

# Modules in Futhark

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## 0.1 Abstract

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# 1 Introduction

This report describes the efforts in defining a concrete method of extending the Futhark Programming Language with a module system. A module system is here defined as a programming feature, which allows the programmer to package functionality into a discrete code package.

After a module has been defined, it can then be shared and utilized in any other program, by including it in these programs.

## 1.1 Problem definition

Is it possible to implement a module system in Futhark, which displays features comparable to the module system implemented in Standard ML? [?]

## 2 Motivation

The implementation of a module system, will greatly expand the usability of Futhark, without having side effects on the performance of the compiled Futhark code. The following subsections contain the features this project is implementing, and examples of why these features are desirable.

To be able to claim, that modules increases usability, we must first setup some metrics of usability.

### **Readability increases usability:**

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### **Compartmentalization of functionality increases usability:**

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### **A module system increases the ability to share code between developers.**

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## 2.1 Abstraction increases readability.

In example, type aliasing lets us abstract from the actual definition of data types. When writing our source code, we can define our type aliases before writing the rest of our program. If we want to define a sphere in a three dimensional space, we want to define it with **1) a radius**, **2) a position**, and **3) a direction** that it is moving. Let us define a function that multiplies the speed of the sphere by a factor k.

```
fun {f32, {f32, f32, f32}, {f32, f32, f32}} multiply_velocity(  
    {f32 r,  
     {f32 x_pos , f32 y_pos , f32 z_pos} ,  
     {f32 x_dir, f32 y_dir, f32 z_dir}} , f32 k)  
    =  
    {r, {x_pos, y_pos, z_pos}, {k * x_dir , k * y_dir, k * z_dir }}
```

---

With type aliasing, we can compartmentalize the data type, and remove the need for explicitly typing out every parameter of function input. Coupled with helper functions, we can now multiply the speed of the sphere like this:

```
type vec3 = {f32, f32, f32}
type position, direction = vec3
type radius = f32
type sphere = {radius, position, direction}

fun vec3 multiply_vector(vec3 {pos_x, pos_y, pos_z}, f32 k) =
  {k * pos_x , k * pos_y , k * pos_z}

fun sphere multiply_velocity(
  sphere {radius, position, direction}, f32 factor) =
  let new_direction = multiply_vector(direction, factor)
  in {radius, position, new_direction}
```

As we are now using the sphere type as the function argument, we can pattern match on the type aliased values contained in the sphere type. Most importantly, the vectors of the sphere are abstractized into a single variables instead of tuples.

The end result is a shorter, more readable program 3.5.4

## 2.2 Approximating higher order functionality whilst keeping performance

It is possible to express higher-order functionality in Futhark, without taking a performance hit in the compiled Futhark code. We will reiterate on the concept of modules<sup>4</sup>, by introducing the concept of functors. First we repeat the three-dimensional vector module from earlier, but without declaring any particular primitive 3.1 type as the contained type of the vector:

```
structure Vec3 =
  struct
    type vector = {t, t, t}
    fun vector add( ... ) = ...
    fun vector subtract( ... ) = ...
    fun vector multiply ( ... ) = ...
    fun vector divide ( ... ) = ...
  end
```

The structure above cannot be used on its own. Type `t` is not instantiated, and the module cannot be type checked, which causes an error.

We can solve that problem, by instantiating the abstract structure, using a simple functor; the where-clause.

```
structure IntVec3 = Vec3 where type t = int
```

We can now access the structure `IntVec3` throughout the rest of the program. The structure `IntVec3` is `Vec3`, except all instances of type `t` in `Vec3` is exchanged with type `int`.

**To recap:** functors allows us to define an abstract implementation of some structure **ONCE**, and lets us instantiate this structure any number of times, each time with our own type. This functionality will be elaborated on in a later section `Functors??`.

From a performance-concerned point of view, the module system is desirable. Every function in the written program, whether it is inside a structure or defined in the top level program, is ultimately accumulated into the same scope. 4.9.1

The entire program is then compiled, and the optimizations that makes `FUTHARK GO FAST` are applied to the included modules as well as the top level declarations.

## 2.3 Scope of project

The scope of the project is

- to implement a type aliasing system in Futhark
- to implement a module system in Futhark, which has:
  - The definition of structures containing types and functions
  - Nested modules; meaning that any structure can contain a structure
  - A well defined way of referring to types, structures and functions contained in a structure
- to implement functor functionality, so that Futhark supports the definition of abstract structures and concretizations of these structures.

## 2.4 Success definitions

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### 3 Type aliases

To make an implementation of functors?? in Futhark, it was necessary to implement type aliases first.

#### 3.1 The language

The initial type system in Futhark supported the following type definitions:

$Type =$	Primitive type
	$(Types)$
	$[Type]$
$Types =$	$Type , Types$
	$Type$
$Primitivetype =$	$UnsignedInteger$
	$SignedInteger$
	$FloatType$
	$Boolean$

Implementing type aliases expands our type constructions as following:

$Type :$	Primitive type
	$(Types)$
	$[Type]$
	$TypeAlias$
$Types :$	$Type , Types$
	$Type$
$Primitivetype :$	UnsignedInteger
	SignedInteger
	FloatType
	Boolean
$TypeAlias :$	<code>type <math>strid</math> = <math>Type</math></code>

where  $type\_id$  is short for  $string\ id$ , a string identifier.

### 3.2 Interference rules

A type alias in a Futhark program is done like this: `type  $Strids$  =  $Type$`  , where  $Type$  is as defined in the grammar above.



Given an environment  $\Gamma : \{FE, TE\}$ ,  $Types = \{strid \rightarrow Type\}$ ,  $Type \rightarrow \tau^a$  we can define the following inference rules for using type aliases:

$$\frac{\Gamma \vdash typedecl \Rightarrow \Gamma' \quad \Gamma \vdash Type \rightarrow \tau \quad \Gamma \not\vdash type\_id \rightarrow Type}{\Gamma \vdash \text{type } type\_id = Type \Rightarrow \{\emptyset, \{type\_id \rightarrow Type\}\} \oplus \Gamma}$$

where

$$\begin{aligned} & \{\emptyset, \{strid \rightarrow Type\}\} \oplus \Gamma \\ \Rightarrow & Types(\Gamma) \leftarrow Types(\Gamma) \cup \{strid \rightarrow Type\} \end{aligned}$$

iff the type alias declaration does not clash with the current environment. There is no clash, if the declaration obeys the three rules in the implementation subsection 3.3.

We can assign the same type to several string ids simultaneously, as long as we don't declare the same type alias twice.

$$\frac{\Gamma \vdash typedecl \Rightarrow \Gamma' \quad Strids = strid_1, \dots, strid_n}{\Gamma \vdash \text{type } Strids = Type \Rightarrow \{strid_1 \rightarrow Type, \dots, strid_n \rightarrow Type\} \oplus \Gamma \Rightarrow \Gamma'}$$

where

$$\begin{aligned} & \{strid_1 \rightarrow Type, \dots, strid_n \rightarrow Type, \} \oplus \Gamma \\ \Rightarrow & Types(\Gamma) \rightarrow Types(\Gamma) \cup \{strid_1 \rightarrow Type, \dots, strid_n \rightarrow Type\} \end{aligned}$$

---

<sup>a</sup>We will let  $\tau$  designate a primitive type in Futhark

### 3.3 Implementation

A type alias  $strid_i \rightarrow Type$  declaration is succesful, if three rules are followed:

1. The alias  $strid_i$  is not already declared<sup>1</sup> in the current local environment. I.e. the example below:

```
type foo = i32
type foo = f32
```

---

<sup>1</sup>This is by convention of only being able to define a value *once*

2. The alias *strid<sub>i</sub>* refers to a type (or a type alias), that is either already defined in the current environment (including structure environments), or is in the same declaration chunk as *strid<sub>i</sub>*. In the example below, `foo` refers to `bar`, but `bar` is in the same chunk as `foo`.

Therefore, `foo` can be resolved by resolving `bar`. The implementation of this described in a later subsection 3.4.2.

```
type foo = bar
type bar = {f32, i32}
```

There are no hard limit to the number of type aliases that has to be checked, before a type alias is resolved:

```
type foo = bar
type bar = {f32, baz, i32}
type baz = [{bee, bang, boo}]
type boo ...

...

type bep = i32
```

Such a chain of type aliases is allowed, as long as the last of the three rules is obeyed:

3. The alias being resolved cannot be cyclically defined.  
Imagine that some type `type my_type = foo` is in the chain in the type aliasing example 2 above.

In this case, the compiler is first trying to resolve `foo` by resolving `bar`, and trying to resolving `bar` by resolving `baz`, et cetera.

At some point, the compiler encounters `my_type`, and must resolve `foo` to continue - which creates a cycle, because `foo` is resolved by `bar`, and so on.

To keep track of this, the compiler maintains the set of aliases that has been visited in the attempt to resolve some type alias. Every time another type alias has to be checked, the compiler first checks the set to find out, whether this alias is already on the list of aliases that needs to be resolved.

If so, the compiler returns an error. The implementation of this can be read here 3.3

## 3.4 Parsing a type alias

### 3.4.1 Data types for describing a type

Initially, the types parsed in a Futhark program were always represented as instances of the following datatype `TypeBase`:

```
data TypeBase shape as vn = Prim PrimType
    | Array (ArrayTypeBase shape as vn)
    | Tuple [TypeBase shape as vn]

data ArrayTypeBase shape as vn =
    PrimArray PrimType (shape vn) Uniqueness (as vn)
    -- ^ An array whose elements are primitive types.
    | TupleArray [TupleArrayElemTypeBase shape as vn] (shape vn) Uniqueness
    -- ^ An array whose elements are tuples.

data TupleArrayElemTypeBase shape as vn =
    PrimArrayElem PrimType (as vn) Uniqueness
    | ArrayArrayElem (ArrayTypeBase shape as vn)
    | TupleArrayElem [TupleArrayElemTypeBase shape as vn]
```

---

These are also the data types that are used in the internal Futhark program.

Before this project, Futhark parsed types from Futhark source code directly to `TypeBases`. However, we decided to add an intermediate data type between raw source code and `TypeBases`, to make type aliases available.

```
data UserType vn = UserPrim PrimType SrcLoc
    | UserArray (UserType vn) (DimDecl vn) SrcLoc
    | UserTuple [UserType vn] SrcLoc
    | UserTypeAlias LongName SrcLoc
    | UserUnique (UserType vn) SrcLoc

deriving (Show)
```

---

The parser was changed, so type declarations in Futhark source would now be parsed as `UserTypes` and not `typebases`.

### 3.4.2 Adding resolved types to scope

```
1  type TypeAliasTableM =
2    ReaderT (HS.HashSet LongName) (StateT Scope TypeM)
3
4  typeAliasTableFromProg :: [TypeDefBase NoInfo VName]
5                        -> Scope
6                        -> TypeM Scope
7  typeAliasTableFromProg defs scope = do
8    checkForDuplicateTypes defs
9    execStateT (runReaderT (mapM_ process defs) mempty) scope
10 where
11     findDefByName name = find ((==name) . typeAlias) defs
12
13     process :: TypeDefBase NoInfo VName
14             -> TypeAliasTableM (StructTypeBase VName)
15     process (TypeDef name (TypeDecl ut NoInfo) _) = do
16       t <- expandType typeOfName ut
17       modify $ (addType name t)
18       return t
19
20     typeOfName :: LongName -> SrcLoc
21             -> TypeAliasTableM (StructTypeBase VName)
22     typeOfName (prefixes, name) loc = do
23       inside <- ask
24       known <- get
25       case typeFromScope (prefixes, name) known of
26         Just t -> return t
27         Nothing
28           | (prefixes, name) `HS.member` inside ->
29             throwError $ CyclicalTypeDefinition loc name
30           | Just def <- findDefByName name ->
31             local (HS.insert (prefixes, name)) $ process def
32           | otherwise ->
33             throwError $ UndefinedAlias loc name
```

TypeAliasTableM is a monad stack that is used to resolve a list of type alias declarations in a declaration chunk.

It is a reader monad transformer that contains a state monad transformer, that contains the TypeM monad.

The reader monad is used to contain a HashSet as its environment. This environment is used to keep check of cyclical definitions as described in 3.

For each type alias, we use the function process to resolve the type, and modify the scope contained within the state monad of the ReaderT.

Resolving a type from a type aliasing is done using the function typeOfName in the code snippet3.4.2. typeOfName tries to resolve the type by name by retrieving the scope contained in the transformed State monad inside the reader.

If this is not immediately possible, we must either continue our search for the type, throw an error due to a cyclical type definition, or throw an error because the type alias has not been defined yet.

If our attempt to resolve  $strid_a$  leads to another alias  $strid_b$ <sup>2</sup>, we add  $strid_a$  to our reader environment using the function `local`, and process  $alias_b$  instead.

**Acknowledgement:** The initial design of `expandType` and the addition of type aliases to the scope was initially much larger, but was reduced in size by Troels Henriksen, who rewrote the process to use monads, and reduced some code duplication.

### 3.4.3 Converting UserType to TypeBase

```

1  expandType :: (Applicative m, MonadError TypeError m) =>
2      (LongName -> SrcLoc -> m (StructTypeBase VName))
3      -> UserType VName
4      -> m (StructTypeBase VName)
5
6  expandType look (UserTypeAlias longname loc) =
7      look longname loc
8  expandType _ (UserPrim prim _) =
9      return $ Prim prim
10 expandType look (UserTuple ts _) =
11     Tuple <$> mapM (expandType look) ts
12 expandType look (UserArray t d _) = do
13     t' <- expandType look t
14     return $ arrayOf t' (ShapeDecl [d]) Nonunique
15 expandType look (UserUnique t loc) = do
16     t' <- expandType look t
17     case t' of
18         Array{} -> return $ t' `setUniqueness` Unique
19         _        -> throwError $ InvalidUniqueness loc $ toStructural t'

```

### 3.4.4 Why we added UserType instead of extending TypeBase

Adding `UserType` and then resolving these into `TypeBases` whilst running the program through the `TypeChecker`, removes the need of handling `UserAliases` after the type check, where these aliases are resolved.

Furthermore, not all information about a `TypeBase` declaration can actually be claimed already at program parse time. Some information about i.e. IKKE SIKKER HER array dimensionality in regards to aliased arrays, is decided within the type checker as well.

---

<sup>2</sup>line 27-33

### 3.4.5 The slip from type aliases to realized types

Since every type alias is resolved in the type checker, the `UserTypes` are not used after the type check.

The data type for a type declaration is this:

```
data TypeDeclBase f vn =
  TypeDecl { declaredType :: UserType vn
            -- ^ The type declared by the user.
            , expandedType :: f (StructTypeBase vn)
            -- ^ The type deduced by the type checker.
            }
```

---

An unresolved type looks like this:

```
TypeDecl userType NoInfo
```

After resolve, `NoInfo` has been filled out with a variable of type `Info TypeBase`, giving us the following `TypeDecl`:

```
TypeDecl usertype (Info typebase).
```

At any point after the type check, only the `expandedType` of `TypeDecl` is used.

## 3.5 Results

The addition of type aliases works without any issues. To verify this, Futhark has been tested to pass all of the tests in futharks test suite<sup>3</sup>.

However, it was also necessary to write tests to specifically confirm, that the implementation respects the rules defined in 3.3

### 3.5.1 A type cannot be defined twice in the same environment

From `alias-error3.fut` in `futhark/src/data/tests/types`:

```
-- You may not define the same alias twice.
--
-- ==
-- error: Duplicate.*mydup

type mydup = int
type mydup = f32

fun int main(int x) = x
```

This program fails as expected.

---

<sup>3</sup>located in folder `futhark/data/tests`

### 3.5.2 A type alias cannot be defined, if it refers to a type alias that has not been defined

From alias-error4.fut in futhark/src/data/tests/types:

```
-- No undefined types!
--
-- ==
-- error: .*not defined.*
```

```
type foo = bar
```

```
fun foo main(foo x) = x
```

This program fails as expected.

### 3.5.3 A type alias cannot be cyclically defined

From alias-error5.fut in futhark/src/data/tests/types:

```
-- No tricky circular types!
--
-- ==
-- error: .*cycl.*
```

```
type t0 = [t1]
type t1 = {int, float, t2}
type t2 = t0
```

```
fun t1 main(t1 x) = x
```

This program fails as expected.

### 3.5.4 Example of planet simulations being simplified by type aliases

A nice example of the benefits of type aliasing, is the N-body simulation (`nbody`), which is a simulation over the n-body problem<sup>4</sup>.

The original Futhark implementation of the simulation contained function arguments of tuples with arity up to 10. Whilst it is still necessary to bring all the arguments throughout the program, type aliasing makes the program itself much more readable:

## 3.6 Future work

To make functors work, it must be possible to declare an abstract type alias in a structure.

That will allow for a structure definition, where a type variable<sup>5</sup> has been declared, but not defined, until the containing structure is instantiated using a functor.

This has not been implemented yet.

---

<sup>4</sup>[https://en.wikipedia.org/wiki/N-body\\_problem](https://en.wikipedia.org/wiki/N-body_problem)

<sup>5</sup>in the form of a *strid*



## 4 Structures

**Introduction to structures:** Without structures, every function and type in futhark shares the same scope. Implementing modules lets us create functions that are alike, but keeps distinctions between them. Take this example of a program with two different vector types:

```
type vec3 = {f32, f32, f32}
type vec4 = {f32, f32, f32, f32}

fun vec3 vec3_plus(
  vec3 {a_1, ... , a_3},
  vec3 {b_1, ..., b_3}
) = {a_1 + b_1, ... , a_3 + b_3}

fun vec4 vec4_plus(
  vec4 {a_1, ... , a_4},
  vec4 {b_1, ..., b_4}
) = {a_1 + b_1, ... , a_4 + b_4}
```

Let us try compartmentalizing a vector type and its functions into a structure. In the following example, we have defined two different modules, each containing a structure, and a futhark program which includes and utilizes these modules:

```
Vec3Float.fut:
  structure Vec3Float =
    struct
      type vector = {f32, f32, f32}
      fun vector plus( ... ) = ...
      fun vector minus( ... ) = ...
      fun vector multiply ( ... ) = ...
    end

Vec4Float.fut:
  structure Vec4Float =
    struct
      type vector = {f32, f32, f32, f32}
      fun vector plus( ... ) = ...
      fun vector minus( ... ) = ...
      fun vector multiply ( ... ) = ...
    end
```

```

myprogram.fut:

include Vec3Float
include Vec4Float

type vec3 = Vec3Float.vector
type vec4 = Vec4Float.vector

fun vec4 foo(vec3 vector) =
  let {a, b, c} = Vec3.plus(vector, vector)
  in Vec4.multiply({a, b, c, 1.0f} , 4.0f)

```

Whilst it *is* possible to create libraries without a module implementation <sup>6</sup>, the user runs a risk of running into errors like `MulipleDefinitionError`<sup>7</sup>, if any of the library functions uses any of the names, that the local user is using as well.

The module system removes this hazard, as application of functions and types are done using **longnames**, which adds prefixes to names. This way, functions can have the same name, as long as they do not share the same prefix.

#### Longnames:

A longname consists of any amount of prefixes followed by a dot, followed by the string id of the desired function or type:

$$\begin{array}{rcl}
 \textit{LongName} : & & \textit{prefix}.\textit{LongName} \\
 | & & \textit{identifier}
 \end{array}$$

In Futhark we will be using string ids for declarations, and longnames for the accessing types and functions in structures.

## 4.1 Accessing types and functions within structures

To work with type aliases and modules, we need to define the internal environment of Futhark during compile time.

Before starting this project, the environment of Futhark could be described like this:

$\Gamma = (FE)$  , where  $FE$  is a function environment, mapping function ids to functions:  $\{funid \rightarrow funexpr\}$ .

It is the goal of this project to expand the environment of Futhark, so that

$\Gamma = (FE, TE, SIGE, STRUCTE, FUNCTE)$ , where the additions are a type alias environment, a signature environment, a structure environment and a functor environment.

<sup>6</sup>By including libraries that adds functions to the top level environment

<sup>7</sup>Multiple functions of same name defined

Type aliases, signatures, structures and functors are described in their respective sections. A structure can be regarded as a structure name and an environment contained in the structure, so that  $\{strid \rightarrow \Gamma_{strid}\}$  is a mapping in *STRUCTE*.

$$\frac{\Gamma \vdash fundecl \Rightarrow \Gamma' \quad \Gamma \not\vdash fun\_id \rightarrow funexpr}{\Gamma \vdash fun \ fun\_id = funexpr \Rightarrow \{\{fun\_id \rightarrow funexpr\}, \emptyset, \emptyset, \emptyset, \emptyset\} \oplus \Gamma}$$

iff the function the function body is otherwise well-formed<sup>a</sup>.

<sup>a</sup>I will not go further into the functionality of functions, as it is not within scope of this project.

Figure 1: Rule for adding a function to the environment

$$\frac{\Gamma \vdash Decl_1 \Rightarrow \Gamma' \quad \Gamma \not\vdash id \rightarrow decltype \ Decl_2}{\Gamma \vdash decltype \ id = decl \Rightarrow \{id \rightarrow Decl\}}$$

Any  $\Gamma_{strid}$  contains its own function-, typealias-, signature-, structure- and functor declarations.

## 4.2 Interference rules

Now that we can discern between the global environment, and any number of environments in structures, we must redefine the environment behaviour of Futhark at compile time.

After expanding the environment to contain the four new elements, we can define the addition of any of these four elements follow a generalized rule: Please note, that several declarations can actually share name  $id$ , as long as they don't share declaration type.

### 4.2.1 Interference rules for adding multiple declarations

Finally, we want to create a rule for which environment we get, when we state multiple declarations in a row. This rule is covers the run of an entire program, since we can set the initial declaration in the program as  $Decl_1$ , use rule ?? to handle the first declaration in the pair  $Decl_1$  and  $Decl_2$ , and afterwards using the same rule again on  $Decl_2$ ,  $Decl_3$ .

This procedure is repeated until the rule reaches the two last statements,  $Decl_{n-1}$ ,  $Decl_n$ .

$$\frac{\Gamma \vdash Decl_1 \Rightarrow \Gamma_1 \quad \Gamma \oplus \Gamma_1 \vdash Decl_2 \Rightarrow \Gamma_2}{\Gamma \vdash Decl_1, Decl_2 \Rightarrow \Gamma_1 \oplus \Gamma_2}$$

For any  $\Gamma$ ,  $\Gamma_a \oplus \Gamma_b \Rightarrow \Gamma_a \cup \Gamma_b$

### 4.3 Interference rules for structure definitions

We must take a closer look at what happens, when a structure is defined. The definition of a structure in Futhark has the following behaviour.

#### 4.3.1 Rule for adding a structure to the local environment

$$\frac{\Gamma \vdash \text{strdecl} \Rightarrow \Gamma' \quad \Gamma \vdash \text{strdecls} \Rightarrow \Gamma_{\text{strdecls}} \quad \Gamma \oplus \Gamma_{\text{strdecls}} \vdash \Gamma_{\text{strid}}}{\Gamma \vdash \text{struct } \text{strid} \{ \text{strdecls} \} \Rightarrow \{ \text{strid} \rightarrow \Gamma_{\text{strid}} \}}$$

where  $\Gamma \oplus \Gamma_{\text{strdecls}} \Rightarrow \Gamma \cup \Gamma_{\text{strdecls}}$ . If there is a shared identifier in both  $\Gamma$  and  $\Gamma_{\text{strdecls}}$ , the definition in  $\Gamma_{\text{strdecls}}$  is the definition in  $\Gamma_{\text{strid}}$ .

The consequence of defining  $\Gamma_{\text{strid}}$  as a union of new declarations together with the old environment means, that Futhark exhibits variable shadowing, where definitions in a structure are valid in its nested structures, unless they are redefined in the nested structure.

### 4.4 Interference rules for interpreting functions and types in an environment with structures

There are three cases where it is necessary to resolve a function or a type from a longname:

1) When applying a function as an expression in a function expression; i.e.

```
let myNumber = Constants.numberFour()
```

2) When using a function as an argument in a currying function; i.e.

```
let numbers = [1, 2, 3, 4] in
let sum = reduce(MathLib.plus , 0 , numbers)
```

3) When using a type definition from a structure; i.e.

```
type int_pair = Pairs.Int.t
```

We can define the interference rule for using a longname in the three different cases. As all three cases handles resolving a longname the same way, the rule will only be called for the first case:

### 4.5 Implementation

A program in the Futhark type checker is defined as a list of unchecked declarations. I will not describe the process in details, but will instead explain specific parts of the code. I will specifically focus on parts of the code, that corresponds to the interference rules described earlier.

$$\frac{\Gamma \vdash \text{longname} \rightarrow \text{fun\_definition}}{\Gamma \vdash \text{let } x = \text{longname}() \Rightarrow x := \downarrow \text{longname}}$$

where

$$\downarrow \text{longname} = \text{returngetFromEnv}(\text{longname}, \Gamma)$$

#### 4.5.1 The Scope datatype

To describe the environment in the Futhark compiler, we use the Scope datatype:

```
data Scope = Scope { envVtable :: HM.HashMap VName Binding
                    , envFtable :: HM.HashMap Name FunBinding
                    , envTatable :: HM.HashMap Name TypeBinding
                    , envModTable :: HM.HashMap Name Scope
                    , envBreadcrumb :: LongName
                    }
```

Given  $\Gamma = (\text{Functions}, \text{TypeAliases}, \text{Structures})$ , then *Functions* is implemented in `envFtable`, *TypeAliases* in `envTatable` and *Structures* in `envModTable`.

#### 4.5.2 checkProg

The type check is initiated in the functions `checkProg` and `checkProg'`. `checkProg'` checks the initial top level of declaratations for duplicates.

```
checkProg :: UncheckedProg -> Either TypeError (Prog, VNameSource)
checkProg prog = do
  checkedProg <- runTypeM initialScope src $ Prog <$> checkProg' (progDecs
    prog')
  return $ flattenProgFunctions checkedProg
  where
    (prog', src) = tagProg' blankNameSource prog
\\
checkProg' :: [DecBase NoInfo VName] -> TypeM [DecBase Info VName]
checkProg' decs = do
  _ <- checkForDuplicateDecs decs
  (_, decs') <- checkDecs decs
  return decs'
```

---

#### 4.5.3 Checking for duplicates

As described in rule ??, we allow multiple declarations of same name, as long as they don't share type: In example, checking for two function definitions of the same type is done in the first case of `checkForDuplicateDecs`:

```

    fun int four() = 4
  \
  structure M0 =
    struct
      fun int double(int a) = a + a
    end
  \
  fun int main() =
    let x = four() in
      M0.double(x, x)

```

---

four () does not have M0 available in its local environment, but main () does, because M0 HAS already been parsed, at the point where main () is declared

---

```

checkForDuplicateDecs :: [DecBase NoInfo VName] -> TypeM ()
checkForDuplicateDecs decs = do
  _ <- foldM_ f HM.empty decs
  return ()
  where
    f known dec@(FunOrTypeDec (FunDec (FunDef _ (name,_) _ _ _ loc))) =
      case HM.lookup name known of
        Just dec'@(FunOrTypeDec (FunDec FunDef{})) ->
          throwError $ DupDefinitionError name loc $ decLoc dec'
        _ -> return $ HM.insert name dec known

```

---

## 4.6 Dividing a Futhark program into chunks

To facilitate variable shadowing, it was necessary to split a parsed Futhark program into chunks.

A structure declaration alters the environment drastically, by enabling the following program declarations following the structure to access the environment of the structure. Therefore we divide any list of declaration into type- and function declarations, and structure declarations. An example is given in figure??

This chunking is done recursively on a list of declarations:

```

chompDecs :: [DecBase NoInfo VName]
          -> ([FunOrTypeDecBase NoInfo VName], [DecBase NoInfo VName])
chompDecs decs = f ([], decs)
  where f (foo , FunOrTypeDec dec : xs ) = f (dec:foo , xs)
        f (foo , bar) = (foo, bar)

```

---

## 4.7 Checking function- and type declarations

```

1 checkDecs :: [DecBase NoInfo VName] -> TypeM (Scope, [DecBase Info VName])
2 checkDecs decs = do
3   \

```

```

4      let (funOrTypeDecs, rest) = chompDecs decs
5      scopeFromFunOrTypeDecs <- buildScopeFromDecs funOrTypeDecs
6      local (const scopeFromFunOrTypeDecs) $ do
7          checkedeDecs <- checkFunOrTypeDec funOrTypeDecs
8          (scope, rest') <- checkDecs rest
9          return (scope , checkedeDecs ++ rest')
10  \\\
11  checkDecs [] = do
12      scope <- ask
13      return (scope, [])

```

---

- 1) The current chunk of declarations is built using `chompDecs`.
  - 2) A scope is built from this chunk, by using the three inference rules in section ??.
  - 3) The scope from step 2 is now the local scope in the following function execution.
  - 4) `checkFunOrTypeDec` does the actual typechecking of the function declaration.
  - 5) The remainder of the declarations chunked in `chompDecs` is checked using `checkDecs`.
- Note, that the first element of `rest` must be a structure declaration, due to the implementation of `chompDecs`.

## 4.8 Checking structure declarations

Please note that structures are currently called modules inside Futharks compiler.

```

checkDecs (ModDec modd:rest) = do
    (modscope, modd') <- checkMod modd
    local (addModule modscope) $
    do
        (scope, rest') <- checkDecs rest
        return (scope, ModDec modd' : rest' )

```

---

When `checkDecs` encounters a `ModDec`, `checkMod` is called to resolve the `ModDec`. The `ModDec` defines an environment of functions and type declarations called `modscope`, which we add to the local environment by calling `local (addModule modscope)`

This part of the code is the implementation of rule??.

```

checkMod :: ModDefBase NoInfo VName -> TypeM (Scope , ModDefBase Info VName)
checkMod (ModDef name decs loc) =
    local ('addBreadcrumb' name) $ do
        _ <- checkForDuplicateDecs decs
        (scope, decs') <- checkDecs decs
        return (scope, ModDef name decs' loc)

```

---

`checkMod` first adds a “breadcrumb” of the structure’s name (*strid*) to the local environment. This is where the transformation in rule ?? of  $\Gamma_{strdecls} \Rightarrow \Gamma_{strid}$  is implemented. After the environment has been given its name, the internal declaration of the structure is read using `checkDecs`.

## 4.9 Resolving the application of a longname

Resolving the application for a longname is done through two short recursive functions. Figure ?? shows the implementation of the longname rule ??:



```

type LongName = ([Name], name)

typeFromScope :: LongName -> Scope -> Maybe TypeBinding
typeFromScope (prefixes, name) scope = do
  scope' <- envFromScope prefixes scope
  let taTable = envTTable scope'
  HM.lookup name taTable

funFromScope :: LongName -> Scope -> Maybe FunBinding
funFromScope (prefixes, name) scope = do
  scope' <- envFromScope prefixes scope
  let taTable = envFtable scope'
  HM.lookup name taTable

envFromScope :: [Name] -> Scope -> Maybe Scope
envFromScope (x:xs) scope =
  case HM.lookup x (envModTable scope) of
    Just scope' -> envFromScope xs scope'
    Nothing -> Nothing
envFromScope [] scope = Just scope

```

---

#### 4.9.1 Including structures, functions and types from other files

The Futhark `include`-statement lets the user include other files into the current program.

This is implemented by letting the Futhark compiler combine the source code from all the included files, with the declarations in the main program. The combined program is then sent passed on through the rest of the compiler.

As the Futhark program with an arbitrary number of includes is merged into a single program within the compiler, we can compartmentalize a program into discrete files.

However, this creates a hazard: there might be declarations in the included code, which has names that overlap with names in the remaining code, so that a `DuplicateDefinitionError` is triggered. the included code has any declarations that results in a duplicate definition error. Refer to ?? for a possible future solution.

#### 4.9.2 Keeping track of function names

Futharks variable shadowing made it necessary to extend the type of the function declaration type.

Initially, a function had only the declared name of the function as an identifier. However, it was necessary to extend this name into a pair:

```

type FunName = (declaredName :: Name, expandedName :: LongName).

```

When a function is added to the function table during `buildFtable`, it is added to the function table together with it's longname. The function's longname contains the function's name, and the name of the (potentially) nested structures it was defined in.

## 4.10 Tests

To verify that the implementation of structures was correct, a series of test programs were written to verify, whether the inference rules defined for the structures, actually held in an executed Futhark program.

Having writting the structure implementation myself, I have been able to consider my testing options. I have had the choice of writing either *white box* tests, *black box* tests, or both.

In the context of these futhark structures, white box tests are defined as tests that are designed by a programmer, who has intricate knowledge of the code implementation of the program features, he is supposed to test.

I.e., this means trying to write programs, which tests whether there can be ambiguities in the parsing of the program, or other general errors in the code.

Black box tests are written without knowledge of the code behind the program. In the context of Futhark structures, this limits the test writer to test, whether the rules which are specified in 4.2 holds in the actual implementation.

In the following tests, programs which break the rules are expected to return with an error.

### 4.10.1 Testing rule for multiple declarations [...]

The following tests are implemented to test whether rule ?? holds. `duplicate_def0.fut`:

```
-- This test is written to ensure that the same name can be used
-- for different declarations in the same local environment, as long as their
-- types does not overlap.
-- ==
-- input { 4 }
-- output { 4 }
```

```
type foo = int
fun foo foo(int a) = a + a
struct foo
{
  fun int one() = 1
}
```

```
fun int main(int x) = x
```

---

This test passes.

In the following test, the structure `foo` contains declarations of name `foo` also. This is a white box test to ensure, that the implementation handles `type`, `struct` and `fun` declarations in separete tables. `duplicate_def1.fut`:

```
-- The struct opens a new environment, which lets us use names again, which
-- were used
-- in a previous scope.
-- ==
```

```
-- input { }
-- output { (1 , 2.0) }
```

```
type foo = int
struct foo
{
  fun int foo() = 1
  struct foo
  {
    type foo = float
    fun foo foo() = 2.0
  }
}
```

```
fun (foo, foo.foo.foo) main() = ( foo.foo() , foo.foo.foo())
```

---

This test passes.

In the following tests, the programmer has attempted to declare functions and types in the same environment, twice.

duplicate\_error0.fut:

```
-- This test fails with a DuplicateDefinition error.
-- ==
-- error: .*Dup.*
```

```
fun int bar() = 1
struct foo
{
  fun foo foo() = 1
}
fun int bar() = 2

fun int main() = 0
```

---

This test passes.

In the following test, the structure foo contains declarations of name foo also. duplicate\_error1.fut:

```
-- This test fails with a DuplicateDefinition error.
-- ==
-- error: .*Dup.*
```

```
type foo = int

struct foo
{
  fun foo foo() = 1
}
type foo = float

fun int main() = 0
```

---

This test passes.

#### 4.10.2 Testing structures can be called as expected

The following tests are implemented to test that calling structures works as defined in rule ??.

```
calling_nested_module.fut:

-- ==
-- input {
--   10 21
-- }
-- output {
--   6
-- }

type t = int
struct NumLib {
  fun t plus(t a, t b) = a + b
  struct BestNumbers
  {
    fun t four() = 4
    fun t seven() = 42
    fun t six() = 41
  }
}

fun int localplus(int a, int b) = NumLib.plus (a,b)

fun int main(int a, int b) =
  localplus (NumLib.BestNumbers.four() , 2)
```

---

This test passes.

Currying functions such as map and reduce works as expected:

map\_with\_structure0.fut:

```
-- Testing whether it is possible to use a function
-- from a struct in a curry function (map)
-- ==
-- input {
--   [1, 2, 3 ,4, 5, 6, 7, 8, 9, 10]
-- }
-- output {
--   55
-- }

struct f {
  fun int plus(int a, int b) = a+b
}

fun int main([int] a) = reduce(f.plus , 0 , a)
```

---

This test passes.

Using structures from an include works as expected:

Vec3.fut:

```

-- ==
-- tags { disable }
struct Vec3
{
    struct F32
    {
        type t = ( f32 , f32 , f32 )
        fun t add(t a , t b) =
            let (a1, a2, a3) = a in
            let (b1, b2, b3) = b in
            (a1 + b1, a2 + b2 , a3 + b3)

        fun t subtract(t a , t b) =
            let (a1, a2, a3) = a in
            let (b1, b2, b3) = b in
            (a1 - b1, a2 - b2 , a3 - b3)

        fun t scale(f32 k , t a) =
            let (a1, a2, a3) = a in
            (a1 * k, a2 * k , a3 * k)

        fun f32 dot(t a , t b) =
            let (a1, a2, a3) = a in
            let (b1, b2, b3) = b in
            a1*b1 + a2*b2 + a3*b3
    }

    struct Int
    {
        type t = ( int , int , int )
        fun t add(t a , t b) =
            let (a1, a2, a3) = a in
            let (b1, b2, b3) = b in
            (a1 + b1, a2 + b2 , a3 + b3)

        fun t subtract(t a , t b) =
            let (a1, a2, a3) = a in
            let (b1, b2, b3) = b in
            (a1 - b1, a2 - b2 , a3 - b3)

        fun t scale(int k , t a) =
            let (a1, a2, a3) = a in
            (a1 * k, a2 * k , a3 * k)

        fun int dot(t a , t b) =
            let (a1, a2, a3) = a in
            let (b1, b2, b3) = b in
            a1*b1 + a2*b2 + a3*b3
    }
}

}

-- ==
-- input {
--     (10, 21, 21) (19, 12, 5)

```

```

-- }
-- output {
--   547
-- }

include Vec3

type vec3 = Vec3.Int.t
fun int main(vec3 a, vec3 b) = Vec3.Int.dot(a , b)

```

---

This test passes.

### 4.10.3 Testing rule for variable shadowing

The following tests are implemented to test whether the rule ?? holds: Simple shadowing for functions holds:

shadowing0.fut:

```

-- M1.foo() calls the most recent declaration of number, due to M0.number()
-- being brought into scope of M1, overshadowing the top level definition of
-- number()
-- ==
-- input {
-- }
-- output {
--   2
-- }

fun int number() = 1
struct M0
{
  fun int number() = 2
  struct M1
  {
    fun int foo() = number()
  }
}

fun int main() = M0.M1.foo()

```

---

Simple shadowing for types holds:

shadowing1.fut:

```

-- M1.foo() calls the most recent declaration of number, due to M0.number()
-- being brought into scope of M1, overshadowing the top level definition of
-- number()
-- ==
-- input {
-- }
-- output {
--   (6.0, 6, 6)
-- }

type best_type = float

```

```

fun best_type best_number() = 6.0
struct M0
{
  type best_type = int
  fun best_type best_number() = 6
  struct M1
  {
    fun best_type best_number() = 6
  }
}

fun (float, int, int) main() = (best_number() , M0.best_number() , M0.M1.
  best_number())

```

---

This test shows, that structures are only read into scope, after they are fully parsed. shadowing2.fut:

```

-- M0.foo() changes meaning inside M1, after the previous declaration of M0
-- is overshadowed.
--
-- ==
-- input {
-- }
-- output {
-- 12
-- }

struct M0
{
  fun int foo() = 1
}

struct M1
{
  fun int bar() = M0.foo()
  struct M0
  {
    fun int foo() = 10
  }
  fun int baz() = M0.foo()
}

fun int main() = M0.foo() + M1.bar() + M1.baz()

```

---

and undefined\_structure\_err0.fut:

```

-- We can not access a struct before it has been defined.
-- ==
-- error: .*Unknown.*

fun int try_me() = M0.number()
struct M0
{
  fun int number() = 42
}

```

```
fun int main() = try_me()
```

---

## 4.11 Results

The implementation of structures works, and I am satisfied with the results. As shown in the tests, the structures behave as prescribed in the rules4.2.

To answer the problem definition, it is definitely possible to add module functionality to Futhark, in terms of structure support.

## 4.12 Future work

As mentioned in ??, it is possible to trigger a `DuplicateDefinitionError` from including files into the main program.

Given more time, I would like to extend the Futhark `include` statement to support an `as` statement, so that the inclusion

```
include some_module as M0
```

will include the declarations from `some_module` into the futhark program, but not into the top level declarations. Instead, the declarations will be loaded into a structure `M0`, which can then be accessed throughout the rest of the program as any other module.

Another potential effort worth of mentioning is the commence the definition of a standard library for futhark. Futhark already has built-ins mathematical functions like `log10(float a)`, so defining a standard library of structures which define i.e. Integer-functions and Float-functions, could be the next step.



## 5 Signatures

Just like a type signature of a function is the definitions of the argument- and return types of a function, we can define module signatures similarly:

A module signature specifies the type signatures for a set of declarations.

Let us define a signature called `hasPlus`:

```
sig hasPlus {  
  type t  
  val plus : (t,t) -> t  
}
```

---

Then, after this declaration, we can define some structure, and append this structure declaration with a signature:

```
include hasPlus  
  
struct M0 {  
  ...  
} : hasPlus
```

---

With this signature, we promise that the structure `M0` matches the declarations in signature `hasPlus`. In the instance of `M0`, we now require, that `M0` has a declaration, where alias `t` is assigned. `M0` must also have a function, that takes two arguments of type `t`, and returns a value of type `t`.

In the following example, two structures are written. `M0` adheres to the specifications in `hasPlus`, but `M1` does not, which ultimately results in an error at compile time:

```
include hasPlus  
  
struct M0 {  
  type t = int  
  fun t plus(t a, t b) = a + b  
} : hasPlus  
  
struct M1 {  
  type t = (int, int)  
  fun int plus(t tuple) =  
    let (a, b) = tuple  
    in a + b  
} : hasPlus
```

---

### 5.1 Purpose of module signatures

By themselves, the ability to add a signature to a structure does not add any significant usefulness to a language. Signatures are useful together with *functors*??, so I will refrain from doing further explanation outside of functor context.

## 5.2 Implementation

*This section is not complete, as I have not implemented a complete and working signature system at this point.*

We need a data type which represents a signature. I chose to define it as a table from some declaration name to a signature binding:

```
type Signature = HM.HashMap Name SigBinding

data SigBinding = FunSignature (Maybe (TypeDeclBase Info VName)) (TypeDeclBase
  Info VName)
    -- ^ A function may take arguments, but might also be a
    -- constant.
    -- It always has a return type.

    | TypeSignature (TypeDeclBase Info VName)
    -- A type signature must resolve to a type.

    | ModSignature LongName
    -- A structure of signature sig, can itself be demanded to
    -- contain a structure of some ModSignature
```

---

With the Signature defined, we need to extend the data type of the structure to accommodate the possibility of adding a signature to a structure:

```
data ModDefBase f vn = ModDef { modName :: Name
    , modDecls :: [DecBase f vn]
    , modDefLocation :: SrcLoc
    , modSignature :: Maybe LongName
  }
```

---

The parsed structure from the futhark program now has either `modSignature Nothing`, or `modSignature (Just signature)`.

We update `checkMod` from 4.8 accordingly, by calling an extra function `verifyMod`:

```
checkMod :: ModDefBase NoInfo VName -> TypeM (Scope , ModDefBase Info VName)
checkMod (ModDef name decs loc sig) =
  local ('addBreadcrumb' name) $ do
    checkForDuplicateDecls decs
    (scope, decs') <- checkDecls decs
    verifyMod decs' sig loc
    return (scope, ModDef name decs' loc sig)

verifyMod :: [DecBase Info VName] -> Maybe LongName -> TypeM ()
verifyMod _ Nothing _ = return ()
verifyMod decs (Just sig) loc = do
  sigtable <- asks (sigFromScope sig)
  case sigtable of
    Nothing -> throwError $ UndefinedLongName sig loc
    Just t ->
      foldM_ verifyDec decs $ HM.toList t
```

---

For each entry in the signature table, we call `verifyDec` on this entry to verify, that decs contain a declaration that satisfies the signature table.

As I have not implemented `verifyDec` completely yet, this implementation is not finished.

### 5.3 Results

As the signature implementation is *incomplete*, I cannot claim that it is possible to implement module signatures in Futhark, by referring to tests.

However, the current implementation lacks **only** a correct implementation of `verifyDec` before it works, barring some corrections of bugs that might show up during tests. The ongoing work on this feature can be followed on the `signatures-branch` in the public Futhark GitHub repo.

To answer the question whether it is possible to implement module signatures in Futhark: Yes, it is.

## 6 Functors

A function takes a number of arguments of specified types, and returns a value of some return type. A **functor** takes a structure?? of a specified signature, and returns a structure of some signature.

The concept is best explained through an example:

```
--
--==
-- input {}
-- output { 4 }

sig NumType {
  type t
  val add : (t,t) -> t
  val subtract : (t,t) -> t
  val divide : (t,t) -> t
  val multiply : (t,t) -> t
}

struct Int {
  type t = int
  fun t add(t a, t b) = a + b
  fun t subtract(t a, t b) = a - b
  fun t divide(t a, t b) = a / b
  fun t multiply(t a, t b) = a * b

} : NumType

functor Doubler(number : NumType){
  type t = number.t
  fun t add(t a, t b) = number.add
  fun t double(t a) = add(a,a)
}

struct IntDoubler = Doubler(Int)

fun int main() = IntDoubler.double(2)
```

---

We first declare a signature `NumType`, which defines the number type. We declare it so that there is a type `t`, that has four standard operations (add, subtract, divide and multiply.).

We then declare a structure `Int`, which adheres to the `NumType` signature: The `add` function in `Int` uses the built-in functionality of the plus operator in Futhark, but could in principle be defined as whatever the programmer would like *as long as* the type checks correctly.

Then, the functor is defined: The functor takes a structure as an argument, and can then refer to this structure in the declarations of the functor, using dot-notation. The functor contains a list of type-, function- and structure declarations, just like the structure does. However, none of these declarations has to be defined inside the functor:

Any of the declarations can be loaded from the functor's argument instead!

In the example above, the `Doubler` functor contains a function called `double`. But

`double` is not defined inside the functor, but will instead become whatever the `double` function is defined as, inside the `number` argument of the functor. At the application of the `Doubler` functor, creating `IntDoubler`, we can guarantee, that our types still hold. This is because the function definitions in the functor has an abstract type<sup>8</sup>, which is not instantiated until the functor can get the type definition from the `structure` argument.

To boot, we can even bind a signature to functor if we want to. In that case, we will have to create a functor, that returns a structure of signature *sig*, when it is applied.

## 6.1 The reason for functors

## 6.2 Implementing functors

Implementing a simple but

---

<sup>8</sup>type `t`

## **7 Final section**

### **7.1 Discussion of results**

### **7.2 Discussion of method**

### **7.3 Conclusion**

### **7.4 Acknowledgements**

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Language part	language construct	definition	example
Core:	dec ::=	<i>type t = type_def</i>	<i>(* type Status = int *)</i>
		<i>fun f args = exp</i>	<i>(* fun foo n = n + 5 *)</i>
		<i>val x = exp</i>	<i>(* val eleven = 11 *)</i>
Module:	topdec ::=	<i>sigdec</i>	
		<i>moddec</i>	
		<i>topdec topdec</i>	
	sigdec ::=	signature <i>X = sigexp</i>	<i>(* signature foo = ... *)</i>
	sigexp ::=	sig <i>sigspec</i> end	<i>(* sig val bar : int end *)</i>
		<i>X</i>	<i>(*{foo, bar, One, ... }*)</i>
	sigspec	val <i>x</i> : type	<i>(* val bar : string *)</i>
		module <i>X</i> : <i>sigexp</i>	<i>(* module Bar : Numberable *)</i>
		sigspec sigspec	
	moddec ::=	<i>dec</i>	
		module <i>X</i> = <i>modexp</i>	<i>(* module One = ... *)</i>
		moddec moddec	<i>(* val x = 1 val y = 2 *)</i>
	modexp ::=	struct <i>moddec</i> end	<i>(* struct module Adder = PlusOp val one = 1 end *)</i>
		<i>modexp</i> : <i>sigexp</i>	<i>(* struct val bar = 1 end : foo *)</i>
		<i>X</i>	<i>(* {One, Numberable, Counter, Queue, ... } *)</i>
Module access:	LongIdent ::=	<i>module.field</i>	<i>(* One.bar *)</i>
		<i>module.LongIdent</i>	<i>(* Numberable.One.bar *)</i>

-- N-body simulation based on the one from Accelerate:

```

-- https://github.com/AccelerateHS/accelerate-examples/tree/master/examples/n-
  body
--
-- Type descriptions:
--
-- type mass = f32
-- type position = {f32, f32, f32}
-- type acceleration = {f32, f32, f32}
-- type velocity = {f32, f32, f32}
-- type body = (position, mass, velocity, acceleration)
--           =~ {f32, f32, f32, -- position
--               f32,           -- mass
--               f32, f32, f32, -- velocity
--               f32, f32, f32} -- acceleration
--
fun {f32, f32, f32}
  vec_add({f32, f32, f32} v1,
          {f32, f32, f32} v2) =
  let {x1, y1, z1} = v1
  let {x2, y2, z2} = v2
  in {x1 + x2, y1 + y2, z1 + z2}

fun {f32, f32, f32}
  vec_subtract({f32, f32, f32} v1,
               {f32, f32, f32} v2) =
  let {x1, y1, z1} = v1
  let {x2, y2, z2} = v2
  in {x1 - x2, y1 - y2, z1 - z2}

fun {f32, f32, f32}
  vec_mult_factor(f32 factor,
                  {f32, f32, f32} v) =
  let {x, y, z} = v
  in {x * factor, y * factor, z * factor}

fun f32
  dot({f32, f32, f32} v1,
      {f32, f32, f32} v2) =
  let {x1, y1, z1} = v1
  let {x2, y2, z2} = v2
  in x1 * x2 + y1 * y2 + z1 * z2

fun {f32, f32, f32}
  accel(f32 epsilon,
        {f32, f32, f32} pi,
        f32 mi,
        {f32, f32, f32} pj,
        f32 mj) =
  let r = vec_subtract(pj, pi)
  let rsqr = dot(r, r) + epsilon * epsilon
  let invr = 1.0f32 / sqrt32(rsqr)
  let invr3 = invr * invr * invr
  let s = mj * invr3

```



```

in vec_mult_factor(s, r)

fun {f32, f32, f32}
  accel_wrap(f32 epsilon,
    {f32, f32, f32, f32, f32, f32, f32, f32, f32, f32} body_i,
    {f32, f32, f32, f32, f32, f32, f32, f32, f32, f32} body_j) =
  let {xi, yi, zi, mi, _, _, _, _, _, _} = body_i
  let {xj, yj, zj, mj, _, _, _, _, _, _} = body_j
  let pi = {xi, yi, zi}
  let pj = {xj, yj, zj}
  in accel(epsilon, pi, mi, pj, mj)

fun {f32, f32, f32}
  move(f32 epsilon,
    [{f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}] bodies,
    {f32, f32, f32, f32, f32, f32, f32, f32, f32, f32} body) =
  let accels = map(fn {f32, f32, f32} ({f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}
    , f32, f32) body_other) =>
    accel_wrap(epsilon, body, body_other),
    bodies)
  in reduceComm(vec_add, {0f32, 0f32, 0f32}, accels)

fun [{f32, f32, f32}]
  calc_accels(f32 epsilon,
    [{f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}] bodies) =
  map(move(epsilon, bodies), bodies)

fun {f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}
  advance_body(f32 time_step,
    {f32, f32, f32, f32, f32, f32, f32, f32, f32, f32} body) =
  let {xp, yp, zp, mass, xv, yv, zv, xa, ya, za} = body
  let pos = {xp, yp, zp}
  let vel = {xv, yv, zv}
  let acc = {xa, ya, za}
  let pos' = vec_add(pos, vec_mult_factor(time_step, vel))
  let vel' = vec_add(vel, vec_mult_factor(time_step, acc))
  let {xp', yp', zp'} = pos'
  let {xv', yv', zv'} = vel'
  in {xp', yp', zp', mass, xv', yv', zv', xa, ya, za}

fun {f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}
  advance_body_wrap(f32 time_step,
    {f32, f32, f32, f32, f32, f32, f32, f32, f32, f32} body,
    {f32, f32, f32} accel) =
  let {xp, yp, zp, m, xv, yv, zv, _, _, _} = body
  let accel' = vec_mult_factor(m, accel)
  let {xa', ya', za'} = accel'
  let body' = {xp, yp, zp, m, xv, yv, zv, xa', ya', za'}
  in advance_body(time_step, body')

fun [{f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}, n]
  advance_bodies(f32 epsilon,
    f32 time_step,

```

```

        [{f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}, n]
        bodies) =
    let accels = calc_accels(epsilon, bodies)
    in zipWith(advance_body_wrap(time_step), bodies, accels)

fun [{f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}, n]
    advance_bodies_steps(i32 n_steps,
        f32 epsilon,
        f32 time_step,
        [{f32, f32, f32, f32, f32, f32, f32, f32, f32, f32}, n]
        bodies) =
    loop (bodies) = for i < n_steps do
        advance_bodies(epsilon, time_step, bodies)
    in bodies

fun [{[f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [
    f32, n], [f32, n], [f32, n]}]
    main(i32 n_steps,
        f32 epsilon,
        f32 time_step,
        [f32, n] xps,
        [f32, n] yps,
        [f32, n] zps,
        [f32, n] ms,
        [f32, n] xvs,
        [f32, n] yvs,
        [f32, n] zvs,
        [f32, n] xas,
        [f32, n] yas,
        [f32, n] zas) =
    let bodies = zip(xps, yps, zps, ms, xvs, yvs, zvs, xas, yas, zas)
    let bodies' = advance_bodies_steps(n_steps, epsilon, time_step, bodies)
    in unzip(bodies')

```

---

### **nbody using type aliases:**

```

type mass = f32
type vec3 = (f32, f32, f32)
type position, acceleration, velocity = vec3

type body = (position, mass, velocity, acceleration)

fun vec3 vec_add(vec3 v1, vec3 v2) =
    let (x1, y1, z1) = v1
    let (x2, y2, z2) = v2
    in (x1 + x2, y1 + y2, z1 + z2)

fun vec3 vec_subtract(vec3 v1, vec3 v2) =
    let (x1, y1, z1) = v1
    let (x2, y2, z2) = v2
    in (x1 - x2, y1 - y2, z1 - z2)

fun vec3 vec_mult_factor(f32 factor, vec3 v) =
    let (x, y, z) = v
    in (x * factor, y * factor, z * factor)

```

```

fun f32 dot(vec3 v1, vec3 v2) =
  let (x1, y1, z1) = v1
  let (x2, y2, z2) = v2
  in x1 * x2 + y1 * y2 + z1 * z2

fun velocity accel(f32 epsilon, vec3 pi, f32 mi, vec3 pj, f32 mj) =
  let r = vec_subtract(pj, pi)
  let rsqr = dot(r, r) + epsilon * epsilon
  let invr = 1.0f32 / sqrt32(rsqr)
  let invr3 = invr * invr * invr
  let s = mj * invr3
  in vec_mult_factor(s, r)

fun vec3 accel_wrap(f32 epsilon, body body_i, body body_j) =
  let (pi, mi, _, _) = body_i
  let (pj, mj, _, _) = body_j
  in accel(epsilon, pi, mi, pj, mj)

fun position move(f32 epsilon, [body] bodies, body this_body) =
  let accels = map(fn acceleration (body other_body) =>
    accel_wrap(epsilon, this_body, other_body),
    bodies)
  in reduceComm(vec_add, (0f32, 0f32, 0f32), accels)

fun [acceleration] calc_accels(f32 epsilon, [body] bodies) =
  map(move(epsilon, bodies), bodies)

fun body advance_body(f32 time_step, body this_body) =
  let (pos, mass, vel, acc) = this_body
  let pos' = vec_add(pos, vec_mult_factor(time_step, vel))
  let vel' = vec_add(vel, vec_mult_factor(time_step, acc))
  let (xp', yp', zp') = pos'
  let (xv', yv', zv') = vel'
  in (pos', mass, vel', acc)

fun body advance_body_wrap(f32 time_step, body this_body, acceleration accel)
  =
  let (pos, mass, vel, acc) = this_body
  let accel' = vec_mult_factor(mass, accel)
  let body' = (pos, mass, vel, accel')
  in advance_body(time_step, body')

fun [body, n] advance_bodies(f32 epsilon, f32 time_step, [body, n] bodies) =
  let accels = calc_accels(epsilon, bodies)
  in zipWith(advance_body_wrap(time_step), bodies, accels)

fun [body, n] advance_bodies_steps(i32 n_steps, f32 epsilon, f32 time_step,
  [body, n] bodies) =
  loop (bodies) = for i < n_steps do
    advance_bodies(epsilon, time_step, bodies)
  in bodies

fun body wrap_body (f32 posx, f32 posy, f32 posz,

```

```

        f32 mass,
        f32 velx, f32 vely, f32 velz,
        f32 accx, f32 accy, f32 accz) =
    ((posx, posy, posz), mass, (velx, vely, velz), (accx, accy, accz))

fun (f32, f32, f32, f32, f32, f32, f32, f32, f32, f32) unwrap_body(body
    this_body) =
    let ((posx, posy, posz), mass, (velx, vely, velz), (accx, accy, accz)) =
        this_body
    in (posx, posy, posz, mass, velx, vely, velz, accx, accy, accz)

fun ([f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [
    f32, n], [f32, n], [f32, n])
main(i32 n_steps,
    f32 epsilon,
    f32 time_step,
    [f32, n] xps,
    [f32, n] yps,
    [f32, n] zps,
    [f32, n] ms,
    [f32, n] xvs,
    [f32, n] yvs,
    [f32, n] zvs,
    [f32, n] xas,
    [f32, n] yas,
    [f32, n] zas) =
    let bodies = map(wrap_body, zip(xps, yps, zps, ms, xvs, yvs, zvs, xas, yas,
        zas))
    let bodies' = advance_bodies_steps(n_steps, epsilon, time_step, bodies)
    let bodies'' = map(unwrap_body, bodies')
    in unzip(bodies'')

```

---