Intergenerational mobility or gender inequality: What are rank correlations measuring?

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Abstract

The rank correlation between parent and child has become a central measure of intergenerational income mobility in recent years. A great advantage of the rank correlation, compared to other measures such as elasticities, is the invariance with respect to the shape of income distributions and therefore intragenerational inequality. This facilitates comparisons of societies over time and across space. However, I show that changes in inequality between genders *within* generations directly affect the rank correlation; diminishing gender inequality leads to a fall in mobility. The implication is that the same rank correlation can map into very different societies. By estimating American income distributions and assuming constant within-gender mobility, I show that the rank-correlation should have risen by almost 25 percent over the last 40 years, solely due to the narrowing gender income gap. An arguably benign decrease in gender inequality will therefore register as an adverse development in measured mobility. The findings underscore the importance of being explicit about the definition of income concepts and the importance of group-inequality when comparing rank correlations between countries and across time.

1 Introduction

Economic inequality and intergenerational mobility has taken center stage in recent years.[1] This renewed public interest, as well as access to new data sources, have reinvigorated the inequality research agenda. The most popular measure of mobility is the intergenerational income elasticity (henceforth abbreviated IGE),

[1]As then American president, Barack Obama, stated on December 4, 2013: *"The combined trends of increased inequality and decreasing mobility pose a fundamental threat to the American Dream, our way of life, and what we stand for around the globe."*

which measures the link between parent and child income. Recent research has documented close links between inequality and this measure of mobility. The "Great Gatsby-curve" may be the best know example.[2]

However, the IGE is itself a function of inequality. There may, therefore, be mechanical linkages between inequality within generations and the IGE. If this is the case, interpretation of a difference over time or between societies becomes difficult. Furthermore, the IGE has been shown to be quite sensitive to the choice of income concepts and modeling choices, see Chetty et al. (2014b) for an example. To address this problem, part of the literature has shifted focus from the actual income to *positions* in the income distribution. By only comparing the intergenerational association between positions, also called ranks, and not actual incomes, one may disregard differences in the shape of income distributions. Rank-based mobility measures, therefore, facilitate comparisons of mobility across time and between countries while disregarding inequality. In other words, mobility and inequality become decoupled into two separate concepts. Such a measure of association between parent and child rank is the rank correlation, also known as the Spearman correlation coefficient.

Chetty et al. (2014a) show that the rank correlation in America has been remarkably stable for the last decades, despite sizable increases in cross-sectional inequality. These considerations could lead one to conclude that the rank-correlation is superior to the IGE as a measure of intergenerational mobility. However, I argue in this paper that the robustness of the rank-correlation may be overstated. I point to an often-overlooked issue of inequality between groups but within the same generation. When income distributions differ between groups, the interpretation of the correlation becomes ambiguous.

In this paper, I show that a society with high gender inequality will have higher rank mobility than a society with low gender inequality. This is the case even if the two countries exhibit the same relative mobility levels within gender. Intuitively, the importance of gender naturally depends on the level of gender inequality. If a society has no gender inequality, the gender of a child will matter little when we measure the association between the positions of parent and child in society. On the other hand, if a society has high gender inequality, then the sorting into gender becomes important. As the gender of a child is essentially

[2] Corak (2006) documented that countries with high cross-sectional inequality also tend to have lower intergenerational mobility. In other words, a high level of inequality is associated with a stronger dependence between parent and child income. The term "Great Gatsby curve" came later than the finding. It was first introduced in 2012 by Alan Krueger, then chairman of the Council of Economic Advisors.

random and uncorrelated with income, the gender gap will register as mobility from one generation to the next. This matters for comparisons over time and space. If a society manages to remove some structural obstacles for women in the labor market and this leads to less gender inequality, this will also lead to an apparent *fall* in mobility.

Formally, I exploit that rich and poor households alike share the randomness of a child's gender. I, therefore, conceptualize gender as a random sorting mechanism into two distinct groups; male and female. I then proceed to describe theoretically how the link between the mobility within the two genders and the mobility in society as a whole is a function of the level of gender inequality. I approximate the link as a linear relationship, where the factor is a function of the income distributions of men and women. Under reasonable assumptions on the shape of the income distributions of men and women, I show that this factor approaches one *from below* when the gender gap narrows. A given rank correlation can thereby map into infinitely many combinations of gender inequality and within-gender intergenerational mobility. Consequently, when gender is not taken into account we cannot know how to draw out policy implications from a change in the rank correlation.

To gauge whether this issue matters in practice, I take the results to American survey data. Using the developed formulas, I show that the rank-correlation based on individual incomes should have risen by almost 25 percent due to the narrowing gender income gap alone. In other words, without any change in mobility *within* genders, the apparent mobility would have fallen substantially due to the increase in gender equality. As I use Taylor-approximations to develop the link between within-gender and aggregate mobility, there may be approximation errors. To assess this, I perform simulations of the link between within-gender mobility and aggregate mobility. I show that results from theoretically derived formulas and simulations align and that the overall conclusions are insensitive to the approach taken. The simulation tools I present in this paper may also serve as a tool for researchers to perform *ceteris paribus* analysis of the importance of inequality for explaining changes in mobility, and I make these tools freely available.[3]

As mentioned, the advantage of the rank correlation, as a measure of intergenerational mobility, is its supposed invariance to the shape of income distributions. My findings, however, show that while the aggregate income distribution may not matter, the shapes of the gender-specific income distributions certainly

[3]Software to perform copula simulation is available from my website.

do. The issue highlighted in this paper directly concerns rank correlations when individual incomes are used and both genders are included in the same calculation. A way to bypass this problem is to estimate rank correlations for sons and daughters separately. Fortunately, this is often done when investigating changes over time. Chadwick et al. (2002) conjecture that the focus on sons might partly be due to unrealized sexism and the recognition that the entrance of women into the labor force present one of the most salient and fundamental developments of the latter half of the twentieth century.[4] For comparisons between countries, it might more have been due to luck than intent that the rank correlation is only calculated for sons. However, I have not found a thorough discussion on the implication of group inequality for the rank correlation.[5] An alternative strategy is to use household incomes and the unit of analysis. If household income is used one naturally does not observe the same degree of gender inequality. However, with rising trends in single adult households across the developed world and the substantial variation in household sizes across countries, gender inequality continues to pose problems for comparisons of rank correlations over time and space.

The issues presented in this paper directly ties into the discussion of how intergenerational mobility should be conceptualized. An extreme position is that gender inequality *actually is* mobility. The gender of a child is a lottery where the odds are the same for poor and rich households alike. As I show, this lottery decreases the importance of parental background characteristics. In other words, the randomness of gender reduces the importance of the accident of birth. However, the policy implication is that in order to make a society more mobile, gender inequality should be *increased*. By implication, rank-based measures cannot ignore gender inequality. The issues presented in this paper should, therefore, lead to thorough sensitivity analysis and methodological considerations of the choice of income concepts and unit of analysis when using rank-correlations in practice. In this way, the rank-correlation may not be easier to use than the traditional measures of mobility.

The paper proceeds as follows. Section 2 presents the canonical conceptual framework for studying income mobility along with measures of mobility with a focus on the rank correlation coefficient. I then introduce the issue of gender in-

[4]A related feature may be the IGE's inability to handle zeros due to its' logarithmic nature, the sensitivity to year-to-year fluctuations and bias from mismeasurement of lifetime incomes, see Mitnik et al. (2015) and Haider and Solon (2006) for a discussion of some of these issues.

[5]An example, where such considerations are not taken, is Landersø and Heckman (2017), where rank measures are employed for both genders jointly with individual incomes.

equality into this framework. Section 3 shows how gender inequality affects the measure of mobility while section 4 quantifies the effect of gender inequality on intergenerational mobility. Section 5 discusses the generalization of the results while section 6 concludes.

2 Measuring intergenerational mobility

I begin by introducing a standard framework in which to conceptualize inequality and intergenerational mobility. This framework follows Jäntti and Jenkins (2015) closely. Let a family be defined by a parent and a child income, x_i and y_i respectively. Most measures of intergenerational mobility can be thought of as describing the joint distribution of (x_i, y_i). Denote this bivariate joint distribution as $H(x, y)$ with the corresponding marginal distributions, $F(x)$ and $G(y)$. The marginal distributions are the income distributions of the two generations. Thus all measures of inequality, such as the Gini coefficient, can be calculated from F and G to describe inequality in the parent and child generation respectively.

Typical measures of mobility are based on a normalized covariance between parent and child income or log income. Pearson's linear correlation coefficient is defined as $\frac{Cov(x,y)}{\sigma_x \sigma_y}$ and the intergenerational income elasticity (IGE) is given by $\frac{Cov(\log x, \log y)}{Var(\log x)}$. The latter, to an approximation, describes the percentage change of child income when parent income is raised by one percent. Intuitively, the higher the IGE, the higher is the dependency between parent and child, and thus the lower is the intergenerational mobility. In practice, one usually obtains the IGE by regressing the logarithm of child income on the logarithm of parent income. The IGE has long been the preferred measure of income mobility, see Jäntti and Jenkins 2015 for an exhaustive review and Mitnik et al. (2015) for a recent application.

From the definitions of the correlation coefficient and the IGE, one can see that, if incomes are measured in logs, the two measures only differ by a rescaling.[6] The presence of the standard deviation of child income in the denominator of the correlation coefficient can be seen as a normalization, such that one may compare societies with different levels of inequality. Even so, Chetty et al. (2014b) make a convincing case for why these measures are difficult to interpret

[6]To see this, let the Pearson correlation of log income be given as $\rho = \frac{Cov(log(x), log(y)}{\sigma_{log(y)}\sigma_{log(x)}} = \frac{\sigma_{log(x)}}{\sigma_{log(y)}} IGE$.

in practice. When using actual incomes as the unit of analysis, one cannot have a change in the income distributions without directly affecting the joint distribution and thus measures of mobility based on income. An example could be general economic growth or rising inequality over time. This is a well-known property of mobility measures and has been shown to be empirically of great importance, see Jäntti and Lindahl (2012).

However, comparisons are not only made over time but across countries. Thus, differences in economic development and economic institutions between countries may directly enter into the calculation of the IGE. This property of the IGE is not necessarily a problem; the effect of institutions on mobility may truly be the object of interest in an analysis. Nevertheless, it is difficult to know whether the effect of institutions go through affecting inequality, i.e. the marginal distributions of $H(x, y)$ or through the dependence structure between parent and child.[7]

2.1 Disentangling mobility and inequality

In order to disentangle inequality and mobility, a branch of the mobility literature has moved towards a focus on the *position* in the income distributions, i.e. the ranks of the individuals. Denote the ranks of parent and child as u_i and v_i respectively. The ranks are most often calculated by software simply by sorting the data. Ranks can, however, be related to the marginal distributions by the *probability integral transform*; Assuming the income distributions are continuous, a rank is simply the cumulative distribution function applied to the income, $u_i = F(x_i)$ and $v_i = G(y_i)$. It follows that both u_i and v_i are always uniformly distributed. The rank is therefore invariant to monotonic positive transformations of the income distributions (i.e. changes in inequality) and changes general income levels.[8]

The rank correlation, at times referred to as Spearman's ρ, is simply the linear

[7]This point also relates to the interpretation of the Great Gatsby curve shoving a negative association between inequality and mobility measured by the IGE. Both measures are based on marginal distributions, and we may suspect a somewhat mechanical relationship. Berman (2017) investigates this point in setting with log-normal income distribution. He finds, that there may be such a relationship present. I have simulated the relationship between mobility and inequality with copulas and find that this relationship is somewhat mechanical. The sign of the slope, however, depends on the shape of the copula. Simulation results are available upon request.

[8]An issue concerns mass points in the income distributions such as zeros. I abstract from these in the present context, as I seek to establish a general point. How to treat zeroes is however massively important in applied research, see Chetty et al. (2014b) for an example.

correlation coefficient between parent and child rank;

$$\rho^S = \frac{Cov(u,v)}{Var(u)} = \frac{Cov(u,v)}{\sigma_u \sigma_v} \tag{1}$$

A link between the intergenerational dependence structure of incomes and ranks is provided by Sklar's theorem stating that any joint multivariate distribution can be described by the marginal distributions and a distribution describing the dependence of the ranks.[9] The latter distribution is called a copula and is defined on the unit square.[10] Formally $H(x,y) = C(F(x), G(y)) = C(u,v)$. It is, therefore, possible to apply rank-based measures of mobility while abstracting from the shape of income distributions. This facilitates comparisons of mobility over time and between countries regardless of differing income distributions. In other words; with the rank correlation, we can take inequality out of the picture and focus exclusively on mobility.

Besides rank correlations, other measures can also be derived from the copula. An example is transition matrices, which can be conceptualized as discretized representations of the copula. These types of measures also exploit the uniform marginals in order to make comparisons across societies. However, the rank correlation is the measure of interest for the rest of this paper.

2.2 The role of gender inequality

The framework described above is purely descriptive and is uninformative about mechanisms creating a given dependence structure. In this paper, I investigate one of these mechanisms, namely gender. Gender is a salient and well-defined characteristic and a gender wage gap is well documented in most modern societies.[11] The gender gap in incomes has been greatly reduced since the middle of the twentieth century both through women entering into the labor force and a rise in relative education levels for women. In the following, I am agnostic as to what has caused this narrowing income gap and focus on the implications for measures of mobility.

In what follows, I assign the superscript m to families with a male child and f to families with a female child. Furthermore I assume that the gender of a child is independent of income, and that the gender is assigned by the random

[9]The theorem was introduced in Sklar (1959).

[10]For a proper proof see Nelsen (2006)

[11]By well-defined I imply no value judgment as to gender politics. Biological gender is here seen as a binary variable observable in data.

variable s_i, such that child i is a boy if $s_i = 1$.[12] Let $E[s] = \mu$, that is the share of males in a cohort. I assume this to be stable at 0.5. Gender assigns the child to an income distribution. Family i draws two sets of incomes where only one set is realized depending on gender: $(x_i, y_i) = s_i(x^m, y^m) + (1 - s_i)(x^f, y^f)$. With these assumptions, we can rewrite the joint distribution H:

$$H(x, y) = \mu H^m(x, y) + (1 - \mu)H^f(x, y)$$

and the marginal distributions F and G:

$$F(x) = \mu F^m(x) + (1 - \mu)F^f(x)$$
$$G(x) = \mu G^m(y) + (1 - \mu)G^f(y)$$

The copula of $H(x, y)$ is now given by:

$$C(F(x), G(y)) = \mu C^m(F^m(x), G^m(y)) + (1 - \mu)C^f(F^f(x), G^f(y)), \qquad (2)$$

where C^m and C^f are the "subcopulas" for m and f.[13] Notice that the marginal distributions of the subgroups enter into the copula. The implication of thinking of the joint distribution as a mixture distribution is that changes in marginal distributions of subgroups influence the measurement of mobility. This holds regardless of whether measured are based on the joint distribution or the copula. If full gender equality is achieved, that is $F^m = F^f$ and $G^m = G^f$, then (2) collapses to the usual formulation.

As the irrelevance of marginal distributions has been touted as a great advantage of rank-based measures, this highlight an important drawback; while aggregate inequality does not influence the measure, the inequality between groups does. Equation (2) is however uninformative of how the measures are affected by this type of inequality. This is the focus of the following section.

[12]There is evidence that gender might not be completely uncorrelated with income through differing mortality rates between male and female fetuses and the mother's circumstances and lifestyle. See Orzack et al. (2015) for an analysis of the prenatal gender ratio. Furthermore, there is evidence that the gender of the child might alter parent behavior. Lundberg et al. (2007) present evidence that father involvement and fragility of families may be affected by the gender of the child. Nonetheless, the assumption of orthogonality between gender and parent income is maintained throughout the rest of the analysis.

[13]By the following derivation: $C(F(x), G(y)) = H(x, y) = \mu H^m(x, y) + \mu H^f(x, y) = \mu C^m(F^m(x), G^m(y)) + (1 - \mu)C^f(F^f(x), G^f(y))$

3 Mobility measures with gender inequality

All mobility measures presented above make use of the covariance either between income in base, logs or in ranks. Using the incomes in base (or logs), the aggregate covariance is easily decomposed into male and female covariances, and the aggregate measure is thus a simple mean of the two gender-specific measures. This is not the case with rank correlations. The rank of a child, by definition, depends on a comparison group. In this setup, one may either compare the child to other children of the *same gender* or to *all* children. I will refer to the former as the within-gender rank and the latter as the total or aggregate rank.

In order to understand the importance of gender inequality for the measurement of mobility, we need a way to describe how the dependence structure between parents and children, *given* the gender of the child, affects the dependence of parent and child when using total ranks, thereby disregarding gender. In order to elicit this link, I set up a very simple data-generating process.

I assume a data-generating process where families draw ranks rather than incomes. To simplify, I assume that the distribution of ranks is independent of gender. Gender is a random variable, s_i, and orthogonal to the parent rank. In other words, a family with a given rank will draw a female child with a rank in the female distribution which corresponds to the counterfactual rank of a male child in the male distribution. Families are therefore defined by the tuple (u_i, v_i, s_i).

Ranks are the related to income through the gender and marginal distributions:[14]

$$y = sG^{m^{-1}}(v) + (1-s)G^{f^{-1}}(v) \tag{3}$$

In this simplified setup, gender is only important insofar as the gender-specific income distributions differ, that is $G^f \neq G^m$. The aggregate income distribution of the child generation is now given by

$$G(y) = \mu G^m(y) + (1-\mu)G^f(y) \tag{4}$$

Let v^A be the *total* rank of the child, i.e. the income rank of the child when she is compared to every other child regardless of gender. Using (3) and (4), the

[14] I need to assume invertible income distributions. This assumption may be empirically problematic as a mass point at zero is common. I return to this point in section 4.

aggregate rank can be expressed as a function of the *within gender* rank and the gender indicator:

$$\begin{aligned} v^A = \Lambda(v,s) =& \mu \left[G^m \left(sG^{m^{-1}}(v) + (1-s)G^{f^{-1}}(v) \right) \right] \\ &+ (1-\mu) \left[G^f \left(sG^{m^{-1}}(v) + (1-s)G^{f^{-1}}(v) \right) \right] \end{aligned}$$

Full gender equality implies that the aggregate rank is equal to the within-gender rank, $G^m = G^f \to v = v^A$. However, this will not be the case whenever men and women have different income distributions. In most societies a rank will often translate into a lower income for women than for men.

The assumption of orthogonality between child gender and parent income implies that $F^m = F^f$. A given parent rank thus corresponds to the same income regardless of the gender of the child. Under the assumption of strictly increasing cumulative distribution functions (*cdf*), there is a direct mapping from parent income to parent rank $x = F^{-1}(u)$.

Since both v_i and v_i^A are uniformly distributed, the aggregate rank correlation can be described by the covariance $Cov(U, V^A) = Cov(U, \Lambda(V, S))$.[15] While a fully analytical solution would require assuming specific distributions I show in the appendix that the first order Taylor-approximation of this covariance around a given rank, $\hat{v}$ can be expressed as:

$$Cov(U, V^a) \approx A(\hat{v}) \times Cov(U, V), \tag{5}$$

$$A(v) = \left\{ \frac{1}{2} + \frac{1}{4} \left[\lambda^m(v) + \lambda^f(v) \right] \right\} \tag{6}$$

$$\lambda^m(v) = \frac{g^f(G^{m^{-1}}(v))}{g^m(G^{m^{-1}}(v))}, \tag{7}$$

$$\lambda^f(v) = \frac{g^m(G^{f^{-1}}(v))}{g^f(G^{f^{-1}}(v))}, \tag{8}$$

where I assume equal arrival probability of sons and daughters.

The adjustment term in (5), $A(v)$, is a function of λ^m and λ^f. These two functions are likelihood ratios evaluated at different incomes. For a given rank, v $\lambda^m(v)$ is a likelihood ratio evaluated at the income corresponding to that rank in the male income distribution. Conversely, $\lambda^f(v)$ is the reciprocal likelihood ratio evaluated at the income corresponding to the rank in the female distribution. To

[15]A uniformly distributed variable on the unit interval has a variance of 1/12. The rank correlation is therefore simply a rescaling of the covariance.

evaluate the magnitude and sign of the adjustment term in (5) is equivalent to evaluating these two likelihood ratios.

As densities are always positive, both λ^m and λ^f are positive for all possible ranks of evaluation. This implies that the adjustment term is never negative. In other words, the total correlation will always have the same sign as the within-gender correlation. When the marginal distributions are the same, then $\lambda^m = \lambda^f = 1$. This implies that the adjustment factor equals one. In other words, when there is no gender inequality, the within-gender correlation equals the total correlation. What remains to be determined is the magnitude, and, maybe most pressing, when the ratio is larger or smaller than one.

3.1 Evaluation of adjustment factor

In order to investigate the size of the adjustment term, I need to make assumptions concerning the income distributions. I assume that the likelihood ratio, $\frac{g^m(y)}{g^f(y)}$, increases monotonically. Intuitively this means that the higher the income, the larger is the ratio of men to women. From this follows that the male distribution stochastically dominates the female distribution, $G^m(y) < G^f(y)$ and that densities cross only once.

An example of such a situation can be seen in Figure 1. Take two distributions, one for women (in red) and one for men (in blue). The top plot in Figure 1 displays the cumulative distribution functions of such two artificial distribution. The two functions are chosen such that they exhibit a monotonic likelihood ratio and therefore exhibit stochastic dominance and single crossing.[16]

The density functions are displayed in the middle of Figure 1. One can intuitively see that the blue density divided by the red rises monotonically with the income level and cross only once. Denote the point of crossing y^*, that is $g^m(y^*)$=$g^f(y^*)$. The single crossing implies the following:

$$\frac{g^m(y)}{g^f(y)} < 1 \text{ if } y < y^*$$
$$\frac{g^m(y)}{g^f(y)} > 1 \text{ if } y > y^*$$

In other words, the male density is lower than the female density when the income is below the crossing point, and higher when income is above. Using this observation we can bound the interval, where $\lambda^m(v)$ and $\lambda^f(v)$ are *both*

[16]In the empirical analysis in section 4 I show that the assumption of monotonic likelihood ratios is reasonable in an American context.

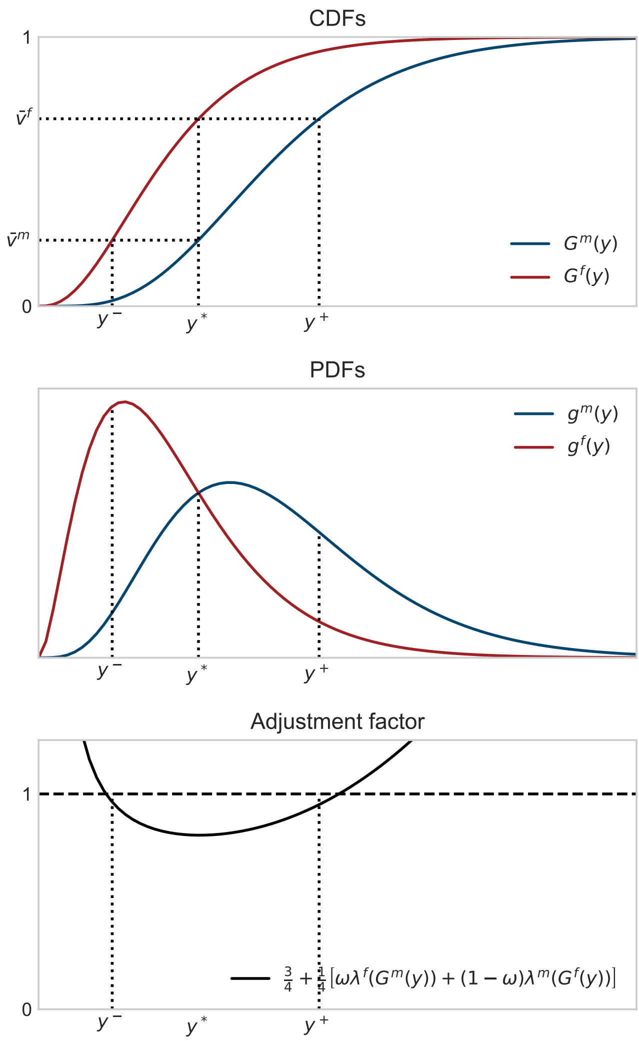


Figure 1: Range where aggregate covariance $<$ within-gender covariance

The figure depicts artificial distributions which exhibit monotonic likelihood ratios and $\frac{g^f(v)}{g^m(v)}$ is decreasing. The top figure displays the cumulative distribution functions (CDFs). From these functions one can identify $\bar{v}^m = G^m(y^*)$ and $\bar{v}^f = G^f(y^*)$ where the likelihood ratios are evaluated, where y^* is the crossing of densities. The ranks are transformed back to incomes, $y^- = {G^f}^{-1}(\bar{v}^m)$ and $y^+ = {G^m}^{-1}(\bar{v}^f)$. Evaluating the covariance at any income level in the interval (y^-, y^+) the aggregate rank correlation is smaller than the within gender correlation. With the distributions specified in this example, y^- and y^+ correspond to the 13th and the 82nd percentile in the aggregate income distribution.

smaller than one:

$$\lambda^m(v) < 1 \Leftrightarrow \frac{g^f\left(G^{m-1}(v)\right)}{g^m\left(G^{m-1}(v)\right)} < 1 \Leftrightarrow G^{m-1}(v) < y^* \Leftrightarrow v < G^m(y^*)$$
$$\lambda^f(v) < 1 \Leftrightarrow \frac{g^m\left(G^{f^{-1}}(v)\right)}{g^f\left(G^{f^{-1}}(v)\right)} < 1 \Leftrightarrow G^{f^{-1}}(v) > y^* \Leftrightarrow v > G^m(y^*),$$

where the second inequality follows from the single crossing property.

Therefore, the correction term is less than one for $\hat{v} \in \left[G^m(y^*), G^f(y^*)\right]$. Theses intervals can be read off Figure 1 in the following way. Find the crossing point, y^*, in the middle figure. Now evaluate this value in the CDFs in the top figure to find the corresponding bounding ranks, $\bar{v}^m = G^m(y^*)$ and $\bar{v}^f = G^f(y^*)$. These bounds can, in turn, be transferred back into income levels, denoted by y^- and y^+. The adjustment term is less than one for every income level within these bounds.

A calculation of the adjustment term is performed in the bottom plot of Figure 1. Note that these bounds are a sufficient condition. The adjustment term can be smaller in cases where either λ^m or λ^f are larger than one, as long as the other ratio is sufficiently small. We see, however, that the size of the adjustment term depends on income level at which one evaluates the approximation. Thus for practical application the adjustment term should be a function of an income level rather than a rank. I turn to the choice of evaluation in the next section.

3.1.1 Evaluation at an income level

The approximated relationship in Equation (5) is evaluated at a *rank* level. However, if we want to approximate the adjustment factor at an *income* level this may correspond to two different ranks, one for women and one for men. This gives rise to two different adjustment factors. In order to collapse the factors into a simple evaluation, I suggest taking the density-weighted mean, where the densities are evaluated at the income level of the overall approximation.

Denote $\bar{y}$ as the income of evaluation and $\omega = \frac{g^m(\bar{y})}{g^m(\bar{y})+g^f(\bar{y})}$. Now define the two ranks corresponding to $\bar{y}$ for men as v^m and for women v^f. We can now calculate the weighted mean of the adjustment term:

$$
\begin{aligned}
&\omega A(\bar{v}^m) + (1-\omega)A(v^f) \\
=&\frac{1}{2} + \frac{1}{4}\left[\omega\left\{\lambda^m(v^m) + \lambda^f(v^m)\right\} + (1-\omega)\left\{\lambda^m(v^f) + \lambda^f(v^f)\right\}\right] \\
=&\frac{1}{2} + \frac{1}{4}\left[\omega\left\{\frac{g^f(\bar{y})}{g^m(\bar{y})} + \lambda^f(v^m)\right\} + (1-\omega)\left\{\lambda^m(v^f) + \frac{g^m(\bar{y})}{g^f(\bar{y})}\right\}\right] \\
=&\frac{3}{4} + \frac{1}{4}\left(\omega\lambda^f(v^m) + (1-\omega)\lambda^m(v^f)\right),
\end{aligned} \tag{9}
$$

where I have assumed equal overall shares of men and women.[17]

The density-weighted adjustment term is now a function of a single income level. An example of such an evaluation at the mean of the income distributions is displayed in Appendix Figure A.1. Taking the density weighted mean is a convenient choice as it simplifies the terms considerably. Other approaches are also possible. As long as the income level of evaluation is such that the corresponding male rank translates into an income for females below the crossing point for the densities, then $\lambda^f(v^m) < 1$. Likewise, if the corresponding female rank corresponds to an income for men above the crossing point, then $\lambda^m(v^f) < 1$. This implies that other weighted means where these conditions are fulfilled also will evaluate to a mean adjustment factor less than one.

3.2 Interpretation

The exercise above is somewhat technical and does not offer much intuition. However, it is useful to think of the gender assignment as a lottery. If the child is male, then the child rank translates into income via the male distribution and likewise for females. In a society with full gender equality, this lottery doesn't matter. In that case, income maps to the same rank, regardless of gender. Any correlation between parent and within-gender child rank, therefore, goes straight through to the total rank.

This is not the case in societies with gender inequality. In that case, the

[17]The more general case is given by:

$$
\begin{aligned}
&\omega A(\bar{v}^m) + (1-\omega)A(v^f) \\
=&\mu^2 + (1-\mu)^2 + \mu(1-\mu)\left(\omega\left[\lambda^m(v^m) + \lambda^f(v^m)\right] + (1-\omega)\left[\lambda^m(v^f) + \lambda^f(v^f)\right]\right) \\
=&\mu^2 + (1-\mu)^2 + \mu(1-\mu)\left(1 + \omega\lambda^f(v^m) + (1-\omega)\lambda^m(v^f)\right).
\end{aligned}
$$

When I assume equal shares of males and females in the economy, $\mu = 1/2$ this term collapses to 9.

random sorting matters. Take an extreme case where all women earn a lower income than any man. In that case, the random sorting may almost completely dominate the link from parent to child. In other words, given the same within-gender mobility the unequal society would be measured as much more mobile than the equal society. In Figure 1 is can be seen by the differences in CDFs. When the horizontal difference is large, the mapping matters greatly. This horizontal difference is equivalent to what Bayer and Charles (2018) calls the earnings gap. The lower the inequality the smaller the earnings gap, and therefore the less important is the gender.

4 Empirical Analysis

To illustrate the empirical relevance of my findings I calibrate the developed adjustment factors to empirical income distributions. When estimating empirical distributions, there are a number of issues such as top-coding, weighting and sampling error. As the calculations are made for an illustrative purpose, I ignore most of these things. That means that I do not regard my quantitative results as definitive, though highly suggestive of magnitudes.[18] This section proceeds as follows: First, I describe the data and estimation techniques. I then turn to the calibration of the approximated weights developed in the previous section. Lastly, I compare them briefly to results obtained from copula-simulations.

4.1 Estimation of income distributions

I estimate the income distributions on data provided by IPUMS-USA (Ruggles et al. 2017), from the largest sample available in each year from 1970 to 2016. I restrict attention to individuals between the ages of 30 to 39. The variable of interest is total individual income (*totinc*). Only individuals with positive earnings are kept in the data.[19] The sample is weighted and I draw artificial samples from the original sample according to these weights. I then apply KDE to obtain densities, cumulative distributions and quantile functions for each gender

[18]Full code used to generate the results is available on my website. The program is written in Python and extensively documented. One can therefore easily change assumptions to investigate the stability of results.

[19]This is not an innocuous restriction as it affects individuals along the external margin. The quantitative results become more dramatic is gender-inequality increases by including individuals with no income. However including zero-incomes also breaks with the assumption of smooth distributions. As the purpose of this section is to serve as an illustration, I find the drawbacks of exclusion of zero-incomes acceptable.

in each year. I smooth the densities in order to interpolate values for incomes not present in the data.[20]

4.2 Results

Figure 2a displays the quantile-ratios of the estimated distributions. For each rank, it shows the corresponding income for males divided by the income for women. Total lack of gender inequality would imply a flat line at one, corresponding to the black dashed line. If the ratio is above 1 the rank of a man corresponds to a higher income than the corresponding rank for a woman.[21] As can be seen, the lines are consistently above 1, indicating that the male income distribution stochastically dominates the female distribution in all years. In other words, in all years a given within gender rank will translate into a higher income for males than for females. As time has progressed, the ratios have tended downwards toward 1. This provides evidence that gender equality has improved substantially since 1970.

Figure 2b shows the estimated likelihood ratios. In order to compare the functional shape of the ratios over time the likelihood ratios are evaluated at incomes corresponding to male ranks in the given year and indexed to the likelihood ratio at the 90th percentile of the male income distribution. The likelihood ratios are in general increasing for all years. This provides the empirical justification for the assumption of monotonic likelihood ratios, which was instrumental in developing the theoretical bounds on where the adjustment factor was smaller than one in section 2.[22]

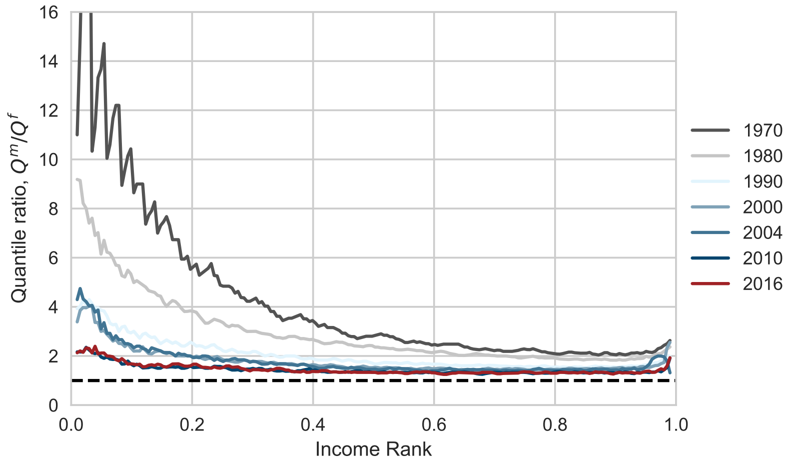
4.3 Estimated adjustment factors

Using the estimated densities, I can calibrate the adjustment factor, developed in section 2. Recall, that one needs to choose an income level at which to evaluate

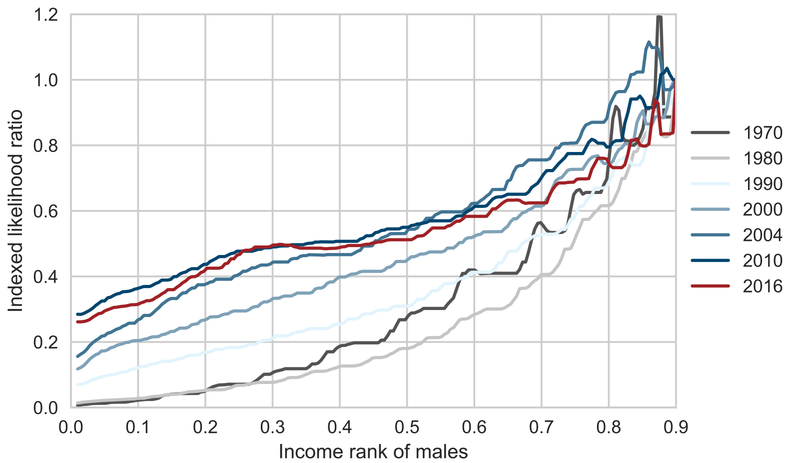
[20] Note, that I use income in a single year. It is recognized across the literature that this is problematic as it may be a poor proxy for lifetime income, Chen et al. (2017) and Guvenen et al. (2017) for recent discussions. However, Chetty et al. (2014b) find that the rank-correlation in the US is insensitive to the number of years over which to average income levels. The estimated distributions are only used to calculate ratios in the same year. Hence, there is no need to deflate the distributions, as the deflation cancels out.

[21] The quantile ratio is closely related to the earnings gap described in section 3.2. Formally, the earnings gap is defined as difference ${F^m}^{-1}(v) - {F^f}^{-1}(v)$, whereas the quantile ratio is given by the ratio ${F^m}^{-1}(v)/{F^f}^{-1}(v)$.

[22] Again, stochastic dominance follows from the monotonic likelihood property. Thus the stochastic dominance in Figure 2a is a necessary condition for the monotonic likelihood property to be a reasonable assumption.



(a) Quantile ratios



(b) Likelihood ratios

Figure 2: Estimated distributions

Figure 2a shows the male quantile distribution divided by the female, and will thus take the value 1 when the quantiles are equal. I omit ratios at the very bottom, as these are very volatile due to division by values close to zero. It is evident that first-order stochastic dominance is maintained for all years. Figure 2b show the density of males divided by the density of women, i.e. likelihood ratios. For comparison, the ratios are indexed at the 90th percentile of the male income distribution in a given year and evaluated at incomes corresponding to male ranks.

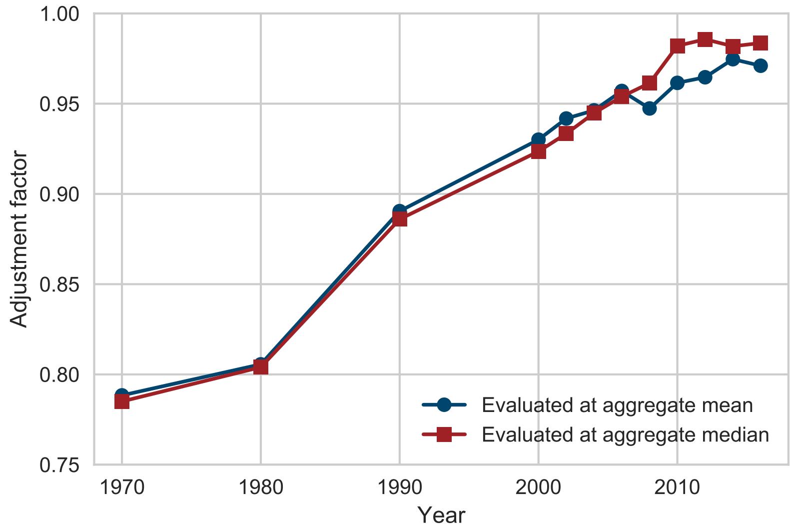


Figure 3: Calibrated adjustment factor

The figure shows the calibrated density weighted adjustment factors according to Equation (9) evaluated at the aggregate mean and median. The weights are not sensitive to choice of the income level at which to evaluate the adjustment factors. For the value of the adjustment factor for other income levels I refer to Appendix Figure B.2.

the adjustment factor. The density-weighted mean adjustment factor evaluated at the aggregate mean and median is shown in Figure 3. The general trend confirms the intuition that the increased gender equality should in an of itself increase the total rank correlation.

From Figure 3 we see that the adjustment factor rises from 0.79 to 0.97 between 1970 and 2016. Assuming constant inter-generational mobility within gender the aggregate mobility would have *fallen* by 23 percent solely due to greater gender equality.[23] In other words, the greater gender inequality achieved in the last half of the twentieth century decreased mobility considerably. In appendix Figure B.2 I calibrate the adjustment factor for incomes corresponding to male ranks between the first and the 99th percentile. I find that it in practice matters little at which point in the income distribution the adjustment factor is calculated, as long as it is not in the extreme tails.

4.4 Simulation

The adjustment factor is developed using a Taylor approximation and one should not expect it to fit the data perfectly. Another way to approach the question of

[23]Calculated as 0.97/0.79-1=0.23.

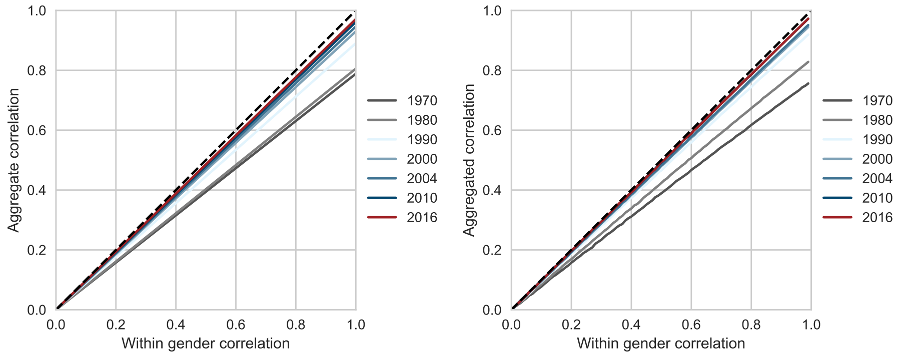
the significance of gender inequality is through simulation. As mentioned in section 2, a joint distribution can be described by a copula and a set of marginal distributions. The income distributions have been estimated, which leaves only the copula unknown. I assume a functional form of the copula and specifically choose a Gaussian copula, as it only contains a single parameter, which maps one-to-one to the rank correlation.[24]

The simulation exercise runs as follows: For a range of rank correlations between 0 and 1, I draw ranks for children and parents from the Gaussian copula corresponding to that rank correlation along with a random binary gender indicator. I then map the ranks into incomes according to the gender dummy and the estimated gendered quantile functions. The total rank correlation is then calculated as the correlation between the ranked child income and parent rank. I repeat the exercise a hundred times and take the mean of the resulting hundred rank correlation. The simulation protocol is described in Appendix C, and code is available online.

Figure 4 compares the results from calibrating the adjustment factor and the copula simulations. The linear relationship in Figure 4a is mechanical, as I just plot the within gender rank on the x-axis and the same correlation multiplied by the calibrated adjustment factor. However, I have not imposed the linear relationship in Figure 4b, where I plot the mean total rank correlations from the simulations as a function of the imposed within-gender rank correlation. The figure shows that the linear relationship between the within-gender and total rank correlation is an extremely good approximation. Appendix B shows that the approximation performs well when compared directly to the simulations using Gaussian copulas. Though the relationship is not exactly one, they follow each other closely.

Figure 4 also presents another way to interpret the importance of gender inequality for the measurement of mobility. A given rank correlation can map into many different societies. An aggregate rank correlation of 0.4 can thus map into a within gender rank correlation between 0.4 today and 0.5 in 1970. This can be seen by following a horizontal line from 0.4 on the y-axis in 4a. Furthermore, due to the proportional relationship, the larger the underlying rank correlation,

[24] The Gaussian copula has been controversial as it cannot describe tail dependence. This is relevant in mobility studies, as one often observe lower mobility in tails of the distribution. In other words, children of high very income parents are much more likely to become very high earners themselves, see Chetty et al. (2014b) and Boserup et al. (2014) for examples of such a pattern in the US, Canada and Denmark. However, as I do not concern myself with the copula itself, but rather the link between within gender and total mobility, I find these issues to be of minor importance in the present setting.



(a) Calibrated adjustment factor (b) Simulated relationship

Figure 4: Calibrated and simulation association between within-gender and aggregate rank correlations.

Figure 4a shows the relationship between within-gender and aggregate rank correlation with calibrated density weighted adjustment factor as described in Equation (9) evaluated at the aggregate mean. The aggregate gender correlation is simply the adjustment factor are multiplied by the within gender rank correlation. The dashed black line represent full gender equality. Figure 4b shows the simulated aggregate rank correlation as a function of a known within-gender rank correlation. The simulation assumes a Gaussian copula and follows the protocol described in appendix C. Note that no linear relationship has been imposed. The relationship with the chosen copula, therefore, seems to be almost perfectly linear. For a direct comparison between the calibration and simulation see Appendix Figure B.1.

the larger is the absolute difference between the aggregate and within-gender correlations.

These results show that the rank correlation is not as robust as often assumed. Interpretation and comparison of rank correlations over time are not straightforward when changes in underlying societal factors occur at the same time. Time is however only one dimension, another is space. One should expect the same issues when comparing across countries or other geographic entities where gender equality may differ.

5 Discussion and further issues

Lastly, I briefly touch upon some other issues of mobility measurement which relate to the findings of this paper. These are all important issues but are outside the scope of the present analysis.

Unit of analysis This analysis has exclusively focused on individuals as the unit of analysis. Many studies have instead used household measures. The inequality between genders is obfuscated by the coupling of male and female income, but issues with intragroup inequality remain. The weighting of single income households against dual earners will be directly affected by the dynamics presented here, though not to a full degree. Trends in assortative mating and household-level optimization also affect rank-based measures in ways, which cannot be expected to remain constant over time.[25]

Definitions of parents Another issue the present paper has not touched upon is the definition of parents. Whether parents should be defined as households or the individual parent is an issue that only complicates the interpretation of rank-based measures further. These considerations will always be present, but as long as gender is random, the definition of parents should not affect the main conclusions of this paper.

Other measures beside income Though the intuition and results in this paper are developed using income as a measure of social position, the findings may carry over to other measures such as wealth and educational attainment. The theoretical results are developed under the assumption of continuous marginal distributions, but I conjecture that the overall logic does not hinge on this assumption. However, it is naturally crucial that one is able to rank the measures in order to calculate the rank correlation.

Other types of group inequality An important question is whether insights of this paper translates to other forms of group inequality, the most present being ethnicity and race. Gender is characterized by the fact that all types of families have sons and daughters. This feature is not present with race or ethnicity. The interpretation of gender as a random sorting mechanism does therefore not carry over to ethnicity. To see why, note that parent income will correlate with the assignment to groups. Thus, it is perfectly plausible that the total rank correlation will be *higher* than the within-race rank correlation. This could be the case if the races are so unequal that their income distributions have little overlap *in both generations*. The aggregate and within-race rank correlations will almost surely differ but in ways different from the case of gender. Thus, while this paper documents the inequality within groups is important for the rank-correlation, the exact results are specific to gender.

[25] Chadwick et al. (2002) discuss these issues in the context of estimating IGEs.

Gender discrimination and equality of opportunity One can be perfectly content with using an aggregated rank measure as all the above-stated phenomena reflect economic mobility. Nevertheless, the choice of measure directly affects the interpretation and political implications. This article has shown that it is not meaningful to compare a society with large gender inequality with a society that has succeeded in alleviating gender inequality, unless one is willing to argue that, indeed, gender inequality should be interpreted as a means to increase intergenerational mobility. How to define intergenerational mobility is closely intertwined with the discussion of equality of opportunity. Gender is exogenously given and does not depend on the effort of the child. Ensuring gender equality is therefore often seen as a pathway to ensuring equality of opportunity.[26] In light of this, I claim that it makes little sense to have gender discrimination be a policy tool to ensure mobility.

6 Conclusion

The potential for confusing increased gender equality with decreased mobility shows how the interpretation of rank-based mobility measures is less obvious than it might initially appear. Copulas and ranks greatly increase the scope for research on intergenerational mobility. However, too little focus has been afforded the process of transformation of income to ranks and the corresponding changes in measures of mobility. No one correct ranking procedure exist, nor should there, but this paper has illustrated the perils of comparing mobility in societies over time and across space.

These issues are not new in the mobility literature, but rank-based measured have not yet received the careful inspection that has been afforded the measures such as the inter-generational income elasticity. This paper has provided a stepping stone towards this goal.

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A Taylor approximation of covariance

As in the main text, I assume that a family can be described by 3 variables:

$$(U_i, V_i, S_i),$$

where U_i and V_i are parent and child rank and S_i is an indicator variable which evaluates to one if the child is male. For simplicity, I assume that the covariance between parent and child rank is not a function of gender, $E(V_i|U_i, S_i) = E(V_i|U_i)$. This is easily changed, though the resulting equations will be less neat. Furthermore, I assume that parent income does not depend on the gender of the child, $E(U_i|S_i) = E(U_i)$. The income distributions are gender-specific and taken for granted. The translation from child rank to child income is described by the following relation:

$$y(v, s) = sG^{m-1}(v) + (1 - s)G^{f^{-1}}(v),$$

where G^m and G^f are the cumulative income distribution functions of male and female children respectively. I assume these to be invertible, thereby excluding mass-points, including zero income. The *aggregate* income distribution of children is then a mixture distribution given by $G(y) = \mu G^m(y) + (1 - \mu)G^f(y)$, where $\mu = E[S_i]$.

Given these definition, the aggregate rank, v^A can now be described as a function of the within-gender rank, v. Using the *probability integral transform*:

$$v^A(v, s) = G(y(v, s)) = \mu G^m(y(v, s)) + (1 - \mu)G^f(y(v, s)) \tag{10}$$

Define $v^A = \Lambda(v, s)$. All results in this section will relate to the functional properties of $\Lambda(v, s)$.

We want to approximate the covariance between parent rank and child aggregate rank, $Cov(U, V^a)$. Due to the law of total covariance:

$$\begin{aligned} Cov(U, V^a) =& Cov(U_i, \Lambda(V, S)) \\ =& \underbrace{E\left[Cov(U, \Lambda(V, S)|S)\right]}_{\text{within gender}} + \underbrace{Cov\left(E\left[U|S\right], E\left[\Lambda(V, S)|S\right]\right)}_{\text{between gender=0}}. \end{aligned} \tag{11}$$

The decomposition shows that we can handle the description of the covariance in two separate parts. The first term is the mean of the covariance between parent rank and child aggregate rank *conditional on gender*. The second term is the covariance between means of parent and child rank for families of sons and daughters. But since I have assumed that $E(U_i|S_i) = E(U_i)$, the latter term evaluates to zero. We therefore only need to focus of the within-gender covariance.

Since S is a binary variable, it follows that (11) can be written as:

$$E\left[Cov(U,\Lambda(V,S)|S)\right] = \mu \underbrace{Cov(U,\Lambda(V,1))}_{(i)} + (1-\mu)\underbrace{Cov(U,\Lambda(V,0))}_{(ii)}. \tag{12}$$

As will become clear, it is easier to handle gender-specific versions of $\Lambda(v,s)$. Therefore define:

$$\Lambda^m(v) = \Lambda(v,1) = \mu v + (1-\mu)G^f\left(G^{m^{-1}}(v)\right) \tag{13}$$

$$\Lambda^f(v) = \Lambda(v,0) = \mu G^m\left(G^{f^{-1}}(v)\right) + (1-\mu)v \tag{14}$$

We first focus on (i) in (12):

$$Cov(U,\Lambda(V,1)) = E\left[U\Lambda(V,1)\right] - E\left[U\right]E\left[\Lambda(V,1)\right] \tag{15}$$

Insert (13):

$$\begin{aligned} Cov(U,\Lambda(V,1)) =& E\left[U\left(\mu V + (1-\mu)G^f\left(G^{m^{-1}}(V)\right)\right)\right] \\ &- E\left[U\right]E\left[\mu V + (1-\mu)G^f\left(G^{m^{-1}}(V)\right)\right] \\ =& \mu \underbrace{Cov(U,V)}_{\text{Within-gender covariance}} + (1-\mu)\underbrace{Cov\left(U,G^f\left(G^{m^{-1}}(V)\right)\right)}_{\text{“Adjusted covariance”}} \end{aligned} \tag{16}$$

While the within-gender covariance is known, we cannot yet describe the “adjusted covariance”. We, therefore, reformulate the second term:

$$\begin{aligned} &Cov\left(U,G^f\left(G^{m^{-1}}(V)\right)\right) = \\ &E\left\{(U-E[U])\left(G^f\left(G^{m^{-1}}(V)\right) - E\left[G^f\left(G^{m^{-1}}(V)\right)\right]\right)\right\} \end{aligned} \tag{17}$$

Now perform a first order Taylor expansion of $G^f\left(G^{m-1}(V)\right)$ around $\hat{v}$:

$$G^f\left(G^{m-1}(v)\right) \approx G^f\left(G^{m-1}(\hat{v})\right) + \underbrace{\left.\frac{g^f\left(G^{m-1}(v)\right)}{g^m\left(G^{m-1}(v)\right)}\right|_{\hat{v}}}_{\lambda^m(\hat{v})}(v-\hat{v}) \tag{18}$$

Reinserting equation (18) into equation (17):

$$\begin{aligned}&Cov\left(U, G^f\left(G^{m-1}(V)\right)\right) \approx \\ &\lambda^m(\hat{v})E\left\{(U-E[U])\left([(V-\hat{v})-E[V-\hat{v}]]\right)\right\}\end{aligned} \tag{19}$$

Reinserting equation (19) into equation (16):

$$\begin{aligned}Cov(U,\Lambda(V,1)) \approx & \mu Cov(U,V) \\ & + (1-\mu)\lambda^m(\hat{v})E\left\{(U-E[U])\left([(V-\hat{v})-E[V-\hat{v}]]\right)\right\} \\ = & \mu Cov(U,V) + (1-\mu)\lambda^m(\hat{v})Cov(U,V-\hat{v}) \\ = & \left\{\mu + (1-\mu)\lambda^m(\hat{v})\right\}Cov(U,V)\end{aligned}$$

We perform the exact same operation on (ii) in equation (12) which yields:

$$Cov(U,\Lambda(V,0)) \approx \left\{\mu\lambda^f(\hat{v}) + (1-\mu)\right\}Cov(U,V)$$

where $\lambda^f(v) = \frac{g^m\left(G^{f^{-1}}(v)\right)}{g^f\left(G^{f^{-1}}(v)\right)}$. Reinserting into equation (12) and inserting into equation (11) yields:

$$Cov(U,V^a) \approx \left\{\mu^2 + (1-\mu)^2 + \mu(1-\mu)\left[\lambda^m(\hat{v}) + \lambda^f(\hat{v})\right]\right\}Cov(U,V) \tag{20}$$

Assuming equal share of men and women, $\mu = 1/2$, the expression simplifies to:

$$Cov(U,V^a) \approx \left\{\frac{1}{2} + \frac{1}{4}\left[\lambda^m(\hat{v}) + \lambda^f(\hat{v})\right]\right\}Cov(U,V),$$

which is identical to equation (5) in the main text.

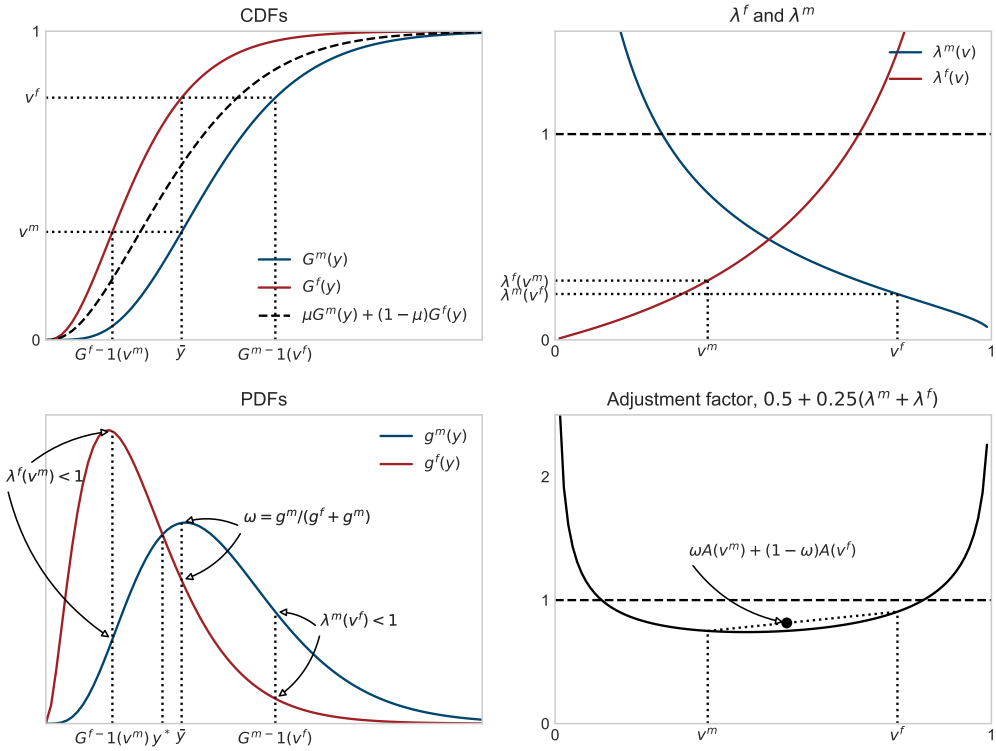

Figure A.1: Example of evaluation of the adjustment factor at the mean income

The figure depicts artificial distributions which exhibit monotonic likelihood ratios. In this example the adjustment factor is evaluated at the mean of the aggregate distribution, $\bar{y}$. The top left figure displays the cumulative distribution functions (CDFs). From these functions one can identify $v^m = G^m(\bar{y})$ and $v^f = G^f(E(y))$ where the likelihood ratios are evaluated. In this example ${G^f}^{-1}(v^m) < y^* < {G^f}^{-1}(v^f)$, where y^* is the crossing of densities. This implies that both $\lambda^f(v^m)$ and $\lambda^m(v^f)$ are below one. This can be read off the top right figure, which plots the values for λ^f and λ^m for all values of the rank, v. The adjustment factor for all values of v is shown in the bottom right figure, where the density weighted mean between the two adjustment factors are shown as a black circle.

B Additional empirical and simulation results

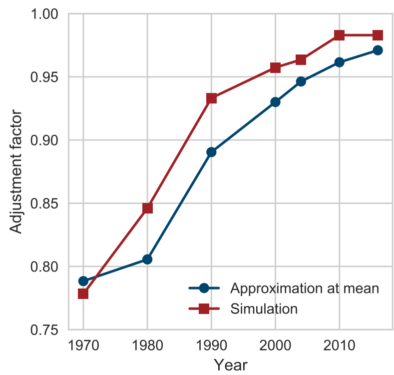

Figure B.1: Calibrated and simulated adjustment factor

The figure shows the calibrated adjustment factors according to Equation (9) in blue. It is evaluated at the aggregate mean. In red I have taken the average ratio between the total and the within-gender rank correlation across all values of the with-gender rank correaltion. As seen in Figure B.1 the realtionship between the two rank correalations is approximately linear and thus have an almost constant ratio.

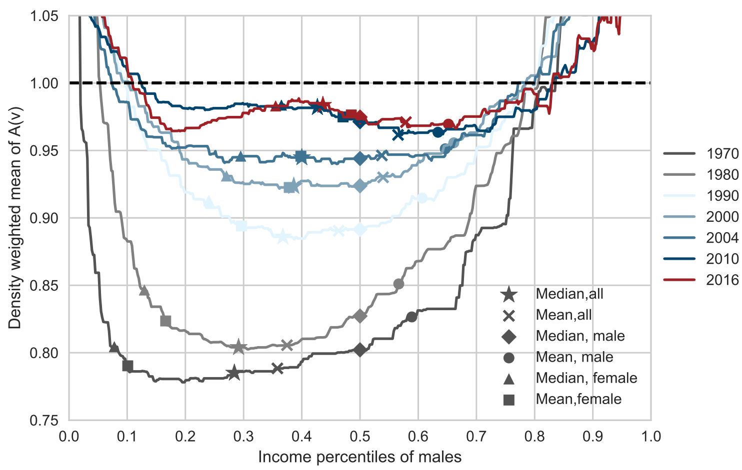

Figure B.2: Evaluation of the adjustment factor at different levels of income

The figure shows the importance of the choice of evaluation point. For each year I calculate the density weighted adjustment factor at an income corresponding to the rank on the x-axis for males. The ranks serve as a normalisation of the income distributions, such that one can compare across years. The very flat lines indicate that it matters little where the adjustment factor is approximated, less it not be in the tails. The figure also displays the adjustment factors for means and medians for the two genders and for the total income distribution. These are represented by the symbols on the line placed at the corresponding ranks in the male income distribution. By definition, the median of males are always located at 0.5 on the x-axis.

C Simulation protocol

The data is generated from a Gaussian copula, which only take one parameter which maps 1:1 with the rank correlation. Marginal distributions for parents and children have to continuous and invertible. No gender is attributed to parents and thus it is assumed that the marginal distribution of parents is uniform, $F_p(x) = x$. As described in the text children are divided into to groups m and f having different marginal distributions, G^m and G^f respectively. Let G be the income distribution containing both group m and f. The protocol for simulating is as follows:

For a given ρ repeat R times:

- Draw two vectors K_p and K_c from a bivariate normal distribution[27], $K_p, K_c \sim N\left([0,0], \begin{bmatrix} 1 & \rho_G \\ \rho_G & 1 \end{bmatrix}\right)$
- Apply the cdf of the standard normal distribution to K_p and K_c to obtain U and V: $U = \Phi(K_p), V = \Phi(K_c)$
- Depending on gender:
  - If no gender:
    - Apply quantile functions of the marginal distributions to ranks to obtain income variables:
      - For parents: $Y_p = F_p^{-1}(U)$
      - For children: $Y_c = G_c^{-1}(V)$
  - If gender:
    - Draw a binary variable, S with probability μ.
    - Apply quantile functions of the marginal distributions to ranks to obtain income variables.
      - For parents: $Y_p = F_p^{-1}(U)$
      - For children: $Y_c = S \times G^{m^{-1}}(V) + (1 - S) \times G^{f^{-1}}(V)$

Finally take the mean of the desired statistic over the R realizations.

[27] In the Gaussian copula there is a 1:1-mapping between the correlation in the Gaussian copula, ρ, and the rank correlation coefficient,ρ_s:$\rho_G = 2\sin(\rho\frac{\pi}{6})$. See Meyer (2013) for details.