# Modules in Futhark

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# 0.1 Abstract

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# 1 Introduction

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# 1.1 Problem definition

Is it possible to implement a module system in Futhark, which displays features comparable to the module system implemented in Standard ML? [?]

# 2 Motivation

The implementation of a module system, will greatly expand the usability of Futhark, without having side effects on the performance of the compiled Futhark code. The following subsections contain the features this project is implementing, and examples of why these features are desirable.

To be able to claim, that modules increases usability, we must first setup some metrics of usability.

### Readability increases usability:

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Compartmentalization of functionality increases usability:

skriv mere her

A module system increases the ability to share code between developers. skriv mere her

# 2.1 Abstraction increases readability.

In example, type aliasing lets us abstract from the actual definition of data types. When writing our source code, we can define our type aliases before writing the rest of our program. If we want to define a sphere in a three dimensional space, we want to define it with 1) a radius, 2) a position, and 3) a direction that it is moving. Let us define a function that multiplies the speed of the sphere by a factor k.

With type aliasing, we can compartmentalize the data type, and remove the need for explicitally typing out every parameter of function input. Coupled with helper functions, we can now multiply the speed of the sphere like this:

```
type vec3 = {f32, f32, f32}
type position, direction = vec3
type radius = f32
type sphere = {radius, position, direction}

fun vec3 multiply_vector(vec3 {pos_x, pos_y, pos_z}, f32 k) =
    {k * pos_x , k * pos_y , k * pos_z}

fun sphere multiply_velocity(
    sphere {radius, position, direction}, f32 factor) =
        let new_direction = multiply_vector(direction, factor)
        in {radius, position, new_direction}
```

As we are now using the sphere type as the function argument, we can pattern match on the type aliased values contained in the sphere type. Most importantly, the vectors of the sphere are abstracized into a single variables instead of tuples.

The end result is a shorter, more readable program 3.5.4

# 2.2 Approximating higher order functionality whilst keeping performance

It is possible to express higher-order functionality in Futhark, without taking a performance hit in the compiled Futhark code. We will reiterate on the concept of modules4, by introducing the concept of functors. First we repeat the three-dimensional vector module from earlier, but without declaring any particular primitive 3.1 type as the contained type of the vector:

```
structure Vec3 =
  struct
  type vector = {t, t, t}
  fun vector add( ... ) = ...
  fun vector subtract( ... ) = ...
  fun vector multiply ( ... ) = ...
  fun vector divide ( ... ) = ...
end
```

The structure above cannot be used on its own. Type t is not instantiated, and the module cannot be type checked, which causes an error.

We can solve that problem, by instantiating the abstract structure, using a simple functor; the where-clause.

```
structure IntVec3 = Vec3 where type t = int
```

We can now access the structure IntVec3 throughout the rest of the program. The structure IntVec3 is Vec3, except all instances of type t in Vec3 is exchanged with type int.

**To recap:** functors allows us to define an abstract implementation of some structure **ONCE**, and lets us instantiate this structure any number of times, each time with our own type. This functionality will be elaborated on in a later section Functors??.

From a performance-concerned point of view, the module system is desirable. Every function in the written program, whether it is inside a structure or defined in the top level program, is ultimately accumulated into the same scope. 4.9.1

The entire program is then compiled, and the optimizations that makes FUTHARK GO FAST are applied to the included modules as well as the top level declarations.

# 2.3 Scope of project

The scope of the project is

- to implement a type aliasing system in Futhark
- to implement a module system in Futhark, which has:
  - The definition of structures containing types and functions
  - Nested modules; meaning that any structure can contain a structure
  - A well defined way of referring to types, structures and functions contained in a structure
- to implement functor functionality, so that Futhark supports the definition of abstract structures and concretizations of these structures.

#### 2.4 Success definitions

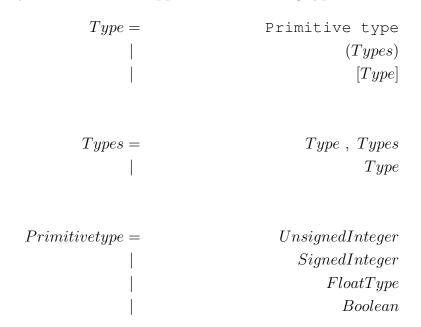
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# 3 Type aliases

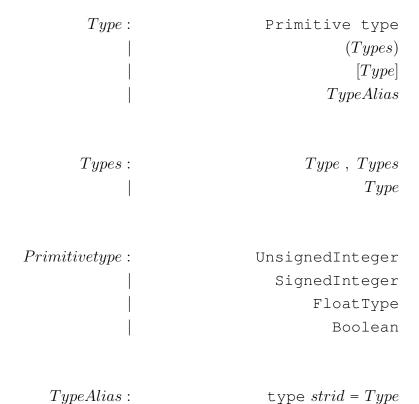
To make an implementation of functors?? in Futhark, it was necessary to implement type aliases first.

# 3.1 The language

The initial type system in Futhark supported the following type definitions:



Implementing type aliases expands our type constructions as following:



where  $type\_id$  is short for  $string\ id$ , a string identifier.

# 3.2 Interference rules

A type alias in a Futhark program is done like this:  $type\ Strids = Type$ , where Type is as defined in the grammar above.

Given an environment  $\Gamma: \{FE, TE\}, Types = \{strid \to Type\}, Type \to \tau^a$  we can define the following interference rules for using type aliases:

$$\frac{\Gamma \vdash typedecl \Rightarrow \Gamma' \qquad \Gamma \vdash Type \rightarrow \tau \qquad \Gamma \not\vdash type\_id \rightarrow Type}{\Gamma \vdash type \ type\_id = Type \Rightarrow \{\emptyset, \{type\_id \rightarrow Type\}\}}$$

where

$$INDS \not\equiv TUNION\{strid \rightarrow Type\} \oplus \Gamma$$
  
$$\Rightarrow Types(\Gamma) \leftarrow Types(\Gamma) \cup \{strid \rightarrow Type\}$$

iff the type alias declaration does not clash with the current environment. There is no clash, if the declaration obeys the three rules in the implementation subsection 3.3.

We can assign the same type to several string ids simultaneously, as long as we don't declare the same type alias twice.

$$\frac{\Gamma \vdash typedecl \Rightarrow \Gamma' \quad \text{Strids} = strid_1, ..., strid_n}{\Gamma \vdash \text{type} \ \textit{Strids} = \ \textit{Type} \Rightarrow \{strid_1 \rightarrow \textit{Type}, .., strid_n \rightarrow \textit{Type}\} \oplus \Gamma \Rightarrow \Gamma'\}}$$

where

$$\{strid_1 \to Type, \cdots, strid_n \to Type, \} \oplus \Gamma$$
  
$$\Rightarrow Types(\Gamma) \to Types(\Gamma) \cup \{strid_1 \to Type, \cdots, strid_n \to Type\}$$

# 3.3 Implementation

A type alias  $strid_i \to Type$  declaration is successful, if three rules are followed:

1. The alias  $strid_i$  is not already declared<sup>1</sup> in the current local environment. I.e. the example below:

type foo = 
$$i32$$
  
type foo =  $f32$ 

 $<sup>^</sup>a$ We will let au designate a primitive type in Futhark

<sup>&</sup>lt;sup>1</sup>This is by convention of only being able to define a value *once* 

2. The alias  $strid_i$  refers to a type (or a type alias), that is either already defined in the current environment (including structure environments), or is in the same declaration chunk as  $strid_i$ . In the example below, foo refers to bar, but bar is in the same chunk as foo.

Therefore, foo can be resolved by resolving bar. The implementation of this described in a later subsection 3.4.2.

```
type foo = bar
type bar = \{f32, i32\}
```

There are no hard limit to the number of type aliases that has to be checked, before a type alias is resolved:

```
type foo = bar
type bar = {f32, baz, i32}
type baz = [{bee, bang, boo}]
type boo ...

type bep = i32
```

Such a chain of type aliases is allowed, as long as the last of the three rules is obeyed:

3. The alias being resolved cannot be cyclically defined.

Imagine that some type type my\_type = foo is in the chain in the type aliasing example2 above.

In this case, the compiler is first trying to resolve foo by resolving bar, and trying to resolving bar by resolving baz, et cetera.

At some point, the compiler encounters my\_type, and must resolve foo to continue - which creates a cycle, because foo is resolved by bar, and so on.

To keep track of this, the compiler maintains the set of aliases that has been visited in the attempt to resolve some type alias. Every time another type alias has to be checked, the compiler first checks the set to find out, whether this alias is already on the list of aliases that needs to be resolved.

If so, the compiler returns an error. The implementation of this can be read here 3.3

# 3.4 Parsing a type alias

### 3.4.1 Data types for describing a type

Initially, the types parsed in a Futhark program were always represented as instances of the following datatype TypeBase:

These are also the data types that are used in the internal Futhark program.

Before this project, Futhark parsed types from Futhark source code directly to TypeBases. However, we decided to add an intermediate data type between raw source code and TypeBases, to make type aliases available.

The parser was changed, so type declarations in Futhark source would now be parsed as UserTypes and not typebases.

### 3.4.2 Adding resolved types to scope

```
1
     type TypeAliasTableM =
2
     ReaderT (HS.HashSet LongName) (StateT Scope TypeM)
3
4
   typeAliasTableFromProg :: [TypeDefBase NoInfo VName]
5
                           -> Scope
6
                           -> TypeM Scope
7
   typeAliasTableFromProg defs scope = do
8
     checkForDuplicateTypes defs
9
     execStateT (runReaderT (mapM_ process defs) mempty) scope
10
     where
            findDefByName name = find ((==name) . typeAlias) defs
11
12
13
           process :: TypeDefBase NoInfo VName
14
                    -> TypeAliasTableM (StructTypeBase VName)
           process (TypeDef name (TypeDecl ut NoInfo) _) = do
15
             t <- expandType typeOfName ut
16
17
             modify $ (addType name t)
18
             return t
19
20
           typeOfName :: LongName -> SrcLoc
21
                      -> TypeAliasTableM (StructTypeBase VName)
22
           typeOfName (prefixes, name) loc = do
23
              inside <- ask
24
             known <- get
25
             case typeFromScope (prefixes, name) known of
26
               Just t -> return t
27
               Nothing
28
                  | (prefixes, name) 'HS.member' inside ->
29
                      throwError $ CyclicalTypeDefinition loc name
30
                  | Just def <- findDefByName name ->
31
                      local (HS.insert (prefixes, name)) $ process def
32
                  | otherwise ->
33
                      throwError $ UndefinedAlias loc name
```

TypeAliasTableM is a monad stack that is used to resolve a list of type alias declarations in a declaration chunk.

It is a reader monad transformer that contains a state monad transformer, that contains the TypeM monad.

The reader monad is used to contain a HashSet as its environment. This environment is used to keep check of cyclical definitions as described in 3.

For each type alias, we use the function process to resolve the type, and modify the scope contained within the state monad of the ReaderT.

Resolving a type from a type aliasing is done using the function typeOfName in the code snippet3.4.2. typeOfName tries to resolve the type by name by retrieving the scope contained in the transformed State monad inside the reader.

If this is not immediately possible, we must either continue our search for the type, throw an error due to a cyclical type definition, or throw an error because the type alias has not been defined yet.

If our attempt to resolve  $strid_a$  leads to another alias  $strid_b^2$ , we add  $strid_a$  to our reader environment using the function local, and process  $alias_b$  instead.

**Acknowledgement**: The initial design of expandType and the addition of type aliases to the scope was initially much larger, but was reduced in size by Troels Henriksen, who rewrote the process to use monads, and reduced some code duplication.

### 3.4.3 Converting UserType to TypeBase

```
expandType :: (Applicative m, MonadError TypeError m) =>
2
                   (LongName -> SrcLoc -> m (StructTypeBase VName))
3
               -> UserType VName
               -> m (StructTypeBase VName)
4
5
6
   expandType look (UserTypeAlias longname loc) =
7
     look longname loc
8
   expandType _ (UserPrim prim _) =
9
     return $ Prim prim
10
   expandType look (UserTuple ts ) =
     Tuple <$> mapM (expandType look) ts
11
12
   expandType look (UserArray t d ) = do
     t' <- expandType look t
13
     return $ arrayOf t' (ShapeDecl [d]) Nonunique
14
   expandType look (UserUnique t loc) = do
15
16
     t' <- expandType look t
17
     case t' of
18
       Array{} -> return $ t' `setUniqueness` Unique
19
                -> throwError $ InvalidUniqueness loc $ toStructural t'
```

### 3.4.4 Why we added UserType instead of extending TypeBase

Adding UserType and then resolving these into TypeBases whilst running the program through the TypeChecker, removes the need of handling UserAliases after the type check, where these aliases are resolved.

Furthermore, not all information about a TypeBase declaration can actually be claimed already at program parse time. Some information about i.e. IKKE SIKKER HER array dimensionality in regards to aliased arrays, is decided within the type checker as well.

 $<sup>^{2}</sup>$ line 27-33

### 3.4.5 The slip from type aliases to realized types

Since every type alias is resolved in the type checker, the UserTypes are not used after the type check.

The data type for a type declaration is this:

An unresolved type looks like this:

```
TypeDecl userType NoInfo
```

After resolve, NoInfo has been filled out with a variable of type Info TypeBase, giving us the following TypeDecl:

```
TypeDecl usertype (Info typebase).
```

At any point after the type check, only the expandedType of TypeDecl is used.

### 3.5 Results

The addition of type aliases works without any issues. To verify this, Futhark has been tested to pass all of the tests in futharks test suite<sup>3</sup>.

However, it was also necessary to write tests to specifically confirm, that the implementation respects the rules defined in 3.3

### 3.5.1 A type cannot be defined twice in the same environment

From alias-error3.fut in futhark/src/data/tests/types:

```
-- You may not define the same alias twice.
--
-- ==
-- error: Duplicate.*mydup

type mydup = int
type mydup = f32

fun int main(int x) = x
```

This program fails as expected.

 $<sup>^3</sup>$ located in folder futhark/data/tests

# 3.5.2 A type alias cannot be defined, if it refers to a type alias that has not been defined

From alias-error4.fut in futhark/src/data/tests/types:

```
-- No undefined types!
--
-- ==
-- error: .*not defined.*

type foo = bar

fun foo main(foo x) = x

This program fails as expected.
```

## 3.5.3 A type alias cannot be cyclically defined

From alias-error5.fut in futhark/src/data/tests/types:

```
-- No tricky circular types!
--
-- ==
-- error: .*cycl.*

type t0 = [t1]
type t1 = {int, float, t2}
type t2 = t0

fun t1 main(t1 x) = x
```

This program fails as expected.

### 3.5.4 Example of planet simulations being simplified by type aliases

A nice example of the benefits of type aliasing, is the N-body simulation (nbody), which is a simulation over the n-body problem<sup>4</sup>.

The original Futhark implementation of the simulation contained function arguments of tuples with arity up to 10. Whilst it is still necessary to bring all the arguments throughout the program, type aliasing makes the program itself much more readable:

```
-- N-body simulation based on the one from Accelerate:
-- https://github.com/AccelerateHS/accelerate-examples/tree/master/examples/n-
-- Type descriptions:
-- type mass = f32
-- type position = {f32, f32, f32}
-- type acceleration = \{f32, f32, f32\}
-- type velocity = {f32, f32, f32}
-- type body = (position, mass, velocity, acceleration)
             =~ {f32, f32, f32, -- position
                 f32,
                                 -- mass
                 f32, f32, f32, -- velocity
                 f32, f32, f32} -- acceleration
fun {f32, f32, f32}
  vec_add({f32, f32, f32} v1,
               \{f32, f32, f32\} v2) =
  let \{x1, y1, z1\} = v1
  let \{x2, y2, z2\} = v2
  in \{x1 + x2, y1 + y2, z1 + z2\}
fun {f32, f32, f32}
  vec_subtract({f32, f32, f32} v1,
               \{f32, f32, f32\} v2) =
  let \{x1, y1, z1\} = v1
  let \{x2, y2, z2\} = v2
  in \{x1 - x2, y1 - y2, z1 - z2\}
fun {f32, f32, f32}
  vec_mult_factor(f32 factor,
                  \{f32, f32, f32\} v) =
  let \{x, y, z\} = v
  in {x * factor, y * factor, z * factor}
fun f32
  dot({f32, f32, f32} v1,
      \{f32, f32, f32\} v2) =
  let \{x1, y1, z1\} = v1
  let \{x2, y2, z2\} = v2
  in x1 * x2 + y1 * y2 + z1 * z2
```

<sup>4</sup>https://en.wikipedia.org/wiki/N-body\_problem

```
fun {f32, f32, f32}
 accel(f32 epsilon,
     {f32, f32, f32} pi,
     f32 mi,
     {f32, f32, f32} pj,
     f32 mj) =
 let r = vec_subtract(pj, pi)
 let rsqr = dot(r, r) + epsilon * epsilon
 let invr = 1.0f32 / sqrt32(rsqr)
 let invr3 = invr * invr * invr
 let s = mj * invr3
 in vec_mult_factor(s, r)
fun {f32, f32, f32}
 accel_wrap(f32 epsilon,
        let {xi, yi, zi, mi, _, _, _, _, _} = body_i
 let {xj, yj, zj, mj, _, _, _, _, _} = body_j
 let pi = \{xi, yi, zi\}
 let pj = \{xj, yj, zj\}
 in accel (epsilon, pi, mi, pj, mj)
fun {f32, f32, f32}
 move(f32 epsilon,
    , f32, f32} body_other) =>
              accel_wrap(epsilon, body, body_other),
            bodies)
 in reduceComm(vec_add, {0f32, 0f32, 0f32}, accels)
fun [{f32, f32, f32}]
 calc_accels(f32 epsilon,
         map(move(epsilon, bodies), bodies)
advance_body(f32 time_step,
          let \{xp, yp, zp, mass, xv, yv, zv, xa, ya, za\} = body
 let pos = \{xp, yp, zp\}
 let vel = \{xv, yv, zv\}
 let acc = \{xa, ya, za\}
 let pos' = vec_add(pos, vec_mult_factor(time_step, vel))
 let vel' = vec_add(vel, vec_mult_factor(time_step, acc))
 let \{xp', yp', zp'\} = pos'
 let \{xv', yv', zv'\} = vel'
 in {xp', yp', zp', mass, xv', yv', zv', xa, ya, za}
advance_body_wrap(f32 time_step,
```

```
\{f32, f32, f32\} accel) =
 let {xp, yp, zp, m, xv, yv, zv, _, _, _} = body
 let accel' = vec_mult_factor(m, accel)
 let \{xa', ya', za'\} = accel'
 let body' = \{xp, yp, zp, m, xv, yv, zv, xa', ya', za'\}
 in advance_body(time_step, body')
advance_bodies(f32 epsilon,
              f32 time_step,
              bodies) =
 let accels = calc_accels(epsilon, bodies)
 in zipWith(advance_body_wrap(time_step), bodies, accels)
advance_bodies_steps(i32 n_steps,
                   f32 epsilon,
                   f32 time_step,
                   bodies) =
 loop (bodies) = for i < n_steps do</pre>
   advance_bodies(epsilon, time_step, bodies)
 in bodies
fun {[f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [
  f32, n], [f32, n], [f32, n]}
 main(i32 n_steps,
     f32 epsilon,
     f32 time_step,
     [f32, n] xps,
     [f32, n] yps,
     [f32, n] zps,
     [f32, n] ms,
     [f32, n] xvs,
     [f32, n] yvs,
     [f32, n] zvs,
     [f32, n] xas,
     [f32, n] yas,
     [f32, n] zas) =
 let bodies = zip(xps, yps, zps, ms, xvs, yvs, zvs, xas, yas, zas)
 let bodies' = advance_bodies_steps(n_steps, epsilon, time_step, bodies)
 in unzip(bodies')
```

to

### nbody using type aliases:

```
type mass = f32
type vec3 = (f32, f32, f32)
type position, acceleration, velocity = vec3
type body = (position, mass, velocity, acceleration)
fun vec3 vec_add(vec3 v1, vec3 v2) =
 let (x1, y1, z1) = v1
 let (x2, y2, z2) = v2
 in (x1 + x2, y1 + y2, z1 + z2)
fun vec3 vec_subtract(vec3 v1, vec3 v2) =
  let (x1, y1, z1) = v1
 let (x2, y2, z2) = v2
 in (x1 - x2, y1 - y2, z1 - z2)
fun vec3 vec_mult_factor(f32 factor, vec3 v) =
 let (x, y, z) = v
  in (x * factor, y * factor, z * factor)
fun f32 dot(vec3 v1, vec3 v2) =
 let (x1, y1, z1) = v1
 let (x2, y2, z2) = v2
 in x1 * x2 + y1 * y2 + z1 * z2
fun velocity accel(f32 epsilon, vec3 pi, f32 mi, vec3 pj, f32 mj) =
  let r = vec_subtract(pj, pi)
 let rsqr = dot(r, r) + epsilon * epsilon
 let invr = 1.0f32 / sqrt32(rsqr)
 let invr3 = invr * invr * invr
 let s = mj * invr3
  in vec mult factor(s, r)
fun vec3 accel_wrap(f32 epsilon, body body_i, body body_j) =
  let (pi, mi, _ , _) = body_i
  let (pj, mj, _ , _ ) = body_j
 in accel (epsilon, pi, mi, pj, mj)
fun position move(f32 epsilon, [body] bodies, body this_body) =
  let accels = map(fn acceleration (body other_body) =>
                     accel_wrap(epsilon, this_body, other_body),
                   bodies)
  in reduceComm(vec_add, (0f32, 0f32, 0f32), accels)
fun [acceleration] calc_accels(f32 epsilon, [body] bodies) =
  map(move(epsilon, bodies), bodies)
fun body advance_body(f32 time_step, body this_body) =
 let (pos, mass, vel, acc) = this_body
  let pos' = vec_add(pos, vec_mult_factor(time_step, vel))
  let vel' = vec_add(vel, vec_mult_factor(time_step, acc))
  let (xp', yp', zp') = pos'
```

```
let (xv', yv', zv') = vel'
 in (pos', mass, vel', acc)
fun body advance_body_wrap(f32 time_step, body this_body, acceleration accel)
 let (pos, mass, vel, acc) = this_body
 let accel' = vec_mult_factor(mass, accel)
 let body' = (pos, mass, vel, accel')
 in advance_body(time_step, body')
fun [body, n] advance_bodies(f32 epsilon, f32 time_step, [body, n] bodies) =
 let accels = calc_accels(epsilon, bodies)
 in zipWith(advance_body_wrap(time_step), bodies, accels)
fun [body, n] advance_bodies_steps(i32 n_steps, f32 epsilon, f32 time_step,
                                  [body, n] bodies) =
 loop (bodies) = for i < n steps do
    advance_bodies(epsilon, time_step, bodies)
 in bodies
fun body wrap_body (f32 posx, f32 posy, f32 posz,
                   f32 mass,
                   f32 velx, f32 vely, f32 velz,
                   f32 \ accx, f32 \ accy, f32 \ accz) =
  ((posx, posy, posz), mass, (velx, vely, velz), (accx, accy, accz))
this_body) =
 let ((posx, posy, posz), mass, (velx, vely, velz), (accx, accy, accz)) =
     this_body
 in (posx, posy, posz, mass, velx, vely, velz, accx, accy, accz)
fun ([f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [f32, n], [
   f32, n], [f32, n], [f32, n])
 main(i32 n_steps,
      f32 epsilon,
      f32 time_step,
      [f32, n] xps,
      [f32, n] yps,
      [f32, n] zps,
      [f32, n] ms,
      [f32, n] xvs,
      [f32, n] yvs,
      [f32, n] zvs,
      [f32, n] xas,
      [f32, n] yas,
      [f32, n] zas) =
 let bodies = map(wrap_body, zip(xps, yps, zps, ms, xvs, yvs, zvs, xas, yas,
 let bodies' = advance_bodies_steps(n_steps, epsilon, time_step, bodies)
 let bodies'' = map(unwrap_body, bodies')
  in unzip(bodies'')
```

# 3.6 Future work

To make functors work, it must be possible to declare an abstract type alias in a structure.

That will allow for a structure definition, where a type variable<sup>5</sup> has been declared, but not defined, until the containing structure is instantiated using a functor. This has not been implemented yet.

<sup>&</sup>lt;sup>5</sup>in the form of a *strid* 

# 4 Structures

Introduction to structures: Without structures, every function and type in futhark shares the same scope. Implementing modules lets us create functions that are alike, but keeps distinctions between them. Take this example of a program with two different vector types:

```
type vec3 = {f32, f32, f32}
type vec4 = {f32, f32, f32, f32}

fun vec3 vec3_plus(
  vec3 {a_1, ..., a_3},
  vec3 {b_1, ..., b_3}
  ) = {a_1 + b_1, ..., a_3 + b_3}

fun vec4 vec4_plus(
  vec4 {a_1, ..., a_4},
  vec4 {b_1, ..., b_4}
  ) = {a_1 + b_1, ..., a_4 + b_4}
```

Let us try compartmentalizing a vector type and its functions into a structure. In the following example, we have defined two different modules, each containing a structure, and a futhark program which includes and utilizes these modules:

```
Vec3Float.fut:
  structure Vec3Float =
    struct
      type vector = \{f32, f32, f32\}
      fun vector plus ( \dots ) = \dots
      fun vector minus ( \dots ) = \dots
      fun vector multiply ( \dots ) = \dots
    end
Vec4Float.fut:
  structure Vec4Float =
    struct
      type vector = \{f32, f32, f32, f32\}
      fun vector plus ( \dots ) = \dots
      fun vector minus ( \dots ) = \dots
      fun vector multiply ( \dots ) = \dots
    end
```

```
myprogram.fut:
  include Vec3Float
  include Vec4Float

  type vec3 = Vec3Float.vector
  type vec4 = Vec4Float.vector

fun vec4 foo(vec3 vector) =
  let {a, b, c} = Vec3.plus(vector, vector)
  in Vec4.multiply({a, b, c, 1.0f}, 4.0f)
```

Whilst it is possible to create libraries without a module implementation <sup>6</sup>, the user runs a risk of running into errors like MulipleDefinitionError<sup>7</sup>, if any of the library functions uses any of the names, that the local user is using as well.

The module system removes this hazard, as application of functions and types are done using **longnames**, which adds prefixes to names. This way, functions can have the same name, as long as they do not share the same prefix.

### Longnames:

A longname consists of any amount of prefixes followed by a dot, followed by the string id of the desired function or type:

In Futhark we will be using string ids for declarations, and longnames for the accessing types and functions in structures.

# 4.1 Accessing types and functions within structures

To work with type aliases and modules, we need to define the internal environment of Futhark during compile time.

Before starting this project, the environment of Futhark could be described like this:  $\Gamma = (FE)$ , where FE is a function environment, mapping function ids to functions:  $\{funid \rightarrow funexpr\}$ .

It is the goal of this project to expand the environment of Futhark, so that  $\Gamma = (FE, TE, SIGE, STRUCTE, FUNCTE)$ , where the additions are a type alias environment, a signature environment, a structure environment and a functor environment.

<sup>&</sup>lt;sup>6</sup>By including libraries that adds functions to the top level environment

<sup>&</sup>lt;sup>7</sup>Multiple functions of same name defined

Type aliases, signatures, structures and functors are described in their respective sections. A structure can be regarded as a structure name and an environment contained in the structure, so that  $\{strid \to \Gamma_{strid}\}$  is a mapping in STRUCTE.

$$\begin{array}{ccc} \Gamma \vdash fundecl \Rightarrow \Gamma' & \Gamma \not\vdash fun\_id \rightarrow funexpr \\ \hline \Gamma \vdash \text{fun } fun\_id = funexpr \Rightarrow \{\{fun\_id \rightarrow funexpr\}, \emptyset, \emptyset, \emptyset, \emptyset\} \oplus \Gamma \end{array}$$

where

$$funde f_{strid} \oplus \Gamma_{local}$$
  
$$\Rightarrow \Gamma_{local} \leftarrow Functions(\Gamma_{local}) \cup \{strid \rightarrow fundef\})$$

iff the function the function body is otherwise well-formed $^{a}$ .

Figure 1: Rule for adding a function to the environment

$$\frac{\Gamma_{local} \vdash typedecl \Rightarrow \Gamma'_{local}}{\Gamma_{local} \vdash type \; strid = Type \Rightarrow \{strid \rightarrow Type\} \oplus \Gamma_{local} \Rightarrow \Gamma'_{local}\}}$$

where

$$\{strid \to Type\} \oplus \Gamma_{local}$$
  
 $\Rightarrow \Gamma_{local} \leftarrow Types(\Gamma) \cup \{strid \to Type'\}$ 

and

$$Type' \leftarrow Type \ resolved \ in\Gamma_{local}$$

as defined in 3.2.

Any  $\Gamma_{strid}$  contains its own function-, typealias-, signature-, structure- and functor declarations.

### 4.2 Interference rules

Now that we can discern between the glocal environment, and any number of environments in structures, we must redefine the environment behaviour of Futhark at compile time.

After expanding the environment to contain the three new elements, we can define the addition of any of these three elements follow the following rule:

<sup>&</sup>lt;sup>a</sup>I will not go further into the functionality of functions, as it is not within scope of this project.

$$\frac{\Gamma_{local} \vdash fundecl \Rightarrow \Gamma'_{local}}{\Gamma_{local} \vdash \text{fun } strid = Function \Rightarrow \{strid \rightarrow Function\} \oplus \Gamma_{local} \Rightarrow \Gamma'_{local}\}}$$

where  $\{strid \rightarrow Function\} \oplus \Gamma_{local} = \Gamma_{local} := Functions_{\Gamma} \cup \{strid \rightarrow Function\}$  and  $Function' := Function resolved in \Gamma_{local}$  as defined in 3.2.

$$\Gamma_{local} \vdash sigdecl \Rightarrow \Gamma'_{local}$$
$$\Gamma_{local} \vdash \text{fun } strid = Signature \Rightarrow \{strid \rightarrow Signature\} \oplus \Gamma_{local} \Rightarrow \Gamma'_{local} \}$$

where  $\{strid \rightarrow Signature\} \oplus \Gamma = \Gamma := Signatures_{\Gamma} \cup \{strid \rightarrow Signature\}$  as defined in 3.2.

Note, that Signatures should be independently defined, and not refer to any other signatures.

### 4.2.1 Adding declarations to the environment

### 4.3 Interference rules for structure definitions

We must take a closer look at what happens, when a structure is defined. The definition of a structure in Futhark has the following behaviour.

#### 4.3.1 Rule for multiple declarations of same name in same local environment:

Building a structure from a list of *structdecls* allows for several different declarations in the same environment of the same name, as long as it is different types of declarations:

For  $decl:(name,type),type \in \{Function,TypeAlias,Structure,Signature\}$ :

$$\frac{\Gamma_{local} \vdash (decl :: {}^{a}decls) \Rightarrow \Gamma'_{local}}{\Gamma_{local} \vdash structdecl \Rightarrow structdecl \oplus \Gamma_{local}\Gamma'_{local}}$$

where  $decl(name, type) \oplus \Gamma_{local}$  adds the declaration to  $\Gamma_{local}$ , iff  $(name, type) \notin decls$ .

This works, because Functions, TypeAliases, Structures and Signatures are stored in their respective tables, instead of storing all of them in the same table.

<sup>&</sup>lt;sup>a</sup>this is the list constructor operator

$$\Gamma_{local} \vdash structdecl \Rightarrow \Gamma'_{local} \qquad \{structdecl_{strid} \rightarrow decls_{strid}\}$$

 $\frac{\Gamma_{local} \vdash structdecl \Rightarrow \Gamma'_{local} \quad \{structdecl_{strid} \rightarrow decls_{strid}\}}{\Gamma_{local} \vdash struct \; strid = Declarations \; end \Rightarrow \Gamma_{strid} = \Gamma_{local} \oplus Declarations; \; \Gamma'_{local} = \Gamma_{strid} \otimes \Gamma_{local}}$ 

where  $\Gamma_{local} \oplus structdecls_{strid}$  builds an environment  $\Gamma_{strid}$ , from  $structdecls_{strid}$ interpreted in the environment  $\Gamma_{local}$ .

If structdecls<sub>strid</sub> contains a new function, type or structure, with a name that is already used in  $\Gamma_{local}$ , the fun- or type definition in  $\Gamma_{strid}$  is will be the defined in  $structdecls_{strid}$ 

and  $\Gamma_{strid} \otimes \Gamma_{local} = \Gamma_{local} Structures := \Gamma_{local} Structures \cup strid \rightarrow \Gamma_{strid}$ . Please note, that  $\Gamma_{local}$  does not have a reference  $Structure\{strid \to \Gamma_{strid}\}$ , until the entire structure  $structdef_{strid}$  has been parsed.

For  $decl: (name, type), type \in \{Function, TypeAlias, Structure, Signature\}:$ 

$$\frac{\Gamma_{local} \vdash decl_{new}(name, type) \Rightarrow \Gamma'_{local} \quad (name, type) \downarrow \Gamma_{local} \rightarrow decl_{current}}{\Gamma_{local} \vdash decl \Rightarrow \Gamma' := decl_{new} \oplus \Gamma}$$

where  $(name, type) \downarrow \Gamma_{local}$  returns the declaration of (name, type) as defined in  $\Gamma_{local}$ and  $decl_{new} \oplus \Gamma$  replaces the current declaration of (name, type) in  $\Gamma_{local}$ 

#### 4.3.2Rule for adding a structure to the local environment

#### 4.3.3 Rule for variable shadowing

The following interference rule defines the variable shadowing behaviour of the Futhark module system.

#### Interference rules for interpreting functions and types in an 4.4 environment with structures

There are three cases where it is necessary to resolve a function or a type from a longname:

1) When applying a function as an expression in a function expression; i.e.

```
let myNumber = Constants.numberFour()
```

2) When using a function as an argument in a currying function; i.e.

```
let numbers = [1, 2, 3, 4] in
let sum = reduce(MathLib.plus , 0 , numbers)
```

3) When using a type definition from a structure; i.e.

```
type int pair = Pairs.Int.t
```

```
\Gamma_{local} \vdash \text{apply} longname \Rightarrow \text{apply} function

\Gamma_{local} \vdash letx = longname() \Rightarrow x := \downarrow longname
```

where

```
\downarrow longname = returngetFromEnv(longname, \Gamma_{local})
```

.

We can define the interference rule for using a longname in the three different cases. As all three cases handles resolving a longname the same way, the rule will only be called for the first case:

### 4.5 Implementation

A program in the Futhark type checker is defined as a list of unchecked declarations. I will not describe the process in details, but will instead explain specific parts of the code. I will specifically focus on parts of the code, that corresponds to the interference rules described earlier.

### 4.5.1 The Scope datatype

To describe the environment in the Futhark compiler, we use the Scope datatype:

Given  $\Gamma = (Functions, TypeAliases, Structures)$ , then Functions is implemented in envFtable, TypeAliases in envTAtable and Structures in envModTable.

### 4.5.2 checkProg

The type check is initiated in the functions checkProg and checkProg'. checks the initial top level of declaratations for duplicates.

```
fun int four() = 4

\\

structure M0 =
    struct
    fun int double(int a) = a + a
    end

\\

fun int main() =
    let x = four() in
    M0.double(x, x)
```

four () does not have MO available in its local environment, but main () does, because MO HAS already been parsed, at the point where main () is declared

### 4.5.3 Checking for duplicates

As described in rule ??, we allow multiple declarations of same name, as long as they don't share type: In example, checking for two function definitions of the same type is done in the first case of checkForDuplicateDecs:

```
checkForDuplicateDecs :: [DecBase NoInfo VName] -> TypeM ()
checkForDuplicateDecs decs = do
   _ <- foldM_ f HM.empty decs
   return ()
   where
     f known dec@(FunOrTypeDec (FunDec (FunDef _ (name,_) _ _ _ loc))) =
        case HM.lookup name known of
        Just dec'@(FunOrTypeDec (FunDec FunDef{})) ->
             throwError $ DupDefinitionError name loc $ decLoc dec'
        _ -> return $ HM.insert name dec known
```

# 4.6 Dividing a Futhark program into chunks

To facilitate variable shadowing, it was necessary to split a parsed Futhark program into chunks.

A structure declaration alters the environment drastically, by enabling the following program declarations following the structure to access the environment of the structure. Therefore we divide any list of declaration into type- and function declarations, and structure declarations. An example is given in figure??

This chunking is done recursively on a list of declarations:

## 4.7 Checking function- and type declarations

```
checkDecs :: [DecBase NoInfo VName] -> TypeM (Scope, [DecBase Info VName])
2
   checkDecs decs = do
3
   \\
4
       let (funOrTypeDecs, rest) = chompDecs decs
5
        scopeFromFunOrTypeDecs <- buildScopeFromDecs funOrTypeDecs</pre>
6
       local (const scopeFromFunOrTypeDecs) $ do
7
          checkedeDecs <- checkFunOrTypeDec funOrTypeDecs</pre>
8
          (scope, rest') <- checkDecs rest
9
          return (scope , checkedeDecs ++ rest')
10 \\
11
  checkDecs [] = do
12
     scope <- ask
13
     return (scope, [])
```

- 1) The current chunk of declarations is built using chompDecs.
- 2) A scope is built from this chunk, by using the three interference rules in section 4.2.1.
- 3) The scope from step 2 is now the local scope in the following function execution.
- 4) checkFunOrTypeDec does the actual typechecking of the function declaration.
- 5) The remainder of the declarations chunked in chompDecs is checked using checkDecs. Note, that the first element of rest must be a structure declaration, due to the implementation of chompDecs.

# 4.8 Checking structure declarations

Please note that structures are currently called modules inside Futharks compiler.

```
checkDecs (ModDec modd:rest) = do
  (modscope, modd') <- checkMod modd
  local (addModule modscope) $
    do
        (scope, rest') <- checkDecs rest
    return (scope, ModDec modd' : rest' )</pre>
```

When checkDecs encounters a ModDec, checkMod is called to resolve the ModDec. The ModDec defines an environment of functions and type declarations called modscope, which we add to the local environment by calling local (addModule modscope) This part of the code is the implementation of rule??.

```
checkMod :: ModDefBase NoInfo VName -> TypeM (Scope , ModDefBase Info VName)
checkMod (ModDef name decs loc) =
  local ('addBreadcrumb' name) $ do
   _ <- checkForDuplicateDecs decs
  (scope, decs') <- checkDecs decs
  return (scope, ModDef name decs' loc)</pre>
```

```
type LongName = ([Name], name)
typeFromScope :: LongName -> Scope -> Maybe TypeBinding
typeFromScope (prefixes, name) scope = do
  scope' <- envFromScope prefixes scope</pre>
  let taTable = envTAtable scope'
  HM.lookup name taTable
funFromScope :: LongName -> Scope -> Maybe FunBinding
funFromScope (prefixes, name) scope = do
  scope' <- envFromScope prefixes scope</pre>
  let taTable = envFtable scope'
  HM.lookup name taTable
envFromScope :: [Name] -> Scope -> Maybe Scope
 envFromScope (x:xs) scope =
    case HM.lookup x (envModTable scope) of
      Just scope' -> envFromScope xs scope'
      Nothing -> Nothing
envFromScope [] scope = Just scope
```

checkMod first adds a "breadcrumb" of the structure's name (strid) to the local environment. This is where the transformation in rule ?? of  $\Gamma_{local} \Rightarrow \Gamma_{strid}$  is implemented. After the environment has been given its name, the internal declaration of the structure is read using checkDecs.

# 4.9 Resolving the application of a longname

Resolving the application for a longname is done through two short recursive functions. Figure ?? shows the implementation of the longname rule ??:

### 4.9.1 Including structures, functions and types from other files

The Futhark include-statement lets the user include other files into the current program. This is implemented by letting the Futhark compiler combine the source code from all the included files, with the declarations in the main program. The combined program is then sent passed on through the rest of the compiler.

As the Futhark program with an arbitrary number of includes is merged into a single program within the compiler, we can compartmentalize a program into discrete files.

However, this creates a hazard: there might be declarations in the included code, which has names that overlap with names in the remaining code, so that a DuplicateDefinitionError is triggered. the included code has any declarations that results in a duplicate definition error. Refer to ?? for a possible future solution.

### 4.9.2 Keeping track of function names

Futharks variable shadowing made it necessary to extend the type of the function declaration type.

Initially, a function had only the declared name of the function as an identifier. However, it was necessary to extend this name into a pair:

```
type FunName = (declaredName :: Name, expandedName :: LongName).
```

When a function is added to the function table during buildFtable, it is added to the function table together with it's longname. The function's longname contains the function's name, and the name of the (potentially) nested structures it was defined in.

#### 4.10 Tests

To verify that the implementation of structures was correct, a series of test programs were written to verify, whether the interference rules defined for the structures, actually held in an executed Futhark program.

In the following tests, programs which break the rules are expected to return with an error.

### 4.10.1 Testing rule for multiple declarations [...]

The following tests are implemented to test whether rule ?? holds. duplicate\_def0.fut:

```
-- This test is written to ensure that the same name can be used
-- for different declarations in the same local environment, as long as their
    types does not overlap.
-- ==
-- input { 4 }
-- output { 4 }

type foo = int
fun foo foo(int a) = a + a
struct foo
    {
        fun int one() = 1
     }

fun int main(int x) = x
```

This test passes.

In the following test, the structure foo contains declarations of name foo also. duplicate\_def1.fut:

```
-- The struct opens a new environment, which lets us use names again, which
    were used
-- in a previous scope.
-- ==
-- input { }
-- output { (1 , 2.0) }

type foo = int
struct foo
```

```
fun int foo() = 1
struct foo
{
    type foo = float
    fun foo foo() = 2.0
}
fun (foo, foo.foo.foo) main() = ( foo.foo() , foo.foo.foo())
```

This test passes.

In the following tests, the programmer has attempted to declare functions and types in the same environment, twice.

```
duplicate_error0.fut:
-- This test fails with a DuplicateDefinition error.
-- ==
-- error: .*Dup.*

fun int bar() = 1
struct foo
{
  fun foo foo() = 1
}
fun int bar() = 2

fun int main() = 0
```

This test passes.

In the following test, the structure foo contains declarations of name foo also. duplicate\_error1.fut:

```
-- This test fails with a DuplicateDefinition error.
-- ==
-- error: .*Dup.*

type foo = int

struct foo
{
   fun foo foo() = 1
}

type foo = float

fun int main() = 0
```

This test passes.

### 4.10.2 Testing structures can be called as expected

The following tests are implemented to test that calling structures works as defined in rule ??.

calling\_nested\_module.fut:

```
-- ==
-- input {
-- 10 21
-- }
-- output {
-- 6
-- }
type t = int
struct NumLib {
   fun t plus(t a, t b) = a + b
  struct BestNumbers
      fun t four() = 4
     fun t seven() = 42
     fun t six() = 41
    }
  }
fun int localplus(int a, int b) = NumLib.plus (a,b)
fun int main(int a, int b) =
  localplus(NumLib.BestNumbers.four() ,
```

This test passes.

Currying functions such as map and reduce works as expected: map\_with\_structure0.fut:

```
-- Testing whether it is possible to use a function
-- from a struct in a curry function (map)
-- ==
-- input {
-- [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
-- }
-- output {
-- 55
-- }
struct f {
   fun int plus(int a, int b) = a+b
   }
fun int main([int] a) = reduce(f.plus, 0, a)
```

This test passes.

Vec3.fut:

Using structures from an include works as expected:

```
-- ==
-- tags { disable }
struct Vec3
{
struct F32
```

```
type t = (f32, f32, f32)
        fun t add(t a , t b) =
         let (a1, a2, a3) = a in
         let (b1, b2, b3) = b in
          (a1 + b1, a2 + b2, a3 + b3)
        fun t subtract(t a , t b) =
          let (a1, a2, a3) = a in
          let (b1, b2, b3) = b in
          (a1 - b1, a2 - b2, a3 - b3)
       fun t scale(f32 k , t a) =
         let (a1, a2, a3) = a in
          (a1 * k, a2 * k, a3 * k)
        fun f32 dot(t a , t b) =
         let (a1, a2, a3) = a in
         let (b1, b2, b3) = b in
         a1*b1 + a2*b2 + a3*b3
     }
   struct Int
     {
       type t = ( int , int , int )
        fun t add(t a , t b) =
         let (a1, a2, a3) = a in
         let (b1, b2, b3) = b in
          (a1 + b1, a2 + b2, a3 + b3)
        fun t subtract(t a , t b) =
         let (a1, a2, a3) = a in
         let (b1, b2, b3) = b in
          (a1 - b1, a2 - b2, a3 - b3)
        fun t scale(int k , t a) =
         let (a1, a2, a3) = a in
          (a1 * k, a2 * k, a3 * k)
       fun int dot(t a , t b) =
         let (a1, a2, a3) = a in
         let (b1, b2, b3) = b in
         a1*b1 + a2*b2 + a3*b3
     }
 }
-- ==
-- input {
-- (10, 21, 21) (19, 12, 5)
-- }
-- output {
-- 547
-- }
```

```
type vec3 = Vec3.Int.t
fun int main(vec3 a, vec3 b) = Vec3.Int.dot(a , b)
```

This test passes.

### 4.10.3 Testing rule for variable shadowing

The following tests are implemented to test whether the rule ?? holds: Simple shadowing for functions holds:

```
shadowing0.fut:
```

```
-- M1.foo() calls the most recent declaration of number, due to M0.number()
-- being brought into scope of M1, overshadowing the top level definition of
-- number()
-- ==
-- input {
-- }
-- output {
-- 2
-- }
fun int number() = 1
struct M0
 {
    fun int number() = 2
    struct M1
        fun int foo() = number()
fun int main() = M0.M1.foo()
```

Simple shadowing for types holds:

```
shadowing1.fut:
```

```
-- M1.foo() calls the most recent declaration of number, due to M0.number()
-- being brought into scope of M1, overshadowing the top level definition of
-- number()
-- ==
-- input {
-- }
-- output {
-- (6.0, 6, 6)
-- }

type best_type = float
fun best_type best_number() = 6.0
struct M0
{
    type best_type = int
    fun best_type best_number() = 6
```

```
struct M1
     {
        fun best_type best_number() = 6
  }
fun (float, int, int) main() = (best_number() , M0.best_number() , M0.M1.
   best_number())
This test shows, that structures are only read into scope, after they are fully parsed.
shadowing2.fut:
-- M0.foo() changes meaning inside M1, after the previous declaration of M0
-- is overshadowed.
-- input {
-- }
-- output {
-- 12
-- }
struct M0
   fun int foo() = 1
struct M1
   fun int bar() = M0.foo()
   struct M0
       fun int foo() = 10
   fun int baz() = M0.foo()
fun int main() = M0.foo() + M1.bar() + M1.baz()
and undefined_structure_err0.fut:
-- We can not access a struct before it has been defined.
-- ==
-- error: .*Unknown.*
fun int try_me() = M0.number()
struct M0
   fun int number() = 42
```

fun int main() =  $try_me()$ 

# 4.11 Results

The implementation of structures works, and I am satisfied with the results. As shown in the tests, the structures behave as prescribed in the rules 4.2.

# 4.12 Future work

As mentioned in ??, it is possible to trigger a DuplicateDefinitionError from including files into the main program.

Given more time, I would like to extend the Futhark include statement to support an as statement, so that the inclusion

include some\_module as M0

will include the declarations from some\_module into the futhark program, but not into the top level declarations. Instead, the declarations will be loaded into a structure MO, which can then be accessed throughout the rest of the program as any other module.

```
definition
Language language
                                              example
part
         construct
Core:
         dec :=
                        type t = type\_def
                                               (* type Status = int *)
                        fun f args = exp
                                               (* fun foo n = n + 5 *)
                        val x = exp
                                               (* val eleven = 11 *)
Module:
         topdec ::=
                        sigdec
                        moddec
                        topdec topdec
         sigdec :=
                        signature X = sigexp
                                               (* signature foo = ... *)
                        sig sigspec end
         sigexp :=
                                               (* sig val bar : int end
                        X
                                               (*{foo, bar, One, ... }*)
         sigspec
                        val x : type
                                               (* val bar : string *)
                        module X : sigexp
                                               (* module Bar :
                                              Numberable *)
                        sigspec sigspec
         moddec ::=
                        dec
                        module X = modexp
                                               (* module One = ...
                        moddec moddec
                                               (* val x = 1 val y = 2 *)
                        struct moddec end
         modexp :=
                                               (* struct
                                              module Adder = PlusOp
                                              val one = 1
                                              end *)
                        modexp: sigexp
                                               (* struct
                                               val bar = 1
                                               end: foo *)
                        X
                                               (* {One, Numberable,
                                              Counter, Queue, ... }
                                               *)
Module
access:
         LongIdent ::=
                        module.field
                                               (* One.bar *)
                        module . LongIdent
                                               (* Numberable.One.bar *)
```