Review Exercise

1. List the first five terms of each of the sequences whose general terms are given below where n is a positive integer:

   a. $a_n = \dfrac{n^n}{n!}$ b. $a_n = (-1)^n + (-1)^n \sin(n\pi)$

   c. $a_n = Sgn(3-n)$

2. List the first five terms of each of the sequences whose general terms are given below where n is a positive integer:

   a. $a_n = 3^{\frac{1}{n}}(-1)^n$ b. $a_n = ne^{-2n}$ c. $a_n = \dfrac{(-2)^n + 6}{(n-1)!}$

   d. $a_n = (-1)^n - \dfrac{1}{n^2}$ e. $a_n = \cos\left(\dfrac{n\pi}{2}\right)$

3. Determine whether the sequences with the following general terms are arithmetic.

   a. $a_n = \dfrac{n^2 + 5n + 6}{n+2}$ b. $a_n = \sqrt{4n+1}$

4. Find the arithmetic mean between 1 and 9.

5. Find the 10^{th} and n^{th} terms of the geometric progression 5, 25, 125,…

6. Find three numbers in geometric progression such that their sum is $\dfrac{13}{3}$ and the sum of their squares is $\dfrac{91}{3}$.

7. Find the sum of

   a. $\sum_{k=1}^{8}(k^3 + 2k^2 - 3k + 5)$ b. $\sum_{k=2}^{5} 3$ c. $\sum_{k=1}^{5} \dfrac{1}{k^2 + 5k + 6}$

   d. $\sum_{m=1}^{10} \ln\left(\dfrac{m}{m+1}\right)$

8. Let $\sum_{i=1}^{5} x_i = 37$, $\sum_{i=1}^{5} y_i = 127$, $\sum_{i=1}^{5} x_i^2 = 303$, $\sum_{i=1}^{5} y_i^2 = 50$ and $\sum_{i=1}^{5} x_i y_i = 105$.

Evaluate the following

a. $\sum_{i=1}^{5}(2x_i - 3y_i)$ b. $\sum_{i=1}^{5}(2x_i + 3y_i)$ c. $\sum_{i=1}^{5}(2x_i - 3y_i)^2$

d. $\left(\sum_{i=1}^{5} x_i\right)^2$ e. $\sum_{i=1}^{5}(2x_i - 5y_i + 3)$

9. Find the sum of all the natural numbers between 100 and 1000 which are multiples of 5.

10. In an arithmetic progression the 1st term is 2 and the sum of the 1st five terms is one-quarter of the next five terms. Show that the 20^{th} term is –112.

11. How many terms of the arithmetic progression $-6, -\frac{11}{2}, -5, \ldots$ are needed to give the sum –25.

12. The sum of a certain number of terms of the arithmetic progression 25, 22, 19, … is 116. Find the last term.

13. If the sum of n terms of an arithmetic progression is $(pn + qn^2)$, where p and q are constants, find the common difference.

14. If the n^{th} partial sum of an arithmetic sequence $\{A_n\}$ is $4n^2$, find A_n and A_{10}.

15. Convert this mixed recurring decimal $0.3\overline{17}$ to fraction.

16. Convert the mixed recurring decimal to fraction.

a. $0.3\overline{7}$ b. $3.23\overline{54}$

17. The first three terms of a convergent geometric series are: $x+1; x-1; 2x-5$.

a. Find the values of $x (x \neq 1 \text{ or } -1)$.

b. Find sum to infinity of the series.

18. Find $p: \sum_{k=1}^{\infty} 27p^k = \sum_{x=1}^{12}(24 - 3x)$.

19. Find the product $4 \times 4^{\frac{1}{2}} \times 4^{\frac{1}{4}} \times 4^{\frac{1}{8}} \times \ldots \times 4^{\frac{1}{2^n}} \times \ldots$

20. If $\sum_{k=1}^{\infty} 5^{kt} = \frac{1}{24}$, find the values of t.

21. If the product $5^k \cdot 5^{k^2} \cdot 5^{k^3} \ldots = 5$, find k.

22. Evaluate $\sum_{k=1}^{11} (2 + 3^k)$.

23. The sum of the first three terms of a geometric progression is $\frac{39}{10}$ and their product is 1. Find the common ratio and the three terms.

24. How many terms of geometric progression $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

25. The sum of first three terms of a geometric progression is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the geometric progression.

26. Given a geometric progression with $a = 729$ and 7^{th} term 64, determine S_7.

27. If the 4^{th}, 10^{th} and 16^{th} terms of a geometric progression are x, y and z, respectively. Prove that x, y, z are in geometric progression.

28. Find the sum to n terms of the sequence 8, 88, 888, 8888....

29. Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots ar^{n-1}$ and $A, AR, AR^2, \ldots AR^{n-1}$ form a geometric progression, and find the common ratio.

30. If the p^{th}, q^{th} and r^{th} terms of a geometric progression are a, b and c, respectively. Prove that $a^{q-r} b^{r-p} c^{p-q} = 1$

31. If the first and the n^{th} term of a geometric progression are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$

32. If a, b, c and d are in geometric progression. Show that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$

33. Insert two numbers between 3 and 81 so that the resulting sequence is geometric progression.

34. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$.

35. If A and G are an arithmetic mean and geometric mean, respectively between two positive numbers,

    prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

36. 150 workers were hired to finish a job in a certain number of days. 4 workers dropped out on second day, four more workers dropped out on third day and so on. It took eight more days to finish the work. Find the number of days in which the work was completed.