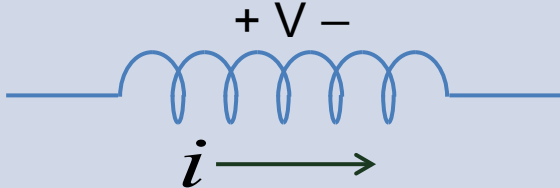
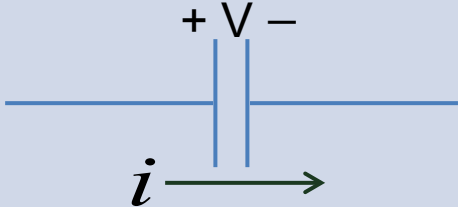


Introduction to Inductors and Capacitors

Inductors and capacitors are passive devices which store (and release) energy (energy is not created/generated by a passive device).

	Inductor	Capacitor
Circuit Symbol		
Fundamental I/V relationship	$v = L \frac{di}{dt}$	$i = C \frac{dv}{dt}$
Units	L=inductance (Henrys)	C=capacitance (Farads)

Properties of Inductors and Capacitors

From the fundamental I/V relationship we can infer several properties of inductors and capacitors.

	Inductor	Capacitor
Fundamental I/V relationship	$v = L \frac{di}{dt}$	$i = C \frac{dv}{dt}$
Instantaneous changes	The current in an inductor cannot change instantaneously. To do so would cause an infinite voltage to appear.	The voltage across a capacitor cannot change instantaneously. To do so would cause an infinite current to flow.
Steady State (when things are not changing with time)	When a circuit with an inductor reaches steady state, the voltage across the inductor will be zero. → An inductor acts like a short circuit in steady state.	When a circuit with a capacitor reaches steady state, the current through the capacitor will be zero. → An capacitor acts like an open circuit in steady state.

Inverse to Fundamental I/V Relationships

From the fundamental I/V relationships we can also derive inverse relationships.

Inductors

$$v = L \frac{di}{dt}$$

$$\int_{t_0}^t v(u) du = L \int_{t_0}^t \frac{di}{du} du$$

$$\int_{t_0}^t v(u) du = L(i(t) - i(t_0))$$

$$i(t) = i(t_0) + \frac{1}{L} \int_{t_0}^t v(u) du$$

Capacitors

$$i = C \frac{dv}{dt}$$

$$\int_{t_0}^t i(u) du = C \int_{t_0}^t \frac{dv}{du} du$$

$$\int_{t_0}^t i(u) du = C(v(t) - v(t_0))$$

$$v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^t i(u) du$$

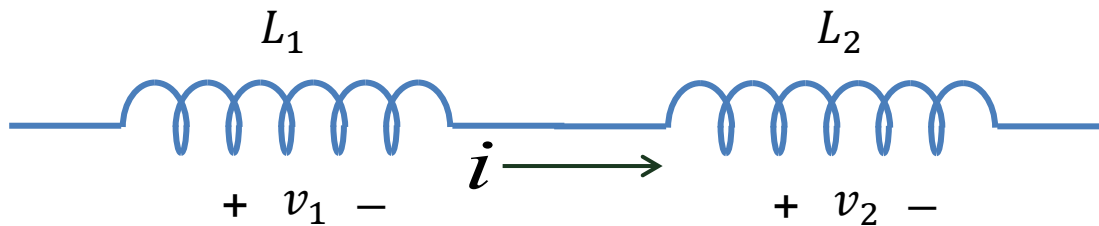
Power/Energy in Inductors and Capacitors

From the fundamental I/V relationship we can infer several properties of inductors and capacitors.

	Inductor	Capacitor
Fundamental I/V relationship	$v = L \frac{di}{dt}$	$i = C \frac{dv}{dt}$
Power	$p(t) = i(t)v(t) = Li(t) \frac{di}{dt}$ $= \frac{L}{2} \frac{d}{dt} (i^2(t))$	$p(t) = i(t)v(t) = Cv(t) \frac{dv}{dt}$ $= \frac{C}{2} \frac{d}{dt} (v^2(t))$
Energy	$p(t) = \frac{dw}{dt}$ $w(t) = \frac{L}{2} i^2(t)$	$p(t) = \frac{dw}{dt}$ $w(t) = \frac{C}{2} v^2(t)$

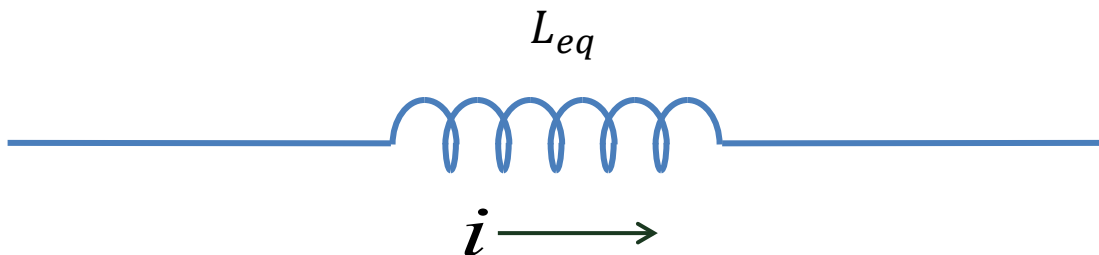
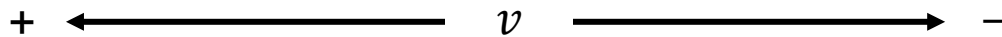
Inductors in Series

Inductors combine in series exactly like resistors.



$$v_1 = L_1 \frac{di}{dt} \quad v_2 = L_2 \frac{di}{dt}$$

$$v = v_1 + v_2 = (L_1 + L_2) \frac{di}{dt}$$

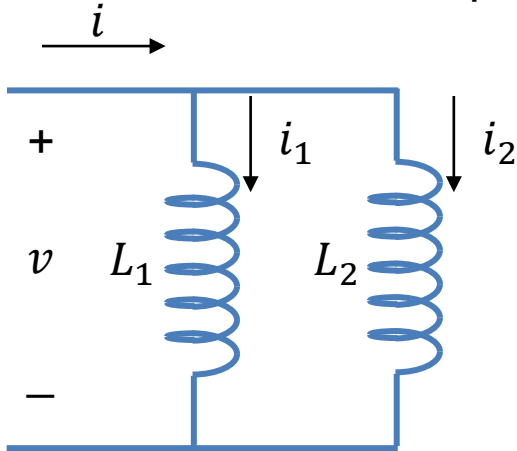


$$v = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2$$

Inductors in Parallel

Inductors combine in parallel exactly like resistors.



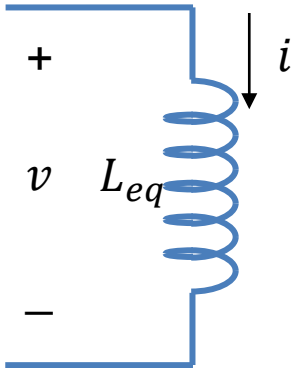
$$v = L_1 \frac{di_1}{dt}$$

$$v = L_2 \frac{di_2}{dt}$$

Using KCL: $i = i_1 + i_2$

Differentiating: $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

Substituting: $\frac{v}{L_{eq}} = \frac{v}{L_1} + \frac{v}{L_2}$



$$v = L_{eq} \frac{di}{dt}$$

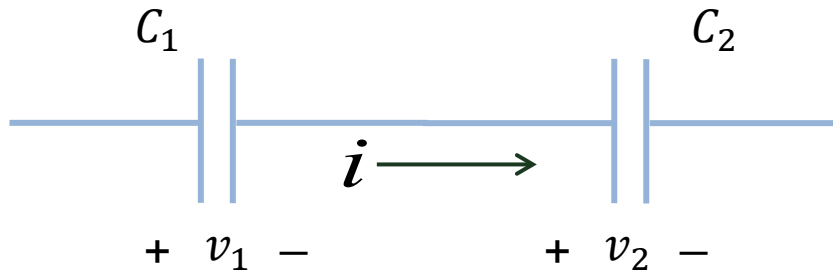
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Capacitors in Series

Capacitors in series combine like resistors in parallel.

$$i = C_1 \frac{dv_1}{dt}$$

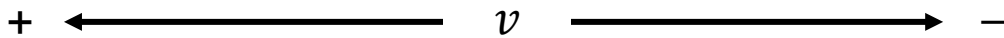
$$i = C_2 \frac{dv_2}{dt}$$



Using KVL: $v = v_1 + v_2$

Differentiating: $\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt}$

Substituting: $\frac{i}{C_{eq}} = \frac{i}{C_1} + \frac{i}{C_2}$

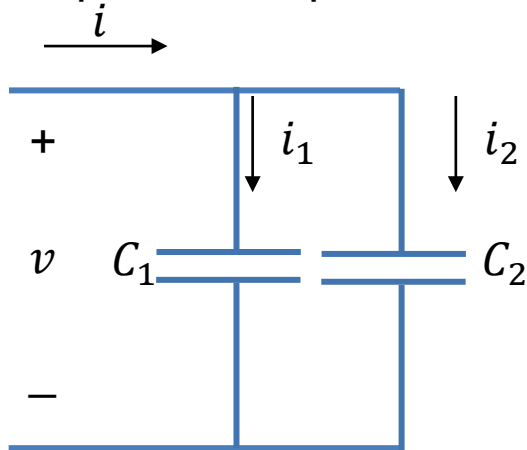


$$i = C_{eq} \frac{dv}{dt}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in Parallel

Capacitors in parallel combine like resistors in series.



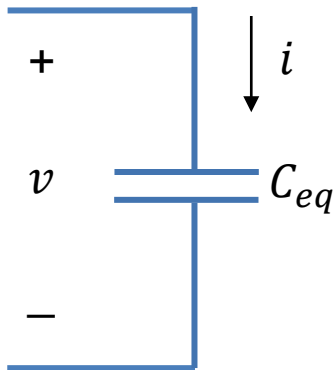
$$i_1 = C_1 \frac{dv}{dt}$$

$$i_2 = C_2 \frac{dv}{dt}$$

Using KCL: $i = i_1 + i_2$

Substituting:

$$\begin{aligned} C_{eq} \frac{dv}{dt} &= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} \\ &= (C_1 + C_2) \frac{dv}{dt} \end{aligned}$$



$$i = C_{eq} \frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2$$