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"Perm State National Research University"

The Faculty of Economics

Department of Information Systems and Mathematical Methods in Economics

Report
Financial Markets Modelling

Performers: groupe of 1st year student full–time training
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Fenfen Sun
A. Sazykin
M. Solodovnikov
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Supervisor:
Dr.
V. Filimonov
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1 Problem #3

1.1 Task

For the 30 stocks that constitute the Dow Jones Industrial Average (Yahoo: DJIA) at the present day: using daily data, estimate the cross correlation over the following periods: 2006 — 2007, 2008 — 2009, 2010 — 2011 and 2012 — 2013. In all four periods select two stocks that have minimal correlation, construct the equally-weighted portfolio of them and calculate the Sharpe ratio the portfolio.

1.2 Problem description

The Dow Jones Industrial Average also called the Industrial Average, the Dow Jones, the Dow Jones Industrial, the Dow 30, or simply the Dow, is a stock market index, and one of several indices created by Wall Street Journal editor and Dow Jones & Company co-founder Charles Dow. The industrial average was first calculated on May 26, 1896. Currently owned by S&P Dow Jones Indices, which is majority owned by McGraw-Hill Financial, it is the most notable of the Dow Averages, of which the first (non-industrial) was first published on February 16, 1885. The averages are named after Dow and one of his business associates, statistician Edward Jones. It is an index that shows how 30 large publicly owned companies based in the United States have traded during a standard trading session in the stock market. It is the second oldest U.S. market index after the Dow Jones Transportation Average, which was also created by Dow.

The Industrial portion of the name is largely historical, as many of the modern 30 components have little or nothing to do with traditional heavy industry. The average is price-weighted, and to compensate for the effects of stock splits and other adjustments, it is currently a scaled average. The value of the Dow is not the actual average of the prices of its component stocks, but rather the sum of the component prices divided by a divisor, which changes whenever one of the component stocks has a stock split or stock dividend, so as to generate a consistent value for the index. Since the divisor is currently less than one, the value of the index is larger than the sum of the component

prices. Although the Dow is compiled to gauge the performance of the industrial sector within the American economy, the index's performance continues to be influenced by not only corporate and economic reports, but also by domestic and foreign political events such as war and terrorism, as well as by natural disasters that could potentially lead to economic harm.

Along with the NASDAQ Composite, the S&P 500 Index, and the Russell 2000 Index, the Dow is among the most closely watched U.S. benchmark indices tracking targeted stock market activity. Components of the Dow trade on both the New York Stock Exchange and the NASDAQ. Derivatives of the Dow trade on the Chicago Board Options Exchange and through the CME Group, the world's largest futures exchange company. CME Group owns 24.4% of S&P Dow Jones Indices, which maintains the Industrial Average.

To calculate the DJIA, the sum of the prices of all 30 stocks is divided by a divisor, the Dow Divisor. The divisor is adjusted in case of stock splits, spinoffs or similar structural changes, to ensure that such events do not in themselves alter the numerical value of the DJIA. Early on, the initial divisor was composed of the original number of component companies; which made the DJIA at first, a simple arithmetic average. The present divisor, after many adjustments, is less than one (meaning the index is larger than the sum of the prices of the components). That is:

$$DJIA = \frac{\sum p}{d} \tag{1.1}$$

where p are the prices of the component stocks and d is the Dow Divisor.

Events like stock splits or changes in the list of the companies composing the index alter the sum of the component prices. In these cases, in order to avoid discontinuity in the index, the Dow Divisor is updated so that the quotations right before and after the event coincide:

$$DJIA = \frac{\sum p_{old}}{d_{old}} = \frac{\sum p_{new}}{d_{new}}. \tag{1.2}$$

The Dow Divisor is currently 0.15571590501117 as of September 27, 2013. Presently, every $1 change in price in a particular stock within the average,

equates to a 6.42 (1/0.15571590501117) point movement. [1]

Cross-correlation. In time series analysis, as applied in statistics and signal processing, the cross correlation between two time series describes the normalized cross covariance function.

Let (X_t, Y_t) represent a pair of stochastic processes that are jointly wide sense stationary. Then the cross correlation is given by

$$\gamma_{xy}(\tau) = E[(X_t - \mu_x)(Y_{t+\tau} - \mu_y)], \tag{1.3}$$

where μ_x and μ_y are the means of X_t and Y_t respectively.

The cross correlation of a pair of jointly wide sense stationary stochastic process can be estimated by averaging the product of samples measured from one process and samples measured from the other (and its time shifts). The samples included in the average can be an arbitrary subset of all the samples in the signal (e.g., samples within a finite time window or a sub-sampling of one of the signals). For a large number of samples, the average converges to the true cross-correlation. [2]

In this research will be used 3 correlation coefficients: Pearson's correlation coefficient, Spearman's correlation coefficient, Kendall's correlation coefficient.

Pearson's correlation coefficient between two variables is defined as the covariance of the two variables divided by the product of their standard deviations. The form of the definition involves a "product moment", that is, the mean (the first moment about the origin) of the product of the mean-adjusted random variables; hence the modifier product-moment in the name.

Pearson's correlation coefficient when applied to a sample is commonly represented by the letter r and may be referred to as the sample correlation coefficient or the sample Pearson correlation coefficient. We can obtain a formula for r by substituting estimates of the covariances and variances based on a sample into the formula above [3]. That formula for r is:

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}} \tag{1.4}$$

In statistics, **Spearman's** rank correlation coefficient or Spearman's rho, named after Charles Spearman and often denoted by the Greek letter ρ (rho) or as r_s, is a nonparametric measure of statistical dependence between two variables. It assesses how well the relationship between two variables can be described using a monotonic function. If there are no repeated data values, a perfect Spearman correlation of $+1$ or -1 occurs when each of the variables is a perfect monotone function of the other.

The Spearman correlation coefficient is defined as the Pearson correlation coefficient between the ranked variables. For a sample of size n, the n raw scores X_i, Y_i are converted to ranks x_i, y_i, and ρ is computed from these:

$$\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}} \tag{1.5}$$

Identical values (rank ties or value duplicates) are assigned a rank equal to the average of their positions in the ascending order of the values [4].

In statistics, the **Kendall** rank correlation coefficient, commonly referred to as Kendall's tau (τ) coefficient, is a statistic used to measure the association between two measured quantities. A tau test is a non-parametric hypothesis test for statistical dependence based on the tau coefficient.

Specifically, it is a measure of rank correlation, i.e., the similarity of the orderings of the data when ranked by each of the quantities. It is named after Maurice Kendall, who developed it in 1938, though Gustav Fechner had proposed a similar measure in the context of time series in 1897.

Let (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n) be a set of observations of the joint random variables X and Y respectively, such that all the values of (x_i) and (y_i) are unique. Any pair of observations $(x_i,\ y_i)$ and $(x_j,\ y_j)$ are said to be concordant if the ranks for both elements agree: that is, if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$. They are said to be discordant, if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant.

The Kendall τ coefficient is defined as:

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{\frac{1}{2}n(n-1)}. \tag{1.6}$$

The denominator is the total number pair combinations, so the coefficient must be in the range $-1 <= \tau <= 1$.

- If the agreement between the two rankings is perfect (i.e., the two rankings are the same) the coefficient has value 1.
- If the disagreement between the two rankings is perfect (i.e., one ranking is the reverse of the other) the coefficient has value -1.
- If X and Y are independent, then we would expect the coefficient to be approximately zero [5].

Equally-weighted portfolio in our case consist only of 2 assets, which has minimal correlation. That's why weights will be equal 50% for first asset, and 50% for second.

Sharpe ratio is a way to examine the performance of an investment by adjusting for its risk. The ratio measures the excess return (or risk premium) per unit of deviation in an investment asset or a trading strategy, typically referred to as risk (and is a deviation risk measure), named after William Forsyth Sharpe.

Since its revision by the original author, William Sharpe, in 1994, the ex-ante Sharpe ratio is defined as:

$$S_p = \frac{E[R_a - R_b]}{\sigma_p} = \frac{E[R_a - R_b]}{\sqrt{\text{var}[R_a - R_b]}}, \tag{1.7}$$

where R_a is the asset return, R_b is the return on a benchmark asset, such as the risk free rate of return or an index such as the S&P 500. $E[R_a - R_b]$ is the expected value of the excess of the asset return over the benchmark return, and σ is the standard deviation of this excess return. This is often confused with the information ratio, in part because the newer definition of the Sharpe ratio matches the definition of information ratio within the field of finance. Outside of this field, information ratio is simply mean over the standard deviation of a series of measurements.

The Sharpe ratio characterizes how well the return of an asset compensates the investor for the risk taken. When comparing two assets versus a common

benchmark, the one with a higher Sharpe ratio provides better return for the same risk (or, equivalently, the same return for lower risk). However, like any other mathematical model, it relies on the data being correct. Pyramid schemes with a long duration of operation would typically provide a high Sharpe ratio when derived from reported returns, but the inputs are false. When examining the investment performance of assets with smoothing of returns (such as with-profits funds) the Sharpe ratio should be derived from the performance of the underlying assets rather than the fund returns [6].

1.3 Solution

Dow Jones Industrial Average consists of companies with next tickers: AXP, BA, CAT, CSCO, CVX, DD, DIS, GE, GS, HD, IBM, INTC JNJ, JPM, KO, MCD, MMM, MRK, MSFT, NKE, PFE, PG, T, TRV, UNH, UTX, V, VZ, WMT, XOM. Transcription of these tickers (components of DJIA) you can find here. There are no data for Visa until 19th of March 2008. Visa was removed from this list for simplicity.

2 Yeears Treasury Bonds were choosen as risk free asset. Yields were upload frome www.treasury.gov.Yield, were choosen for last date of each period.

Special function was written on R language, to solve this problem. Input data consists of:

- assets returns
- risk free asset returns
- vector of periods
- method to calculate cross-correlations (pearson, spearman, kendall)

On the outpup:

- heat maps (cross-correalation matrixes) for 4 periods, saved as pdf files
- cross-correalation matrixes for 4 periods, saved as xlsx-tables
- 4 summary text files (for each period), which include tickers for 2 assets with minimal correlation, expected returns of each assets, portfolio expected return, portfolio variance, portfolio Sharpe ratio
  File with code and all data you can find in folder "Problem3".

1.4 Results

From the tab. 1.1, tab. 1.2, tab. 1.3 you can see portfolios which were constructed for each period and different methods cross–correlation estimation. These assets has minimal covariance for each period. Companies from these portfolios introduce five sectors of economy: finance (GS, JPM), technology (CSCO, VZ), healthcare (UNH), services (WMT), Basic Materials (XOM); and seven industries: Investment Brokerage - National (GS); Networking & Communication Devices (CSCO); Discount, Variety Stores (WMT); Health Care Plans (UNH); Money Center Banks(JPM); Major Integrated Oil & Gas (XOM); Telecom Services - Domestic (VZ). The smallest number of full time emloyees has The Goldman Sachs Group, Inc. (32 600 persons). The lagest company for this indicator is Wal-Mart Stores Inc. (2 000 000 persons).

As you can see, all companies in each portfolio introduce different sectors (it is quite logical, cause the reason we choose these portfolio was minimal covariance between assets). Cross–correalation matrices as a heat maps introduce in fig. 1.1 — fig. 1.12

Table 1.1. Portfolio based on Pearson's cross-correlation

Period	Asset 1	Asset 2
2006–2007	UnitedHealth Group Incorporated (UNH)	Cisco Systems, Inc.(CSCO)
2008–2009	UnitedHealth Group Incorporated (UNH)	JPMorgan Chase & Co. (JPM)
2010–2011	Wal-Mart Stores Inc. (WMT)	The Goldman Sachs Group, Inc. (GS)
2012–2013	Wal-Mart Stores Inc. (WMT)	Cisco Systems, Inc. (CSCO)

From fig. 1.1, fig. 1.2 you can see that UnitedHealth Group Incorporated (UNH) uncorrelated almost all companies in DJIA.

For periods which described by fig. 1.3, fig. 1.4 (2010 – 2014), you can see changes in cross-correlation matrices. The Goldman Sachs Group, Inc. (GS), Cisco Systems, Inc. (CSCO) and Wal-Mart Stores Inc. (WMT), uncorrelatad with companies in DJIA.

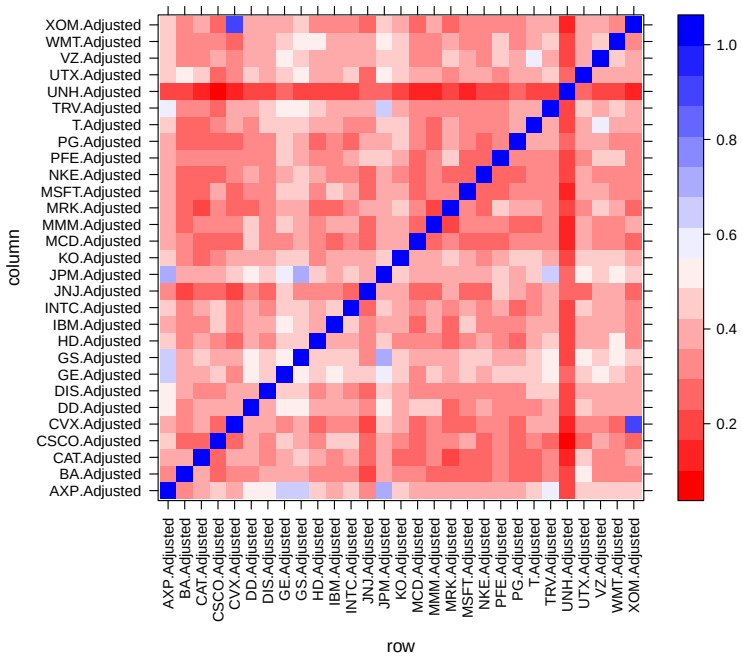

Figure 1.1. Pearson's cross–correlation matrix for period since 2006-01-01 to 2007-12-31

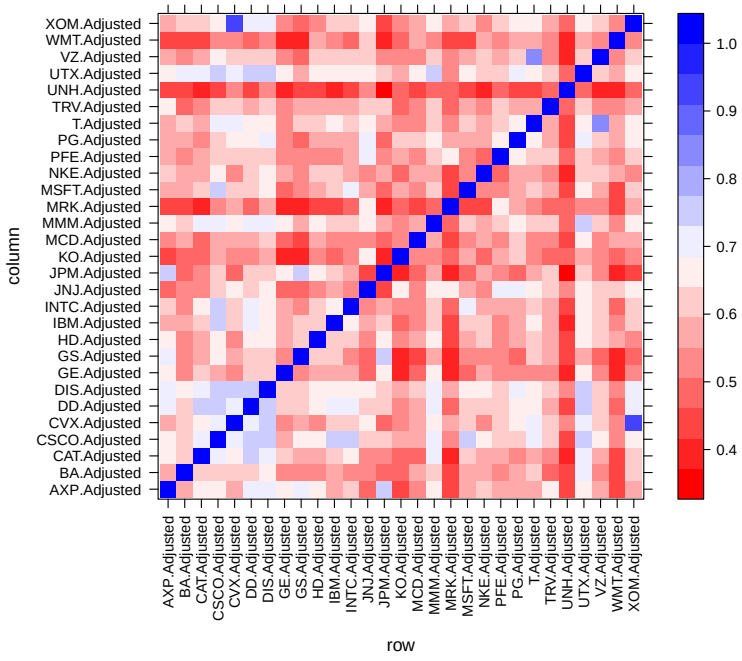

Figure 1.2. Pearson's cross–correlation matrix for period since 2008-01-01 to 2009-12-31

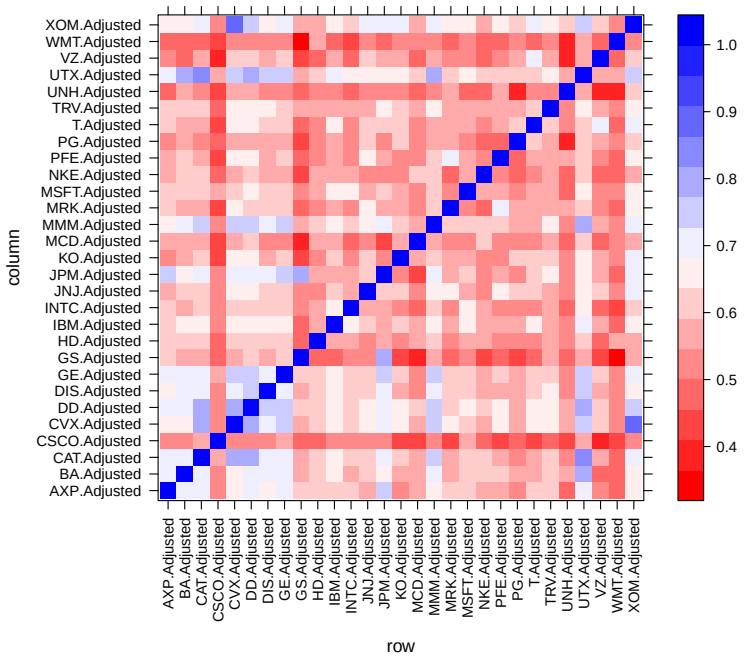

Figure 1.3. Pearson's cross–correlation matrix for period since 2010-01-01 to 2011-12-31

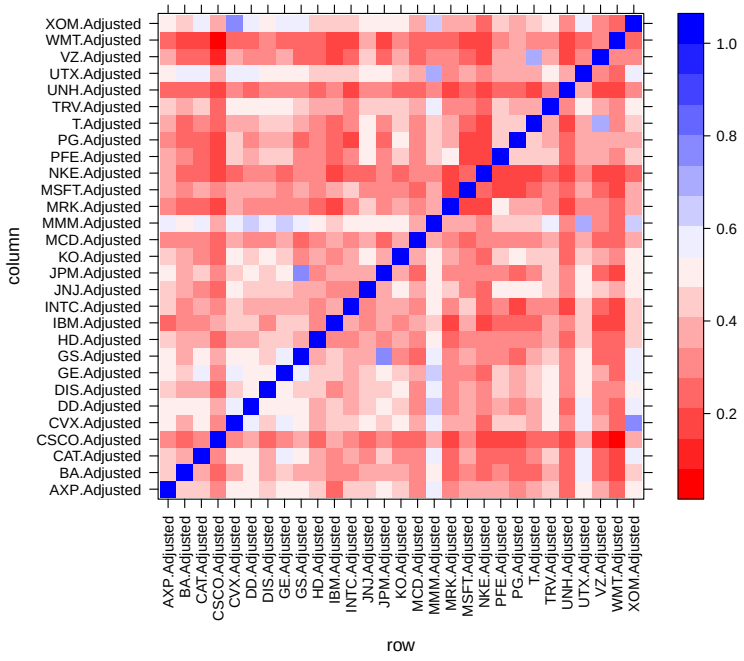

Figure 1.4. Pearson's cross–correlation matrix for period since 2012-01-01 to 2013-12-31

Table 1.2. Portfolio based on Spearman's cross-correlation

Period	Asset 1	Asset 2
2006–2007	UnitedHealth Group Incorporated (UNH)	Exxon Mobil Corporation (XOM)
2008–2009	UnitedHealth Group Incorporated (UNH)	Verizon Communications Inc. (VZ)
2010–2011	Wal-Mart Stores Inc. (WMT)	The Goldman Sachs Group, Inc. (GS)
2012–2013	Verizon Communications Inc. (VZ)	Cisco Systems, Inc. (CSCO)

Portfolio, which based on Spearman's cross-correlation save properties of the Pearson's cross–correlatin based portfolio for first two periods (fig. 1.5, fig. 1.6). UnitedHealth Group Incorporated (UNH) uncorrelated almost all companies in DJIA.

Third period (2010–2011) only Wal-Mart Stores Inc. (WMT), was differ much from all list of DJIA index. Correlation was higher beetween all companies this period. The reason of it, we think, is consequences of the World Financia Crisis which begans at 2008. Economies of big amount of countries were in recession.

The global recession resulted in a sharp drop in international trade, rising unemployment and slumping commodity prices. Several economists predicted that recovery might not appear until 2011 and that the recession would be the worst since the Great Depression of the 1930s. Paul Krugman, who won the Nobel Memorial Prize in Economics, once commented on this as seemingly the beginning of "a second Great Depression. "The conditions leading up to the crisis, characterised by an exorbitant rise in asset prices and associated boom in economic demand, are considered a result of the extended period of easily available credit and inadequate regulation and oversight. [7]

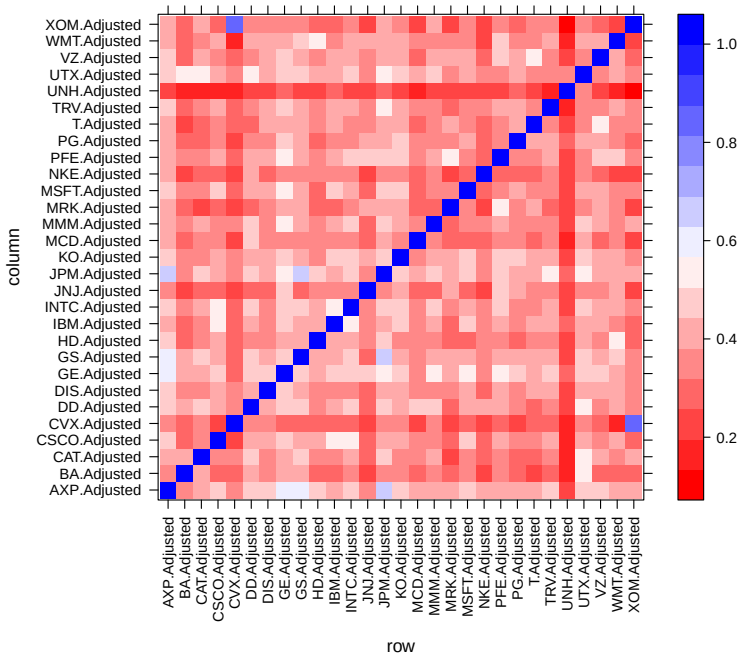

Figure 1.5. Spearman's cross–correlation matrix for period since 2006-01-01 to 2007-12-31

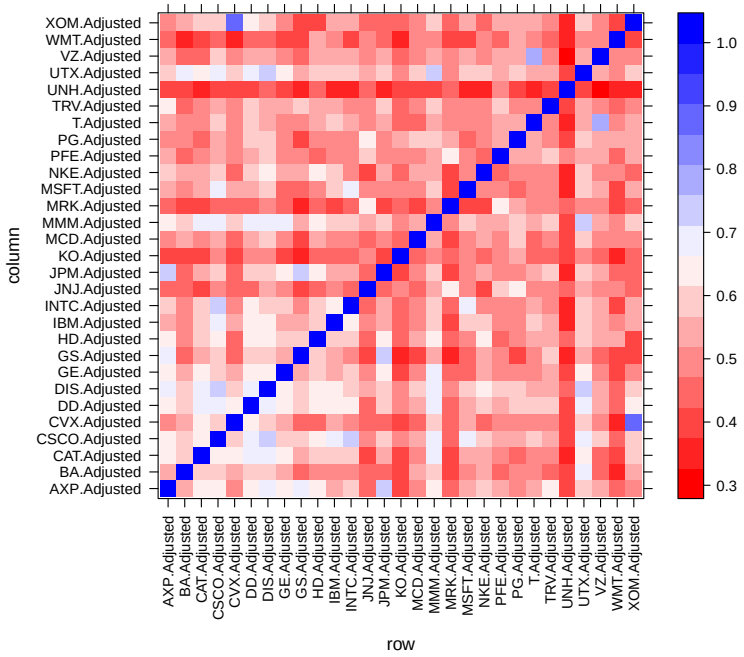

Figure 1.6. Spearman's cross–correlation matrix for period since 2008-01-01 to 2009-12-31

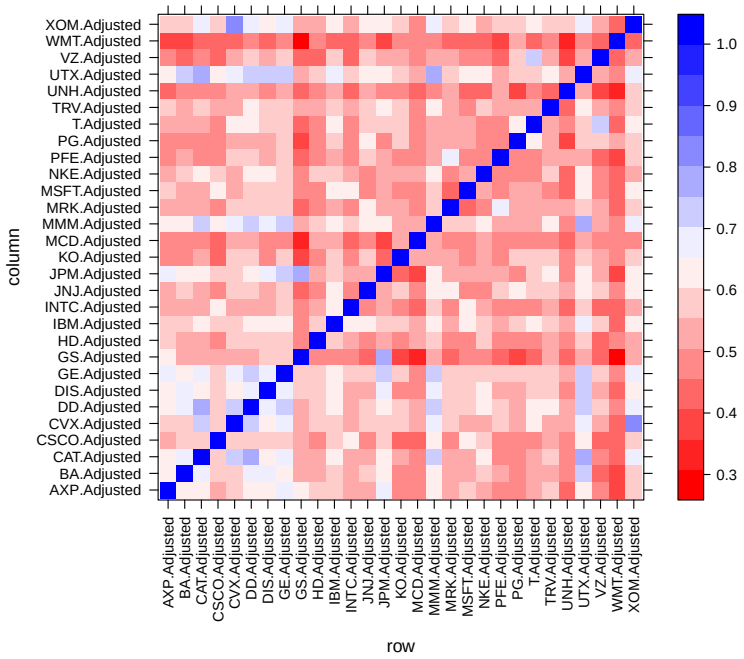

Figure 1.7. Spearman's cross–correlation matrix for period since 2010-01-01 to 2011-12-31

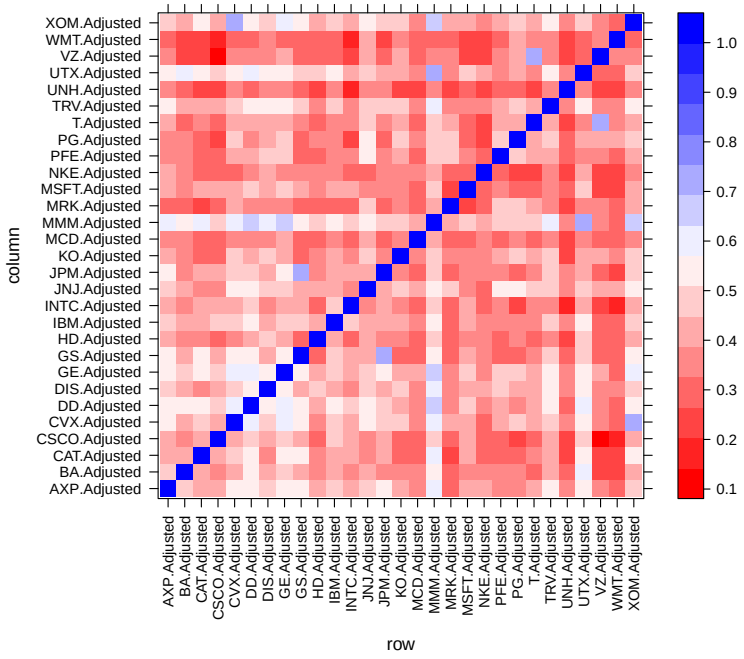

Figure 1.8. Spearman's cross–correlation matrix for period since 2012-01-01 to 2013-12-31

Table 1.3. Portfolio based on Kendall's cross-correlation

Period	Asset 1	Asset 2
2006–2007	UnitedHealth Group Incorporated (UNH)	Exxon Mobil Corporation (XOM)
2008–2009	UnitedHealth Group Incorporated (UNH)	Wal-Mart Stores Inc. (WMT)
2010–2011	Wal-Mart Stores Inc. (WMT)	The Goldman Sachs Group, Inc. (GS)
2012–2013	Verizon Communications Inc. (VZ)	Cisco Systems, Inc. (CSCO)

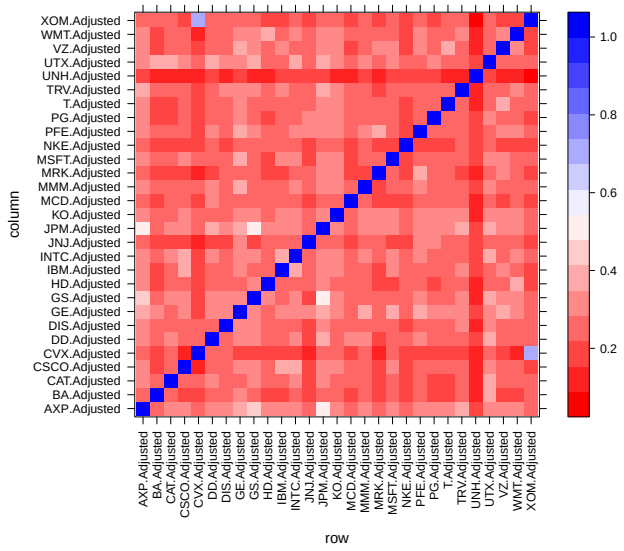

Figure 1.9. Kendall's cross–correlation matrix for period since 2006-01-01 to 2007-12-31

Protfolio, which build with Kendall's cross-correlation very close to portfolio which build with Sperman's cross-correlation. Differ is on period from 2008–2009. If you see to heet maps fig. 1.9 – fig. 1.12, the asset more uncorrelated versus Pearson and Spearman correlation.

If you see to tab. 1.4, portfolios which calculates by Spearman's and Kendall's are equall. But portfolios which calculates by Pearson method. It means that last portfolio has the same yield with less risk. But we should understand, that Person cross–correlation **not robust** and it isn't efficient if returns has not normal distribution.

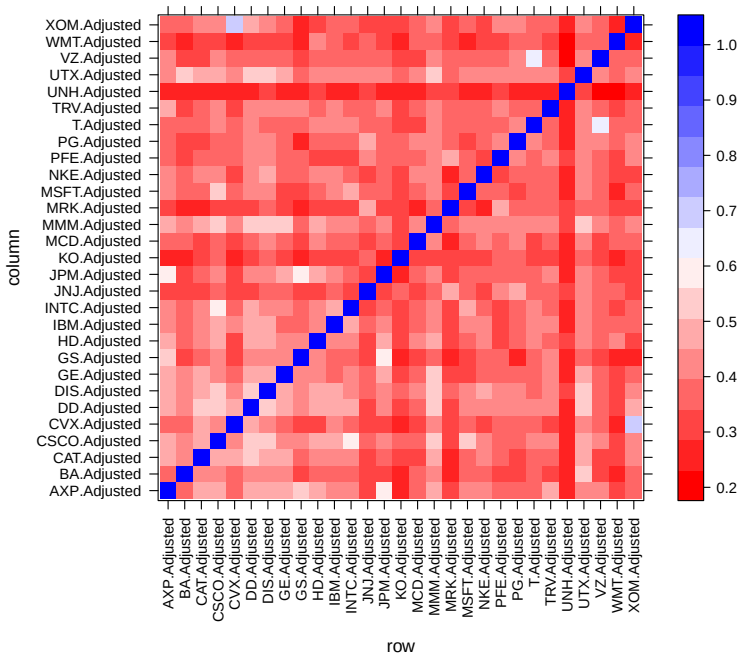

Figure 1.10. Kendall's cross–correlation matrix for period since 2008-01-01 to 2009-12-31

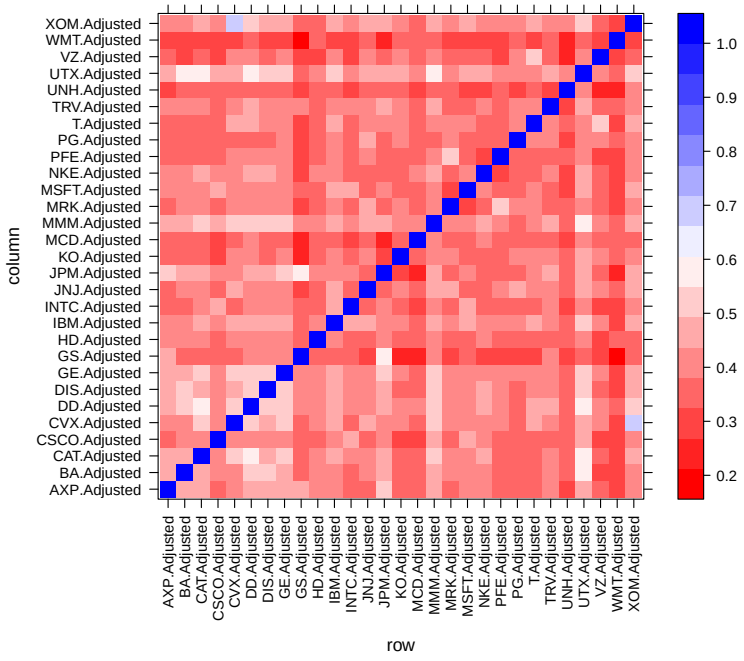

Figure 1.11. Kendall's cross–correlation matrix for period since 2010-01-01 to 2011-12-31

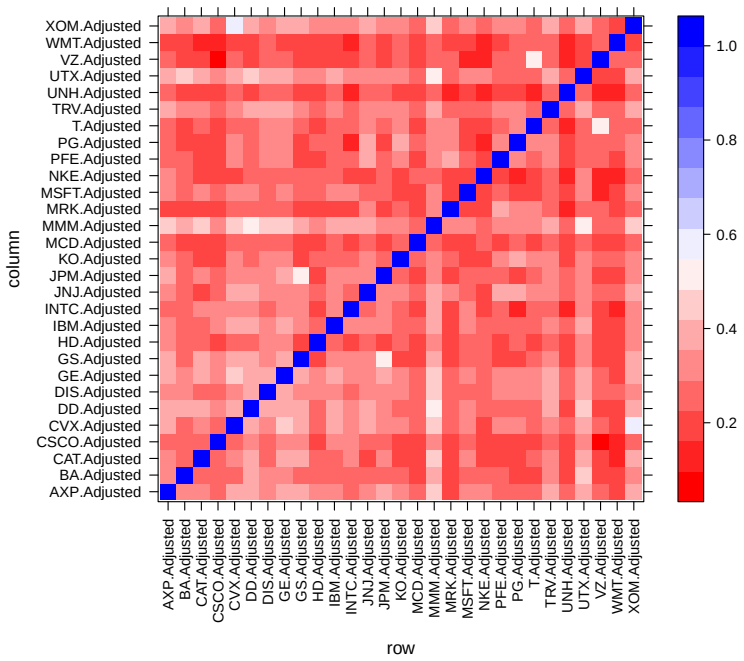

Figure 1.12. Kendall's cross–correlation matrix for period since 2012-01-01 to 2013-12-31

Table 1.4. Portfolio's Sharpe ratio for four periods

Period	Pearson	Speaman	Kendall
2006–2007	-0.044	-0.048	-0.048
2008–2009	-0.006	-0.009	-0.009
2010–2011	-0.004	-0.005	-0.005
2012–2013	-0.004	-0.004	-0.004

2 Problem #4

2.1 Task

Construct the minimum variance portfolio and tangent portfolio using 15 best performing stocks in US at the present day (use Yahoo! Finance: Market Data -> Market Stats -> Market Movers -> % Gainers). Assume that short-selling is allowed. For the historical data consider the horizon of the last 2 years. Estimate the tail exponent for the returns of this portfolio. Compare with the tail exponent of the S&P 500 index considered at the same horizon of 2 years.

2.2 Problem description

Minimum-variance portfolio — the portfolio of risky assets with lowest variance [8].

A portfolio of individually risky assets that, when taken together, result in the lowest possible risk level for the rate of expected return. Such a portfolio hedges each investment with an offsetting investment; the individual investor's choice on how much to offset investments depends on the level of risk and expected return he/she is willing to accept. The investments in a minimum variance portfolio are individually riskier than the portfolio as a whole. The name of the term comes from how it is mathematically expressed in Markowitz Portfolio Theory, in which volatility is used as a replacement for risk, and in which less variance in volatility correlates to less risk in an investment [9].

Modern Portfolio Theory — a theory of investing stating that every rational investor, at a given level of risk, will accept only the largest expected return. More specifically, modern portfolio theory attempts to account for risk and expected return mathematically to help the investor find a portfolio with the maximum return for the minimum about of risk. A Markowitz efficient portfolio represents just that: the most expected return at a given amount of risk (sometimes excluding zero risk). Harry Markowitz first began developing this theory in an article published in 1952 and received the Nobel prize for

economics for his work in 1990 [10].

Tangent Portfolios — are portfolios of stocks and bonds designed for long-term investors. Since people hate losing money about twice as much as they enjoy making it, the Tangent Portfolios start by asking how much you would be prepared to lose in a worst-case scenario without bailing out of the market: 20%, 25%,or 33%? Once the maximum loss level is selected, the Tangent Portfolios attempt to deliver a high rate of return for that amount of risk. Most investors tend to take on too much risk during good times ("buy high"), and then sell out during bad times ("sell low"), ruining their returns in the process. The Tangent Portfolios are designed to let you do well enough during both good times and bad to keep you in the markets throughout. This lets you reap the long-term benefits form investing in stocks and bonds with a simple, low-maintenance solution [11].

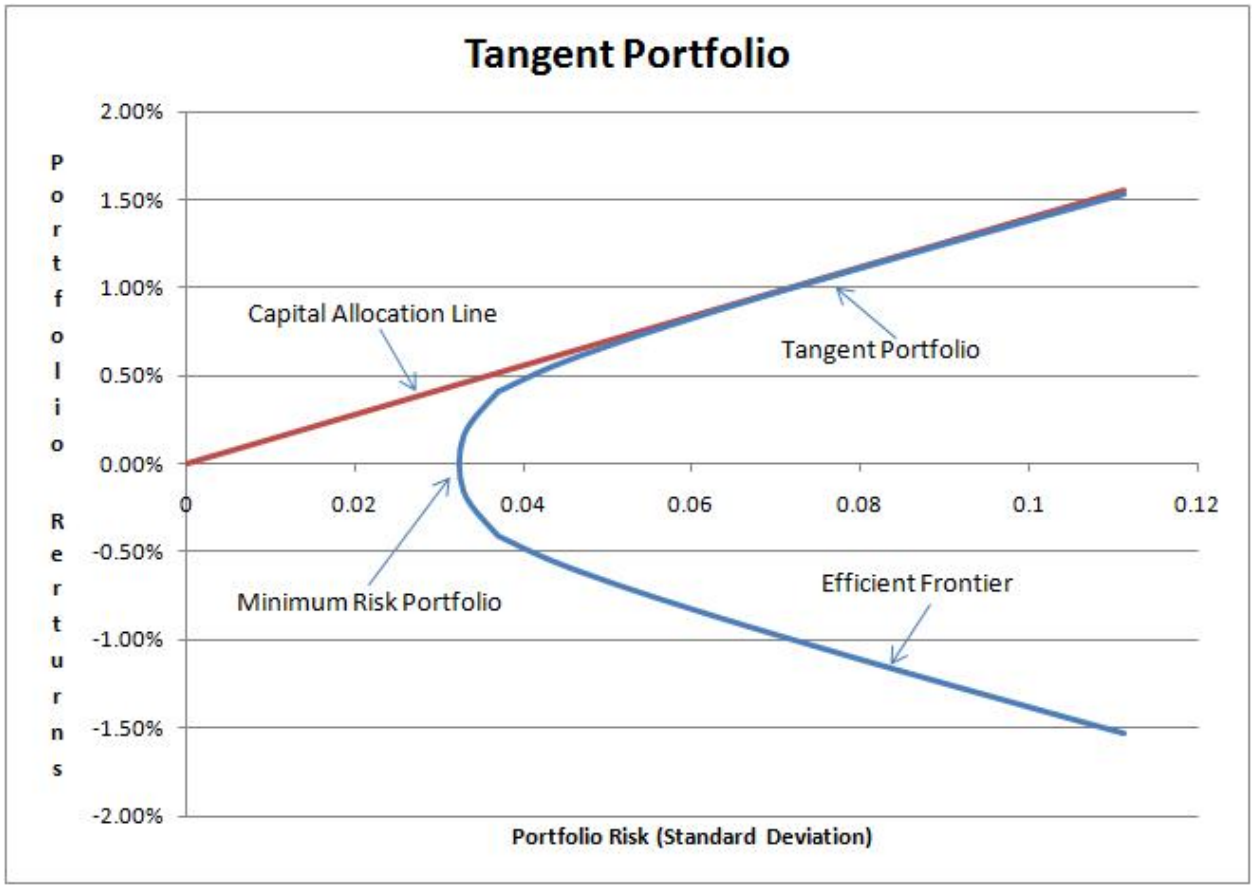

Figure 2.1. Tangent portfolio defenition

S&P 500 or the Standard & Poor's 500, is a stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. The S&P 500 index components and their weightings are determined by S&P Dow Jones Indices. It differs from other U.S. stock market indices, such as the Dow Jones Industrial Average or the Nasdaq Composite index, because of its diverse constituency and weighting methodology. It is one of the most commonly followed equity indices, and many consider it one of the best representations of the U.S. stock market, and a bellwether for the U.S. economy.The National Bureau of Economic Research has classified common stocks as a leading indicator of business cycles.

The S&P 500 was developed and continues to be maintained by S&P Dow Jones Indices, a joint venture majority-owned byMcGraw Hill Financial. S&P Dow Jones Indices publishes many stock market indices, such as the Dow Jones Industrial Average, S&P MidCap 400, the S&P SmallCap 600, and the S&P Composite 1500. It is a free-float capitalization-weighted index, and has many ticker symbols, such as: GSPC, INX,and $SPX [12].

2.3 Solution

2.4 Results

3 Problem #5

3.1 Task

For the 30 stocks that constitute the Dow Jones Industrial Average (Yahoo: DJIA) at the present day, find (a) the stock with the maximum P/E ratio, (b) the stock with the minimum P/E ratio and (c) the stock, which P/E ratio is the closest to the average value among all 30 components. Calculate annual returns, annual volatility, daily VaR and daily ES (using historical simulation method) for these stocks for the periods of 2010—2012 and 2005—2007. Calculate correlations between these 3 stocks in both of periods.

3.2 Problem description

The price-to-earnings ratio, or P/E ratio — an equity valuation multiple. It is defined as market priceper share divided by annual earnings per share. There are multiple versions of the P/E ratio, depending on whether earnings are projected or realized, and the type of earnings.

- **"Trailing P/E"** uses net income for the most recent 12 month period, divided by the weighted average number of common shares in issue during the period. This is the most common meaning of "P/E" if no other qualifier is specified. Monthly earnings data for individual companies are not available, and in any case usually fluctuate seasonally, so the previous four quarterly earnings reports are used and earnings per share are updated quarterly. Note, each company chooses its own financial year so the timing of updates varies from one to another.
- **"Trailing P/E from continued operations"** uses operating earnings, which exclude earnings from discontinued operations, extraordinary items (e.g. one-off windfalls and write-downs), and accounting changes.
- **"Forward P/E"**: Instead of net income, this uses estimated net earnings over next 12 months. Estimates are typically derived as the mean of those published by a select group of analysts (selection criteria are rarely cited) [13].

Annual Return — the return an investment provides over a period of time, expressed as a time-weighted annual percentage. Sources of returns can include dividends, returns of capital and capital appreciation. The rate of annual return is measured against the initial amount of the investment and represents a geometric mean rather than a simple arithmetic mean.

Annual return is the de facto method for comparing the performance of investments with liquidity, which includes stocks, bonds, funds, commodities and some types of derivatives. Different asset classes are considered to have different strata of annual returns [13].

Volatility — a measure for variation of price of a financial instrument over time. Historic volatility is derived from time series of past market prices. An implied volatility is derived from the market price of a market traded derivative (in particular an option). The symbol σ is used for volatility, and corresponds to standard deviation, which should not be confused with the similarly named variance, which is instead the square, σ^2.The annualized volatility σ is the standard deviation of the instrument's yearly logarithmic returns. The generalized volatility σT for time horizon T in years is expressed as [15]:

$$\sigma_T = \sigma\sqrt{T} \tag{3.1}$$

Value at risk (VaR) — a widely used risk measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value (assuming normal markets and no trading in the portfolio) is the given probability level [16].

Expected shortfall (ES) — a risk measure, a concept used in finance (and more specifically in the field of financial risk measurement) to evaluate themarket risk or credit risk of a portfolio. It is an alternative to value at risk that is more sensitive to the shape of the loss distribution in the tail of the distribution. The "expected shortfall at q% level" is the expected return on the portfolio in

the worst % of the cases. Expected shortfall is also called conditional value at risk (CVaR), average value at risk (AVaR), and expected tail loss (ETL). ES evaluates the value (or risk) of an investment in a conservative way, focusing on the less profitable outcomes. For high values of q it ignores the most profitable but unlikely possibilities, for small values of q it focuses on the worst losses. On the other hand, unlike the discounted maximum loss even for lower values of q expected shortfall does not consider only the single most catastrophic outcome. A value of q often used in practice is 5%. Expected shortfall is a coherent, and moreover a spectral, measure of financial portfolio risk. It requires a quantile-level q, and is defined to be the expected loss of portfolio value given that a loss is occurring at or below the q-quantile [17].

3.3 Solution

3.4 Results

References

[1] http://en.wikipedia.org/wiki/Dow_Jones_Industrial_Average

[2] http://en.wikipedia.org/wiki/Cross-correlation

[3] http://en.wikipedia.org/wiki/Pearson_product-moment_correlation_coefficient

[4] http://en.wikipedia.org/wiki/Spearman's_rank_correlation_coefficient

[5] http://en.wikipedia.org/wiki/Kendall_tau_rank_correlation_coefficient

[6] http://en.wikipedia.org/wiki/Sharpe_ratio

[7] http://en.wikipedia.org/wiki/Great_Recession

[8] http://www.nasdaq.com/investing/glossary/m/minimum-variance-portfolio

[9] http://financial-dictionary.thefreedictionary.com/Minimum-Variance+Portfolio

[10] http://financial-dictionary.thefreedictionary.com/Markowitz+Portfolio+Theory

[11] http://www.tangentportfolio.com/

[12] http://en.wikipedia.org/wiki/S&P_500

[13] http://en.wikipedia.org/wiki/Price%E2%80%93earnings_ratio

[14] http://www.investopedia.com/terms/a/annual-return.asp

[15] http://en.wikipedia.org/wiki/Volatility_(finance)

[16] http://en.wikipedia.org/wiki/Value_at_risk

[17] http://en.wikipedia.org/wiki/Expected_shortfall