Ecx1

Wednesday, 4 September 2024 10.20

Problem 1.1.

A hypothetical isotropic antenna is radiating in freespace. At a distance of 100 meters from the antenna the total electrical field (E_Θ) is measured to be 5 V/m. Find the

a) Power density (W_{rad})

b) Power radiated (P_{rad})

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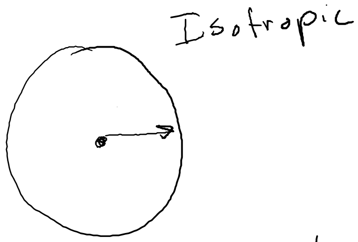

a) Power density

$$U(\theta,\varphi) = \frac{r^2}{2\eta}|E(r,\theta,\varphi)|^2 = U = \frac{r^2}{2\eta}|5[V/m]|^2 = \frac{3125}{3\pi} \approx 331.58 \left[\frac{W}{sterad}\right]$$

$\eta = 120\pi$ $\theta_1 = 0,\ \varphi = 0$ da den er isotopisk

$$U = r^2 \cdot W_{rad}$$

$$W_{rad} = \frac{U}{r^2} = \frac{5}{100^2} = 0.033 \left[\frac{W}{m^2}\right]$$

b)

$$P_{rad} = P_{av} = \oiint W_{rad}\, ds$$

$$P_{rad} = W_{rad}\, 4\pi r^2 = 0.033 \cdot 4\pi \cdot 100^2 = \frac{12500}{3} = 4166.7\ [W]$$

Problem 1.2

The maximum radiation intensity of a 90% efficient antenna is 200 mW/unit solid angle.
Find the directivity and gain (dimensionless and in dB) when the:

a) Input power is 125.66 mW
b) Radiated power is 125.66 mW

A)

$$D_0 = \frac{U_{max}}{\frac{P_{rad}}{4\pi}} = \frac{U_{max}}{U_0}$$

$$P_{rad} = P_{in} \cdot e_t$$

$P_{in} = 125.66$ [mW]
$e_t = 90\% = 0.90$

$$P_{rad} = 125.66 \cdot 0.9 = 113.01 \text{ [mW]}$$

$$D_0 = \frac{200 \text{ [mW]}}{\frac{113.01}{4\pi}} = \frac{200 \text{ [mW]} \cdot 4\pi}{113.01} = \underline{\underline{22.22}} \text{ [-]}$$

same, bare i dB

$$10 \cdot \log_{10}(22.22) = \underline{\underline{13.15}} \text{ dB}$$

$$G_0 = e_t \cdot D_0 = 0.90 \cdot 22.22 = \underline{\underline{20}}$$

$$10 \cdot \log_{10}(20) = \underline{\underline{13 \text{ dB}}}$$

B

For radiating er det vel effektivt og derfor ingen 0.9 faktor på.

$$D_0 = \frac{200 \cdot 10^{-3} \; 4\pi}{125{,}66 \cdot 10^{-3}} = 20 = 13.01 \text{ dB}$$

$$G_0 = e_{cd} \cdot D_0 = 0.90 \cdot 20 = 18 = 12.55 \text{ dB}$$

Problems

Problem 1.3

In target-search ground-mapping radars it is desirable to have ecco power received from target, of constant cross section, to be independent of its range. For one such application, the desireble radiation intensity of the antenna is given by

$U(\Theta,\Phi) = 1$ for $0^\circ \leq \Theta < 20^\circ$

$U(\Theta,\Phi) = 0{,}342 \csc(\Theta)$ for $20^\circ \leq \Theta < 60^\circ$

$U(\Theta,\Phi) = 0$ for $60^\circ \leq \Theta \leq 180^\circ$

Find the directivity in dB using the exact formula.

$$P_{rad} = 2\pi \left[\int_0^{20^\circ} \sin(\theta) \cdot d\theta + \int_{20^\circ}^{60^\circ} 0.346 \cdot \csc(\theta) \cdot \sin\theta \, d\theta \right] = 1.879$$

$$D_0 = \frac{4\pi}{P_{rad}} = 6.68 = 8.25 \text{ dB}$$

restart

OBS: remember to convert degree bounds to rad, csc() = 1/sin()

$$U := \begin{cases} 1 & 0 \le \theta < 20 \cdot \frac{\pi}{180} \\ \begin{cases} 0.342 \cdot \frac{1}{\sin(\theta)} & 20 \cdot \frac{\pi}{180} \le \theta < 60 \cdot \frac{\pi}{180} \\ 0 & \textit{otherwise} \end{cases} & \textit{otherwise} \end{cases} :$$

If U is given for a normalized radiation intensity, U_max = 1

$U_{max} := 1 :$

When calculating directivity, A_0 = U_max

$A_0 := U_{max} :$

$$P_{rad} := \left| A_0 \cdot \int_0^{2 \cdot \pi} \int_0^{\frac{\pi}{2}} U \cdot \sin(\theta) \, d\theta \, d\phi \right| = 1.879102308$$

Unitless directivity:

$$D_0 := \frac{4 \cdot \pi \cdot U_{max}}{P_{rad}} = 6.687432908$$

Directivity in dBi:

$$D_{0-dBi} := 10 \cdot \log_{10}(D_0) = 8.252594380 \xrightarrow{\text{at 5 digits}} 8.2526$$

Sol1

Thursday, 2 January 2025 11.29

Lecture1SolutionsCS

Problem 1.1

a) $$\underline{W}_{rad} = \frac{1}{2}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta} \cdot \vec{a}_r = \frac{5^2\,\vec{a}_r}{2 \cdot 120 \cdot \pi} = 0.03315\,\vec{a}_r \;\; Watt/m^2$$

b) $$P_{rad} = \oint_s W_{rad} \cdot dS = \int_0^{2\pi}\int_0^{\pi} 0.03315 \cdot (r^2 \sin(\theta) \cdot d\theta \cdot d\varphi =$$

$$\int_0^{2\pi}\int_0^{\pi} 0.03315 \cdot (100^2 \sin(\theta) \cdot d\theta \cdot d\varphi =$$

$$2\pi \cdot 0.03315 \cdot 100^2 \cdot \int_0^{\pi} \sin(\theta) \cdot d\theta = 4165.75 \quad Watt$$

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Problem 1.2

a)

$$D_0 = \frac{4\pi \cdot U_{\max}}{P_{rad}} = \frac{4\pi(200 \cdot 10^{-3})}{0.9 \cdot (125.66 \cdot 10^{-3})} = 22.22 = 13.47dB$$

$$G_0 = e_{cd} \cdot D_0 = 0.9 \cdot 22.22 = 20 = 13.01dB$$

b)

$$D_0 = \frac{4\pi \cdot U_{\max}}{P_{rad}} = \frac{4\pi(200 \cdot 10^{-3})}{125.66 \cdot 10^{-3}} = 20 = 13.01dB$$

$$G_0 = e_{cd} \cdot D_0 = 0.9 \cdot 20 = 18 = 12.55dB$$

Problem 1.3

$$U(\theta,\phi)=\begin{Bmatrix} 1 & 0^\circ \le & \theta & \le 20^\circ \\ 0.342 & 20^\circ \le & \theta & \le 60^\circ \\ 0 & 60^\circ \le & \theta & \le 180^\circ \end{Bmatrix} \qquad 0^\circ \le \phi \le 360^\circ$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\cdot\sin(\theta)\cdot d\theta\cdot d\phi = 2\pi[\int_0^{20^\circ}\sin(\theta)\cdot d\theta + \int_0^{20^\circ} 0.342\cdot\csc(\theta)\cdot\sin(\theta)\cdot d\theta]$$

$$= 2\pi\left\{-\cos(\theta)\ \Big|_0^{\pi/9} + 0.342\cdot\theta\ \Big|_{\pi/9}^{\pi/3}\right\} = 1.87912$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi}{1.87912} = 6.68737 = 8.25dB$$

Problem 1.4

a) $$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\cdot\sin(\theta)\cdot d\theta\cdot d\phi = \int_0^{2\pi}\sin^2(\phi)\cdot d\phi\cdot\int_0^{\frac{\pi}{2}}\cos^4(\theta)\cdot\sin(\theta)\cdot d\theta = \frac{\pi}{5}$$

$$U_{\max} = U(\theta=0,\phi=\frac{\pi}{2}) = 1$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi}{\pi/5} = 20 = 13.0dB$$

b) Elevation plane: theta varies, phi fixed. => choose $\phi = \frac{\pi}{2}$

$$U(\theta,\phi=\pi/2) = \cos^4(\theta) \qquad 0 \le \theta \le \pi/2$$

$$\cos^4\left[\frac{HPBW(elevation)}{2}\right] = \frac{1}{2}$$

$$HPBW(elevation) = 2\cdot\cos^{-1}(\sqrt{0.5}) = 65.5^\circ$$

Problems

2.1 For an X-band (8.2 – 12.4 GHz) rectangular horn, with aperture dimension of 5,5 cm and 7,4 cm , find its maximum effective aperture in cm^2 when its gain over isotropic is:

a) 14,8 dB

b) 16,5 dB

c) 18,0 dB

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Communication Systems 5. semester.
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$$A_r = \underbrace{e_t D_r(\theta_r, \phi_r)}_{\text{Gain}} \frac{\lambda^2}{4\pi}$$

$$\lambda_1 = \frac{3 \cdot 10^8}{8.2 \cdot 10^9} = 0.036 \ [m]$$

$$A_r = 10^{1.48} \cdot \frac{\lambda^2}{4\pi}$$

$$\lambda_2 = \frac{3 \cdot 10^8}{12.4 \cdot 10^9} = 0.024 \ [m]$$

$$14.8 \ dB_i \longrightarrow 10^{1.48}$$

$$A_{r_1} = 10^{1.48} \cdot \frac{(0.036)^2}{4\pi} = 3{,}22 \cdot 10^{-3} \ [m]$$

$$A_{r_2} = 10^{1.48} \frac{(0.024)^2}{4\pi} = 4{,}09 \cdot 10^{-3} \ [m]$$

$$A_{r_1} = 10^{1.65} \frac{(0.036)^2}{4\pi} =$$

$$A_{r_2} = 10^{1.65} \frac{(0.024)^2}{4\pi} =$$

$$A_{r_2} = 10^{1.65}\frac{(0.024)^2}{4\pi} =$$

And so on for each of them

bla bla bla

Problem 2.72, Balanis 3nd edition

$$A_{em} = \frac{\lambda^2}{4\pi}e_t D_0 = \frac{\lambda^2}{4\pi}G_0$$

(a) $G_0 = 14.8dB \Rightarrow G_0(power\ ratio) = 10^{1.48} = 30.2$

$f = 8.2GHz \Rightarrow \lambda = 3.6585cm$

$$A_{em} = \frac{(3.6585)^2}{4\pi}\cdot 30.2 = 32.167cm^2$$

$A_p = 5.5\cdot 7.4 = 40.7cm^2$

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Problem 2.72, Balanis 3nd edition

(b) $G_0 = 16.5dB \Rightarrow G_0(power\ ratio) = 10^{1.65} = 44.668$

$f = 10.3GHz \Rightarrow \lambda = 2.912cm$

$$A_{em} = \frac{(2.912)^2}{4\pi}\cdot 44.668 = 30.142cm^2$$

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Problem 2.72, Balanis 3nd edition

(c) $G_0 = 18.0dB \Rightarrow G_0(power\ ratio) = 10^{1.8} = 63.096$

$f = 12.4GHz \Rightarrow \lambda = 2.419cm$

$$A_{em} = \frac{(2.419)^2}{4\pi}\cdot 63.096 = 29.389cm^2$$

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Problems

2.2 A communication satellite is in the stationary orbit about the earth (22.300 statute miles ~ 36.000 km). Its transmitter generates 8 Watt. Assume the transmitting antenna is isotropic. Its signal is received by a 210 foot diameter tracking parabol antenna on the earth. Also assume no resistive losses in either antenna, perfect polarization match and perfect impedance matching at both antennas. At a frequency of 2 GHz, determinate the:

a) Power density in Watts/m² incident on the receiving antenna.

b) Power received by the ground based antenna whose gain is 60 dBi

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Okay så her ved vi at den er isotropisk og derfor er der 8Watt fordelt på en FUCKER stor kugle! Vi skal bare finde svaret per m^2 for den fucker store kugle

$$dist = 36.000\ km = 36\cdot 10^6\ [m]$$

Isotropic, with 8 watt

area of circle

Parabol area = 210 feet = 64 [m] → 3217 [m²]

fra opgaven

8 ... 10^{-18} [W/...]

Parabol area = 210 feet ...

a) $W_T = \frac{8}{\underbrace{4\pi(36\cdot 10^6)^2}_{\text{surface area of sphere}}} = 491.22\cdot 10^{-18}$ [W/m²]

fra oppgaven

b) plugg fra formlen fra før

$\lambda = \frac{3\cdot 10^8}{2\cdot 10^9} = 0{,}15$ [m]

$A_r = 10^6 \frac{(0{,}15)^2}{4\pi} = 1790$ [m²]

$P_r = W_T \cdot A_r = \underline{\underline{879\cdot 10^{-15}}}$ [W]

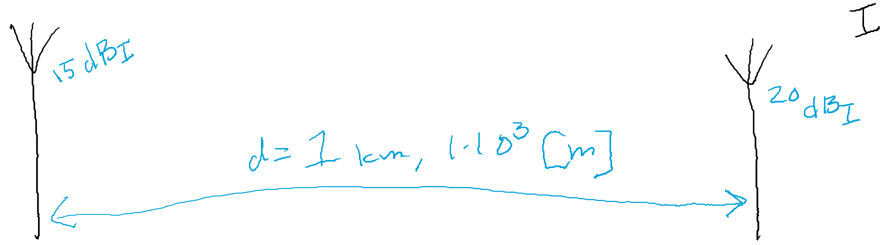
Problems

2.3 Transmitting and receiving antennas operating at 1 GHz with gains of 20 and 15 dBi respectively, are separated by a distance of 1 km.

Find the maximum power delivered to the load when the input power is 150 W. You can assume the antennas are polarization matched.

R = reciver
T = transmitter

freq = $1\cdot 10^6$ Hz
Input P = 150 W



$$\overset{\text{Power received}}{P_r} = \underbrace{(1-|\Gamma_r|^2)(1-|\Gamma_t|^2)}_{\text{Ignor as Ideal}}\overbrace{G_r(\theta_r,\phi_r)}^{\text{Gain R}}\overbrace{G_t(\theta_t,\phi_t)}^{\text{Gain T}}\frac{\lambda^2}{(4\pi R)^2}P_t\overbrace{|\hat{\rho}_r\cdot\hat{\rho}_t|^2}^{=1\text{ as parallel}}$$

Transmitted power

distance imellem

$$P_r = \underset{G_T}{10^{1.5}}\cdot\underset{G_R}{10^{2.0}}\cdot \overset{T_R}{150}\cdot\frac{\overset{\lambda}{0.3^2}}{\underset{\text{dist}}{(4\pi\cdot 10^3)^2}} = \underline{\underline{270\cdot 10^{-3}\ [W]}}$$

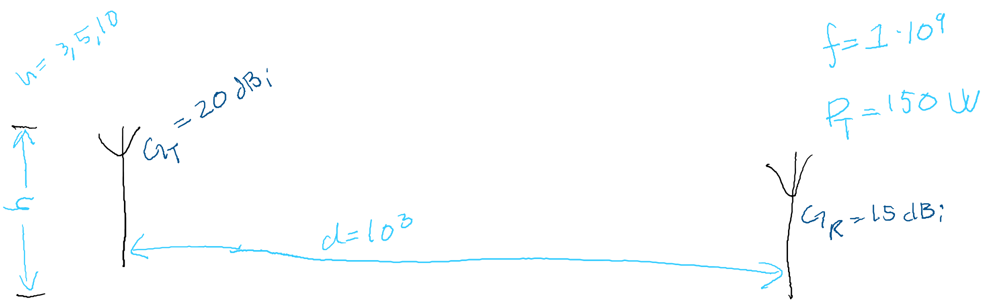
Problems

2.4 Repeat Problem 3 for the case of a reflecting ground and antenna height of both the receiver and transmitter of:

I. 3 meters

II. 5 meters

III. 10 meters



$$G_R \cdot G_T \left(\frac{h_T \cdot h_R}{d^2}\right)^2 P_T$$

a) $h = 3$

$$10^{20} \cdot 10^{15} \left(\frac{3 \cdot 3}{(10^3)^2}\right)^2 150 = \underline{\underline{38\ [\mu W]}}$$

b) $h = 5$

$$10^{20} \cdot 10^{15} \left(\frac{5 \cdot 5}{(10^3)^2}\right)^2 150 = \underline{\underline{276\ [\mu W]}}$$

c) $h = 10$

$$10^{20} \cdot 10^{15} \left(\frac{10 \cdot 10}{(10^3)^2}\right)^2 150 = \underline{\underline{4{,}7\ [mW]}}$$

Sol2

Thursday, 2 January 2025 11.29

Lecture14SolutionsCi...

Problem 2.72, Balanis 3nd edition

$$A_{em} = \frac{\lambda^2}{4\pi} e_t D_0 = \frac{\lambda^2}{4\pi} G_0$$

$$(a) \quad G_0 = 14.8dB \Rightarrow G_0\left(power\ ratio\right) = 10^{1.48} = 30.2$$

$$f = 8.2GHz \Rightarrow \lambda = 3.6585cm$$

$$A_{em} = \frac{(3.6585)^2}{4\pi} \cdot 30.2 = 32.167cm^2$$

$$A_p = 5.5 \cdot 7.4 = 40.7cm^2$$

Problem 2.72, Balanis 3nd edition

$$(b) \quad G_0 = 16.5dB \Rightarrow G_0\left(power\ ratio\right) = 10^{1.65} = 44.668$$

$$f = 10.3GHz \Rightarrow \lambda = 2.912cm$$

$$A_{em} = \frac{\left(2.912\right)^2}{4\pi} \cdot 44.668 = 30.142cm^2$$

Problem 2.72, Balanis 3nd edition

$$(c) \quad G_0 = 18.0dB \Rightarrow G_0\left(power\ ratio\right) = 10^{1.8} = 63.096$$

$$f = 12.4GHz \Rightarrow \lambda = 2.419cm$$

$$A_{em} = \frac{\left(2.419\right)^2}{4\pi} \cdot 63.096 = 29.389cm^2$$

Problem 2.81, Balanis 3nd edition

$$1\ status\ mile = 1.609.3\ meter, \quad 22300\ status\ mile = 3.588739 \cdot 10^{7}\ m$$

$$(a) \quad P_i = \frac{P_{rad}}{4\pi R^2} = \frac{8 \cdot 10^{-14}}{4\pi \cdot 3.58874} = 4.943 \cdot 10^{-16} Watts / m^2$$

$$(b) \quad A_{em} = \frac{\lambda^2}{4\pi} e_t D_0, \quad D_0 = 60dB = 10^6, \quad \lambda = 0.15m$$

$$A_{em} = \frac{(0.15)^2}{4\pi} 10^6 = 1790.493 m^2$$

$$P_{received} = A_{em} P_i = 1790.493 \cdot 4.943 \cdot 10^{-16} = 8.85 \cdot 10^{-13}\ watts$$

Problem 2.85, Balanis 3nd edition

$$\frac{P_r}{P_t} = \left|\hat{\rho}_t \bullet \hat{\rho}_r\right|^2 \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t} G_{0r}$$

$$G_{0t} = 20dB \Rightarrow G_{0t}\left(power\ ratio\right) = 10^2 = 100$$

$$G_{0r} = 15dB \Rightarrow G_{0r}\left(power\ ratio\right) = 10^{1.5} = 31.623$$

$$f = 1GHz \Rightarrow \lambda = 0.3m$$

$$R = 10^3 m$$

$$(a) \quad for \left|\hat{\rho}_t \bullet \hat{\rho}_r\right|^2 = 1$$

$$P_r = \left(\frac{0.3}{4\pi \cdot 10^3}\right)^2 (100)(31.623)\left(150 \cdot 10^3\right) = 270.344 \mu Watts$$

Problem 2.85, Balanis 3nd edition

(b) *when transmitting antenna is circularly polarized and receiving antenna is linearly polarized, the PLF =*

$$\left|\hat{\rho}_t \bullet \hat{\rho}_r\right|^2 = \left|\left(\frac{\hat{a}_x \pm j\hat{a}_y}{\sqrt{2}} \cdot \hat{a}_x\right)\right|^2 = \frac{1}{2}$$

$$Thus \quad P_r = \frac{1}{2}\left(270.344 \cdot 10^{-6}\right) = 135.172 \cdot 10^{-6} = 135.172 \mu Watts$$

Problem 4.

Repeat Problem 3 for the case of a reflecting ground and antenna height of both the receiver and transmitter of:

I. 3 meters

II. 5 meters

III. 10 meters

$$(a) \quad h_T = h_R = 3m$$

$$P_R = P_T \cdot G_T \cdot G_R \left(\frac{h_T h_R}{d^2} \right)^2 \quad eq\ 2.22,\ d \gg h_T, h_R$$

$$P_R = 150 \cdot 100 \cdot 31 \left(\frac{3 \cdot 3}{1000^2} \right) \sim 38 \mu W$$

$$(b) \quad h_T = h_R = 5m$$

$$P_R = P_T \cdot G_T \cdot G_R \left(\frac{h_T h_R}{d^2} \right)^2 \quad eq\ 2.22,\ d \overset{?}{\gg} h_T, h_R$$

$$P_R \sim 270 \mu W \Rightarrow same\ as\ \ free\ space$$

Problem 4.

$$use\ equation\ 2.21\ as\ d\ is\ not \gg h_T, h_R$$

$$P_R = 4P_T\left(\frac{\lambda}{4\pi d}\right)^2 G_T G_R \cdot \sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right)$$

$$P_R = 4\cdot 150\left(\frac{0.3}{4\pi\cdot 1000}\right)^2 \cdot 100\cdot 31\cdot \sin^2\left(\frac{2\pi\cdot 5\cdot 5}{0.3\cdot 1000}\right) \sim 296\mu W$$

explained by Fig 2.5, close to the breaking point!

i.e. Friis eq = flat reflecting surface

$$eq\quad 2.119\ Balanis = 2.22\ Parson$$

Problem 4.

$$(c) \quad h_T, h_R = 10m$$

$$P_R = 4P_T\left(\frac{\lambda}{4\pi d}\right)^2 G_T G_R \cdot \sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right) = 4.74mW$$

$$\Rightarrow compare\ to\ free\ space, Wrong\,?\,or$$

$$why\,? \quad No\ grazing\ angle,\ \rho \neq -1$$

Compare to freespace, wrong or?

a) Max power will be 4x freespace due to doubel E-field. => Could be OK

b) Or reflection coefficient diffrent from -1 due to wrong assumption of grasing angle

Sol3

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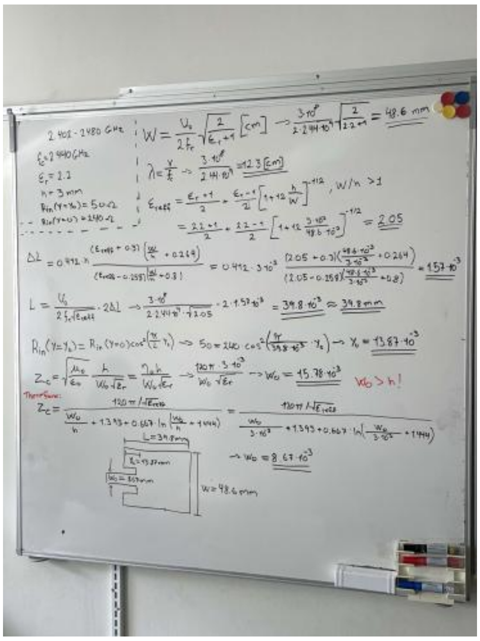

Lecture3SolutionsCS

Problem 3.1

$$(a) \quad D_0 = 4N\left(\frac{d}{\lambda}\right), eq.\, 6.49 \; or \; table \; 6.8$$

$$20 = 10\log_{10}(D_0) \Rightarrow D_0 = 100$$

$$100 = 4N\left(\frac{\lambda}{4\lambda}\right) = N \Rightarrow N = 100$$

$$(b) \quad L = 99\left(\frac{\lambda}{4}\right) = 24.75\lambda$$

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Problem 3.1

$$(c) \quad \theta_{3dB} = \theta_h = 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right), tab.\, 6.4$$

$$\theta_{3dB} \simeq 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi\left(\frac{\lambda}{4}\right)100}\right) \simeq 2\cos^{-1}\left(1 - \frac{1.391\cdot 4}{\pi\cdot 100}\right)$$

$$\theta_{3dB} \simeq 2\cos^{-1}(1 - 0.01771) \simeq 2\cdot 10.799° \simeq 21.6°$$

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Problem 3.1

$$(d) \quad sidelobe(dB) \simeq -13.5dB$$ (Sinc function)

$$(e) \quad \beta = \pm kd = \pm\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \pm\frac{\pi}{2} = \pm 90°, eq.\, 6.20$$

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Problem 3.2

$$f_r = 2.441 GHz$$
$$\varepsilon_r = 2.2\,(pp - plastic)$$
$$h = 3mm$$
$$k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$
$$W = \frac{\lambda}{2}\sqrt{\frac{2}{2.2+1}} \simeq 48.6mm,\ eq.14.6$$

Problem 3.2

$$\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\cdot\frac{1}{\sqrt{1+\frac{12h}{W}}} \simeq 2.005,\ eq.14.1$$
$$\Delta L = h\cdot 0.412\frac{\left(\varepsilon_{reff}+0.3\right)\left(\frac{W}{h}+0.264\right)}{\left(\varepsilon_{reff}-0.258\right)\left(\frac{W}{h}+0.8\right)} \simeq 1.569mm,\ eq.14.2$$

Problem 3.2

$$L = \frac{1}{2f_r\sqrt{\varepsilon_{reff}}\cdot\sqrt{\mu_0\varepsilon_0}} = \frac{c}{2f_r\sqrt{\varepsilon_{reff}}} = \frac{\lambda_0}{2\sqrt{\varepsilon_{reff}}} = 42.87mm,\ eq.14.4$$
$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$
$$L_{eff} = L - 2\Delta L = 39.73mm,\ eq.14.3$$
$R_{in0} = 240\Omega$ Feed point $for\ 50\Omega \Rightarrow y_0 = 13.88mm, eq.14.20a$

Problem 3.2

$$Z_c = \frac{60}{\sqrt{\varepsilon_{reff}}}\ln\left[\frac{8h}{W_0}+\frac{W_0}{4h}\right],\ for\ \frac{W_0}{h} \le 1, eq.14.19a$$
$$e^{\left(\frac{Z_c\sqrt{\varepsilon_{reff}}}{60}\right)} = \left[\frac{8h}{W_0}+\frac{W_0}{4h}\right] \Rightarrow$$

Problem 3.2

$$\Rightarrow 8hW_0^{-1} + \frac{1}{4h}W_0 = e^{\left(\frac{Z_c\sqrt{\varepsilon_{reff}}}{60}\right)} \Rightarrow$$
$$\Rightarrow \frac{1}{4h}W_0^2 - e^{\left(\frac{50\sqrt{2.05}}{60}\right)}W_0 + 24 = 0 \Rightarrow$$
$$\Rightarrow \frac{1}{12}W_0^2 - 3.36W_0 + 24 = 0 \Rightarrow$$

Problem 3.2

$$W_0=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{3.3\pm\sqrt{3.3^2-4\cdot\frac{1}{12}\cdot 24}}{2\cdot\frac{1}{12}}=9.6mm\vee 30mm$$

$$wrong\ formula!\ \frac{W}{h}\ is\ not\leq 1!$$

Problem 3.2

$$Z_c=\frac{120\pi}{\sqrt{\varepsilon_{reff}}\left[\frac{W_0}{h}+1.393+0.667\ln\left(\frac{W_0}{h}+1.444\right)\right]},\ for\ \frac{W_0}{h}>1,\ eq.14.19b$$

$$Z_c\sqrt{\varepsilon_{reff}}\frac{1}{3}\cdot\ln\left(\frac{W_0}{h}+1.444\right)=\eta-\frac{Z_c\sqrt{\varepsilon_{reff}}W_0}{h}-Z_c\sqrt{\varepsilon_{reff}}\cdot 1.393\Rightarrow$$

Problem 3.2

$$\Rightarrow c_1\ln\left(\frac{W_0}{h}+1.444\right)+c_2W_0+c_3=0$$

optimize numerically or try

$$W_0=9.3mm\Rightarrow Z_c=50\Omega,\ OK!$$

- See matlab program Imp.m