# Distributionally Robust Co-Optimization of Energy and Reserve Dispatch of Integrated Electricity and Heat System - Online Appendix


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## Nomenclature

### A. Parameters

| | |
|---|---|
| $\underline{P}^G, \overline{P}^G$ | Vector of the minimum and maximum real power output of generators [MW] |
| $\underline{P}^{CHP}, \overline{P}^{CHP}$ | Vector of the minimum and maximum power supply from CHP unit [MW] |
| $\overline{R}^G$ | Vector of the generators reserve limits [MW] |
| $\overline{R}^{CHP}$ | Vector of the CHP reserve limits [MW] |
| $P_W^f$ | Wind power forecast vector [MW] |
| $P^D$ | Electric demand vector [MW] |
| $P^l$ | Line transmission capacity vector [MW] |
| $\underline{H}_s^{HS}, \overline{H}_s^{HS}$ | Minimum and maximum heat supply from HS unit [MW] |
| $H_l^L$ | Heat demand [MW] |
| $c_2, c_1, c_0$ | Cost coefficients of generators [$/MWh] |
| $c_e, c_h$ | Cost coefficients for CHP [$/MWh] |
| $\overline{c_G}, \underline{c_G}$ | Cost coefficients of generators for providing reserves [$/MWh] |
| $\overline{c_c}, \underline{c_c}$ | Cost coefficients of CHP for providing reserves [$/MWh] |
| $\rho_s^H, /\rho_s^E$ | Heat/Electricity-to-fuel ratio of the extraction CHP |
| $\rho_s$ | Heat-to-Electricity output ratio of the extraction CHP |
| $\eta^{WP}$ | Water pump efficiency |
| $COP_s$ | Coefficient of performance of the heat pump |
| $\overline{F}_s$ | Maximum fuel consumption of CHP unit [MW] |
| $\underline{P}_s^{WP}, \overline{P}_s^{WP}$ | Technical limits of the water pump at the heat station |
| $\underline{m}_p^S, \overline{m}_p^S$ | Lower and upper limits of mass flow rate of the pipeline $p$ in supply network [kg/s] |
| $\underline{m}_p^R, \overline{m}_p^R$ | Lower and upper limit of mass flow rate of the pipeline $p$ in return network [kg/s] |
| $\underline{m}_s^{HS}, \overline{m}_s^{HS}$ | Lower and upper limit of mass flow rate of the HS $s$ [kg/s] |
| $\underline{m}_l^{HL}, \overline{m}_l^{HL}$ | Lower and upper limit of mass flow rate of the heat load $l$ [kg/s] |
| $\underline{T}_n^S, \overline{T}_n^S$ | Minimum/maximum temperature at node $n$ in the supply network [°C] |
| $\underline{T}_n^R, \overline{T}_n^R$ | Minimum/maximum temperature at node $n$ in the return network [°C] |
| $\underline{T}_p^S, \overline{T}_p^S$ | Minimum/maximum temperature at pipe $p$ in the supply network [°C] |
| $\underline{T}_p^R, \overline{T}_p^R$ | Minimum/maximum temperature at pipe $p$ in the return network [°C] |
| $\underline{pr}_n^S, \overline{pr}_n^S$ | Minimum/maximum pressure at node $n$ in the supply network [kPa] |
| $\underline{pr}_n^R, \overline{pr}_n^R$ | Minimum/maximum pressure at node $n$ in the return network [kPa] |
| $\underline{pr}_l^{HL}$ | Minimum pressure difference at the heat load [kPa] |
| $C_p$ | Specific water capacity [J/kg°C] |
| $\lambda$ | Heat transfer coefficient per unit length [W/m°C] [MW] |
| $K_p$ | Pipe resistance coefficient [$m^{-1}kg^{-1}$] |
| $L_p$ | Length of pipe $p$ [m] |
| $D_p$ | Diameter of a pipe $p$ [$m$] |
| $\nu_p$ | Absolute roughness of a pipe $p$ [$m$] |

**Notation:** Index denotes an element of the vector with the corresponding dimension.

## Case Study Data

Electrical demand is 200 MW and 100 MW at buses 4 and 5. We assume that reserves from generator are more expensive than CHP reserves $\overline{c_G} = \underline{c_G} = 1.2c_1$, $\overline{c_c} = \underline{c_c} = 1.1c_e$ In this study water density is $\rho = 1000$ kg/$m^3$ and kinematic viscosity of water is $\mu = 0.4736 \times 10^{-6}$ m/$s^2$. Darcy friction factor $f_D = 0.0118$. The characteristics of the generation units, transmission lines parameters and DHN parameters are presented in Table I,II,III and IV.

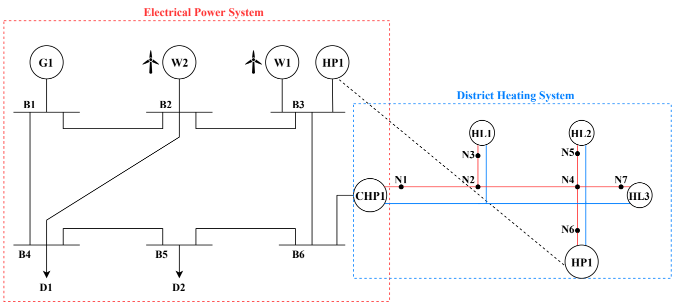


Fig. 1. Configuration of the six-bus and seven-node integrated system.

TABLE I
GENERATION UNITS

| | | $G_1$ | $CHP_1$ | $HP_1$ |
|---|---|---|---|---|
| $\overline{P}$ | MW | 230 | 208.3 | - |
| $\underline{P}$ | MW | 10 | 15 | - |
| $\overline{R}$ | MW | 92 | 41.66 | - |
| $\underline{R}$ | MW | 0 | 0 | - |
| $\underline{H}$ | MW | - | 0 | 5 |
| $\overline{H}$ | MW | - | 250 | 100 |
| $\underline{m}_s^{HS}$ | kg/s | - | 300 | 300 |
| $\overline{m}_s^{HS}$ | kg/s | - | 700 | 700 |
| $\overline{F}$ | MW | - | 500 | - |
| COP | - | - | - | 2.5 |
| $r$ | - | - | 0.5 | - |
| $\rho^E$ | - | - | 2.4 | - |
| $\rho^H$ | - | - | 0.25 | - |
| $\eta$ | - | - | 0.9 | 0.9 |
| $\overline{P}_{WP}$ | MW | - | 20 | 20 |
| $\underline{P}_{WP}$ | MW | - | 0 | 0 |
| $c_2$ | $\$/\ \mathrm{MW}h^2$ | 0.00125 | - | - |
| $c_1$ | $\$/\ \mathrm{MWh}$ | 40.622 | - | - |
| $c_0$ | $\$$ | 0 | - | - |
| $c_e$ | $\$/\ \mathrm{MWh}$ | - | 3.6 | - |
| $c_h$ | $\$/\ \mathrm{MWh}$ | - | 0.06 | - |

TABLE II
ELECTRICAL TRANSMISSION LINES

| | | $l_{12}$ | $l_{14}$ | $l_{23}$ | $l_{24}$ | $l_{36}$ | $l_{45}$ | $l_{56}$ |
|---|---|---|---|---|---|---|---|---|
| $\overline{P}_l$ | MW | 200 | 250 | 250 | 200 | 250 | 250 | 250 |
| $X$ | p.u. | 0.17 | 0.0586 | 0.1 | 0.072 | 0.0625 | 0.16 | 0.085 |

TABLE III
DISTRICT HEATING PIPES

| | | $p_{12,24,46}$ | $p_{23}$ | $p_{45,47}$ |
|---|---|---|---|---|
| $L_p$ | m | 800 | 600 | 500 |
| $D_p$ | m | 0.8 | 0.8 | 0.8 |
| $\lambda$ | W/m°C | 0.2 | 0.2 | 0.2 |
| $\nu_p$ | $m \times 10^{-3}$ | 0.045 | 0.045 | 0.045 |
| $K_p$ | $1/[m \times kg]$ | 0.0233 | 0.0175 | 0.0146 |
| $\underline{m}_p^S, \underline{m}_p^R$ | $kg/s$ | 300 | 300 | 300 |
| $\overline{m}_p^S, \overline{m}_p^R$ | $kg/s$ | 700 | 700 | 700 |
| $\overline{T}_p^R$ | °C | 45 | 45 | 45 |
| $\underline{T}_p^R$ | °C | 25 | 25 | 25 |
| $\overline{T}_p^S$ | °C | 65 | 65 | 65 |
| $\underline{T}_p^S$ | °C | 50 | 50 | 50 |

TABLE IV
DISTRICT HEATING NETWORK PARAMETERS

| | | $N_{1,2,4,6}$ | $N_3(L)$ | $N_5(L)$ | $N_7(L)$ |
|---|---|---|---|---|---|
| $\underline{m}_l^{HL}$ | kg/s | - | 300 | 300 | 300 |
| $\overline{m}_l^{HL}$ | kg/s | - | 700 | 700 | 700 |
| $H_l^L$ | MW | - | 45 | 40 | 50 |
| $\underline{T}_n^S$ | °C | 50 | 50 | 50 | 50 |
| $\overline{T}_n^S$ | °C | 65 | 65 | 65 | 65 |
| $\underline{T}_n^R$ | °C | 25 | 25 | 25 | 25 |
| $\overline{T}_n^R$ | °C | 45 | 45 | 45 | 45 |
| $\underline{pr}_n^S, \underline{pr}_n^R$ | kPa | 0 | 0 | 0 | 0 |
| $\overline{pr}_n^S, \overline{pr}_n^R$ | kPa | 30000 | 30000 | 30000 | 30000 |
| $\underline{pr}_l^{HL}$ | kPa | - | 50 | 50 | 50 |