$$\sum_{k=0}^{n} q^k = \frac{1-q^{n-1}}{1-q} \qquad q \neq 1 \qquad\qquad a_n = a q^n \quad a \in \mathbb{R} \quad q \in \mathbb{R}$$

$$0,\bar{3} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots$$

prog. geom.

$$[0,3]_n = \frac{3}{10} + \frac{3}{10^2} + \dots \frac{3}{10^n} = \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{10^n} + \dots + \frac{1}{10^{n-1}}\right)$$

$$n=0 \qquad \sum_{k=0}^{0} q^k = q^0 = 1 \qquad\qquad \left.\frac{1-q^{n+1}}{1-q}\right|_{n=0} = \frac{1-q^{0+1}}{1-q} = 1$$

supponiamo che la formula sia vero per un certo $N \in \mathbb{N}$

$\Rightarrow P(n+1)$ è vero

$$✓ \quad \sum_{k=0}^{N} q^k = \frac{1-q^{N+1}}{1-q} \overset{?}{\Rightarrow} \sum_{k=0}^{N+1} q^k = \frac{1-q^{N+2}}{1-q} \; ?$$

$$\sum_{k=0}^{N+1} q^k = (1 + q + q^2 + \dots + q^N) + q^{N+1} = \left(\sum_{k=0}^{N} q^k\right) + q^{N+1}$$

$$= \frac{1-q^{N+1}}{1-q} + q^{N+1} = \frac{1-q^{N+1} + (1-q)q^{N+1}}{1-q}$$

$$= \frac{1-\cancel{q^{N+1}} + \cancel{q^{N+1}} - q^{N+2}}{1-q}$$

$$= \frac{1-q^{N+2}}{1-q}$$

$$[0,\bar{3}]_n = \frac{3}{10}\left(1 + \frac{1}{10} + \dots + \frac{1}{10^{n-1}}\right) = \frac{3}{10}\left[\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right]$$

$$\sum_{k=0}^{n-1} q^k \qquad q = \frac{1}{10}$$

$$= \frac{3}{10}\,\frac{10}{9}\left[1-\left(\frac{1}{10}\right)^n\right] = \frac{1}{3}\left[1-\left(\frac{1}{10}\right)^n\right]$$

$$\frac{1}{3} > [0,\bar{3}]_n \qquad\qquad \frac{1}{3} - [0,\bar{3}]_n = \frac{1}{3} - \frac{1}{3}\left[1-\left(\frac{1}{10}\right)^n\right] = \cancel{\frac{1}{3}} - \cancel{\frac{1}{3}} + \frac{1}{3}\left(\frac{1}{10}\right)^n$$

$$\frac{1}{3}\,\frac{1}{10^n} < \frac{1}{10^6} \qquad\qquad 10^n > \frac{10^6}{3}$$