Algebra

Notes and Practice

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Contents

1 Introduction

This document contains concise notes and worked examples.

2 Algebra Basics

2.1 Constants

A **constant** is a fixed value that does not change. Unlike a **variable**, which can represent different numbers, a **constant** always has the same value.

Examples of **constant** include:

- Specific numbers like 2, -7, or $\frac{1}{2}$
- Mathematical constants like $\pi \approx 3.1416$ or $e \approx 2.718$

In the equation:

$$x + 3 = 7$$

the number 3 and 7 are constants — they stay the same, while x is the variable we solve for.

2.2 Variables

A **variable** is a symbol, usually a letter like x, y, or z, that represents an unknown or changeable value.

For example, in the equation:

$$x + 3 = 7$$

the variable x represents a number. Solving the equation means finding the value of x that makes the equation true in this case, $x = 4$.

Variables are fundamental in algebra because they allow us to generalize problems and create formulas.

2.3 Coefficients

A **coefficient** is the numerical factor multiplied by a variable in an algebraic expression.

For example, in the term:

$$5x$$

the number 5 is the coefficient of the variable x. It tells us how many times x is being counted or scaled.

More examples:

- In $-3y$, the coefficient is -3
- In $\frac{1}{2}a$, the coefficient is $\frac{1}{2}$
- In z, the coefficient is implicitly 1, since $z = 1 \cdot z$

Coefficients help determine the slope of a line in linear equations and play a major role in simplifying and solving expressions.

3 Equations vs. Expressions

Understanding the difference between expressions and equations is essential in algebra.

3.1 Expressions

An **expression** is a combination of numbers, variables, and operations (like addition or multiplication), but it does **not** contain an equals sign.

Examples:

$$2x + 5, \quad 3a^2 - 4, \quad \frac{1}{2}y$$

Expressions represent a value, but not a complete statement to solve. You can simplify or evaluate expressions, but you cannot "solve" them unless they're part of an equation.

3.2 Equations

An **equation** is a mathematical statement that two expressions are equal. It always contains an equals sign (=) and usually involves finding the value of a variable that makes the equation true.

Examples:

$$2x + 5 = 11, \quad a^2 = 16, \quad \frac{1}{2}y = 3$$

Solving an **equation** means determining the value(s) of the variable(s) that make both sides equal.

Summary Table

Expression	**Equation**
No equals sign	Has an equals sign
Represents a value	Represents a relationship
Can be simplified or evaluated	Can be solved
Example: $3x + 2$	Example: $3x + 2 = 11$

4 The Associative Property

The **associative property** refers to the grouping of terms using parentheses in addition or multiplication. It tells us that the way numbers are grouped does not change the result — only the order of operations inside parentheses changes, not the outcome.

4.1 Associative Property of Addition

The associative property of addition states:

$$(a + b) + c = a + (b + c)$$

You can add numbers in any grouping, and the sum will stay the same.

Example:

$$(2 + 3) + 4 = 5 + 4 = 9$$

$$2 + (3 + 4) = 2 + 7 = 9$$

So, $(2 + 3) + 4 = 2 + (3 + 4)$.

4.2 Associative Property of Multiplication

The associative property of multiplication states:

$$(a \times b) \times c = a \times (b \times c)$$

You can multiply in any grouping, and the product remains unchanged.

Example:

$$(2 \times 3) \times 4 = 6 \times 4 = 24$$

$$2 \times (3 \times 4) = 2 \times 12 = 24$$

So, $(2 \times 3) \times 4 = 2 \times (3 \times 4)$.

5 The Commutative Property

The **commutative property** describes how the order of numbers does not affect the result when adding or multiplying. It applies only to **addition and multiplication** — not subtraction or division.

5.1 Commutative Property of Addition

The commutative property of addition states:

$$a + b = b + a$$

You can change the order of the numbers being added without changing the sum.

Example:

$$4 + 7 = 11 \quad \text{and} \quad 7 + 4 = 11$$

So, $4 + 7 = 7 + 4$

5.2 Commutative Property of Multiplication

The commutative property of multiplication states:

$$a \times b = b \times a$$

You can change the order of the numbers being multiplied without changing the product.

Example:

$$6 \times 5 = 30 \quad \text{and} \quad 5 \times 6 = 30$$

So, $6 \times 5 = 5 \times 6$

6 Like Terms

Like terms are terms that have the same variable(s) raised to the same power(s). Only the numerical coefficients can be different. Like terms can be combined using addition or subtraction.

6.1 Addition and Subtraction of Like Terms

To combine like terms, simply add or subtract their coefficients.

Example 1:

$$3x + 5x = (3 + 5)x = 8x$$

Example 2:

$$7a^2 - 2a^2 = 5a^2$$

Note: You *cannot* combine terms that are not like terms.

$$4x + 2x^2 \neq 6x^2 \quad \text{(not like terms)}$$

6.2 Multiplication and Division of Like Terms

When multiplying or dividing like terms, you combine coefficients and apply exponent rules to the variables.

Multiplication Example:

$$(3x)(2x) = 6x^2$$

$$(4a^2)(-2a^3) = -8a^5$$

Division Example:

$$\frac{10x^3}{2x} = 5x^2$$

$$\frac{-6y^4}{3y^2} = -2y^2$$

Key Idea

- **Addition/Subtraction:** Combine only like terms (same variables and exponents)
- **Multiplication/Division:** Use exponent rules, even for unlike terms

7 The Distributive Property

The **distributive property** connects multiplication and addition or subtraction. It allows you to multiply a number or variable by each term inside parentheses.

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

This property is used frequently in algebra to expand expressions and solve equations.

Examples

Example 1 (with numbers):

$$3(4+5) = 3 \cdot 9 = 27$$

$$3 \cdot 4 + 3 \cdot 5 = 12 + 15 = 27$$

Example 2 (with variables):

$$x(2+y) = 2x + xy$$

Example 3 (with subtraction):

$$5(a-3) = 5a - 15$$

Why It Matters

The distributive property helps you:

- Expand expressions like $2(x+3)$
- Simplify algebraic expressions
- Solve equations more efficiently
- Factor expressions in reverse

More on the Distributive Property

The distributive property is also essential when working with variables, negative numbers, and factoring expressions.

Example with Variables and Negatives

$$-2(x-4) = -2 \cdot x + (-2) \cdot (-4) = -2x + 8$$

Notice:

- The negative sign distributes to both terms
- Be careful with signs: $-2 \cdot -4 = +8$

Common Mistake to Avoid

Incorrect:

$$3(x+2) = 3x + 2 \quad \text{(Only distributed to } x\text{)}$$

Correct:

$$3(x+2) = 3x + 6$$

Always distribute to *every* term inside the parentheses.

Using the Distributive Property to Factor

The distributive property also works *in reverse*, which is how we factor expressions.

$$6x + 12 = 6(x + 2)$$

Here, we pulled out the common factor of 6 — essentially undoing the distribution.

Factoring is the process of writing an expression as a product using the distributive property in reverse.

Tip: Look for a greatest common factor (GCF) before factoring!

8 Polynomials

A **polynomial** is an expression made up of variables, constants, and exponents, combined using addition, subtraction, and multiplication — but no variables in the denominator or under radicals.

Examples of Polynomials

$$3x^2 + 2x - 5, \quad x^3 - 4x + 7, \quad 2a^2b + 3ab^2$$

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8.1 Addition and Subtraction of Polynomials

To add or subtract polynomials:

- Combine like terms (same variables raised to the same powers)
- Add/subtract the coefficients of like terms

Example 1 — Addition:

$$(2x^2 + 3x + 1) + (x^2 + 4x - 5)$$
$$= (2x^2 + x^2) + (3x + 4x) + (1 - 5) = 3x^2 + 7x - 4$$

Example 2 — Subtraction:

$$(5x^2 - 2x + 6) - (3x^2 + x - 4)$$
$$= (5x^2 - 3x^2) + (-2x - x) + (6 + 4) = 2x^2 - 3x + 10$$

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8.2 Multiplication of Polynomials

To multiply polynomials:

- Use the distributive property (FOIL for binomials)
- Multiply each term in the first polynomial by each term in the second
- Combine like terms

Example:

$$(x+2)(x+5) = x(x+5) + 2(x+5) = x^2 + 5x + 2x + 10 = x^2 + 7x + 10$$

Another Example:

$$(2x-3)(x^2+x-4) = 2x(x^2+x-4) - 3(x^2+x-4)$$

$$= 2x^3 + 2x^2 - 8x - 3x^2 - 3x + 12 = 2x^3 - x^2 - 11x + 12$$

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8.3 Division of Polynomials (Intro)

Dividing polynomials can be done using:

- Long division
- Synthetic division (when dividing by linear terms like $x - a$)

Basic Example:

$$\frac{6x^2 + 9x}{3x} = \frac{6x^2}{3x} + \frac{9x}{3x} = 2x + 3$$

Note: More complex division techniques will be covered in a later section.

Polynomial Operations Summary

Polynomials are expressions with variables and constants using only addition, subtraction, and multiplication (no variables in denominators or exponents).

Addition/Subtraction:
- Combine like terms (same variable and exponent)
- Only coefficients are added or subtracted

Multiplication:
- Use the distributive property or FOIL
- Multiply each term in one polynomial by each term in the other
- Combine like terms

Division:
- Simplify each term if possible
- For complex division, use long division or synthetic division (covered later)

8.4 Long Division

Long division is a step-by-step method for dividing numbers or algebraic expressions.

- **Dividend:** the number or expression being divided
- **Divisor:** the number or expression you are dividing by
- **Quotient:** the result of the division (goes on top)

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Example 1: Long Division with Whole Numbers

Divide:

$$125 \div 5$$

Here:

- Dividend = 125
- Divisor = 5
- Quotient = 25

$$\begin{array}{c|c} 5 & 125 \\ \hline & 25 \end{array}$$

Steps:

1. Divide 12 by 5 $\rightarrow$ 2 (since $5 \times 2 = 10$)
2. Subtract: $12 - 10 = 2$, bring down the 5 $\rightarrow$ 25
3. Divide 25 by 5 $\rightarrow$ 5 (since $5 \times 5 = 25$)
4. Final result: 25

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Example 2: Polynomial Long Division

Divide:

$$\frac{x^2 + 3x + 2}{x + 1}$$

Here:

- Dividend $= x^2 + 3x + 2$
- Divisor $= x + 1$
- Quotient = the expression that results from division

Step 1: Divide leading terms:

$$x^2 \div x = x$$

Step 2: Multiply and subtract:

$$(x^2 + 3x + 2) - (x)(x + 1) = x^2 + 3x + 2 - (x^2 + x) = 2x + 2$$

Step 3: Divide leading terms again:

$$2x \div x = 2$$

Step 4: Multiply and subtract:

$$(2x + 2) - 2(x + 1) = 2x + 2 - (2x + 2) = 0$$

So the division is exact.

Final Answer:

$$\frac{x^2 + 3x + 2}{x + 1} = x + 2$$

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Terminology Review

- **Dividend:** what you're dividing (e.g., $x^2 + 3x + 2$)
- **Divisor:** what you're dividing by (e.g., $x + 1$)
- **Quotient:** result of the division (e.g., $x + 2$)

8.5 Multivariable Polynomial Long Division

Polynomial long division can also be performed with expressions that include more than one variable. The process is similar, but care must be taken to match like terms correctly and order terms consistently by degree.

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Example: Divide $6x^2y + 9xy^2$ by $3xy$

Here:

- Dividend: $6x^2y + 9xy^2$
- Divisor: $3xy$

Step 1: Divide the first term of the dividend by the first term of the divisor

$$\frac{6x^2y}{3xy} = 2x$$

Step 2: Multiply the entire divisor by $2x$

$$2x \cdot (3xy) = 6x^2y$$

Step 3: Subtract

$$(6x^2y + 9xy^2) - 6x^2y = 9xy^2$$

Step 4: Divide next term

$$\frac{9xy^2}{3xy} = 3y$$

Step 5: Multiply and subtract

$$3y \cdot (3xy) = 9xy^2$$

$$9xy^2 - 9xy^2 = 0$$

Final Answer:

$$\frac{6x^2y + 9xy^2}{3xy} = 2x + 3y$$

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Tips for Multivariable Long Division

- Organize terms in descending order of one variable (typically x)
- Divide one term at a time, matching both variable parts and coefficients
- Use standard subtraction to cancel each step before proceeding

9 Quadratic Polynomials

A **quadratic polynomial** is a polynomial of degree 2, meaning the highest power of the variable is 2.

General Form

$$ax^2 + bx + c$$

where:

- a, b, c are real numbers and $a \neq 0$
- a is the **leading coefficient**
- b is the **linear coefficient**
- c is the **constant term**

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Examples

- $x^2 + 5x + 6$
- $3x^2 - 2x + 1$
- $-x^2 + 4$

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Graphing Quadratics

Quadratic functions graph as a **parabola**. The shape of the parabola depends on the sign of a:

- If $a > 0$: the parabola opens **upward** (like a smile)
- If $a < 0$: the parabola opens **downward** (like a frown)

The highest or lowest point on the parabola is called the **vertex**.

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Solving Quadratic Equations

Quadratics can be solved using several methods:

- **Factoring**

- **Completing the square**
- **Quadratic formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Quick Facts About Quadratics

- Degree = 2 (because of x^2)
- Graph = parabola
- May have 0, 1, or 2 real roots depending on the discriminant $b^2 - 4ac$
- Coefficients tell you the shape and position of the graph

9.1 Factoring a Quadratic Polynomial

Factoring is one of the most common ways to solve a quadratic equation when it can be written as a product of two binomials.

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Example: Factor $x^2 + 5x + 6$

Step 1: Look for two numbers that multiply to 6 (the constant term) and add to 5 (the linear coefficient).

$$\text{Factors of 6: } (1, 6),\ (2, 3)$$
$$2 + 3 = 5 \quad \Rightarrow \quad \text{Use 2 and 3}$$

Step 2: Write the factored form:

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Step 3: Check by expanding:

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

Confirmed!

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Factoring Tips

- Always look for a greatest common factor (GCF) first
- Use a factoring method like:
  - Simple guess-and-check
  - Box or area method
  - AC method (for trinomials where $a \neq 1$)
- Always double-check by expanding!

9.2 Difference of Squares

The **difference of squares** is a special factoring pattern that applies when a binomial consists of two perfect squares being subtracted.

$$a^2 - b^2 = (a-b)(a+b)$$

This works because when expanded:

$$(a-b)(a+b) = a^2 + ab - ab - b^2 = a^2 - b^2$$

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Examples

- $x^2 - 9 = (x-3)(x+3)$
- $4x^2 - 25 = (2x-5)(2x+5)$
- $49y^2 - 1 = (7y-1)(7y+1)$
- $x^4 - 16 = (x^2)^2 - 4^2 = (x^2-4)(x^2+4) = (x-2)(x+2)(x^2+4)$

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Important Notes:

- Works only with **subtraction**, not addition
- Both terms must be perfect squares
- Often used to simplify expressions or solve equations

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Difference of Squares Summary

$$a^2 - b^2 = (a-b)(a+b)$$

Examples:

- $x^2 - 16 = (x-4)(x+4)$
- $9a^2 - 1 = (3a-1)(3a+1)$

9.3 Completing the Square

To solve a quadratic equation of the form:

$$ax^2 + bx + c = 0$$

we can complete the square — a method that rewrites the expression as a perfect square trinomial.

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What is a Trinomial?

A **trinomial** is a polynomial with three terms.
In quadratic form:

$$ax^2 + bx + c$$

is a trinomial.
When completing the square, we turn:

$$x^2 + bx \quad \text{(2 terms)}$$

into:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 \quad \text{(3 terms — a perfect square trinomial)}$$

Steps to Complete the Square (when $a = 1$)

1. Move the constant to the other side:

$$x^2 + bx = -c$$

2. Take half of the coefficient of x, square it, and add to both sides:

$$\left(\frac{b}{2}\right)^2$$

3. Now the left-hand side is a perfect square:

$$\left(x + \frac{b}{2}\right)^2$$

4. Solve by taking the square root of both sides
5. Solve for x

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Example: Solve $x^2 + 6x + 4 = 0$

$$\begin{aligned} x^2 + 6x + 4 &= 0 \\ x^2 + 6x &= -4 \\ \left(\frac{6}{2}\right)^2 &= 9 \\ x^2 + 6x + 9 &= -4 + 9 \\ (x+3)^2 &= 5 \\ x + 3 &= \pm\sqrt{5} \\ x &= -3 \pm \sqrt{5} \end{aligned}$$

So the solution is:

$$x = -3 \pm \sqrt{5}$$

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Tip for Completing the Square

When $a \neq 1$, divide the whole equation by a first. Completing the square is also how we derive the quadratic formula!

Nature of Solutions: Real vs Complex

After completing the square (when $a = 1$), we get:

$$x^2 + bx + c = 0 \quad \Rightarrow \quad \left(x + \frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$$

The number and type of solutions depend on the value of:

$$\left(\frac{b}{2}\right)^2 - c$$

- If $\left(\frac{b}{2}\right)^2 - c > 0$, then x has **two real** solutions
- If $\left(\frac{b}{2}\right)^2 - c = 0$, then x has **one real** solution (a repeated root)
- If $\left(\frac{b}{2}\right)^2 - c < 0$, then x has **two complex** (nonreal) solutions

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Real vs Complex Solutions from Completing the Square

Given:

$$x^2 + bx + c = 0 \Rightarrow \left(x + \frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$$

Then:

- $\left(\frac{b}{2}\right)^2 - c > 0 \Rightarrow 2$ distinct real solutions
- $\left(\frac{b}{2}\right)^2 - c = 0 \Rightarrow 1$ real solution
- $\left(\frac{b}{2}\right)^2 - c < 0 \Rightarrow 2$ complex solutions

Example: Completing the Square with Imaginary Solutions

Solve the equation:

$$x^2 + 4x + 8 = 0$$

Step 1: Move the constant to the other side.

$$x^2 + 4x = -8$$

Step 2: Complete the square. Take half of 4 and square it:

$$\left(\frac{4}{2}\right)^2 = 4$$

Add 4 to both sides:

$$x^2 + 4x + 4 = -8 + 4$$
$$(x + 2)^2 = -4$$

Step 3: Take the square root of both sides.

$$x + 2 = \pm\sqrt{-4}$$

$$x + 2 = \pm 2i$$

Step 4: Solve for x.

$$x = -2 \pm 2i$$

Final Answer:

$$\boxed{x = -2 \pm 2i}$$

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Imaginary Solutions

When completing the square leads to a negative number under the square root, the equation has **two complex conjugate solutions**, involving the imaginary unit $i = \sqrt{-1}$.

Complex Conjugate Solutions and the Discriminant (When $a = 1$)

Consider the quadratic equation:

$$x^2 + 2x + 5 = 0$$

Completing the square:

$$x^2 + 2x = -5$$

Take half of 2 and square it:

$$\left(\frac{2}{2}\right)^2 = 1$$

Add 1 to both sides:

$$x^2 + 2x + 1 = -5 + 1$$
$$(x + 1)^2 = -4$$

Taking the square root:

$$x + 1 = \pm\sqrt{-4} = \pm 2i$$

Therefore,

$$x = -1 \pm 2i$$

Complex Conjugates: These solutions come in pairs called complex conjugates:

$$a + bi \quad \text{and} \quad a - bi$$

Here, $a = -1$ and $b = 2$.

Discriminant: The discriminant is:

$$\Delta = b^2 - 4ac$$

For the equation:

$$a = 1, \quad b = 2, \quad c = 5$$

$$\Delta = 2^2 - 4 \cdot 1 \cdot 5 = 4 - 20 = -16 < 0$$

Since $\Delta < 0$, the solutions are complex conjugates.

Summary

- $\Delta > 0$: Two distinct real solutions
- $\Delta = 0$: One repeated real solution
- $\Delta < 0$: Two complex conjugate solutions

The Quadratic Formula and the Discriminant

Any quadratic equation of the form:

$$ax^2 + bx + c = 0$$

can be solved using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: The expression under the square root, denoted by:

$$\Delta = b^2 - 4ac$$

is called the **discriminant**. It determines the type of solutions:

Discriminant Summary

- If $\Delta > 0$: two distinct real solutions
- If $\Delta = 0$: one real solution (a repeated root)
- If $\Delta < 0$: two complex conjugate solutions

Example: Solve $x^2 - 2x - 3 = 0$ using the quadratic formula.

Here, $a = 1$, $b = -2$, and $c = -3$. Compute the discriminant:

$$\Delta = (-2)^2 - 4(1)(-3) = 4 + 12 = 16$$

Now apply the formula:

$$x = \frac{-(-2) \pm \sqrt{16}}{2(1)} = \frac{2 \pm 4}{2}$$

So the two solutions are:

$$x = \frac{2+4}{2} = 3 \quad \text{and} \quad x = \frac{2-4}{2} = -1$$

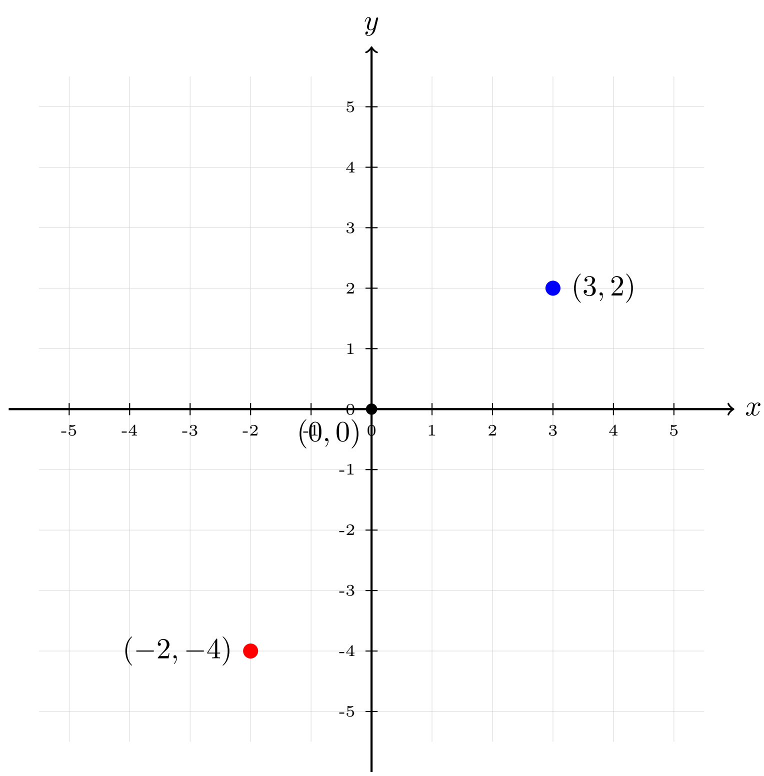
Conclusion: The quadratic formula is a universal method for solving any quadratic equation, and the discriminant tells you what kind of solutions to expect.

10 The Cartesian Plane

The Cartesian Plane is a two-dimensional coordinate system defined by a horizontal number line called the **x-axis**, and a vertical number line called the **y-axis**. These axes intersect at the **origin**, denoted as $(0, 0)$.

Each point on the plane is represented by an ordered pair (x, y), where:

- x is the horizontal value (left/right),
- y is the vertical value (up/down).



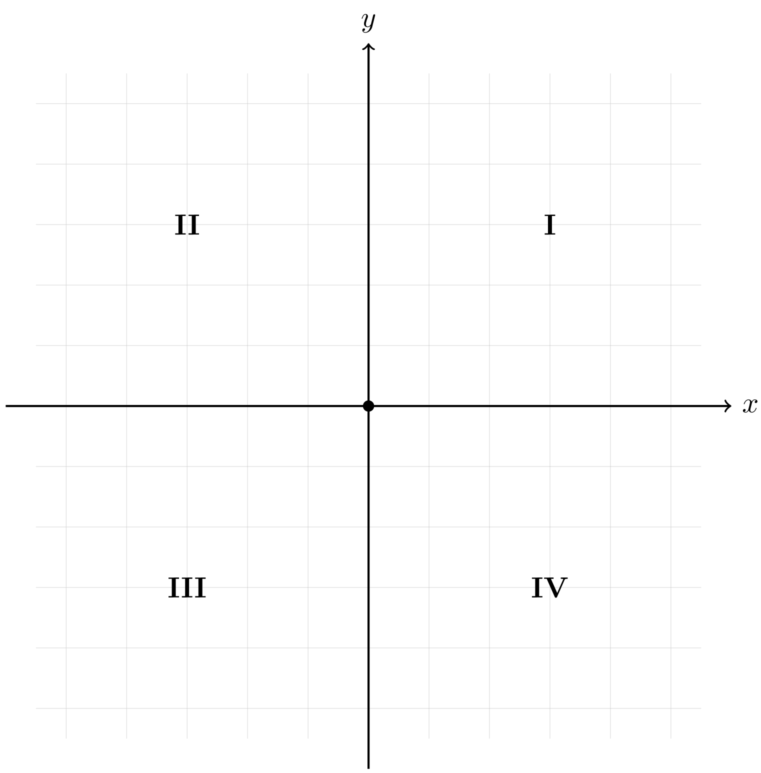
Summary

- The Cartesian Plane has two axes: x-axis (horizontal), y-axis (vertical).
- The point where they meet is the origin $(0, 0)$.
- Each point is identified by an ordered pair (Called a coordinate) (x, y).

10.1 The Four Quadrants

The Cartesian Plane is divided into four regions called **quadrants**. These are numbered in a counterclockwise direction starting from the upper-right:

- **Quadrant I**: $(+x, +y)$
- **Quadrant II**: $(-x, +y)$
- **Quadrant III**: $(-x, -y)$
- **Quadrant IV**: $(+x, -y)$



Summary of Quadrants

- Points in each quadrant have characteristic signs for x and y.
- The quadrants are labeled I through IV, moving counterclockwise.

10.2 Slope of a Line

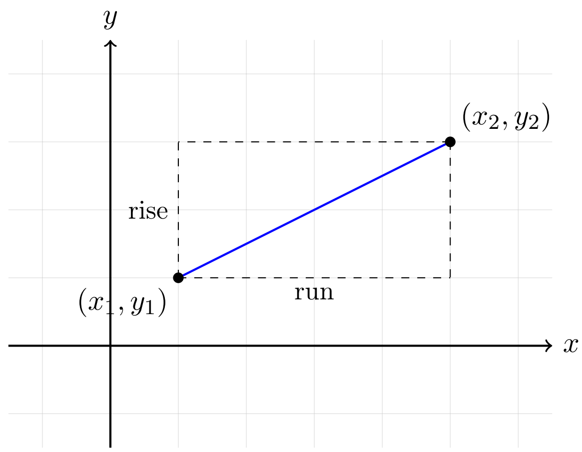
The **slope** of a line is a measure of its steepness. It tells us how much the y-value changes for a given change in the x-value.

Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope m is calculated using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This is also known as "rise over run":

- **Rise**: the vertical change $(y_2 - y_1)$
- **Run**: the horizontal change $(x_2 - x_1)$



Summary: Slope of a Line

- Slope measures how steep a line is.
- Use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate it.
- A positive slope rises left to right; negative slope falls.

10.3 Point-Slope Form of a Line

If you know the slope m of a line and a point (x_1, y_1) it passes through, you can write the line's equation using the **point-slope form**:

$$y - y_1 = m(x - x_1)$$

Requirements:

- A single point (x_1, y_1)
- The slope m

Example: Given the point $(2, 3)$ and slope $m = 4$, the equation becomes:

$$y - 3 = 4(x - 2)$$

This can be left in point-slope form or simplified to slope-intercept form.

Point-Slope Summary

- Use when you know one point and the slope.
- Formula: $y - y_1 = m(x - x_1)$

10.4 Slope-Intercept Form of a Line

The **slope-intercept form** expresses a linear equation as:

$$y = mx + b$$

Where:

- m is the slope of the line.
- b is the **y-intercept** — the value of y when $x = 0$.

Requirements:

- Slope m
- Y-intercept b

Example: If a line has slope $m = -2$ and y-intercept $b = 5$, the equation is:

$$y = -2x + 5$$

This form is especially useful for graphing.

Slope-Intercept Summary

- Use when slope and y-intercept are known.
- Formula: $y = mx + b$
- b tells you where the line crosses the y-axis.

10.5 Converting Between Forms of a Linear Equation

Linear equations can be written in multiple forms. It's often helpful to convert between them depending on the context (e.g., graphing, solving, or analyzing).

1. Point-Slope to Slope-Intercept

Start with the point-slope form:

$$y - y_1 = m(x - x_1)$$

Distribute the slope and solve for y to convert to slope-intercept form:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y &= m(x - x_1) + y_1 \quad \text{(Slope-Intercept Form: } y = mx + b\text{)} \end{aligned}$$

Example: Convert $y - 2 = 3(x + 1)$ to slope-intercept form:

$$\begin{aligned} y - 2 &= 3(x + 1) \\ y - 2 &= 3x + 3 \\ y &= 3x + 5 \end{aligned}$$

2. Two Points to Any Form

Given two points (x_1, y_1) and (x_2, y_2):

1. Find the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
2. Use point-slope form with one point
3. Convert to slope-intercept form if needed

Summary: Converting Forms

- Point-slope to slope-intercept: distribute and solve for y
- Two points $\rightarrow$ slope $\rightarrow$ point-slope $\rightarrow$ slope-intercept
- Use the form that best suits the problem (graphing, solving, etc.)

10.6 Undefined Slope and Vertical Lines

A **vertical line** goes straight up and down and has an **undefined slope**. This is because its run (change in x) is zero:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

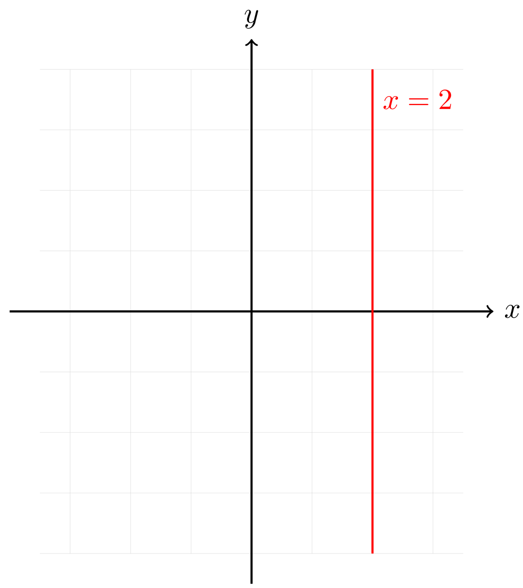
If $x_2 = x_1$, then the denominator becomes 0, and division by zero is undefined.

Equation of a Vertical Line:

A vertical line through $x = a$ is written as:

$$x = a$$

- It has an **undefined slope**.
- It has an **x-intercept only** — it does not cross the y-axis.



Important Notes on Vertical Lines

- Vertical lines have the form $x = a$
- Their slope is undefined.
- They do not cross the y-axis — no y-intercept exists.