LOGIC AND PROOFS

EXCLUSIVE - OR

Let p and q he propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

TRUTH TABLE:

p	**q**	**$p \oplus q$**
F	F	F
F	T	T
T	F	T
T	T	F

- When the **exclusive or** is used to connect the propositions p and q, the proposition "p or q (but not both)" is obtained.
- It is **true when p is true and q is false, and when p is false and q is true.**
- It is **false** when **both** p and q are false and when **both** are true.

EXAMPLE:

When we say,

"Students who have taken calculus or computer science, but not both, can enroll in this class."

we mean that students who have taken both calculus and a computer science course cannot take the class. Only those who have taken exactly one of the two courses can take the class.

Similarly, when a menu at a restaurant states,

"Soup or salad comes with an entrée, "

the restaurant almost always means that customers can have either soup or salad, but not both. Hence, this is an exclusive, rather than an inclusive, or.

CONVERSE, CONTRAPOSITIVE AND INVERSE

We can form some new conditional statements starting with a conditional statement
$p \rightarrow q$

- The proposition **$q \rightarrow p$** is called the **converse** of $p \rightarrow q$.
- The **contrapositive** of $p \rightarrow q$ is the proposition **$\neg q \rightarrow \neg p$**.
- The proposition **$\neg p \rightarrow \neg q$** is called the **inverse** of $p \rightarrow q$.

TRUTH TABLE:

- Conditional Statement $p \rightarrow q$ and Contrapositive $\neg q \rightarrow \neg p$

p	**q**	**$p \rightarrow q$**	**$\neg q$**	**$\neg p$**	**$\neg q \rightarrow \neg p$**
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

TRUTH TABLE:

- Inverse $\neg p \rightarrow \neg q$ and Converse $q \rightarrow p$

p	**q**	**$q \rightarrow p$**	**$\neg p$**	**$\neg q$**	**$\neg p \rightarrow \neg q$**
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

EXAMPLE:

Find the contrapositive, the converse, and the inverse of the conditional statement:

"The home team wins whenever it is raining."

SOLUTION:

Because " q whenever p" is one of the ways to express the conditional statement p q, the original statement can be rewritten as:

"If it is raining, then the home team wins."

Consequently, the contrapositive of this conditional statement is:

"If the home team does not win, then it is not raining."

The converse is

"If the home team wins, then it is raining."

The inverse is

"If it is not raining, then the home team does not win."

TRUTH TABLES OF COMPOUND PROPOSITIONS

EXAMPLE:

Construct the truth table of the compound proposition:

$(p \lor \neg q) \rightarrow (p \land q)$

SOLUTION:

The truth table for the proposition **$(p \lor \neg q) \rightarrow (p \land q)$** is:

p	**q**	**¬ q**	**p ∨ ¬ q**	**p ∧ q**	**(p ∨ ¬ q) → (p ∧ q)**
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

BIT

True – **1** False – **0**

Operations of BIT:

1. NEGATION
2. CONJUNCTION
3. DISJUNCTION
4. EXCLUSIVE OR

x	**y**	**x ∨ y**	**x ∧ y**	**x ⊕ y**
1	1	1	1	0
1	0	1	0	1
0	1	1	0	1
0	0	0	0	0

x	**y**	**~ x**	**x ∨ y**
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

x	**y**	**z**	**~ x**	**y ∧ z**	**~ x ∨ z**
1	1	1	0	1	1
1	1	0	0	0	0
1	0	1	0	0	1
1	0	0	0	0	0
0	1	1	1	1	1
0	1	0	1	0	1
0	0	1	1	0	1
0	0	0	1	0	1

BIT SPRING

True – **1** False – **0**

Operations of BIT SPRING:

1. Bitwise OR
2. Bitwise AND
3. Bitwise XOR

A: 1101 0011
B: 0011 1011
OR: 1111 1011

A: 0010 0011 0110 1110
B: 1100 0111 0110 0001
XOR: 1110 0100 0000 1111

A: 1110 1011 1010
B: 1110 1011 0101
AND: 1110 1011 0000