

Q1.6.

let λ be a real eigenvalue of A . and v be a correspondingly real eigenvector, so that $Av = \lambda v$

$$\text{Because } v^T A v \geq 0$$

$$v^T \lambda v \geq 0.$$

$$\lambda \|v\|^2 \geq 0.$$

Since $\|v\|^2$ is a positive number, λ must be non-negative.

Q1.7

$$\textcircled{1} A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

$$A^2 + 2AB + B^2 = \begin{bmatrix} 6 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow (A+B)^2 \neq A^2 + 2AB + B^2.$$

$$\textcircled{2} A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = 0 \text{ but } A \neq 0, B \neq 0.$$