$\sin_a cos.pdf$

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1 Introduction

The graph of $\sin(\cos^{-1}(x))$ and $-\sqrt{1-x^2}$ are reflections of each other about the x-axis. They each form a half circle, upper and lower half respectively.

Proof of this relationship: Set $\theta = \cos^{-1}(x)$. Then the expression $\sin(\cos^{-1}(x))$ becomes $\sin(\theta)$. From $\theta = \cos^{-1}(x)$, we have $\cos(\theta) = x$. We can write this as $\cos(\theta) = \frac{x}{1}$, which represents the adjacent side over the hypotenuse in a right triangle. Consider a right triangle with an angle θ, an adjacent side of x, and a hypotenuse of 1. By the Pythagorean theorem, the opposite side is $\sqrt{1^2 - x^2} = \sqrt{1-x^2}$. Then, $\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$. So, $\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$.

The function $y = \sqrt{1-x^2}$ represents the upper half of a unit circle centered at the origin (since $y \geq 0$). When reflected about the x-axis, this function becomes $y = -\sqrt{1-x^2}$, which represents the lower half of the same unit circle. Thus, $\sin(\cos^{-1}(x))$ and $-\sqrt{1-x^2}$ are indeed reflections of each other about the x-axis. Q.E.D.