

1 Communication systems

1.1 Antenna

1.1.1 Basic

$\lambda = \frac{c}{f}, [m]$
 $c = 3 \cdot 10^8, [m/s]$
dBm to Watt: $P = 10^{(dBm-30)/10}, [W]$;
 $E_{field} = 20 \cdot \log_{10}(x)$, for vovres E felt ;
 $10 \cdot \log_{10}(x)$, for power ;
 $dB_i = 10 \cdot \log_{10}(x)$;

1.1.2 Gain

$G = \frac{4\pi U(\theta, \phi)}{P_{in}(losslessisotropicsource)}$;
 $G(\theta, \phi) = E_{cd} D(\theta, \phi)$;
 $G_0 = 10 \log_{10}(E_{cd} D_0)$;
For more see, Balanis p 61

1.1.3 Regions

$R_{reActive} < 0.62 \sqrt{\frac{D^3}{\lambda}} [m]$;
 $R_{radiating} < \frac{2D^2}{\lambda} [m]$;
 $Farfield \geq \frac{2D^2}{\lambda}$; [m]
where D = the largest dimension of your antenna ;
for 2d $D_2 = \sqrt{l^2 + w^2} [m]$;

1.1.4 Directivity

$U = r^2 W_{rad}$ where U = radiation intensity [W] ,
 W_{rad} = Radiation density $[W/m^2]$;
 $P_{rad} = A_0 \int_0^{\phi_b} \int_0^{\theta_b} U \cdot \sin(\theta) d\theta d\phi$; [W]
 $D_0 = \frac{4\pi \cdot U_{max}}{abs(P_{rad})}$; [·]
 $E_r \approx H_r = H_\theta = E_\phi = 0$;
 $H_\phi \approx j \frac{K I_0 \sin(\theta)}{4\pi r} e^{-jkr}$;

$E_\theta \approx j\eta \frac{K I_0 \sin(\theta)}{4\pi r} e^{-jkr}$;
 $\Rightarrow Z = \frac{E_\theta}{H_\phi} \approx \eta$;
HPBW in (rad or degress) depends on U:
 $\frac{1}{2} = U(\theta) \leftrightarrow \theta = \theta_{HPBW}$;
 $HPMW, \theta = 2 \cdot \theta_{HPBW}$;

1.1.5 Friis & Transmissions

$D = \frac{4\pi A}{\lambda^2}, [\cdot]$ Directivity;
 $A = l \cdot w, [m^2]$;
 $\frac{P_r}{P_t} = \lambda^2 \frac{D_t \cdot D_r}{(4\pi R)^2} P_Q$;
 $R = \sqrt{\frac{P_t}{P_r} \cdot \frac{D_t \cdot D_r \cdot \lambda^2}{(4\pi R)^2} \cdot P_Q}, [m]$;
 $\frac{P_r}{P_t} = D_t D_r \cdot \left(\frac{h_t h_r}{R^2}\right)^2$ where h_t & h_r is the height from the antenna to the ground ;
 P_Q = projection quality: [·]
ideal lin \leftrightarrow lin = 1
circ. pol. \leftrightarrow lin pol, $P_Q = \frac{1}{2}$
lin \leftrightarrow lin, ang. dif, ρ , $P_Q = \cos(\rho)$
 $A_r = e_t \cdot D_r(\theta_r, \phi_r) \frac{\lambda^2}{4\pi}, [m^2]$;
 $A_r = G_0 \cdot \frac{\lambda^2}{4\pi}, [m^2]$
 $P_r = G_t \cdot G_r \cdot P_t \frac{\lambda}{(4\pi R)^2}, [W]$;
 $P_{refl} = G_r \cdot G_t \left(\frac{h_t h_r}{d^2}\right)^2 P_t, [W]$;

1.1.6 Reflection

Flat:
 $\frac{P_r}{P_t} = D_t D_r \cdot \left(\frac{h_t h_r}{R^2}\right)^2$;
 $P_r = 4P_t \left(\frac{\lambda}{4\pi R}\right)^2 G_r G_t \sin^2 \left(\frac{2\pi h_r h_t}{R\lambda}\right)$

here h_R is height of receiver from ground and h_T is height of transmitter from ground.
IF $R \geq \frac{4 \cdot h_t \cdot h_r}{\lambda}$ then below can be used instead.
 $\frac{P_r}{P_t} = D_r \cdot D_t \left(\frac{h_t \cdot h_r}{R^2}\right)^2 \leftrightarrow$
 $R = \sqrt[4]{\frac{P_t}{P_r} D_t D_r (h_t h_r)^2 [m]}$;

1.1.7 Isotopic

$W_{rad} = \frac{U}{r^2}, [W/m^2]$
 $P_{rad} = R_{rad} \cdot 4\pi r^2, [W]$

1.1.8 Build-a-patch

if needed, se mm3, designAffPatch

1.1.9 Rewrite & DUM DUM

$dB_i(x) = 10 \cdot \log_{10} \cdot (x)$
 $x = 10^{dB_i(x)/10}$
 $A_r + jB_r$;
 $A_p = \sqrt{A_p^2 + B_p^2}$; $B_p = \angle = \arctan(\frac{b}{a}) \cdot \frac{360}{2 \cdot \pi}$;
HUSK FOR GUDS SKYLD: Cos og Sin med
start i MAPLE! og med with(Gym):
 $\circ \cdot \frac{\pi}{180} = rad$; $rad \cdot \frac{180}{\pi} = \circ$;
 $A_p \angle B_p \leftrightarrow A_r + jB_r = A_p \cdot (\cos(B_p) + j\sin(B_p))$;
 $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} X = (y_1 z_2) - (z_1 y_2) \\ Y = (x_1 z_2) - (z_1 x_2) \\ Z = (x_1 y_2) - (y_1 x_2) \end{bmatrix}$;
Peta, P = 10^{15} ; Tera, T = 10^{12} ; Giga, G = 10^9 ;
Mega, M = 10^6 ; Kilo, k = 10^3 ;
Milli, m = 10^{-3} ; micro, μ = 10^{-6} ; Nano, n = 10^{-9} ;
Pico, p = 10^{-12} , femto, f = 10^{-15}
For more prefixes see slides "ISQ" (mm8) slide 14.

1.2 Networking

1.2.1 Basic

Throughput = successful transmitted data rate
Goodput = $\frac{\text{effectiveDataSize}}{\text{totalFrameSize}}$
Probability Bit Error Rate; $P_{BER} = \frac{\text{ErrorCount}}{\text{TotalCount}}$
Packet Error Ratio; $P_{PER} = 1 - (1 - P_{BER})^N$,
where N is number of packet bits
 $T_{av.time} = \frac{T_{data} + T_{ack}}{1 - P_{PER}}$
 $T_{time} = \frac{\text{data}[b]}{\text{speed}[b/s]} + \text{delay}[ms], [ms]$

1.2.2 ALOHA

RANDOM SLOTTED

$S_{tp,slot} = \lambda T \exp(-\lambda T)$ where λ mean N, number of slots

FRAMED

Probability of successful transmission; $P(S) = \frac{K}{S} \left(1 - \frac{1}{S}\right)^{K-1}$, where K is user count and S is slot count

$S_{pt,framed} = \lambda T \exp(-2\lambda T)$;

1.2.3 Acronyms

Network

ARQ = Automatic Retransmission Request;
TDD = Time Division Duplex;
FDD = Frequency Duplex Division;
FD = Full Duplex (simultaneous Rx/Tx);
HD = Half Duplex (non-simultaneous Rx/Tx);

Medium sharing

FDMA = Frequency Division Multiple Access;
TDMA = Time Division Multiple Access;
CSMA = Carrier-sense multiple access;
CDMA = Code division multiple access;
OFDMA = Orthogonal frequency-division multiple access;

NOMA = Non-orthogonal multiple access;
WDMA = Wavelength Division Multiple Access;
CWDM = Coarse Wavelength Division Multiplexing;
DWDM = Dense Wavelength Division Multiplexing;

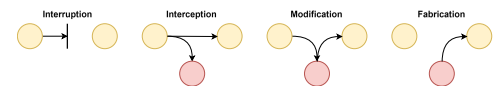
Spread Spectrum

SS = Spread Spectrum;

FHSS = Frequency-hopping Spread Spectrum;
CSS = Chirp Spread Spectrum;
DSSS = Direct Sequence Spread Spectrum;
Security
MITM = Man-in-the-middle attack;
DDoS = Distributed Denial-of-Service;
AES = Advanced Encryption Standard;
DES = Data Encryption Standard;
PKI = Public Key Infrastructure;
CA = Certificate Authority;

1.2.4 Security

symmetric = sender/receiver, same key
asymmetric = sender/receiver, different key



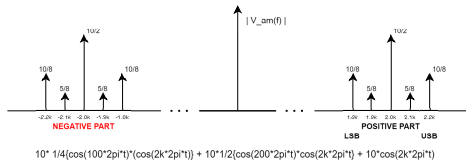
1.3 Modulation

1.3.1 Code rate

$R = \frac{b}{n} [\text{bits/channel}]$, where b = bits, n = channel use;
 $R = \frac{\log_2(M)}{n}$, where M is bits, n is channel use;
 $T_u = \frac{b}{b+c} (1 - p_u)^{(b+c)}$, where c = check bits;
 $T_c = T_u \frac{1}{3} (\frac{1-p_c}{1-p_v})^{b+c}$;
 $BW = f_{mark} - f_{space} + 2R_{sym} [Hz]$;

1.3.2 Amplitude modulation

$\cos(a) \cdot \cos(b) = 1/2 \cdot \cos(a+b) + 1/2 \cdot \cos(a-b)$;
 $\cos(a-b) = \cos(b-a)$;
 $k = \log_2(M)$, where k is bit/symbol and M is symbols;
 $\mu = K_a \cdot A_m$, modulation factor where k_a is amplitude sensitivity and A_m is amplitude of modulating signal;
 $A_c = \sqrt{P_c/2} [V]$;
SINGLE
 $V_{am}(t) = s_{am}(t) = A_c \cdot [1 + k_a \cdot v(t)] \cdot v_c(t)$ where A_c is the carrier amplitude, k_a is amplitude sensitivity, $v(t)$ is the base signal and $v_c(t)$ is the carrier frequency;
 $S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a \cdot A_c}{2} [M(f - f_c) + M(f + f_c)]$
REMEMBER! when drawing, the mirrored amplitudes $f_c = 1/2, v(t) = 1/4$ for each.



DSB-SC

Pros: Energy efficient, all power is in the sidebands;
 Cons: Cannot be detected (demodulated) with a simple envelope detector;
 $V_{am} = A_c \cdot k_a \cdot v_c(t) \cdot v(t)$;

PAM

$s_{m_{baseband}}(t) = A_m \cdot g_T(t)$, where A_m is amplitude, and G_T is pulse shape;

$u_{m_{pandpass}} = A_m \cdot G_T(t) \cos(f_c 2\pi t)$;

QPSK

$M_{baseband}$ = point constellation in 2D

$u_m(t) = s_t(t) \cos(2\pi f_c t) = g_T(t) \cos(2\pi f_c t \frac{2\pi m}{M})$;

QAM

$M_{baseband}$ = point constellation in 2D ;

$u_{mn}(t) = A_m \cdot g_T(t) \cos(f_c 2\pi t + \theta_n)$ for $m = 1 \dots M_1$,
 $n = 1 \dots M_2$, $M = M_1 + M_2$;

Noise in PAM:

$\varepsilon_g = \int_0^T g_T^2(t) dt$, where ε_g is the energy of the pulse and g_T is your cos function ;

$\psi(t) = \frac{2}{\sqrt{\varepsilon_g}} g_T(t)$;

$\int_0^T r(t) \psi(t) dt = A_m \frac{2}{\sqrt{\varepsilon_g}} \int_0^T g_T^2(t) \cos^2(2\pi t) dt +$

$\frac{2}{\sqrt{\varepsilon_g}} \int_0^T n(t) g_T(t) dt$;

$\int_0^T r(t) \psi(t) dt = A_m \sqrt{\varepsilon_g/2} + n$;

1.3.3 Bandwidth transmission

BR = bit rate;

$R_b = N R_{sym} = R_{sym} \log_2(M) = BR$;

$B = 2 R_{sym}$;

FSK: $2 \cdot f_{d2f_c} + 2 \cdot DS$;

$BW_{M_n} = \frac{2 R_{sym}}{\log_2(M)}$;

OOK: $M = 2$;

QPSK, QAM: $M = 4$;

16-PSK: 16-QAM: $M = 16$;

512-QAM: $M = 512$;

1.3.4 Amplitude DEModulation

Known pahse:

$s(t) = m(t) \cos(\omega_c t)$;

$v(t) = s(t) \cos(\omega_c t) = m(t) [\frac{1}{2} + \frac{1}{2} \cos(2\omega_c t)]$;

Unknown phase:

$s(t) = A_c \cdot m(t) \cos(\omega_c t)$;

$v(t) = s(t) A'_c \cos(\omega_c t + \phi) =$

$1/2 m(t) A_c A'_c (\cos(2\omega_c t + \phi) + \cos(\phi))$;

After lowpass

$v_0(t) = 1/2 A_c A'_c \cos(\phi) m(t)$

constant phase $\neq \pm\pi/2$ $v_0(t)$ is proportional to $M(t)$

if the phase = $\pm\pi/2$ $v_0(t) = 0$

1.3.5 Phase and frequency modulation

Phase modulation

$s_m(t) = A_c \cos(\theta_i(t))$ where, A_c is the amp. of the

modulated signal, $\theta_i(t)$ is the variable instantenous angle of the modulated signal, $s_m(t)$ is the modulated signal ;

$s_p(t) = A_c \cos(f_c 2\pi t)$, where $s_p(t)$ is with no modulating signal ;

$\theta_i(t) = 2\pi f_c t + k_p m(t)$ where $\theta_i(t)$ change linearly as a function of $m(t)$ and k_p is the phase sensitivity of the modulator;

$s_m(t) = A_c \cos(2\pi f_c t + k_p m(t))$ IF $m(t)$ is a first order function:

$\theta_i(t) = 2\pi f_c t + k_p a t = 2\pi (f_c \frac{k_p a}{2\pi}) t$

$= 2\pi (f_c + f_m) t$;

frequency modulation

$f_i(t) = f_c + k_f m(t)$ where k_f is the frequency sensitivity of the modulator ;

$\theta_i(t) = \int_0^t f_i(\tau) d\tau = 2\pi (f_c t + k_f \int_0^t m(\tau) d\tau)$;

$s_m(t) = A_c \cos[2\pi (f_c t + k_p f \int_0^t m(\tau) d\tau)]$;

constant propeties

$P = \frac{A_c^2}{2}$, is transmitted power ;

$m(t) = m_1(t) + m_2(t)$;

$s(t) = A_c \cos[2\pi f_c t + k_p (m_1 t + m_2 t)]$;

$s_1(t) = A_c \cos[2\pi f_c t + k_p m_1(t)]$;

$s_2(t) = A_c \cos[2\pi f_c t + k_p m_2(t)]$;

$s(t) \neq s_1(t) + s_2(t)$;

1.3.6 Acronyms

ASK = Amplitude-shift key;

OOK = On-Off Keying;

M-ASK = M-array Amplitude-shift keying (e.g. 4-ASK);

PSK = Phase-shift key;

BPSK = Binary Phase-shift key;

M-PSK = M-array Phase-shift keying (e.g. 4-PSK);

QPSK = Quadrature-PSK - Like 4-PSK but rotated $\pi/4$ (see s. 37 LEC 11);

FSK = Frequency-shift key;

BFSK = Binary Frequency-shift key;

QAM = Quadrature amplitude modulation;

DSB-SC = Double-sideband suppressed-carrier;

PAM = Pulse Amplitude Modulation;

BSC = Binary Symmetric Channel;

FEC = Forward Error Correction;

CRC = Cyclic redundancy check;

SSB = Single Sideband;

USB = Upper Sideband;

LSB = Lower Sideband;

DSB = Double Sideband;

2 HighSpeed

2.0.1 SmithCharts, LEC12

$$\lambda = \frac{v}{f}, [m];$$

$$Z_0 = \sqrt{\frac{L}{C}}, [\Omega];$$

$$v = \sqrt{\frac{1}{L \cdot C}} \leftrightarrow \frac{1}{Z_0 \cdot C} \leftrightarrow \frac{Z_0}{L}, [\frac{m}{s}];$$

$$R||L \rightarrow Z_L = \frac{1}{\frac{1}{R} + \frac{j}{\omega L}};$$

$$R + L \rightarrow Z_L = R + j\omega L;$$

$$R||C \rightarrow Z_L = \frac{1}{\frac{1}{R} + j\omega C};$$

$$R + C \rightarrow Z_L = R + \frac{1}{j\omega C}, Z_L, [\Omega]$$

$$Z_n = \frac{Z_L}{Z_0} [:];$$

$$Z_{stub} = j - \frac{Z_0}{B_{stub}}, [\Omega];$$

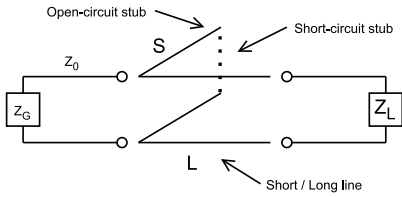
FIND DIT KOMPONENT:

$$Z_{stub} = +/\text{ovre del} = \text{spole}$$

$$Z_{stub} = -/\text{nedre del} = \text{condensator}$$

$$C = \frac{-1}{\omega \cdot \text{Im}(Z_{stub})}, [F];$$

$$L = \frac{\text{Im}(Z_{stub})}{\omega}, [H];$$



SmithChart: Slides MM12, slide 20-22.

2.0.2 Iron cores, LEC3&4

$$\vec{B} = \mu \cdot \vec{H}, [\frac{Wb}{m^2} = \frac{V \cdot s}{m^2}];$$

$$\vec{H} = \frac{I}{\ell}, [\frac{A}{m}];$$

$$\vec{F} = \vec{\ell} \times \vec{B}, [N];$$

$$\mu = \mu_0 \cdot \mu_r, [\frac{H}{m}]; \mu_0 = 4\pi \cdot 10^{-7};$$

$$\mu_r(\text{air}) = 1; \mu_r(\text{iron}) = 3000;$$

$$F = N \cdot I, F = I \cdot \mathcal{R}, [A];$$

$$\phi = \frac{F}{\mathcal{R}}, \phi = \frac{F}{\mathcal{R}_1 + \mathcal{R}_2}, \phi = \frac{|V|}{\omega \cdot N}, [Wb];$$

$$\omega = 2 \cdot \pi \cdot f;$$

$$\mathcal{R} = \frac{\ell}{\mu \cdot A}, [H^{-1}, \frac{A}{Wb}];$$

$$I = \frac{N \cdot I}{\mathcal{R}_1 + \mathcal{R}_2}, [\frac{A}{Wb}, H^{-1}];$$

$$A = \text{area}, [m^2];$$

$$\ell = \text{lengthFromTheCENTER!}, [m];$$

2.0.3 Beam on line, LEC5

CHECK BLACKBOARDS!

$$F = B \cdot I \ell, [N];$$

$$\vec{F} = I \cdot \vec{\ell} \times \vec{B}, [N]; a = \frac{F}{m}, [\frac{m}{s^2}];$$

$$v = a \cdot t, [\frac{m}{s}];$$

P is the effect:

$$P_{el} = \frac{v^2}{R}$$

$$P_{mec} = v \cdot F, [W];$$

$$P_{mec} = P_{el} = V \cdot I, [W];$$

dot means towards us
x means away from us

2.0.4 Turning frame, LEC5

CHECK BLACKBOARDS!

$$\vec{\mu} = I \cdot N \cdot A \cdot \hat{n}, [Am^2];$$

$$N = \text{turns}; A = \text{Area}, [m^2];$$

finding \hat{n} :

cos = horizontal line
sin = vertical line
 $\vec{\tau} = \vec{\mu} \times \vec{B}, [Nm];$

2.0.5 Reflections, LEC7&10& 13

$$K_L = \frac{Z_L - Z_0}{Z_L + Z_0}, [\Omega];$$

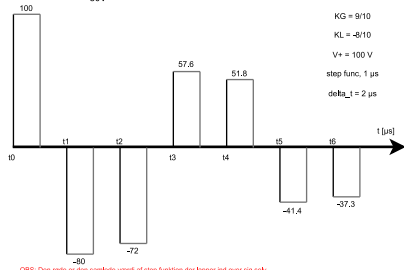
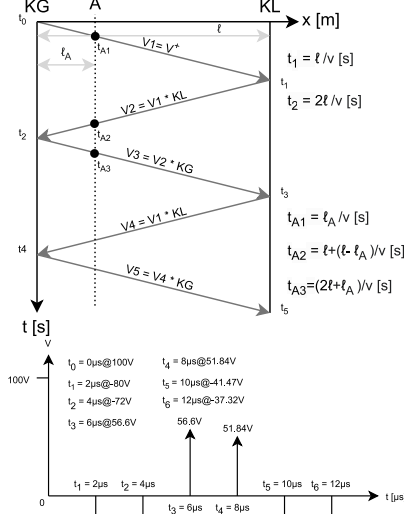
$$K_G = \frac{Z_G - Z_0}{Z_G + Z_0}, [\Omega];$$

$$V_+ = V_G \cdot \frac{Z_0}{Z_0 + Z_G}, [V]$$

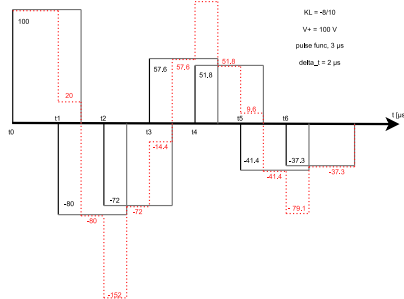
$$\Delta T = \frac{\ell}{v}, [s];$$

$$V_\infty = V_G \cdot \frac{Z_L}{Z_G + Z_L}, [V];$$

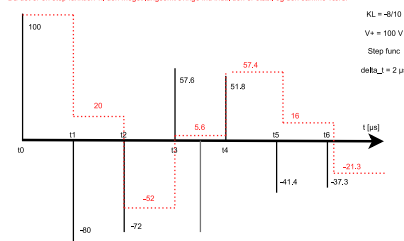
If it is current flip the sign on KG and KL otherwise, carry on.



OBS: Den røde er den samlede værdi af step funktion der lægger ind over sig selv.



OBS: Den røde er den samlede værdi af step funktion der lægger ind over sig selv.



Important note to the figures. They are made each of the reflections. If asked to do at the GENER-

ATOR, it would be $V_1 \cdot V^+, (V_2 + V_3) \cdot V^+, (V_4 + V_5) \cdot V^+$ and so on. If it is at the LOAD it would be $(V_1 + V_2) \cdot V^+, (V_3 + V_4) \cdot V^+$ and so on. If it is on the middle it would be for each separat.

2.0.6 Standing waves, LEC11

$$\omega = 2 \cdot \pi \cdot f, [\frac{rad}{s}]; \gamma = \alpha - j\beta, [m^{-1}];$$

$$\beta = \omega \sqrt{L \cdot C}, [\frac{rad}{m}]; \alpha = 0, [\frac{Np}{m}]$$

$$\lambda = \frac{2 \cdot \pi}{\beta} = \frac{v}{f}, [m]; v = \frac{1}{\sqrt{LC}} = \frac{\omega}{\beta}, [\frac{m}{s}]$$

$$SWR = \frac{max}{min};$$

$$K(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0} [:]; Z(x) = Z_0 \frac{1 + K(x)}{1 - K(x)}, [\Omega];$$

$$K_L = \frac{Z_L - Z_0}{Z_L + Z_0}, [\Omega]; K_L = -(\frac{Z_0 - Z_L}{Z_0 + Z_L}), [:];$$

$$abs(K_L) = \frac{SWR - 1}{SWR + 1}, [:];$$

$$V_{min}/I_{max} = V^+ / I^+ \cdot 1 + abs(K_L), [VorA];$$

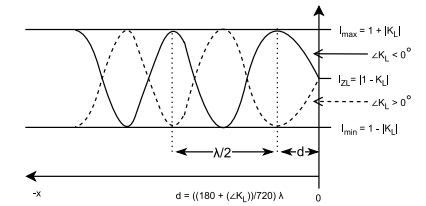
$$V_{min}/I_{min} = V^+ / I^+ \cdot 1 - abs(K_L), [VorA];$$

$$V_{ZL} = V^+ \cdot abs(1 + K_L), [V]$$

$$I_{ZL} = I_{(0)} = I^+ \cdot abs(1 - K_L), [A];$$

$$d = \lambda \frac{\varphi}{720 \text{ deg}}; \varphi = 180 \text{ deg} + \angle(K_L);$$

$$Abs(K_L) = |K_L|$$



2.0.7 Point charges, LEC1

$$Q_1 = -F < x, y, z >; \hat{d} = \frac{\vec{d}}{|\vec{d}|}, [:];$$

$$\vec{d} = < x, y, z >, [m]; |\vec{d}| = d = \sqrt{x^2 + y^2 + z^2}, [m];$$

$$\vec{AB} = < X_b - X_a, Y_b - Y_a >, [m];$$

$$\epsilon_0 = \frac{10^{-9}}{36\pi} [\frac{F}{m}];$$

$$\vec{E}_{QP} = \frac{Q_b}{4\pi\epsilon_0 \cdot d^2} \cdot \hat{d}, [\frac{V}{m}];$$

$$\vec{D} = \epsilon \cdot \vec{E}, [\frac{C}{m^2}];$$

$$\vec{E}_{QP(FULL)} = \vec{E}_{QP(1)} + \vec{E}_{QP(2)} + \vec{E}_{QP(3)}, [\frac{V}{m}];$$

$$V_{pot} = \frac{Q_b}{4\pi\epsilon_0 \cdot x}, [V]; x = \text{dist}, [m];$$

$$V_{pot(FULL)} = V_{pot(1)} + V_{pot(2)} + V_{pot(3)}, [V];$$

$$\vec{F} = Q_a \cdot -\vec{E}_{QP(FULL)}, [N];$$

$$\vec{a} = \frac{\vec{F}}{m}, [\frac{m}{s^2}]; m = \text{mass}, [kg];$$

$$\vec{F} := \frac{Q_1 \cdot Q_2}{4 \cdot \pi \epsilon d} \cdot \hat{d};$$

2.0.8 DETRIMENTAL formulas

$$A_r + jB_r;$$

$$A_p = \sqrt{A_r^2 + B_r^2}; B_p = \angle = \arctan(\frac{b}{a}) \cdot \frac{360}{2 \cdot \pi};$$

HUSK FOR GUDS SKYLD: Cos og Sin med stort i MAPLE! og med with(Gym):

$$\circ \cdot \frac{\pi}{180} = \text{rad}; \text{rad} \cdot \frac{180}{\pi} = \circ;$$

$$A_p \angle B_p \leftrightarrow A_r + jB_r = A_p \cdot (\cos(B_p) + j\sin(B_p));$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} X = (y_1 z_2) - (z_1 y_2) \\ Y = (x_1 z_2) - (z_1 x_2) \\ Z = (x_1 y_2) - (y_1 x_2) \end{bmatrix};$$

Tera, T = 10^{12} ; Giga, G = 10^9 ; Mega, M = 10^6 ; Kilo, k = 10^3 ; Milli, m = 10^{-3} ; micro, μ = 10^{-6} ; Nano, n = 10^{-9} ; Pico, p = 10^{-12}

For more prefixes see slides "ISQ"(mm8) slide 14.

3 Analog Electronic

3.1 THE BJTS

3.1.1 Basic simple ones

- FIND IC
 $I_C = \frac{V_{RC}}{R_C} = \beta \cdot I_B = e^{\frac{V_{BE}}{V_T}} [A]$
 - FIND IB
 $I_B = \frac{I_C}{\beta} [A]$
 - FIND gm
 $gm = \frac{I_C}{V_T} = \frac{\beta}{R_{\pi}} [S] [\Omega^{-1}]$
 - FIND β
 $\beta = gm \cdot R_{\pi} = \frac{I_C}{I_B} [.]$
 - FIND r 's
 $r_{\pi} = \frac{\beta}{gm}; r_e = \frac{1}{gm}; r_o = \frac{V_A}{I_C}$
 where $V_A = 15 < V_A < 200$, Early Voltage effect.
 - FIND V_{BE}
 $V_{BE} = \ln\left(\frac{I_C}{I_S}\right) \cdot V_T [V]$
 - FIND Q Point
 Q (I_{BQ}, I_{CQ}, V_{CEQ})
Without (RE & RB2):
 $I_{BQ} = \frac{V_{BQ} - V_{BEQ}}{R_B} [A]$
 $I_{CQ} \approx \beta \cdot I_{BQ} [A]$
 $V_{CEQ} = V_{CC} - I_{CQ} \cdot R_C [V]$
With (RE & RB2):
 $V_{BQ} \approx \frac{R_{B2}}{R_{B1} + R_{B2}} \cdot V_{CC} [V]$
 $I_{CQ} \approx I_{EQ} = \frac{V_{BQ} - V_{BEQ}}{R_e} [A]$
 $I_{BQ} = \frac{I_{CQ}}{\beta} [A]$
 $V_{CEQ} = V_{CC} - I_{CQ} \cdot (R_C + R_E) [V]$
 - SMALL SIGNAL MODEL
 $R_i = R_{B1} || R_{B2} || (r_{\pi} + (1 + \beta) \cdot R_E)$
 $r_{be} = \frac{\beta}{gm}$
 $gm = \frac{I_C}{V_T}$
 $A_V = \frac{\beta \cdot (R_C || R_L)}{r_{\pi} \cdot (R_C + R_L)}$
 FOR MANY MORE SEE, BJT MAPLE DOC

3.1.2 Things you can assume

$r_{\pi} = r_{be}$ $V_{BE} \approx < 2.4, 3 > [V]$
 $V_{BEQ} = 0.6 < V_{BEQ} < 0.8$
 $V_T \approx 26 \cdot 10^{-3} [V]$ temp coef for BJT
 OR
 $V_T = \frac{K \cdot T_K}{q} = \frac{1.38 \cdot 10^{-23} \cdot 273 + \text{currentTemp}}{1.6 \cdot 10^{-19}} [V]$
 $R_B \approx \frac{\beta \cdot R_E}{10}$
 $I_C \approx I_B$ for simple model (Lec 5.3)

3.1.3 FL from values, or desired FL

- SOM BASIS R-equivalantes.
MAY NOT BE THE ONES YOU HAVE!
 $R_{base} = \frac{1}{\frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{r_{\pi}}} + R_S$
 $R_{collector} = R_C + R_L$
 $R_{emitterBBL} = \frac{1}{gm} + \frac{\frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_S}}{\beta + 1}$
 $R_{emitter} = \frac{1}{\frac{1}{R_E} + \frac{1}{R_{emitterBBL}}}$
 - FIND FL FROM KNOWN C's
 $FL_x = \frac{1}{2\pi \cdot C_x \cdot R_{eqX}}$
 - FIND C's FROM DESIRED F-L
 - We need these for later.
 $f_{base} = 0.1 \cdot f_{desired}$
 $f_{collector} = 0.1 \cdot f_{desired}$
 $f_{emitter} = 0.8 \cdot f_{desired}$
 - FOR BASE and COLLECTOR
 $C_{base} = \frac{1}{2\pi \cdot f_{base} \cdot R_{base}}$
 $C_{collector} = \frac{1}{2\pi \cdot f_{collector} \cdot R_{collector}}$
 - FOR EMITTER
 $C_{emitter} = \frac{1}{2\pi \cdot f_{emitter} \cdot R_{emitter}}$

3.1.4 THD

$$R_{S_{prime}} = \frac{1}{\frac{1}{R_S} + \frac{1}{R_{B1}} + \frac{1}{R_{B2}}}$$

$$R_{e_{prime}} = \frac{1}{\frac{1}{R_E} + \frac{1}{R_e}}$$

$$A_{v_{prime}} = - \frac{\left(\frac{1}{R_C + R_L} \right)}{\frac{1}{gm} + R_{e_{prime}} + \frac{R_{S_{prime}}}{\beta}}$$

$$V_{S_{prime}} = \frac{V_{op}}{A_{v_{prime}}}$$

Harmonic distortion term F:
 $F = 1 + gm \cdot \left(\frac{R_{S_{prime}}}{\beta} + R_{e_{prime}} \right) [.]$

$$THD = \frac{\frac{1}{4} \cdot \frac{abs(V_{S_{prime}})}{V_T}}{F^2} [.]$$

3.2 THE DIODES

3.2.1 PN diode

$V_T \approx 26 \cdot 10^{-3} [V]$ temp coef for BJT
 OR
 $V_T = \frac{K \cdot T_K}{q} = \frac{1.38 \cdot 10^{-23} \cdot (273 + \text{currentTemp})}{1.6 \cdot 10^{-19}} [V]$
 $I_D = I_S \cdot \left(e^{\frac{V_D}{n \cdot V_T}} - 1 \right)$
 $I_D \approx I_S \cdot e^{\frac{V_D}{n \cdot V_T}}$
 $I_S \approx I_D \cdot e^{-\frac{V_D}{n \cdot V_T}}$
 When in forward basis mode
 $I_D \approx I_S \cdot e^{\frac{V_D}{V_T}}$
 $I_S \approx I_D \cdot e^{-\frac{V_D}{V_T}}$
 where:
 I_S = reverse saturation (find in datasheet)
 V_D = Voltage across junction
 n = ideal factor, $1 < n < 2$, ideal = 1
 V_T = Thermal voltage
 See Lec 1 for example:
 $n = \frac{V_{D2} - V_{D1}}{V_T \cdot \ln\left(\frac{I_{D2}}{I_{D1}}\right)}$
 $V_{D1} = n \cdot V_T \cdot \ln\left(\frac{I_{D1}}{I_S}\right)$
 $V_{D2} = n \cdot V_T \cdot \ln\left(\frac{I_{D2}}{I_S}\right)$
 Get the equivalent resistance of a diode:
 $r_D = \frac{V_T}{I_{DQ}} [\Omega]$

3.2.2 Rectifiers

- HALF RECTIFIER
 $A_{V_{ripple}} = \frac{V_{out} - V_{D_{on}}}{f \cdot R \cdot C}$
 $V_{reverse} = 2 \cdot V_{out} - V_{D_{on}}$
 - FULL RECTIFIER
 $A_{V_{ripple}} = \frac{V_{out} - 2V_{D_{on}}}{2 \cdot f \cdot R \cdot C}$
 $V_{reverse} = V_{out} - V_{D_{on}}$
 - FOR BOTH APPLIES, where:
 V_{out} = output voltage
 $V_{D_{on}}$ When the diode turns on ≈ 0.7
 f = the frequency
 R = the resistor value
 C = the capacitor value

3.2.3 Constant voltage drop

$$V_{CC} = \frac{R1 + R2}{R2} \cdot V_{D_{on}}$$

3.3 THE MOSFETS

3.3.1 Basics

- CONSTANTS
 $V_{TH} = 0.3 < V_{TH1} [V]$ (Voltage Threshold)
 $k_n = 0.9 \cdot 10^{-3} [A/V^2]$ transconductance parameter
 $V_{DD} = V_{CC} [V]$, (kært barn, mange navne)

- FIND GM r_o and AV
 $gm = 2 \cdot \frac{I_{DQ}}{V_{GSQ} - V_{TH}}$
 $gm = k_n \cdot (V_{GSQ} - V_{TH})$
 $A_V = -gm \cdot \frac{1}{\frac{1}{R_D} \frac{1}{r_o}}$
 $r_o = \frac{1}{I_{DQ} \cdot \lambda}$
 $\lambda = \frac{L - L'}{V_{DS} \cdot L}$
 where:
 L' = actually channel length
 - V's and D $V_{DS} = V_{DD} - R_D \cdot I_D$
 $I_D = \frac{1}{2} k_n \cdot (V_{GS} - V_{TH})^2$
 $V_{GS} = \sqrt{\frac{2 \cdot I_D}{k_n}} + V_{TH}$

3.3.2 Signal swing

- Max output swing
 $V_{DS_{max}} = V_{DD} [V]$
 $V_{DS_{min}} = V_{GS} - V_{TH} [V]$
 $maxSwing = \min(V_{DS} - V_{DS_{min}}, V_{DD} - V_{DS}) [V_{pp}]$
 - Optimize RD for max output swing
 $V_{range} = V_{DD} - V_{DS_{min}} [V]$
 $V_{DSQ} = \frac{V_{range}}{2} + V_{DS_{min}} [V]$
 $V_{RD} = V_{DD} - V_{DSQ} [V]$
 $R_{D_{optimized}} = \frac{V_{RD}}{I_D} [\Omega]$

3.3.3 THD

$$THD = HD_2 = \frac{V_{pp_{input}}}{4(V_{GS} - V_{TH})} [\%]$$

3.3.4 FL from values, or desired FL

- SOME BASIS R-equivalantes.
MAY NOT BE THE ONES YOU HAVE!
 $R_{gate} = R_s + \frac{1}{\frac{1}{R_{G1}} + \frac{1}{R_{G2}}}$
 $R_{drain} = R_D + R_L$
 $R_{source} = \frac{1}{\frac{1}{R_S} + gm}$
 - FIND FL FROM KNOWN C's
 $FL_x = \frac{1}{2\pi \cdot C_x \cdot R_{eqX}}$
 - FIND C's FROM DESIRED F-L
 - We need these for later.
 $f_{gate} = f_{drain} = 0.1 \cdot f_{desired}$
 $f_{source} = 0.8 \cdot f_{desired}$
 - FOR GATE AND DRAIN
 $C_{gate} = \frac{1}{2\pi \cdot f_{gate} \cdot R_{gate}}$
 $C_{drain} = \frac{1}{2\pi \cdot f_{drain} \cdot R_{drain}}$
 $C_{source} = \frac{1}{2\pi \cdot f_{source} \cdot R_{source}}$

3.4 Others

3.4.1 Miller equivalents

$$C_{in_{Miller}} = C_f \cdot \left(1 - A_V \right) [F]$$

$$C_{out_{Miller}} = C_f \cdot \left(1 - \frac{1}{A_V} \right) [F]$$

3.4.2 Spice Commands

.op (giver værdier over komponenter)
 .four <test-frequency> [Nharmonics] [-1]
 <outNetName> (THD directive)

3.4.3 The 3 Golden Triangles

