

**Problem 3 (15%)**

The normalised radiation intensity of a given antenna is given by:  $U = \cos(2\theta)$ . The intensity exists only in the  $0 <= \theta <= \pi/2$ ,  $0 <= \phi <= 2\pi$  region and is zero elsewhere. Find

- (10%) the exact directivity (dimensionless and in dB).
- (5%) the azimuthal and elevation plane half-power beamwidths in degrees (hint: an elevation plane is for a fixed  $\phi$ ).

**Problem 4 (35%)**

A Bluetooth receiver operating at 2500 MHz needs 50 micro Watt for error-free transmission and receive its signal from a 0 dBm transmitter by a half-wave dipole antenna connected to the receiver. The transmitter can use a 50 by 50 mm antenna area.

4.a (15%) Find the maximum distance of communication if free-space propagation can be assumed.

4.b (5%) Give under which conditions the maximum distance is obtained.

4.c (5%) Find the maximum distance if the reader uses circular polarization.

4.d (10%) If the receiver and transmitter are operated above a perfect reflecting surface, what is then the maximum distance. Assume the height of both the receiver and transmitter is one meter above the reflection surface.

P4)

$$T_x = 50 \text{ mm} \times 50 \text{ mm} \quad 1 \text{ mW} = 2500 \text{ MHz}$$

$$R_x = D_r = 1.643 \quad \text{min power} = 50 \cdot 10^{-12}$$

$$D_t = \frac{U \pi A}{\lambda} = 2.18$$

A)

$$R^2 = \frac{P_t}{P_r} \cdot \frac{D_t \cdot D_r \cdot \lambda^2}{(4\pi)^2} \cdot P_Q$$

for this case

$$R = \sqrt{\frac{P_t}{P_r} \cdot \frac{D_t D_r \lambda^2}{(4\pi)^2}} = \underline{\underline{80.85}}$$

B)

- Perfect polarization
- Perfect line of sight
- Impedance matched
- 

C)

$$R = \sqrt{\frac{P_t}{P_r} \cdot \frac{D_t D_r \lambda^2}{(4\pi)^2} \cdot P_Q} = 57.17 \Big|_{P_Q = \frac{1}{2}}$$

A)

$$\cos(2\theta) \Rightarrow U_{max} = 1$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}}$$

$$P_{rad} = A_0 \int_0^{\pi} \int^{2\pi} U \cdot \sin(\theta) d\phi d\theta = \frac{2\pi}{3}$$

$$D_0 = \frac{4\pi}{2\pi/3} = 6 \text{ [dBi]}$$

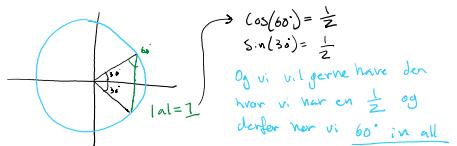
$$\text{in } dBi = 10 \cdot \log_{10}(6) = 7.82 \text{ [dB]}$$

B)

$$\text{for } \phi, \text{Azimuth} = 360^\circ$$

$$\text{for } \theta, \text{Solve } U(\theta) = \frac{1}{2} U(\theta), \theta = \frac{\pi}{6} = \frac{\pi}{6} \cdot \frac{180}{\pi} = 30^\circ \text{ sin } 60^\circ$$

eller i Johans resonans:



$$D) \frac{P_r}{P_f} = D_f D_r \left( \frac{h_f h_r}{R^2} \right)^2 =$$

$$D_f D_r \frac{P_r}{P_f} h_f h_r^2 = R^4$$

$$R = \sqrt[4]{D_f P_r \frac{P_r}{P_f} (h_f h_r)^2}$$

# Sol-Antenna

Thursday, 2 January 2025 08.23



SolutionExa  
mExampl...

## Answers

### Problem 3

The radiation intensity of a given antenna is :  $U = |\cos(2\theta)|$  and the intensity exists in the region defined by  $0 \leq \theta \leq \frac{\pi}{2}$  and  $0 \leq \varphi \leq 2\pi$ .

3.a. Directivity :

To find the total radiated power the radiation intensity is integrated over its closed surface S.

$$S = \begin{cases} 0 \leq \theta \leq \frac{\pi}{2} \\ 0 \leq \varphi \leq 2\pi \\ 0 \text{ elsewhere} \end{cases}$$

$$\begin{aligned} P_{rad} &= \iint_S U dS = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} U \sin\theta d\theta d\varphi = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} |\cos(2\theta)| \sin\theta d\theta \\ P_{rad} &= \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} |1 - 2\sin^2\theta| \sin\theta d\theta = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} |\sin\theta - 2\sin^3\theta| d\theta \quad (\text{because } \sin\theta \geq 0 \text{ here}) \\ P_{rad} &= [1]_0^{2\pi} * \text{abs} \left( [-\cos\theta]_0^{\frac{\pi}{2}} + \left[ \frac{2}{3}\sin^2\theta \cos\theta + \frac{4}{3}\cos\theta \right]_0^{\frac{\pi}{2}} \right) = 2\pi * \text{abs} \left( 1 + \left( -\frac{4}{3} \right) \right) = \frac{2}{3}\pi \end{aligned}$$

Note : Power must be positive.

The maximum radiation is directed along  $\theta = 0$ . Thus,  $U_{max} = 1$ .

The maximum directivity is equal to :

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = 6$$

$$D_0(\text{dBt}) = 10 * \log_{10}(6) = 7.8 \text{ dBt}$$

3.b. HPBW :

The half-power beamwidth can be found with the function equal to half of  $U_{max}(\theta)$ .

$$\text{Thus, } \frac{1}{2} U_{max}(\theta) = \frac{1}{2} \Rightarrow \cos(2\theta_h) = \frac{1}{2} \Rightarrow \theta_h = \frac{\pi}{6} = 0.5236 \text{ rad}$$

Because U is symmetric around  $\theta = 0$ , HPBW(el) =  $2 * 0.5236 = 1.0472 \text{ rad} = 60^\circ$

Because U doesn't depend on  $\varphi$ , HPBW(az) =  $2 * \pi = 6.2832 \text{ rad} = 360^\circ$

#### Problem 4

Data:

$$f = 2500 \text{ MHz} \Rightarrow \lambda = \frac{c}{f} = 0.12 \text{ m}$$

$$P_r = 50 \cdot 10^{-12} \text{ W}, P_t = 0 \text{ dBm} = 1 \text{ mW}$$

$$A = 2,50 \cdot 10^{-3} \text{ m}^2$$

4.a. Free space propagation :

Friis transmission equation can be applied assuming reflection and polarization – matched lossless antennas:

$$\frac{P_r}{P_t} = \lambda^2 \frac{D_t * D_r}{(4\pi R)^2}$$

Where  $D_t = \frac{4\pi A}{\lambda^2} = 2,181$  and  $D_r = 1,643$  for the half wave dipole.

The maximum distance of communication in free space is  $\mathbf{R = 80,85 \text{ m}}$

4.b. The maximum distance in free space is achieved under the following conditions :

- Perfect impedance matching
- Perfect polarization matching
- Lossless antennas

4.c. In this case the reader uses circular polarization resulting in a mismatched polarization and thus losses. The polarization mismatch is 50 % with one circularly polarized antenna.

Friis law gives  $R^2 = \frac{P_t D_t * D_r * \lambda^2}{P_r (4\pi)^2} * \frac{1}{2}$ .  $\mathbf{R = 57,17 \text{ m}}$

4.d. Perfect reflecting surface:

In the case where a LOS plus a ground reflection exist the following equation applies for lossless antennas:

$$\frac{P_r}{P_t} = D_t * D_r \left( \frac{h_t h_r}{R^2} \right)^2,$$

and is valid for  $R \geq \frac{4h_t h_r}{\lambda} = 33,33 \text{ m}$  ( $h_t = h_r = 1 \text{ m}$ )

*Note: This equation, which replaces the Friis' law for  $R > 33 \text{ m}$ , implies that the received power is independent of frequency and increases with the square of the height for both the MS and the BS.*

The maximum distance for polarization matched antennas is  $\mathbf{R = 92,01 \text{ m}}$ .

# Antenna-Quick-Fix

Thursday, 2 January 2025 10.34

**TABLE 4.3** Summary of Important Parameters and Associated Formulas and Equation Numbers for a Dipole in the Far Field

Parameter	Formula	Equation Number
<i>Infinitesimal Dipole</i> $(l \leq \lambda/50)$		
Normalized power pattern	$U = (E_{\theta n})^2 = C_0 \sin^2 \theta$	(4-19)
Radiation resistance $R_r$	$R_r = \eta \left( \frac{2\pi}{3} \right) \left( \frac{l}{\lambda} \right)^2 = 80\pi^2 \left( \frac{l}{\lambda} \right)^2$	(4-19)
Input resistance $R_{in}$	$R_{in} = R_r = \eta \left( \frac{2\pi}{3} \right) \left( \frac{l}{\lambda} \right)^2 = 80\pi^2 \left( \frac{l}{\lambda} \right)^2$	(4-19)
Wave impedance $Z_w$	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377 \text{ ohms}$	
Directivity $D_0$	$D_0 = \frac{3}{2} = 1.761 \text{ dB}$	(4-31)
Maximum effective area $A_{em}$	$A_{em} = \frac{3\lambda^2}{8\pi}$	(4-32)
Vector effective length $\ell_e$	$\ell_e = -\hat{a}_\theta l \sin \theta$ $ \ell_e _{max} = \lambda$	(2-92) Example 4.2
Half-power beamwidth	$\text{HPBW} = 90^\circ$	(4-65)
Loss resistance $R_L$	$R_L = \frac{l}{P} \sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{l}{2\pi b} \sqrt{\frac{\omega\mu_0}{2\sigma}}$	(2-90b)
<i>Small Dipole</i> $(\lambda/50 < l \leq \lambda/10)$		
Normalized power pattern	$U = (E_{\theta n})^2 = C_1 \sin^2 \theta$	(4-36a)
Radiation resistance $R_r$	$R_r = 20\pi^2 \left( \frac{l}{\lambda} \right)^2$	(4-37)
Input resistance $R_{in}$	$R_{in} = R_r = 20\pi^2 \left( \frac{l}{\lambda} \right)^2$	(4-37)
Wave impedance $Z_w$	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377 \text{ ohms}$	(4-36a), (4-36c)
Directivity $D_0$	$D_0 = \frac{3}{2} = 1.761 \text{ dB}$	
Maximum effective area $A_{em}$	$A_{em} = \frac{3\lambda^2}{8\pi}$	
Vector effective length $\ell_e$	$\ell_e = -\hat{a}_\theta \frac{l}{2} \sin \theta$ $ \ell_e _{max} = \frac{l}{2}$	(2-92) (4-36a)
Half-power beamwidth	$\text{HPBW} = 90^\circ$	(4-65)
<i>Half Wavelength Dipole</i> $(l = \lambda/2)$		
Normalized power pattern	$U = (E_{\theta n})^2 = C_2 \left[ \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right]^2 \simeq C_2 \sin^3 \theta$	(4-87)
Radiation resistance $R_r$	$R_r = \frac{\eta}{4\pi} C_{in}(2\pi) \simeq 73 \text{ ohms}$	(4-93)

(continued overleaf)

TABLE 4.3 (continued)

Parameter	Formula	Equation Number
Input resistance $R_{in}$	$R_{in} = R_r = \frac{\eta}{4\pi} C_{in}(2\pi) \approx 73 \text{ ohms}$	(4-79), (4-93)
Input impedance $Z_{in}$	$Z_{in} = 73 + j42.5$	(4-93a)
Wave impedance $Z_w$	$Z_w = \frac{E_\theta}{H_\phi} \approx \eta = 377 \text{ ohms}$	
Directivity $D_0$	$D_0 = \frac{4}{C_{in}(2\pi)} \approx 1.643 = 2.156 \text{ dB}$	(4-91)
Vector effective length $\ell_e$	$\ell_e = -\hat{a}_\theta \frac{\lambda}{\pi} \frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta}$	(2-91)
	$ \ell_e _{\max} = \frac{\lambda}{\pi} = 0.3183\lambda$	(4-84)
Half-power beamwidth	HPBW = $78^\circ$	(4-65)
Loss resistance $R_L$	$R_L = \frac{l}{2P} \sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{l}{4\pi b} \sqrt{\frac{\omega\mu_0}{2\sigma}}$	Example (2-13)
<i>Quarter-Wavelength Monopole</i> ( $l = \lambda/4$ )		
Normalized power pattern	$U = (E_{\theta n})^2 = C_2 \left[ \frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]^2 \approx C_2 \sin^3 \theta$	(4-87)
Radiation resistance $R_r$	$R_r = \frac{\eta}{8\pi} C_{in}(2\pi) \approx 36.5 \text{ ohms}$	(4-106)
Input resistance $R_{in}$	$R_{in} = R_r = \frac{\eta}{8\pi} C_{in}(2\pi) \approx 36.5 \text{ ohms}$	(4-106)
Input impedance $Z_{in}$	$Z_{in} = 36.5 + j21.25$	(4-106)
Wave impedance $Z_w$	$Z_w = \frac{E_\theta}{H_\phi} \approx \eta = 377 \text{ ohms}$	
Directivity $D_0$	$D_0 = 3.286 = 5.167 \text{ dB}$	
Vector effective length $\ell_e$	$\ell_e = -\hat{a}_\theta \frac{\lambda}{\pi} \cos\left(\frac{\pi}{2} \cos \theta\right)$	(2-91)
	$ \ell_e _{\max} = \frac{\lambda}{\pi} = 0.3183\lambda$	(4-84)

Power (dBm)	Power (W)
-30 dBm	0,000001 W
-20 dBm	0,00001 W
-10 dBm	0,0001 W
0 dBm	0,001 W
1 dBm	0,0012589 W
2 dBm	0,0015849 W
3 dBm	0,0019953 W
4 dBm	0,0025119 W
5 dBm	0,0031628 W
6 dBm	0,0039811 W
7 dBm	0,0050119 W
8 dBm	0,0063096 W
9 dBm	0,0079433 W
10 dBm	0,01 W
20 dBm	0,1 W
30 dBm	1 W
40 dBm	10 W
50 dBm	100 W

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Prefix		Base 10	Decimal	Adoption [nb 1]
Name	Symbol			
quetta	Q	$10^{30}$	1 000 000 000 000 000 000 000 000 000 000 000	2022 <sup>[3]</sup>
ronna	R	$10^{27}$	1 000 000 000 000 000 000 000 000 000 000 000	
yotta	Y	$10^{24}$	1 000 000 000 000 000 000 000 000 000 000 000	1991
zetta	Z	$10^{21}$	1 000 000 000 000 000 000 000 000 000 000 000	
exa	E	$10^{18}$	1 000 000 000 000 000 000 000 000 000 000 000	1975 <sup>[4]</sup>
peta	P	$10^{15}$	1 000 000 000 000 000 000 000 000 000 000 000	
tera	T	$10^{12}$	1 000 000 000 000 000 000 000 000 000 000 000	1960

Prefix		Base 10	Decimal	Adoption [nb 1]
Name	Symbol			
quetta	Q	$10^{30}$	1 000 000 000 000 000 000 000 000 000 000 000 000	2022 <sup>[3]</sup>
ronna	R	$10^{27}$	1 000 000 000 000 000 000 000 000 000 000 000 000	
yotta	Y	$10^{24}$	1 000 000 000 000 000 000 000 000 000 000 000 000	1991
zetta	Z	$10^{21}$	1 000 000 000 000 000 000 000 000 000 000 000 000	
exa	E	$10^{18}$	1 000 000 000 000 000 000 000 000 000 000 000 000	1975 <sup>[4]</sup>
peta	P	$10^{15}$	1 000 000 000 000 000 000 000 000 000 000 000 000	
tera	T	$10^{12}$	1 000 000 000 000 000 000 000 000 000 000 000 000	1960
giga	G	$10^9$	1 000 000 000 000 000 000 000 000 000 000 000 000	
mega	M	$10^6$	1 000 000 000 000 000 000 000 000 000 000 000 000	1873
kilo	k	$10^3$	1 000 000 000 000 000 000 000 000 000 000 000 000	
hecto	h	$10^2$	100 000 000 000 000 000 000 000 000 000 000 000	1795
deca	da	$10^1$	10 000 000 000 000 000 000 000 000 000 000 000	
—	—	$10^0$	1 000 000 000 000 000 000 000 000 000 000 000	—
deci	d	$10^{-1}$	0.1 000 000 000 000 000 000 000 000 000 000 000	
centi	c	$10^{-2}$	0.01 000 000 000 000 000 000 000 000 000 000 000	1795
milli	m	$10^{-3}$	0.001 000 000 000 000 000 000 000 000 000 000 000	
micro	μ	$10^{-6}$	0.000 001 000 000 000 000 000 000 000 000 000 000	1873
nano	n	$10^{-9}$	0.000 000 001 000 000 000 000 000 000 000 000 000	
pico	p	$10^{-12}$	0.000 000 000 001 000 000 000 000 000 000 000 000	1960
femto	f	$10^{-15}$	0.000 000 000 000 001 000 000 000 000 000 000 000	
atto	a	$10^{-18}$	0.000 000 000 000 000 001 000 000 000 000 000 000	1964
zepto	z	$10^{-21}$	0.000 000 000 000 000 000 001 000 000 000 000 000	
yocto	y	$10^{-24}$	0.000 000 000 000 000 000 000 001 000 000 000 000	1991
ronto	r	$10^{-27}$	0.000 000 000 000 000 000 000 000 001 000 000 000	
quecto	q	$10^{-30}$	0.000 000 000 000 000 000 000 000 000 000 000 000	2022 <sup>[3]</sup>

S. No.	Dielectric Material	Relative Permittivity	Loss Tangent
1	Air	1.0006	0
2	FR4 epoxy	4.4	0.02
3	Bakelite	4.8	0.0002
4	Duroid	2.2	0.0009
5	Quartz glass	3.78	0
6	Foam	1.03	0
7	Polystyrene	2.55	0
8	Plexiglas	2.59	0.0068
9	Fused quartz	3.78	0
10	E glass	6.22	0.0023
11	RO4725JXR	2.55	0.0022
12	RO4730JXR	3	0.0023
13	Rogers RT/duroid 5870/5880	2.33/2.2	0.0012/0.0009
14	Teflon	2.1	0.001
15	Taconic CER-10	10	0.0035
16	Taconic RF-30	3	0.0014
17	Taconic RF-35	3.5	0.0018

**TABLE 26.1** Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant $\kappa$	Dielectric Strength <sup>a</sup> (V/m)
Air (dry)	1.000 59	$3 \times 10^6$
Bakelite	4.9	$24 \times 10^6$
Fused quartz	3.78	$8 \times 10^6$
Neoprene rubber	6.7	$12 \times 10^6$
Nylon	3.4	$14 \times 10^6$
Paper	3.7	$16 \times 10^6$
Polystyrene	2.56	$24 \times 10^6$
Polyvinyl chloride	3.4	$40 \times 10^6$
Porcelain	6	$12 \times 10^6$
Pyrex glass	5.6	$14 \times 10^6$
Silicone oil	2.5	$15 \times 10^6$
Strontium titanate	233	$8 \times 10^6$
Teflon	2.1	$60 \times 10^6$
Vacuum	1.000 00	—
Water	80	—

<sup>a</sup> The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

# Ecx-Networking

Friday, 3 January 2025 08.15



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**Exam problems examples (networking part with TKM).**

**Problem 1.**

Download a file cap3.pcapng. Open it with wireshark and answer the following questions:

1. a) Find the IP address assigned by DHCP to the client computer b) What is the MAC address of the client computer?
2. The same computer makes an ARP request. What is the Target's IP address and MAC address?
3. NTP (Network Time Protocol) is used to synchronize time and dates between computers. The capture contains some NTP packets. a)How many NTP packets does the capture contain? b) What date does the NTP server say it is?

**Problem 2.**

Assume that it takes 1 ms from a bit leaves A to the bit is received in B. Assume also that the bandwidth of the line is 5Mbit/s. How long does it take to transfer the frame of 1500 bytes from A to B? (round to nearest ms).

**Problem 3.**

What is the difference between error detection and error correction. Give an example where it is more suitable to use error correction mechanisms.

**Problem 4.**

Which layer header contains

- a) MAC address
- b) Destination IP address
- c) SYN bits when a connection is established

**Problem 5.**

What is the purpose of Inter Frame Space in 802.11 MAC protocol?

**Problem 6.**

- a) Why authentication is needed? b) given that two devices share a common secret, how authentication can be done?

### Problem 1.

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2. The same computer makes an ARP request. What is the Target's IP address and MAC address?
3. NTP (Network Time Protocol) is used to synchronize time and dates between computers. The capture contains some NTP packets. a) How many NTP packets does the capture contain? b) What date does the NTP server say it is?

No.	Time	Source	Destination	Protocol	Length	Info
0	0.000000000	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
1	0.000169847	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
2	0.000239614	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
3	1.000000000	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
4	1.0000053942	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
5	2.343495934	0.0.0.0	255.255.255.255	DHCP	355	DHCP Discover - Transaction ID 0xb0bc02e
6	2.343618497	192.168.122.1	192.168.122.43	DHCP	342	DHCP Offer - Transaction ID 0xb0bc02e
7	2.343618497	192.168.122.1	255.255.255.255	DHCP	347	DHCP Request - Transaction ID 0xb0bc02e
8	2.343618497	192.168.122.1	192.168.122.43	DHCP	348	DHCP ACK - Transaction ID 0xb0bc02e
9	3.082276847	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
10	3.082614385	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
11	3.768636291	192.168.122.1	192.168.122.250	UDP	86	57621 - 57621 Len=40
12	3.768636291	192.168.122.1	192.168.122.250	ARP	44	ARP who has 192.168.122.250? Tell 192.168.122.43
13	5.699405518	52.54.00.00:63:d3	192.168.122.43	ARP	42	192.168.122.1 is at 52:54:00:26:63:d3
14	5.699599813	192.168.122.43	192.168.122.1	DNS	87	Standard query 0x4c5 A apl.snapcraft.io OPT
15	5.699599813	192.168.122.43	192.168.122.1	DNS	87	Standard query 0x4c48 AAAA apl.snapcraft.io OPT
16	5.699803865	192.168.122.43	192.168.122.43	DNS	87	Standard query 0x4c48 AAAA apl.snapcraft.io OPT
17	5.699803865	192.168.122.43	192.168.122.43	DNS	189	192.168.122.43 is at 52:54:00:26:63:d3
18	5.8240957213	192.168.122.43	192.168.122.43	DNS	87	Standard query 0x4c48 AAAA apl.snapcraft.io OPT
19	5.824134484	192.168.122.43	192.168.122.43	DNS	87	Standard query response 0x9eaa AAAA apl.snapcraft.io OPT
20	5.824134484	192.168.122.43	192.168.122.43	DNS	87	Standard query 0x4c48 AAAA apl.snapcraft.io OPT
21	54.104592861	192.168.122.43	192.168.122.43	DNS	87	Standard query 0x4c48 AAAA apl.snapcraft.io OPT
22	54.104592861	192.168.122.43	192.168.122.43	DNS	149	192.168.122.43 is at 52:54:00:26:63:d3
23	54.176844112	192.168.122.43	192.168.122.43	DNS	141	Standard request 0x4e48 AAAA http://ubuntu.com A 91.189.91.157 A 91.189.91.4 A 91.189.89.198 A 91.189.89.199 OPT
24	54.176844112	192.168.122.43	192.168.122.43	NTP	90	NTP Version 4, client
25	54.176844112	192.168.122.43	192.168.122.43	NTP	90	NTP Version 4, server
26	54.176844112	192.168.122.43	192.168.122.43	NTP	90	NTP Version 4, client
27	66.65758615	192.168.122.43	91.189.91.157	NTP	90	NTP Version 4, client
28	66.758738234	91.189.91.157	192.168.122.43	NTP	90	NTP Version 4, server
29	78.772128824	192.168.122.43	192.168.122.1	UDP	72	738384 - 5678 Len=38
30	114.104595531	192.168.122.43	192.168.122.1	DHCP	87	Standard query 0x0000 PTR _spotify-connect._tcp.local, "QH" question
31	114.104595531	192.168.122.43	192.168.122.1	DHCP	167	Standard query 0x0000 PTR _spotify-connect._tcp.local, "QH" question
32	117.002885229	192.168.122.43	192.168.122.1	DNS	208	M-SEARCH * HTTP/1.1
33	118.004196914	192.168.122.43	192.168.122.43	DNS	208	M-SEARCH * HTTP/1.1
34	119.005782894	192.168.122.43	192.168.122.43	DNS	208	M-SEARCH * HTTP/1.1
35	123.512286807	192.168.122.43	192.168.122.43	DNS	208	M-SEARCH * HTTP/1.1
36	123.512286807	192.168.122.43	192.168.122.43	DNS	208	M-SEARCH * HTTP/1.1
37	130.987767292	192.168.122.43	91.189.91.157	NTP	90	NTP Version 4, Client
38	131.807764586	91.189.91.157	192.168.122.43	NTP	90	NTP Version 4, server
39	132.174240801	192.168.122.43	192.168.122.255	UDP	86	57621 - 57621 Len=44
40	132.174240801	192.168.122.43	192.168.122.255	UDP	86	57621 - 57621 Len=44
41	134.574768088	192.168.122.43	224.0.0.251	DHCP	87	Standard query 0x0000 PTR _spotify-connect._tcp.local, "QH" question
42	236.398572721	192.168.122.43	224.0.0.251	DHCP	167	M-SEARCH * HTTP/1.1
43	237.005672600	192.168.122.1	239.255.255.250	SSDP	208	M-SEARCH * HTTP/1.1
44	237.005672600	192.168.122.1	239.255.255.250	SSDP	208	M-SEARCH * HTTP/1.1
45	239.007723992	192.168.122.1	239.255.255.250	SSDP	208	M-SEARCH * HTTP/1.1
46	240.009269948	192.168.122.1	239.255.255.250	SSDP	208	M-SEARCH * HTTP/1.1
47	259.158861508	192.168.122.43	91.189.91.157	NTP	90	NTP Version 4, server
48	259.258975524	91.189.91.157	192.168.122.43	NTP	90	NTP Version 4, server

Hops: 0  
 Transaction ID: 0xb0bc02e  
 Seconds elapsed: 1  
 Bootp flags: 0x0000 (Unicast)  
 Client IP address: 0.0.0.0  
 Your (client) IP address: 192.168.122.43 *a*  
 Next server IP address: 192.168.122.1  
 Relay agent IP address: 0.0.0.0  
 Client MAC address: 52:54:00:07:f7:e3:df *b*  
 Client hardware address padding: 00000000000000000000000000000000  
 Server host name not given  
 Boot file name not given

*default gateway. This value must be zero*

2) ARP

9 3.002276047	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
10 3.002614385	192.168.122.1	224.0.0.251	MDNS	82	Standard query 0x0000 PTR _googlecast._tcp.local, "QH" question
11 3.002614385	192.168.122.1	239.255.255.250	SSDP	86	57621 - 57621 Len=44
12 5.699377885	52.54:00:07:f7:e3:df	192.168.122.43	ARP	42	192.168.122.1 is at 52:54:00:26:63:d3
13 5.699405518	52.54:00:07:f7:e3:df	192.168.122.43	ARP	42	192.168.122.1 is at 52:54:00:26:63:d3
14 5.699599813	192.168.122.43	192.168.122.1	DNS	87	Standard query 0x4c5 A apl.snapcraft.io OPT
15 5.699599813	192.168.122.43	192.168.122.1	DNS	87	Standard query 0x4c48 AAAA apl.snapcraft.io OPT
16 5.699803865	192.168.122.43	192.168.122.43	DNS	87	Standard query response 0x9eaa AAAA apl.snapcraft.io OPT

Så:

42 Who has 192.168.122.1? Tell 192.168.122.43  
 42 192.168.122.1 is at 52:54:00:26:63:d3

Dig på 192. m fordel det til 19.2

192, her med MACadr, 52:54:

### 3) NTP

No.	ntp_time	Source	Destination	Protocol	Length	Info
24	34.380693425	192.168.122.43	91.189.91.157	NTP	98	NTP Version 4, client
25	34.488819221	91.189.91.157	192.168.122.43	NTP	98	NTP Version 4, server
27	66.657850615	192.168.122.43	91.189.91.157	NTP	98	NTP Version 4, client
28	66.758738234	91.189.91.157	192.168.122.43	NTP	98	NTP Version 4, server
37	130.907827292	192.168.122.43	91.189.91.157	NTP	98	NTP Version 4, client
38	131.007764506	91.189.91.157	192.168.122.43	NTP	98	NTP Version 4, server
47	259.158805185	192.168.122.43	91.189.91.157	NTP	98	NTP Version 4, client
48	259.258975514	91.189.91.157	192.168.122.43	NTP	98	NTP Version 4, server

Ber er 8 styks.

```
▼ Network Time Protocol (NTP Version 4, server)
  ▶ Flags: 0x24, Leap Indicator: no warning, Version number: NTP Version 4, Mode: server
  [Request In: 27]
  [Delta Time: 0.100887619 seconds]
  Peer Clock Stratum: secondary reference (2)
  Peer Polling Interval: 3 (8 seconds)
  Peer Clock Precision: -24 (0,000000060 seconds)
  Root Delay: 0,046432 seconds
  Root Dispersion: 0,023010 seconds
  Reference ID: 132.163.96.1
  Reference Timestamp: Dec 14, 2021 13:19:45.270827755 UTC
  Origin Timestamp: Dec 14, 2021 13:26:49.219124305 UTC
  Receive Timestamp: Dec 14, 2021 13:26:49.992623017 UTC
  Transmit Timestamp: Dec 14, 2021 13:26:49.992645397 UTC
```

Dato er er Dec 14 2021

Ref. timestamp er sidst vi spurte om klokken

#### Problem 2.

Assume that it takes 1 ms from a bit leaves A to the bit is received in B. Assume also that the bandwidth of the line is 5Mbit/s. How long does it take to transfer the frame of 1500 bytes from A to B? (round to nearest ms).



$$1500 \text{ byte} = 12000 \text{ bits}$$

$$5 \text{ Mbit/s} = 5 \cdot 10^6$$

$$\frac{1500 \cdot 8}{5 \cdot 10^6} = 0.0024 \text{ [s]} = 2,4 \text{ ms [s]}$$

$$\text{Med et delay på en ms, } = 3,4 = 4 \text{ ms [s]}$$

### Problem 3.

What is the difference between error detection and error correction. Give an example where it is more suitable to use error correction mechanisms.

I error detection kan man se at der er en fejl, men du kan ikke gøre noget ved det, men du kan spørge igen om det.

I correction, kan du se det OG FIXE det selv.

Det er smart der hvor det tager lang tid at sende frem og tilbage, så det er besværligt at skulle vente på et svar tilbage.

### Problem 4.

Which layer header contains

- a) MAC address L<sup>2</sup>
- b) Destination IP address L<sup>3</sup>
- c) SYN bits when a connection is established L<sup>4</sup>

OSI (Open Source Interconnection) 7 Layer Model

Layer	Application/Example	Central Device/Protocols	DOD4 Model
<b>Application (7)</b> Serves as the window for users and application processes to access the network services.	<b>End User layer</b> Program that opens what was sent or creates what is to be sent Resource sharing • Remote file access • Remote printer access • Directory services • Network management	User Applications SMTP	Process
<b>Presentation (6)</b> Formats the data to be presented to the Application layer. It can be viewed as the "Translator" for the network.	<b>Syntax layer</b> encrypt & decrypt (if needed) Character code translation • Data conversion • Data compression • Data encryption • Character Set Translation	JPEG/ASCII EBDIC/TIFF/GIF PICT	
<b>Session (5)</b> Allows session establishment between processes running on different stations.	<b>Synch &amp; send to ports</b> (logical ports) Session establishment, maintenance and termination • Session support - perform security, name recognition, logging, etc.	Logical Ports RPC/SQL/NFS NetBIOS names	
<b>Transport (4)</b> Ensures that messages are delivered error-free, in sequence, and with no losses or duplications.	<b>TCP</b> Host to Host, Flow Control Message segmentation • Message acknowledgement • Message traffic control • Session multiplexing	F P A C K E T T R A N S F I L T E R E R I N G TCP/SKP/UDP	Host to Host
<b>Network (3)</b> Controls the operations of the subnet, deciding which physical path the data takes.	<b>Packets</b> ("letter", contains IP address) Routing • Subnet traffic control • Frame fragmentation • Logical-physical address mapping • Subnet usage accounting	Routers IP/IPX/ICMP	Internet
<b>Data Link (2)</b> Provides error-free transfer of data frames from one node to another over the Physical layer.	<b>Frames</b> ("envelopes", contains MAC address) [NIC card — Switch — NIC card] (end to end) Establishes & terminates the logical link between nodes • Frame traffic control • Frame sequencing • Frame acknowledgement • Frame delimiting • Frame error checking • Media access control	Switch Bridge WAP PPP/SLIP	Can be used on all layers
<b>Physical (1)</b> Concerned with the transmission and reception of the unstructured raw bit stream over the physical medium.	<b>Physical structure</b> Cables, hubs, etc. Data Encoding • Physical medium attachment • Transmission technique - Baseband or Broadband • Physical medium transmission Bits & Volts	Hub	Land Based Layers

### Problem 6.

- a) Why authentication is needed? b) given that two devices share a common secret, how authentication can be done?

Authentication er dit ID-kort.

Authorazation er at du har adgang til det sted med dit ID-kort

- a) Så det er vigtigt at vi kun sender vores information der hen hvor at den skal, til dem vi stoler på.
- b) A, sender en random besked. B, incrypter og sender den retur. A ack. Så kan B sende random besked, A incrypter retur, og B ack. Så er alle sikre på at alle har den key, som man beder om.

# Sol-Networking

Friday, 3 January 2025 08.15



examExam  
pleNetwo...

Answers to the example problems.

Problem 1.

1. a) 192.168.122.43 b) 52:54:00:7f:e3:df
2. 192.168.122.1, 52:54:00:26:63:d3
3. a) 8 packets b) 2. December 14.

Problem 2.

Answer: 3,4ms

$$8 * 1500 \text{ bits} / (5 * 10^6) \text{ bits/sec} = 2,4 * 10^{-3} \text{ s} = 2,4 \text{ ms}$$

Time to send the frame 2,4ms + propagation time 1 ms = 3,4ms

Problem 4.

- a) L2
- b) L3
- c) L4

Problem 5.

To introduce priorities and to give higher priorities to the packets like ACK

# Ecx-mod1

Friday, 3 January 2025 08.15



old\_modula  
tion\_exam

### Problem 1

$v(t)$  denotes a baseband signal to be amplitude modulated:

$$v(t) = 0.5 \cdot \cos(200 \cdot \pi \cdot t) + 1.0 \cdot \cos(400 \cdot \pi \cdot t)$$

- a. The coefficients for an ordinary AM-modulation are:

1. amplitude sensitivity: 0,5
2. carrier frequency: 2 kHz
3. carrier amplitude: 10

Calculate the AM-signal  $v_{AM}(t)$  and plot the spectrum  $V_{AM}(f)$

- b. Determine the result of a double-sideband-suppressed-carrier modulation (DSB-SC) of  $v(t)$  – with a carrier frequency and a carrier amplitude similar to “a”.

I.e. calculate the signal  $v_{dsb-SC}(t)$  and plot the spectrum  $V_{dsb-SC}(f)$

- c. Compare the spectrum  $V_{AM}(f)$  with the spectrum  $V_{dsb-SC}(f)$

### Problem 2

Consider the signals  $x(t)$  and  $y(t)$ :

$$\begin{aligned}x(t) &= A_x(\sin(2\pi \cdot 5.000 \cdot t) + \text{sinc}(2 \cdot W_x \cdot t)); & \text{Bandwidth } W_x &\sim 8 \text{ kHz} \\y(t) &= A_y(\sin(2\pi \cdot 14.000 \cdot t) + \text{sinc}(2 \cdot W_y \cdot t)); & \text{Bandwidth } W_y &\sim 12 \text{ kHz}\end{aligned}$$

Determine the Nyquist sampling rate for:

- a. the signals  $x(t)$  and  $y(t)$
- b. the signals  $x^2(t)$  and  $y^2(t)$
- c. the signal  $x(t) \cdot y(t)$

### Problem 3

A FM signal with modulation index  $\beta = 2$  is transmitted through an ideal bandpass filter with center frequency  $f_c$ , (carrier amplitude 1 volt) and bandwidth  $7 \cdot f_m$ , where  $f_c$  is the carrier frequency and  $f_m$  is the frequency of the sinusoidal modulating wave.

Determine the magnitude spectrum of the filter output.

**Problem 1**

$v(t)$  denotes a baseband signal to be amplitude modulated:

$$v(t) = 0.5 \cdot \cos(200 \cdot \pi \cdot t) + 1.0 \cdot \cos(400 \cdot \pi \cdot t)$$

- a. The coefficients for an ordinary AM-modulation are:

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Calculate the AM-signal  $v_{AM}(t)$  and plot the spectrum  $V_{AM}(f)$

- b. Determine the result of a double-sideband-suppressed-carrier modulation (DSB-SC) of  $v(t)$  – with a carrier frequency and a carrier amplitude similar to "a".

i.e. calculate the signal  $v_{dsb-sc}(t)$  and plot the spectrum  $V_{dsb-sc}(f)$

- c. Compare the spectrum  $V_{AM}(f)$  with the spectrum  $V_{dsb-sc}(f)$

$$A) V(t) = 0.5 \cos(200 \cdot \pi \cdot t) + 1.0 \cos(400 \cdot \pi \cdot t)$$

$f=100$                                      $f=200$

$$V_{AM}(t) = A_c \cdot f_c \cdot V(t) = 10 \cdot \cos(2k \cdot 2\pi \cdot t) \cdot V(t)$$

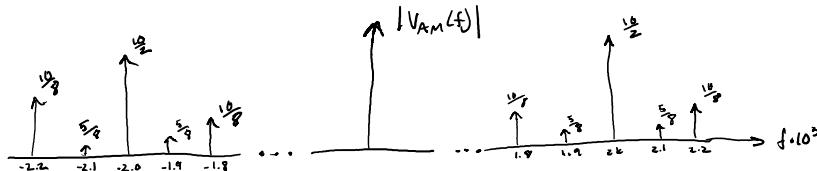
$A_c$                                      $f_c$                                      $V(t)$

$$V_{AM} = A_c [1 + k_a (V(t))] \cdot f_c$$

$$= 10 [1 + 0.5 (0.5 \cos(100 \cdot 2\pi t) + 1.0 \cos(200 \cdot 2\pi t))] \cos(2k \cdot 2\pi t)$$

$$10 \cos(2k \cdot 2\pi t) + 10 \frac{1}{2} \cos(100 \cdot 2\pi t) \cdot \cos(2k \cdot 2\pi t)$$

$$+ 10 \frac{1}{2} \cos(200 \cdot 2\pi t) \cdot \cos(2k \cdot 2\pi t)$$



$$\text{for } \cos(A) \cdot \cos(B) = \frac{\cos(A+B)}{2} + \frac{\cos(A-B)}{2}$$

aka, du får halvdelen af den positive og negative

B)

$$V_{AM} = A_c \cdot V_c(t) \cdot V(t)$$

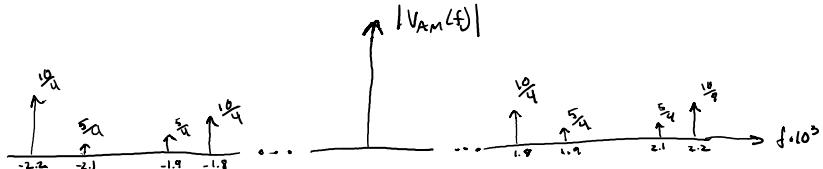
$$= 10 \cdot \cos(2k \cdot 2\pi t) \cdot \left[ \frac{1}{2} \cos(100 \cdot 2\pi t) + \cos(200 \cdot 2\pi t) \right]$$

$$= \frac{10}{2} \cos(100 \cdot 2\pi t) \cos(2k \cdot 2\pi t) + 10 \cos(200 \cdot 2\pi t) \cos(2k \cdot 2\pi t)$$

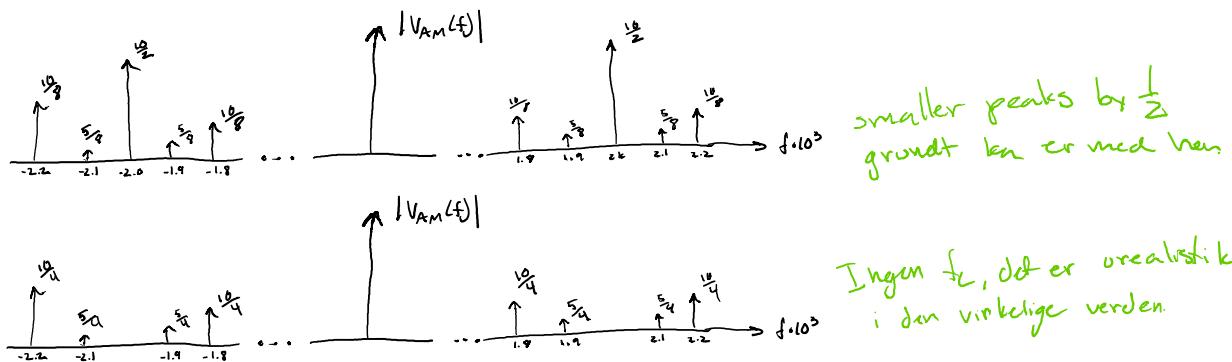
$$\frac{10}{4} \cos(2100 \cdot 2\pi t) + \frac{10}{4} \cos(1400 \cdot 2\pi t) +$$

$$\frac{10}{2} \cos(2200 \cdot 2\pi t) + \frac{10}{2} \cos(1800 \cdot 2\pi t)$$

og de findes  
for negative og pos  
med  $\pm$  amp af det



c)



### Problem 2

Consider the signals  $x(t)$  and  $y(t)$ :

$$x(t) = A_x(\sin(2\pi \cdot 5.000 \cdot t) + \text{sinc}(2 \cdot W_x \cdot t));$$

$$y(t) = A_y(\sin(2\pi \cdot 14.000 \cdot t) + \text{sinc}(2 \cdot W_y \cdot t));$$

Bandwidth  $W_x \sim 8 \text{ kHz}$   
Bandwidth  $W_y \sim 12 \text{ kHz}$

Determine the Nyquist sampling rate for:

- a. the signals  $x(t)$  and  $y(t)$
- b. the signals  $x^2(t)$  and  $y^2(t)$
- c. the signal  $x(t) \cdot y(t)$

$$A_x (\sin(5k \cdot 2\pi t) + \text{sinc}(8k \cdot 2\pi t))$$

$$A_x (\sin(14k \cdot 2\pi t) + \text{sinc}(12k \cdot 2\pi t))$$

A) for  $x$ ,  $8k > 5k$ , hence,  $8k \cdot 2 = 16 \text{ kHz}$

for  $y$   $14k > 12k$  hence  $14k \cdot 2 = 28 \text{ kHz}$

(B)  $x^2$  &  $y^2$   $f \cdot f = \underline{\underline{H_2 + H_2}}$

for  $x$   $8k \cdot 2 = 16k$   $16 \cdot 2 = \underline{\underline{32 \text{ kHz}}}$

for  $y$   $14k \cdot 2 = 28k$   $28k \cdot 2 = \underline{\underline{56 \text{ kHz}}}$

$$\text{for } \gamma \quad 14k\omega_2 = 28k \quad 28k \cdot 2 = \underline{\underline{56k \text{ Hz}}}$$

C)  $x \cdot y$

$$8k + 14k = 22k \quad 22k \cdot 2 = \underline{\underline{44k \text{ Hz}}}$$

# Sol-mod1

Saturday, 4 January 2025 08.09



old\_modula  
tion\_exa...

(1)

# EXAM : INTRODUCTION TO WIRELESS COMMUNICATION

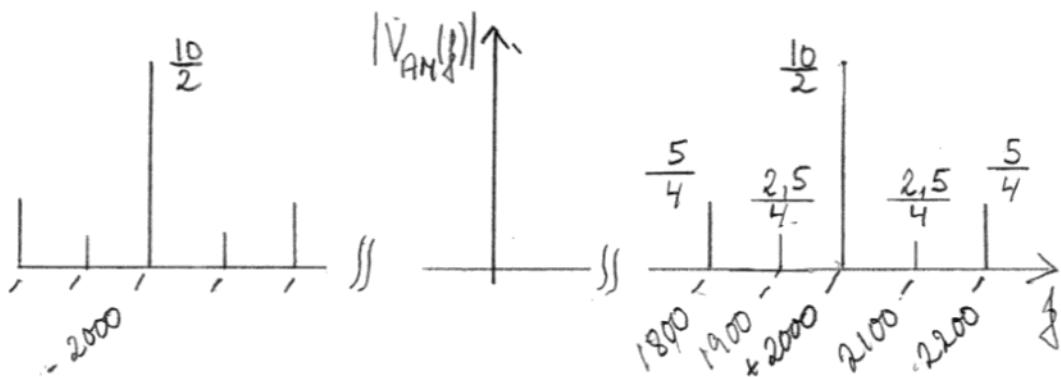
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## PROBLEM 1 :

a)  $\underline{v_{AM}(t)} = A_c [1 + k_a \cdot n(t)] \cdot \cos(2\pi f_c t)$

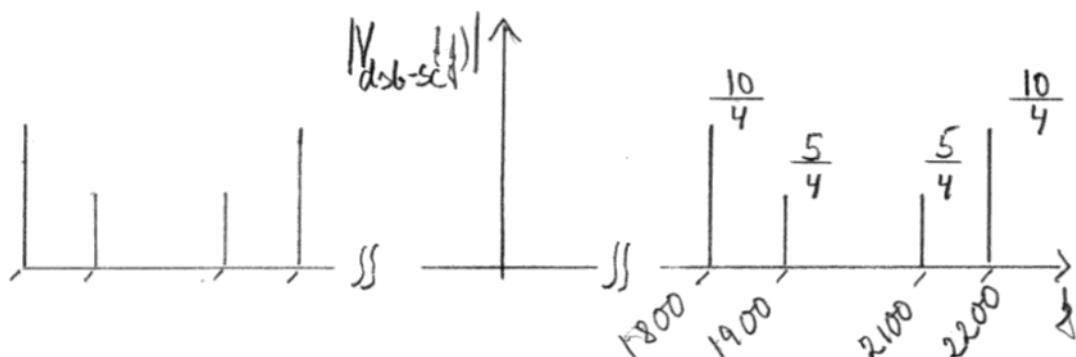
$$= 10 [1 + 0,5(0,5 \cdot \cos(200\pi t) + \cos(400\pi t))] \cdot \cos(2\pi \cdot 2 \cdot 10^3 t)$$

$$= 10 [1 + 0,25 \cos(2\pi \cdot 100t) + 0,5 \cdot \cos(2\pi \cdot 200t)] \cdot \cos(2\pi \cdot 2 \cdot 10^3 t)$$



b)  $\underline{n_{dsb-sc}(t)} = A_c \cdot n(t) \cdot \cos(2\pi \cdot f_c \cdot t)$

$$= 10 \cdot (0,5 \cdot \cos(2\pi \cdot 100t) + \cos(2\pi \cdot 200t)) \cdot \cos(2\pi \cdot 2 \cdot 10^3 t)$$

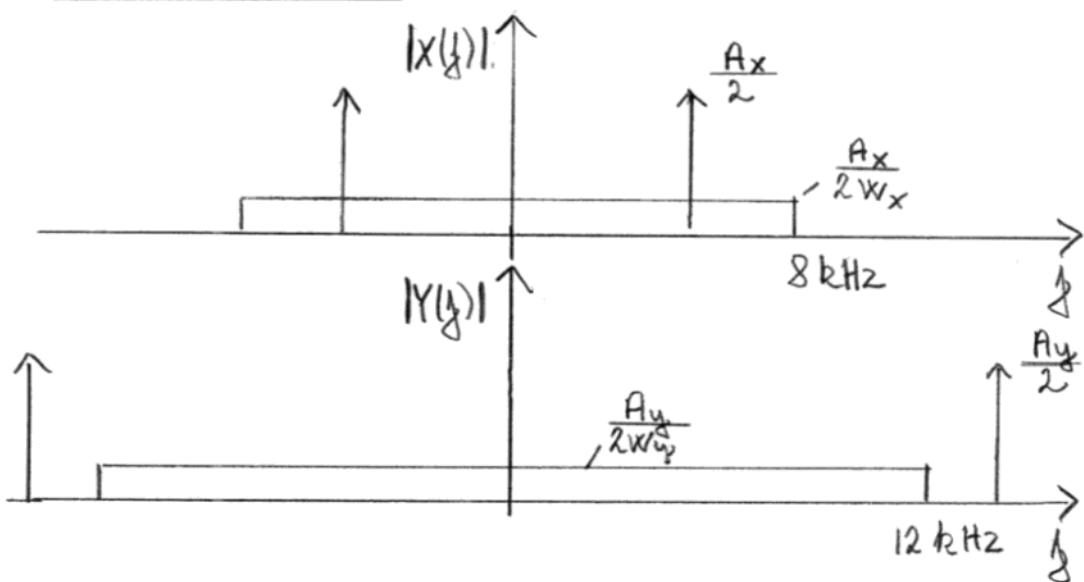


c)  $\dots$

---

PROBLEM 2:

(2)



The Nyquist sampling rates:

a)  $X(t) : 16 \text{ kHz} \wedge Y(t) : 28 \text{ kHz}$

b)  $X^2(t) : 32 \text{ kHz} \wedge Y^2(t) : 56 \text{ kHz}$

c)  $X(t) \cdot Y(t) : 44 \text{ kHz}$

(b and c solved by convolution in the frequency-domain)

=====

PROBLEM 3:

(3)

For  $\beta = 2$  ( $= \frac{\Delta f}{f_m}$ , wideband FM) we have:

$$J_0(2) \sim 0,27$$

$$J_1(2) \sim 0,57$$

$$J_2(2) \sim 0,36$$

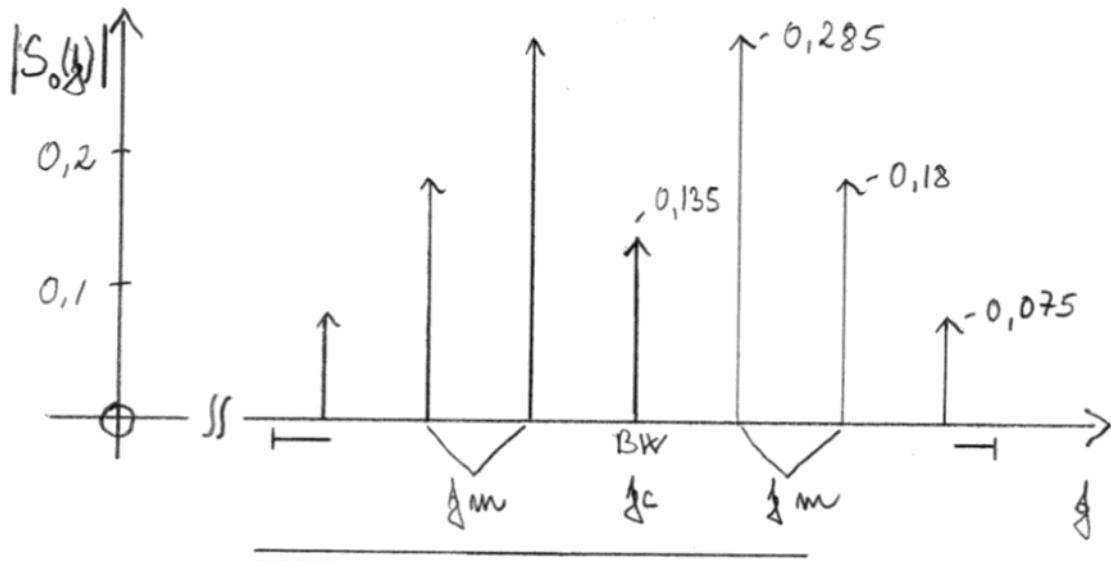
$$J_3(2) \sim 0,15$$

$$(J_4(2) \sim 0,03)$$

The output of the bandpass filter with the bandwidth  $7 \cdot f_m$  is approximately (J-values):

$$\begin{aligned} S_o(t) &= 0,27 \cos(2\pi f_c t) \\ &+ 0,57 (\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)) \\ &+ 0,36 (\cos(2\pi(f_c + 2f_m)t) + \cos(2\pi(f_c - 2f_m)t)) \\ &+ 0,15 (\cos(2\pi(f_c + 3f_m)t) + \cos(2\pi(f_c - 3f_m)t)) \end{aligned}$$

The magnitude spectrum of  $S_o(t)$  for positive frequencies:





## Extra exercises

### Exercise 1

Your friend has bought two Icom IC-9100, a multiple band HF/VHF/UHF radio capable of doing FM modulation and demodulation. He is using them to transmit a modulating signal  $m(t) = 2 \cos(2\pi 100t)$ . The power of the FM signal is 50 W, and it has a carrier frequency of  $f_c = 100$  MHz and a peak frequency deviation  $\Delta f = 10$  kHz.

- Write the waveform expression for the FM waveform without any integral
- Determine the approximated bandwidth of the FM waveform. Is the modulated signal a Narrow-Band or a Wide-Band Signal? (explain)

Your friend is very curious about the possibilities of the Icom IC-9100, and he is now considering the option of digitalizing the information and using BFSK as modulation scheme, with a space frequency of  $f_{space} = 99.95$  MHz and a mark frequency of  $f_{mark} = 100.05$  MHz.

- If the digital signal is transmitted at a rate of  $R_b = 3$  kbps, how much bandwidth is needed for the BFSK transmission?

### Exercise 2

Consider the baseband signal  $m(t) = 2 \cos(2\pi 100t) - \cos(2\pi 500t)$ . The signal is AM-modulated by a carrier wave  $c(t)$ .

- Determine the resulting (traditional) AM-signal  $v_{AM}(t)$  and plot the amplitude spectrum  $V_{AM}(f)$  when  $f_c = 1500$  kHz, the carrier amplitude is  $A_c = 10$  and the amplitude sensitivity is  $k_a = 0.2$ .
- Determine the result of a double-sideband-suppressed-carrier modulation (DSB-SC) of  $m(t)$  – with the carrier amplitude decreased by a factor of five compared to the amplitude modulated (AM) signal: calculate the signal  $v_{dsb-SC}(t)$  and plot the amplitude spectrum  $V_{dsb-SC}(f)$ .
- In view of the results, explain briefly the advantages and disadvantages of DSB-SC versus standard AM.

### Exercise 3

Consider the following baseband signal  $v(t) = 3 \sin(2\pi 200t) - 2 \sin(2\pi 800t)$

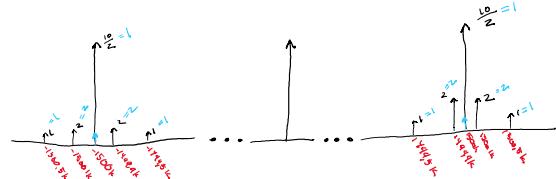
The signal is modulated by the carrier wave  $c(t)$  in an AM-transmitter:

$c(t) = A_c \cos(2\pi f_c t)$ . The power of the carrier signal is 8 W.

$$V_{AM}(t) = A_c [1 + k_a \cdot V(t)] v_c(t)$$

$$\begin{aligned} &= 10 [1 + 0.2 (2 \cos(100 \cdot 2\pi t) - \cos(500 \cdot 2\pi t)) \cos(1500 \cdot 2\pi t)] \\ &= 10 \cos(1500 \cdot 2\pi t) + 10 \frac{2}{5} \cos(100 \cdot 2\pi t) \cos(1500 \cdot 2\pi t) - \\ &\quad 10 \frac{1}{5} \cos(500 \cdot 2\pi t) \cos(1500 \cdot 2\pi t) \\ &= 10 \cos(1500 \cdot 2\pi t) + \frac{20}{10} \cos(1500 \cdot 2\pi t) + \frac{20}{10} \cos(1499.4 \cdot 2\pi t) - \\ &\quad \frac{10}{10} \cos(1500.5 \cdot 2\pi t) - \frac{10}{10} \cos(1499.5 \cdot 2\pi t) \end{aligned}$$

Og i b1 med  $\frac{1}{5}$  suppression da de blå tall på tegningen nede under



- Determine the maximum usable amplitude sensitivity  $k_a$  for an ordinary AM-modulation
- Determine the resulting AM-signal  $v_{AM}(t)$  and plot the amplitude spectrum  $V_{AM}(t)$  for  $f_c = 1500$  kHz.

#### Exercise 4

An FM signal with carrier frequency  $f_c = 750$  kHz is generated in an FM system with frequency sensitivity  $k_f = 15$  kHz/V. For a sinusoidal modulating signal of the form  $m(t) = A_m \cos(2\pi f_m t)$  with  $f_m = 25$  kHz and  $A_m = 5$  V, the FM signal is given by:

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

where  $\beta$  is the modulation index.

- Determine the values of the frequency deviation  $\Delta f$ , the modulation index  $\beta$ , and plot/sketch the amplitude spectrum
- Via an ideal multiplicator, the FM signal  $s(t)$  is now squared. Determine the values of the frequency deviation and the modulation index of the resulting signal  $s^2(t)$ , and plot/sketch the amplitude spectrum of this signal

#### Exercise 5

A communication system transmits at 100 kbps. For each of the following modulation types, determine the bandwidth of the transmission.

- FSK, with frequency deviation 200 kHz.
- OOK
- QPSK
- 16-PSK
- 16-QAM
- 512-QAM

a)  
 $F_{6k} = 2 \cdot R_{sym} + 2 \cdot B_R \rightarrow 2 \cdot 200k + 2 \cdot 100k \text{ Hz} = 600k \text{ Hz}$

b)  
 $600k$ , has 2 modes.

c)  
 $BW_{ook} = \frac{2R_b}{\log_2(2)} = \frac{200k}{1} = 200k \text{ Hz}$

c)  
 $BW_{qpsk} = \frac{200k}{\log_2(4)} = 100k \text{ Hz}$

d)  
 $BW_{16psk} = \frac{200k}{\log_2(16)} = 50k \text{ Hz}$

e)  
 $BW_{16qam} = \frac{200k}{\log_2(16)} = 222k \text{ Hz}$

# Sol-mod2

Saturday, 4 January 2025 08.09



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# EIT5 solutions to extra exercises

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## EXERCISE 1

We are given  $m(t) = 2 \cos(2\pi 1000t)$ ,  $f_c = 100$  MHz, and  $\Delta f = 10$  kHz. Therefore:

- $A_m = 2$  V
- $f_m = 1000$  Hz
- $P_c = 50$  W and  $P_c = A_c^2/2$ . Hence,  $A_c = \sqrt{P_c/2} = 5$  V
- $k_f = 10/2 = 5$  kHz/V
- $\beta = \Delta f/f_m = 10$

a) The FM waveform is given as

$$s_{\text{FM}}(t) = A_c \cos \left[ 2\pi \left( f_c t + k_f \int_0^t m(\tau) d\tau \right) \right] \quad (1)$$

Then, we calculate the integral

$$\int_0^t m(\tau) d\tau = \int_0^t 2 \cos(2\pi 1000\tau) d\tau = \frac{\sin(2\pi 1000t)}{\pi 1000} \quad (2)$$

And so, we get the FM waveform

$$s_{\text{FM}}(t) = 5 \cos \left[ 2\pi \left( 10^7 t + \frac{5 \sin(2\pi 1000t)}{\pi 1000} \right) \right] \quad (3)$$

b) The bandwidth of an FM signal can be approximated using the Carson's rule as

$$B_{\text{FM}} \approx 2\Delta f + 2f_m^* \quad (4)$$

Since we have a single modulating carrier  $f_m^* = f_m$  and we get

$$B_{\text{FM}} \approx 2 \times 10^4 + 2 \times 10^3 = 22 \text{ kHz} \quad (5)$$

Since  $\beta = 10$ , we have a wideband FM signal

c) We are given  $f_{\text{space}} = 99.95$  MHz and  $f_{\text{mark}} = 100.05$  MHz and  $R_b = 3$  kbps.

The bandwidth of a BFSK signal is approximated as

$$B_{\text{BFSK}} \approx f_{\text{mark}} - f_{\text{space}} + 2R_{\text{sym}} \quad (6)$$

So, we need to find  $R_{\text{sym}} = R_b / \log_2 M$ . Since the signal is binary (BFSK), the number of symbols  $M = 2$  and we get  $R_{\text{sym}} = 3$  bauds.

$$B_{\text{BFSK}} \approx 100.05 \cdot 10^6 - 99.95 \cdot 10^6 + 2 \cdot 3 \cdot 10^3 = 106 \text{ kHz} \quad (7)$$

### EXERCISE 2

We are given  $m(t) = 2 \cos(1\pi 100t) - \cos(2\pi 500t)$ .

a) The carrier is  $c(t) = 10 \cos(2\pi 1.5 \cdot 10^6 t)$  and the amplitude sensitivity is  $k_a = 0.2$ .

The traditional AM (DSBAM) modulated signal is

$$\begin{aligned} v_{\text{AM}}(t) &= A_c (1 + k_a m(t)) \cos(2\pi f_c t) \\ &= 10 \left( 1 + 0.4 \cos(2\pi 100t) - 0.2 \cos(2\pi 500t) \right) \\ &\quad \times \cos(2\pi 1.5 \cdot 10^6 t) \end{aligned} \quad (8)$$

And, hence, its spectrum is

$$\begin{aligned} V_{\text{AM}}(f) &= 5 (\delta(f - 1.5 \cdot 10^6) + \delta(f + 1.5 \cdot 10^6)) \\ &\quad + (\delta(f - 1.5 \cdot 10^6 - 100) + \delta(f + 1.5 \cdot 10^6 + 100)) \\ &\quad + (\delta(f - 1.5 \cdot 10^6 + 100) + \delta(f + 1.5 \cdot 10^6 - 100)) \\ &\quad - 0.5 (\delta(f - 1.5 \cdot 10^6 - 500) + \delta(f + 1.5 \cdot 10^6 + 500)) \\ &\quad - 0.5 (\delta(f - 1.5 \cdot 10^6 + 500) + \delta(f + 1.5 \cdot 10^6 - 500)) \end{aligned} \quad (9)$$

b) Now the amplitude of the carrier is  $A_c = 2$  V

The DSB-SC AM signal is

$$\begin{aligned} v_{\text{DSB-SC}}(t) &= A_c k_a m(t) \cos(2\pi f_c t) \\ &= (0.8 \cos(2\pi 100t) - 0.4 \cos(2\pi 500t)) \cos(2\pi 1.5 \cdot 10^6 t) \end{aligned} \quad (10)$$

An its spectrum is

$$\begin{aligned} V_{\text{DSB-SC}}(f) &= 0.2 (\delta(f - 1.5 \cdot 10^6 - 100) + \delta(f + 1.5 \cdot 10^6 + 100)) \\ &\quad + 0.2 (\delta(f - 1.5 \cdot 10^6 + 100) + \delta(f + 1.5 \cdot 10^6 - 100)) \\ &\quad - 0.1 (\delta(f - 1.5 \cdot 10^6 - 500) + \delta(f + 1.5 \cdot 10^6 + 500)) \\ &\quad - 0.1 (\delta(f - 1.5 \cdot 10^6 + 500) + \delta(f + 1.5 \cdot 10^6 - 500)) \end{aligned} \quad (11)$$

c) DSB-SC is much more power efficient: all the power is in the sidebands (no carrier). On the other hand,

$$\frac{5^2}{5^2 + 1^2 + 1^2 + (-0.5)^2 + (-0.5)^2} = 0.909 \quad (12)$$

of the power is in the carrier for DSBAM. On the downside, the simple envelope detector cannot be used for DSB-SC.

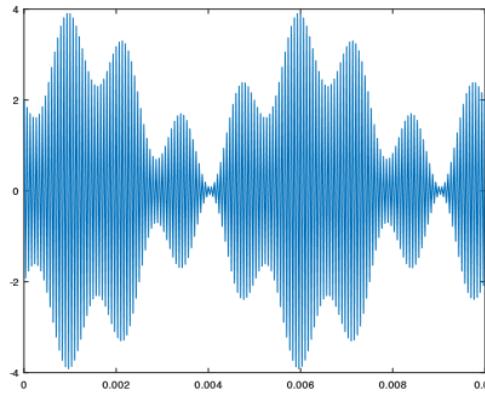


Fig. 1. Signal for exercise 3 with  $k_a = 1/5$ .

### EXERCISE 3

**This type of exercise will not be in the exam**

We are given (changed the signal's name)  $m(t) = 3 \sin(2\pi 200t) - 2 \sin(2\pi 800t)$ .

The signal is modulated with  $c(t) = A_c \cos(2\pi f_c t)$  with power  $P_c = 8$  W.

1. As in the first exercise, we get the amplitude of the carrier as  $A_c = \sqrt{P_c/2} = 2$  V.

For ordinary AM modulation we need to avoid phase reversals, which occur when  $\mu = k_a A_m > 1$ . In other words, we need to ensure that

$$|k_a m(t)| < 1 \quad \text{for all } t \quad (13)$$

We can rewrite  $k_a m(t)$  as

$$x(t) = 3k_a \sin(2\pi 200t) - 2k_a \sin(2\pi 800t) \quad (14)$$

We can approximate the maximum of  $|x(t)| \approx 4.79$  using numerical methods. Therefore, we have that  $k_a \leq 1/4.79$ .

2. We will make  $k_a = 1/5$ . Hence, the modulated signal is given by

$$\begin{aligned} v_{\text{AM}}(t) &= A_c (1 + k_a m(t)) \cos(2\pi f_c t) \\ &= 2 \left( 1 + \frac{3}{5} \sin(2\pi 200t) - \frac{2}{5} \sin(2\pi 800t) \right) \\ &\quad \times \cos(2\pi 1.5 \cdot 10^6 t) \end{aligned} \quad (15)$$

Then, the spectrum of this signal is

$$\begin{aligned} V_{\text{AM}}(f) &= (\delta(f - 1.5 \cdot 10^6) + \delta(f + 1.5 \cdot 10^6)) \\ &\quad + \frac{6}{20j} (\delta(f - 1.5 \cdot 10^6 - 200) + \delta(f + 1.5 \cdot 10^6 + 200)) \\ &\quad + \frac{6}{20j} (\delta(f - 1.5 \cdot 10^6 + 200) + \delta(f + 1.5 \cdot 10^6 - 200)) \\ &\quad - \frac{4}{20j} (\delta(f - 1.5 \cdot 10^6 - 800) + \delta(f + 1.5 \cdot 10^6 + 800)) \\ &\quad - \frac{4}{20j} (\delta(f - 1.5 \cdot 10^6 + 800) + \delta(f + 1.5 \cdot 10^6 - 800)) \end{aligned} \quad (16)$$

#### EXERCISE 4

We are given  $f_c = 750$  kHz and  $k_f = 15$  kHz/V plus the modulating signal

$$m(t) = 5 \cos(2\pi 25 \cdot 10^3 t) \quad (17)$$

and the modulated signal

$$s(t) = A_c \cos(2\pi 750 \cdot 10^3 t + \beta \sin(2\pi 25 \cdot 10^3 t)) \quad (18)$$

a. The frequency deviation is  $\Delta f = k_f A_m = 75$  kHz, the modulation index is  $\beta = \Delta f / f_m = 3$ . Due to the Carson's rule, 98% of the spectrum is contained within

$$B_{\text{FM}} \approx 2\Delta f + 2f_m^* = 150 \cdot 10^3 + 20 \cdot 10^3 = 170 \text{ kHz} \quad (19)$$

centered at frequency  $f_c = 750$  kHz.

b) To square the signal we need

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 1 - 2\sin^2(A) = 2\cos^2(A) - 1 \quad (20)$$

Then

$$\cos^2(A) = \frac{\cos(2A) + 1}{2} \quad (21)$$

The term  $1/2$  is simply a DC shift.

$$s^2(t) = \frac{A_c}{2} \cos(2\pi(2 \cdot 750 \cdot 10^3)t + 2\beta \sin(2\pi 25 \cdot 10^3 t)) + \frac{1}{2} \quad (22)$$

We have that  $\Delta f = 2\beta f_m = 6 * 25 \cdot 10^3 = 150$  kHz and the modulation index is  $\beta' = 2\beta = 6$ . The spectrum now has a delta at frequency 0 due to the DC shift.

## EXERCISE 5

We have that

$$R_b = NR_{\text{sym}} = R_{\text{sym}} \log_2 M = 100 \text{ kbps} \quad (23)$$

Then, for all the modulations except FSK, the bandwidth is  $B = 2R_{\text{sym}}$ .

a)  $M = 2$ , so, the bandwidth for BFSK is

$$B_{\text{BFSK}} \approx f_{\text{mark}} - f_{\text{space}} + 2R_{\text{sym}} = 2 \cdot 200 \cdot 10^3 + 2 \cdot 100 \cdot 10^3 = 600 \text{ kHz} \quad (24)$$

b)  $M = 2$  so, the bandwidth is

$$B_{\text{OOK}} = 2R_{\text{sym}} = \frac{2R_b}{\log_2 M} = 200 \text{ kHz} \quad (25)$$

c)  $M = 4$  so, the bandwidth is

$$B_{\text{QPSK}} = 2R_{\text{sym}} = \frac{2R_b}{\log_2 4} = 100 \text{ kHz} \quad (26)$$

d)  $M = 16$  so, the bandwidth is

$$B_{\text{16-PSK}} = 2R_{\text{sym}} = \frac{2R_b}{\log_2 16} = 50 \text{ kHz} \quad (27)$$

e) Same as in d)

f)  $M = 512$  so, the bandwidth is

$$B_{\text{512-QAM}} = 2R_{\text{sym}} = \frac{2R_b}{\log_2 512} = 22.222 \text{ kHz} \quad (28)$$