MATRICES

1. If the matrix A is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is

$$A = \begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

a) 0
b) 5
c) 10
d) 25

2. Given that $A^{-2} = \frac{1}{7} \cdot \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is.

a) $A = 7 \cdot \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$
b) $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$
c) $A = \frac{1}{7} \cdot \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$
d) $A = \frac{1}{49} \cdot \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

3. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then find the value of $I - A + A^{-2} - A^{-3} + \ldots$.

a) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$
b) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$
c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then find the value of $|A(\text{adj}.A)|$.

a) 1001
b) 101

c) 10

d) 1000

5. Given that $\begin{bmatrix} 1 & x \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, the value of x is:

a) -4

b) -2

c) 2

d) 4

6. If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

(a) $2x + y - 3z = 13$

(b) $3x + 2y + z = 4$

(c) $x + 2y - z = 8$

COORDINATE GEOMETRY

1. The line $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to

2. Solve

a) Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

b) If the lines $\frac{x}{2} = \frac{2y-6}{4} = \frac{z-6}{-7}$ and $\frac{x-1}{3k} = \frac{y-1}{3k} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$.

DIFFERENTIATION

1. Derivative of e^{2x} with respect to e^{x} is

a) e^{x}

b) $2e^{x}$

c) $2e^{2x}$

d) $2e^{3x}$

2. Determine the value of k for which the following function is continuous at $x = 0$:

$$f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

a) 0
b) $\frac{1}{4}$
c) 1
d) 4

3. The general solution of the differential equation $x\,dy + y\,dx = 0$ is

a) $xy = c$
b) $x + y = c$
c) $x^2 + y^2 = e^2$
d) $\log y = \log x + c$

4. Solve

a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$
b) If $y = e^{x-y}$, prove that $\frac{dy}{dt} = \frac{\log x}{(1+\log x)^2}$

5. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

6. The volume of a cube is increasing at the rate of $6\,\text{cm}^3/\text{s}$. How fast is the surface area increasing when the length of an edge is 8 cm?

7. Given that $y = (\sin x)^x \cdot x^{\sin x} + a^x$, find $\frac{dy}{dx}$.

8. Solve

a) Find the particular solution of the differential equation $\frac{dy}{dx} = y\cot(2x)$, given that $y\left(\frac{\pi}{4}\right) = 2$.
b) Find the particular solution of the differential equation $(xe^x + y)\,dx = x\,dy$ given that $y = 1$ when $x = 1$.

TRIGONOMETRY

Solve

a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$, in its simplest form.

OR

b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

INTEGRATION

1. The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ is:

a) $\frac{\pi}{6}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{2}$
d) $\frac{\pi}{18}$

2. The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx} = y \quad (y > 0)$ is:

a) $\frac{1}{x}$
b) x
c) y
d) $\frac{1}{y}$

3. Find: $\int \frac{1}{x(x^2-1)}\, dx$

4. Solve

a) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x+\sin 2x}$
or
b) Find: $\int e^x \left[\frac{1}{(1+x^2)^{5/2}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

5. Find: $\int \frac{3x+5}{\sqrt{x^2+2x+4}}\, dx$

6. Solve

a) Sketch the graph of $y = x\,|x|$ and hence find the area bounded by this curve, X-axis, and the ordinates $x = -2$ and $x = 2$, using integration
or
b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$ and the lines $x = -2$ and $x = 2$

RELATIONS

1. Assertion (A) : The relation R = (x,y) : (x+y) is a prime number and x,y $x \in A$ N is not a reflexive relation.
Relation (R) : The number '2n' is composite for all natural numbers n.

PROBABILITY

1. **The probability distribution of a random variable X is:**

X	0	1	2	3	4
P(X)	0.1	k		$2k$	0.1

where k is some unknown constant. The probability that the random variable X takes the value 2 is:

   a) $\frac{1}{5}$
   b) $\frac{2}{5}$
   c) $\frac{4}{5}$
   d) 1

2. **Solve:**

   a) A card from a well-shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

   OR

   b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

3. Rohit, Jaspreet, and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$, and Alia's selection is $\frac{1}{4}$. The events of selection are independent of each other.

   Based on the above information, answer the following questions:

   i) What is the probability that at least one of them is selected?
   ii) Find $P(G \mid \bar{H})$ where G is the event of Jaspreet's selection and $\bar{H}$ denotes the event that Rohit is not selected.
   iii) i) Find the probability that exactly one of them is selected. or
   ii) Find the probability that exactly two of them are selected.

ALGEBRA

1. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 1$, $\left|\vec{b}\right| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is:

   a) $\frac{\pi}{6}$
   b) $\frac{\pi}{3}$
   c) $\frac{5\pi}{6}$

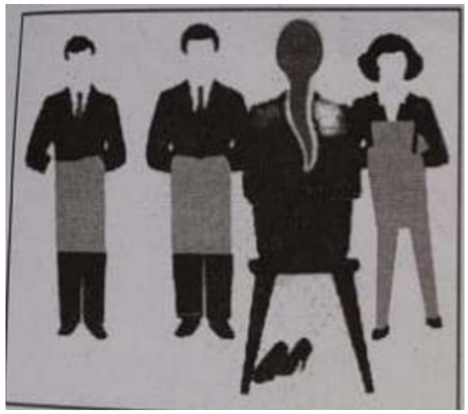

Figure 1: Enter Caption

d) $\frac{11\pi}{6}$

2. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$, and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represent the sides of:

a) an equilateral triangle
b) an obtuse-angled triangle
c) an isosceles triangle
d) a right-angled triangle

3. Let $\vec{a}$ be any vector such that $\left|\vec{a}\right| = a$. The value of

$$\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^2$$

is:

a) a^2
b) $2a^2$
c) $3a^2$
d) 0

4. The vector equation of a line passing through the point (1, -1, 0) and parallel to the Y-axis is:

a) $\vec{r} = \vec{i} - \vec{j} + \lambda(\hat{i} - \hat{j})$
b) $\vec{r} = \vec{i} - \vec{j} + \lambda\hat{j}$

c) $\vec{r} = \vec{i} - \vec{j} + \lambda\hat{k}$

d) $\lambda\hat{j}$

5. An instructor at the astronomical center shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O(0,0,0)$ and the three stars have their locations at the points D, A, and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}$, $7\hat{i}+5\hat{j}+8\hat{k}$, and $-3\hat{i}+7\hat{j}+11\hat{k}$, respectively.

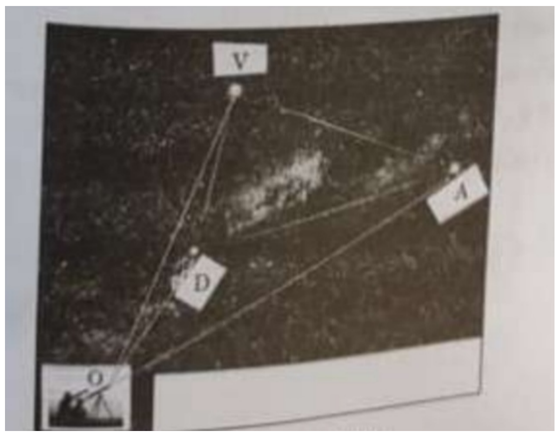

Figure 2: (a) Example of star positions.

FUNCTIONS

1. The function $f(x) = kx - \sin x$ is strictly increasing for

A) $k > 1$

B) $k < 1$

C) $k > -1$

D) $k < -1$

2. Solve

a) Let A = R- {5} and B = R- {1} . Consider the function f :$A \rightarrow B$, defined by $f(x) = \frac{x-3}{x-5}$.Show that f is one-one and onto.

OR

b) Check whether the relation S in the set of real numbers R defined by S=(a,b) : where $a - b + sqrt2$ is an irrational number is reflexive,symmetric or transitive.

3. A store has been selling calculators at ¤350 each. A market survey indicates that a reduction in price p of the calculator increases the number of units x sold. The relation between the price and quantity sold is given by the demand function:

$$p = 450 - \frac{1}{2}x$$

Based on the above information, answer the following questions:

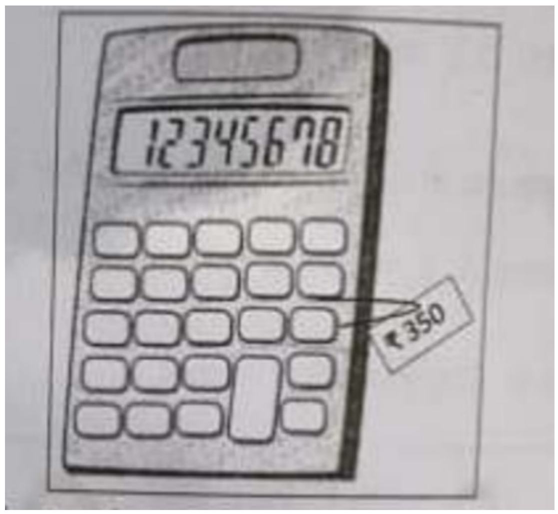

Figure 3: (a) Example of star positions.

(a) Determine the number of units x that should be sold to maximize the revenue $R(x) = x \cdot p(x)$. Also, verify the result.

(b) What rebate in the price of the calculator should the store give to maximize the revenue?

LINEAR PROGRAMMING PROBLEM

1. The maximum value of $Z = 4x + y$ for a linear programming problem (L.P.P) whose feasible region is given below is:

   a) 50
   b) 110
   c) 120
   d) 170

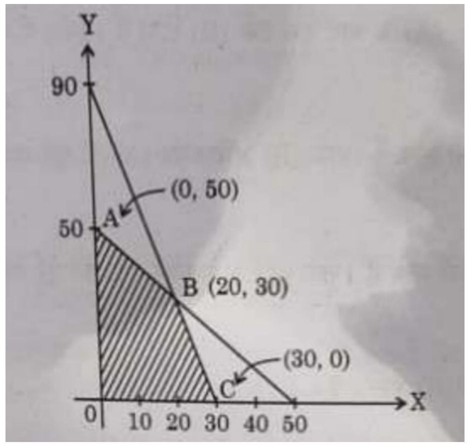

Figure 4: Feasible Region

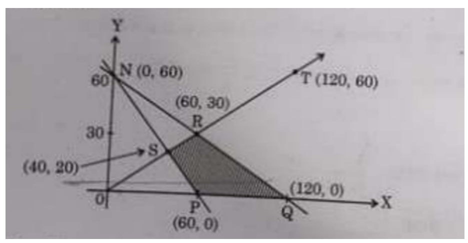

Figure 5: .2

2. Assertion (A): The corner points of the bounded feasible region of an LPP are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.

   Reason (R): The optimal solution of an LPP having a feasible region must occur at corner points.

3. Solve the following linear programming problem graphically:

$$\begin{aligned} \text{Maximize} \quad & Z = 2x + 3y \\ \text{Subject to:} \quad & x + y \leq 6 \\ & x \geq 2 \\ & y \leq 3 \\ & x, y \geq 0 \end{aligned}$$