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ISBA 2400 TTh 05:45PM
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Case Project: Optimization of Support Staffing at Tesla

***Note:** In practice, the number of agents must be an integer. The optimization model produces ideal (continuous) staffing levels that may include decimals. Rounding down can result in insufficient staffing, leading to unmet call demands, while rounding up may increase operating costs. Therefore, additional analysis is required to evaluate the cost-service trade-offs and determine the most practical hiring strategy.

a) How many full-time English-speaking agents, full-time Spanish-speaking agents, and part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls? (Please round each number to the nearest integer.)

Based on the conditions, we set x = the number of full-time agents and y = the number of part-time agents. Full-time agents may start their shift at 7 am while part-time agents are restricted to start during 3-5pm and 5-7pm. Accordingly, we index the decision variables by time period. (E.g., x1 represents full-time agents who start the shift from 7-9am, y1: part-time agents who start from 3pm to 5pm.)

			The shifts (time slots) that an agent is hired for					
	Full time agents		x1	x2	x3	x4	x5	
	Part time agents						y1	y2
Hourly Payment		Time	7-9 am	9-11 am	11 - 1 pm	1-3 pm	3-5 pm	5-7 pm
30	1	7-9 am	x1					
30	2	9-11 am		x2				
30	3	11 - 1 pm	x1		x3			
30	4	1-3 pm		x2		x4		
30	5	3-5 pm			x3		x5 + y1	
45	6	5-7 pm				x4	y1	y2
45	7	7-9 pm					x5	y2
		Cost	120	120	120	150	150	180

After creating a framework to compute the number of agents, the Both full-time and part-time agents earned $30/hour before 5 P.M. and $45/hour afterward. Each of them will contribute to the calls for at least 4 hours/day. So we conclude that:

- Those who start at 7am -9am, 9am -11am and 11am -1am complete all four working hours before 5:00 p.m. and therefore earn 4×$30=$120 per day. These agents are represented by the full-time decision variables x1, x2 and x3.
- Full-time agents who start at 1:00pm (x4) will resume handling calls after 5 pm, when the hourly wage rises into $45.The same wage structure applies to agents starting at 3:00 p.m., including both full-time agents(x5) and part-time agents (y1). For these agents, the first two working hours are paid at $30 per hour, while the remaining two hours are paid at $45 per hour, resulting in a total daily wage of 2×30+2×45=$150.

- Finally, agents(y2) whose shifts begin after 5:00 p.m work entirely during the higher-wage period and therefore earn 4×45 =$180 per day.

Based on the above finding:

$$\text{Total Cost} = 120 * (x1 + x2 + x3) + 150 * (x4 + x5 + y1) + 180 * y2$$

In addition, agents are divided into English- and Spanish-speaking groups. Since part-time agents are only qualified to handle English calls, no part-time Spanish agents are included in the model. As a result, the corresponding entries for part-time Spanish agents marked as not applicable (N/A) in the table.

To minimize the total cost, we introduce capacity constraints to determine the optimal placement of agents across time intervals. Based on the available information(e.g., specifically, each support agent can efficiently handle an average of six calls per hour, the expected call volumes vary by time period with insights showing that 20% of calls came from non-English speakers.), we translate demand requirements into these capacity constraints which ensure that, in each time interval, the total call-handling capacity provided by the assigned agents is sufficient to meet the average number of incoming calls.

Constraint for English
6 * x1 >= 32
6 * x2 >= 68
6 *(x1 + x3) >= 56
6*(x2 + x4) >= 76
6(x3 + x5 + y1) >= 64
6(x4 + y1 + y2) >= 28
6*(x5 + y2) >= 8

Constraint for spanish
6 * x1 >= 8
6 * x2 >= 17
6 *(x1 + x3) >= 14
6*(x2 + x4) >= 19
6(x3 + x5) >= 16
6(x4) >= 7
6*(x5) >= 2

After combining the total cost function and the constraint into the R Studio, we have obtained the following results:

Agent	English	Spanish
x1	5.33	1.33
x2	11	2.83
x3	5.89	2.33
x4	1.22	1.66
x5	1.33	0.33
y1	3.44	N/A
y2	0	N/A

The total number of full-time English agents is 24.77, full-time Spanish agents is 8.48 and part-time agents is 3.44

b) What is the minimum cost for the optimization model to assist Alex's decision in hiring all agents that she needs? (Please round to two decimal places, e.g., 123.45.)

From our R results:

English = $3620.051

Spanish = $1005.001

All agents = $3520.051 + $1005.001 = **$4,525.05**

```r
#English Speaking Agents
f = function(x) 120 * (x[1] + x[2] + x[3]) + 150 * (x[4] + x[5] + x[6]) + 180 * x[7]
inequality_constraint = function(x){
  g = 0
  g[1] = 6 * x[1] - 32
  g[2] = 6 * x[2] - 68
  g[3] = 6 * (x[1] + x[3]) - 56
  g[4] = 6 * (x[2] + x[4]) - 76
  g[5] = 6 * (x[3] + x[5] + x[6]) - 64
  g[6] = 6 * (x[4] + x[6] + x[7]) - 28
  g[7] = 6 * (x[5] + x[7]) - 8
  g[8] = x[1]
  g[9] = x[2]
  g[10] = x[3]
  g[11] = x[4]
  g[12] = x[5]
  g[13] = x[6]
  g[14] = x[7]

  return(g)

}

p0 = rep(12,7)
p0
answer = constrOptim.nl(p0,f,hin = inequality_constraint)
answer$value
answerApprox = (answer$par)
format(answerApprox, scientific = FALSE)
f(answerApprox)
```

```
30:1    (Top Level)    R Script
Console   Terminal   Background Jobs
R 4.5.1 · ~/Documents/2025-2026/Math with R/
> f(answerApprox)
[1] 3620.051
> answerApprox
[1] 5.333333e+00 1.144368e+01 5.887999e+00 1.222991e+00 1.334069e+00 3.444599e+00 1.648012e-10
> f(answerApprox)
[1] 3620.051
> format(answerApprox, scientific = FALSE)
[1] " 5.3333333962025185" "11.4436822591800418" " 5.8879986332831118"
[4] " 1.2229910776167137" " 1.3340690082425197" " 3.4445991850674194"
[7] " 0.0000000001648012"
```

```r
#Spanish Speaking Agents
f = function(x) 120 * (x[1] + x[2] + x[3]) + 150 * (x[4] + x[5])
inequality_constraint = function(x){
  g = 0
  g[1] = 6 * x[1] - 8
  g[2] = 6 * x[2] - 17
  g[3] = 6 * (x[1] + x[3]) - 14
  g[4] = 6 * (x[2] + x[4]) - 19
  g[5] = 6 * (x[3] + x[5]) - 16
  g[6] = 6 * x[4] - 7
  g[7] = 6 * x[5] - 2
  g[8] = x[1]
  g[9] = x[2]
  g[10] = x[3]
  g[11] = x[4]
  g[12] = x[5]


  return(g)

}

p0 = rep(3,5)
p0
answer = constrOptim.nl(p0,f,hin = inequality_constraint)
answer$value
answerApprox = (answer$par)
answerApprox
format(answerApprox, scientific = FALSE)
f(answerApprox)
```

```
31:1    (Top Level)
Console   Terminal   Background Jobs
R 4.5.1 · ~/Documents/2025-2026/Math with R/
> answerApprox
[1] 1.3333338 2.8333333 2.3333246 1.1666670 0.3333435
> format(answerApprox, scientific = FALSE)
[1] "1.3333338" "2.8333333" "2.3333246" "1.1666670" "0.3333435"
> f(answerApprox)
[1] 1005.001
```

Due to a preference among full-time agents to avoid late evening shifts, Alex can find only one qualified English-speaking agent willing to start work at 1 P.M. and 3 P.M. Given this new constraint:

Following the same framework as above, since there is a new constraint of one qualified English-speaking agent, therefore we will set x4 and x5 as 1, where x4 is the agent who starts at 1 P.M. and x5 is the agent who starts at 3 P.M..

Constraint for English
$6 * x1 >= 32$
$6 * x2 >= 68$
$6 * (x1 + x3) >= 56$
$6 * (x2 + 1) >= 76$
$6 * (x3 + 1 + y1) >= 64$
$6 * (1 + y1 + y2) >= 28$
$6 * (1 + y2) >= 8$
$x1, x2, x3, y1, y2 >= 0$
$x4, x5 = 1$

Since this only affects full-time agents who speak English, therefore, the number of Spanish-speaking agents remains the same as part (a).

c) How many full-time English-speaking agents, full-time Spanish-speaking agents, and part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls? (Please round each number to the nearest integer.)

Agent	English	Spanish
x1	5.33	1.33
x2	11.66	2.83
x3	6.33	2.33
x4	1	1.66
x5	1	0.33
y1	3.33	N/A
y2	0.33	N/A

The total number of full-time English agents is 25.32, full-time Spanish agents is 8.48 and part-time agents is 3.66

d) What is the minimum cost for the optimization model to assist Alex's decision in hiring all agents that she needs? (Please round to two decimal places.)

Using the above framework, we input the total cost function:

$$\text{Total cost} = 120 * (x1 + x2 + x3) + 150 * (1 + 1 + y1) + 180 * y2$$

In R, since we set x4 and x5 to 1, therefore we had to change x6 and x7 (which is actually y1 and y2) to x4 and x5 so that R runs, or else there is incoherence in the code. Furthermore, we had to change rep(12,7) to rep(12,5) since there are only 5 decision variables to repeat now.

As a result from R, the following numbers were given to answer the minimum cost:
English = $3660.006
Spanish = $1005.001
All agents = $3660.006 + $1005.001 = $4,665.007
Cost (Mono) = **$4,665.01**

```r
# only one qualified english speaking agent starting 1pm
f = function(x) 120 * (x[1] + x[2] + x[3]) + 150 * (1 + 1 + x[4]) + 180 * x[5]
inequality_constraint = function(x){
  g = 0
  g[1] = 6 * x[1] - 32
  g[2] = 6 * x[2] - 68
  g[3] = 6 * (x[1] + x[3]) - 56
  g[4] = 6 * (x[2] + 1) - 76
  g[5] = 6 * (x[3] + 1 + x[4]) - 64
  g[6] = 6 * (1 + x[4] + x[5]) - 28            #x[4] is actually y[1]
  g[7] = 6 * (1 + x[5]) - 8                    #x[5] is actually y[2]
  g[8] = x[1]
  g[9] = x[2]
  g[10] = x[3]
  g[11] = x[4]
  g[12] = x[5]

  return(g)

}

p0 = rep(12,5)
p0
answer = constrOptim.nl(p0,f,hin = inequality_constraint)
answer$value
answerApprox = (answer$par)
answerApprox
format(answerApprox, scientific = FALSE)
f(answerApprox)
```

```
1:57    (Top Level)    R Script
Console   Terminal   Background Jobs
R 4.5.1 · ~/Documents/2025-2026/Math with R/
> answerApprox = (answer$par)
> format(answerApprox, scientific = FALSE)
[1] " 5.3333339" "11.6666667" " 6.3331475" " 3.3335195" " 0.3333337"
> f(answerApprox)
[1] 3660.006
```

Alex is now exploring the possibility of hiring bilingual agents instead of monolingual agents. If all agents are bilingual:
e) How many full-time and part-time agents should Alex hire for each 2-hour shift to minimize operating costs while attending to all calls? (Please round each number to the nearest integer.)

In this scenario, Alex considers replacing monolingual agents with fully bilingual agents who can handle both English and Spanish calls. Because bilingual agents can serve all calls, the problem becomes simpler: we only need to satisfy the total call demand in each 2-hour time block, rather than separate English and Spanish requirements.

We define seven decision variables:

- x1: Full-time bilingual agents starting at 7 A.M.
- x2: Full-time bilingual agents starting at 9 A.M.
- x3: Full-time bilingual agents starting at 11 A.M.
- x4: Full-time bilingual agents starting at 1 P.M.
- x5: Full-time bilingual agents starting at 3 P.M.
- x6: Part-time bilingual agents working 3–7 P.M.
- x7: Part-time bilingual agents working 5–9 P.M.

Because each agent handles 6 calls per hour, every time-block capacity constraint is written as:

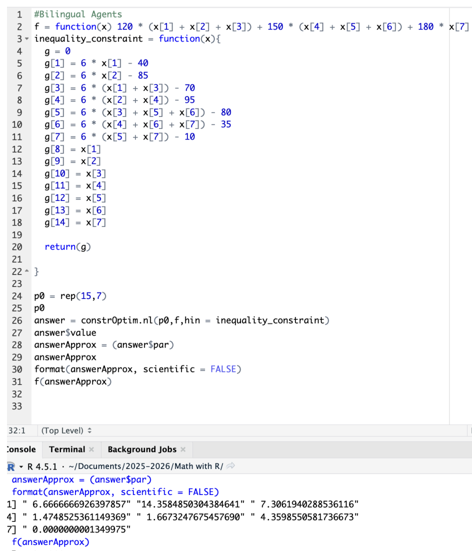
$$\text{Total hourly capacity} \geq \text{Required calls}$$

The total cost is:

$$\text{Total Cost} = 120\,(x1 + x2 + x3) + 150\,(x4 + x5 + x6) + 180\,(x7)$$

The table below shows the optimal number of bilingual agents needed for each shift, based on the staffing model we created. The numbers come from the solution produced by our R optimization model, and each value is rounded to two decimal places for better clarity.

Agent	Bilingual
x1	6.66
x2	14.36
x3	7.31
x4	1.47
x5	1.67
y1	4.36
y2	0



```r
#Bilingual Agents
f = function(x) 120 * (x[1] + x[2] + x[3]) + 150 * (x[4] + x[5] + x[6]) + 180 * x[7]
inequality_constraint = function(x){
  g = 0
  g[1] = 6 * x[1] - 40
  g[2] = 6 * x[2] - 85
  g[3] = 6 * (x[1] + x[3]) - 70
  g[4] = 6 * (x[2] + x[4]) - 95
  g[5] = 6 * (x[3] + x[5] + x[6]) - 80
  g[6] = 6 * (x[4] + x[6] + x[7]) - 35
  g[7] = 6 * (x[5] + x[7]) - 10
  g[8] = x[1]
  g[9] = x[2]
  g[10] = x[3]
  g[11] = x[4]
  g[12] = x[5]
  g[13] = x[6]
  g[14] = x[7]

  return(g)

}

p0 = rep(15,7)
p0
answer = constrOptim.nl(p0,f,hin = inequality_constraint)
answer$value
answerApprox = (answer$par)
answerApprox
format(answerApprox, scientific = FALSE)
f(answerApprox)
```

```
32:1   (Top Level)
Console   Terminal   Background Jobs
R 4.5.1 · ~/Documents/2025-2026/Math with R/
 answerApprox = (answer$par)
 format(answerApprox, scientific = FALSE)
1] " 6.6666666926397857" "14.3584850304384641" " 7.3061940288536116"
4] " 1.4748525361149369" " 1.6673247675457690" " 4.3598550581736673"
7] " 0.0000000001349975"
 f(answerApprox)
1] 4525.066
```

f) What is the minimum cost for the optimization model to assist Alex's decision in hiring all agents that she needs? (Please round to two decimal places.)

According to our results from R, the minimum cost when all agents are bilingual is:

Cost(bi) = **$4,525.07**

g) What is the maximum percentage increase in the hourly wage rate that Alex can offer to bilingual agents over monolingual agents without increasing the total operating costs? (Please round to one decimal place, e.g., 8.7%.)

The minimum daily cost under this constraint is:

$$\text{Cost (mono)} = \$4{,}665.01$$

If bilingual wages increase by a proportion α\alphaα, the new operating cost becomes:

$$(1+\alpha)\ \text{Cost (bi)}$$

To ensure total cost does not exceed the constrained monolingual system, we require:

$$(1+\alpha)\ \text{Cost (bi)} \leq \text{Cost (mono)}$$

Solving for α:

$$\alpha(\text{max}) = \text{Cost (bi)} / \text{Cost (mono)} - 1 = 4525.07 / 4665.01 - 1 \approx 0.0309$$

This corresponds to:

Maximum allowable wage increase: 3.1%