

Sinusoids in Resistors

Suppose the current in a resistor is sinusoidal,

$$i(t) = I_o \cos(\omega t + \theta).$$

Since $v(t) = Ri(t)$, the voltage will also be a sinusoid,

$$v(t) = RI_o \cos(\omega t + \theta).$$

In terms of phasors,

$$\mathbf{I} = I_o e^{j\theta} \text{ and } \mathbf{V} = RI_o e^{j\theta}.$$

Therefore, the voltage and current phasors in a resistor are related by

$$\mathbf{V} = \mathbf{IR}.$$

That is, Ohm's Law still applies in the phasor domain

Sinusoids in Inductors

Suppose the current in an inductor is sinusoidal,

$$i(t) = I_o \cos(\omega t + \theta).$$

Since $v(t) = L \frac{di}{dt}$, the voltage will be,

$$\begin{aligned} v(t) &= -\omega L I_o \sin(\omega t + \theta) \\ &= -\omega L I_o \cos(\omega t + \theta - 90^\circ) \end{aligned}$$

In terms of phasors,

$$I = I_o e^{j\theta} \text{ and } V = -\omega L I_o e^{j(\theta-90^\circ)} = -\omega L I_o e^{-j90^\circ} e^{j\theta} = j\omega L I_o e^{j\theta}$$

Therefore, the voltage and current phasors in an inductor are related by

$$V = j\omega L I.$$

Note that in the phasor domain, there is no longer a derivative in this relationship. It looks just like Ohm's Law with R replaced with $j\omega L$.

Sinusoids in Capacitors

Suppose the voltage in a capacitor is sinusoidal,

$$v(t) = V_o \cos(\omega t + \theta).$$

Since $i(t) = C \frac{dv}{dt}$, the current will be,

$$\begin{aligned} i(t) &= -\omega C V_o \sin(\omega t + \theta) \\ &= -\omega C V_o \cos(\omega t + \theta - 90^\circ) \end{aligned}$$

In terms of phasors,

$$V = V_o e^{j\theta} \text{ and } I = -\omega C V_o e^{j(\theta-90^\circ)} = -\omega C V_o e^{-j90^\circ} e^{j\theta} = j\omega C V_o e^{j\theta}$$

Therefore, the voltage and current phasors in a capacitor are related by

$$I = j\omega C V \text{ or } V = \frac{I}{j\omega C}$$

Note that in the phasor domain, there is no longer a derivative in this relationship. It looks just like Ohm's Law with R replaced with $1/j\omega C$.

Complex Impedance

For the purposes of analyzing circuits with R_s , L_s and C_s that are driven by sinusoidal (AC) sources, we can analyze the circuit using phasors in which case the voltage/current relationship for all three types of elements are basically the same:

$$\mathbf{V} = \mathbf{I}Z.$$

The quantity Z is known as the (complex) impedance.

Element	Time Domain	Phasor Domain	Z
Resistor	$v(t) = R i(t)$	$\mathbf{V} = \mathbf{I}R$	R
Inductor	$v(t) = L \frac{di(t)}{dt}$	$\mathbf{V} = j\omega L \mathbf{I}$	$j\omega L$
Capacitor	$i(t) = C \frac{dv(t)}{dt}$	$\mathbf{V} = \frac{1}{j\omega C} \mathbf{I}$	$\frac{1}{j\omega C}$

Complex Impedance

Some nomenclature associated with the complex impedance Z is as follows.

$$Z = x + jy$$

where

x = resistance,

y = reactance.

$$Y = \frac{1}{Z} = g + jh$$

where

g = conductance,

h = susceptance.

	Resist- ance	React- ance	Conduct- ance	Suscept- ance
R	R	0	$\frac{1}{R}$	0
L	0	ωL	0	$-\frac{1}{\omega L}$
C	0	$-\frac{1}{\omega C}$	0	ωC

Inductors and capacitors have no resistive component and are called reactive elements. Inductors have a positive reactance while capacitors have a negative reactance.

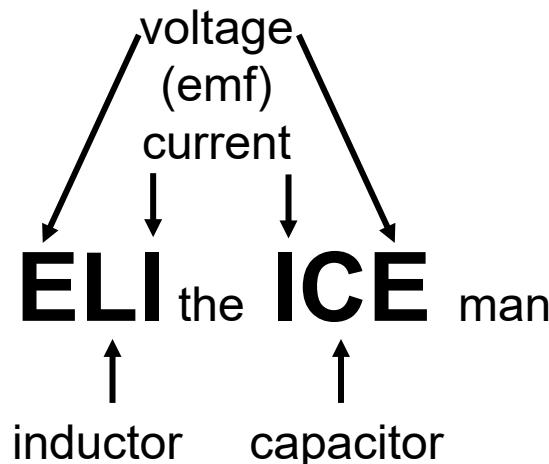
Phase Relationships

It is common to talk about the phase relationships between the voltage and current sinusoids in various circuit elements.

Resistor: $V = IR$ Voltage phase = Current phase

Inductor: $V = j\omega LI$ Voltage phase = Current phase + 90°.
(voltage leads current)

Capacitor: $V = \frac{1}{j\omega C} I$ Voltage phase = Current phase - 90°.
(voltage lags current)



Complex Impedance

Example $v(t)$ is an 80kHz sinusoid with a peak amplitude of 25mV and zero phase. When $v(t)$ is applied across a capacitor, $i(t)$ has a peak amplitude of $628.32\mu\text{A}$. Find:

- (a) The current frequency in rad/sec.
- (b) The current phase.
- (c) The reactance of the capacitor.
- (d) The capacitance of the capacitor.
- (e) The impedance of the capacitor.