第4次作业. 快速排序+线性时间排序　郑源泽 19307130077

7.4.3

$f(q) = q^2 + (n-q-1)^2,\ q \in [0, n-1],\ q \in N$

$f'(q) = 4q - 2n + 2$

$f'(q) = 0 \quad f''(q) = 4 > 0$

$q = \frac{1}{2}n - \frac{1}{2}$ ∴ 对于一级导数,恒增,$(0, \frac{1}{2}n - \frac{1}{2})$区间小于0,$(\frac{1}{2}n - \frac{1}{2}, n-1)$区间大于0

∴ $f(q)$ 在 $(0, \frac{1}{2}n - \frac{1}{2}) \searrow$,在 $(\frac{1}{2}n - \frac{1}{2}, n-1) \nearrow$

$f(0) = (n-1)^2$ ∴原命题得证

$f(n-1) = (n-1)^2$

7.4.4

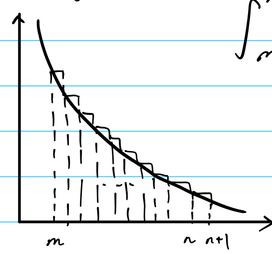
证明随机化快排期望运行时间为 $\Omega(n\lg n)$

由引理7.1,求第四行比较次数 X

$$E(X) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} Pr\{z_i 与 z_j 进行比较\}$$

$$= \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{2}{j-i+1} \quad k = j-i \text{ 变换}$$

$$= \sum_{i=1}^{n-1}\sum_{k=1}^{n-i} \frac{2}{k+1} \quad \text{对于函数 } \frac{1}{x+1}:$$



$$\int_m^{n+1} \frac{1}{x+1}\,dx \le \sum_{x=m}^{n} \frac{1}{x+1}\,dx$$

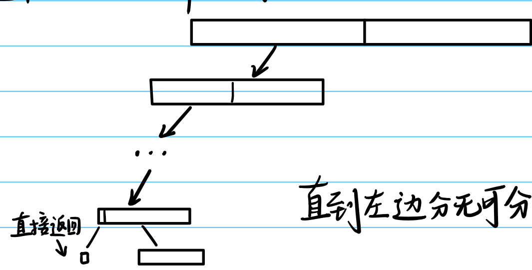
$$\therefore E(X) \geq \sum_{i=1}^{n-1} 2\int_1^{n-i} \frac{1}{k+1} = \sum_{i=1}^{n-1} 2(\ln(n-i+2) - O(1)) \geq 2\int_1^{n} \ln(n-i+2)\,di - O(n)$$

$$= \int_2^{n+1} \ln(x)\,dx - O(n) = 2n\ln(n+1) - O(n) = \Omega(n\lg n)$$

7-4

对于这个尾递归的算法,每次进入循环,首先分割两个子数组,而后对左边递归调用

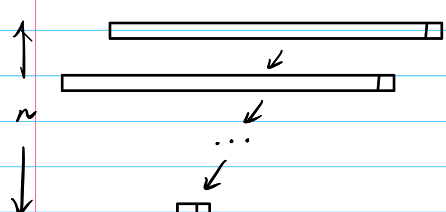
a.



直到左边分无可分,直接返回,进入同一递归while的下一次循环

也就是说每次都把左子数组排好,再下一层循环处理右子数组

b. 输入已经排好序,PARTITION每次分成0个和n-1个元素的子数组



这样,在左侧子数组 while 循环条件

不满足之前已经积累n个栈

而 while 后右侧直接返回,不会压新的栈

c. 每次选择较小的子数组迭代

```
TAIL-RECURSIVE-QUICKSORT(A, p, r)
   while p < r
        q = PARTITION(A,p,r)
        if q < ⌊(r-p)/2⌋ Then
            TAIL-RECURSIVE-QUICKSORT(A, p, q-1)
             P=q+1
        Else TAIL-RECURSIVE-QUICKSORT(A, q+1, r)
             R=q-1
```

8.1.4 每个子数组排列可能性: $k!$

共 n/k 个子数组

故最终结果可能有 $(k!)^{\frac{n}{k}}$ 种

$$(k!)^{\frac{n}{k}} \leq 2^h$$

$$h \geq \frac{n}{k}\lg(k!)$$

$$\lg(k!) \geq \lg\left(\frac{k}{e}\right)^k = k\lg k - k\lg e = k\frac{\ln k}{\ln 2} - k\frac{1}{\ln 2}$$

$$\therefore h \geq \frac{n}{k}\cdot k\left(\frac{\ln k - 1}{\ln 2}\right) = \frac{n\ln k - n}{\ln 2} = \Omega(n\lg k)$$