

Summary

1. Sequence

- A sequence $\{a_n\}$ is a function whose domain is the set of positive integers or a subset of consecutive integers starting with 1.
- The sequence $\{a_1, a_2, a_3, \dots\}$ is denoted by $\{a_n\}$ or $\{a_n\}_{n=1}^{\infty}$.
- A sequence that has a last term is called a **finite sequence**. Otherwise it is called **infinite sequence**.
- **Recursion formula** is a formula that relates the general term a_n of a sequence to one or more of the terms that come before it.

2. Arithmetic and geometric progression

- An **arithmetic sequence** is one in which the difference between consecutive terms is a constant, and this constant is called the common difference.
- If $\{A_n\}$ is an arithmetic progression with the first term A_1 and the common difference d , then the n^{th} term is given by:

$$A_n = A_1 + (n-1)d.$$

- A **geometric progression** is one in which the ratio between consecutive terms is a constant, and this constant is called the common ratio.
- If $\{G_n\}$ is a geometric progression with the first term G_1 and a common ratio r , then the n^{th} term is given by: $G_n = G_1 r^{n-1}$.

3. Partial sums:

- The sum of the first n terms of the sequence $\{a_n\}_{n=1}^{\infty}$, denoted by S_n is called the partial sum of the sequence.
- The sum S_n of the first n terms of an arithmetic sequence with first term A_1 , and common difference d is:

$$S_n = \sum_{k=1}^n A_k = \frac{n}{2}[2A_1 + (n-1)d] \text{ or } S_n = \sum_{k=1}^n A_k = \frac{n}{2}[A_1 + A_n].$$

- In a geometric sequence, $\{G_n\}_{n=1}^{\infty}$, with common ratio r , the sum of the first n

terms S_n is given by:
$$S_n = \begin{cases} nG_1, & \text{if } r = 1 \\ \frac{G_1(1-r^n)}{1-r}, & \text{if } r \neq 1 \end{cases}$$

4. Convergent series and divergent series

- In a sequence if $\{a_n\}_{n=1}^{\infty}$, S_n is the n^{th} partial sum such that, as $n \rightarrow \infty$, $S_n \rightarrow s$ where s is a real number, we say the infinite series

$$\sum_{n=1}^{\infty} a_n \text{ converges to } s, \text{ otherwise the series diverges.}$$