)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Friction factor f
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