Digital Signal Processing Frequency Sampling Method

P5 Miniproject

ES22-ESD5-510

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1 DSP mini project

The given assignment for this DSP mini project is:

1. Make a theoretical and mathematical examination and description of the frequency sampling method for designing a FIR filter
2. The method is then used for construction of a LP-filter, that best possibly complies with the design specifications for the filter, that was introduced in the exercises of course 4, ex.1c.
3. There should be experimented with different values for the frequency samples in a) the interval of 0 to 2π and b) in the transition band in between the passband and stop band.
4. The found results should then be compared to a filter design using the ordinary window method used for FIR filters, where there of course is allowed to experiment with the different window functions and filter orders, that you would find interesting
5. There should then be made a off-line simulation of the filter (Matlab, Pyhton, ...) Where it is shown its ability to "clean" a (self constructed) signal superimposed with noise illustrated.

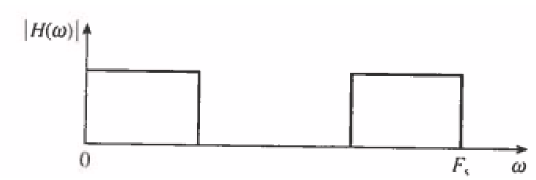
This little report will go through each of these points

1.1 Frequency Sampling Method

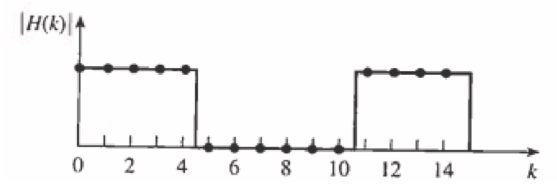
The frequency sampling method, (FSM) can be used for either non-recursive or recursive FIR filters. The method allows to design non-recursive FIR filters in both standard frequency selective filters (low-pass, high-pass, band-pass, stop-band) and filters with arbitrary frequency response. What makes the method unique is that it allows recursive implementation, meaning better computational efficiency, of FIR filters, with the restriction of the coefficients being simple integers.

1.1.1 Non-Recursive frequency sampling filters

The non-recursive frequency sampling filter can be used for both standard frequency selective (LP, HP, BP, SB) and arbitrary frequency response filters. To express and understand the formulas, an ideal low-pass filter will be used for explaining the different processes. The ideal low-pass filter is represented in figure 1.1a and on the other figure 1.1b the samples of it is shown. The reason for it looking like a stop-band comes from the symmetry that occurs in the discrete time domain of the middle frequency π. That can be described by positive and negative frequencies. Where the positive frequencies relate to the points before $N/2$ and the negative is the points after, up to $N-1$, where N = samples.



(a) The filter simulated, showing the amplitude.



(b) The impulse response of the filter, k = sampling index [1]

Figure 1.1: Filter build from requirements, with 8 samples from 0 to π [1]

As can be seen on figure 1.1b there is a total of 15 samples where 5 samples are in the pass-band and 3 in the stop-band. If it is taken into account that it is symmetrical and it is possible just to use half of the samples. Therefore $|H(k)|$ can be represented as.

$$|H(k)| = \begin{cases} 1 & k = 0, 1, 2, 3, 4 \\ 0 & k = 5, 6, 7 \end{cases}$$

From this two important factors can be explained for understanding the principles of the method, that being the normalized cutoff frequency and the amount of samples having an importance in the distance between the samples.

First, the importance of the sampling rate, f_s, is examined when making a FIR filter using the non-recursive frequency sampling method. Figure 1.2 and figure 1.3 show this on a unit circle. This unit circle represents one cycle in the discrete time domain from 0 to 2π, with different sampling rates, one using 15 samples and the other having 30 samples. This clearly shows that more samples in the same area respond to the samples having a smaller frequency period, f_p, which is going to be important to remember when getting into the transition width theory.

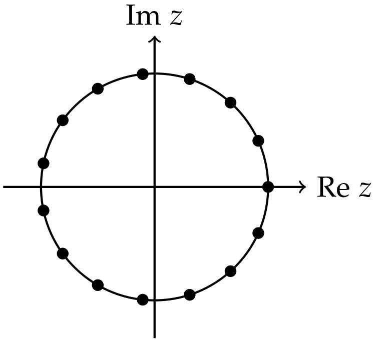


Figure 1.2: Type 1, N odd, with 15 samples.

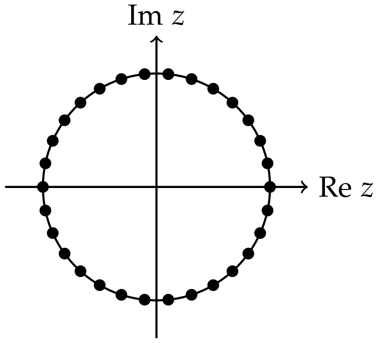


Figure 1.3: Type 1, N even with 30 samples.

Looking into how the normalized cutoff frequency can be used and its importance. It is used to get the actual cutoff frequency, with the use of the normalized cutoff frequency expressed as $f_N = \frac{k}{N}$, and the sampling rate f_s. Where k is the index that tells the position where the cutoff happens. The equation for the actual cutoff frequency can be seen in the equation below.

$$f_k = f_N \cdot f_s \tag{1.1}$$

This gives the opportunity to either design the discrete or frequency domain cutoff point if the factors are known from either, which will be necessary for designing the filter.

The next step is looking into how the non-recursive frequency sampling filter actually is reconstructed based on the ideal low-pass. The $h(n)$ is constructed after the samples taken in the frequency-domain $H(k)$ using Inverse Discrete Fourier Transform, IDFT, as seen in the equation below [1]:

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) \cdot e^{j(2\pi/N)nk} \tag{1.2}$$

where $H(k)$ is the frequency response sample at a specific k value, k is the frequency index, n is the time-domain index and N is the number of samples in the DFT. The twiddle factor is also included as $e^{j(2\pi/N)} = W_N$, it represents the fundamental frequency of the Fourier Transform, and it defines the phase shift associated with one step in the frequency-domain representation.

Expanding on equation 1.2, using Euler's formula and due to the linear phase, knowing that its entirely real, means $h(n)$ can be expressed as [1]:

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{N/2-1} |H(k)| \cos\left[2\pi k \frac{(n-\alpha)}{N}\right] + H(0)\right] \tag{1.3}$$

where the summation for N even is $N/2 - 1$ and for N odd it is $\frac{N-1}{2}$, and $\alpha = \frac{N-1}{2}$ in general. $H(0)$ is the frequency response at $\omega = 0$, that being the gain of the filter at DC. As an example, typically $H(0)$ would be desired at 1 (no attenuation at DC) with a low-pass filter.

It makes it real-valued by using a sum of cosines rather than complex exponentials, by using Eulers law which is $cos(x) + jsin(x)$ and therefore only using the real-valued part and removing the imaginary part. The reason it is possible to remove the complex part is due to the symmetry in the frequency domain when dealing with a real-valued signal. The Fourier coefficients for positive and negative frequencies come in complex conjugate pairs. This means the imaginary parts have opposite signs and cancel each other out. The formula assumes that the frequency samples $H(k)$ is symmetrical.

After constructing the filter it is possible that a lot of ripples occur if the transition width is thin, depending on the amount of samples used in the filter. An example from the analog world could be Chebyshev and Butterworth, where Butterworth has a wider transition band and good amplitude response, while the Chebyshev has a sharp transition band.

Therefore when an improved amplitude response is needed it can be traded off with a wider transition width. Improved refering to less ripples in the amplitude response. This can be done by either reducing the amount of samples causing the samples to have further distance between them or introducing additional frequency samples in the transition band. More transition samples corresponds to getting a better amplitude response, (smoother transition and smaller ripples). The transition band samples can be seen in figure 1.4.

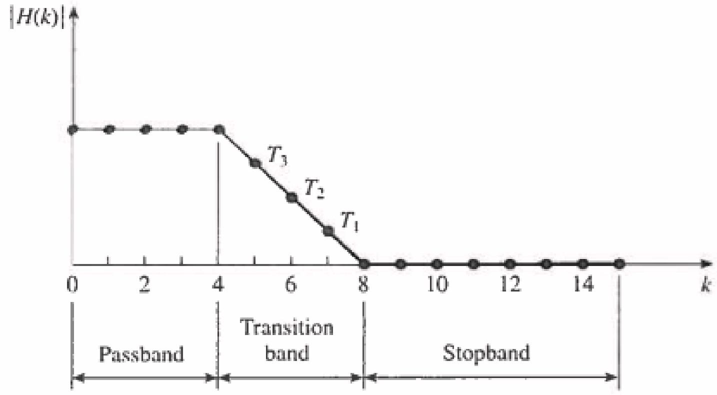


Figure 1.4: Shows the transition band, and how there can be placed transition samples [1].

The results of adding transition band samples can be seen when taking a low-pass filter. As an example, the stop-band attenuation increases, approximately by 20 dB for each transition band frequency sample, with a corresponding increase in the transition width as well, described by [1]:

approximate stopband attenuation	$(25 + 20M)dB$
approximate transition width	$(M+1)F_s/N$

where M is the number of transition band frequency samples and N is the filter length. These transition samples are represented by $T_1, T_2, ...T_M$, the amount of transition samples necessary depends on the criteria set. To get the optimum stop-band attenuation the objective is to find the values that minimize the peak stop-band ripple values for samples. A sample of these optimal values are given in figure 1.5, but can also be computed using a hybrid genetic algorithm [1].

BW	Stopband attenuation (dB)	T_1	T_2	T_3
One transition band frequency sample, N = 15				
1	42.309 322 83	0.433 782 96		
2	41.262 992 86	0.417 938 23		
3	41.253 337 86	0.410 473 63		
4	41.949 077 13	0.404 058 84		
5	44.371 245 38	0.392 681 89		
6	56.014 165 88	0.357 665 25		
Two transition band frequency samples, N = 15				
1	70.605 405 85	0.095 001 22	0.589 954 18	
2	69.261 681 56	0.103 198 24	0.593 571 18	
3	69.919 734 95	0.100 836 18	0.589 432 70	
4	75.511 722 56	0.084 074 93	0.557 153 12	
5	103.460 783 00	0.051 802 06	0.499 174 24	
Three transition band frequency samples, N = 15				
1	94.611 661 91	0.014 550 78	0.184 578 82	0.668 976 13
2	104.998 130 80	0.010 009 77	0.173 607 13	0.659 515 26
3	114.907 193 18	0.008 734 13	0.163 973 10	0.647 112 64
4	157.292 575 84	0.003 787 99	0.123 939 63	0.601 811 54

Figure 1.5: Optimum transition band frequency samples for type 1 lowpass frequency sampling filters for N 15. [1]

The bandwidth, BW, value in the left column of the table describes how many samples that are in the filters pass band. A relation exists that when the coefficients have a lower value, the filter has a wider bandwidth which results in more stop-band attenuation as said above. When an improved amplitude response is needed it can be traded off with a wider transition width. When saying the stop-band attenuation increases it means that reducing the magnitude of the ripple in the stopband, as these frequencies are unwanted.

To get the full idea an example with 3 samples in the pass band and 5 in the stop band to get the 15 samples when mirrored, which is represented as:

$$|H(k)| = \begin{cases} 1 & k = 0,1,2 \\ 0 & k = 4,5,6,7,8 \end{cases}$$

When implementing different amounts of transition samples from 1 to 3, using the tabular for 15 samples in the filter, it is represented as:

$$|H(k)| = \begin{cases} 1 & k = 0,1,2 \\ 0.41047363 & k = 4 \\ 0 & k = 5,6,7,8 \end{cases} \begin{cases} 1 & k = 0,1,2 \\ 0.58943270 & k = 4 \\ 0.10083618 & k = 5 \\ 0 & k = 6,7,8 \end{cases} \begin{cases} 1 & k = 0,1,2 \\ 0.64711264 & k = 4 \\ 0.16397310 & k = 5 \\ 0.00873413 & k = 6 \\ 0 & k = 7,8 \end{cases}$$

1.2 The Window Method

Another method that can be used for creating an FIR filter instead of the frequency sampling method is the window method. It is designed by using the ideal impulse response of the desired filter, from which it is possible to multiply the impulse response with the window chosen to get the finite impulse response. Represented by the equation below, where $H_d(n)$ is the ideal impulse response and $\omega[n]$ is the window function.

$$h(n) = h_d(n) * \omega[n] \tag{1.4}$$

However, before getting into the ideal impulse response, the ideal frequency response is usually found, from which it is then transformed into the time domain by taking its inverse Fourier transform, where the result of this process is an infinite sequence. It is then truncated to get the impulse response by selecting a finite number of samples.
The Gibbs phenomenon happens when truncating, so to smooth out these ripples, the window function is applied to the truncated sequence. The window gradually reduces the values at the edges of the impulse response to zero, which reduces the abrupt transitions that cause spectral leakage and ringing.
Common window functions include [2]:

- Rectangular: $\omega[n] = \begin{cases} 1 & 0 \leq n < M \\ 0 & \text{otherwise} \end{cases}$
- Bartlett: $\omega[n] = \begin{cases} \frac{2n}{M} & 0 \leq n < M/2 \\ 2 - \frac{2n}{M} & M/2 \leq n < M \\ 0 & \text{otherwise} \end{cases}$
- Hanning: $\omega[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right) & 0 \leq n < M \\ 0 & \text{otherwise} \end{cases}$
- Hamming: $\omega[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{M}\right) & 0 \leq n < M \\ 0 & \text{otherwise} \end{cases}$
- Blackman: $\omega[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right) & 0 \leq n < M \\ 0 & \text{otherwise} \end{cases}$

Each of the window functions has its own different properties that balance main-lobe width (affecting the filter's transition bandwidth) and side-lobe levels (affecting stop-band attenuation). As the window function has been chosen and the ideal impulse response is calculated, the FIR filter coefficients can be calculated using equation 1.4. Here the important factor to take into consideration is which window function is used, as this is what decides the Transition bandwidth, Stop-band attenuation, and Pass-band ripple.
Let us go through an example with a low-pass FIR filter, where f_c = cutoff frequency, where the ideal frequency response would be represented as:

$$|H(k)| = \begin{cases} 1 & |f| <= f_c \\ 0 & |f| >= f_c \end{cases}$$

First, the impulse response is found and then the inverse Fourier transform of it is calculated.

$$h_d[n] = \int_{-f_c}^{f_c} e^{j2\pi f n} df \tag{1.5}$$

The ideal impulse response is known to be a sinc function for this specific case, described as:

$$h_d(n) = \frac{\sin(2\pi f_c n)}{\pi n} \tag{1.6}$$

When aiming to design the ideal impulse response for a low-pass filter, it is often desired to center the response around a specific number of samples. Given N samples, where N is typically odd cause the data point is at zero, the midpoint M is defined as $M = N - 1$. This midpoint M ensures that the impulse response is symmetric around $M/2$. The expression for the impulse response is then given by [2]:

$$h_d(n) = \frac{sin(2\pi f_c(n - (M)/2))}{\pi(n - (M)/2)} \tag{1.7}$$

1.3 Build-a-filter

This section will describe the construction and test of different filters.

1.3.1 Specification

From the exercise, a low-pass filter with the requirements has been asked to be designed using FSM that roughly fits within these demands:

DC	750 Hz	1000 Hz	1500 Hz
0 dB	< -1 dB	$\approx$ -3 dB	> -10 dB

Table 1.1: Requirments for the filter.

From this, a few basic things can be seen. A slow and smooth transition is wanted between the passband and stop band. There should be a transition zone instead of a hard transition as pass-band ripple is not desired. A basic rule of thumb when choosing T values can be seen in the table 1.2. There are ways to calculate the exact values, depending on the number of samples N, and the desired number of samples in the transition band T_n. However, this is only for a smooth transition; in theory, the transition samples could be placed wherever you desire.

While illogical, T_1 is placed closest to 0 and T_3 closest to 1 for $T_n = 3$

	For 1 sample	For 2 samples	For 3 samples
T_1	$0.250 < T_1 < 0.450$	$0.040 < T_1 < 0.150$	$0.003 < T_1 < 0.035$
T_2		$0.450 < T_2 < 0.650$	$0.100 < T_2 < 0.300$
T_3			$0.550 < T_3 < 0.750$

Table 1.2: Table of T_n as a rule of thumb.

1.3.2 Filter design

From the requirements, and for ease, the spacing between each sample, will be set to $fd = 250[Hz/n]$ as this allows for easy placement of frequency samples given by the specification. From table 1.1 the three points make a straight line when converted into magnitude. This gives $T_1 = -15dB = 0.178$, $T_3 = -3dB = 0.708$ and $T_4 = 0.891$. From this, a linear regression is made, resulting in an equation for the transition samples:

$$T(x) = -0.001 \cdot x + 1.6381 \tag{1.8}$$

Meaning that $T_2 = 0.3881$

$$|H(K)| = \begin{cases} 1 & = 0, 1, 2 \\ 0.891 & = 3 \\ 0.707 & = 4 \\ 0.388 & = 5 \\ 0.178 & = 6 \\ 0 & = 7...9 \end{cases} \tag{1.9}$$

The zero sample points could be increased, for a larger bandstop zone.
This will result in a filter of

- fd = 250 [Hz]
- N = 19
- $fs = fd \cdot N = 4750$
- $T_n = 4$

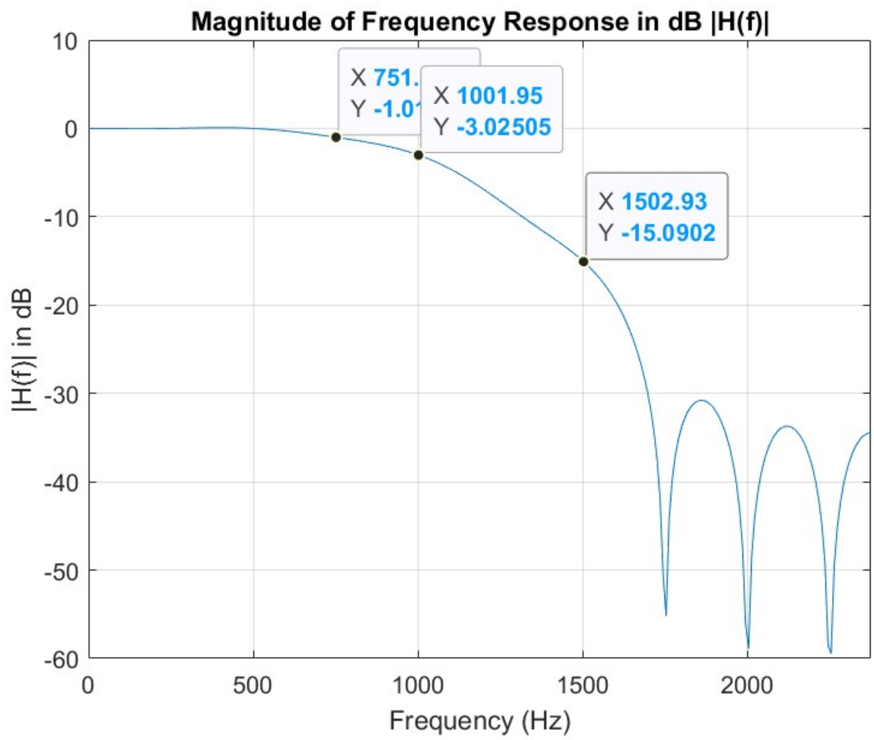
Using equation 1.3 which also can be seen below will give the filter coefficients of:

$$h(n) = \frac{1}{N}\left[\sum_{k=1}^{(N-1)/2} |H(k)| \cos\left[2\pi k \frac{(n-\alpha)}{N}\right] + H(0)\right] \tag{1.10}$$

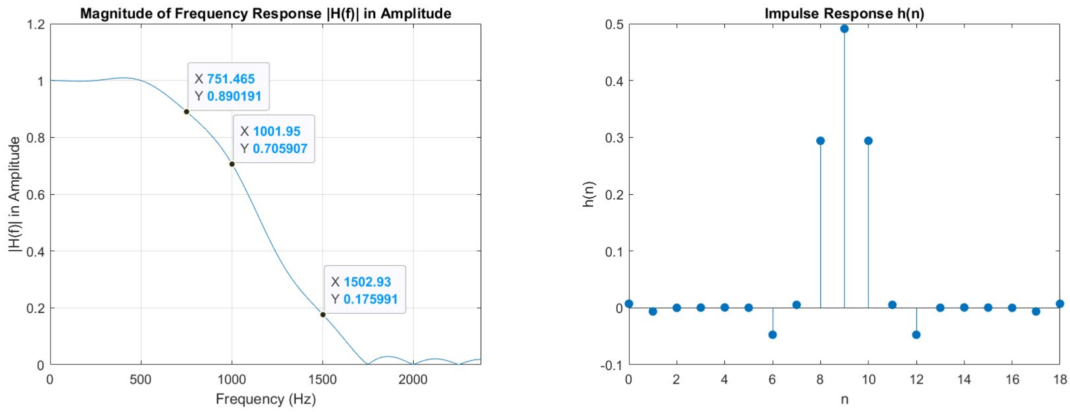
Where $\alpha = (\frac{N-1}{2})$. They are displayed below, and are symmetrical around H[9].

$$\begin{aligned} H[0] &= 0.0073 \\ H[1] &= -0.0063 \\ H[2] &= 0.0001 \\ H[3] &= 0.0005 \\ H[4] &= 0.0007 \\ H[5] &= 0.0002 \\ H[6] &= -0.0473 \\ H[7] &= 0.0053 \\ H[8] &= 0.2940 \\ H[9] &= 0.4911 \end{aligned} \tag{1.11}$$

The filter plotter through MatLab can be seen in figure 1.6



(a) The filter, with a dB scale and the frequencies as desired.



(b) The filter simulated, showing the amplitude.

(c) The impulse response of the filter.

Figure 1.6: Filter build from requirements.

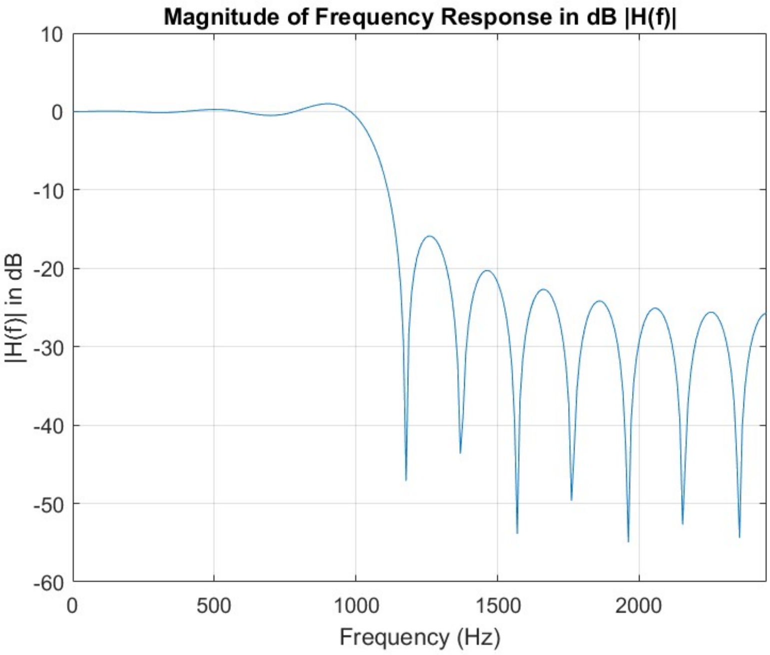
As can be seen, the filter works as desired and fulfills the requirements. But as many parameters can affect your filer design how it affects it should be tested. It is also shown in figure 1.6b that it is not the ideal regular curvature as T_n has been placed outside the guideline parameters.

1.3.3 Affect of N

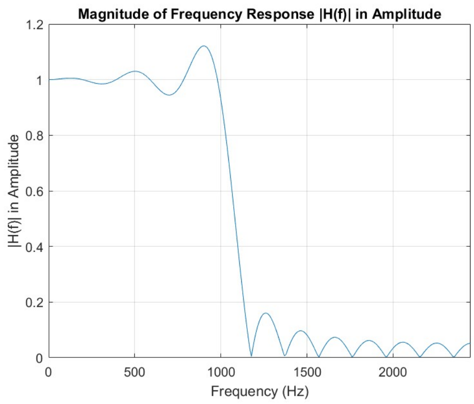
There are a few parameters when designing a filter, one of them being N, which is the number of samples. A smaller N should give a smooth and less hard transition while increasing it, will give a much harder cutoff. Changing N = 15, and setting the parameter to:

$$|H(K)| = \begin{cases} 1 & = 0...5 \\ 0 & = 6..12 \end{cases} \tag{1.12}$$

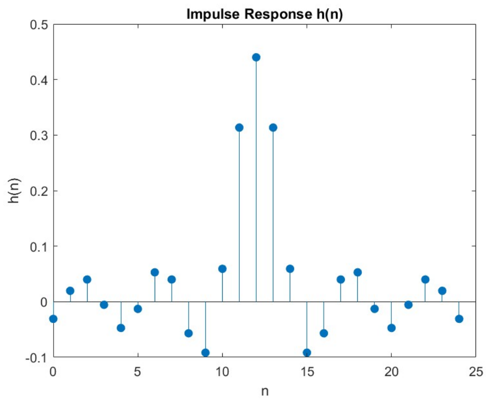
Using N = 25 gives a sample spacing of 196 [Hz] per sampling, and the passband will stop after, 980 [Hz]



(a) The filter, with a dB scale and the frequencies as desired.



(b) The filter simulated, showing the amplitude.



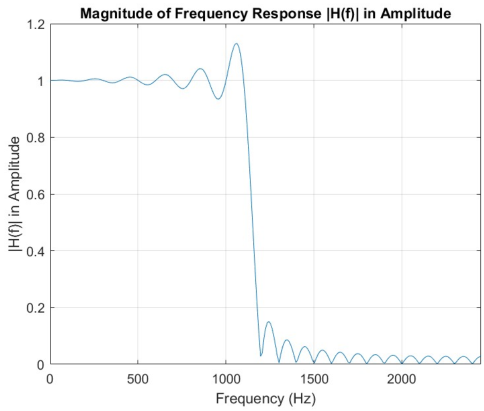
(c) The impulse response of the filter.

Figure 1.7: Filter given $T_n = 0$, N = 25, fs = 4900.

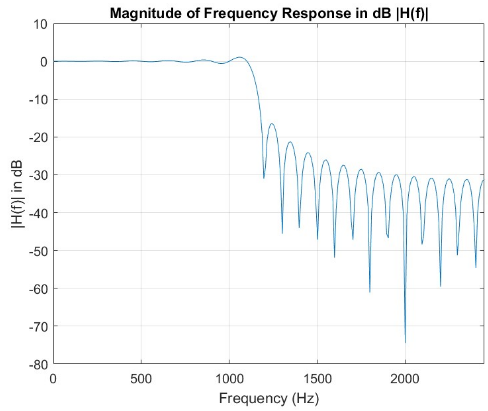
Increasing the sample size for N = 49 gives a filter off:

$$|H(K)| = \begin{cases} 1 & = 0...11 \\ 0 & = 12..24 \end{cases} \tag{1.13}$$

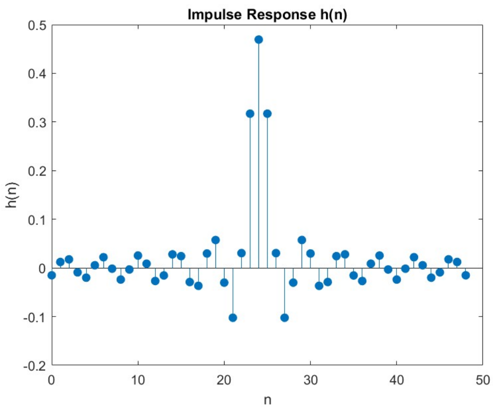
This would mean a 100 [Hz] spacing between each sample and the passband will end after 1100 [Hz] instead.



(a) The filter in dB.



(b) he filter simulated, showing the amplitude.



(c) The impulse response of the filter.

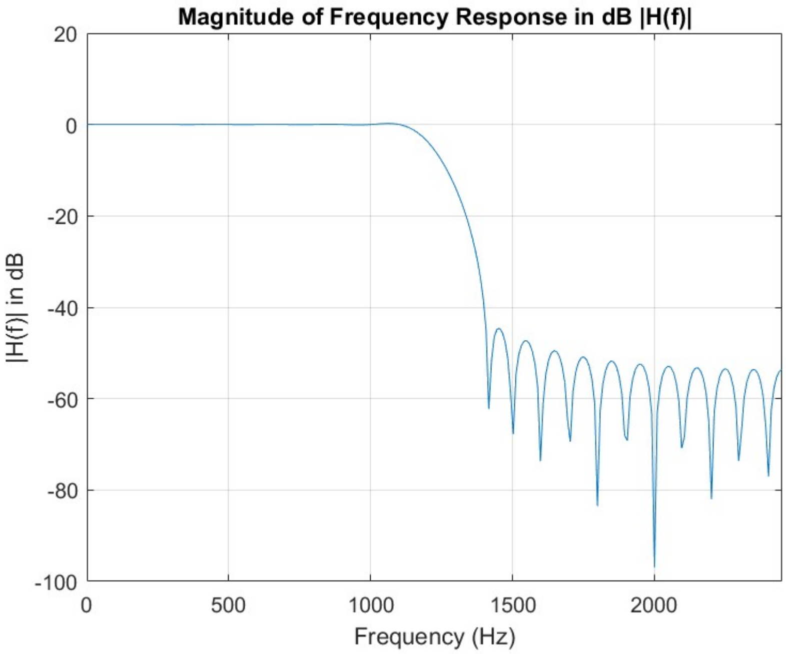
Figure 1.8: Filter given $T_n = 0$, N = 49, fs = 4900.

Here it is clear that increasing N means a much sharper drop where the passband frequency stops and a very large pass-band ripple.

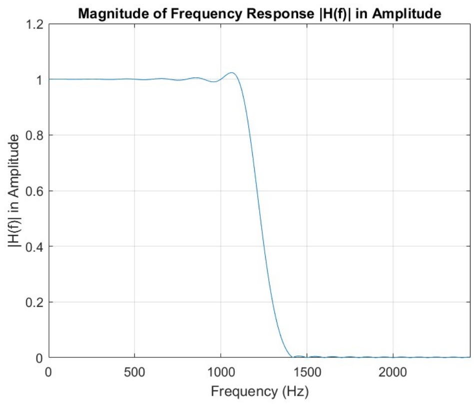
1.3.4 Transitions samples effect on the value

Decreasing this ripple with a few transition samples would be very beneficial as the ripple in 1.7 is very high. This is where samples in the transition band are very useful. This gives a filter of:

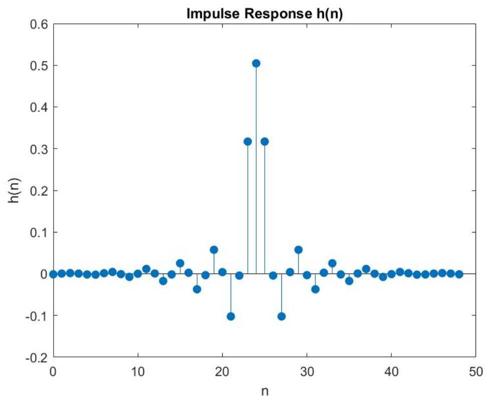
$$|H(K)| = \begin{cases} 1 & = 0...5 \\ 0.650 & = 6 \\ 0.200 & = 7 \\ 0.01 & = 8 \\ 0 & = 9...12 \end{cases} \tag{1.14}$$



(a) The filter, with a dB scale and the frequencies as desired.



(b) The filter simulated, showing the amplitude.



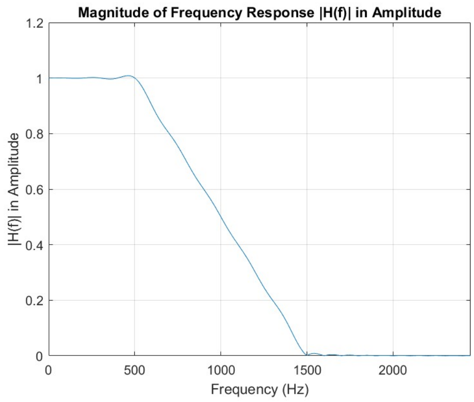
(c) The impulse response of the filter.

Figure 1.9: Filter given $T_n = 3$, N = 49, fs = 4900.

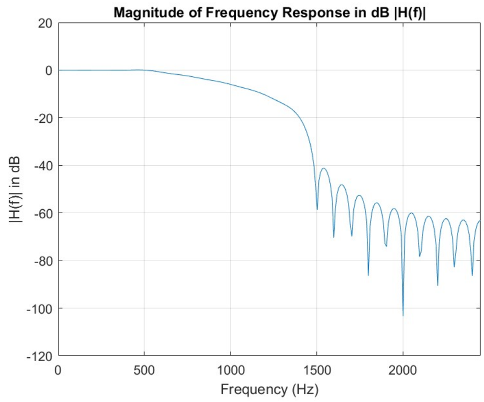
As can be seen in 1.9 this decreased the ripple that had been added when N had been increased. Going further with 8 samples in the transition gives:

$$|H(K)| = \begin{cases} 1 & = 0...6 \\ 0.9 & = 7 \\ 0.8 & = 8 \\ 0.7 & = 9 \\ 0.6 & = 10 \\ 0.5 & = 11 \\ 0.4 & = 12 \\ 0.3 & = 13 \\ 0.2 & = 14 \\ 0.1 & = 15 \\ 0 & = 16...24 \end{cases} \tag{1.15}$$

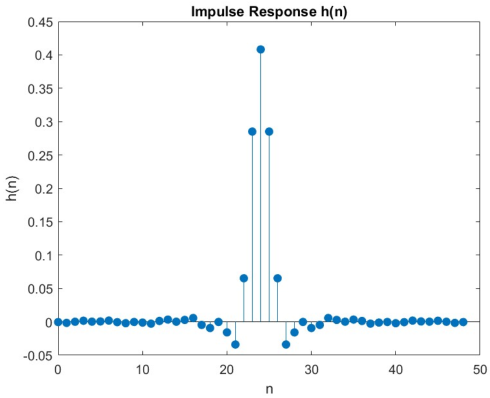
This can be seen in figure 1.10



(a) The filter, in linear scale.



(b) The filter simulated, in dB scale.



(c) The impulse response of the filter.

Figure 1.10: Filter given $T_n = 8$, N = 49, fs = 4900.

While it is hard to see why a filter like this is desired, it can be created.

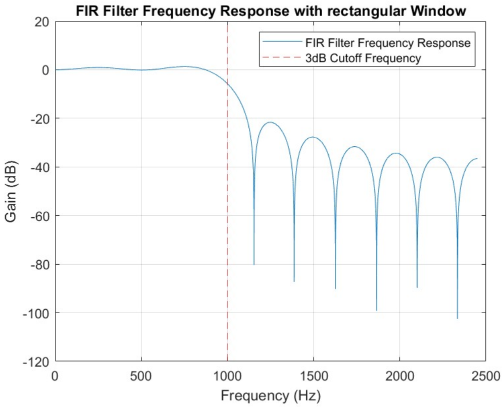
1.3.5 Comparison of Frequency Sampling Method and the Window Method

Now that the FSM has been described and sought out, it should be quickly compared with the window method. The window method is also based on trial and testing to get a filter that matches the desired values. Matlab has several functions for doing so, which will be used.

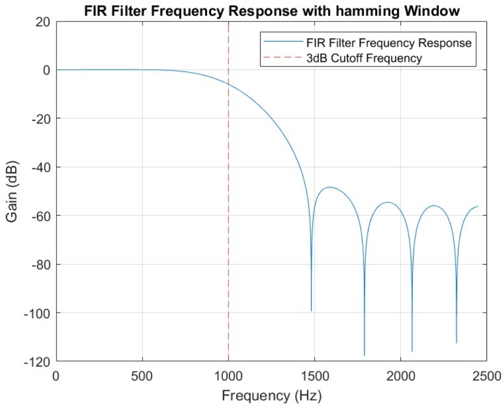
What we know from the classical use of the window method are the different types of windows that can be used to get different properties.

- **Rectangular:** Sharp cutoff, but with ripples in the passband. Not a high damping.
- **Hamming:** Slow cutoff, no ripples in the passband, decent damping.
- **Hanning:** Slow cutoff, no ripples in the passband, decent damping.
- **Blackman:** Very slow cutoff, no ripples, but high damping.
- **Kaiser:** Decent cutoff, no ripples, decent damping.

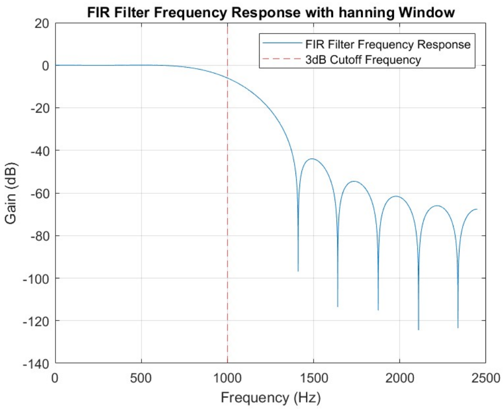
They can also be seen in the figure below, using the same parameters of $fs = 4750$ and $N = 19$.



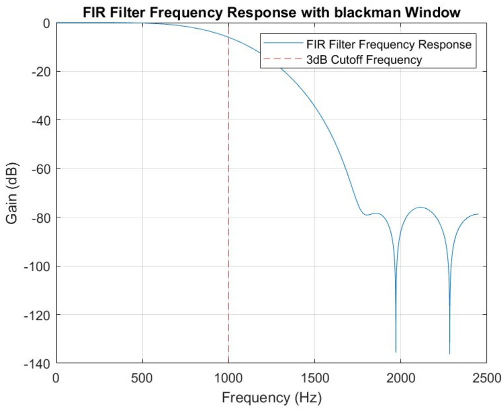
(a) Rectangular window.



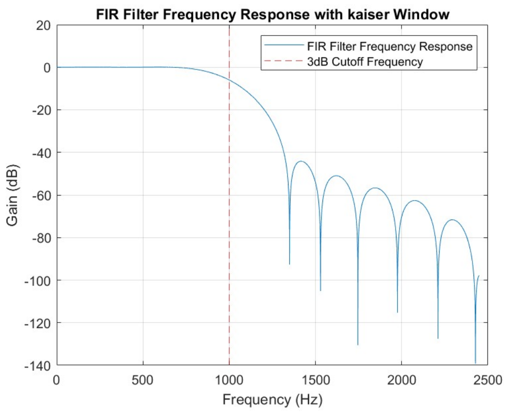
(b) Hamming window.



(c) Hanning window.



(d) Blackman window.



(e) Kaiser window.

Figure 1.11: Diffrent types of window functions.

Of course, getting the "optimal" filter which fits with the requirements, is more of a trial-and-error method. Using the blackman window, with a high sampling frequency should make a filter close to the one made by the frequency sampling method. By experimenting $N = 11$ and $fs = 9500[Hz]$ and keeping $fc = 1000[Hz]$ is a filter getting somewhat close.

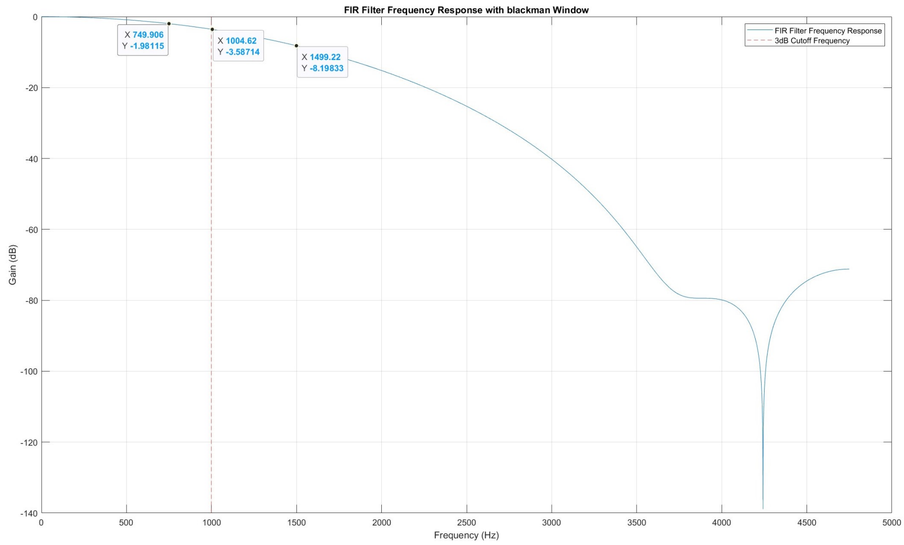


Figure 1.12: Filter using the Blackman window function.

While this is not as close as the other filter design it somewhat fulfills, but a trial and error approach is not as smart as being able to place the transition samples allowing a filter that completely fits.

Using a window function, especially not the rectangular window, is smart when no ripple is desired, as ripples can still occur even when adding transition samples when using FSM to lessen this. To make a hard distinction, the FSM can be used when specific frequencies have a wanted damping and the window method is good when specific qualities are wanted, where the overall characteristics can be designed more specifically after what is wanted.

1.3.6 Testing of the filters

To test the filter's performance, it will be simulated with different types of noisy signals. Here the filter that was asked to be designed is tested. The characteristics can be seen in subsec 1.3.2 if it had been forgotten.

The signal which will be tested is:

- Carrier: 500 [Hz] with an amplitude of 1.
- Noise components: 1500, 1750, and 1900 [Hz], with an amplitude of 0.5 each.

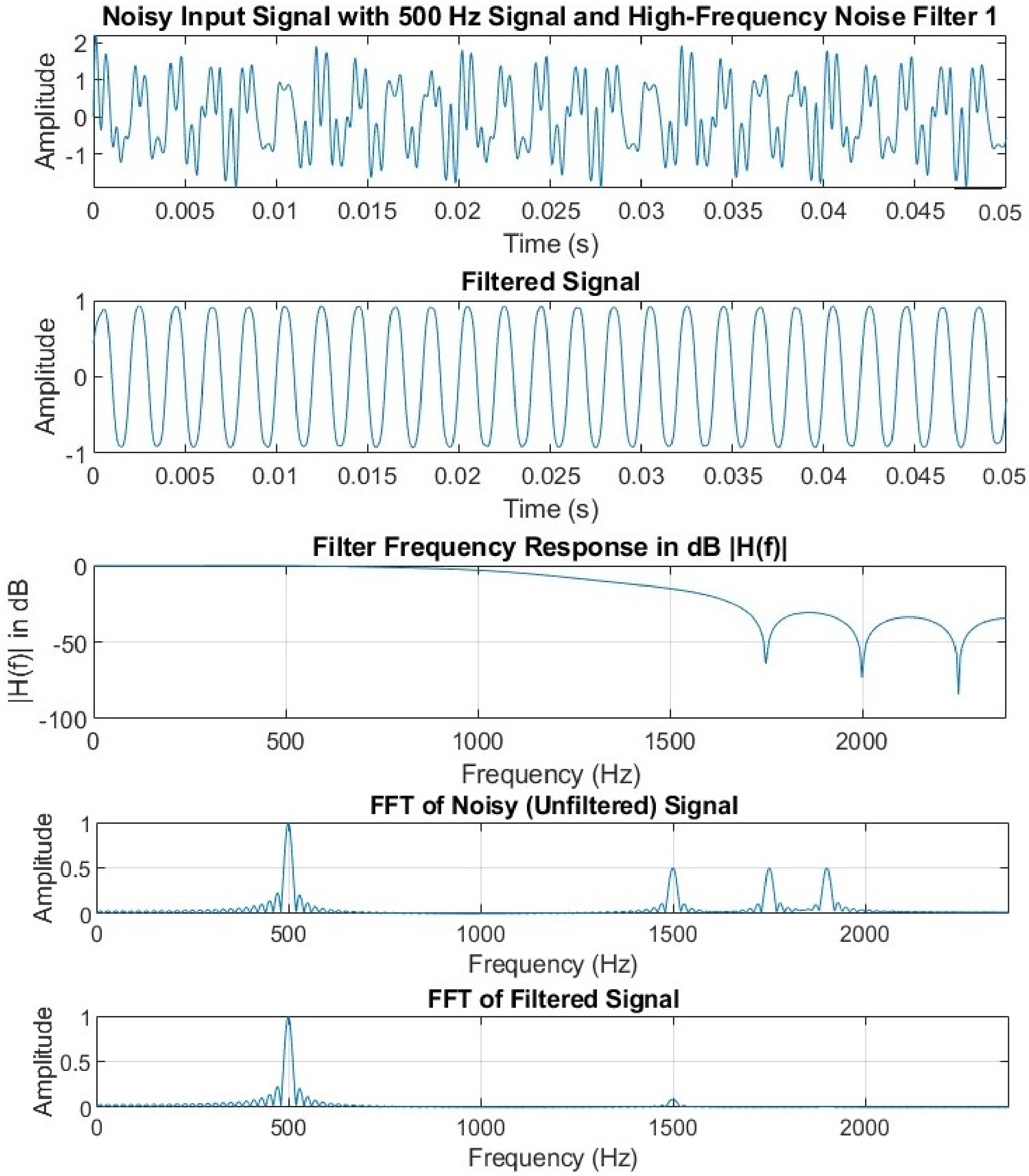


Figure 1.13: For noises at 1500, 1750, and 1900 [Hz]

But as designed, and as can be seen in the frequency response frequencies, and as desired, frequencies around 1000 will not be damped. To test it further these signal components will be used instead.

- Carrier: 500 [Hz] with an amplitude of 1
- Noise components: 750, 888, 1000, [Hz], with an amplitude of 0.5 each

Which gives a response of:

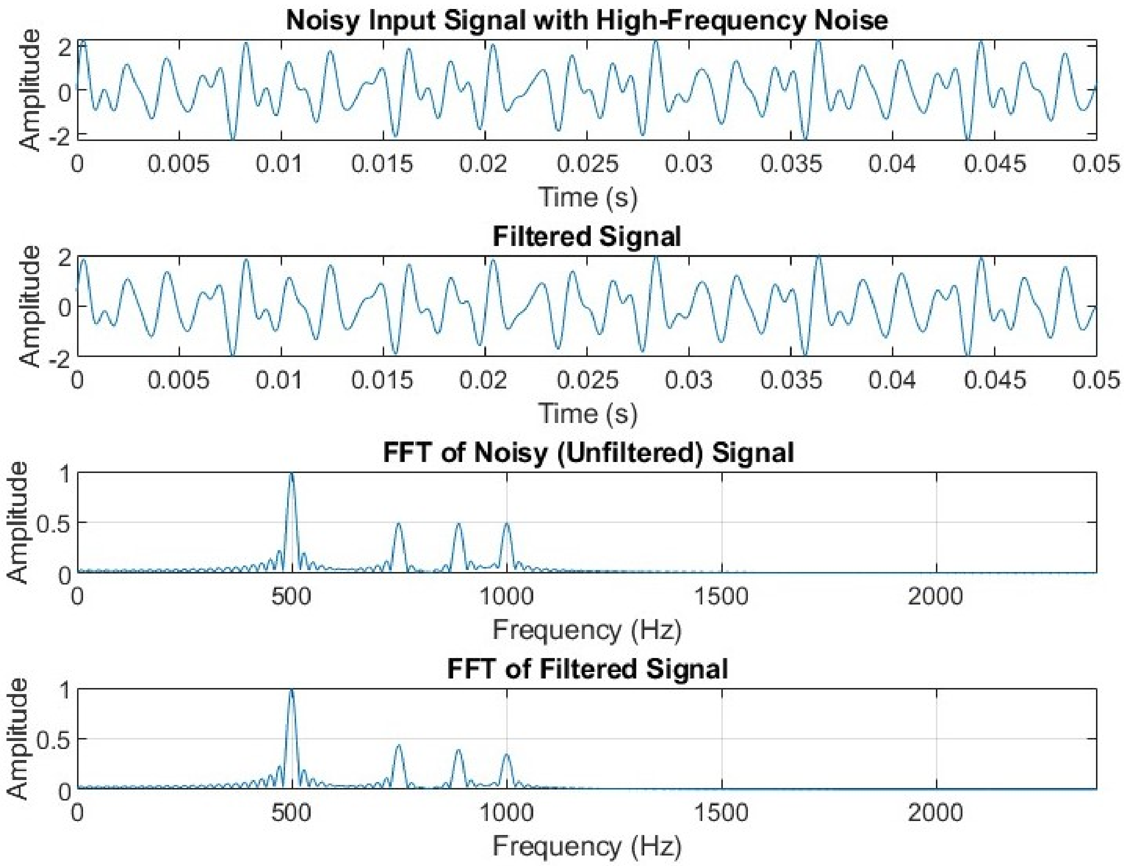


Figure 1.14: For noises at 750, 888, 1000 [Hz].

As can be seen, 750, 888, and 1000 go through, with some slight damping for the signal of a 1000 [Hz] signal.

Testing of a sharper filter

Just to see, making a sharper filter, with N = 49, placing the cutoff at 1000 [Hz] gives

$$|H(K)| = \begin{cases} 1 & = 0...10 \\ 0 & = 11...24 \end{cases} \tag{1.16}$$

And setting noise parameters to:

- Carrier: 900 [Hz] with an amplitude of 1
- Noise components: 1200, 1300, [Hz], with an amplitude of 0.5 each

Which should still give a pure wave out, and it can be seen that it does in fig 1.15

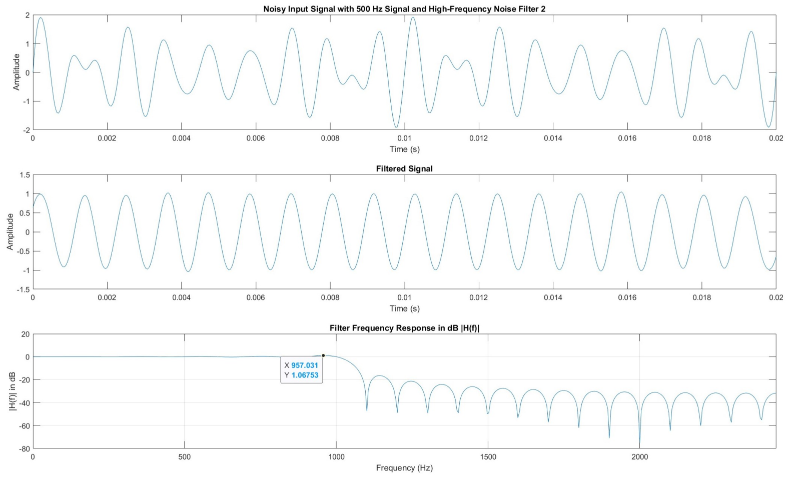


Figure 1.15: Carrier of 900 [Hz] and noise at 1200 and 1300 [Hz]

Removing the transition samples has increased the ripple quite a lot but it does give a very sharp filter.

Testing of a different filter, just for fun

- N = 25
- $fs = 4900[Hz]$
- $T_n = 3$
- $fd = 196[Hz/n]$

These values have been chosen as to get a sharp filter as it is easier to test, but without too much ripple.

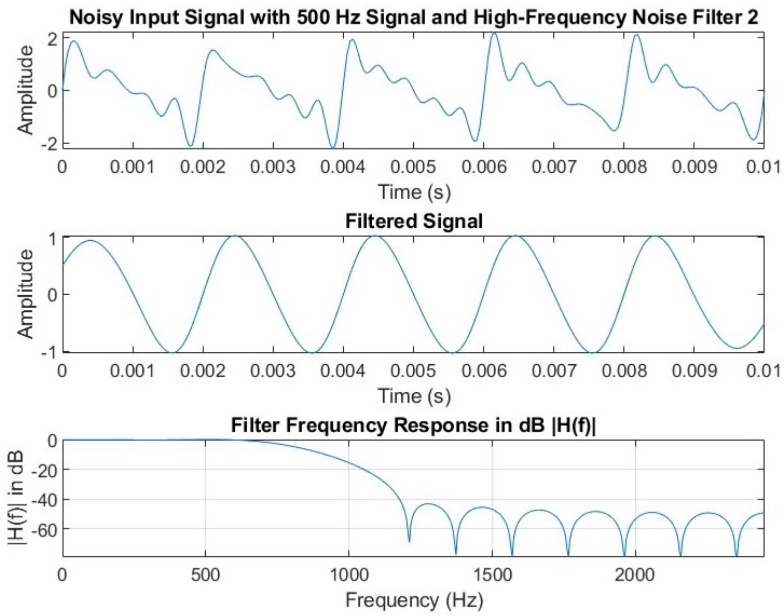


Figure 1.16: With carrier at 500 [Hz] and noise at 1000, 1500, 2000, 2500 [Hz].

Where it all gets filtered out.

Adding a frequency at 4500 should alias as the filter is symmetrical. This can be seen in 1.19

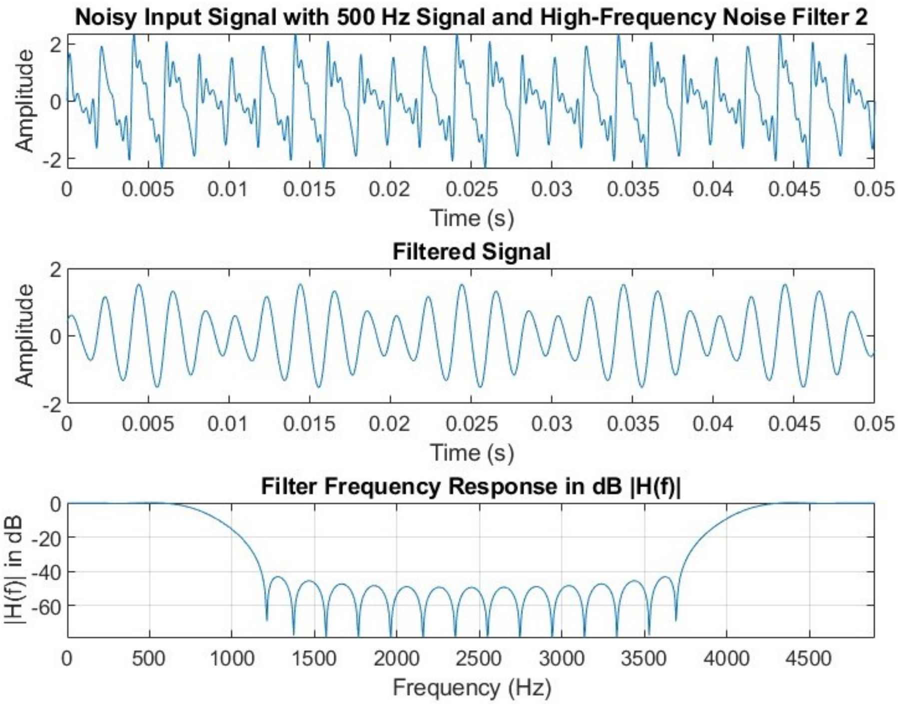


Figure 1.17: With carrier at 500 [Hz] and noise at 1000, 1500, 2000, 2500, and 4500 [Hz]

And the FFT looks like:

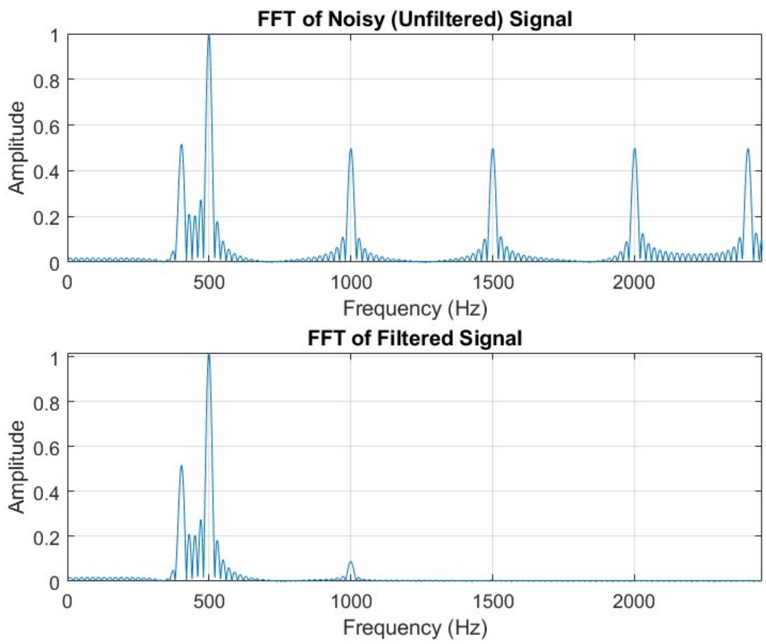


Figure 1.18: The FFT of carrier at 500 [Hz] and noise at 1000, 1500, 2000, 2500, and 4500 [Hz]

Just as adding a 100 [Hz] signal should not be filtered out either.

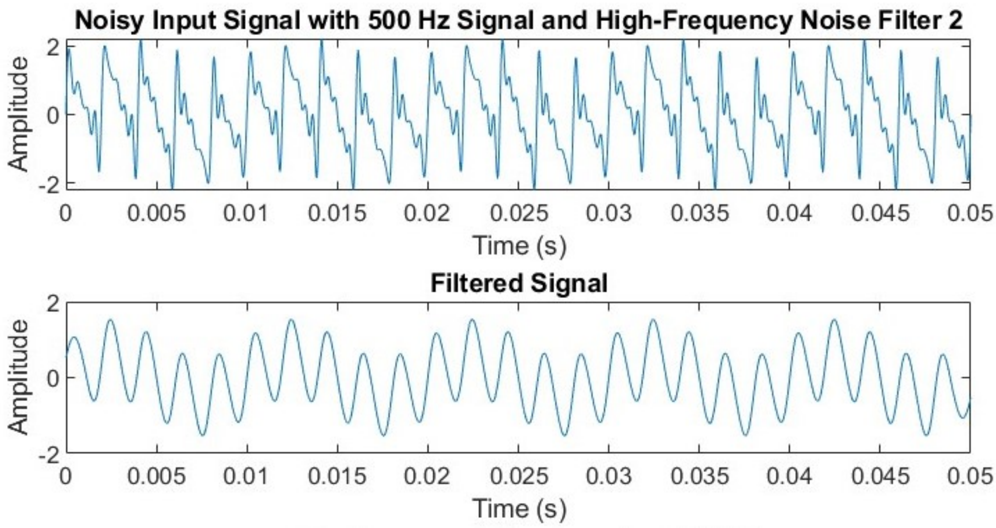


Figure 1.19: With carrier at 500 [Hz] and noise at 100, 1000, 1500, 2000, 2500 [Hz]

Where the FFT looks like:

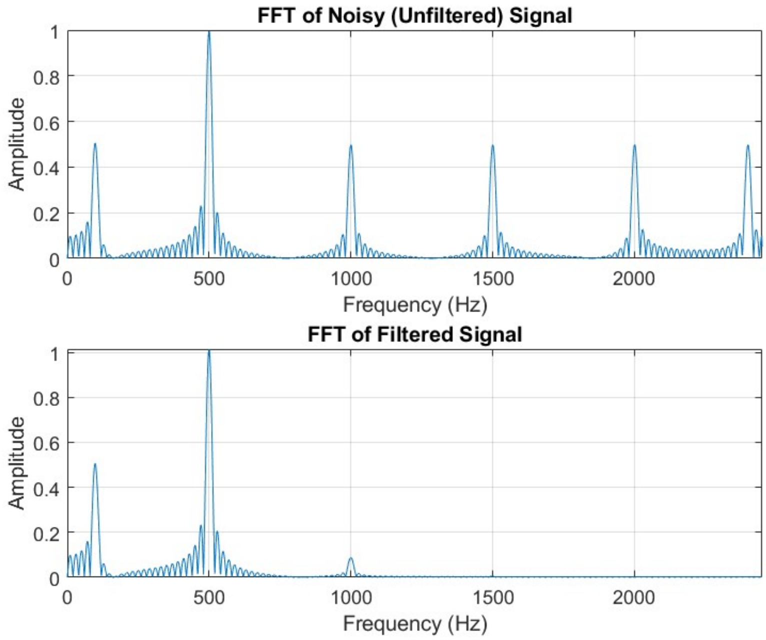


Figure 1.20: FFT of a carrier at 500 [Hz] and noise at 100, 1000, 1500, 2000, 2500 [Hz]

1.3.7 Conclusion

This therefore concludes how a filter can be designed and how the different parameters affect the performance of the filter that has been tested.