

# Algebra

Notes and Practice

Michael leibbrandt

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## 1 Introduction

This document contains concise notes and worked examples.

## 2 Algebra Basics

### 2.1 Constants

A **constant** is a fixed value that does not change. Unlike a **variable**, which can represent different numbers, a **constant** always has the same value.

Examples of **constant** include:

- Specific numbers like 2,  $-7$ , or  $\frac{1}{2}$
- Mathematical constants like  $\pi \approx 3.1416$  or  $e \approx 2.718$

In the equation:

$$x + 3 = 7$$

the number 3 and 7 are constants — they stay the same, while  $x$  is the variable we solve for.

### 2.2 Variables

A **variable** is a symbol, usually a letter like  $x$ ,  $y$ , or  $z$ , that represents an unknown or changeable value.

For example, in the equation:

$$x + 3 = 7$$

the variable  $x$  represents a number. Solving the equation means finding the value of  $x$  that makes the equation true in this case,  $x = 4$ .

**Variables** are fundamental in algebra because they allow us to generalize problems and create formulas.

### 2.3 Coefficients

A **coefficient** is the numerical factor multiplied by a variable in an algebraic expression.

For example, in the term:

$$5x$$

the number 5 is the coefficient of the variable  $x$ . It tells us how many times  $x$  is being counted or scaled.

More examples:

- In  $-3y$ , the coefficient is  $-3$
- In  $\frac{1}{2}a$ , the coefficient is  $\frac{1}{2}$
- In  $z$ , the coefficient is implicitly 1, since  $z = 1 \cdot z$

Coefficients help determine the slope of a line in linear equations and play a major role in simplifying and solving expressions.

### 3 Equations vs. Expressions

Understanding the difference between expressions and equations is essential in algebra.

#### 3.1 Expressions

An **expression** is a combination of numbers, variables, and operations (like addition or multiplication), but it does **not** contain an equals sign.

Examples:

$$2x + 5, \quad 3a^2 - 4, \quad \frac{1}{2}y$$

Expressions represent a value, but not a complete statement to solve. You can simplify or evaluate expressions, but you cannot "solve" them unless they're part of an equation.

#### 3.2 Equations

An **equation** is a mathematical statement that two expressions are equal. It always contains an equals sign (=) and usually involves finding the value of a variable that makes the equation true.

Examples:

$$2x + 5 = 11, \quad a^2 = 16, \quad \frac{1}{2}y = 3$$

Solving an **equation** means determining the value(s) of the variable(s) that make both sides equal.

#### Summary Table

Expression	Equation
No equals sign	Has an equals sign
Represents a value	Represents a relationship
Can be simplified or evaluated	Can be solved
Example: $3x + 2$	Example: $3x + 2 = 11$

### 4 The Associative Property

The **associative property** refers to the grouping of terms using parentheses in addition or multiplication. It tells us that the way numbers are grouped does not change the result — only the order of operations inside parentheses changes, not the outcome.

#### 4.1 Associative Property of Addition

The associative property of addition states:

$$(a + b) + c = a + (b + c)$$

You can add numbers in any grouping, and the sum will stay the same.

**Example:**

$$(2 + 3) + 4 = 5 + 4 = 9$$

$$2 + (3 + 4) = 2 + 7 = 9$$

So,  $(2 + 3) + 4 = 2 + (3 + 4)$ .

## 4.2 Associative Property of Multiplication

The associative property of multiplication states:

$$(a \times b) \times c = a \times (b \times c)$$

You can multiply in any grouping, and the product remains unchanged.

**Example:**

$$(2 \times 3) \times 4 = 6 \times 4 = 24$$

$$2 \times (3 \times 4) = 2 \times 12 = 24$$

So,  $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ .

## 5 The Commutative Property

The **commutative property** describes how the order of numbers does not affect the result when adding or multiplying. It applies only to **addition and multiplication** — not subtraction or division.

### 5.1 Commutative Property of Addition

The commutative property of addition states:

$$a + b = b + a$$

You can change the order of the numbers being added without changing the sum.

**Example:**

$$4 + 7 = 11 \quad \text{and} \quad 7 + 4 = 11$$

So,  $4 + 7 = 7 + 4$

### 5.2 Commutative Property of Multiplication

The commutative property of multiplication states:

$$a \times b = b \times a$$

You can change the order of the numbers being multiplied without changing the product.

**Example:**

$$6 \times 5 = 30 \quad \text{and} \quad 5 \times 6 = 30$$

So,  $6 \times 5 = 5 \times 6$

## 6 Like Terms

**Like terms** are terms that have the same variable(s) raised to the same power(s). Only the numerical coefficients can be different. Like terms can be combined using addition or subtraction.

### 6.1 Addition and Subtraction of Like Terms

To combine like terms, simply add or subtract their coefficients.

**Example 1:**

$$3x + 5x = (3 + 5)x = 8x$$

**Example 2:**

$$7a^2 - 2a^2 = 5a^2$$

**Note:** You *cannot* combine terms that are not like terms.

$$4x + 2x^2 \neq 6x^2 \quad (\text{not like terms})$$

### 6.2 Multiplication and Division of Like Terms

When multiplying or dividing like terms, you combine coefficients and apply exponent rules to the variables.

**Multiplication Example:**

$$(3x)(2x) = 6x^2$$

$$(4a^2)(-2a^3) = -8a^5$$

**Division Example:**

$$\frac{10x^3}{2x} = 5x^2$$

$$\frac{-6y^4}{3y^2} = -2y^2$$

**Key Idea**

- **Addition/Subtraction:** Combine only like terms (same variables and exponents)
- **Multiplication/Division:** Use exponent rules, even for unlike terms

## 7 The Distributive Property

The **distributive property** connects multiplication and addition or subtraction. It allows you to multiply a number or variable by each term inside parentheses.

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

This property is used frequently in algebra to expand expressions and solve equations.

**Examples****Example 1 (with numbers):**

$$3(4 + 5) = 3 \cdot 9 = 27$$

$$3 \cdot 4 + 3 \cdot 5 = 12 + 15 = 27$$

**Example 2 (with variables):**

$$x(2 + y) = 2x + xy$$

**Example 3 (with subtraction):**

$$5(a - 3) = 5a - 15$$

**Why It Matters**

The distributive property helps you:

- Expand expressions like  $2(x + 3)$
- Simplify algebraic expressions
- Solve equations more efficiently
- Factor expressions in reverse

**More on the Distributive Property**

The distributive property is also essential when working with variables, negative numbers, and factoring expressions.

**Example with Variables and Negatives**

$$-2(x - 4) = -2 \cdot x + (-2) \cdot (-4) = -2x + 8$$

Notice:

- The negative sign distributes to both terms
- Be careful with signs:  $-2 \cdot -4 = +8$

**Common Mistake to Avoid**

Incorrect:

$$3(x + 2) = 3x + 2 \quad (\text{Only distributed to } x)$$

Correct:

$$3(x + 2) = 3x + 6$$

Always distribute to *every* term inside the parentheses.

## Using the Distributive Property to Factor

The distributive property also works *in reverse*, which is how we factor expressions.

$$6x + 12 = 6(x + 2)$$

Here, we pulled out the common factor of 6 — essentially undoing the distribution.

Factoring is the process of writing an expression as a product using the distributive property in reverse.

**Tip:** Look for a greatest common factor (GCF) before factoring!

## 8 Polynomials

A **polynomial** is an expression made up of variables, constants, and exponents, combined using addition, subtraction, and multiplication — but no variables in the denominator or under radicals.

### Examples of Polynomials

$$3x^2 + 2x - 5, \quad x^3 - 4x + 7, \quad 2a^2b + 3ab^2$$

### 8.1 Addition and Subtraction of Polynomials

To add or subtract polynomials:

- Combine like terms (same variables raised to the same powers)
- Add/subtract the coefficients of like terms

#### Example 1 — Addition:

$$\begin{aligned} & (2x^2 + 3x + 1) + (x^2 + 4x - 5) \\ &= (2x^2 + x^2) + (3x + 4x) + (1 - 5) = 3x^2 + 7x - 4 \end{aligned}$$

#### Example 2 — Subtraction:

$$\begin{aligned} & (5x^2 - 2x + 6) - (3x^2 + x - 4) \\ &= (5x^2 - 3x^2) + (-2x - x) + (6 + 4) = 2x^2 - 3x + 10 \end{aligned}$$

### 8.2 Multiplication of Polynomials

To multiply polynomials:

- Use the distributive property (FOIL for binomials)
- Multiply each term in the first polynomial by each term in the second
- Combine like terms

**Example:**

$$(x + 2)(x + 5) = x(x + 5) + 2(x + 5) = x^2 + 5x + 2x + 10 = x^2 + 7x + 10$$

**Another Example:**

$$\begin{aligned}(2x - 3)(x^2 + x - 4) &= 2x(x^2 + x - 4) - 3(x^2 + x - 4) \\ &= 2x^3 + 2x^2 - 8x - 3x^2 - 3x + 12 = 2x^3 - x^2 - 11x + 12\end{aligned}$$

### 8.3 Division of Polynomials (Intro)

Dividing polynomials can be done using:

- Long division
- Synthetic division (when dividing by linear terms like  $x - a$ )

**Basic Example:**

$$\frac{6x^2 + 9x}{3x} = \frac{6x^2}{3x} + \frac{9x}{3x} = 2x + 3$$

**Note:** More complex division techniques will be covered in a later section.

#### Polynomial Operations Summary

**Polynomials** are expressions with variables and constants using only addition, subtraction, and multiplication (no variables in denominators or exponents).

##### Addition/Subtraction:

- Combine like terms (same variable and exponent)
- Only coefficients are added or subtracted

##### Multiplication:

- Use the distributive property or FOIL
- Multiply each term in one polynomial by each term in the other
- Combine like terms

##### Division:

- Simplify each term if possible
- For complex division, use long division or synthetic division (covered later)

### 8.4 Long Division

Long division is a step-by-step method for dividing numbers or algebraic expressions.

- **Dividend:** the number or expression being divided
- **Divisor:** the number or expression you are dividing by
- **Quotient:** the result of the division (goes on top)



**Example 1: Long Division with Whole Numbers**

Divide:

$$125 \div 5$$

Here:

- Dividend = 125
- Divisor = 5
- Quotient = 25

$$\begin{array}{r|l} 5 & 125 \\ & 25 \end{array}$$

Steps:

1. Divide 12 by 5  $\rightarrow 2$  (since  $5 \times 2 = 10$ )
2. Subtract:  $12 - 10 = 2$ , bring down the 5  $\rightarrow 25$
3. Divide 25 by 5  $\rightarrow 5$  (since  $5 \times 5 = 25$ )
4. Final result: 25

**Example 2: Polynomial Long Division**

Divide:

$$\frac{x^2 + 3x + 2}{x + 1}$$

Here:

- Dividend =  $x^2 + 3x + 2$
- Divisor =  $x + 1$
- Quotient = the expression that results from division

**Step 1:** Divide leading terms:

$$x^2 \div x = x$$

**Step 2:** Multiply and subtract:

$$(x^2 + 3x + 2) - (x)(x + 1) = x^2 + 3x + 2 - (x^2 + x) = 2x + 2$$

**Step 3:** Divide leading terms again:

$$2x \div x = 2$$

**Step 4:** Multiply and subtract:

$$(2x + 2) - 2(x + 1) = 2x + 2 - (2x + 2) = 0$$

So the division is exact.

**Final Answer:**

$$\frac{x^2 + 3x + 2}{x + 1} = x + 2$$

#### Terminology Review

- **Dividend:** what you're dividing (e.g.,  $x^2 + 3x + 2$ )
- **Divisor:** what you're dividing by (e.g.,  $x + 1$ )
- **Quotient:** result of the division (e.g.,  $x + 2$ )