Q2.1

$$P(y|x) = \frac{1}{1+e^{-\theta^T x}} \quad \#$$

Q2.2

$$P(y=1|x) = \frac{P(x|y=1)\,P(y=1)}{P(x|y=1)\,P(y=1) + P(x|y=0)\,P(y=0)} \quad (\text{Bayes})$$

$$P(y) = \gamma^{(y^{(i)})}(1-\gamma)^{(1-y^{(i)})} \Rightarrow P(y=0) = 1-\gamma;\ P(y=1) = \gamma.$$

$$P(x_i|y=0) = N(\mu_j^0, \sigma_j^2), \quad P(x_i|y=1) = N(\mu_j^1, \sigma_j^2)$$

$$= \frac{1}{1 + \frac{P(x|y=0)\,P(y=0)}{P(x|y=1)\,P(y=1)}}$$

$$= \frac{1}{1 + e^{\ln \frac{P(x|y=0)P(y=0)}{P(x|y=1)P(y=1)}}}$$

$$= \frac{1}{1 + e^{\left(\ln \frac{P(y=0)}{P(y=1)} + \ln \frac{P(x|y=0)}{P(x|y=1)}\right)}}$$

$$= \frac{1}{1 + e^{\left(\ln \frac{1-\gamma}{\gamma} + \ln \frac{P(x|y=0)}{P(x|y=1)}\right)}} \quad \ldots \ (A)$$

∵ Assume conditional independance (Naive Baye's)

$$\ln \frac{P(x|y=0)}{P(x|y=1)} = \cancel{\ln(\prod_i P(x_i|y=0))},$$

$$= \ln \frac{\prod_{i=1}^{m} P(x_i|y=0)}{\prod_{i=1}^{m} P(x_i|y=1)}$$

$$= \sum_{i=1}^{m} \ln \frac{P(x_i|y=0)}{P(x_i|y=1)}$$

$$= \sum_{i=1}^{m} \ln \left[\frac{\frac{1}{\cancel{\sqrt{2\pi\sigma_j^2}}} e^{\left(\frac{-(x_i-\mu_j^0)^2}{2\sigma_j^2}\right)}}{\frac{1}{\cancel{\sqrt{2\pi\sigma_j^2}}} e^{\frac{-(x_i-\mu_j^1)^2}{2\sigma_j^2}}} \right]$$

$$= \sum_{i=1}^{m} \ln e^{\frac{-(x_i-\mu_j^0)^2+(x_i-\mu_j^1)^2}{2\sigma_j^2}}$$

$$= \sum_{i=1}^{m} \frac{(x_i-\mu_j^1)^2-(x_i-\mu_j^0)^2}{2\sigma_j^2}$$

$$= \sum_{i=1}^{m} \frac{\cancel{x_i^2}+\mu_j^{1\,2}-2x_i\mu_j^1-\cancel{x_i^2}-\mu_j^{0\,2}+2x_i\mu_j^0}{2\sigma_j^2}$$

$$= \sum_{i=1}^{m} \left[\frac{(\cancel{2}\mu_j^0-\cancel{2}\mu_j^1)x_i}{\cancel{2}\sigma_j^2} + \frac{(\mu_j^{1\,2}-\mu_j^{0\,2})}{2\sigma_j^2} \right] \quad \cdots (B)$$

∴ Plugging (B) back to (A)

$$P(y=1|x) = \frac{1}{1 + e^{\ln\frac{1-\gamma}{\gamma} + \left[\sum_{i=1}^{m}\left(\frac{\mu_j^0 - \mu_j^1}{\sigma_j^2} x_i + \frac{\mu_j^{1^2} - \mu_j^{0^2}}{2\sigma_j^2}\right)\right]}}$$ #

Q2.3

$\Rightarrow$ Define
$$\begin{cases} w_0 \equiv \ln\frac{1-\gamma}{\gamma} + \sum_{i=1}^{m}\frac{\mu_j^{1^2} - \mu_j^{0^2}}{2\sigma_j^2} \\ w_i = \frac{\mu_j^0 - \mu_j^1}{\sigma_j^2} \end{cases}$$

$$\Rightarrow \frac{1}{1 + e^{(w_0 + \sum_{i=1}^{m} w_i x^{(i)})}}$$ #

$$P(y=0|x) = \frac{e^{(w_0 + \sum_{i=1}^{m} w_i x_i)}}{1 + e^{(w_0 + \sum_{i=1}^{m} w_i x_i)}}$$ #