All cycle decompositions of Symmetric grp S_5 with required swaps to return to original state with x & y added.

$(1)(2)(3)(4)(5)$ = no swaps

$(1\ 2)(3)(4)(5) = (x\,1)(y\,2)(x\,2)(y\,1)(x\,y) = 5.$

2cycle leads to ~~8~~4swaps + 1 if odd number of cycles of order >1.

$(1\ 2)(3\ 4)(5) = (x\,1)(y\,2)(x\,2)(y\,1)(x\,3)(y\,4)(x\,4)(y\,3) = 8.$

2 2cycles $= 4 \times 2 + 0$ since even.

$(1\ 2\ 3)(4)(5) = (x\,1)(x\,2)(y\,3)(x\,3)(y\,1)(x\,y) = 6.$

3 cycle = 6 = 5 + 1 since odd number of cycles.

$(1\ 2\ 3)(4\ 5) = 5 + 4 = 9$ ~~cycles~~ swaps.

$(1\ 2\ 3\ 4)(5) = (x\,1)(x\,2)(x\,3)(y\,4)(x\,4)(y\,1)(x\,y)$
= 7 swaps.

$(1\ 2\ 3\ 4\ 5) = (x\,1)(x\,2)(x\,3)(x\,4)(y\,5)(x\,5)(y\,1)(x\,y)$
= 8 swaps.

Formula for swaps: N people.
(π is arrangement we wish to fix by utilising x & y)

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & \cdots & N \\ a & b & c & d & \cdots & k \end{pmatrix}.$$

isolate cycles & add x & y to each.
$(1\ a\ \cdots)\ \cdots$

Then number of swaps to return everyone to normal without knowing order in which swaps occurred to reach permutation π.

$$= \left(\sum_{i=1}^{p} o(C_i) + 2\right) + (p \bmod 2)$$

C_i = ith cycle where ~~all~~ cycle ≥ 2 (exclude 1-cycles).
$o(C_i)$ = order of ith cycle. (number of terms in cycle decomposition of ith cycle).

p = number ~~all~~ of distinct ~~no~~ cycles in permutation of order ≥ 2.