Sinusoids in Resistors

Suppose the current in a resistor is sinusoidal,

$$i(t) = I_o \cos(\omega t + \theta).$$

Since $v(t) = Ri(t)$, the voltage will also be a sinusoid,

$$v(t) = RI_o \cos(\omega t + \theta).$$

In terms of phasors,

$$\boldsymbol{I} = I_o e^{j\theta} \text{ and } \boldsymbol{V} = RI_o e^{j\theta}.$$

Therefore, the voltage and current phasors in a resistor are related by

$$\boxed{\boldsymbol{V} = \boldsymbol{I}R.}$$

That is, Ohm's Law still applies in the phasor domain

Sinusoids in Inductors

Suppose the current in an inductor is sinusoidal,

$$i(t) = I_o \cos(\omega t + \theta).$$

Since $v(t) = L\frac{di}{dt}$, the voltage will be,

$$\begin{aligned} v(t) &= -\omega L I_o \sin(\omega t + \theta) \\ &= -\omega L I_o cos(\omega t + \theta - 90°) \end{aligned}$$

In terms of phasors,

$\boldsymbol{I} = I_o e^{j\theta}$ and $\boldsymbol{V} = -\omega L I_o e^{j(\theta - 90°)} = -\omega L I_o e^{-j90°} e^{j\theta} = j\omega L I_o e^{j\theta}$

Therefore, the voltage and current phasors in an inductor are related by

$$\boxed{\boldsymbol{V} = j\omega L \boldsymbol{I}.}$$

Note that in the phasor domain, there is no longer a derivative in this relationship. It looks just like Ohm's Law with R replaced with $j\omega L$.

Sinusoids in Capacitors

Suppose the voltage in a capacitor is sinusoidal,

$$v(t) = V_o \cos(\omega t + \theta).$$

Since $i(t) = C\frac{dv}{dt}$, the current will be,

$$\begin{aligned} i(t) &= -\omega C V_o \sin(\omega t + \theta) \\ &= -\omega C V_o cos(\omega t + \theta - 90°) \end{aligned}$$

In terms of phasors,

$$\boldsymbol{V} = V_o e^{j\theta} \text{ and } \boldsymbol{I} = -\omega C V_o e^{j(\theta - 90°)} = -\omega C V_o e^{-j90°} e^{j\theta} = j\omega C V_o e^{j\theta}$$

Therefore, the voltage and current phasors in a capacitor are related by

$$\boxed{\boldsymbol{I} = j\omega C\boldsymbol{V} \text{ or } \boldsymbol{V} = \frac{\boldsymbol{I}}{j\omega C}}$$

Note that in the phasor domain, there is no longer a derivative in this relationship. It looks just like Ohm's Law with R replaced with $1/j\omega C$.

Complex Impedance

For the purposes of analyzing circuits with Rs, Ls and Cs that are driven by sinusoidal (AC) sources, we can analyze the circuit using phasors in which case the voltage/current relationship for all three types of elements are basically the same:

$$\boxed{\boldsymbol{V} = \boldsymbol{I}Z.}$$

The quantity Z is known as the (complex) impedance.

Element	Time Domain	Phasor Domain	Z
Resistor	$v(t) = Ri(t)$	$\boldsymbol{V} = \boldsymbol{I}R$	R
Inductor	$v(t) = L\frac{di(t)}{dt}$	$\boldsymbol{V} = j\omega L\boldsymbol{I}$	$j\omega L$
Capacitor	$i(t) = C\frac{dv(t)}{dt}$	$\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$	$\frac{1}{j\omega C}$

Complex Impedance

Some nomenclature associated with the complex impedance Z is as follows.

$$Z = x + jy$$

where

x = resistance,
y = reactance.

$$Y = \frac{1}{Z} = g + jh$$

where

g = conductance,
h = susceptance.

	Resist-ance	React-ance	Conduct-ance	Suscept-ance
R	R	0	$\frac{1}{R}$	0
L	0	ωL	0	$-\frac{1}{\omega L}$
C	0	$-\frac{1}{\omega C}$	0	ωC

Inductors and capacitors have no resistive component and are called reactive elements. Inductors have a positive reactance while capacitors have a negative reactance.

Phase Relationships

It is common to talk about the phase relationships between the voltage and current sinusoids in various circuit elements.

Resistor: $\boldsymbol{V} = \boldsymbol{I}R$ Voltage phase = Current phase

Inductor: $\boldsymbol{V} = j\omega L\boldsymbol{I}$ Voltage phase = Current phase + $90°$.
(voltage leads current)

Capacitor: $\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$ Voltage phase = Current phase - $90°$.
(voltage lags current)

voltage
(emf)
current

ELI the **ICE** man

inductor capacitor

Complex Impedance

Example $v(t)$ is an 80kHz sinusoid with a peak amplitude of 25mV and zero phase. When $v(t)$ is applied across a capacitor, $i(t)$ has a peak amplitude of 628.32μA. Find:

(a) The current frequency in rad/sec.
(b) The current phase.
(c) The reactance of the capacitor.
(d) The capacitance of the capacitor.
(e) The impedance of the capacitor.