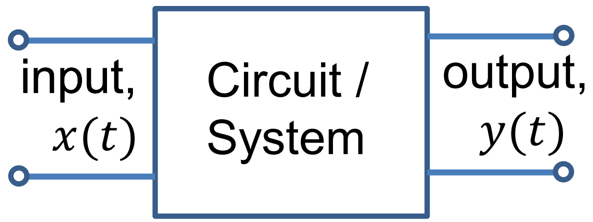
Systems View of Circuits

It is common to study circuits in a generic sense as a system with an input and an output.

- Input – voltage (or current) source
- Output – voltage (or current) on some circuit element



Often we don't concern ourselves with whether or not the inputs/outputs are currents/voltages. Rather we just view them as generic signals.

The circuit/system is described by an equation which relates the input/output. In our case (circuits), this relationship has been described by a differential equation maybe something like

$$b_2 \frac{d^2y}{dt^2} + b_1 \frac{dy}{dt} + b_0 y(t) = a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x(t)$$

The solution to such differential equations can be decomposed into two parts in a few different ways.

Solutions to Diff. Eqs. in Circuits

$$b_2 \frac{d^2 y}{dt^2} + b_1 \frac{dy}{dt} + b_0 y(t) = a_2 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x(t)$$

From a mathematical standpoint, our diff. eq. has a homogeneous solution and a particular solution.

Homogeneous solution

- First order equations
  - decaying exponentials
- Second order equations
  - decaying exponentials, or
  - decaying sinusoids
- Can be found from the roots of the characteristic equation
- Homogeneous solutions will have arbitrary multiplicative constants

Particular solution

- DC Inputs
  - Part. Soln. = constant
  - Can be found easily using method of undetermined coefficients
- AC Inputs
  - Part. Soln. = sinusoid
  - Rather difficult to find using method of undetermined coefficients
  - Can be found using phasors analysis

Solutions to Diff. Eqs. in Circuits

$$b_2 \frac{d^2y}{dt^2} + b_1 \frac{dy}{dt} + b_0 y(t) = a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x(t)$$

The various components to the solution of the diff. eq. can also be characterized as transient or steady state.

Transient solution

- Dies off as $t \to \infty$
- In the context of typical circuits, may last only msec or less
- Maybe irrelevant or of primary importance depending on what the circuit is used for

Steady state solution

- Persists as $t \to \infty$
- Is a result of the input, $x(t)$
- For DC inputs, can be found by
  - C→ open
  - L→ short
- For AC inputs, can be found by phasor analysis.

Solutions to Diff. Eqs. in Circuits

$$b_2 \frac{d^2y}{dt^2} + b_1 \frac{dy}{dt} + b_0 y(t) = a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x(t)$$

From a physical standpoint, the solution to the diff. eq. can be split into a natural response and a forced response.

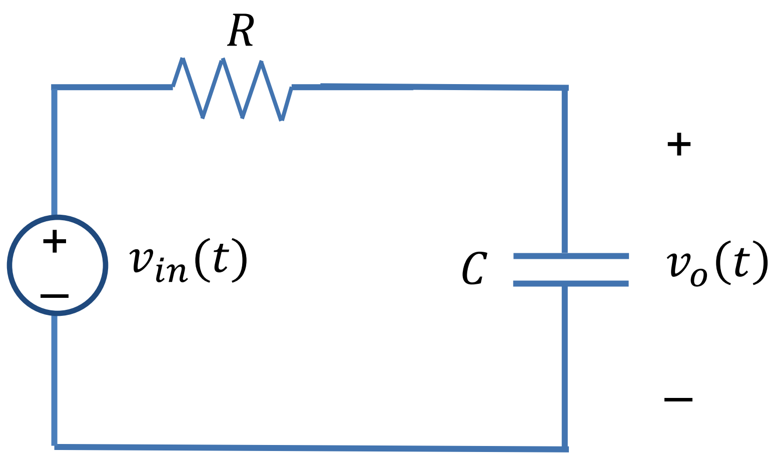
Natural response

- A result of the initial energy stored in the system releasing itself
- In our context (circuits), due to non-zero initial conditions
  - Voltages on capacitors,
  - Currents in inductors,
- Transient in nature

Forced response

- Due to excitation of the circuit/system by the input
- In our context (circuits), due to voltage/current sources
- Generally will have both a transient part and a steady state part

Example: First Order RC Circuit

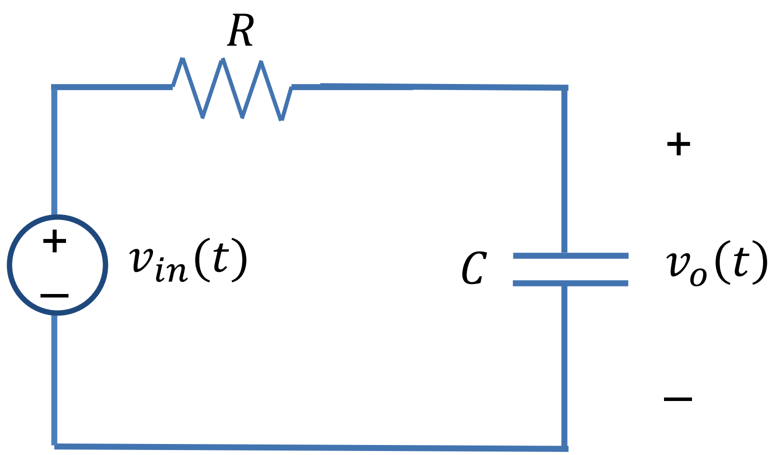


This circuit is described by the differential equation:

$$RC\frac{dv_o}{dt} + v_o(t) = v_{in}(t)$$

- Homogeneous: $v_h(t) = Ae^{-t/RC}$ ← Transient
- Particular:
  - DC: $v_p(t) = v_{in}$
  - AC: $v_p(t) = \frac{|v_{in}|}{\sqrt{1+(\omega RC)^2}}\cos(\omega t + \angle v_{in} - \tan^{-1}(\omega RC))$ ← Steady State

Example: First Order RC Circuit



This circuit is described by the differential equation:

$$RC\frac{dv_o}{dt} + v_o(t) = v_{in}(t)$$

- Natural Response: $(v_{in}(t) = 0,\ v_o(0) \neq 0)$

$$v_o(t) = v_o(0)e^{-t/RC}$$

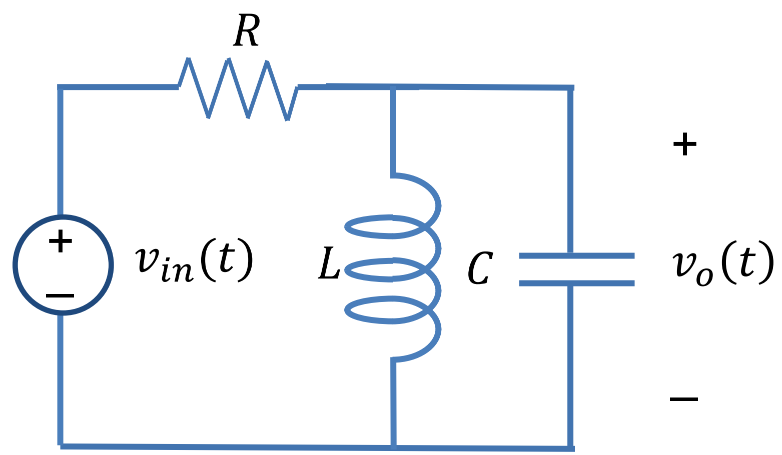
← Transient

- Forced Response: $(v_{in}(t) \neq 0,\ v_o(0) = 0)$
  - DC: $v_p(t) = v_{in}(1 - e^{-t/RC})$
  - AC: $v_p(t) = \frac{|v_{in}|}{\sqrt{1+(\omega RC)^2}}\cos(\omega t + \angle v_{in} - \tan^{-1}(\omega RC))$

$$-\frac{|v_{in}|}{\sqrt{1+(\omega RC)^2}}\cos(\angle v_{in} - \tan^{-1}(\omega RC))\, e^{-t/RC}$$

← Part transient, part steady state

Example:



For the RLC circuit shown, suppose $R = 2k\Omega$, $L = 8$H, $C = 500$nF.

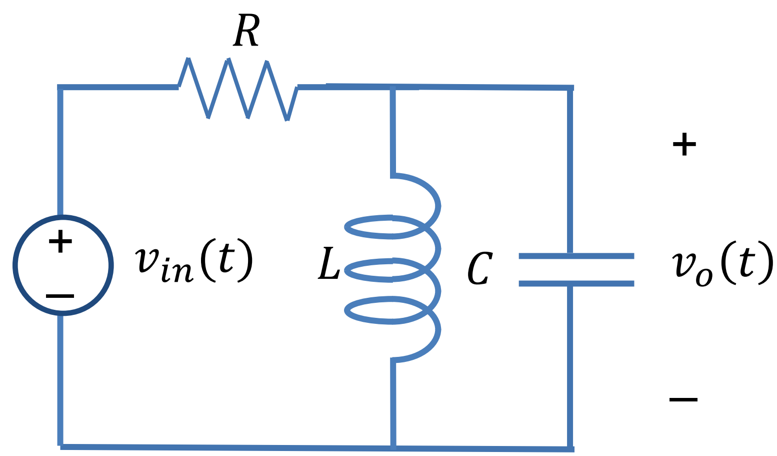
(a) Find the forced response of the circuit if $v_{in} = 6$V (assume no energy in inductor or capacitor at $t = 0$).

(b) Find the natural response of the circuit due to initial current in the inductor (top to bottom) of 25mA at $t = 0$ (assume no input voltage and no energy in the capacitor at $t = 0$).

(c) Find the natural response of the circuit due to initial voltage on the capacitor (top to bottom) of 3V at $t = 0$ (assume no input voltage and no energy in the inductor at $t = 0$).

(d) Find the complete response of the circuit due to $v_{in} = 6$V, $v_C(0) = 3$V, $i_L(0)$ =25mA.

Example:



For the RLC circuit shown, suppose $R = 2k\Omega$, $L = 8$H, $C = 500$nF.

(a) Find the forced response of the circuit if $v_{in} = 6\cos(500t)$V (assume no energy in inductor or capacitor at $t = 0$).

(b) Find the natural response of the circuit due to initial current in the inductor (top to bottom) of 25mA at $t = 0$ (assume no input voltage and no energy in the capacitor at $t = 0$).

(c) Find the natural response of the circuit due to initial voltage on the capacitor (top to bottom) of 3V at $t = 0$ (assume no input voltage and no energy in the inductor at $t = 0$).

(d) Find the complete response of the circuit due to $v_{in} = 6\cos(500t)$ V, $v_C(0) = 3$V, $i_L(0) =$25mA.