

Lab 7: AC Response of a 1st Order RC Circuit

Theory and Introduction

Goals for Lab 7 - The goal of this lab is to explore the response of a 1st order circuit to various periodic inputs.

Theory

A. Response of a first order filter to sinusoidal inputs

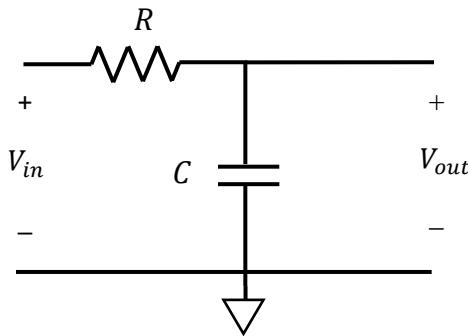


Figure 7.1 – A 1st Order RC Circuit.

The input/output relationship for the first order RC circuit shown in Figure 7.1 is given by the first order differential equation

$$RC \frac{dV_{out}(t)}{dt} + V_{out}(t) = V_{in}(t).$$

Assuming that $V_{in}(t)$ is a sinusoid of the generic form

$$V_{in}(t) = A \cos(\omega t + \theta),$$

the output will be a sinusoid of the same frequency with its amplitude scaled and phase shifted according to

$$V_{out}(t) = \frac{A}{\sqrt{1+(\omega RC)^2}} \cos(\omega t + \theta - \tan^{-1}(\omega RC)).$$

You can easily see that this output does in fact satisfy the differential equation above by plugging these forms of $V_{in}(t)$ and $V_{out}(t)$ into the equation. It is common to define the cutoff frequency of this circuit as $\omega_c = 1/(RC)$. In terms of the cutoff frequency, the output is written as

$$V_{out}(t) = \frac{A}{\sqrt{1+(\omega/\omega_c)^2}} \cos(\omega t + \theta - \tan^{-1}(\omega/\omega_c)).$$

For input frequencies that are small compared to the cutoff frequency, $\omega \ll \omega_c$, both the amplitude scaling and the phase shift are small so that the output is essentially the same as the input. On the other hand, for input frequencies that are large compared to the cutoff frequency, $\omega \gg \omega_c$, the amplitude scaling will be significant (and the phase shift will be near $\pi/2$). As a result the output will be much weaker than the input. We say that the circuit acts as a low-pass filter in that it allows low frequencies to pass but attenuates large frequencies. It is common to describe the filtering action of the circuit in terms of a magnitude response given by

$$\text{Magnitude response} = \frac{|V_{out}|}{|V_{in}|} = \frac{1}{\sqrt{1+(\omega/\omega_c)^2}},$$

and a phase response given by

$$\text{phase response} = \angle V_{out} - \angle V_{in} = -\tan^{-1}(\omega/\omega_c).$$

B. Response of a first order filter to general periodic inputs

When you take ECEN 314, you will learn that any even periodic signal (of period T_o) can be decomposed into a sum of sinusoids of the form

$$x(t) = \sum_{k=0}^{\infty} X_k \cos(k\omega_o t),$$

where $\omega_o = 2\pi/T_o$ is known as the fundamental frequency of the periodic signal $x(t)$. (Odd signals can be represented with sines rather than cosines.) Choosing different forms for the coefficients X_k will result in different shapes for $x(t)$. For example, if we were to choose

$$X_k = \begin{cases} \frac{1}{k^2}, & \text{for } k \text{ odd,} \\ 0, & \text{for } k \text{ even,} \end{cases}$$

then it turns out that $x(t)$ will be a triangle wave. Alternatively, we could build a square wave by choosing

$$X_k = \begin{cases} \frac{(-1)^{(k-1)/2}}{k}, & \text{for } k \text{ odd,} \\ 0, & \text{for } k \text{ even.} \end{cases}$$

You are encouraged to try this for yourself. Use your favorite computer math package (MATLAB, Desmos, etc.) to construct and plot the truncated series

$$x_n(t) = \sum_{m=1}^n \frac{1}{(2m-1)^2} \cos(2\pi(2m-1)t).$$

As you include more and more terms in the series, the signal will look more and more triangular. Figure 7.2 (blue curve) shows the result with keeping only 5 terms in the series.

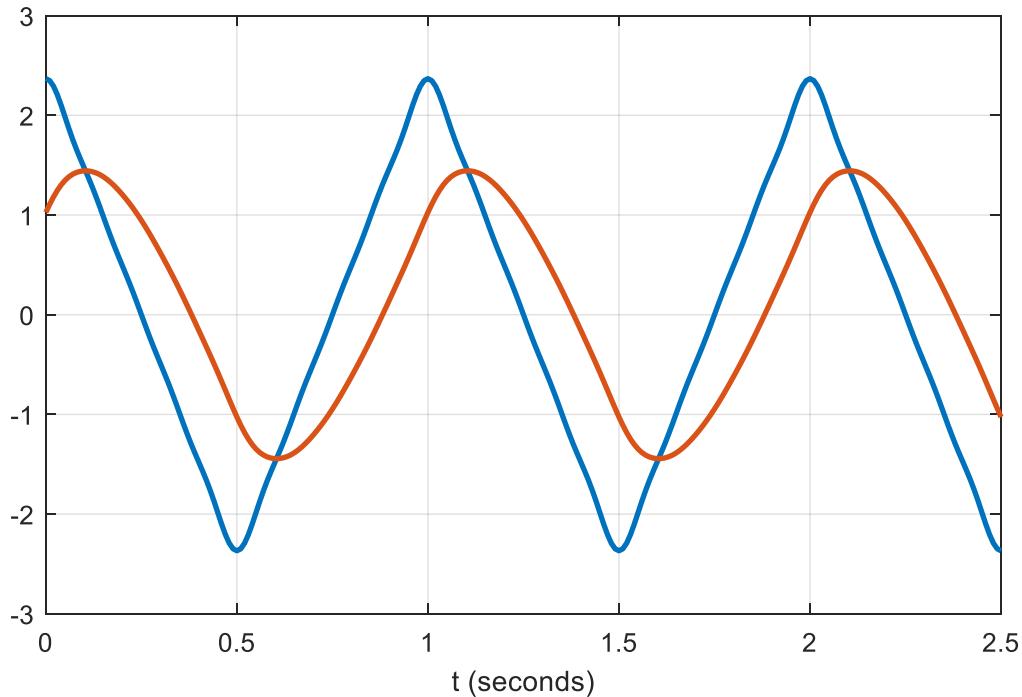


Figure 7.2 – Triangular input (blue) and corresponding output (orange) of a first order RC circuit, $\omega_o = 2\pi$ rad/sec, $\omega_c = 2\pi$ rad/sec.

If a periodic signal $V_{in}(t)$ is input to our first order filter, the output can be determined by representing the input as a sum of sinusoids and then calling on the concept of superposition to determine the output. If the input is written as

$$V_{in}(t) = \sum_{k=0}^{\infty} V_k \cos(k\omega_o t),$$

then the output of the first order low-pass filter will be

$$V_{out}(t) = \sum_{k=0}^{\infty} \frac{V_k}{\sqrt{1+(k\omega_o/\omega_c)^2}} \cos(k\omega_o t - \tan^{-1}(k\omega_o/\omega_c)).$$

That is, the response of the circuit to a sum of sinusoids will be a sum of sinusoids at the same frequencies, each scaled in amplitude and shifted in phase according to their frequencies. The orange curve in Figure 7.2 shows the output that corresponds to the triangular input. Three distinct differences can be seen between the input and the output. The amplitude of the output is smaller than that of the input, due to the amplitude scaling caused by the circuit. The output is a bit delayed relative to the input, due to the phase shifting caused by the circuit. Also, the output is a bit smoother (sinusoidal) looking than the input. This is explained by the low-pass nature of the circuit. With the chosen values of $\omega_o = \omega_c$, the first term in the series (the so-called fundamental frequency which has a frequency of $\omega = \omega_o$) has minimal attenuation. It's amplitude is scaled by a factor of $\frac{1}{\sqrt{2}} = 0.707$. The next term (the third harmonic which has a frequency of $\omega = 3\omega_o$) has it's amplitude scaled by a factor of $\frac{1}{\sqrt{10}} = 0.316$. That's more than double the attenuation of the first term. Higher frequency terms have correspondingly higher attenuation. The RC low-pass filter allows the first term in the series to pass with relatively little attenuation, while the other terms are largely blocked. Thus, the output looks like a sinusoid at a frequency of $\omega = \omega_o$ with a little bit of distortion from the residual higher order terms.

C. Active implementation of first order filter.

At times it is advantageous to implement a first order filter as an active (op-amp) filter. The circuit shown in Figure 7.3 behaves exactly the same as the one in Figure 7.1. To see that, consider performing a KCL at the negative input of the op-amp. Since the current into the negative terminal must be zero, the sum of the currents into that terminal must satisfy

$$\frac{V_{in}}{R} + \frac{V_{out}}{R} + C \frac{dV_{out}}{dt} = 0,$$

or equivalently

$$RC \frac{dV_{out}(t)}{dt} + V_{out}(t) = -V_{in}(t).$$

This is same as the equation that describes the circuit in Figure 7.1 except for the negative sign on the right hand side of the equation. If that matters, the circuit could be preceded or followed by an inverter to compensate.

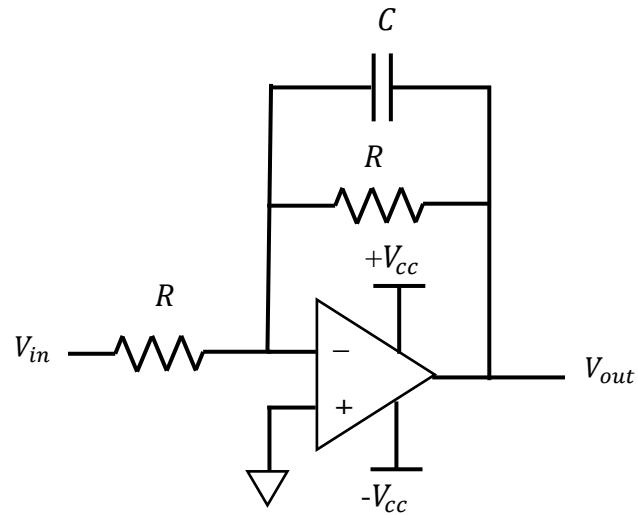


Figure 7.3 – A First Order RC Low-pass Filter

Prelab

1. Find the output (sinusoidal steady state) of the first order RC filter of Figure 7.1 for each of the following inputs:

- a) $V_{in}(t) = 2.5\cos(200\pi t + \frac{\pi}{6})$,
- b) $V_{in}(t) = 2\sin(200\pi t) + \frac{1}{3}\sin(600\pi t)$,
- c) $V_{in}(t) = \cos(200\pi t) - \frac{1}{3}\cos(600\pi t) + \frac{1}{5}\cos(1000\pi t)$.

Assume that the RC time constant of the filter is chosen to be $RC = 1\text{msec}$. For each case provide a plot (computer generated, not hand drawn) of the input and the output on the same graph. Provide three separate plots, one plot for each of the three parts above. How would the output be different if the active filter of Figure 7.3 was used instead of the passive filter of Figure 7.1?

2. The input voltage given by

$$V_{in}(t) = \cos(200\pi t) - \frac{1}{3}\cos(600\pi t) + \frac{1}{5}\cos(1000\pi t) - \frac{1}{7}\cos(1400\pi t) + \frac{1}{9}\cos(1800\pi t)$$

is input to the sequence of back-to-back first order filters shown in Figure 7.4. Find the form of the output voltage and plot (computer generated, not hand drawn) both the input and the corresponding output for each of the following combinations of RC time constants:

- a) $R_1C_1 = 2\text{msec}, R_2C_2 = 8\text{msec}$,
- b) $R_1C_1 = 2\text{msec}, R_2C_2 = 2\text{msec}$,
- c) $R_1C_1 = 2\text{msec}, R_2C_2 = 500\mu\text{sec}$.

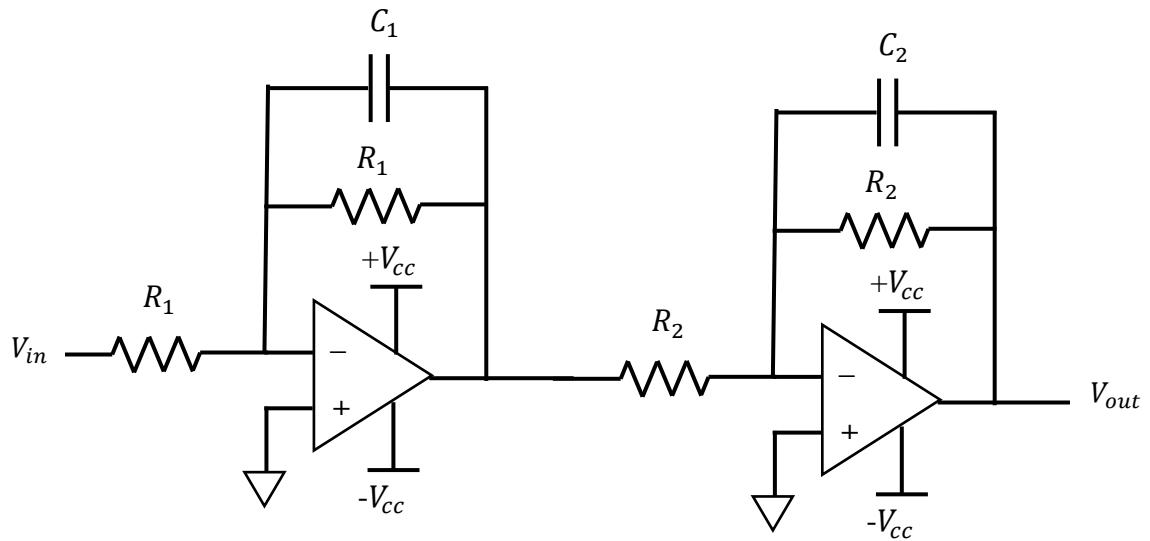


Figure 7.4 Back-to-Back First Order RC Low-pass Filters

Procedure

Equipment and parts needed

- A selection of 1/4-W resistors
- A selection of capacitors
- 2 – 741 OPAMPs

Task 1 – Changing a triangle wave into a sine wave

In this task you will mimic what was outlined in the theory section of the lab. The idea is to input a triangular wave into a first order RC low-pass filter in order to produce an output that looks as sinusoidal as possible.

- A. Build the active first order filter shown in Figure 7.3 on your breadboard. Choose R and C values so that your cutoff frequency somewhere near $\omega_c = \frac{1}{RC} = 2000\pi$ rad/sec (1kHz). The cutoff frequency doesn't have to be exact here, just somewhere in the ballpark. Record in your lab notebook, the component values you chose.
- B. Use either the oscilloscope in the lab or the waveform generator on your AD2 unit to create a triangle wave with a peak-to-peak value of 2 Volts and a frequency of 250Hz. Using the oscilloscope in the lab or the oscilloscope on your AD2 unit, display both the input and the output voltages of the RC circuit on your scope. Since the frequency of the input is somewhat below the cutoff frequency of the circuit, you should find that the input and output look fairly similar. Take a screenshot of your scope display and save it for your lab report.
- C. Vary the frequency of the input triangle wave and watch how the output changes. Find the input frequency at which the output looks the most sinusoidal. This is a subjective measurement here, so just pick what looks best to you. Record the input frequency in your lab notebook and take another screenshot of your input and output signals. **Note:** It is not necessary that the input and output signals have the same amplitude, it is just the sinusoidal shape of the output that you are trying to create here. If the amplitude is not what we want it to be, it is a simple thing to amplify it to where we want it to be.

Task 2 – Changing a square wave into a sine wave

A. Repeat the measurements of Task 1, but this time choose the input to the circuit to be a square wave (still 2V peak-to-peak) rather than a triangle wave. Take screenshots of the input and output signal for the case when the input frequency is 250Hz and the case when the input frequency is chosen to make the output look the most sinusoidal. In this case you will find that the output does not look nearly as sinusoidal as when the input was a triangle wave.

B. In order to improve your ability to change a square wave into a sine wave, this time you will run the square wave through back-to-back first order filters as shown in Figure 7.4. Each stage could have the same cut-off frequency or different ones. Play around with it to see what works best. Again, your goal is to make the output look as sinusoidal as possible. Fix one of the stages to have a cutoff frequency of 1kHz as before. Then try a few different possibilities for the cutoff frequency of the second stage (the same, lower, higher). For each case vary the input frequency and observe the output. Determine what combination of input frequency and cutoff frequencies produces the most sinusoidal looking output. Note the combination in your lab book and take a screenshot of the input and output waveforms.

Task 3 - Build a circuit that simultaneously generates a square wave, a triangle wave, and a sine wave.

Recall in Lab 6 you built an oscillating circuit. Using that circuit as a base and then adding a filter circuit you should be able to create a circuit that (depending on where you grab the output from will generate either a square wave, a triangular wave, or a sine wave. Build the circuit illustrated in Figure 7.5. If you need to, review Lab 6 so that you have an understanding of the nature of the signals produced at nodes 1, 2, and 3. Recall that both V_1 and V_3 will be square waves, while V_2 has a periodic form that looks piecewise exponential. Choose your component values in the circuit in Figure 7.5 as follows:

1. Choose the resistor R_1 and the capacitor C_1 so that the circuit oscillates at a frequency near 250Hz when the potentiometer is set to split the $10\text{k}\Omega$ resistance into two equal $5\text{k}\Omega$ parts (voltage division ratio of $\gamma = \frac{1}{2}$). Record your values chosen in your lab notebook. Then adjust the potentiometer (change the voltage division ratio, γ) until the voltage measured

at node V_2 looks somewhat triangular. Measure and record the frequency at which the circuit oscillates and the voltage division ratio after you are done adjusting the potentiometer. Record these in your lab book.

2. Choose the resistor R_2 and the capacitor C_2 so that RC filter is able to change a triangle wave into a sinusoid (as you did in task 1). When the input is a triangular wave at the frequency you measured in step 1 above, record these component values in your lab book.

Once you have all of your components set, take screenshots of the square wave you get when you measure the voltage at V_1 , the triangle wave you get when you measure the voltage at V_2 , and the sine wave you get when you measure the voltage at V_4 . You have now built yourself a mini function generator complete with square, triangle, and sine wave options.

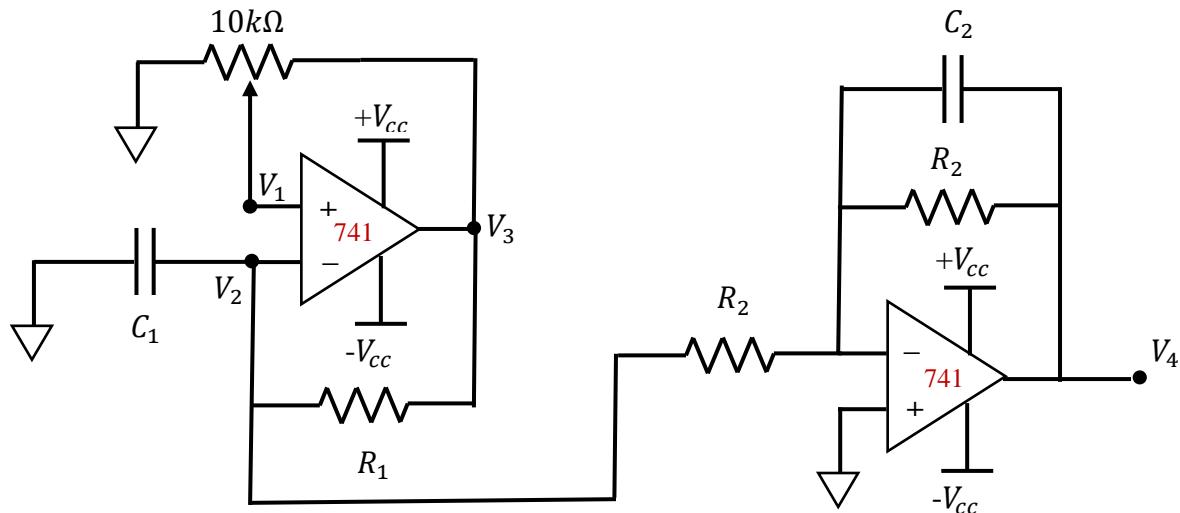


Figure 7.5 – An Op-amp based oscillating circuit

Before you leave the lab...

You should bring your data sheet to be signed by the TA and you should also be prepared to show your TA how you made your various measurements.

Lab Report

1. Title page, as always
2. Report write up:

- A. Procedure – Write in your own words what you did in lab
- B. Measured Data – Include tables of values you measured throughout the lab.
- C. Measured Waveforms -- Be sure that you label your screen shots and intersperse comments and screenshots through your document.
- D. Sample Calculations – Demonstrate how you performed any calculations needed.
- E. Discussion – Comment on any results that were perhaps different than you expected and why that might be.
- F. Conclusion – Write a brief summary.