Q1.2 x, y are two independent variable.

To prove $x+y$ is also a poisson random variable.

$$Pr(x+y=k) = \sum_{i=0}^{k} Pr(x+y=k,\ x=i)$$

Because x and y are independent poisson variable:

$$Pr(x+y=k) = \sum_{i=0}^{k} Pr(y=k-i)\, Pr(x=i)$$

$$= \sum_{i=0}^{k} \frac{e^{-\lambda y} \cdot \lambda y^{k-i}}{(k-i)!} \cdot \frac{e^{-\lambda x} \cdot \lambda x^{i}}{i!}$$

$$= e^{-\lambda y} \cdot e^{-\lambda x} \sum_{i=0}^{k} \frac{\lambda_y^{k-i}}{(k-i)!} \cdot \frac{\lambda_x^{i}}{i!}$$

$$= e^{-\lambda y} \cdot e^{-\lambda x} \sum_{i=0}^{k} \frac{\lambda_y^{k-1}}{(k-1)!} \cdot \frac{\lambda x^{i}}{i!}$$

$$= \frac{e^{-(\lambda x+\lambda y)}}{k!} \sum_{i=0}^{k} \frac{k!}{i!\,(k-i)!} \lambda_y^{k-i} \lambda_x^{i}$$

According to binomial theorem:

$$\sum_{i=0}^{k} \frac{k!}{i!\,(k-i)!} \cdot \lambda_y^{k-i} \cdot \lambda_x^{i} = (\lambda_x + \lambda_y)^k$$

$$Pr(x+y=k) = \frac{e^{-(\lambda x+\lambda y)}}{k!} \cdot (\lambda_x+\lambda_y)^k$$

So that $x+y$ is a possion Variable.