

$n=1, 2, 3, 4, \dots$ but $n \in \mathbb{R}$ works as $f(n)$ is analytic there.

$$f(n) = \left\{ \frac{2}{1}, -\frac{1}{1}, 1, -\frac{1}{2}, \frac{2}{3}, -\frac{1}{3}, \dots \right\}$$

I first thought of two harmonic series and writing

$$f(n) = \left\{ \frac{2}{n}, -\frac{1}{n} \right\} \text{ where } n \in 1, 2, 3, 4, \dots$$

which produces the given sequence. and (triples) but certainly $n \in \mathbb{R} \setminus 0$ works too (analytic)

$$\sum_{n=1}^{1000} f(n) = 2 + (-1) + (1) - \frac{1}{2} + \frac{2}{3} + (-\frac{1}{3}) + \dots$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots = \sum_{n=1}^{500} \frac{1}{n}$$

$$\approx 6.7928$$

using TI-84

However, I will use the equation from Berndt A.I. to evaluate

$$f(-2.7) \approx f(z) = \left(\frac{1 + e^{i\pi(z+1)}}{2} \right) \left(\frac{4}{z+1} \right) + \left(\frac{1 - e^{i\pi(z+1)}}{2} \right) \cdot \left(-\frac{2}{z} \right)$$

though this has two poles $z=0, 1$ (not analytic there)

$$f(-2.7) \approx -1.7154 + 1.252i$$

$f(z)$ is analytic on $\mathbb{R} \setminus \{0, 1\}$.

While I was able to generate the sequence and solve the sum, the analytic closed form for $f(n)$ or on \mathbb{R} will require more investigations, using complex analysis/trigonometry. (if it exists!)