

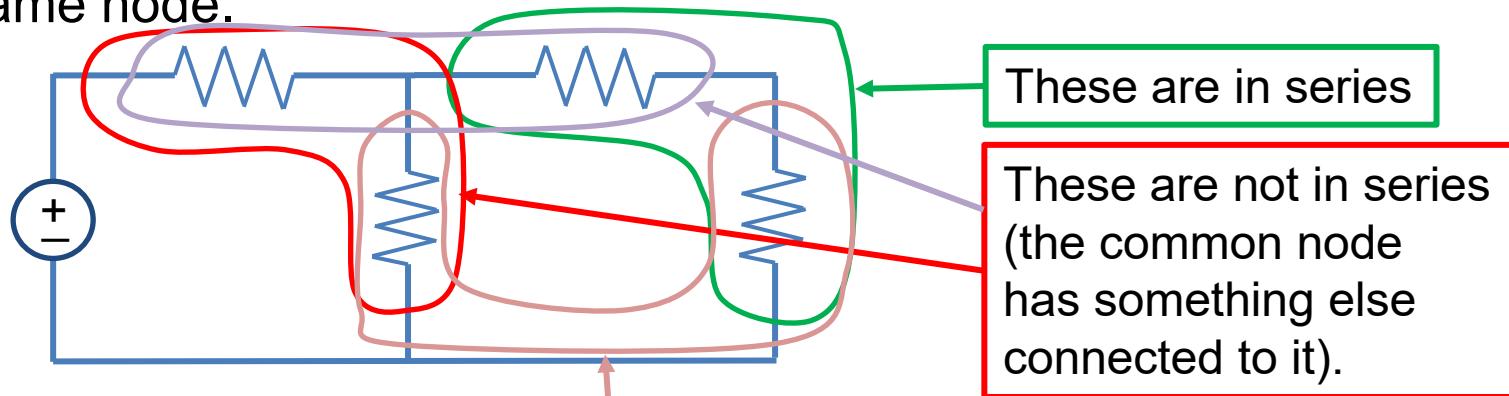


Equivalent Resistances

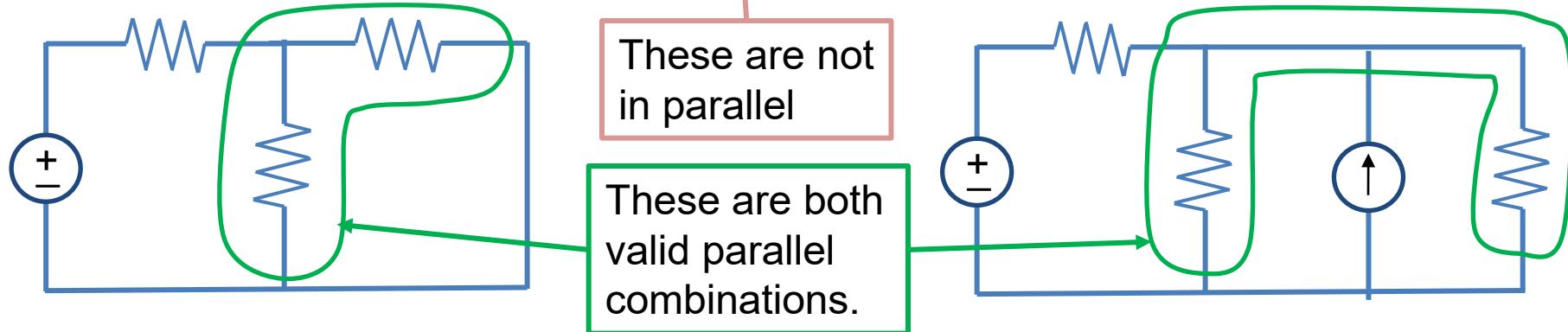
- ✓ Definitions of series and parallel circuit elements
- ✓ How to identify resistors in series/parallel
- ✓ Equivalent resistance for resistors in series
- ✓ Equivalent resistance for resistors in parallel
- ✓ Using series/parallel equivalents to simplify circuits

Series and Parallel

Series Elements - Two circuit elements are in series if one end of each element is connected to a common node and nothing else is connected at the same node.



Parallel Elements - Two circuit elements are in parallel if both ends of each element are connected to two common nodes. It doesn't matter if anything else is connected to either common node.



Resistors in Series

Consider a circuit with two or more resistors connected in series.

The current through each resistor must be the same.

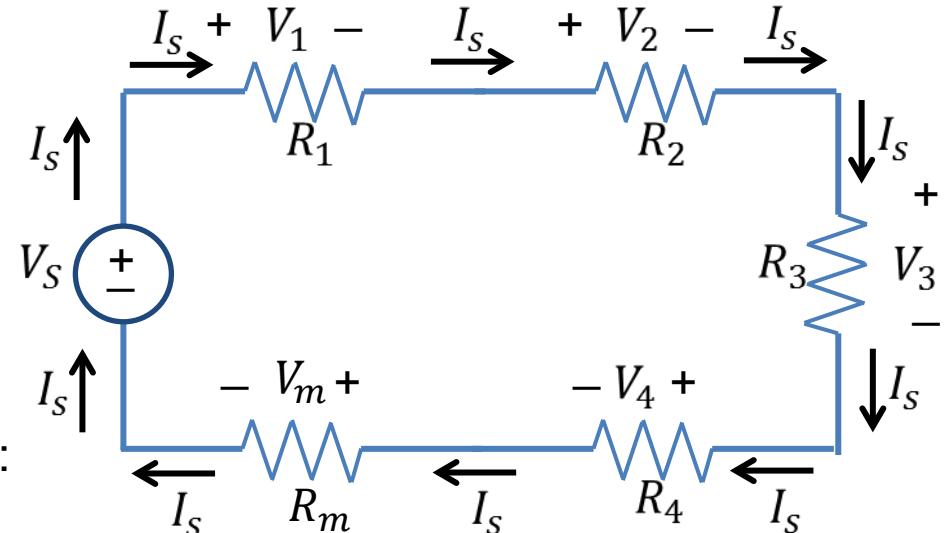
By virtue of Ohm's law, we then get:

$$V_1 = I_s R_1,$$

$$V_2 = I_s R_2,$$

...

$$V_m = I_s R_m.$$



Using Kirchoff's Voltage Law (KVL) around the loop:

$$V_s = V_1 + V_2 + \dots + V_m$$

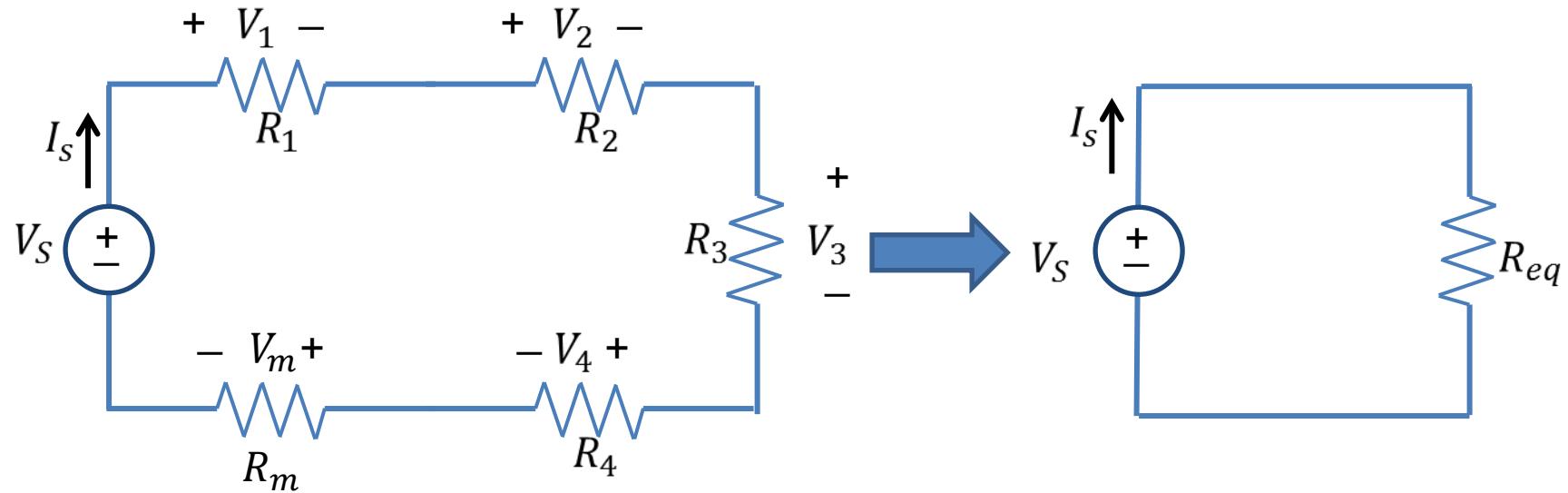
$$= I_s R_1 + I_s R_2 + \dots + I_s R_m$$

$$= I_s (R_1 + R_2 + \dots + R_m)$$

$$= I_s R_{eq}$$

$$R_{eq} = R_1 + R_2 + \dots + R_m$$

Resistors in Series



For the purposes of finding the current flowing through the circuit, the series connection of the multiple resistors can be replaced by a single equivalent resistor whose resistance is the sum.

$$R_{eq} = R_1 + R_2 + \dots + R_m$$

Resistors in Parallel

Consider a circuit with two or more resistors connected in parallel.

Now the current in each branch is different but the voltage across each resistor is the same.

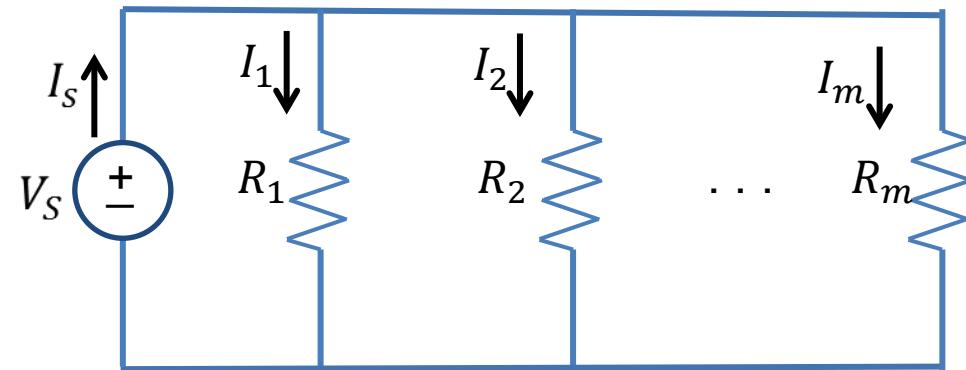
By virtue of Ohm's law, we then get:

$$V_s = I_1 R_1 = I_2 R_2 = \dots = I_m R_m$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_m}$$

or equivalently

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_m}}$$



Using Kirchoff's Current Law (KCL):

$$I_s = I_1 + I_2 + \dots + I_m$$

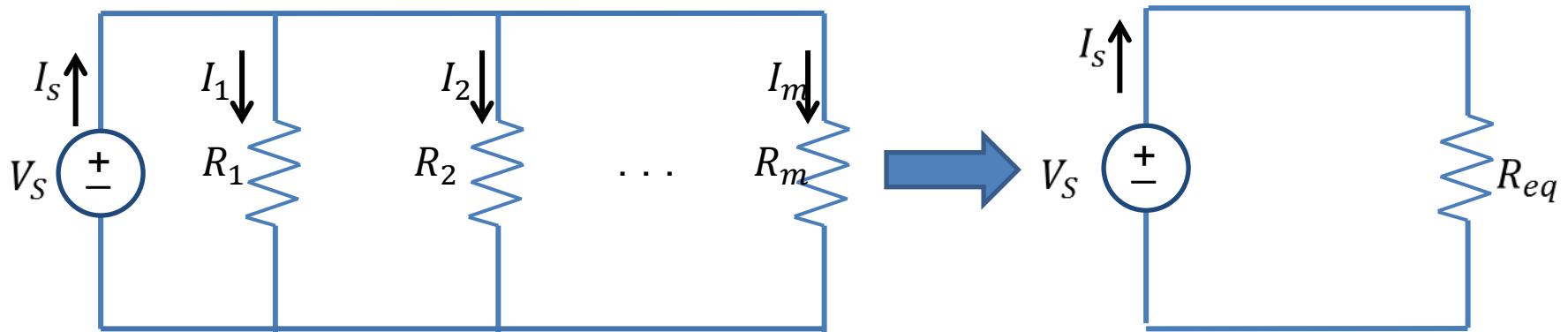
$$= \frac{V_s}{R_1} + \frac{V_s}{R_2} + \dots + \frac{V_s}{R_m}$$

$$= V_s \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_m} \right)$$

$$= \frac{V_s}{R_{eq}}$$

Equivalent Resistances

Resistors in Parallel



For the purposes of finding the current flowing through the source, the parallel connection of resistors can be replaced with a single equivalent resistor such that:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_m} \quad \rightarrow \quad R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_m}}$$

Equivalent Resistances

Most often, we use these concepts with just two resistors at a time. In that case, the formulas simplify to:

Series combination

$$R_{eq} = R_1 + R_2$$

Parallel combination

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Note that:

- 1) Series equivalent resistances are always larger than each individual component, and
- 2) Parallel equivalent resistances are always smaller than each individual component,

Example

For the circuit shown:

- (a) Find the voltage V ,
- (b) Find the power delivered to the circuit by the current source,
- (v) Find the power dissipated in the 10Ω resistor.

