

$$x^T A x \geq 0 \quad \forall x \in \mathbb{R}^n \quad = \text{Positive semi-definite}$$

1. Say λ_i is an eigenvalue of A , then there exists $v_i \in \mathbb{R}^n$ as A 's eigenvector.

$$\text{s.t.} \quad A v_i = \lambda_i v_i$$

2. Given definition of positive semi-definite.

$$v_i^T A v_i = \lambda_i v_i^T v_i \geq 0$$

$$\because v_i^T v_i \geq 0 \quad \forall v_i \in \mathbb{R}^n$$

$$\therefore \lambda_i \geq 0$$

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