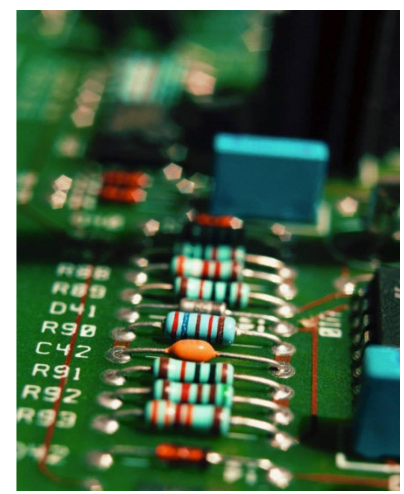
Node-Voltage Circuit Analysis Method

When we attempt to analyze circuits involving several elements, the number of equations and corresponding unknowns we have to solve for can quickly get out of hand.

In this section and the next we will present two systematic techniques for finding voltages and currents in circuits that help to keep the amount of work we have to perform down to a minimum.



Node-Voltage Circuit Analysis Method

The following outlines the node-voltage circuit analysis method.

Step 1 – Identify all essential nodes in the circuit (where three or more elements are connected).

Step 2 – Choose one node as a reference node (usually the one with the most elements connected). Label that node with a and treat the voltage at that point as zero.

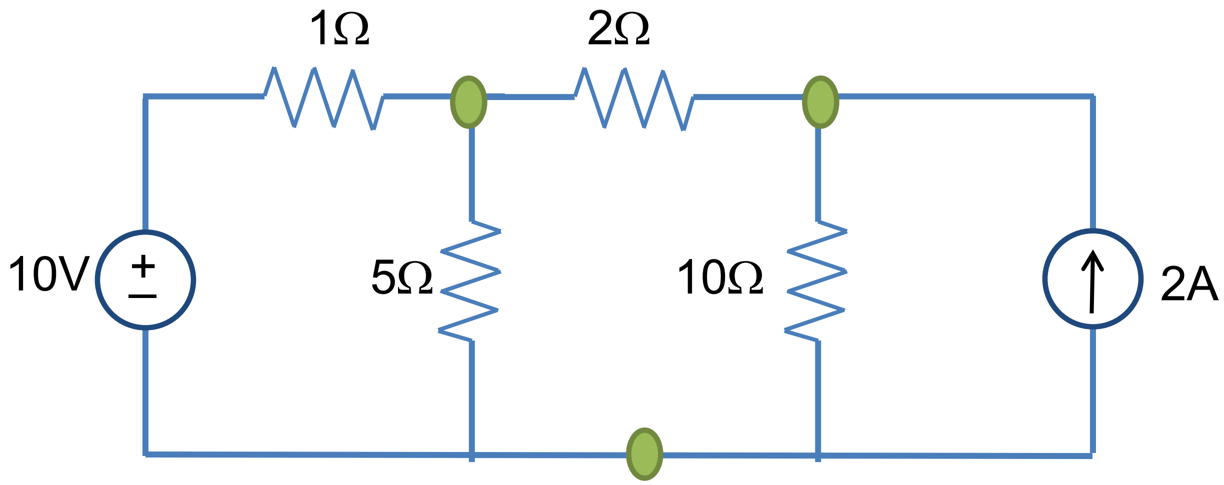
Step 3 – Label unknown voltages at all other essential nodes: V_1, V_2, . . . etc.

Step 4 – Identify currents entering/leaving essential nodes in terms of V_1, V_2, …

Step 5 – Use KCL at each essential node to set up equations to solve for the unknown voltages.

Step 6 – Solve equations for unknown voltages.

Node-Voltage Circuit Analysis Method



Step 1 – Identify all essential nodes in the circuit (where three or more elements are connected).

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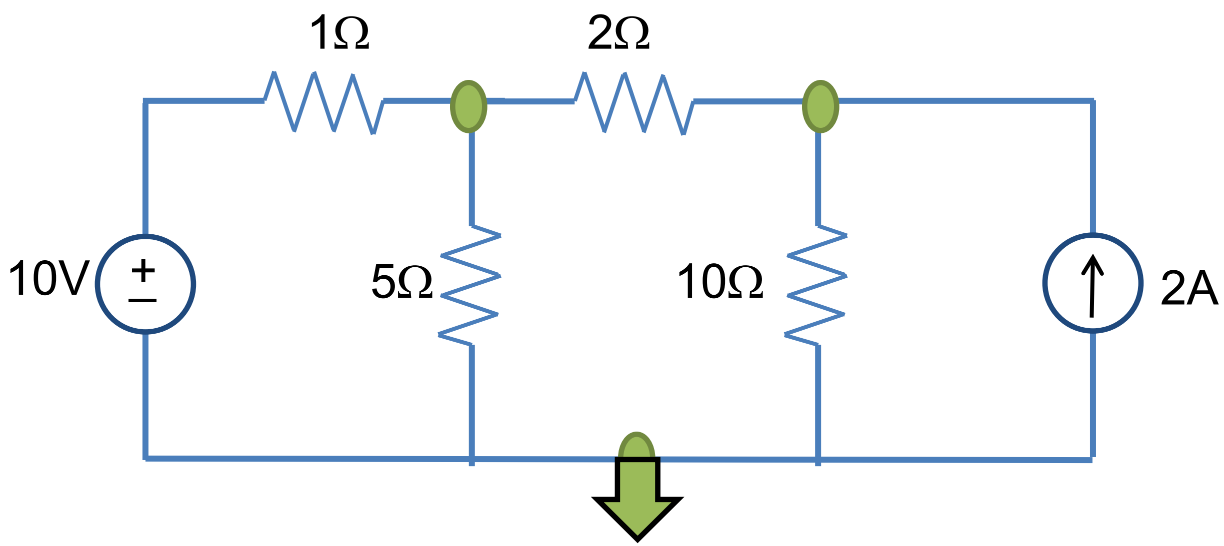

Step 2 – Choose one node as a reference node (usually the one with the most elements connected). Label that node with a and treat the voltage at that point as zero.

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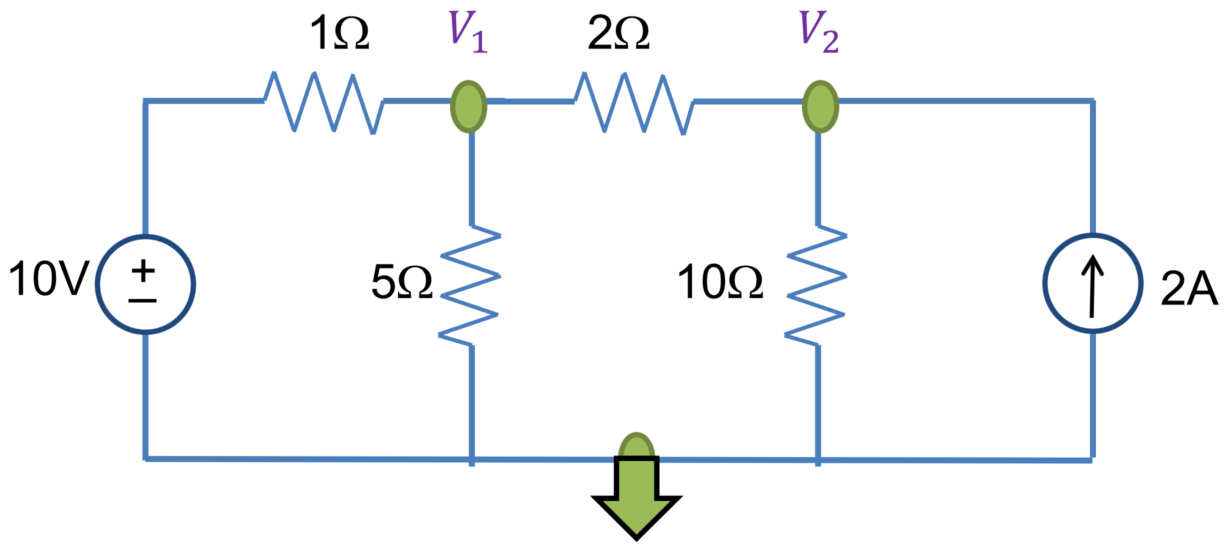
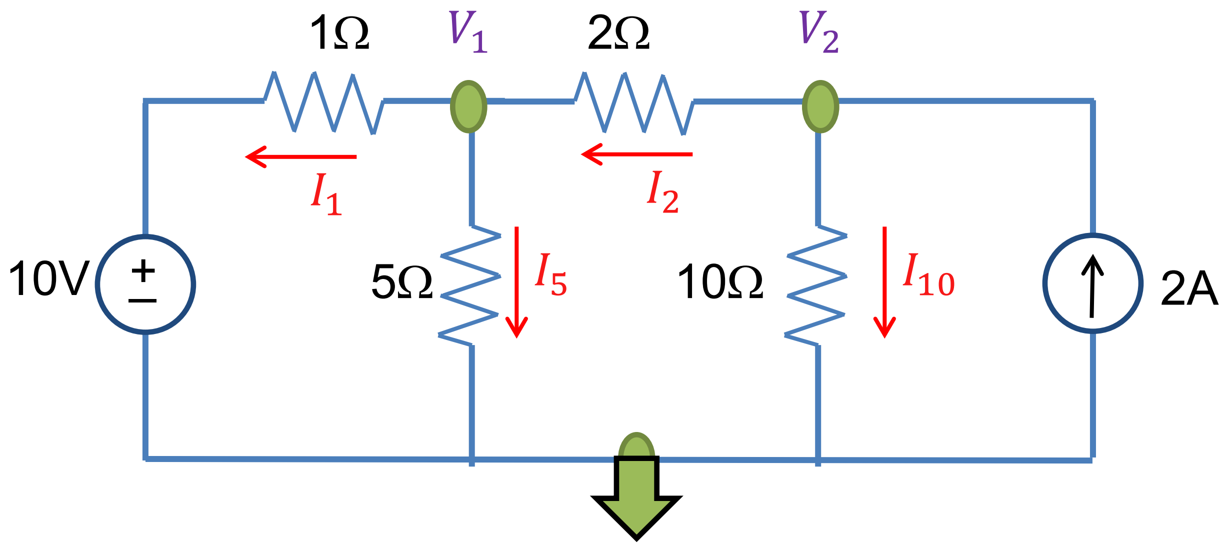


Step 3 – Label unknown voltages at all other essential nodes: V_1, V_2, . . . etc.

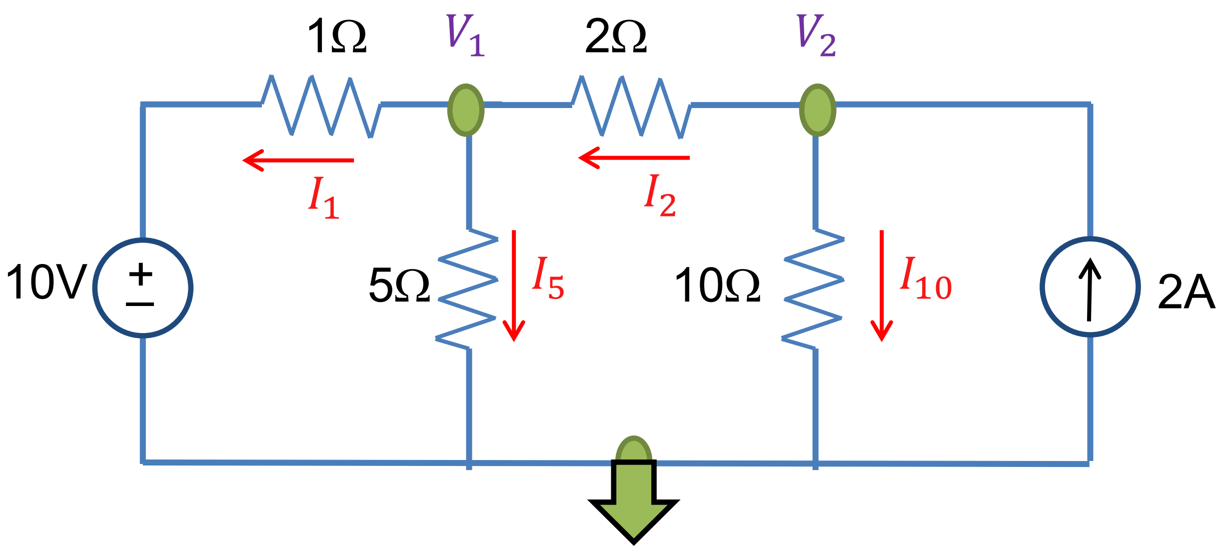
Node-Voltage Circuit Analysis Method



$$I_1 = \frac{V_1 - 10}{1} \qquad I_2 = \frac{V_2 - V_1}{2} \qquad I_5 = \frac{V_1}{5} \qquad I_{10} = \frac{V_2}{10}$$

Step 4 – Identify currents entering/leaving essential nodes in terms of V_1, V_2, …

Node-Voltage Circuit Analysis Method



$$I_1 = \frac{V_1 - 10}{1}$$

$$I_2 = \frac{V_2 - V_1}{2}$$

$$I_5 = \frac{V_1}{5}$$

$$I_{10} = \frac{V_2}{10}$$

At node 1: $I_1 + I_5 = I_2$

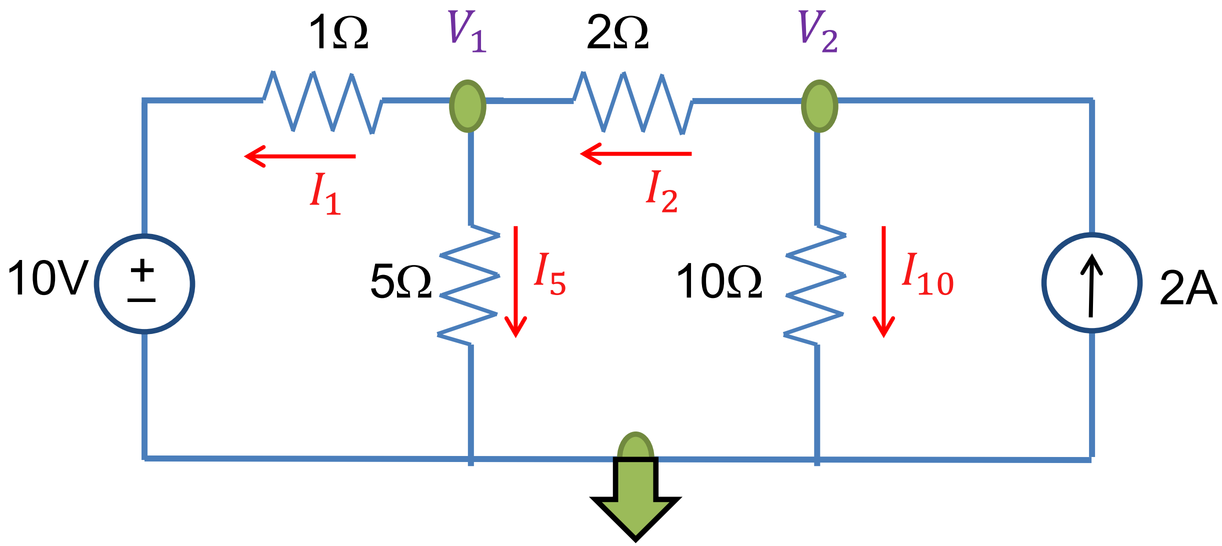
$$V_1 - 10 + \frac{V_1}{5} = \frac{V_2 - V_1}{2}$$

At node 2: $I_2 + I_{10} = 2$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} = 2$$

Step 5 – Use KCL at each essential node to set up equations to solve for the unknown voltages.

Node-Voltage Circuit Analysis Method



$$I_1 = \frac{V_1 - 10}{1}$$

$$I_2 = \frac{V_2 - V_1}{2}$$

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$$I_{10} = \frac{V_2}{10}$$

$$V_1 - 10 + \frac{V_1}{5} = \frac{V_2 - V_1}{2}$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} = 2$$

$$17V_1 - 5V_2 = 100$$

$$-5V_1 + 6V_2 = 20$$

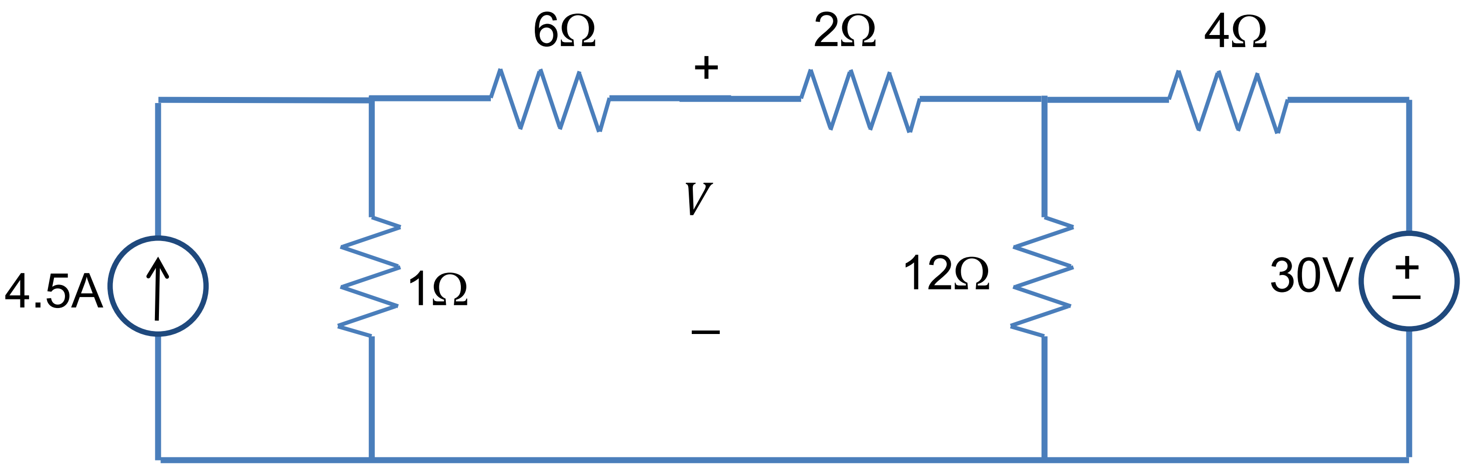
$$V_1 = 9.09$$

$$V_2 = 10.91$$

Step 6 – Solve equations for unknown voltages.

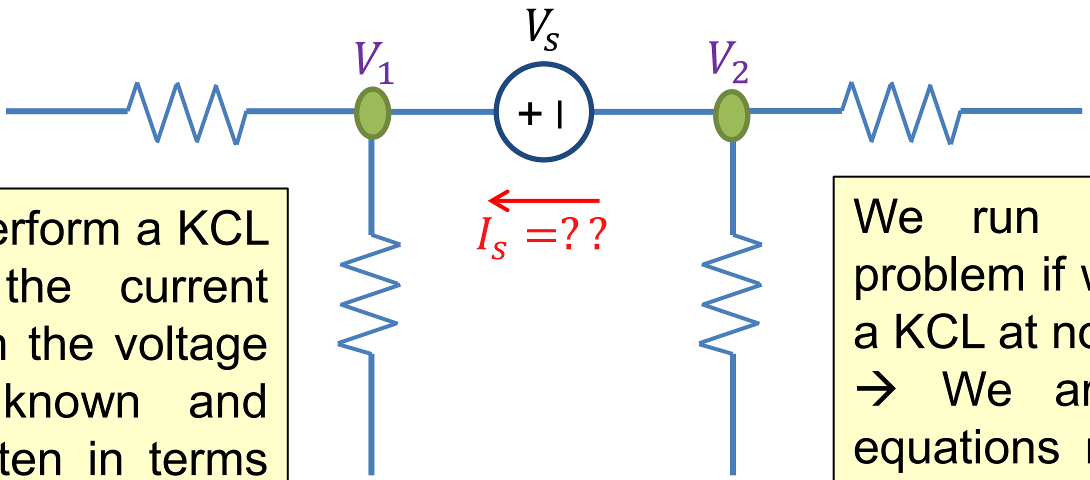
Example

Use the node-voltage method to find V in the circuit.



Node-Voltage Circuit Analysis Method

If a circuit contains a branch with only a voltage source between two nodes, the standard node voltage technique can break down.



If we tried to perform a KCL at node 1, the current flowing through the voltage source is unknown and cannot be written in terms of the various node voltages.

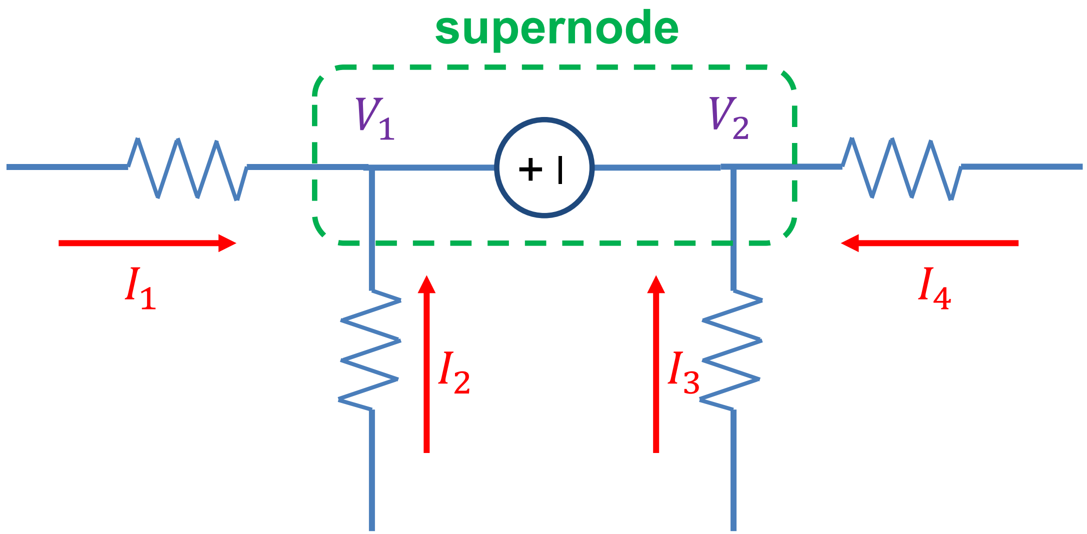
We run into the same problem if we try to perform a KCL at node 2.
→ We are missing two equations needed to solve for the unknown node voltages.

We will replace the missing equations as follows:

1) The voltage source will force a relationship between the two node voltages ($V_1 - V_2 = V_s$ in this case).
2) By introducing the idea of a "supernode", we can construct that last needed equation.

Node-Voltage Circuit Analysis Method

The two nodes connected by the voltage source are combined into what is called a supernode.



A KCL performed at the supernode allows us to specify an equation without needing to know the current through the voltage source.

$$I_1 + I_2 + I_3 + I_4 = 0$$

Example

Use the node-voltage method to find V.