

Q(1.2)

$$\begin{aligned}
 P(D) &= P(D|\theta) \cdot P(\theta) \\
 &= \prod_{i=1}^m \theta^{x^{(i)}} (1-\theta)^{1-x^{(i)}} \cdot \theta^{a-1} (1-\theta)^{b-1} \\
 &= \prod_{i=1}^m \theta^{(x^{(i)} + a - 1)} (1-\theta)^{(1 - x^{(i)} + b - 1)}
 \end{aligned}$$

$$\begin{aligned}
 \log P(D) = LL &= \sum_{i=1}^m \left[(x^{(i)} + a - 1) \log \theta + (1 - x^{(i)} + b - 1) \log(1-\theta) \right] \\
 &= \sum_{i=1}^m \left[x^{(i)} \log \theta + a \log \theta - 1 \cdot \log \theta - x^{(i)} \log(1-\theta) + b \log(1-\theta) - 1 \cdot \log(1-\theta) \right]
 \end{aligned}$$

$$= \sum_{i=1}^m \left(x^{(i)} \log \theta + a \log \theta - 1 \cdot \log \theta - x^{(i)} \log(1-\theta) + b \log(1-\theta) - 1 \cdot \log(1-\theta) \right)$$

$$= \sum_{i=1}^m \left((x^{(i)} + a - 1) \log \theta + (1 - x^{(i)} + b - 1) \log(1-\theta) \right)$$

$$\frac{\partial LL}{\partial \theta} = 0 : \sum_{i=1}^m \left(\frac{x^{(i)} + a - 1}{\theta} + \frac{(b - x^{(i)} - 1) \cdot (-1)}{1 - \theta} \right) = 0$$

$$\sum_{i=1}^m \left(\frac{(x^{(i)} + a - 1)(1 - \theta) + (x^{(i)} - b) \theta}{\theta(1 - \theta)} \right) = 0$$

$$\sum_{i=1}^m \left(\cancel{x^{(i)}} - \cancel{x^{(i)}} \theta + \cancel{a} - \cancel{a} \theta - 1 + \theta + \cancel{x^{(i)}} \theta - \cancel{b} \theta \right) = 0$$

$$\sum_{i=1}^m \cancel{x^{(i)}} + m \cancel{a} - m = m(a - 1 + b) \theta$$

$$\theta_{\text{MAP}} = \left(\frac{\sum_{i=1}^m x^{(i)}}{m} + a - 1 \right) / (a + b - 1) \quad \# \quad \begin{array}{l} \text{when } a=b=1 \\ \text{it becomes MLE} \\ \text{estimates} \end{array}$$