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Chinese Astronomy and Astrophysics 34 (2010) 69–76

CHINESE
ASTRONOMY
AND ASTROPHYSICS

An Accuracy Analysis of the SGP4/SDP4 Model ⋆

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Abstract Based on the latest release of the SGP4/SDP4 (Simplified General Perturbation Version 4/ Simplified Deep-space Perturbation Version 4) model, in this paper we have designed an orbit determination program. Through calculations for the 1120 objects with various types and orbital elements selected from the space objects database, we have obtained the accuracies of the orbit determination prediction dealt with various types of space objects by the SGP4/SDP4 model. The results show that the accuracies of the near-earth objects are in the order of magnitude of 100 meters; the averages of the orbit determination accuracies of the semi-synchronous and geosynchronous orbits are, respectively, 0.7 and 1.9 km. The orbit determination accuracies of the elliptical orbit objects are related to their eccentricities. Except for few elliptical orbit objects with $e > 0.8$, the orbit determination errors of the vast majority of the elliptical orbit objects are all less than 10 km. By using the SGP4/SDP4 model to make 3 days predictions for near-earth objects, 30 days for semi-synchronous orbit objects, 15 days for geosynchronous orbit objects and 1 day for elliptical orbit objects, the errors of prediction generally don't exceed 40 km.

Key words: celestial mechanics: orbit calculations and determination

Received 2008–11–18; revised version 2008–12–01

⋆ A translation of *Acta Astron. Sin.* Vol. 50, No. 3, pp. 332–339, 2009

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doi:10.1016/j.chinastron.2009.12.009

1. INTRODUCTION

The SGP4/SDP4 is one kind of orbit prediction model developed by NORAD (North American Air Defense Command). In combination with the observational materials of SSN (United States Space Surveillance Network), it has generated the largest space objects cataloging database in the world and is released in the form of TLE (two-line element). Since 1980 the mathematical model of the SGP4/SDP4 and the corresponding Fortran codes have been published by DOD (U.S. Department of Defense)[1]. These program codes are revised continuously in actual use and there have appeared many versions. In 2006 Vallado et al.[2] summarized all the versions and provided the latest codes which are compatible with the DOD codes completely.

Because the space objects cataloging database generated by the SGP4/SDP4 model is currently the most complete data source, and the TLE elements cannot achieve the best prediction accuracy without combining with the SGP4/SDP4 model[1], therefore the SGP4/SDP4 model is widely used in all kinds of tasks. But when the officials provide the mathematical model and program codes, they do not give an illustration of the model accuracy, moreover the related references are very few. Using the old version of the SGP4/SDP4 program codes, Han Lei, Chen Lei et al.[3] provided an initial analysis of the SGP4/SDP4 model accuracy through calculating the seven space objects with typical orbits. Taking advantage of the calculation results of 1120 space objects, in this paper an in-depth and detailed study about the model accuracy has been done by adopting the latest release of the SGP4/SDP4 model.

2. DATA FORMATION

In order to all-sidedly reflect the SGP4/SDP4 model accuracy in dealing with various types of space objects, we select 1120 objects which can reflect all the orbital characteristic of space objects from the cataloging database. According to orbital altitudes and eccentricities, the objects can be categorized into several types which possess, respectively, near-circular and near-earth orbits (the orbital altitudes are lower than 5000 km, $e < 0.1$), semi-synchronous orbits (about 20000 km), geosynchronous orbits (about 36000 km) and elliptical orbits ($e > 0.1$)[4].

Using the software of precision track model for orbit prediction, the predicted positions and velocities are taken as the simulated observational data, and the perturbation factors considered in this software include the 20×20-order model of the terrestrial gravitational field, the solar and the lunar gravitational perturbations, the atmospheric drag (the model is DTM 1994), the pressure of sunlight and the perturbation of solid tide[5]. The simulation data are the whole arc observations taken as a point in each 2 minutes. For those objects whose heights of perigee are lower than 500 km, we predict the orbits for 3+30 days, in which the data of the first 3 days are used for orbit determination and the last 30 days are used to compare with the orbits calculated by the SGP4/SDP4 model prediction (in the following we adopts the same definitions). For those objects whose perigee heights are from 500 to 1500 km, we forecast the orbits for 5+60 days. For those objects whose heights of perigee are higher than 1500 km, we predict the orbits for 10+180 days.

3. THE PROGRAM OF ORBIT DETERMINATION

The symbols X and Y express, respectively, the observational and state quantities. The observational quantities are the directly adopted orbital vectors $\vec{r}$ and $\dot{\vec{r}}$ of space objects, and the state quantities include the orbital elements σ and the parameter B^* of the atmospheric drag.

The SGP4/SDP4 model uses the terms containing B^* to simulate the atmospheric damping perturbation. The effect produced by the atmospheric damping perturbation of the low earth orbit objects is comparatively large, however that of the high earth orbit objects is almost negligible. It is reflected in the orbit determination programs that the SGP4 model needs not only to determine the initial orbital elements σ_0, but also determine B^*, while the SDP4 model only requires to determine σ_0.

In this paper, the position and velocity are taken as observational quantities. Adopting the SGP4 model, $\tilde{B}$ matrix is

$$\tilde{B} = (\frac{\partial(\vec{r}, \dot{\vec{r}})}{\partial(\sigma)})(\frac{\partial(\sigma)}{\partial(\sigma_0, B^*)})\,. \tag{1}$$

Using the SDP4 model, $\tilde{B}$ matrix is

$$\tilde{B} = (\frac{\partial(\vec{r}, \dot{\vec{r}})}{\partial(\sigma)})(\frac{\partial(\sigma)}{\partial(\sigma_0)})\,, \tag{2}$$

in which σ represents the elements without singularity of the first category, i.e. $a, i, \Omega, \xi = e\cos\omega, \eta = e\sin\omega, \lambda = M + \omega$. The computational formulae of $\frac{\partial(\vec{r},\dot{\vec{r}})}{\partial(\sigma)}$ and $\frac{\partial(\sigma)}{\partial(\sigma_0)}$ can be found in Ref.[6].

According to Ref.[1], omitting the derivation process we give the computational formulae of $\frac{\partial\sigma}{\partial B^*}$ as follows

$$\begin{cases} \frac{\partial a}{\partial B^*} = 2a_0 {C_2}^2 B^*(t-t_0)^2 - 2a_0 C_2 (t-t_0) \\ \frac{\partial i}{\partial B^*} = 0 \\ \frac{\partial \Omega}{\partial B^*} = 0 \\ \frac{\partial \xi}{\partial B^*} = 0 \\ \frac{\partial \eta}{\partial B^*} = 0 \\ \frac{\partial \lambda}{\partial B^*} = \frac{3}{2} n_0 C_2 (t-t_0)^2 \end{cases}, \tag{3}$$

in which C_2 is a constant, $t - t_0$ is the time starting from the initial moment in units of minutes, a_0 is the semi-major axis and n_0 is the corresponding angular velocity.

4. THE RESULTS OF STATISTICS ON THE ACCURACY OF ORBIT PREDICTION

The precision of orbit determination is expressed as the RMS error σ^* of the position deviations between the standard orbits and the predicted orbits of the orbit determination arcs. The orbits predicted by the software of the precise orbit model are taken as the standard orbits, and the orbit elements determined by the orbit determination program in Section 3 and predicted by the SGP4/SDP4 model are taken as the predicted orbits. Assuming that the number of observational data is k and the position deviations between the predicted orbits and the standard orbits are $y_{\rm j}$, then

$$\sigma^* = \sqrt{U/k}\,, \tag{4}$$

$$U = \sum_{\rm j=1}^{k} (y_{\rm j}^T W_{\rm j} y_{\rm j})\,. \tag{5}$$

The prediction accuracy is the maximum of the position deviations between standard orbits and predicted orbits within the time of prediction.

According to the orbital types we have made statistics and given the computing formlae with a different scope of the orbit determination accuracy and the errors of prediction in n days of the space objects with various types.

4.1 Near-circular and Near-earth Objects

We take the altitudes of the near-earth objects as horizontal axis and the accuracy of orbit determination as vertical axis to draw Fig.1. From this figure it can be seen that the orbit determination accuracies of the near-earth objects are in the order of magnitude of 100 meters. With the increase of the orbital altitude, the errors of orbit determination have a downward trend, but the variation of the orbit determination errors tend to be smooth when the altitudes are higher than 1600 km.

To fit the errors with a formula, we suppose that the orbital altitude is h km, the minimum of the errors of orbit determination is $\Delta r_{\rm min}$ and the maximum is $\Delta r_{\rm max}$ in units of kilometers, so the calculational formula of the scope of orbit determination accuracies of the near-earth objects is expressed as follows:

When the orbital altitudes are less than or equal to 1600 km,

$$\begin{cases} \Delta r_{\rm min} = 0.61 - 2.1 \times 10^{-4} h \\ \Delta r_{\rm max} = 1.03 - 3.6 \times 10^{-4} h \end{cases} (h \leq 1600{\rm km})\,. \tag{6}$$

When the orbital altitudes are larger than 1600 km, the accuracies of orbit determination are from 0.25 km to 0.4 km.

The prediction accuracies of the near-earth objects are relevant to the orbital altitudes. Based on experience the near-earth objects are divided into four categories.

Table 1 The categories of the near-earth objects

Orbital type	A	B	C	D
Altitude(km)	$h < 400$	$400 \leq h < 600$	$600 \leq h < 1200$	$1200 \leq h < 7000$

Tables 2 & 3 show the maximum positional errors of various types of near-earth objects in predicting of n day when the solar 10.7 cm radiation flow ($F_{10.7}$) is 100 and 200 respectively.

Table 2 The orbit prediction accuracies of the near-earth objects ($F_{10.7} = 100$)

		Position differences(km)			
Type	Number	1day	3days	7days	15days
A	57	4	10	60	300
B	168	3	10	50	100
C	267	3	10	20	50
D	90	2	10	10	20

Table 3 The orbit prediction accuracies of the near-earth objects ($F_{10.7} = 200$)

		Position differences(km)			
Type	Number	1day	3days	7days	15days
A	57	10	40	300	1000
B	168	7	30	200	400
C	267	6	15	70	100
D	90	2	10	10	20

We summarize the following conclusions from the above contents:

(1) Dealing with the near-earth objects with the SGP4/SDP4 model, the orbit determination accuracy is in the order of magnitude of 100 meters. And it is related to the orbital altitude. The lower the height is, the worse the orbit determination accuracy is.

(2) In the predictions for the near-earth objects with the SGP4/SDP4 model, the greater the solar radiation flow is, the larger the error of prediction is.

(3) In predicting of 3 days for the near-earth objects, the positional errors are less than 40 km.

4.2 Near-circular and High-orbit Objects

Near-circular and high-orbit objects are mainly concentrated in the geosynchronous and semi-synchronous orbits. The orbits with the altitudes from 18 500 to 21 500 km are classified as semi-synchronous orbits, and those from 33 000 to 38 000 km are classified as synchronous orbits.

Using the orbital altitudes of the high earth orbit objects as abscissa and taking the orbit determination accuracies as ordinate, we draw Fig.2. It can be seen from Fig.2 that the orbit determination accuracies of semi-synchronous orbits are from 0.4 to 1.5 km, however those of synchronous orbit objects are from 1.5 to 3.2 km. With the increase of the orbital altitudes, the orbit determination errors exhibit an increasing trend. And those of the synchronous orbit objects are larger than the errors of the semi-synchronous orbit objects.

The orbit determination and prediction accuracies of the synchronous and semi-synchronous orbit objects are listed in Table 4.

Table 4 The orbit determination and prediction accuracies of the synchronous and semi-synchronous orbits

			Position differences(km)				
	Number		Orbit determination	3days	15days	30days	60days
semi-synchronous orbit	35	Max	1.5	8.6	17.4	39.1	68
		Min	0.4	1.2	3.1	5.1	6.9
		Average	0.7	4.9	10.6	15.6	30.1
geo-synchronous orbit	35	Max	3.2	27.9	40.2	67.2	157
		Min	1.5	8.1	15.8	25.6	27.1
		Average	1.9	13.3	26.0	41.5	70.3

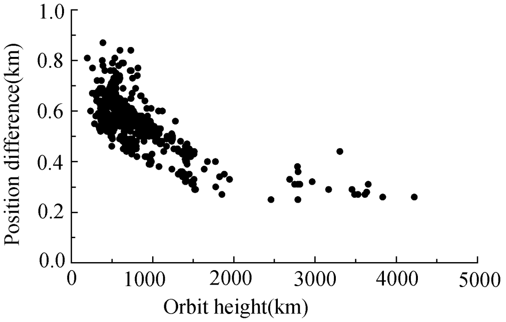

Fig. 1 The relation between the orbit determination accuracies and the orbit altitudes of near-earth objects

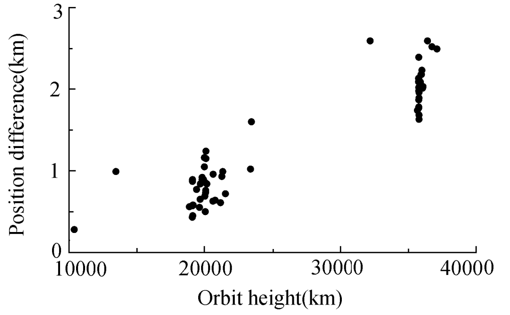

Fig. 2 The relation between the orbit determination accuracies and the orbital altitudes of the near-circular and high-orbit objects

From Table 4 the following conclusions can be summarized:

(1) The orbit determination accuracies of the synchronous orbits are from 1.5 to 3.2 km with an average of 1.9 km. And those of the semi-synchronous orbits are from 0.4 to 1.5 km with an average of 0.7 km.

(2) Using the SGP4/SDP4 model, the prediction errors are less than 40 km when the semi-synchronous and synchronous orbits are predicted, respectively, for 30 days and 15 days.

4.3 Elliptical Orbit Objects

We take the perigee altitudes and eccentricities of the elliptical orbit objects as abscissa and the orbit determination accuracies as ordinate to draw Figs.3 & 4. From these figures we can see that there is no correlation between the orbit determination accuracies and the perigee altitudes of the elliptical orbit objects, but a clear correlation exists between the orbit determination accuracies and eccentricities. The larger the eccentricity, the larger the error of orbit determination.

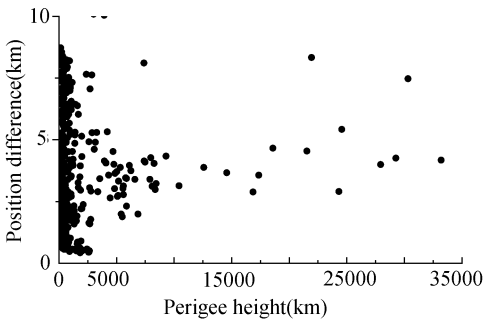

Fig. 3 The relation between the orbit determination accuracies and the altitudes of the perigees of elliptical orbit objects

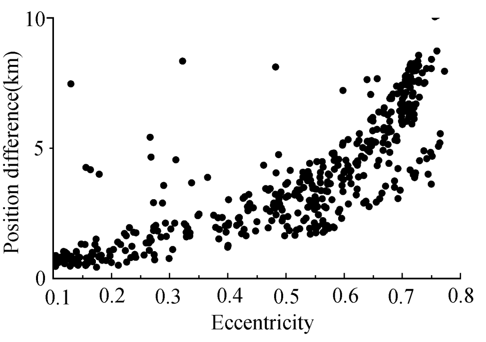

Fig. 4 The relation between the orbit determination accuracies and the eccentricities of elliptical orbit objects

To fit the orbit determination accuracy range of the elliptical orbit objects with a formula, we suppose that the eccentricity is e, the minimum of the errors of orbit determination is $\Delta_{\min}$ and the maximum is $\Delta_{\max}$, then in units of kilometers, the calculational formula is as follows:

When $0.1 \le e \le 0.6$

$$\begin{cases} \Delta r_{\min} = 0.20 + 3.0 \times e \\ \Delta r_{\max} = 0.12 + 8.8 \times e \end{cases} . \tag{7}$$

When $0.6 < e \le 0.8$

$$\begin{cases} \Delta r_{\min} = 2.0 + 10.0 \times (e - 0.6) \\ \Delta r_{\max} = 5.4 + 26.0 \times (e - 0.6) \end{cases} . \tag{8}$$

When $e > 0.8$, all the orbit determination errors are larger than 10 km and the accuracies are in the order of magnitude of 10 km.

There doesn't exist a clear correlation between the prediction accuracies and the eccentricities of the elliptical orbits, but the prediction accuracies are mainly related to the altitudes of perigee. According to the experience, the elliptical orbit objects can be divided into four categories in accordance with the altitudes of perigee. And they are listed in Table 5.

Table 5 The classfication of elliptical orbit objects

Orbit Type	A	B	C	D
Perigee altitude(km)	$h_p < 400$	$400 \le h_p < 600$	$600 \le h_p < 1200$	$1200 \le h_p$

Table 6 shows the maximum position errors of the elliptical orbit objects of various types for predictions in n days.

Table 6 The orbit prediction accuracies of the elliptical orbit objects

		Position differences(km)		
Type	Number	1day	3days	7days
A	178	20	100	500
B	78	20	100	200
C	101	20	100	150
D	109	20	40	50

From Table 6 the following conclusions can be summarized:

(1) The position errors of the elliptical orbit objects are within 20 km for predictions in 1 day and within 100 km for predictions in 3 days.

(2) The orbit prediction accuracies are mainly related to the altitudes of the elliptical orbit objects' perigees, and the lower the altitude is, the larger the prediction error is.

5. CONCLUSIONS

In the processing of space objects of various types with the SGP4/SDP4 model the orbit determination accuracies are different. Those of near-earth objects are in the order of magnitude of 100 meters. The averages of orbit determination accuracies of the semi-synchronous and synchronous orbit objects are, respectively, 0.7 and 1.9 km. Those of the elliptical orbit objects are related to their eccentricities and are in the order of magnitude of kilometers. The larger the eccentricity is, the larger the orbit determination error is, and the orbit determination error is greater than 10 km when the eccentricity is greater than 0.8. The prediction accuracies of the SGP4/SDP4 model are mainly related to the altitudes of objects. For the low earth orbit objects, the lower the orbital altitude is, the larger the prediction error is. For the high-earth objects, the higher the orbital altitude is, the larger the prediction error is. While for the elliptical orbit objects, the lower the perigee altitude is, the larger the prediction error is.

Using the SGP4/SDP4 prediction model to make prediction in 3 days for near-earth objects, semi-synchronous orbit prediction in 30 days, synchronous orbit prediction in 15 days and elliptical orbit prediction in 1 day, the position errors are always less than 40 km.

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