GeekBrains

Курс “Введение в математический анализ”

Тема 6 “Понятие о производной”

1. Найти производную выражения:

a. $\sin x \cdot \cos x = (\sin x \cdot \cos x)' = (\sin x)' \cdot \cos x + \sin x \cdot (\cos x)' =$

$= \cos x \cdot \cos x + \sin x \cdot (-\sin x) = \cos x^2 - \sin x^2$

b. $\ln(2x+1)^3 = (\ln((2x+1)^3))' = (\ln((2x+1)^3))' \cdot ((2x+1)^3)' =$

$= \frac{1}{(2x+1)^{\cancel{3}}} \cdot 3 \cdot \cancel{(2x+1)^2} \cdot 2 = \frac{6}{2x+1}$

c. $\sqrt{\sin^2(\ln(x^3))} = \left((\sin^2(\ln(x^3)))^{\frac{1}{2}}\right)' = \frac{1}{2\sqrt{\sin^2(\ln(x^3))}} \cdot (\sin^2(\ln(x^3)))' =$

$\frac{1}{2\sqrt{\sin^2(\ln(x^3))}} \cdot 2\sin(\ln(x^3)) \cdot (\sin(\ln(x^3)))' =$

$= \frac{2\sin(\ln(x^3))}{2\sqrt{\sin^2(\ln(x^3))}} \cdot \cos(\ln(x^3)) \cdot (\ln(x^3))' =$

$= \frac{2\sin(\ln(x^3))}{2\sqrt{\sin^2(\ln(x^3))}} \cdot \cos(\ln(x^3)) \cdot \frac{1}{x^3} \cdot (x^3)' =$

$= \frac{\cancel{2}\sin(\ln(x^3))}{\cancel{2}\sqrt{\sin^2(\ln(x^3))}} \cdot \cos(\ln(x^3)) \cdot \frac{1}{x^{\cancel{3}}} \cdot 3\cancel{x^2} = \frac{\sin(\ln(x^3))}{\sqrt{\sin^2(\ln(x^3))}} \cdot \cos(\ln(x^3)) \cdot \frac{3}{x}$

d. $\frac{x^4}{\ln(x)} = \left(\frac{x^4}{\ln(x)}\right)' = \frac{(x^4)' \cdot \ln(x) - x^4 \cdot (\ln(x))'}{(\ln(x))^2} = \frac{4x^3 \cdot \ln(x) - x^4 \cdot \frac{1}{x}}{(\ln(x))^2} = \frac{4x^3 \cdot \ln(x) - \frac{x^4}{x}}{(\ln(x))^2} =$

$= \frac{4x^3 \cdot \ln(x) - x^3}{(\ln(x))^2}$

2. Найти выражение производной функции и ее значение в точке:

$f(x) = \cos(x^2 + 3x), \quad x_0 = \sqrt{\pi}$

$$f'(x) = -\sin(x^2+3x)\cdot(x^2+3x)' = -\sin(x^2+3x)(2x+3)$$

$$f'(\sqrt{\pi}) = -\sin\left(\left(\sqrt{\pi}\right)^2 + 3\sqrt{\pi}\right)\left(2\sqrt{\pi}+3\right) = -\sin\left(\pi + 3\sqrt{\pi}\right)\left(2\sqrt{\pi}+3\right)$$

3.* Найти значение производной функции в точке:

$$f(x) = \frac{x^3 - x^2 - x - 1}{1 + 2x + 3x^2 + 4x^3}, \quad x_0 = 0$$

Решение:

$$f'(x) = \left(\frac{x^3 - x^2 - x - 1}{1 + 2x + 3x^2 + 4x^3}\right)' = \frac{u'v - uv'}{v^2} =$$

$$= \frac{(x^3 - x^2 - x - 1)'\cdot(1 + 2x + 3x^2 + 4x^3) - (x^3 - x^2 - x - 1)\cdot(1 + 2x + 3x^2 + 4x^3)'}{(1 + 2x + 3x^2 + 4x^3)^2} =$$

$$= \frac{(3x^2 - 2x - 1)\cdot(1 + 2x + 3x^2 + 4x^3) - (x^3 - x^2 - x - 1)\cdot(2 + 6x + 12x^2)}{(1 + 2x + 3x^2 + 4x^3)^2}$$

$$u' = (x^3 - x^2 - x - 1)' = 3x^2 - 2x - 1$$

$$u(0) = 0^3 - 0^2 - 0 - 1 = -1$$

$$u'(0) = 3\cdot 0^2 - 2\cdot 0 - 1 = -1$$

$$v' = (1 + 2x + 3x^2 + 4x^3)' = 2 + 6x + 12x^2$$

$$v(0) = 1 + 2\cdot 0 + 3\cdot 0^2 + 4\cdot 0^3 = 1$$

$$v'(0) = 2 + 6\cdot 0 + 12\cdot 0^2 = 2$$

$$v^2 = (1 + 2x + 3x^2 + 4x^3)^2 = 1 + 4x^2 + 9x^4 + 16x^6$$

$$v^2(0) = 1 + 4\cdot 0^2 + 9\cdot 0^4 + 16\cdot 0^6 = 1$$

$$f'(x_0) = \frac{u'v - uv'}{v^2} = \frac{(-1\cdot 1) - (-1\cdot 2)}{1^2} = \frac{-1 - (-2)}{1^2} = \frac{1}{1} = 1$$

Проверка:

$$u'v = (x^3 - x^2 - x - 1)'\cdot(1 + 2x + 3x^2 + 4x^3) = (3x^2 - 2x - 1)\cdot(1 + 2x + 3x^2 + 4x^3) =$$

$$= 3x^2 - 2x - 1 + 3x^2 2x - 2x2x - 2x + 3x^2 3x^2 - 2x3x^2 - 3x^2 + 3x^2 4x^3 - 2x4x^3 - 4x^3 =$$

$$= \cancel{3x^2} - 2x - 1 + \cancel{6x^3} - 4x^2 - 2x + \cancel{9}x^4 - \cancel{6x^3} - \cancel{3x^2} + 12x^5 - \cancel{8}x^4 - 4x^3 =$$

$= 12x^5 + x^4 - 4x^3 - 4x^2 - 4x - 1$

$uv' = (x^3 - x^2 - x - 1) \cdot (1 + 2x + 3x^2 + 4x^3)' = (x^3 - x^2 - x - 1) \cdot (2 + 6x + 12x^2) =$

$= 2x^3 - 2x^2 - 2x - 2 + x^3 6x - x^2 6x - x6x - 6x + x^3 12x^2 - x^2 12x^2 - x12x^2 - 12x^2 =$

$= \cancel{2}x^3 - \cancel{2}x^2 - 2x - 2 + \cancel{6}x^4 - \cancel{6}x^3 - \cancel{6}x^2 - 6x + 12x^5 - \cancel{12}x^4 - \cancel{12}x^3 - \cancel{12}x^2 =$

$= 12x^5 - 6x^4 - 16x^3 - 20x^2 - 8x - 2$

$u'v - uv' = (12x^5 + x^4 - 4x^3 - 4x^2 - 4x - 1) - (12x^5 - 6x^4 - 16x^3 - 20x^2 - 8x - 2) =$

$= \cancel{12x^5} + x^4 - 4x^3 - 4x^2 - 4x - 1 - \cancel{12x^5} + 6x^4 + 16x^3 + 20x^2 + 8x + 2 =$

$= 7x^4 + 12x^3 + 16x^2 + 4x + 1$

$v^2 = (1 + 2x + 3x^2 + 4x^3)^2 = 1 + 4x^2 + 9x^4 + 16x^6$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{7x^4 + 12x^3 + 16x^2 + 4x + 1}{1 + 4x^2 + 9x^4 + 16x^6}$$

$$f'(x_0) = \frac{7 \cdot 0^4 + 12 \cdot 0^3 + 16 \cdot 0^2 + 4 \cdot 0 + 1}{1 + 4 \cdot 0^2 + 9 \cdot 0^4 + 16 \cdot 0^6} = \frac{1}{1} = 1$$

4. Найти угол наклона касательной к графику функции в точке:

$f(x) = \sqrt{3x} \cdot \ln x, \quad x_0 = 1$

$$f'(x) = \left(\sqrt{3x} \cdot \ln x\right)' = u'v + uv' = \left((3x)^{\frac{1}{2}}\right)' \cdot \ln x + \sqrt{3x} \cdot (\ln x)' =$$

$$= \frac{1}{2} \cdot (3x)^{-\frac{1}{2}} \cdot 3 \cdot \ln x + \sqrt{3x} \cdot \frac{1}{x} = \frac{3}{2\sqrt{3x}} \cdot \ln x + \sqrt{3x} \cdot \frac{1}{x}$$

$$f'(1) = \frac{3}{2\sqrt{3 \cdot 1}} \cdot \ln 1 + \sqrt{3 \cdot 1} \cdot \frac{1}{1} = \frac{3}{2\sqrt{3}} \cdot 0 + \sqrt{3} \cdot 1 = \sqrt{3}$$

$$\operatorname{tg} \alpha = \sqrt{3} \Longrightarrow \alpha = \operatorname{arctg}(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$$