

$$\textcircled{2} \quad A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -1 \\ 0 & 5 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 1+4 & -2-1 \\ 3+0 & 0+5 \end{pmatrix} = \begin{pmatrix} 5 & -3 \\ 3 & 5 \end{pmatrix}$$

$$A \times B = C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

$$C_{11} = a_{11} \times b_{11} + a_{12} \times b_{21} = 1 \times 4 + (-2) \times 0 = 4$$

$$C_{12} = a_{11} \times b_{12} + a_{12} \times b_{22} = 1 \times (-1) + (-2) \times 5 = -11$$

$$C_{21} = a_{21} \times b_{11} + a_{22} \times b_{21} = 3 \times 4 + 0 \times 0 = 12$$

$$C_{22} = a_{21} \times b_{12} + a_{22} \times b_{22} = 3 \times (-1) + 0 \times 5 = -3$$

$$\Rightarrow C = \begin{pmatrix} 4 & -11 \\ 12 & -3 \end{pmatrix}$$

$$\textcircled{3} \quad A = \begin{pmatrix} 1 & 7 \\ 3 & -6 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 5 \\ 2 & -1 \end{pmatrix}; \quad C = \begin{pmatrix} 2 & -4 \\ 1 & 1 \end{pmatrix}$$

$$3A - 2B + 4C = \begin{pmatrix} 3 & 21 \\ 9 & -18 \end{pmatrix} - \begin{pmatrix} 0 & 10 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 8 & -16 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 11 & -5 \\ 9 & -12 \end{pmatrix}$$

(4)

$$A = \begin{pmatrix} 4 & 1 \\ 5 & -2 \\ 2 & 3 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} 4 & 1 \\ 5 & -2 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 4 & 5 & 2 \\ 1 & -2 & 3 \end{pmatrix} = C = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$$

$$C_{11} = a_{11} \times b_{11} + a_{12} \times b_{21} = 4 \times 4 + 1 \times 1 = 17$$

$$C_{12} = a_{11} \times b_{12} + a_{12} \times b_{22} = 4 \times 5 + 1 \times (-2) = 18$$

$$C_{13} = a_{11} \times b_{13} + a_{12} \times b_{23} = 4 \times 2 + 1 \times 3 = 11$$

$$C_{21} = a_{21} \times b_{11} + a_{22} \times b_{21} = 5 \times 4 + (-2) \times 1 = 18$$

$$C_{22} = a_{21} \times b_{12} + a_{22} \times b_{22} = 5 \times 5 + (-2) \times (-2) = 29$$

$$C_{23} = a_{21} \times b_{13} + a_{22} \times b_{23} = 5 \times 2 + (-2) \times 3 = 4$$

$$C_{31} = a_{31} \times b_{11} + a_{32} \times b_{21} = 2 \times 4 + 3 \times 1 = 11$$

$$C_{32} = a_{31} \times b_{12} + a_{32} \times b_{22} = 2 \times 5 + 3 \times (-2) = 4$$

$$C_{33} = a_{31} \times b_{13} + a_{32} \times b_{23} = 2 \times 2 + 3 \times 3 = 13$$

$$C = \begin{pmatrix} 17 & 18 & 11 \\ 18 & 29 & 4 \\ 11 & 4 & 13 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} 4 & 5 & 2 \\ 1 & -2 & 3 \end{pmatrix} \times \begin{pmatrix} 4 & 1 \\ 5 & -2 \\ 2 & 3 \end{pmatrix} = D = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$$

$$d_{11} = a_{11} \times b_{11} + a_{12} \times b_{21} + a_{13} \times b_{31} = 4 \times 4 + 5 \times 5 + 2 \times 2 = 45$$

$$d_{12} = a_{11} \times b_{12} + a_{12} \times b_{22} + a_{13} \times b_{32} = 4 \times 1 + 5 \times (-2) + 2 \times 3 = 0$$

$$d_{21} = a_{21} \times b_{11} + a_{22} \times b_{21} + a_{23} \times b_{31} = 1 \times 4 + (-2) \times 5 + 3 \times 2 = 0$$

$$d_{22} = a_{21} \times b_{12} + a_{22} \times b_{22} + a_{23} \times b_{32} = 1 \times 1 + (-2) \times (-2) + 3 \times 3 = 14$$

$$\Rightarrow D = \begin{pmatrix} 45 & 0 \\ 0 & 14 \end{pmatrix}$$

⑥ ОПРЕДЕЛИТЕ МАТРИЦЫ

$$\det \begin{pmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{pmatrix} = \sin^2 x - (-\cos^2 x) = 1$$

$$\det \begin{pmatrix} 8 & 4 & 6 \\ 0 & 5 & 1 \\ 0 & 0 & 9 \end{pmatrix} = 8 \times (5 \times 9 - 1 \times 0) - 0 \times (4 \times 9 - 6 \times 0) + 0 \times (4 \times 1 - 6 \times 5) = 360$$

$$\det \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{pmatrix} = 2 \times (6 \times 10 - 7 \times 9) - 5 \times (3 \times 10 - 4 \times 9) + 8 \times (3 \times 7 - 4 \times 6) = 0$$

⑦ $\det(A) = 4$

~~det~~ $\det(A^2) = \det(A) \times \det(A) = 16$

$$\det(A^T) = \det(A) = 4$$

$$\det(2A) = \det(A) \times 2^n = 4 \times 2^n, n - \text{размерность } A$$

⑧ Вспомогательная матрица

$$A = \begin{pmatrix} -2 & 7 & -3 \\ 4 & -14 & 6 \\ -3 & 7 & 13 \end{pmatrix}$$

$$\det(A) = -2(-14 \times 13 - 6 \times 7) - 4(7 \times 13 - (-3) \times 7) - 3(7 \times 13 - (-3) \times 7) = 0$$

$\det(A) = 0 \Rightarrow$ матрица вырождена.

⑨ Rang und Rangfolge

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \text{ Rang } A = 2$$

$$B = \begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 3 \\ 2 & 3 & 5 & 6 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \\ 2 & 3 & 5 & 6 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 3 & 5 & 6 \end{pmatrix} \text{ Rang } B = 3$$