

Теор. Загара.

$$X \in \{x_1, \dots, x_n\}, x_i \in \{1, 2, 3\}$$

$$P_1: \begin{array}{ccc} & 1 & 2 & 3 \\ & \alpha & 1-\alpha & 0 \end{array}$$

$$P_2: \begin{array}{ccc} & 1 & 2 & 3 \\ & 0 & 1-\beta & \beta \end{array}$$

$$X: 30 \text{ "1"}; 20 \text{ "2"}; 60 \text{ "3"}$$

$$\alpha_0 = \beta_0 = \gamma_0 = 0.5$$

$$Z = \{z_1, \dots, z_n\}, z_i = [x_i \in P_1]$$

$$\begin{aligned} P(X, Z \mid \alpha, \beta, \gamma) &= \prod_{i=1}^N p(x_i, z_i \mid \alpha, \beta, \gamma) = \\ &= \prod_{i=1}^N (\gamma P_1(x_i \mid \alpha))^{z_i} ((1-\gamma) \cdot P_2(x_i \mid \beta))^{1-z_i} \end{aligned}$$

E-step

$$q(z) = p(z | x, \alpha, \beta, \gamma) = \frac{p(z, x, \alpha, \beta, \gamma)}{p(x, \alpha, \beta, \gamma)} =$$
$$= \frac{p(z, x | \alpha, \beta, \gamma)}{p(x | \alpha, \beta, \gamma)} = \prod_{i=1}^N \frac{p(x_i, z_i | \alpha, \beta, \gamma)}{p(x_i | \alpha, \beta, \gamma)} = \prod_{i=1}^N q(z_i)$$

$$\prod_{i=1}^N \frac{(\gamma p_1(x_i | \alpha))^{z_i} ((1-\gamma) p_2(x_i | \beta))^{1-z_i}}{\gamma p_1(x_i | \alpha) + (1-\gamma) p_2(x_i | \beta)}$$

$$q(z_1) = \frac{0,5\alpha}{0,5\alpha} = 1$$

$$q(z_2) = \frac{0,5(1-\alpha)}{0,5(1-\alpha) + 0,5(1-\beta)} = \frac{1}{2}$$

$$q(z_3) = \frac{0}{0,5\beta} = 0$$

M-step

$$\mathbb{E}_{q(z)} \log p(x, z | \alpha, \beta, \gamma) = \sum_{i=1}^N \mathbb{E}_{q(z_i)} \log p(x_i, z_i | \alpha, \beta, \gamma)$$

↓
max
 α, β, γ

$$\mathbb{E}_{q(z_i)} \log \left[\prod_{i=1}^3 (\gamma p_1(x_i | \alpha))^{z_i} ((1-\gamma) p_2(x_i | \beta))^{1-z_i} \right] =$$

③ расписываем в сумму
 + нежность про серию

$$\textcircled{3} \frac{30}{110} \cdot \log[(\gamma \cdot p_1(x_{i=1}|\alpha))^1] +$$

$$+ \frac{30}{110} \log[(1-\gamma) \cdot p_2(x_{i=1}|\beta)^{1-1}] +$$

$$+ \frac{20}{110} \cdot \log[(\gamma \cdot p_1(x_{i=2}|\alpha))^{\frac{1}{2}}] +$$

$$+ \frac{20}{110} \log[(1-\gamma) \cdot p_2(x_{i=2}|\beta)^{1-\frac{1}{2}}] +$$

$$+ \frac{60}{110} \log[(\gamma \cdot p_1(x_{i=3}|\alpha))^0] +$$

$$+ \frac{60}{110} \log[(1-\gamma) \cdot p_2(x_{i=3}|\beta)^{1-0}] =$$

она в сумму
 розов + красн
 средн

$$= \frac{30}{110} \cdot \log(\gamma) + \frac{30}{110} \log(\alpha) + \frac{20}{110} \cdot \frac{1}{2} \log(\gamma) +$$

$$+ \frac{20}{110} \cdot \frac{1}{2} \log(1-\alpha) + \frac{20}{110} \cdot \frac{1}{2} \log(1-\gamma) +$$

$$+ \frac{20}{110} \cdot \frac{1}{2} \log(1-\beta) + \frac{60}{110} \cdot \log(1-\gamma) + \frac{60}{110} \log(\beta)$$

мы теперь берем производ и приравн. 0

$$\frac{30}{110\alpha} - \frac{10}{110(1-\alpha)} = 0$$

$$\boxed{\alpha = \frac{3}{4}}$$

$$\frac{60}{110\beta} - \frac{10}{110(1-\beta)} = 0$$

$$\boxed{\beta = \frac{6}{7}}$$

$$\frac{30}{110\gamma} + \frac{10}{110\gamma} - \frac{10}{110(1-\gamma)} - \frac{60}{110(1-\gamma)} = 0$$

$$\boxed{\gamma = \frac{4}{11}}$$