Lab 9: Sinusoidal Steady State Response of a 2nd Order Circuit

Theory and Introduction

Goals for Lab 9 – The goal of this lab is to familiarize students with the response of circuits to sinusoidal (AC) inputs. Students will connect the analysis techniques learned in class using complex phasors to real world sinusoidal signals.

Theory

As we continue to study 2nd order circuits, we will once again use the Sallen-Key circuit from the previous lab which is reproduced in Figure 9.1. However, for this lab, we will use an alternating current (AC) input to the circuit. That is, we will apply a sinusoidal voltage to the circuit and observe the response of the circuit to such inputs. Furthermore, in the last lab we focused on the transient response of the circuit to a step in the applied voltage. This time we will focus on the (sinusoidal) steady-state response.

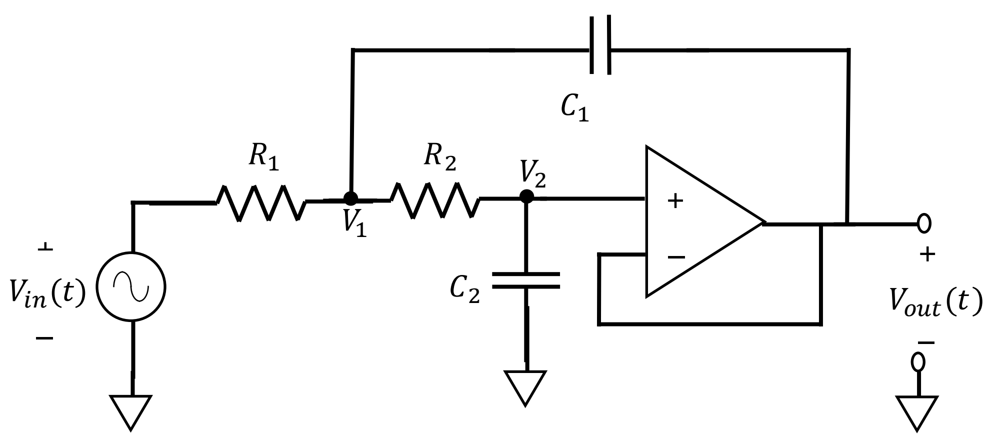

Figure 9.1 – A Sallen-Key 2nd Order Circuit

In class, you learned how to use phasors to find the steady state response of circuits with AC inputs. We will do the same with our Sallen-Key circuit. The phasor representation of the circuit is shown in Figure 9.2. Noting that $V_2 = V_{out}$ and performing KCLs at nodes 1 and 2 leads to the following equations:

$$\frac{V_{in} - V_1}{R_1} + \frac{V_{out} - V_1}{R_2} + \frac{V_{out} - V_1}{1/j\omega C_1} = 0,$$

$$\frac{V_1 - V_{out}}{R_2} = \frac{V_{out}}{1/j\omega C_2}.$$

Solving for V_1 in the second equation and substituting into the first (after a little algebraic manipulation) results in the following relationship between the input and output phasors

$$V_{in}(\omega) = (1 + j\omega(R_1 + R_2)C_2 - \omega^2 R_1 C_1 R_2 C_2)V_{out}(\omega).$$

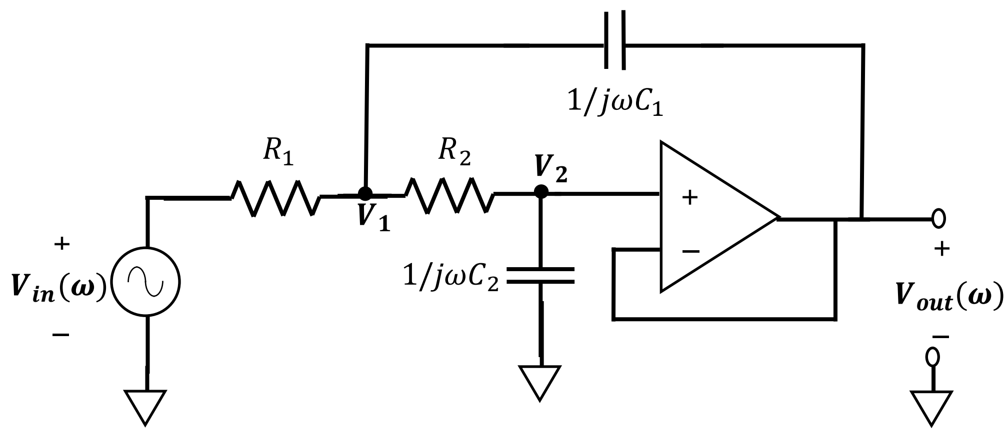

Figure 9.2 – The Sallen-Key Circuit in the Phasor Domain

It is common to describe the sinusoidal response of a circuit in terms of a quantity known as its *transfer function*, $H(\omega)$, which is a function of frequency defined simply as the ratio of the output and input phasors,

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}.$$

Given this definition and the relationship above, our Sallen-Key circuit has a transfer function given by

$$H(\omega) = \frac{1}{1 + j\omega(R_1 + R_2)C_2 - \omega^2 R_1 C_1 R_2 C_2}.$$

Two related quantities which we will measure in this lab are the *magnitude response* of the circuit which is just the absolute value of the transfer function, $|H(\omega)|$, and the *phase response* which is the angle of the transfer function, $\angle H(\omega)$. The magnitude and phase responses of the Sallen-Key circuit are found to be

$$
\begin{aligned}
|H(\omega)| &= \left|\frac{1}{1+j\omega(R_1+R_2)C_2-\omega^2 R_1C_1R_2C_2}\right| \\
&= \frac{|1|}{|1+j\omega(R_1+R_2)C_2-\omega^2 R_1C_1R_2C_2|} \\
&= \frac{1}{\sqrt{(1-\omega^2R_1C_1R_2C_2)^2+(\omega(R_1+R_2)C_2)^2}},
\end{aligned}
$$

$$
\begin{aligned}
\angle H(\omega) &= \angle\left(\frac{1}{1+j\omega(R_1+R_2)C_2-\omega^2 R_1C_1R_2C_2}\right) \\
&= \angle 1 - \angle(1+j\omega(R_1+R_2)C_2-\omega^2R_1C_1R_2C_2) \\
&= 0-\tan^{-1}\left(\frac{\omega(R_1+R_2)C_2}{1-\omega^2R_1C_1R_2C_2}\right) \\
&= -\tan^{-1}\left(\frac{\omega(R_1+R_2)C_2}{1-\omega^2R_1C_1R_2C_2}\right).
\end{aligned}
$$

If you have trouble performing these complex manipulations, please contact your TA or instructor for help. A typical plot of the magnitude and phase responses are shown in Figure 9.3. The frequency variable ω is expressed in units of radians/second. Note that in the plots in Figure 9.3 the frequency axis has been scaled by a factor of 2π so that frequency can be expressed in Hz.

Look at the expression for the magnitude response of this circuit (or look at the plot in Figure 9.3). You should be able to convince yourself that the following statements are true:

$$|H(0)| = 1 \quad \text{and} \quad |H(\infty)| = 0.$$

What this means is that for low frequencies ($\omega \to 0$) the output signal will have the same magnitude as the input signal, while for high frequencies ($\omega \to \infty$) the output signal will have a magnitude of zero. In other words, this circuit acts as a filter that allows low frequencies to pass and attenuates high frequencies. We say that the circuit is a *low-pass filter*. You will demonstrate in the prelab that by changing some of the components around, the circuit can also function as a high-pass filter (i.e., it will pass high frequencies and attenuate low frequencies).

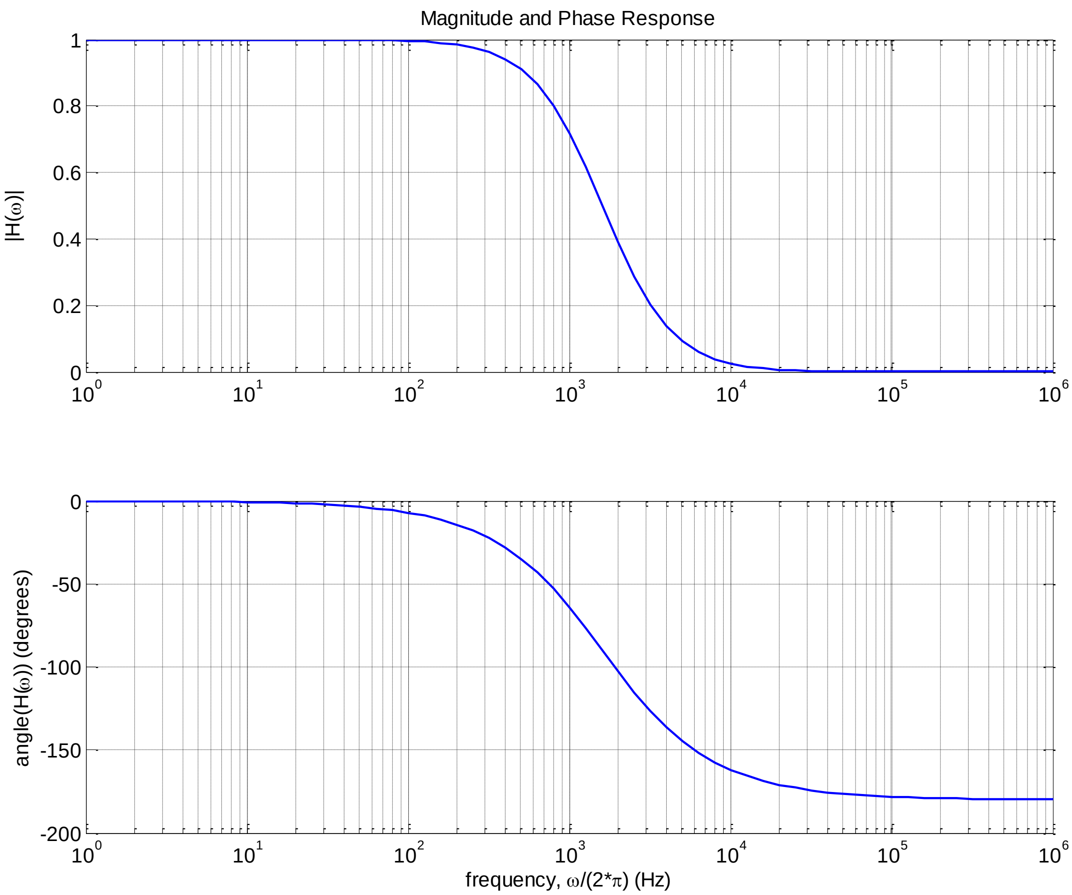

Figure 9.3 – Magnitude and Phase Response of a Typical Sallen-Key Circuit.

Prelab

A. Design a Sallen-Key circuit as shown in Figure 9.1. Choose component values so that the circuit produces an under damped response with a Q-factor of $Q = 3/2$ and a resonant radian frequency of $\omega_o = 1000\pi$ rad/sec ($f_o = 500$Hz). Refer to the previous lab to refresh your memory on these quantities if needed. Be sure to choose component values that are available to you in your lab kit. Also, try to avoid using very small resistors (less than a few hundred Ohms) as that may cause bad behavior from your op amp. **Provide a plot of the magnitude and phase response of your circuit similar to the ones provided in Figure 9.3. In your plot, the frequency should be in a logarithmic scale and should range from 10Hz to 10kHz.**

B. Modify your circuit by exchanging the positions of the resistors and the capacitors as shown in Figure 9.4. Keep the same component values as you used in part A, just move their positions as shown in the Figure. **Develop an equation that provides the relationship between the phasor input and phasor output of the circuit. From this**

relationship develop a formula for the magnitude and phase response of the circuit. Show your derivations. Provide a plot of the magnitude and phase response of your modified circuit.

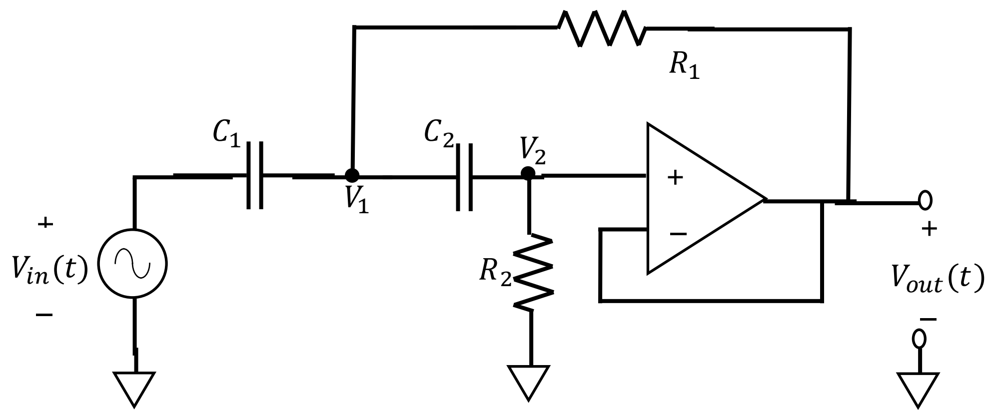

Figure 9.4 – A Modified Sallen-Key 2nd Order Circuit

Procedure

Parts and Equipment needed:

Several resistors

Several (non-polarized) capacitors

1—741 OPAMP

Power Supply (+/- 5V)

Function Generator

Oscilloscope

Task 1 – Sinusoidal Steady-State Response of a 2nd Order Low-Pass Circuit

A. Build the Sallen-Key circuit of Figure 9.1 on your breadboard. Use the component values you designed in Section A of your prelab. Use the function generator to provide a sine wave as the input voltage and display both the input and output voltages on your scope.
    a. For several different frequencies of the input sine wave, measure each of the following quantities:
    - Amplitude of input voltage,
    - Amplitude of output voltage,
    - Phase difference between input and output voltage.

    When measuring amplitudes, you can measure either peak-to-peak voltage or RMS voltage but make sure you are consistent and do the same throughout. To measure phase difference, you can measure the time delay between the two sine waves and then use the known frequency to convert that time difference to a phase difference. You can use radians or degrees to measure phase, whichever you like. You should make these measurements for each of the following frequencies: 10Hz, 18Hz, 32Hz, 56Hz, 100Hz, 178Hz, 316Hz, 562Hz, 1,000Hz, 1,778Hz, 3,162Hz, 5,623Hz, 10kHz. You will find for some frequencies, the output voltage is quite small. In those cases you can increase the amplitude of the input voltage so that the output voltage is a little larger and easier to measure. You can also change the voltage scale on your scope for the channel that has the output so that it appears larger on the screen. This will make it easier to measure relative time delay.

b. Adjust the frequency of the input sine wave until the ratio of the output amplitude to input amplitude is

$$\frac{|v_{out}(t)|}{|v_{in}(t)|} = \frac{1}{\sqrt{2}} = 0.707.$$

This is known as the *cut-off frequency*. Record the frequency at which this condition occurs and also measure the phase difference at this frequency.

Task 2 – Sinusoidal Steady-State Response of a 2nd Order High Pass Circuit

A. Repeat the procedure from Task 1 for the modified circuit of Figure 9.4. Again, use the component values you calculated in the pre-lab.

Before you leave the lab …

Make sure to bring your circuit to your TA and to have the TA sign your data sheet. Your TA may ask you a few questions about how you made various measurements and calculations.

Lab Report Requirements

1. Title Page
2. Procedure – Summarize in your own words what you have done.
3. Data and Results –Include your raw measured data in a table and then provide a plot of the magnitude and phase responses for each of the circuits. In your plots, your frequency axis should be presented on a logarithmic scale (why?) like was done in Figure 9.3. Compare the theoretical responses with the measured responses. Your measured responses should have 14 data points for each plot (the 13 frequencies specified plus the cut-off frequency). Make sure to point out any significant differences you observed and try to explain the most likely causes of those differences.
4. Discussion –
    - Comment on the pass and stop bands of each circuit. That is, what frequencies pass through the circuit relatively unaltered and which frequencies are highly attenuated.
    - Given what you have learned about this circuit, what component values could you change (and how) to adjust the ranges of the pass bands and stop bands?
    - Be sure to point out any changes you could make to the procedure to make your results come out better if you had to do it all over again.