Distributionally Robust Co-Optimization of Energy and Reserve Dispatch of Integrated Electricity and Heat System - Online Appendix

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NOMENCLATURE

A. Indexes and Sets

n	Index of nodes
l	Index of thermal loads
s	Index of heat stations
p	Index of pipes
Λ^N	Set of nodes
Λ^P	Set of pipes
Λ^{N-}	Set of nodes with a single pipe ending
Λ^{N+}	Set of nodes with a single pipe starting
Θ_n^{HS}	Set of heat sources (HS) at node n
Θ_n^{CHP}	Set of CHPs at node n, $\Theta_n^{CHP} \in \Theta_n^{HS}$
Θ_n^{HP}	Set of heat pumps at node n, $\Theta_n^{HP} \in \Theta_n^{HS}$
Θ_n^{HL}	Set of heat demands at node n
$\Theta_n^{P,OUT}$	Set of indices of pipes starting at node n
$\Theta_n^{P,IN}$	Set of indices of pipes ending at node n

B. Parameters

$\underline{P}^G, \overline{P}^G$	Vector of the minimum and maximum real power output of generators [MW]
$\underline{P}^{CHP}, \overline{P}^{CHP}$	Vector of the minimum and maximum power supply from CHP unit [MW]
$\overline{R}^G$	Vector of the generators reserve limits [MW]
$\overline{R}^{CHP}$	Vector of the CHP reserve limits [MW]
P_W^f	Wind power forecast vector [MW]
P^D	Electric demand vector [MW]
P^l	Line transmission capacity vector [MW]
$\underline{H}_s^{HS}, \overline{H}_s^{HS}$	Minimum and maximum heat supply from HS unit [MW]
H_l^L	Heat demand [MW]
c_2, c_1, c_0	Cost coefficients of generators [$/MWh]
c_e, c_h	Cost coefficients for CHP [$/MWh]
$\overline{c_G}, \underline{c_G}$	Cost coefficients of generators for providing reserves [$/MWh]
$\overline{c_c}, \underline{c_c}$	Cost coefficients of CHP for providing reserves [$/MWh]
$\rho_s^H, /\rho_s^E$	Heat/Electricity-to-fuel ratio of the extraction CHP
ρ_s	Heat-to-Electricity output ratio of the extraction CHP
η^{WP}	Water pump efficiency
COP_s	Coefficient of performance of the heat pump
$\overline{F}_s$	Maximum fuel consumption of CHP unit [MW]
$\underline{P}_s^{WP}, \overline{P}_s^{WP}$	Technical limits of the water pump at the heat station
$\underline{m}_p^S, \overline{m}_p^S$	Lower and upper limits of mass flow rate of the pipeline p in supply network [kg/s]
$\underline{m}_p^R, \overline{m}_p^R$	Lower and upper limit of mass flow rate of the pipeline p in return network [kg/s]
$\underline{m}_s^{HS}, \overline{m}_s^{HS}$	Lower and upper limit of mass flow rate of the HS s [kg/s]
$\underline{m}_l^{HL}, \overline{m}_l^{HL}$	Lower and upper limit of mass flow rate of the heat load l [kg/s]
$\underline{T}_n^S, \overline{T}_n^S$	Minimum/maximum temperature at node n in the supply network [°C]
$\underline{T}_n^R, \overline{T}_n^R$	Minimum/maximum temperature at node n in the return network [°C]
$\underline{T}_p^S, \overline{T}_p^S$	Minimum/maximum temperature at pipe p in the supply network [°C]
$\underline{T}_p^R, \overline{T}_p^R$	Minimum/maximum temperature at pipe p in the return network [°C]
$\underline{pr}_n^S, \overline{pr}_n^S$	Minimum/maximum pressure at node n in the supply network [kPa]
$\underline{pr}_n^R, \overline{pr}_n^R$	Minimum/maximum pressure at node n in the return network [kPa]
$\underline{pr}_l^{HL}$	Minimum pressure difference at the heat load [kPa]
C_p	Specific water capacity [J/kg°C]
λ	Heat transfer coefficient per unit length [W/m°C] [MW]
K_p	Pipe resistance coefficient [$m^{-1}kg^{-1}$]
L_p	Length of pipe p [m]

D_p	Diameter of a pipe p $[m]$
ν_p	Absolute roughness of a pipe p $[m]$

C. Decision Variables

m_p^S, m_p^R	Mass flow rate of pipeline p [kg/s]
m_s^{HS}	Mass flow rate of HS s [kg/s]
m_l^{HL}	Mass flow rate of heat load l [kg/s]
$T_p^{S,start}$	Temperature at the start node of pipe p in the supply network [°C]
$T_p^{S,end}$	Temperature at the end node of pipe p in the supply network [°C]
$T_p^{R,start}$	Temperature at the start node of pipe p in the return network [°C]
$T_p^{R,end}$	Temperature at the end node of pipe p in the return network [°C]
T_n^S	Temperature of node n in the supply network [°C]
T_n^R	Temperature of node n in the return network [°C]
H_s^{HS}	Heat supply from HS unit s
P^{CHP}	Day-ahead power dispatch from CHP unit [MW]
P^G	Day-ahead power dispatch of generators [MW]
P_s^{HP}	Power consumption of heat pump [MW]
P_s^{WP}	Power consumption of circulation pump [MW]
$R^{up/dn}$	Upward/Downward reserves for the generators [MW]
$R^{uc/dc}$	Upward/Downward reserve for the CHPs [MW]
Y	Generation participation factor

Notation: Index denotes an element of the vector with the corresponding dimension.

Distributionally robust reformulation

Stochastic constraints that involve random variable could be written as one-side chance-constraints

$$\mathbb{P}(A[C(P+R)+C_W(P^f+\Delta W)-C_DP^D - C_pP^{WP}-C_{HP}P^{HP}] \geq -P^l) \geq 1-\epsilon$$

$$\mathbb{P}(A[C(P+R)+C_W(P^f+\Delta W)-C_DP^D - C_pP^{WP}-C_{HP}P^{HP}] \leq P^l) \geq 1-\epsilon$$

$$\mathbb{P}(P+R \geq \underline{P}) \geq 1-\epsilon$$

$$\mathbb{P}(P+R \leq \overline{P}) \geq 1-\epsilon$$

$$\mathbb{P}(R \leq RT^{up}) \geq 1-\epsilon$$

$$\mathbb{P}(R \geq -RT^{dn}) \geq 1-\epsilon$$

where ϵ - is the violation probability. By substituting $R = -Y\sum_w \Delta W_w$ and the summation into $\sum_w = \mathbf{1}^T_{1\times N_W}$ we can fraction out the uncertain part. Then the inner part of the above mentioned constraints can be reformulated as

$$(A_i^x)^T\xi \leq b_i^x, \forall i=1,\dots m$$

where random variable ξ is wind forecast error ΔW, m - is a number of chance-constraints, matrices A_i^x and b_i^x - deterministic parts of the constraints which depend only on the decision variables. The DR variants of the chance-constraints are formulated as follows

$$\inf_{\mathbb{P}_\xi\in\mathcal{D}_\xi} \mathbb{P}((A_i^x)^T\xi \leq b_i^x) \geq 1-\epsilon_i, \forall i=1\dots m$$

In our case:

$$\inf_{\mathbb{P}_\xi\in\mathcal{D}_\xi} \mathbb{P}(A(CY\mathbf{1}^T_{1\times N_W}-C_W)\Delta W \leq A(CP+C_WP^f - C_DP^D - C_pP^{WP} - C_{HP}P^{HP}) + P_l) \geq 1-\epsilon$$

$$\inf_{\mathbb{P}_\xi\in\mathcal{D}_\xi} \mathbb{P}(A(-CY\mathbf{1}^T_{1\times N_W}+C_W)\Delta W \leq A(-CP-C_WP^f + C_DP^D + C_pP^{WP} + C_{HP}P^{HP}) + P_l) \geq 1-\epsilon$$

$$\inf_{\mathbb{P}_\xi\in\mathcal{D}_\xi} \mathbb{P}((Y\mathbf{1}^T_{1\times N_W})\Delta W \leq P-\underline{P}) \geq 1-\epsilon$$

$$\inf_{\mathbb{P}_\xi\in\mathcal{D}_\xi} \mathbb{P}((-Y\mathbf{1}^T_{1\times N_W})\Delta W \leq -P+\overline{P}) \geq 1-\epsilon$$

$$\inf_{\mathbb{P}_\xi\in\mathcal{D}_\xi} \mathbb{P}((-Y\mathbf{1}^T_{1\times N_W})\Delta W \leq RT^{up}) \geq 1-\epsilon$$

$$\inf_{\mathbb{P}_\xi\in\mathcal{D}_\xi} \mathbb{P}((Y\mathbf{1}^T_{1\times N_W})\Delta W \leq RT^{dn}) \geq 1-\epsilon$$

After calculating from the data samples $\{\xi^l\}_{l=1}^N$ the empirical mean vector $\mu_{N_W\times 1} = \frac{1}{N}\sum_{l=1}^N \xi^l$ and covariance matrix $\Sigma_{N_W\times N_W} = \frac{1}{N}\sum_{l=1}^N(\xi^l-\mu)(\xi^l-\mu)^T$ we can build moment-based ambiguity set $\mathcal{D}_\xi$

$$\mathcal{D}_\xi := \{\mathbb{P}_\xi \in \mathcal{P}' : \mathbb{E}_{\mathbb{P}_\xi}[\xi] = \mu, \mathbb{E}_{\mathbb{P}_\xi}[(\xi-\mu)(\xi-\mu)^T] = \Sigma\}$$

which requires that the true mean and covariance matrix of ξ, given by any distribution in set $\mathcal{D}_\xi$, be exactly the empirical mean μ and covariance Σ. General DR chance-constraints can be reformulated as follows

$$\sqrt{\left(\frac{1-\epsilon}{\epsilon}\right)(A_i^x)^T(\Sigma)A_i^x} \leq b_i^x - (A_i^x)^T\mu, \ \forall i=1\dots m$$

which is a second-order cone constraint:

$$K_\epsilon||(\Sigma)^{\frac{1}{2}}A_i^x||_2 \leq b_i^x - (A_i^x)^T\mu, \ \forall i=1\dots m \qquad (1)$$

where $K_\epsilon = \sqrt{\left(\frac{1-\epsilon}{\epsilon}\right)}$.

Case Study Data

Electrical demand is 200 MW and 100 MW at buses 4 and 5. We assume that reserves from generator are more expensive than CHP reserves $\overline{c_G} = \underline{c_G} = 1.2c_1$, $\overline{c_c} = \underline{c_c} = 1.1c_e$ In this study water density is $\rho = 1000$ kg/m^3 and kinematic viscosity of water is $\mu = 0.4736\times 10^{-6}$ m/s^2. Darcy friction factor $f_D = 0.0118$ that is computed as in [1]. The characteristics of the generation units, transmission lines parameters and DHN parameters are presented in Table I,II,III and IV.

References

[1] D. Clamond, "Efficient resolution of the colebrook equation," *Industrial & Engineering Chemistry Research*, vol. 48, no. 7, pp. 3665–3671, 2009.

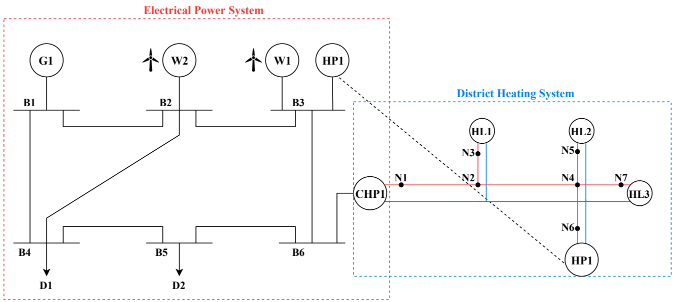

Fig. 1. Configuration of the six-bus and seven-node integrated system.

TABLE I
GENERATION UNITS

		G_1	CHP_1	HP_1
$\overline{P}$	MW	230	208.3	-
$\underline{P}$	MW	10	15	-
$\overline{R}$	MW	92	41.66	-
$\underline{R}$	MW	0	0	-
$\underline{H}$	MW	-	0	5
$\overline{H}$	MW	-	250	100
$\underline{m}_s^{HS}$	kg/s	-	300	300
$\overline{m}_s^{HS}$	kg/s	-	700	700
$\overline{F}$	MW	-	500	-
COP	-	-	-	2.5
r	-	-	0.5	-
ρ^E	-	-	2.4	-
ρ^H	-	-	0.25	-
η	-	-	0.9	0.9
$\overline{P}_{WP}$	MW	-	20	20
$\underline{P}_{WP}$	MW	-	0	0
c_2	\$/ MWh^2	0.00125	-	-
c_1	\$/ MWh	40.622	-	-
c_0	\$	0	-	-
c_e	\$/ MWh	-	3.6	-
c_h	\$/ MWh	-	0.06	-

TABLE II
ELECTRICAL TRANSMISSION LINES

		l_{12}	l_{14}	l_{23}	l_{24}	l_{36}	l_{45}	l_{56}
$\overline{P}_l$	MW	200	250	250	200	250	250	250
X	p.u.	0.17	0.0586	0.1	0.072	0.0625	0.16	0.085

TABLE III
DISTRICT HEATING PIPES

		$p_{12,24,46}$	p_{23}	$p_{45,47}$
L_p	m	800	600	500
D_p	m	0.8	0.8	0.8
λ	W/m°C	0.2	0.2	0.2
ν_p	$m \times 10^{-3}$	0.045	0.045	0.045
K_p	$1/[m \times kg]$	0.0233	0.0175	0.0146
$\underline{m}_p^S, \underline{m}_p^R$	kg/s	300	300	300
$\overline{m}_p^S, \overline{m}_p^R$	kg/s	700	700	700
$\overline{T}_p^R$	°C	45	45	45
$\underline{T}_p^R$	°C	25	25	25
$\overline{T}_p^S$	°C	65	65	65
$\underline{T}_p^S$	°C	50	50	50

TABLE IV
DISTRICT HEATING NETWORK PARAMETERS

		$N_{1,2,4,6}$	$N_3(L)$	$N_5(L)$	$N_7(L)$
$\underline{m}_l^{HL}$	kg/s	-	300	300	300
$\overline{m}_l^{HL}$	kg/s	-	700	700	700
H_l^L	MW	-	45	40	50
$\underline{T}_n^S$	°C	50	50	50	50
$\overline{T}_n^S$	°C	65	65	65	65
$\underline{T}_n^R$	°C	25	25	25	25
$\overline{T}_n^R$	°C	45	45	45	45
$\underline{pr}_n^S, \underline{pr}_n^R$	kPa	0	0	0	0
$\overline{pr}_n^S, \overline{pr}_n^R$	kPa	30000	30000	30000	30000
$\underline{pr}_l^{HL}$	kPa	-	50	50	50