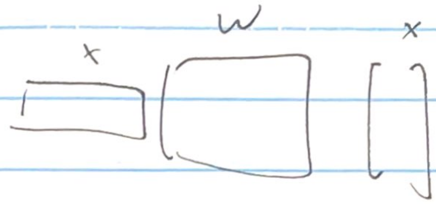
$$J(\theta) = \frac{1}{2} \sum_{i=1}^{m} w^{(i)} \left(\theta^T x^{(i)} - y^{(i)}\right)^2$$



$$\cancel{A^T W A = A^T \sum_{i,k} W_{ij} A_{jk}}$$

$\because$ $\cancel{A^T W}$ $X^T W X = X^T \left(\sum_j W_{ij} X_j \right)$

$$= \sum_i X_i \sum_j W_{ij} X_j = \sum_{ij} W_{ij} X_i X_j$$

$\therefore$ $(X\theta - y)^T W (X\theta - y) = \sum_{ij} W_{ij} (X\theta - y)_i (X\theta - y)_j$

Assume $i = j$. $\rightarrow$ $= \sum_i W_{ii} (X\theta - y)_i^2$

$$= \sum_i W_{ii} (x^i \theta - y^i)^2$$

By setting $W_{ii} = \frac{1}{2} w^{(i)}$, then we arrive at $J(\theta)$.

W: $m \times m$ diagonal matrix, $W_{ii} = \frac{1}{2} w^{(i)}$

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