$k_1(x, x')$, $k_2(x, x')$ are valid kernels.

Q5.1

$k(x,x') = c\, k_1(x, x')$ is valid. $(c > 0)$

$\because$ $k_1(x,x')$ is valid

$\therefore$ $k_1(x, x') = \phi(x)^T \phi(x')$

$\rightarrow$ $k(x, x') = (\sqrt{c}\,\phi(x))^T (\sqrt{c}\,\phi(x'))$ can be expressed as inner product #

Q5.3

$k(x,x') = k_1(x, x') + k_2(x, x')$ ← $\boxed{\text{valid kernel}}$ #

$= \phi_1^T(x)\, \phi_1(x') + \phi_2^T(x)\, \phi_2(x')$

$\left[\phi_1^T\right]\left[\phi_1\right]$

$= \cancel{\langle \alpha \Phi_1(x) + \beta \phi_2(x),\ \alpha \phi_1(x') + \beta \phi_2(x') \rangle}$

$\left[\phi_1^T\ \phi_2^T\right]\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix}$

$\Rightarrow$ Define $\phi_3 = [\ \phi_1(x),\ \phi_2(x)\ ]$, i.e. concatenate $\phi^1_{(i)}$ & $\phi^2_{(i)}$ $\forall$ column i

$= \phi_3^T(x)\, \phi_3(x')$ #