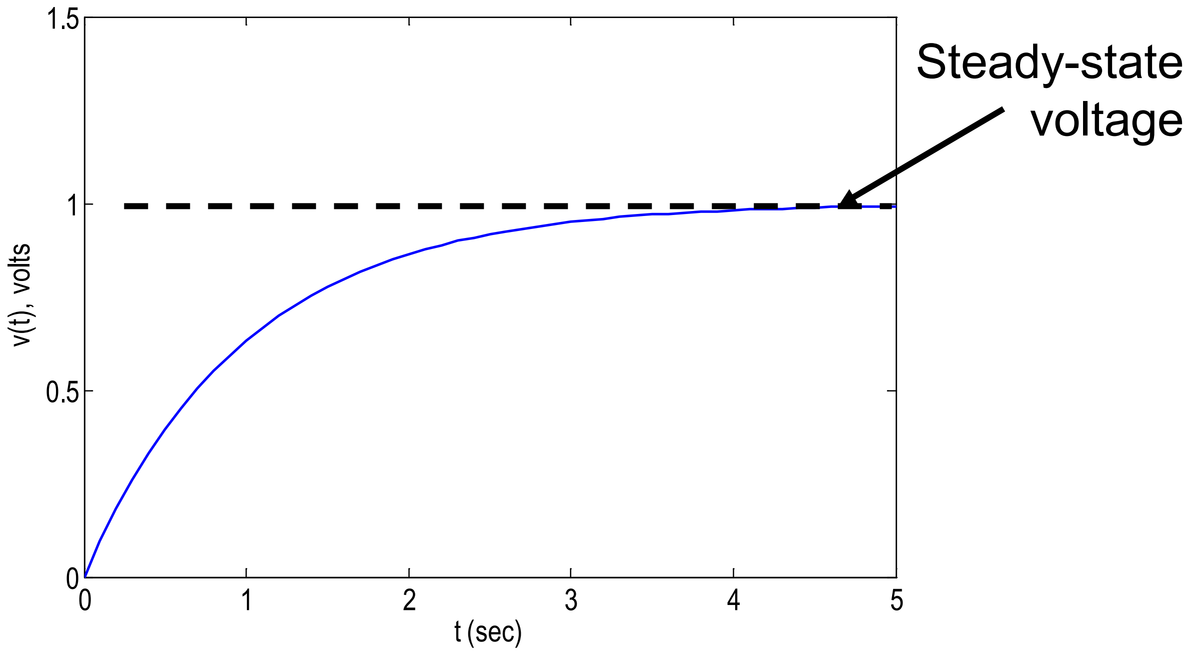
Circuits with Resistors, Capacitors and/or Inductors

We saw in the last module that RL and RC circuits are governed by differential equations. As a result, the solutions to these differential equations are not usually constants. That is, the voltages and currents in the circuit will be time-varying.

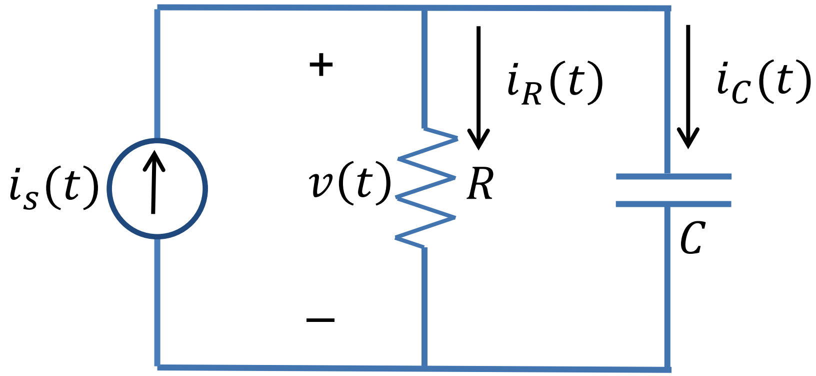
However, after a "long time," the circuits will settle into a stable behavior where things stop changing. We refer to this as "steady state."

In this set of slides we will look at the steady state behavior of RC and RL circuits…

…and how to use the steady state behavior to find initial conditions.



Steady State Behavior of an RC Circuit



Recall that this circuit was governed by the differential equation:

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{i_s}{C}$$

In steady-state, things are no longer changing => derivatives are zero.

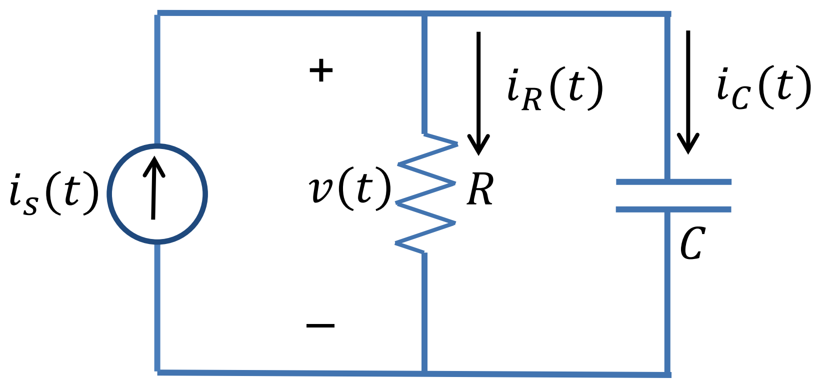
For the above circuit, in steady-state, $\frac{dv}{dt} = 0$, so that:

$$\frac{v}{RC} = \frac{i_s}{C} \Rightarrow v = Ri_s.$$

From this we can also conclude that $i_R = i_s$ and $i_C = 0$. (why?)

Steady State Behavior of an RC Circuit

We can also determine the steady state behavior of the circuit without having to develop the underlying differential equation.



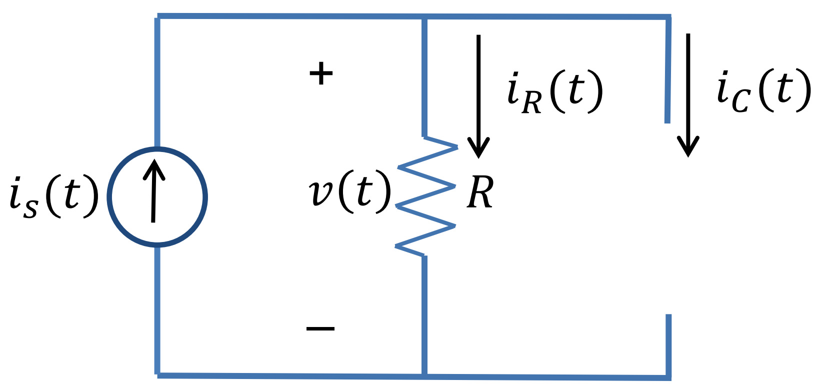
Recall that for a capacitor,

$$i_C(t) = C\,\frac{dv}{dt}.$$

In steady-state, $\frac{dv}{dt} = 0$, so that $i_C = 0$.

The capacitor acts like an open circuit in steady-state (no current flows).

Steady-state equivalent circuit



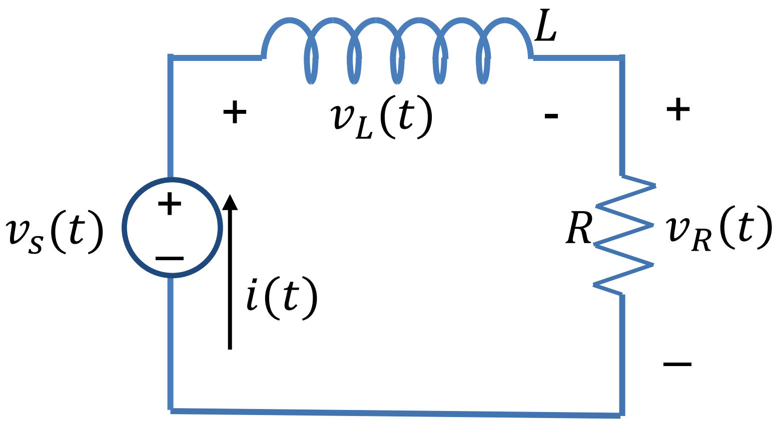
$$i_C = 0$$

$$i_R = i_s$$

$$v = R i_s$$

Steady State Behavior of an RL Circuit

We can follow an identical procedure to determine the steady state behavior of circuits with inductors.



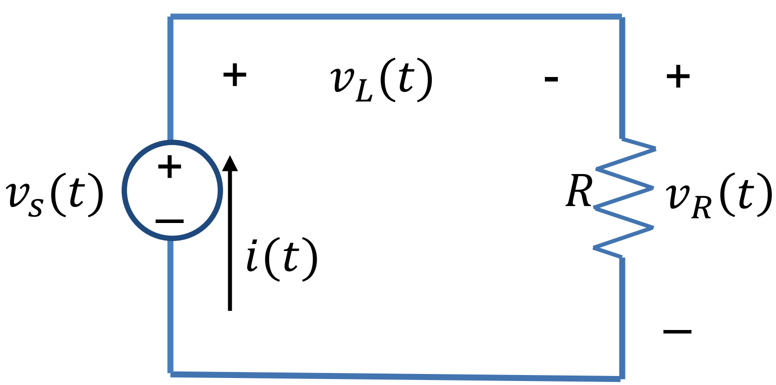
Recall that for an inductor,

$$v_L(t) = L\frac{di}{dt}.$$

In steady-state, $\frac{di}{dt} = 0$, so that $v_L = 0$.

The inductor acts like a short circuit in steady-state (no voltage).

Steady-state equivalent circuit



$$v_L = 0$$

$$v_R = v_s$$

$$i = \frac{v_s}{R}$$

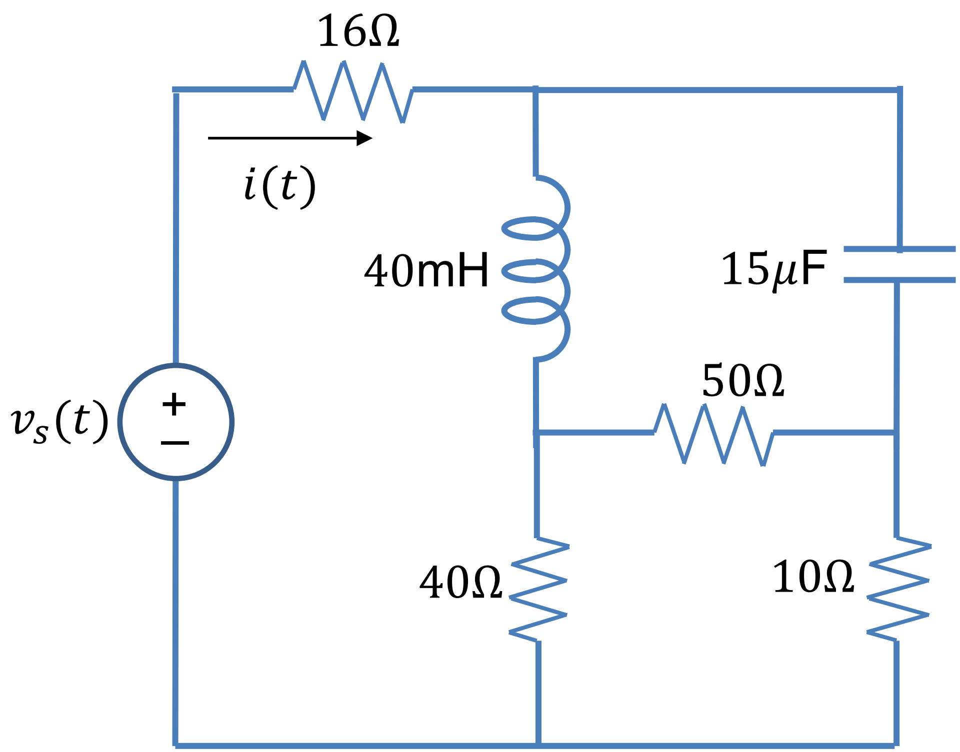
Example:

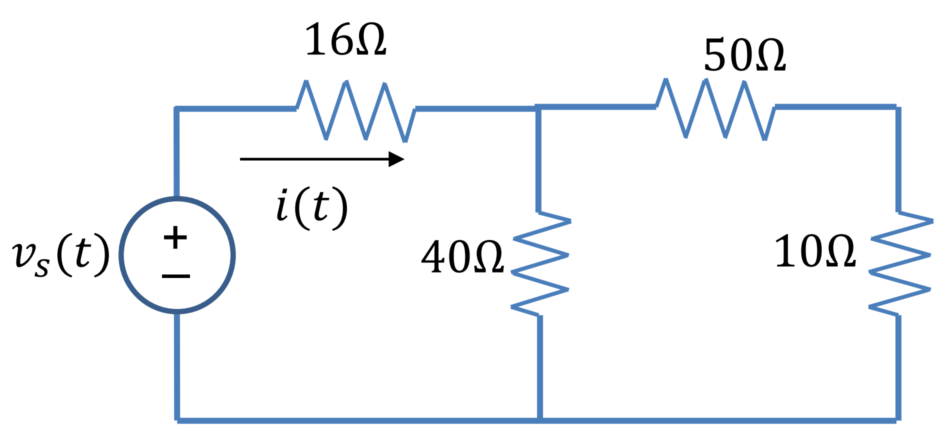
In the circuit shown, the voltage source provides

$$v_s(t) = 12 - 6e^{-4t}.$$

(a) Find the current through the 16Ω resistor, $i(t)$, in steady-state.

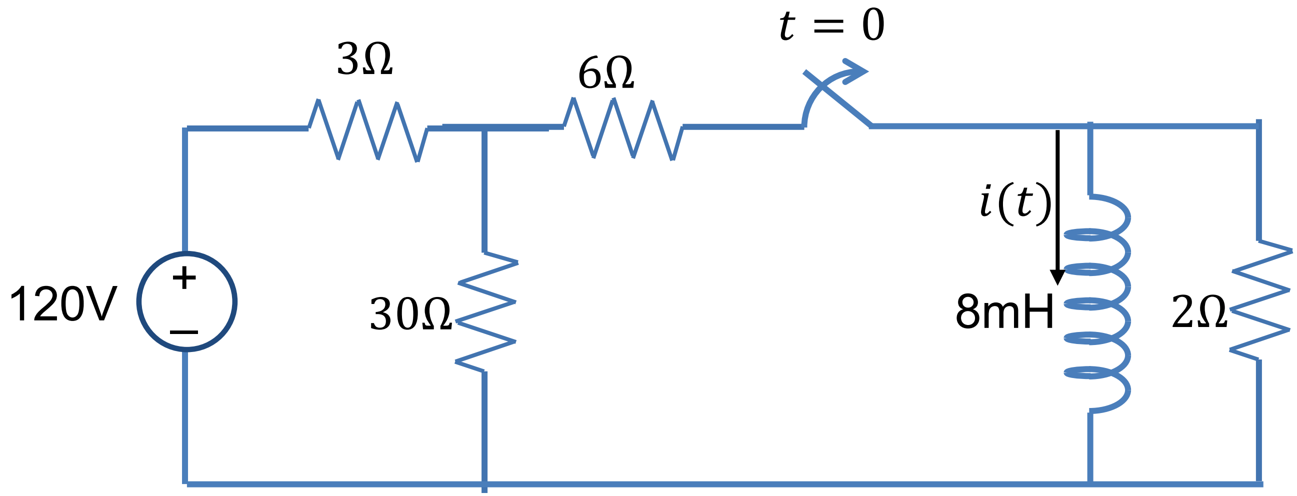
(b) Find the voltage across the capacitor in steady-state.





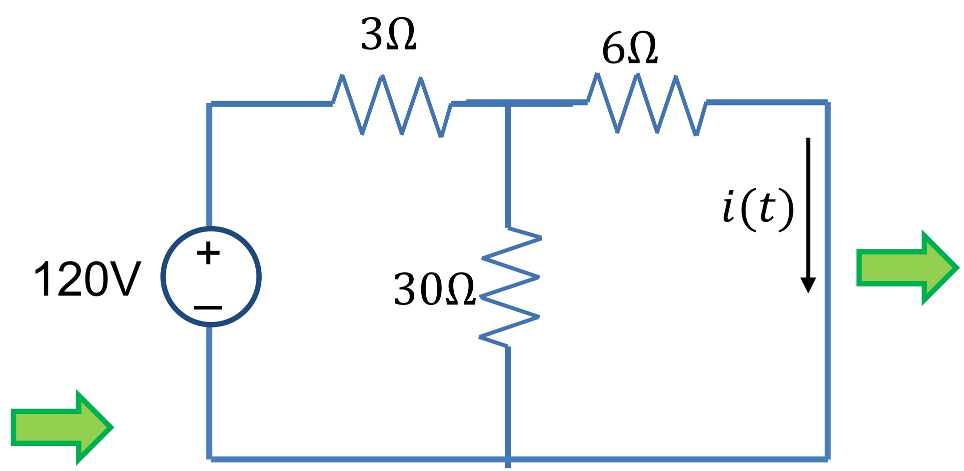
Instantaneous Changes in an RL Circuit

We can use our knowledge about instantaneous changes in capacitors and inductors to help us analyze circuits with switches.



For time, $t < 0$, switch is closed:

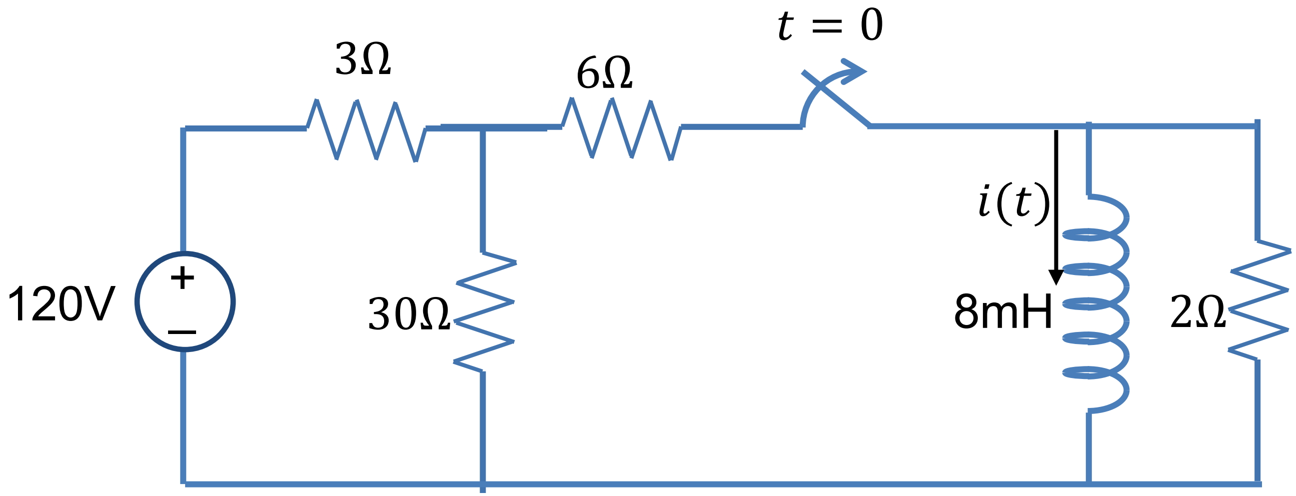
- In steady state ($t = 0^-$) inductor acts like a short
- 2Ω resistor is shorted
- S.S. equivalent circuit is as shown

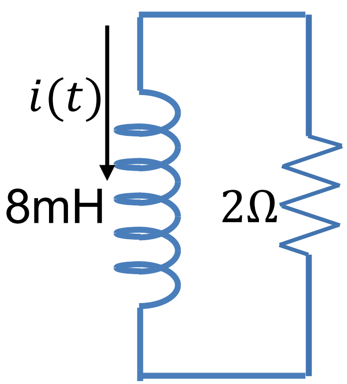


$i(0^-)$
$= 12.5A$

Instantaneous Changes in an RL Circuit

After the switch opens. The left part of the circuit is disconnected leaving just an RL circuit.

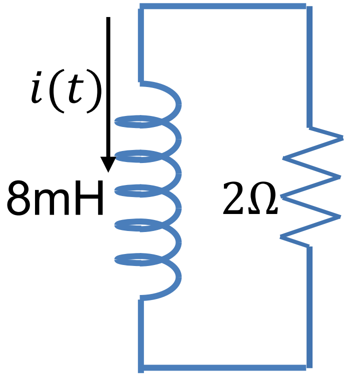




Since the current through an inductor cannot change instantaneously, the current immediately after the switch opens must be the same as the current immediately prior to the switch opening.

$$i(0^+) = i(0^-) = 12.5\text{A}.$$

Instantaneous Changes in an RL Circuit



For $t > 0$, the RL circuit is governed by the differential equation

$$8 * 10^{-3} \frac{di}{dt} + 2i = 0$$

Subject to the initial condition

$$i(0) = 12.5A.$$

The solution to this ODE is:

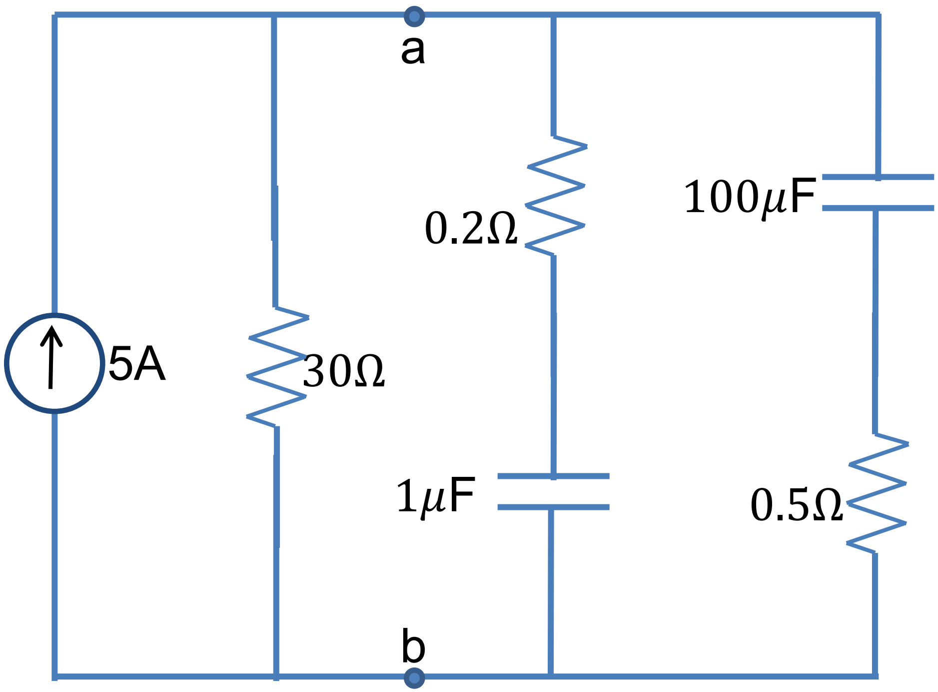
$$i(t) = 12.5e^{-250t} \text{Amps.}$$

The voltage across both the resistor and the inductor is

$$v(t) = 25e^{-250t} \text{ Volts.}$$

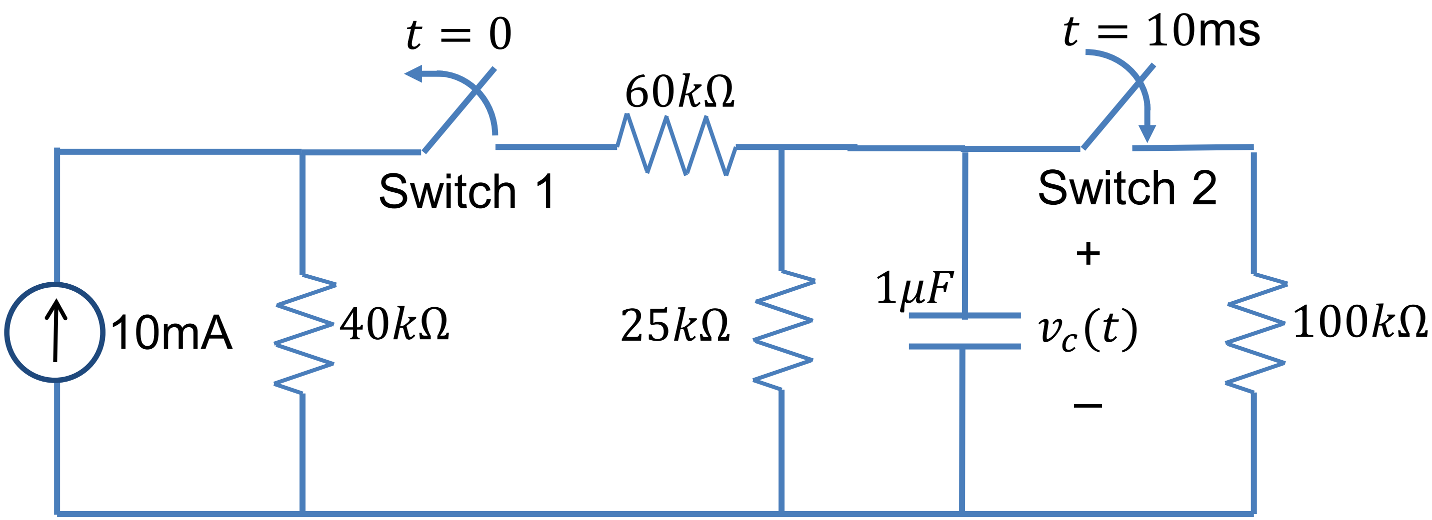
What is the polarity of that voltage?

Example



After the circuit has been in operation for a long time, a screwdriver is inadvertently connected across the terminals a-b at time $t = 0$. Assuming the resistance of the screwdriver is negligible, find the current in the screwdriver at $t = 0^{+}$ and $t = \infty$.

Example

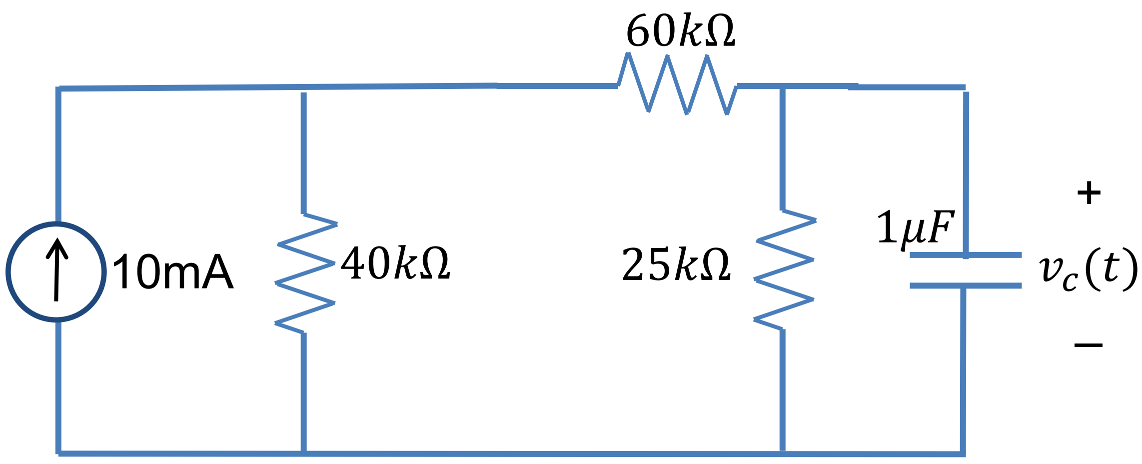


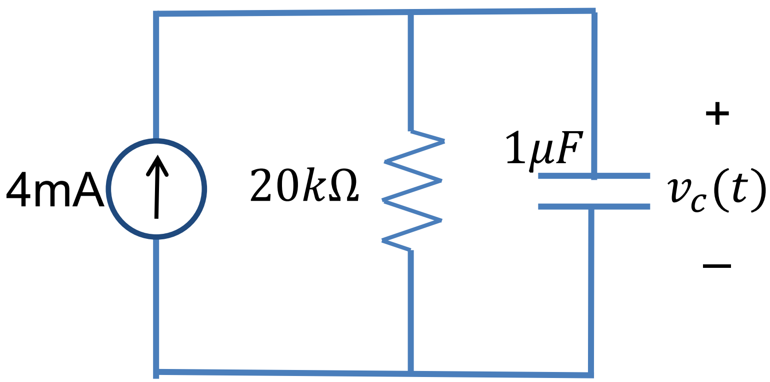
In the circuit shown, switch 1 has been closed and switch 2 has been open for a long time. At $t = 0$ switch 1 is opened. Then 10ms later, switch 2 is closed.

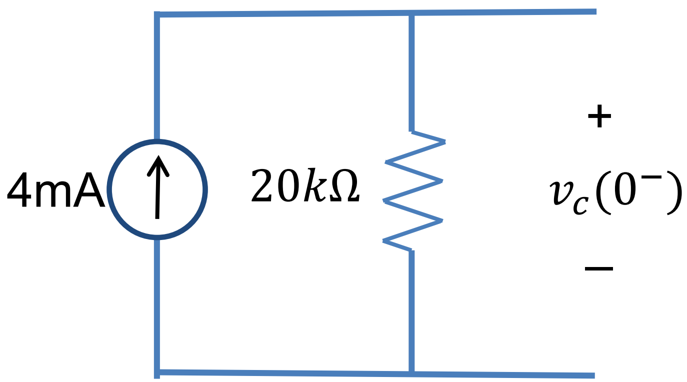
(a) Find $v_c(t)$ for $0 \leq t \leq 10$ms.

(b) Find $v_c(t)$ for $t \geq 10$ms.

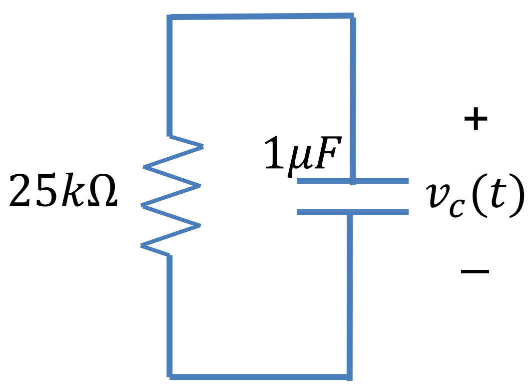
Case 1: $t < 0$ (switch 1 closed, switch 2 open)







Case 2: $0 < t < 10ms$ (switch 1 open, switch 2 open)



Case 3: $0 < t < 10ms$ (switch 1 open, switch 2 closed)