$\theta(1.2)$

$$P(D) = P(D|\theta)\cdot P(\theta)$$

$$= \prod_{i=1}^{m} \theta^{x^{(i)}} (1-\theta)^{1-x^{(i)}} \cdot \theta^{\alpha-1} (1-\theta)^{b-1}$$

$$= \prod_{i=1}^{m} \theta^{(x^{(i)}+\alpha-1)} \cdot (1-\theta)^{(\cancel{1}-x^{(i)}+b-\cancel{1})}$$

$$\log P(D) = LL = \sum_{i=1}^{m} \Big[(x^{(i)}+\alpha-1)\overset{\log}{\theta} + (-x^{(i)}+b)\underset{\log}{(}1-\theta) \Big]$$

$$\cancel{= \sum_{i=1}^{m} \Big[x^{(i)}\theta + \alpha\theta - \theta - x^{(i)} + x^{(i)}\theta + b - b\theta \Big]}$$

$$= \sum_{i=1}^{m} \Big(x^{(i)}\log\theta + \alpha\log\theta - 1\cdot\log\theta + -x^{(i)}\log(1-\theta) + \cancel{b} \, b\log(1-\theta) \Big)$$

$$= \sum_{i=1}^{m} \Big((x^{(i)}+\alpha-1)\log\theta + (-x^{(i)}+b)\log(1-\theta) \Big)$$

$$\frac{\partial LL}{\partial\theta} = 0: \sum_{i=1}^{m} \left(\frac{x^{(i)}+\alpha-1}{\theta} + \frac{(b-x^{(i)})\cdot(-1)}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{m} \left(\frac{(x^{(i)}+\alpha-1)(1-\theta) + (x^{(i)}-b)\theta}{\cancel{\theta(1-\theta)}} \right) = 0$$

$$\sum_{i=1}^{m} \left(x^{(i)} - \cancel{x^{(i)}\theta} + \alpha - \alpha\theta - 1 + \theta + \cancel{x^{(i)}\theta} - b\theta \right) = 0$$

$$\cancel{\sum_{i=1}^{m} x^{(i)} + \alpha - 1} \; \sum_{i=1}^{m} x^{(i)} + m\alpha - m = m(\alpha-1+b)\theta$$

$$\theta_{MAP} = \left(\frac{\sum_{i=1}^{m} x^{(i)}}{m} + \alpha - 1 \right) \Big/ (\alpha+b-1) \quad \#$$

when $\alpha = \beta = 1$ it becomes MLE estimates