



ESD5 – Fall 2024 Problem Set 6

Department of Electronic Systems
Aalborg University
October 21, 2024

Problem 1 – Computing Rates

In this exercise, you will learn how encoding can influence the rate of a channel.

- Consider the simple encoding scheme where we send an 8-bit message using the channel six times and decode it by taking the majority vote. What is the rate of this encoding scheme?
- Consider a BFSK transmitter that sends information at a rate of 10 kbps using a space frequency of 500 kHz and a mark frequency of 700 kHz. How much bandwidth is needed for the transmission?

Problem 2 – Digital Modulation

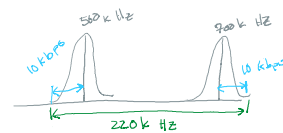
In this exercise, you will learn a digital modulation scheme and define its main properties. Consider the signal constellation shown in Fig. 1. Answer the following.

- What type of modulation is represented?
- How many symbols are represented?
- How many bits does each symbol represent?
- What is the bit rate if the baud rate is 10000 symbols/second?

1)

$$\Rightarrow R = \frac{\log_2(9)}{6} = \frac{1}{2}$$

b)



$$BW = f_{\text{mark}} - f_{\text{space}} + 2K_{\text{sym}} = 500k - 700k + 2 \cdot 10k = 220k \text{ Hz}$$

2)

a) M-Psk

b) 16 (not you count)c) $2^4 = 16$ hence 4d) $10000 = 10k$ $10k \cdot 4 = \underline{\underline{40k}} \left[\frac{b}{s} \right]$

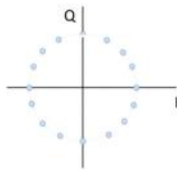


Figure 1: Example of a signal constellation.

Problem 3 – Yet Another Digital Modulation Technique

Consider a communication system that transmits a bit stream using 16-QAM.

- How many bits are transmitted per symbol?
- Suppose four different phases and four different amplitudes are used. Sketch a constellation diagram for the 16-QAM modulation. Label/Indicate the symbols. Is the order important to label them? Are there other ways to label them?

Plus: For the interested, you can play with 16-QAM on MATLAB and try to draw the constellation from there by using: <https://se.mathworks.com/help/comm/gs/examine-16-qam-using-matlab.html>

Problem 4 – An Error Detection Scheme

In this exercise, you will learn an error detection mechanism. In particular, we consider Error detection with Cyclic Redundancy Check (CRC). The message 11011 will be transmitted using CRC polynomial $x^3 + x + 1$ to protect it from errors.

- How many bits are required to apply the CRC?
- Describe the CRC generation process and compute the message that should be transmitted.
- Consider that the transmission is damaged, so the receiver receives 11010001. Will this error be detected?

Hint : Please see, for example, <https://ecomputernotes.com/computernetworkingnotes/communication-networks/cyclic-redundancy-check-to-better-understand-crc/>

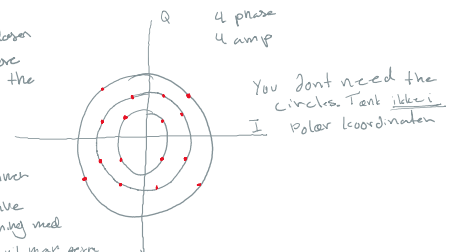
2

3)

a) 4

b) i)

The closer the more likely is the error



ii)

Det hammen til at have en betydning med Hamming v.l man gerne have en stor Hamming dist

4)

$$a) x^3 + 0x^2 + x + 1 = [1 \ 0 \ 1 \ 1] \leftarrow \text{divisor}$$

$$CRC = \text{Len}(\text{divisor}) - 1 = 3$$

b)

11011000	110000
10110000	101100
01101000	011100
00000000	000000
11010000	111100
10110000	101100
01100000	010100
00000000	000000
	1010
	1011
	0001
	0000
	001 ← CRC

hence final packet

11011001
original CRC

c) do it again
hehe not

Problem 5 – Maximum Ratio Combining and Incremental Retransmission

In this exercise, you will see how we can combine baseband bits to improve our communication. Then, you grasp the role of retransmission and how we can make it more efficiency through a thought exercise about incremental redundancy.

- Let $s = (s_1, s_2, \dots, s_n)$ represent the packet sent by Xia to Yoshi through the respective baseband symbols. Yoshi does not receive the first packet transmission correctly, Xia does not receive an ACK, and she retransmits the same s . Let us look at the same received symbol from both packet transmissions:

$$y_{i,1} = hs_1 + n_{i,1}, \quad (1)$$

$$y_{i,2} = hs_2 + n_{i,2}, \quad (2)$$

where the index j stands for the j th packet transmission; $y_{i,j}$ is the i th symbol received by Yoshi and $n_{i,j}$ is the i th noise sample. It is assumed that the channel h stays constant during the transmission and the retransmission.

- Using Chase combining or maximum ratio combining (MRC), what is the message that Yoshi creates? How do the signal-to-noise ratio (SNR) and nominal throughput change in this case?
- The idea of partial retransmission is that instead of retransmitting the same $S_{1,1} = S_{1,2} = S_{1,3} = s$, Xia can retransmit another set of symbols $R_{1,2}$ for the first retransmission, $R_{1,3}$ for the second retransmission, etc. The redundancy of $R_{1,2}, R_{1,3}, \dots$ is not introduced in the first transmission but upon feedback from the receiver. This is why this retransmission method is called *incremental redundancy*. Describe a simple protocol that uses incremental redundancy to show that this approach can improve the throughput without decreasing the reliability.

Extra Problem – Hamming Code with Syndrome Decoding

Based on the file "extra_hamming_code.m", write the code for an encoder that uses the (7,4) Hamming code and a decoder that uses the syndrome decoding algorithm. The decoder must return the decoded message and print the number of erroneous bits detected in the received codeword.

CRC generator	CRC check
11011000	11010001
1011	1011
1101	1100
1011	1011
1100	1110
1011	1011
1110	1010
1011	1011
1010	0111 + 0
1011	
001	
Message: 11011001	

3

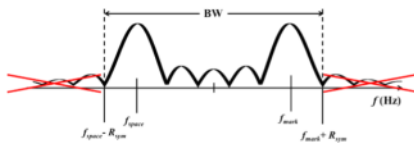


ESD5 – Fall 2024
Problem Set 6 – **Solutions**

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Problem 1 – Computing Rates

- (a) Code rate: $R = \frac{\log_2(8)}{6} = 0.5$.



- (b) The symbol rate is equal to the bit rate (10000 symbols/s).

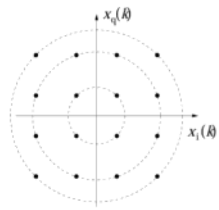
$$BW = f_{mark} - f_{space} + 2R_{sym} = 220 \text{ [kHz]}.$$

Problem 2 – Digital Modulation

- (a) 16 PSK
- (b) 16
- (c) $\log_2(16) = 4$ bits
- (d) $10000 \text{ symbols/second} * 4 \text{ bits/symbol} = 40000 \text{ bits/second}$

Problem 3 – Yet Another Digital Modulation Technique

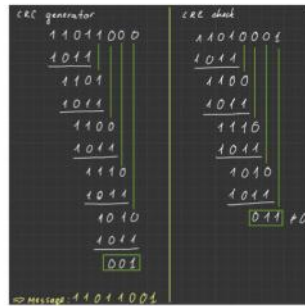
(a) 4 bits per symbol.



(b) Yes, the order in which the symbols are labeled matters. We can use for example Gray labeling. The idea is to reduce the quantity of errors and make them easier to be detected.

Problem 4 – An Error Detection Scheme

- 3 bits.
- Message: 11011001
- Yes, the receiver can detect the error.



Problem 5 – Maximum Ratio Combining and Incremental Retransmission

1. Yoshi creates:

$$y_i = y_{i,1} + y_{i,2} = 2hs_i + n_{i,2} + n_{i,2}. \quad (1)$$

If the noise samples are independent, then MRC makes the SNR of y_i double the original SNR under which the data is attempted to be decoded from the individual $y_{i,j}$. Assuming that the feedback from Yoshi is ideal and instantaneous, then double retransmissions will increase the overall transmission time of the packet two times, which decreases the nominal throughput two times.

2. This could be achieved by rich feedback instead of a simple NACK. After failing to decode the packet, Yoshi sends feedback to Xia, which contains information “I am missing b' bits of information to be able to decode”. The key phrase is “in principle,” as there is a major difficulty in finding coding/decoding methods that enable Yoshi to measure how much information he is missing to decode the packet correctly.

The simplest way to use NACK for incremental redundancy is to retransmit only a subset of the symbols $\mathbf{S}_{1,1} = \mathbf{s}$ sent in the original transmission. For example, Xia retransmits s_1, s_2, s_3 only and during the retransmission, Yoshi receives $y_{i,2}$ for $i = 1, 2, 3$, as given by [eq. (1, 2), Prob. 1]. Then one option is that Yoshi replaces $y_{i,1}$ from the previously received packet with the respective $y_{i,2}$ for $i = 1, 2, 3$, while he reuses the remaining $y_{i,1}$ for $i > 3$ and attempts to decode the packet again.

Extra Problem – Hamming Code with Syndrome Decoding

The solution is in the file “extra_hamming_code_solution.m”.

ESD5 – Fall 2024 Problem Set 7

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Aalborg University
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Problem 1 – From the Analog to the Discrete World

Consider the signals:

$$x(t) = A_x (\sin(2\pi 4000t) + \cos(2\pi a_x t)) \quad (1)$$

$$y(t) = A_y (\sin(2\pi a_y t) + \cos(2\pi b_y t)), \quad (2)$$

where $0 \leq a_x \leq 8$ kHz and $0 \leq b_y \leq 16$ kHz. What are the Nyquist rate (sampling frequency) and sampling period for:

- (a) $x(t)$? $2 \cdot 8 \text{ kHz} = 16 \text{ kHz}$ $\Delta t = \frac{1}{2 \cdot 16000} = 3.125 \cdot 10^{-5} \text{ s}$
 (b) $y(t)$? $2 \cdot 16 \text{ kHz} = 32 \text{ kHz}$
 (c) $x(t) + y(t)$? $\text{Jubæver lægger de sammen, og så har vi } 2 \cdot 16 \text{ kHz}$
 (d) $x(t)y(t)$? $2 \cdot (16 \text{ kHz} + 16 \text{ kHz}) = 4 \cdot 16 \text{ kHz}$

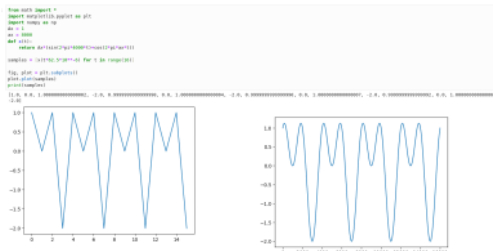
Let us focus now on $x(t)$. Assume $A_x = 1$ and $a_x = 8$ kHz. Imagine that we observe the signal for 1 ms and we start to observe the signal at $T = 0$. Answer the following:

- (e) Using the result from (a), get the number of samples required to reconstruct $x(t)$ for $t \in [0, 1 \cdot 10^{-3}]$ according to the Nyquist rate. $\frac{1 \cdot 10^{-3}}{3.125 \cdot 10^{-5}} = 32$
 (f) Using Matlab or Python, plot the signal $x(t)$ over this 1 ms and the respective sampled points. What are the values of $x(t)$ in the sampled points?

1

$$d) \sin(2\pi 8t) \cos(2\pi 8t) = \frac{1}{2} [\cos(2\pi(8-8)t) + \cos(2\pi(8+8)t)] = \frac{1}{2} [\cos(0) + \cos(32\pi t)]$$

Oppgave 1.f



- (g) The last step to the analog signal $x(t)$ become a completely discrete signal is to discretize its amplitude value. Read more about quantization on [https://en.wikipedia.org/wiki/Quantization_\(signal_processing\)](https://en.wikipedia.org/wiki/Quantization_(signal_processing))¹. If you had access to a 2-bit resolution quantizer, how would you do the quantization of $x(t)$? After quantizing, can you get $x(t)$ back perfectly from the quantized version? No!

(optional) This optional exercise is about estimating the quantization error. Define the quantization error as $e_n = x[nT_s] - \hat{x}[nT_s]$, where $n = 1, 2, \dots, N$ with N obtained in (e), T_s being the sampling rate, and \hat{x} being the quantized version. Compute the average quantization error according to the:

$$\text{MSE} = \frac{1}{N} \sum_{n=1}^N e_n^2, \quad (3)$$

where MSE is the mean-squared error.

Problem 2 – Conventional AM modulation

Let $m(t) = 0.5 \cos(200\pi t) + \cos(400\pi t)$ be the AM baseband modulating signal. Calculate the AM modulated signal and plot its spectrum given amplitude sensitivity $k_a = 0.5$, carrier frequency of $f_c = 2$ kHz, and carrier amplitude of $A_c = 10$ V.

Problem 3 – 4-PAM

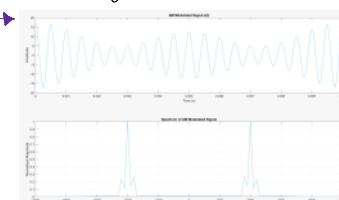
Consider an $M = 4$ PAM modulation. Answer the following:

- (a) Enumerate the binary symbols that this modulation can represent.
 (b) Assume $A_1 = -2, A_2 = -1, A_3 = 1, A_4 = 2$ and $g_T(t) = \cos(2\pi t)$. Draw the signal waveforms and associate them with a symbol enumerated in (a). Please associate the according A_i to the symbol that represents x in the binary numeral system.
 (c) Assume that the transmission period of a signal waveform is $T = 1$ s and $f_c = 1$ Hz. Draw how the transmission of "00101101110010" occurs over time. How long does the transmission of such a sequence take?
 (d) Assume that at the receiver side, the symbol "00" transmitted by the transmitter suffered from a noise component of $n = 0.5$. What is the value of r after the signal demodulator?

¹For a more in-depth and formal treatment of Quantization, consider the book: Proakis, J. G. & Manolakis, D. K. (2006), Digital Signal Processing (4th Edition), Prentice Hall. You can easily find it on Google.

2

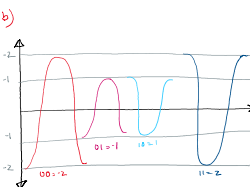
Problem 2)



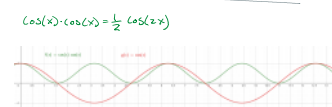
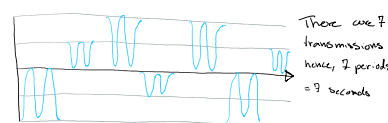
$$\begin{aligned} V_{AM} &= A_c [1 + k_p \cdot V(t)] V_c(t) \\ &= A_c V_c(t) + A_c k_p V(t) V_c(t) \\ &= 10 \cdot \cos(2\pi 2000t) + 10 \cdot \frac{1}{2} \cdot \left(\frac{1}{2} \cos(1000t) + \cos(2000t) \right) \cdot \cos(2\pi 2000t) \\ &= 10 \cos(2\pi 2000t) + \frac{10}{4} (\cos(1000t) \cos(2\pi 2000t) + \cos(2\pi 2000t) \cos(2\pi 2000t)) \\ &= 10 \cos(2\pi 2000t) + \frac{10}{4} (\cos(1000t) \cos(2\pi 2000t) + \cos(2\pi 2000t) \cos(2\pi 2000t)) \\ &= 10 \cos(2\pi 2000t) + \frac{10}{8} \cos(1.9k 2\pi t) + \frac{10}{8} \cos(2.1k 2\pi t) \end{aligned}$$

Problem 3

$$a) \text{ } 00 \ 01 \ 10 \ 11, \quad n = \log_2(M) = \log_2(4) = 2$$



$$c) \ 00 \ 10 \ 11 \ 01 \ 11 \ 00 \ 10$$



$$d) \text{ } 01 \quad 1 \quad 1 \quad \dots \text{ formulas we don't have}$$

- (e) Based on the maximum-a-posteriori principle, how would the signal detector decide for the following sequence received demodulated signals: $[-1.20, -2.40, 1.49, 2.10]$. Assume that the sequence of true transmitted symbols was: $\{00, 00, 11, 11\}$. What is the percentage of error committed by the receiver?



D) Okay, based on formulas we don't have better views

$$E_g = \int_0^T g_T^2 dt = \int_0^T \cos^2(2\pi t) dt = \frac{4\pi T + \sin(4\pi T)}{8\pi} = 0.5 \Big|_T=1$$

$$\psi(t) = \frac{2}{\sqrt{E_g}} g_T(t) = \frac{2}{\sqrt{0.5}} \cos(2\pi t) = 2\sqrt{2}$$

$$\int_0^T r(t) \psi(t) dt = A_m \frac{2}{\sqrt{E_g}} \int_0^T g_T(t) \cos(2\pi t) dt + \frac{2}{\sqrt{E_g}} \int_0^T n(t) g_T(t) dt$$

$\int_0^T g_T(t) \cos(2\pi t) dt = 0.5$ (so, it maps!) $\int_0^T n(t) g_T(t) dt = 0.5$ (from opgaven)

$$\int_0^T r(t) \psi(t) dt = 1.060 A_m + 0.5$$

Q: so for $A_m = -2$

$$1.060 \cdot -2 + 0.5 = \underline{\underline{-1.62}}$$

E) fra opgaven

-1.20	→	01	00	should be	so	hence 50% correction
-2.40	→	00	01		no	
1.49	→	10	10		✓	
2.10	→	11	11		✓	
	maps					

ESD5 – Fall 2024 Problem Set 7 – Solutions

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Problem 1 – From the Analog to the Discrete World

To avoid oversampling, we should carefully consider all the possible cases of a_x and b_y . A complete solution considering those is given below. However, one may argue that since the problem does not specify that oversampling is a problem, we could just assume the worst-case conditions of $a_x = 8$ kHz and $b_y = 16$ kHz, which solution can also be considered right. We denote the sampling frequency as f_s and the sampling period as T_s . For simplicity, note that $T_s = \frac{1}{f_s}$, which is straightforward to be obtained.

(a) $x(t)$?

$$f_s = \begin{cases} 2a_x, & \text{if } a_x \geq 4000, \\ 8000, & \text{o/w.} \end{cases}$$

(b) $y(t)$?

$$f_s = \begin{cases} 2b_y, & \text{if } b_y \geq a_x, \\ 2a_x, & \text{o/w.} \end{cases}$$

(c) $x(t) + y(t)$?

$$f_s = \begin{cases} 2b_y, & \text{if } b_y \geq a_x \text{ and } a_x \geq 4000, \\ 2a_x, & \text{if } a_x \geq b_y \text{ and } b_y \geq 4000, \\ 8000, & \text{o/w.} \end{cases}$$

(d) $x(t)y(t)$? By ignoring the amplitudes, that is, $A_x = A_y = 1$, we have that $x(t)y(t)$ can be written as (use trigonometric identities):

$$x(t)y(t) = \frac{1}{2} \left(\cos(2\pi(a_x - b_y)t) + \cos(2\pi(a_x + b_y)t) + \sin(4\pi a_x t) + \cos(2\pi(4000 - a_x)t) - \cos(2\pi(4000 + a_x)t) + \sin(2\pi(4000 - b_y)t) + \sin(2\pi(4000 + b_y)t) \right).$$

Due to the periodicity of trigonometric functions, it turns out that we can only pay attention to the highest components: $a_x + b_y$, $a_x + 4000$, and $b_y + 4000$. In this case, it becomes easier to visualize graphically the regions, as can be seen in the figure below.

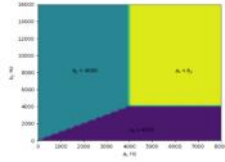


Figure 1: Decision regions for (d). The sampling frequency f_s is twice the frequency of the decision region. For example, $f_s = 2(b_y + 4000)$ in the green area.

OBS: The solutions above are based on the fact that the Fourier transform is linear.

Let us focus now on $x(t)$. Assume $A_x = 1$ and $a_x = 8$ kHz. Imagine that we observe the signal for 1 ms and we start to observe the signal at $T = 0$. Answer the following:

(e) The Nyquist sampling rate is $f_s = 16$ kHz and the sampling period is $T_s = \frac{1}{16}$ ms. Therefore, the number of samples can be computed as:

$$N = \left\lceil \frac{1 \text{ ms}}{1/16 \text{ ms}} \right\rceil = 16 \text{ samples.}$$

(f) The figure below shows the function and its 16 sampled points. The 16 values obtained were:

$$[1., 0.02, 0.71, -1.76, 1.08, 0.37, -0.28, -0.85, 0.64, 0.9, -1.37, 0.26, 0.14, 1.12, -1.97, 1.]^T$$

2

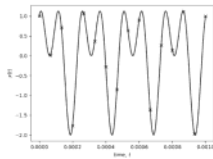


Figure 2: Function over time and its sampled values according to the Nyquist sampling theorem.

(g) One way to do so is the following:

- Since we have 2 bits, we can just represent 4 voltage (magnitude) levels. These are associated with the following: '00', '01', '10', '11'.
- Find the minimum and maximum values of the function $x(t)$: $x_{\min}(t) = -2$ and $x_{\max}(t) = 1.12$. Associate those as follows: '00' $\leftarrow x_{\min}(t)$ and '11' $\leftarrow x_{\max}(t)$.
- Find the remaining points by finding the resolution:

$$\Delta = \frac{1.12 - (-2)}{4 - 1} = 1.04.$$

- Now, '01' $\leftarrow -0.96 = x_{\min} + \Delta$ and '10' $\leftarrow 0.08 = x_{\min} + 2\Delta$.
- The last step is: go through all 16 sampled magnitudes, for each one find which one of the quantization levels is closer to it. Substitute the original value with the quantized one. An illustrative figure of this process is shown below.

(optional)

$$\text{MSE} = \frac{1}{N} \sum_{n=1}^N e_n^2 = 0.0074.$$

3

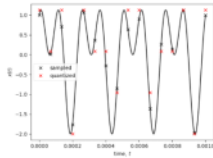


Figure 3: (g) – Quantized samples.

Problem 2 – Conventional AM modulation

The classical AM occurs as follows:

1. Multiply the baseband signal by a sinusoid carrier signal;
2. Add the carrier;

These steps can be summarized as:

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t).$$

By substituting the values, we have:

$$s(t) = 10[1 + 0.5(0.5 \cos(2\pi 100t) + \cos(2\pi 200t))] \cos(2\pi 2000t).$$

The spectrum is given by

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2} [M(f - f_c) + M(f + f_c)]$$

The spectrum of the AM baseband modulating signal is:

$$M(f) = 0.25[\delta(f - 100) + \delta(f + 100)] + 0.5[\delta(f - 200) + \delta(f + 200)].$$

By substituting all values, we have:

$$\begin{aligned} S(f) = & 5[\delta(f - 2000) + \delta(f + 2000)] + \\ & + 2.5\{0.25[\delta(f - 2100) + \delta(f - 1900)] + 0.5[\delta(f - 2200) + \delta(f + 1800)]\} \\ & + 2.5\{0.25[\delta(f + 1900) + \delta(f + 2100)] + 0.5[\delta(f - 1800) + \delta(f + 2200)]\}. \end{aligned}$$

Figures showing the AM modulated signal over time and frequency are available below.

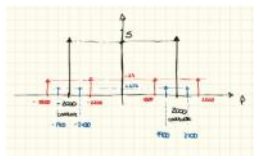


Figure 4: Expected theoretical spectrum.

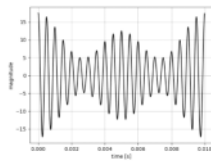


Figure 5: AM signal over time. For the fans of [Artic Monkeys](#), hope you can appreciate the album cover now :)

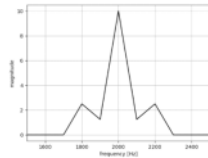


Figure 6: Spectrum obtained numerically via FFT. The power normalization takes into account that there is no left FFT plot, this is why is twice the expected plot.

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Problem 3 – 4-PAM

(a) '00', '01', '10', '11'.

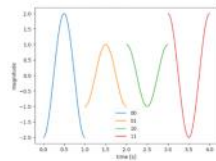


Figure 7: (b) – Signal waveforms.

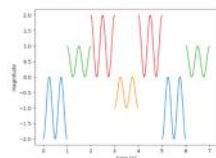


Figure 8: (c) – Sending message "00101101110010". Remember to multiply the signal waveforms by the carrier cosine.

(d) First, we need to determine the energy of the used pulse:

$$\mathcal{E}_p = \int_0^T g_p^2(t) dt = \int_0^T \cos^2(2\pi t) dt = \frac{4\pi T + \sin(4\pi T)}{8\pi}.$$

Since $T = 1$, we have $\mathcal{E}_p = 0.5$. Now, we need to obtain the basis function of the PAM, which process can be found on pg. 374 - Proakis, J. G., Salehi, M. (2001).

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$$\psi(t) = \frac{2}{\sqrt{E_s}} g_T(t).$$

Thus, at the receiver, we need to solve the following:

$$\int_0^T r(t)\psi(t)dt = A_m \frac{2}{\sqrt{E_s}} \int_0^T g_T^2(t) \cos^2(2\pi t)dt + \frac{2}{\sqrt{E_s}} \int_0^T n(t)g_T(t)dt.$$

We divide the solution of the above integral into two: the signal part and the noise part. The signal part yields:

$$A_m \frac{2}{\sqrt{E_s}} \int_0^T g_T^2(t) \cos^2(2\pi t)dt = A_m \frac{2}{\sqrt{E_s}} \int_0^T \cos^4(2\pi t)dt \approx A_m 0.5303.$$

The noise part simply results in $n = 0.5$, which is specified by the exercise. Therefore, we have the following received energy:

$$\int_0^T r(t)\psi(t)dt \approx A_m 0.5303 + 0.5$$

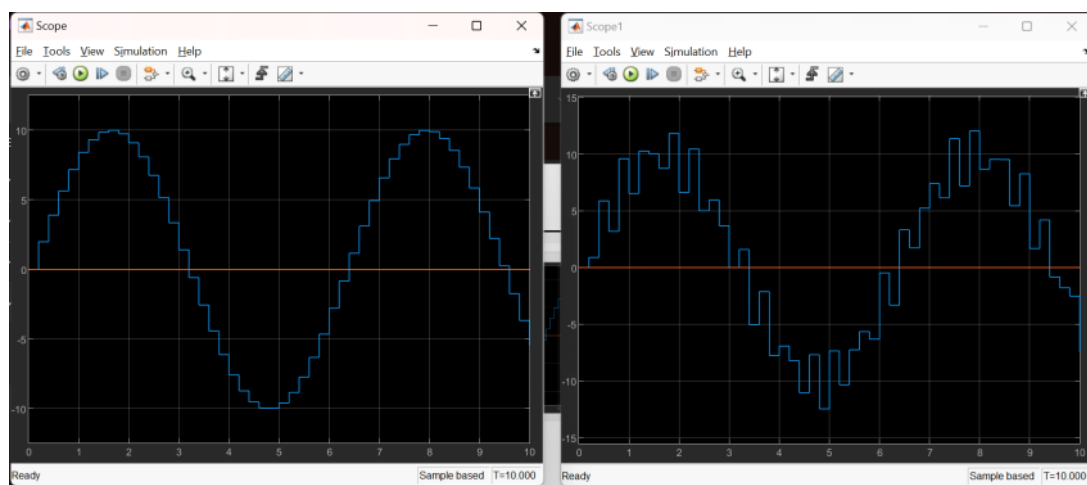
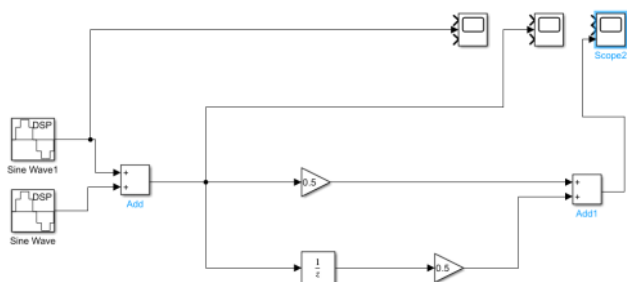
Since the true signal sent was '00', the received value is approximately $-2 \cdot 0.5303 + 0.5 = -1.5606$.

- (e) Based on the maximum-a-posterior principle, the received demodulated signals would be: $[-1.20, -2.40, 1.49, 2.10] \rightarrow [01, 00, 10, 11]$ (just find the closest symbol using the TRUE mapping $[-2, -1, 1, 2] \rightarrow [00, 01, 10, 11]$). Assuming that the sequence of true transmitted symbols was: $[00, 00, 11, 11]$, the percentage of error was 50%.

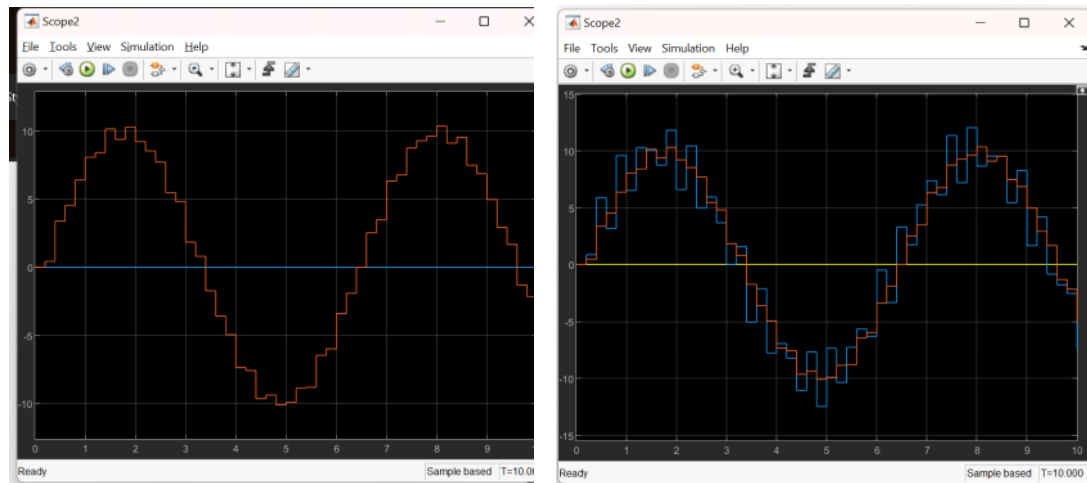
Workshop

Monday, 11 November 2024 12:52

Så det her er det modul vi har lavet

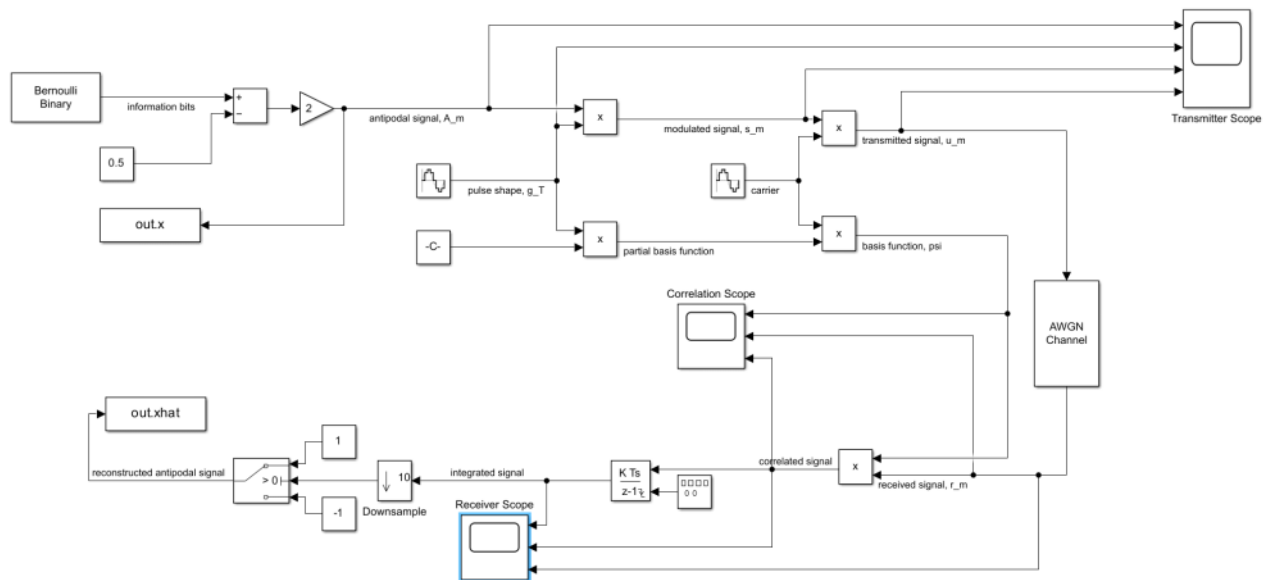


Så den første er det original signal, det andet er uden forsinkelsen

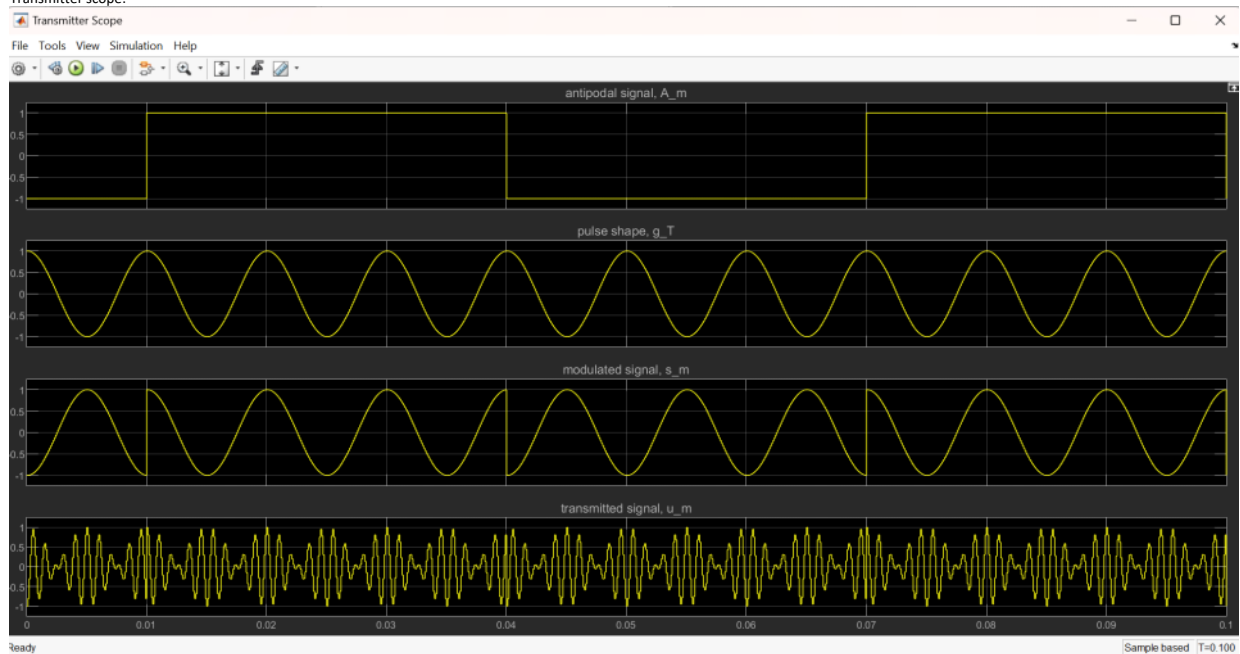


Op det her er med forsinkelse på. Det er kønner, det kan vi godt se. Her har man den så oven i hinanden, og prøv at se hvor pænt det er.

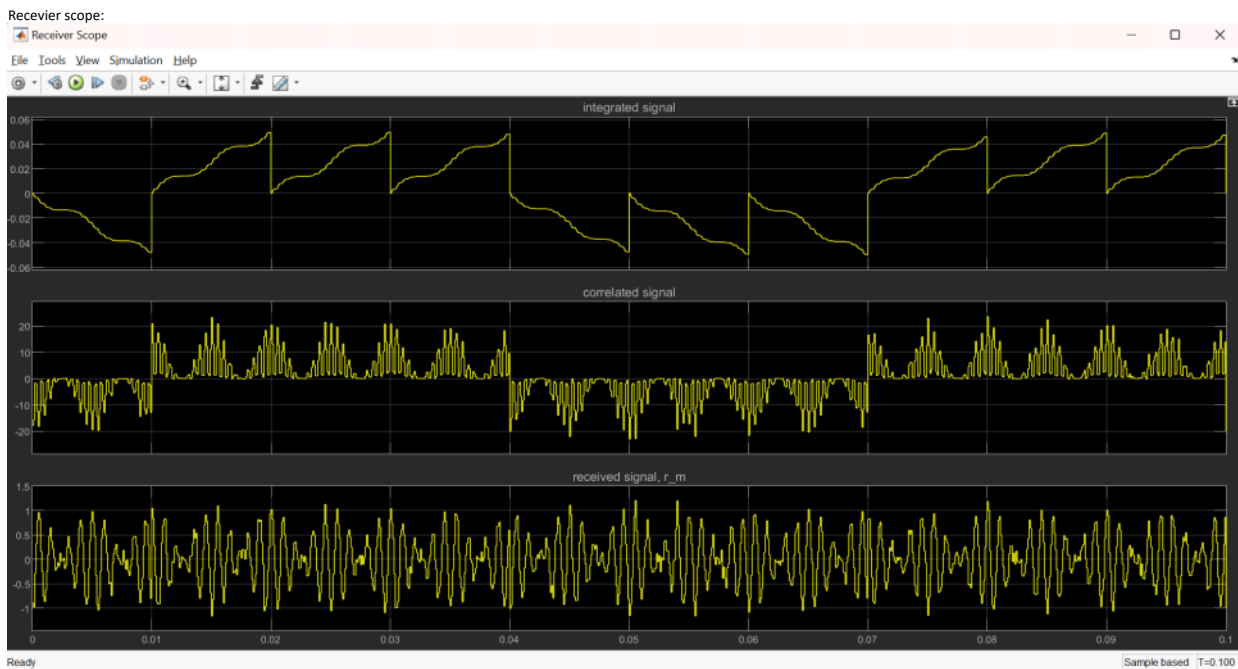
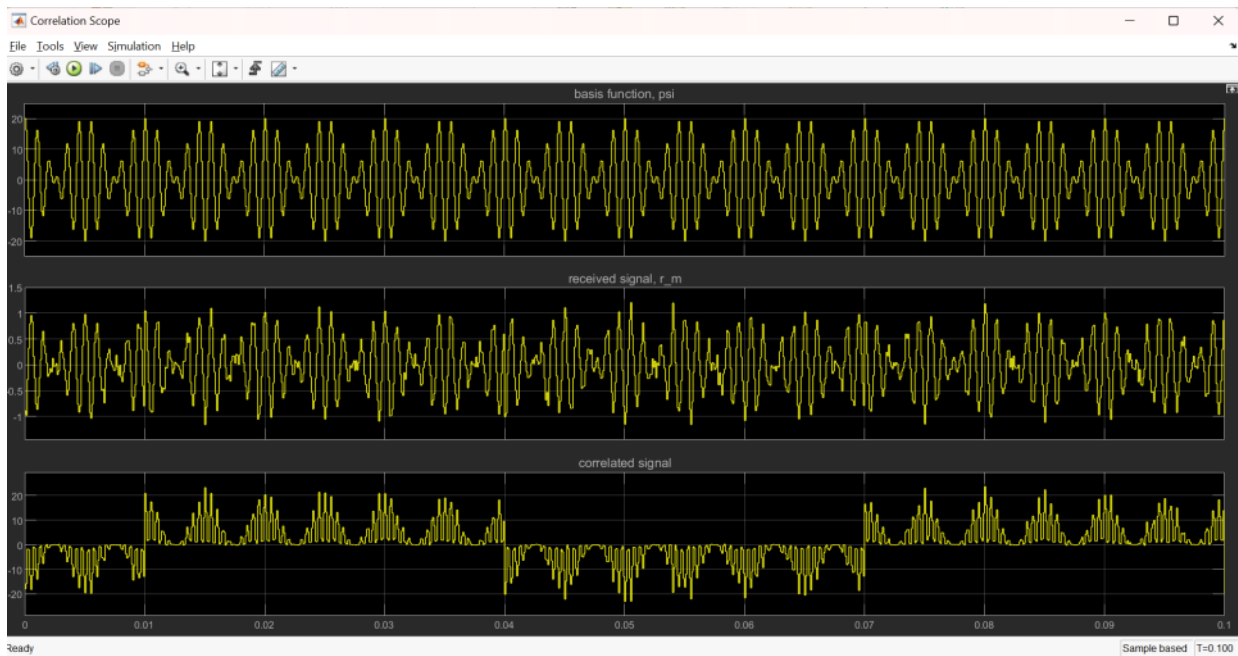
OG NU TIL NOGET NYT OG MERE AVANCERET



Transmitter scope:



Correlation scope:



OG NU TIL SNR.

