

Q3.2

$$\begin{aligned}\text{Var}(\theta) &= E[(\theta - E[\theta])(\theta - E[\theta])^T] \\ &= E\left[(X^T X)^{-1} X^T Y - E[(X^T X)^{-1} X^T Y] \cdot (X^T X)^{-1} X^T Y - E[(X^T X)^{-1} X^T Y]\right] \\ &= (X^T X)^{-1} X^T \cdot E[(Y - E[Y]) \cdot (Y - E[Y])^T] \cdot (X^T X)^{-1} X^T\end{aligned}$$

Because $Y - E[Y] = X\theta + \varepsilon - E[X\theta + \varepsilon] = \varepsilon$.

$$\begin{aligned}&= (X^T X)^{-1} X^T \cdot E[\varepsilon \cdot \varepsilon^T] \cdot (X^T X)^{-1} X^T \\ &= (X^T X)^{-1} X^T \cdot \sigma^2 \cdot (X^T X)^{-1} X^T \\ &= (X^T X)^{-1} X^T \cdot X^T ((X^T X)^{-1})^T \cdot \sigma^2 \\ &= (X^T X)^{-1} (X^T \cdot X) \cdot (X^T X)^{-1} \cdot \sigma^2 \\ &= (X^T X)^{-1} \cdot \sigma^2\end{aligned}$$