Q 1.9.

① $f(x) = x^3, \quad x \geq 0$

$f'(x) = 3x^2$

$f''(x) = 6x \geq 0.$

$\Rightarrow f(x) = x^3$ is convex for $x \geq 0$

② $f(\lambda x + (1-\lambda) y)$

$= \max_{i \in \{1,2\}} (\lambda x_i + (1-\lambda) y_i)$

$\leq \lambda \max_{i \in \{1,2\}} x_i + (1-\lambda) \max_{i \in \{1,2\}} y_i.$

$= \lambda f(x) + (1-\lambda) f(y)$

$\Rightarrow f(x_1, x_2) = \max(x_1, x_2)$ is convex on $\mathbb{R}^2$

③ Because f and g are univariate, according to derivative rule:

$(f+g)'' = f'' + g''$

Because $f'' \geq 0$, $g'' \geq 0$, on set S

$(f+g)'' \geq 0$

$\Rightarrow f+g$ is convex on Set S.

④ $(fg)'' = (f'g + fg')' = f''g + 2f'g' + fg''$

Becuese, f and g are convex and non-negative on S, $f''g \geq 0$,

Because, f and g share same minimum at x' $fg'' \geq 0$.

1. when $x < x'$, $f' \leq 0$ and $g' \leq 0 \Rightarrow f'g' \geq 0$

2. When $x \geq x'$, $f' \geq 0$ and $g' \geq 0 \Rightarrow f'g' \geq 0$.

So all three terms in $(fg)'' \geq 0 \Rightarrow fg$ is convex on S.