School of Mathematics and Statistics
MAST30030
Applied Mathematical Modelling

Problem Sheet 1. Some answers

Question 1

(a) $\bar{x} = k\pi$
(b) $\bar{x} = (2k + 1/2)\pi$
(c) $\ddot{x} = 1/2 \sin 2x$
(d) $x = 2\pi k$

Question 2

(a) $\bar{x} = -2$ (stable); $\bar{x} = 2$ (unstable)
(b)$\bar{x} = 1$ (stable); $\bar{x} = -1$ (unstable)
(c) $\bar{x} = \pm 1$ (stable); $\bar{x} = 0$ (unstable)
(d) $\bar{x} = (2k + 1)\pi$ (stable); $\bar{x} = 2k\pi$ (unstable)
(e) No fixed points
(f) $\bar{x} = (2k + 1/3)\pi$ (stable); $\bar{x} = (2k - 1/3)\pi$ (unstable)

Question 3

$\bar{x} = -1$ (half stable); $\bar{x} = 0$ (stable); $\bar{x} = 1$ (unstable)

Question 4

(a) $\bar{x} = 1$ (stable); $\bar{x} = 0$ (unstable)
(b) $\bar{x} = 1$ (stable); $\bar{x} = 0, 2$ (unstable)
(c) $\bar{x} = k\pi$ (unstable)
(d) $\bar{x} = 6$ (stable); $\bar{x} = 0$ (LSA indeterminate - half stable)
(e) $\bar{x} = 1$ (unstable)

Question 5

Not one-dimensional system

Question 6

(a) $x(t) = x_0 e^{at}$, $y(t) = y_0 e^{-t}$

(b) $y = Ax^{-1/a}$

(d) Along y-axis.

Question 7

(a) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$.

(b) $\lambda_1 = 2$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\lambda_2 = 3$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(c) $\mathbf{x}(t) = C_1 e^{2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(d) Origin is unstable.

(e) $\mathbf{x}(t) = e^{2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Question 8

(a) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$; $\lambda_{1,2} = 1 \pm i$

(b) $\lambda_1 = 1 + i$, $\mathbf{v}_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}$; $\lambda_2 = 1 - i$, $\mathbf{v}_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$.

(c) $\mathbf{x}(t) = e^t \left(C_1 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} + C_2 \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} \right)$.

Question 9

(a) Unstable degenerate node (counter clockwise).

(b) Centre (clockwise).

(c) Saddle node.

(d) Indeterminate.