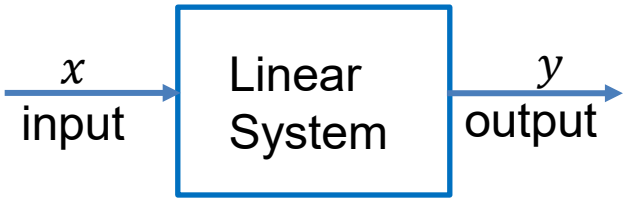
Superposition



A linear system (circuit) is one that has the property that if:

1) input x_1 produces output y_1,

and 2) input x_2 produces output y_2,

then

input $\alpha x_1 + \beta x_2$ produces output $\alpha y_1 + \beta y_2$.

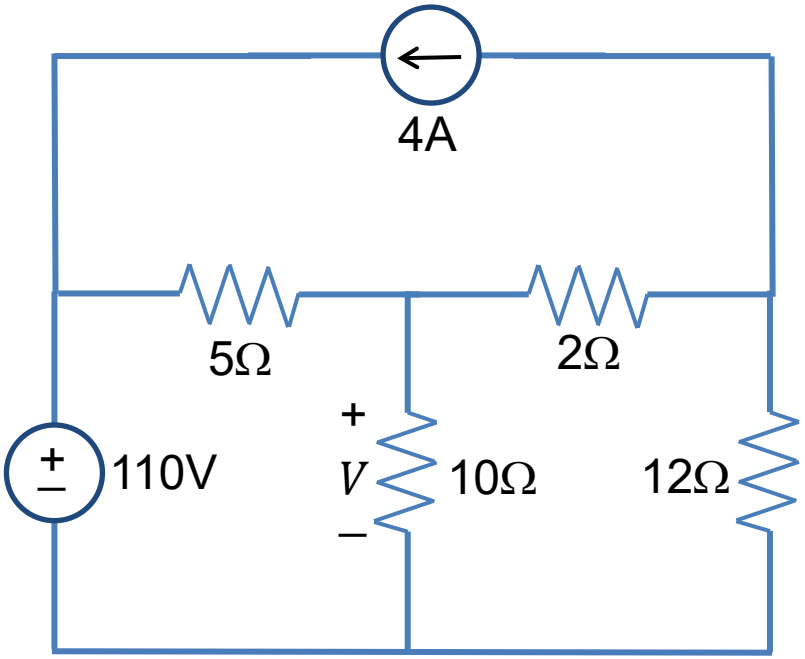
That is, a linear combination of inputs will cause the same linear combination of corresponding outputs.

Any circuit consisting of resistors, capacitors, inductors will behave in a linear fashion.

Superposition

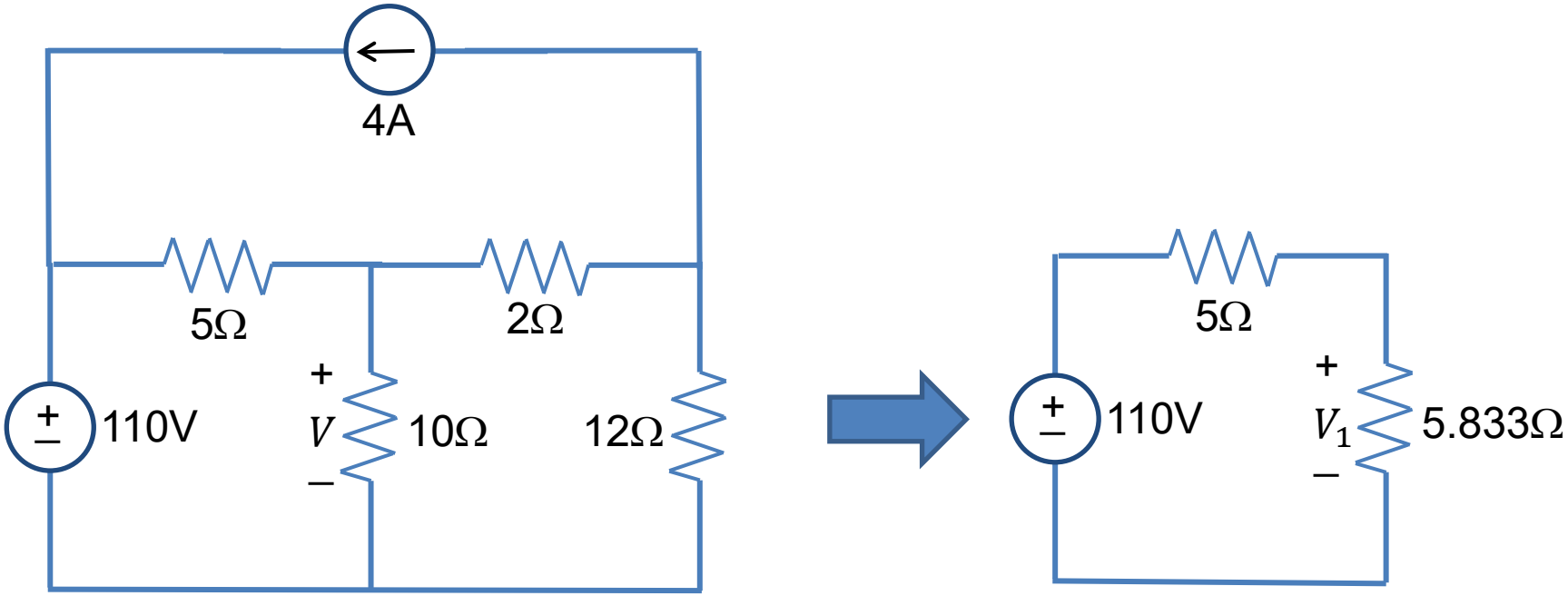
Superposition - For a circuit with multiple independent sources, we can find the response of the circuit by summing the responses to each individual source.

Example: Find V in the circuit shown



Superposition

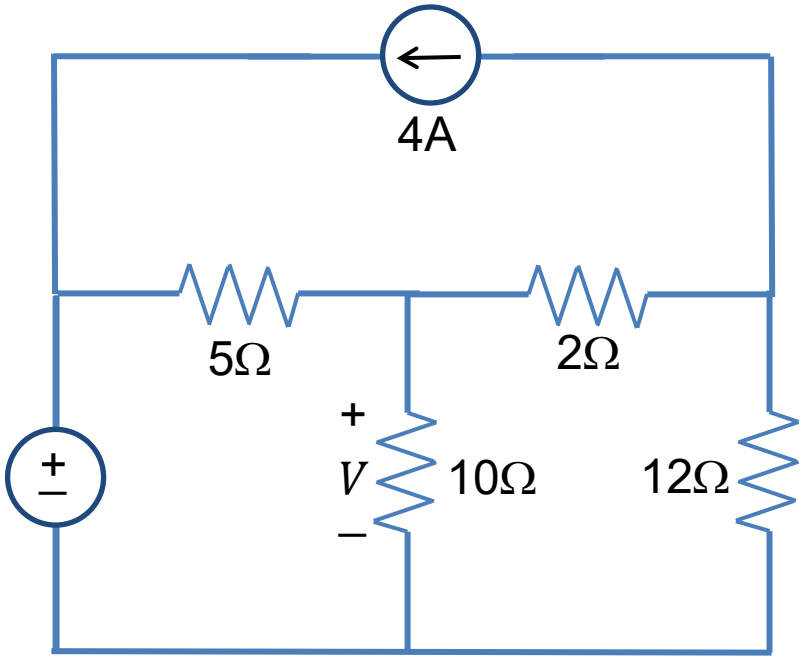
First find the response of the circuit to the 110V source (remove the current source).

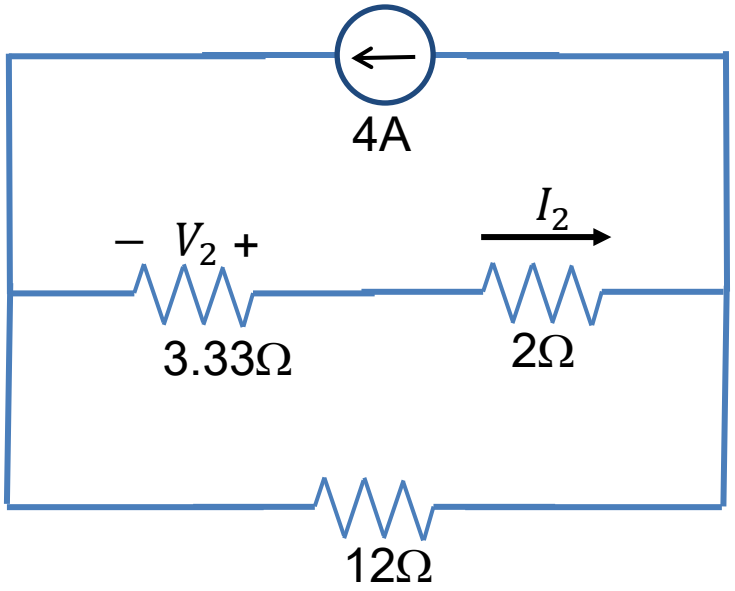


$$V = 110 \cdot \frac{5.833}{5 + 5.833} = 59.23$$

Superposition

Next find the response of the circuit to the 4A source alone (remove the voltage source).





$$I_2 = 4\frac{12}{12+2+3.33} = 2.7692$$

$$V_2 = -(2.7692)\cdot(3.33) = -9.23$$

Superposition

Finally, the desired voltage in the original circuit is just the sum of the voltages found in the two simplified circuits (with only one independent source present).

$$V = V_1 + V_2 = 59.23 - 9.23 = 50$$

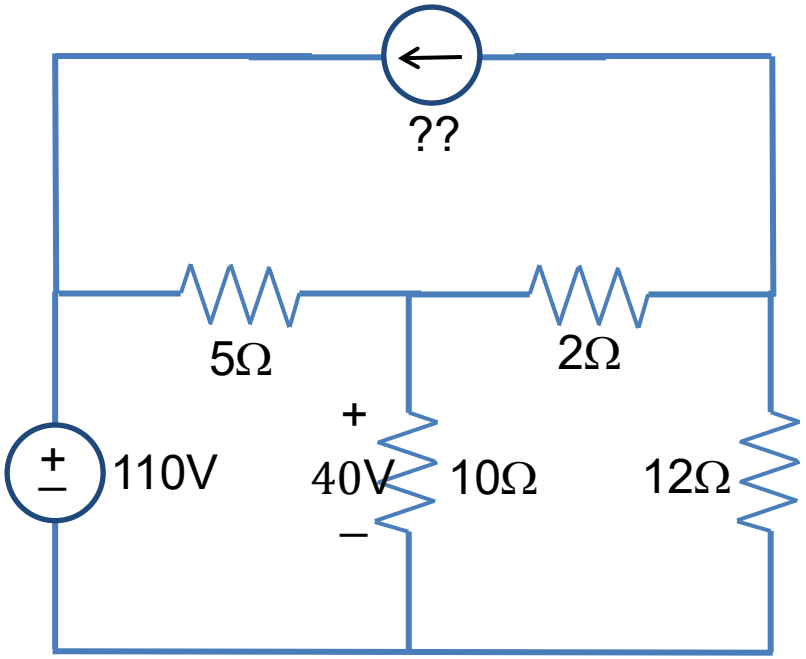
Note 1: When using superposition, we must keep any dependent sources in place.

Note 2: Superposition allows us to analyze a complicated circuit by analyzing several simpler circuits. Often, it is easier to just analyze the single (more complicated) circuit rather than trying to analyze multiple circuits. So…before using superposition, be sure it is really going to make your life easier. Often it does not.

Note 3: The real power of the superposition technique often comes in design type problems as shown in the next slide.

Superposition

Suppose in the circuit from the previous example, we were asked to find the value of the current source so that the voltage across the 10Ω resistor is 40V. The concept of superposition allows us to easily do this.



Input	Output, V
110V	59.23V
4A	-9.23V
$\alpha(110V) + \beta(4A)$	$\alpha(59.23) + \beta(-9.23)$

Since we are only adjusting the current source, we will use $\alpha = 1$ (leave the voltage source alone). Then we want:

$$59.23 - 9.23\beta = 40 \rightarrow \beta = 2.0834$$

Therefore, we should replace the 4A current source with a 2.0834(4A) = 8.33A current source.