Econ 210C Homework 3

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1. Sticky Wage Model

Instead of assuming that prices are sticky for one period, we now assume that nominal wages are sticky for one period,

$$W_1 = W_0$$

The short-run equilibrium is

$$\begin{aligned}
Y_1 &= A_1 N_1 \\
W_1 &= W_0 \\
\frac{W_1}{P_1} &= A_1 \\
Y_1 &= C_1 \\
\frac{M_1}{P_1} &= \zeta^{1/\nu} \left(1 - \frac{1}{Q_1}\right)^{-1/\nu} C_1^{\gamma/\nu} \\
1 &= \beta E_1 \left\{ Q_1 \frac{P_1}{P_2} \frac{C_2^{-\gamma}}{C_1^{-\gamma}} \right\}
\end{aligned}$$

The long-run equilibrium ($t \geqslant 2$) is

$$
\begin{aligned}
Y_t &= A_t N_t \\
\frac{W_t}{P_t} &= A_t \\
\frac{W_t}{P_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
Y_t &= C_t \\
\frac{M_t}{P_t} &= \zeta^{1/\nu}\left(1 - \frac{1}{Q_t}\right)^{-1/\nu} C_t^{\gamma/\nu} \\
1 &= \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}
\end{aligned}
$$

(a) Are firms on their labor curve? Explain.

Solution: Yes. I'm going to assume that the short-run wage is not equal to the long-run wage, i.e., $W_0 \neq W_t$ where $t \geqslant 2$, otherwise this question won't be very interesting.

Suppose, WLOG, that W_0 is lower than what the firm would want it to be. What happens? Well, we know that the real wage is equal to the marginal product of labor, i.e., $\frac{W_1}{P_1} = A_1$, so if W_0 is too low, then, assuming A_1 is exogenous, then P_1 must adjust downward somehow. Regardless, the MPL equation holds, so firms must be on their labor demand curve.

(b) Are households on their labor supply curve? Explain.

Solution: The first thing to notice is that one equation that is present in the long run equilibrium is not present in the short-run equilibrium:

$$
\begin{aligned}
\frac{W_t}{P_t} &= \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \\
C_t^{-\gamma} &= \chi \frac{P_t}{W_t} N_t^{\varphi} \\
\frac{\partial U}{\partial C_t} &= -\frac{P_t}{W_t} \frac{\partial U}{\partial N_t}
\end{aligned}
$$

This is the household labor supply equation. In the long run, households set their marginal consumption equal to the marginal utility of forgoing one unit of labor.

But in the short run, households are not doing that. Instead, they base their consumption decision on the Euler equation (setting marginal utility of consumption today equal to discounted marginal utility of consumption tomorrow) and the money demand equation (setting marginal utility of consumption equal to the marginal utility of holding money). That sets the level of output.

(c) How does the labor market clear?

Solution: Based on the Euler equation, the money demand equation, and (exogenous and endogenous) prices, households make consumption decisions. That determines the level of output. Firms produce to exactly clear the consumption good market, demanding labor sufficient setting wage equal to MPL. Households supply labor exactly to meet this demand, whether they like it or not.

(d) Solve for the long-run steady state.

Solution: Denote long-run parameters without subscripts. First, note that the Euler equation simplifies to

$$\begin{aligned} 1 &= \beta \mathbb{E}\left\{ Q \frac{P}{P} \frac{C^{-\gamma}}{C^{-\gamma}} \right\} \\ &= \beta \mathbb{E}\left\{ Q \right\} \\ Q &= \frac{1}{\beta} \end{aligned}$$

Then we can solve for labor by imposing market clearing $C = Y = AN$.

$$\begin{aligned} A &= \frac{W}{P} = \frac{\chi N^{\varphi}}{C^{-\gamma}} \\ A &= \frac{\chi N^{\varphi}}{(AN)^{-\gamma}} \\ A^{1-\gamma} &= \chi N^{\varphi+\gamma} \\ N &= \left(\frac{A^{1-\gamma}}{\chi} \right)^{\frac{1}{\varphi+\gamma}} \end{aligned}$$

Then we can easily solve for consumption and output.

$$\begin{aligned} C &= Y \\ &= AN \\ &= A\left(\frac{A^{1-\gamma}}{\chi}\right)^{\frac{1}{\varphi+\gamma}} \\ &= A^{1+\frac{1-\gamma}{\varphi+\gamma}}\chi^{\frac{-1}{\varphi+\gamma}} \\ &= A^{\frac{\varphi+1}{\varphi+\gamma}}\chi^{\frac{-1}{\varphi+\gamma}} \end{aligned}$$

And finally, money.

$$\begin{aligned} \frac{M}{P} &= \zeta^{1/\nu}\left(1-\frac{1}{\frac{1}{\beta}}\right)^{-1/\nu}\left(A^{\frac{\varphi+1}{\varphi+\gamma}}\chi^{\frac{-1}{\varphi+\gamma}}\right)^{\gamma/\nu} \\ &= \zeta^{1/\nu}\left(1-\beta\right)^{-1/\nu}\left(\frac{A^{\varphi+1}}{\chi}\right)^{\frac{\gamma}{\nu(\varphi+\gamma)}} \end{aligned}$$

Now we half (almost) every variable in terms of parameters.

(e) Does the Classical Dichotomy hold in the long-run? Explain.

Solution: Yes. Real variables (C, Y, N, M/P, W/P, Q/P) are independent of price levels. So any change in price (P, W, Q) will affect the other prices, but have no effect on any real variable.

(f) Solve for output and the money market equilibrium in the short-run.

Solution: First, solve for prices.

$$\begin{aligned} \frac{W_0}{P_1} &= A_1 \\ P_1 &= \frac{W_0}{A_1} \end{aligned}$$

This means that effectively, P_1 is fixed. I suspect this means that our results will look a lot like our results from class when we fixed $P_1 = P_0$, but let's find out.

Now we impose that $t = 2$ parameters are equal to long-run steady-state parameters. Then we can solve for consumption, noting that all the future paramters are known so we can drop the expectation.

$$\begin{aligned}
1 &= \beta E_1 \left\{ Q_1 \frac{P_1}{P} \frac{C^{-\gamma}}{C_1^{-\gamma}} \right\} \\
C_1^{-\gamma} &= \beta Q_1 \frac{W_0}{A_1 P} C^{-\gamma} \\
C_1 &= \left(\beta Q_1 \frac{W_0}{A_1 P} \right)^{\frac{-1}{\gamma}} C \\
&= \left(\frac{A_1 P}{\beta Q_1 W_0} \right)^{\frac{1}{\gamma}} C \\
&= \left(\frac{1}{\beta Q_1} \frac{P}{P_1} \right)^{\frac{1}{\gamma}} C
\end{aligned}$$

Assuming A_1 is given, then we *almost* have consumption solved for in terms of paramteres (Our only troublemaker is Q_1): C_1 is some scaled version of C that depends on prices and the discount factor — which does, in fact, look like what we had in class. We could try to solve for it. Next, let's do the money demand equation.

$$\begin{aligned}
\frac{M_1}{P_1} = M_1 \frac{A_1}{W_0} &= \zeta^{1/\nu} \left(1 - \frac{1}{Q_1} \right)^{-1/\nu} C_1^{\gamma/\nu} \\
&= \zeta^{1/\nu} \left(1 - \frac{1}{Q_1} \right)^{-1/\nu} \left(\left(\frac{1}{\beta Q_1} \frac{P}{P_1} \right)^{\frac{1}{\gamma}} C \right)^{\gamma/\nu} \\
&= \zeta^{1/\nu} C^{\gamma/\nu} \frac{(1-\beta)^{-1/\nu}}{(1-\beta)^{-1/\nu}} \left(1 - \frac{1}{Q_1} \right)^{-1/\nu} \left(\left(\frac{1}{\beta Q_1} \frac{P}{P_1} \right)^{\frac{1}{\gamma}} \right)^{\gamma/\nu} \\
&= \frac{M}{P} \frac{1}{(1-\beta)^{-1/\nu}} \left(1 - \frac{1}{Q_1} \right)^{-1/\nu} \left(\left(\frac{1}{\beta Q_1} \frac{P}{P_1} \right)^{\frac{1}{\gamma}} \right)^{\gamma/\nu} \\
&= \frac{M}{P} \left(1 - \beta - \frac{1-\beta}{Q_1} \right)^{-1/\nu} \left(\left(\frac{1}{\beta Q_1} \frac{P}{P_1} \right)^{\frac{1}{\gamma}} \right)^{\gamma/\nu}
\end{aligned}$$

Again, we end up with the steady state scaled by some prices and the discount factor, and, again, it looks like what we had in class. Finally, let's solve for labor.

$$\begin{aligned}
N_1 &= \frac{C_1}{A_1} \\
&= \frac{P_1}{W_0} C_1 \\
&= \frac{P_1}{W_0} \left(\frac{1}{\beta Q_1} \frac{P}{P_1} \right)^{\frac{1}{\gamma}} C
\end{aligned}$$

Notably, this looks *nothing* like the steady state labor supply equation (and we didn't solve for N in class). This is because households are off their labor supply curve in the short run — they just supply labor such that $Y_1 = C_1$.

(g) Does the Classical Dichotomy hold in the short-run?

Solution: The fact that W_0 shows up directly in all of our equations for the real variables (C_1, N_1, M_1) is direct evidence that prices matter, i.e., the Classical Dichotemy doesn't hold because we can't separate the real and the nominal variables.

(h) Explain intuitively (in words) how an increase in the money supply affects output in the short-run.

Solution: Increasing the money supply means that, because price P_1 is fixed, that the real value of money held by households must increase. In order to convince households to do this, they must be convinced to forgo investment or consumption. Price of consumption is fixed at $P_1 = W_0/A_1$, so the real interest rate $Q_1(P_1/P)$ must adjust, but (P_1/P) is fixed, so Q_1 must fall.

This ripples through the economy. C_1 increases as a result the Euler equation: becasue households are equilibrating the marginal utility of consumption today and the marginal utility of one unit of consumption tomorrow by forgoing one unit of consumption today, and the latter is an increasing function of the return on investment $Q_1(P_1/P)$ (savings today are scaled up tomorrow by the interest rate), so saving for tomorrow suddenly looks less appealing. The increased consumption needs to be supported by increased output and, therefore, increased labor.

(i) How does productivity affect output? Explain intuitively.

Solution: Increasing productivity decreases the price of the consumption good P_1, so households will want to consume more and save less, due to the Euler equation. Simultaneously, the falling P_1 descreases the real interest rate, making savings less appealing, households want to hold less money, so this further increases consumption. Also, the falling P_1 increases the real wage $\frac{W_0}{P_1}$, but households are not on their labor supply curve, so this does not affect their decisionmaking. Firms have no trouble meeting the increased demand: this is partially through increased productivity and partially through increased labor demand/supply.

Overall, higher productivity means lower prices, lower interest rate, higher consumption, higher output, and higher labor.

(j) Derive the labor wedge. Is it procyclical or countercyclical?

Solution:

$$
\begin{aligned}
1-\tau_1^N &= \frac{MRS_1}{MPL_1} \\
&= \frac{\chi {N_1}^{\varphi} C_1^{\gamma}}{(1-\alpha)Y_1/N_1} \\
&= \frac{1}{(1-\alpha)}\chi {N_1}^{\varphi+1} C_1^{\gamma-1} \\
&= \frac{1}{(1-\alpha)}\chi \left(\frac{C_1}{A_1}\right)^{\varphi+1} C_1^{\gamma-1} \\
&= \frac{1}{(1-\alpha)}\chi {A_1}^{-\varphi-1} C_1^{\gamma+\varphi} \\
&= \frac{1}{(1-\alpha)}\chi {A_1}^{-\varphi-1} \left(\left(\frac{1}{\beta Q_1}\frac{PA_1}{W_0}\right)^{\frac{1}{\gamma}} C\right)^{\gamma+\varphi} \\
&= \frac{1}{(1-\alpha)}\chi A_1^{-\varphi-1} \left(\frac{1}{\beta Q_1}\frac{PA_1}{W_0}\right)^{1+\frac{1}{\gamma}} C^{\gamma+\varphi} \\
&= \frac{1}{(1-\alpha)}\chi A_1^{\frac{1}{\gamma}-\varphi} \left(\frac{1}{\beta Q_1}\frac{P}{W_0}\right)^{1+\frac{1}{\gamma}} C^{\gamma+\varphi}
\end{aligned}
$$

The wedge is procyclical if it increases with A_1. Assuming $\gamma, \varphi, \alpha \in (0,1)$, then all those terms in the above equation are positive and then $(1-\tau_1^N)$ is an increasing function of A_1.

But is the wage $(1-\tau_1^N)$ or just τ_1^N? Assuming it's the latter, then the wedge is decreasing with A_1, so it is countercyclical.

(k) What moments of the data would you use to discriminate between the predictions of the sticky price and the sticky wage model?

Solution:

	Sticky Prices	Sticky Wages
$\partial P/\partial A$	0	$-$
$\partial Y/\partial A$	0	$+$
$\partial N/\partial A$	$-$	$+$
$\partial W/\partial M$	$+$	0