Algebra

Notes and Practice

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July 18, 2025

Contents

1 Introduction

This document contains concise notes, worked examples, and practice problems to help rebuild my algebra foundation.

2 Algebra Basics

2.1 Constants

A **constant** is a fixed value that does not change. Unlike a **variable**, which can represent different numbers, a **constant** always has the same value.

Examples of **constant** include:

- Specific numbers like 2, -7, or $\frac{1}{2}$
- Mathematical constants like $\pi \approx 3.1416$ or $e \approx 2.718$

In the equation:

$$x + 3 = 7$$

the number 3 and 7 are constants — they stay the same, while x is the variable we solve for.

2.2 Variables

A **variable** is a symbol, usually a letter like x, y, or z, that represents an unknown or changeable value.

For example, in the equation:

$$x + 3 = 7$$

the variable x represents a number. Solving the equation means finding the value of x that makes the equation true in this case, $x = 4$.

Variables are fundamental in algebra because they allow us to generalize problems and create formulas.

2.3 Coefficients

A **coefficient** is the numerical factor multiplied by a variable in an algebraic expression.

For example, in the term:

$$5x$$

the number 5 is the coefficient of the variable x. It tells us how many times x is being counted or scaled.

More examples:

- In $-3y$, the coefficient is -3
- In $\frac{1}{2}a$, the coefficient is $\frac{1}{2}$
- In z, the coefficient is implicitly 1, since $z = 1 \cdot z$

Coefficients help determine the slope of a line in linear equations and play a major role in simplifying and solving expressions.

3 Equations vs. Expressions

Understanding the difference between expressions and equations is essential in algebra.

3.1 Expressions

An **expression** is a combination of numbers, variables, and operations (like addition or multiplication), but it does **not** contain an equals sign.

Examples:

$$2x + 5, \quad 3a^2 - 4, \quad \frac{1}{2}y$$

Expressions represent a value, but not a complete statement to solve. You can simplify or evaluate expressions, but you cannot "solve" them unless they're part of an equation.

3.2 Equations

An **equation** is a mathematical statement that two expressions are equal. It always contains an equals sign (=) and usually involves finding the value of a variable that makes the equation true.

Examples:

$$2x + 5 = 11, \quad a^2 = 16, \quad \frac{1}{2}y = 3$$

Solving an **equation** means determining the value(s) of the variable(s) that make both sides equal.

Summary Table

Expression	**Equation**
No equals sign	Has an equals sign
Represents a value	Represents a relationship
Can be simplified or evaluated	Can be solved
Example: $3x + 2$	Example: $3x + 2 = 11$

4 The Associative Property

The **associative property** refers to the grouping of terms using parentheses in addition or multiplication. It tells us that the way numbers are grouped does not change the result — only the order of operations inside parentheses changes, not the outcome.

4.1 Associative Property of Addition

The associative property of addition states:

$$(a + b) + c = a + (b + c)$$

You can add numbers in any grouping, and the sum will stay the same.

Example:

$$(2 + 3) + 4 = 5 + 4 = 9$$

$$2 + (3 + 4) = 2 + 7 = 9$$

So, $(2 + 3) + 4 = 2 + (3 + 4)$.

4.2 Associative Property of Multiplication

The associative property of multiplication states:

$$(a \times b) \times c = a \times (b \times c)$$

You can multiply in any grouping, and the product remains unchanged.

Example:

$$(2 \times 3) \times 4 = 6 \times 4 = 24$$

$$2 \times (3 \times 4) = 2 \times 12 = 24$$

So, $(2 \times 3) \times 4 = 2 \times (3 \times 4)$.

5 The Commutative Property

The **commutative property** describes how the order of numbers does not affect the result when adding or multiplying. It applies only to **addition and multiplication** — not subtraction or division.

5.1 Commutative Property of Addition

The commutative property of addition states:

$$a + b = b + a$$

You can change the order of the numbers being added without changing the sum.

Example:

$$4 + 7 = 11 \quad \text{and} \quad 7 + 4 = 11$$

So, $4 + 7 = 7 + 4$

5.2 Commutative Property of Multiplication

The commutative property of multiplication states:

$$a \times b = b \times a$$

You can change the order of the numbers being multiplied without changing the product.

Example:

$$6 \times 5 = 30 \quad \text{and} \quad 5 \times 6 = 30$$

So, $6 \times 5 = 5 \times 6$

6 The Distributive Property

The **distributive property** connects multiplication and addition (or subtraction). It states that multiplying a number by a sum is the same as multiplying it by each term separately and then adding the results.

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

This property is especially useful for simplifying expressions and solving equations.

Examples

Example 1:

$$3(4 + 5) = 3 \times 9 = 27$$
$$3 \times 4 + 3 \times 5 = 12 + 15 = 27$$

Example 2 (with variables):

$$x(2 + y) = x \cdot 2 + x \cdot y = 2x + xy$$

Example 3 (with subtraction):

$$5(a - 3) = 5a - 15$$

Why It's Important

The distributive property is essential when:

- Expanding expressions like $2(x + 3)$
- Factoring expressions like $3x + 6$
- Solving equations efficiently

7 Like Terms

Like terms are terms that have the same variable(s) raised to the same power(s). Only the numerical coefficients can be different. Like terms can be combined using addition or subtraction.

7.1 Addition and Subtraction of Like Terms

To combine like terms, simply add or subtract their coefficients.

Example 1:

$$3x + 5x = (3 + 5)x = 8x$$

Example 2:

$$7a^2 - 2a^2 = 5a^2$$

Note: You *cannot* combine terms that are not like terms.

$$4x + 2x^2 \neq 6x^2 \quad \text{(not like terms)}$$

7.2 Multiplication and Division of Like Terms

When multiplying or dividing like terms, you combine coefficients and apply exponent rules to the variables.

Multiplication Example:

$$(3x)(2x) = 6x^2$$

$$(4a^2)(-2a^3) = -8a^5$$

Division Example:

$$\frac{10x^3}{2x} = 5x^2$$

$$\frac{-6y^4}{3y^2} = -2y^2$$

Key Idea

- **Addition/Subtraction:** Combine only like terms (same variables and exponents)
- **Multiplication/Division:** Use exponent rules, even for unlike terms