$X \sim \text{Poisson}(\lambda_1)$
$Y \sim \text{Poisson}(\lambda_2)$, claim $X+Y \sim \text{Poisson}(\lambda_1+\lambda_2)$

(pf).

$$\cancel{P(X+Y)} \quad P(Z) = P(Z=z) \leftarrow Z = X+Y$$

$$= \sum_{i=0}^{z} P(X=i \wedge Y=z-i)$$

$$= \sum_{i=0}^{z} P(X=i)\cdot P(Y=z-i) \quad \because \text{iid.}$$

$$= \sum_{i=0}^{z} \frac{e^{-\lambda_1}\lambda_1^{i}}{i!} \cdot \frac{e^{-\lambda_2}\lambda_2^{(z-i)}}{(z-i)!}$$

$$= \sum_{i=0}^{z} \frac{1}{i!(z-i)!} \cdot e^{-\lambda_1}\lambda_1^{i} \cdot e^{-\lambda_2} \cdot \lambda_2^{z-i}$$

$$= \sum_{i=0}^{z} \frac{z!}{i!(z-i)!} \frac{e^{-\lambda_1}\lambda_1^{i} e^{-\lambda_2} \cdot \lambda_2^{z-i}}{z!}$$

$$= \boxed{\sum_{i=0}^{z} \binom{z}{i} \lambda_1^{i} \cdot \lambda_2^{z-i}} \; e^{-(\lambda_1+\lambda_2)} \cdot \frac{1}{z!}$$

$$= \frac{e^{-(\lambda_1+\lambda_2)} \cdot (\lambda_1+\lambda_2)^{z}}{z!}$$

let $\lambda_1+\lambda_2 = \lambda$
$$= \frac{e^{-\lambda} \cdot \lambda^{z}}{z!}$$

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Ref: MIT OCW RES 6-012 Intro to Probability, Spring 2018