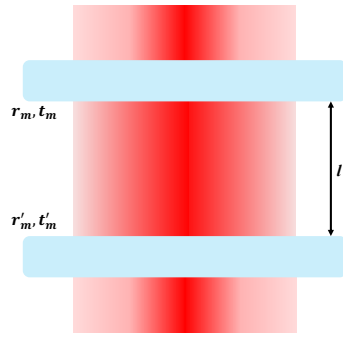


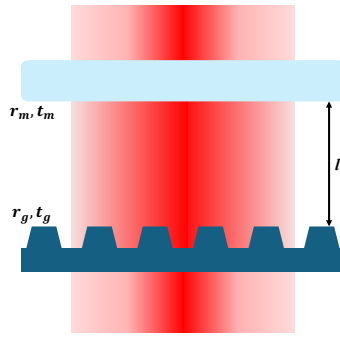
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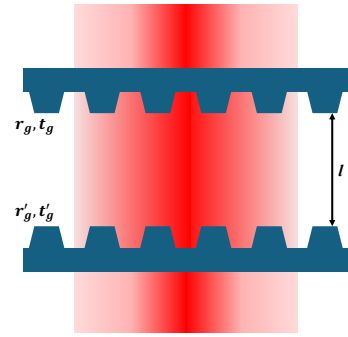
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(a) Broadband cavity.



(b) Single Fano cavity.



(c) Double Fano cavity.

1 Introduction

2 Theory

2.1 The Fabry-Perot interferometer

The Fabry-Perot interferometer, also known as an optical cavity, is generally comprised of two reflective optical resonators. It might make use of the configuration sketched in figure 2.

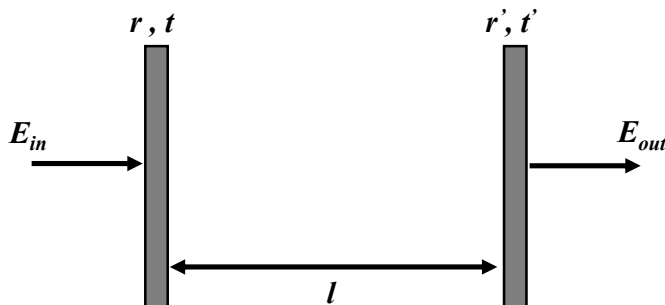


Figure 2: Sketch of a planar Fabry-Perot cavity.

Two planar slabs described by each their respective reflectivity r and transmission t are placed parallel and at a distance l from each other. The transmitted field E_{out} is then given as a super position of all beams coupling into the cavity by E_{in} . In this way the Fabry-Perot cavity gives rise to so-called eigenstates[1] related to it's length l , derived from considering the path difference between successive parallel beams throught the cavity. The allowed modes inside the cavity are generally limited to ones which fulfill the identity[2]

$$2l \cos \theta = m\lambda. \quad (1)$$

Here θ is the incidence angle of the light coupling into the cavity, λ is the wavelength of the light and $m = 1, 2, 3, \dots$ is a positive integer describing the order of the allowed mode. For the rest of this thesis it will be assumed that the laser couples into the cavity at normal incidence, meaning that $\theta = 0^\circ$ and thus $\cos \theta = 1$, as depicted in figure 2.

2.1.1 Transmission

In order to determine the transmission throught the Fabry-Perot interferometer, we once again consider the configuration in figure 2. It is further,

initially, assumed that resonators are both lossless, such that

$$|r|^2 + |t|^2 = |r'|^2 + |t'|^2 = 1. \quad (2)$$

This means that all losses, e.g. absorption, associated with the interaction between the field and the resonators are neglected.

In order to formulate the transmitted field E_{out} in terms of the ingoing one E_{in} , we first assume that the ingoing field can be described well by a plane-wave propagating in the z-direction

$$E_{in} = E_{0,in} e^{ikz}, \quad (3)$$

where $k = 2\pi/\lambda$ is the wave number and $E_{0,in}$ is the amplitude of the field. We then consider E_{out} to be comprised of contributions for each roundtrip inside the cavity. This can be written as an infinite geometrical series given as

$$\begin{aligned} E_{out} &= tt' E_{0,in} e^{ikz} + tt' E_{0,in} e^{ikz} rr' e^{i\delta} \\ &\quad + tt' E_{0,in} e^{ikz} (rr' e^{i\delta})^2 + tt' E_{0,in} e^{ikz} (rr' e^{i\delta})^3 + \dots \\ &= tt' E_{0,in} e^{ikz} \sum_{m=0}^{\infty} (rr' e^{i\delta})^m \end{aligned} \quad (4)$$

where $\delta = 2kl$. The first term of the series corresponds to a direct transmission through the cavity, and each term thereafter corresponds to the respective contribution to the transmission after the m' th round trip.

By evaluating the series it is seen that it indeed converges, and setting $z = l$ the final expression for the transmission through a planar Fabry-Perot cavity is found as

$$E_{out} = E_{0,in} \frac{tt' e^{i\delta/2}}{1 - rr' e^{i\delta}}. \quad (5)$$

Figure 3 shows an example of the normalized transmission $|E_{out}|^2/|E_{0,in}|^2$ of an arbitrary Fabry-Perot cavity of length $l \approx 50\mu m$ with reflectivity

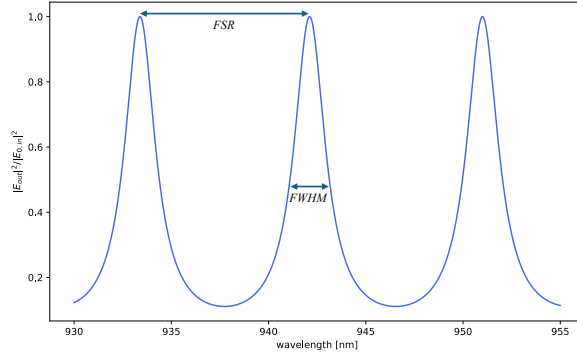


Figure 3: Example of a Fabry-Perot cavity transmission spectrum for cavity length $l \approx 50\mu\text{m}$ and reflectivity $|r|^2 = |r'|^2 = 0.5$. The *Free Spectral Range* and *Full Width at Half Maximum* are indicated in the figure as *FSR* and *FWHM*, respectively.

$|r|^2 = |r'|^2 = 0.5$. The two defining features marked on the figure are the *Free Spectral Range (FSR)* and *Full Width at Half Maximum (FWHM)*. As clearly depicted on the figure, the FSR is defined as the, in this case, spectral distance between two peaks¹, while the FWHM is simply the linewidth of each peak.

2.1.2 Finesse

Another important characteristic of an optical cavity is the so-called *finesse* \mathcal{F} , which generally describes the quality of the cavity by the number of round trips the light makes on average before being transmitted. It can thus be said that the finesse corresponds to the order m in the geometrical series describing the transmitted field E_{out} in eq. (4), making the series, in any practical sense, finite.

There is a variety of ways one can obtain the finesse of a cavity. By examination of the spectrum one can find the finesse in terms of the FSR and FWHM, as seen in figure 3, given as

$$\mathcal{F} = \frac{FSR}{FWHM}, \quad (6)$$

¹The definition of the FSR is not limited to the distance between peaks in wavelength. As seen in eq. (1) the cavity length and the resonant wavelengths are linearly dependent, and the length can thus just as well be varied resulting in a similar spectrum for the cavity transmission.

which indicates the resulting peaks of a cavity of higher finesse are narrower for the same FSR.

The finesse can also be expressed in terms of the reflectivity/transmission of the resonators of the cavity, as the defining physical action is that of the light either being transmitted from the cavity or not. The finesse in terms of r and t , for both resonators, are given as

$$\mathcal{F} = \frac{2\pi}{1 - |r|^2 + 1 - |r'|^2} = \frac{2\pi}{|t|^2 + |t'|^2}. \quad (7)$$

Figure 4 shows an example of the transmission spectra of a high finesse cavity of reflectivity $|r|^2 = 0.9$ together with a cavity of relatively low finesse with reflectivity $|r|^2 = 0.3$. It is clearly seen that the linewidth of the high finesse cavity is much narrower than the low finesse case, which follows from eq. (6).

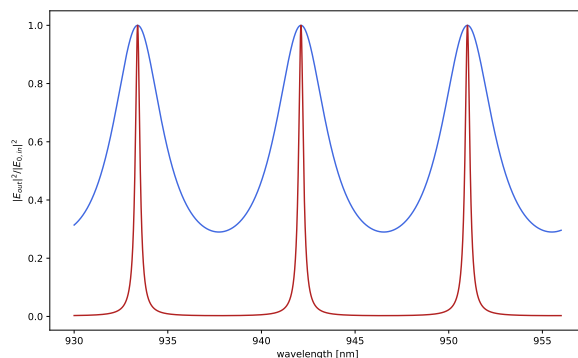


Figure 4: The red line shows the transmission spectrum of a *high* finesse Fabry-Perot cavity of reflectivity $|r|^2 = 0.90$, while the blue line shows the transmission spectrum of a *low* finesse cavity with reflectivity $|r|^2 = 0.30$.

Eq. (7) however is only true for the case of a lossless cavity, i.e. when eq. (2) is fulfilled. In practice, any cavity will have some amount of losses, which would have to be taken into account when calculating the finesse. When losses are present eq. (2) instead generally reads

$$|r|^2 + |t|^2 + L + L' = 1, \quad (8)$$

where L and L' indicates the fractional losses of each resonator.

In this case the finesse would be given as

$$\mathcal{F} = \frac{2\pi}{1 - |r|^2 + 1 - |r'|^2} = \frac{2\pi}{|t|^2 + |t'|^2 + L_{total}}, \quad (9)$$

where $L_{total} = L + L'$ are the total cavity losses.

The effect on the transmission spectrum of a cavity with losses is that the level of the normalized transmission will not reach 1, as some light is lost to i.g. absorption effects for each round trip of the cavity.

2.1.3 Cavity length

So far we have only considered effects that are independent of the length of the Fabry-Perot cavity, and by examining figure 4 it is seen that the only previously discussed feature that does not vary when changing the reflectivity and transmission (and by extension also the finesse) of the cavity, is the FSR. The FSR is defined as the distance between adjacent peaks of the transmission spectrum, and is given as

$$FSR = \frac{c}{2nl}, \quad (10)$$

where c is the speed of light in vacuum, n is the refractive index of the medium inside the cavity and l is the length of the cavity. The inverse proportionality of the cavity length l and the FSR is easily seen, and since the finesse is constant when varying the cavity length, the same must apply for the FWHM. Figure 5 shows a comparison of the transmission through two Fabry-Perot cavities of length $100\mu m$ and $150\mu m$, respectively. It is evident that the FSR depends on the cavity length, as $FSR_{100\mu m} > FSR_{150\mu m}$. It is also seen, that while the FSR becomes smaller with increasing cavity lengths, the linewidth does too, this is a consequence of the finesse \mathcal{F} being constant in terms of the cavity length.

2.1.4 The Fundamental Mode: A Gaussian Beam in the Large Waist Approximation

In order to describe the allowed modes within an optical cavity, it is first assumed that a single-mode field is linearly polarized. We then consider

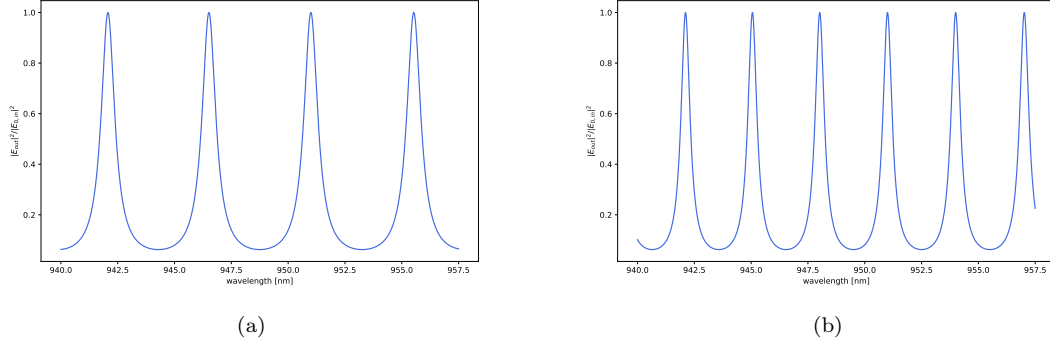


Figure 5: (a) shows the transmission spectrum of a Fabry-Perot cavity of length $l \approx 100\mu m$, while (b) shows the transmission spectrum of a cavity of length $l \approx 150\mu m$.

solutions to the wave equation

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}, \quad (11)$$

given as

$$\vec{E} = E_0(x, y, z) \vec{\epsilon} e^{ikz}. \quad (12)$$

Here $E_0(x, y, z)$ describes the electric field amplitude, $\vec{\epsilon}$ is denoted the polarization vector and $k = 2\pi/\lambda$ is the angular wave number of the field propagating along the z-axis. It is assumed that the electric field has a Gaussian transverse distribution².

This is almost the simplest description of the propagating field, however, as the spacial dependence of the field amplitude still might complicate things, we consider the range in which this can be neglected.

It can be shown from the derivation of the Gaussian distribution that the waist of the beam $w(z)$, which depends on the spacial coordinate in the direction of propagation, can be described as [1]

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2}, \quad (13)$$

where z is the distance from focus, w_0 is the beam waist at focus and z_R is the so-called *Rayleigh range*. The Rayleigh range describes the range in

²When this is the case, the laser is said to operate in the lowest possible mode denoted TEM_{00} . This implies the assumption of ideal lasing conditions.

which the beam diverges slowly, whereas after this has been surpassed, the beam will begin to diverge more rapidly. By quick inspection of eq. (13) it is seen that

$$w(z) = \begin{cases} w_0, & \text{for } z = 0 \\ w_0\sqrt{2}, & \text{for } z = z_R, \end{cases} \quad (14)$$

which shows that the beam waist diverges no more than by a factor of $\sqrt{2}$ from the optimal case, for $0 \leq z \leq z_R$. Considering the case where $z \neq 0$ but however much smaller than the Rayleigh range z_R , we can further inspect eq. (13) and find that this leads to negligible changes in the waist of the beam. Specifically, it can fairly easily be seen that

$$\left(\frac{z}{z_R}\right)^2 \approx 0, \text{ for } z \ll z_R \quad (15)$$

which in turn leads to

$$w(z) \approx w_0. \quad (16)$$

The Rayleigh range can be written as [2]

$$z_R = \frac{\pi w_0^2}{\lambda}, \quad (17)$$

which, through the exponential dependence on w_0 , shows that a large beam waist will result in a long Rayleigh range. As an example, consider a beam of waist $w_0 = 200\mu m$ and wavelength $\lambda = 950nm$. This would result in a Rayleigh range of $z_R = 13.23cm$.

Finally we can conclude, that any optical cavity, for which the total distance travelled inside the cavity³, is significantly shorter than the Rayleigh range of the incident beam, the spacial dependence of the field amplitude inside the cavity is negligible, and the fundamental mode can be described simply by a linearly polarized plane wave

$$\vec{E} = E_0 \vec{e} e^{ikz}. \quad (18)$$

³For any optical cavity the incident light will travel a distance inside the cavity according to, not only the length of the cavity, but also the amount of round trips the light makes when confined inside the cavity.

This is often referred to as the *large waist approximation* of a Gaussian beam, due to the dependence on w_0 of eq. (17).

2.2 The Fano mirror: a sub wavelength grating

2.2.1 Geometric optical analysis

Considering an ideal grating with period d in the sub-wavelength regime, it can be shown that only a single mode of reflection/transmission is supported.

Any grating, of arbitrary dimensions, must comply with the very general *grating equation*[2] given as

$$\sin \theta_m = m \frac{\lambda}{d}, \quad (19)$$

for the special case of a linearly polarized plane wave incident on a grating placed normal to the direction of propagation. Now, inserting the sub-wavelength condition $d \ll \lambda$, it is easily seen that the right side of the equation blows up for any order of reflection $|m| > 0$, effectively showing that this is the aforementioned single supported mode in this regime. Furthermore, it can be equally easily seen that the propagation direction of the 0'th order mode is the same as the incident beam, i.e. normal to the grating.

2.2.2 Reflection/transmission spectra and line shape analysis

2.2.3 Lossless grating

We wish to analytically describe the wavelength-dependent spectra for the transmission and reflectivity of an infinite sub-wavelength grating. By first considering the case where absorption and thermal coupling effects are neglected, i.e. a lossless grating, we can assume conservation of energy and thereby the relations

$$|r_g|^2 + |t_g|^2 = 1 \quad \text{and} \quad |r_d|^2 + |t_d|^2 = 1, \quad (20)$$

where the subscripts g and d indicate the *grating* and *direct* transmissions and reflectivities, respectively. It is implied that the direct coefficients are

constants and describe the transmission and reflectivity when the incident wavelength is significantly detuned from any guided-mode resonance of the grating. Furthermore, it is also implied that the grating coefficients are functions of the incident wavelength.

We now assume a normal incident beam on the grating as a linearly polarized monochromatic plane wave, with a wavelength close to a guided-mode resonance of the grating. In order to describe the coefficients r_g and t_g we follow the formalism presented by Fan and Joannopoulos [3] and consider the likely paths of the incident light through the grating. It is quite intuitive to consider the case where the light is simply transmitted, and this shall be our first case hereafter denoted the *direct pathway*. Another case one might consider is the one where the incident light excites the guided-mode resonance in the grating. This case is denoted the *indirect pathway* and decays more slowly than its direct counterpart.

The interference caused when the guided mode is excited is often referred to as *Fano resonances*, due to its physical similarities to the description of interference between a discrete autoionized state and a bound continuum state first reported by Fano [4]. The cross section of inelastic scattering, when measured as a function of energy, showed characteristic asymmetric peaks. These were described as the aforementioned interference pattern between *direct* (the discrete state) and *indirect* (the continuum state) pathways.

By generalizing the model of Fan and Joannopoulos [3] we describe the transmission and reflectivity coefficient amplitudes as

$$r_g = r_d + \frac{a}{k - k_1 + i\gamma} \quad \text{and} \quad t_g = t_d + \frac{b}{k - k_1 + i\gamma}, \quad (21)$$

where $k = 2\pi/\lambda$ is the incident wave number, $k_1 = 2\pi/\lambda_1$ is the wave number according to the guided-mode resonance and γ is the HWHM (half width at half maximum) of the guided-mode resonance. Complex coefficients a and b describe the interference between the directly transmitted or reflected waves and the guided mode of the grating.

Note that in eq. (21) the right side of the expression for each coefficient corresponds to the continuum state i.e. the indirect pathway, while

the direct transmission and reflection coefficients take the role of the autoionized discrete state, i.e. the direct pathway⁴ [4]

As we are dealing with an ideal, lossless, grating, we assume coefficients a and b to be equal, meaning that we specifically assume vertical symmetry throughout the grating. By considering eq. (20) this in turn leads to

$$a = b = -i\gamma(t_d + r_d), \quad (22)$$

which further yields an expression for the grating transmission amplitude coefficient on the form

$$t_g = t_d \frac{k - k_0}{k - k_1 + i\gamma}. \quad (23)$$

Here, the newly introduced $k_0 = 2\pi/\lambda_0$ is the zero-transmission/unity-reflectivity wave number.

To generalize eq. (23) to include non-unity reflectivity and non-zero transmission, we allow for $a \neq b$ meaning that the case of vertical asymmetry is included in the model. By assuming $r_d, t_d \in \mathbb{R}$, eq. (20) leads to the coupled differential equations

$$\begin{aligned} t_d x_a + r_d x_b &= 0, \quad \text{and} \\ x_a^2 + y_a^2 + x_b^2 + y_b^2 + 2t_d \gamma y_a + 2r_d \gamma y_b &= 0, \end{aligned} \quad (24)$$

where $\{x, y\}_{a,b}$ respectively denotes the real and imaginary parts of the coefficients a and b . Solving eqs. (24) leads to the correct complex reflectivity coefficients and the expression for the transmission coefficient amplitudes now reads

$$t_g = t_d \frac{k - k_0 + i\beta}{k - k_1 + i\gamma}, \quad (25)$$

where k_0 and β are defined from the expression for a found by solving eqs. (24), given as

$$a = t_d(k_1 - k_0 - i\gamma + i\beta). \quad (26)$$

Finally, this allows for non-zero transmission and non-unity reflectivity at wave number k_0 .

⁴The general eigenvector of a state comprised of a super-position between a discrete state and a continuum, i.e. a state vector corresponding to a Fano resonance, is given as $\Psi_E = a\phi + \int dE' b_{E'} \psi_{E'}$, given in eq. (2) in ref. [4], where a and $b_{E'}$ describes the probability of either pathway.

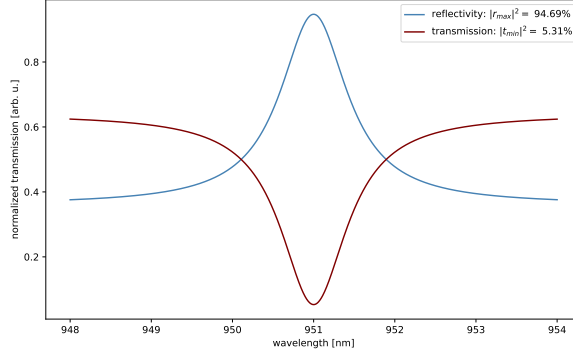


Figure 6: Simulated reflectivity and transmission spectra of an arbitrary lossless grating.

2.2.4 Lossy grating

In order to modify the above model such that losses, e.g. due to absorption or thermal coupling effects, are accounted for, we add a resonant loss term to the energy conservation relation in eq. (20). For this we introduce the resonant loss level L , which must be known in order to accurately calculate the complex reflectivity coefficients. The energy conservation relation is modified such that

$$|t_g|^2 + |r_g|^2 + \frac{c^2}{(k - k_1)^2 + \gamma^2} = 1, \quad (27)$$

where the coefficient $c^2 = L((k - k_1)^2 + \gamma^2)$ includes the resonant loss term L . A new set of coupled differential equations are found, using eq. (27), given as

$$\begin{aligned} t_d x_a + r_d x_b &= 0, \quad \text{and} \\ x_a^2 + y_a^2 + x_b^2 + y_b^2 + c^2 + 2t_d \gamma y_a + 2r_d \gamma y_b &= 0. \end{aligned} \quad (28)$$

It is easily identified that eq. (24) and eq. (28) differ only by the addition of coefficient c^2 , and thereby the losses. Solving eq. (28) leads to the correct complex reflectivity coefficients, except that they now account for any losses associated with the grating.

In conclusion, the complete grating model consists of an expression for the transmission coefficients and a set of coupled differential equations for the reflection coefficients, shown in eq. (25) and eq. (28), respectively. The

model on the form used for this project and subsequent thesis is derived in previous work by A. Darki et al. [5] and more recently T. Mitra et al. [6].

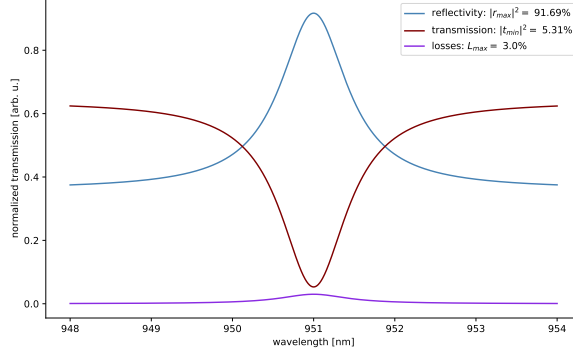


Figure 7: Simulated reflectivity and transmission spectra of an arbitrary lossy grating.

2.3 The Fano cavity

2.3.1 The single Fano cavity model

The single Fano cavity consists of a planar broadband mirror, and a sub-wavelength grating, i.e. a Fano mirror, as described in section 2.2 and seen in figure 8 where schematics of the single Fano and broadband cavity configurations are shown. While the broadband mirror has fixed optical properties, the Fano mirror has transmission and reflection coefficients dependent on the incident wavelength, according to solutions to the coupled differential equations of eq. (28).

In order to model the single Fano cavity transmission spectra, we therefore consider the transmission function of a normal incident and planar Fabry-Perot cavity in eq. (5) with $r, t \rightarrow r_m, t_m$ and $r', t' \rightarrow r_g(\lambda), t_g(\lambda)$. Here the subscript m indicates the broadband *mirror* coefficients, and g is for *grating*, which indicates the coefficients of the Fano mirror. Rewriting eq. (5) such that it describes the normalized transmission amplitude, through the single Fano cavity, $T_{cav} = |E_{out}|^2 / |E_{0,in}|^2$ we get

$$T_{cav} = \left| \frac{t_m t_g(\lambda) e^{i\phi}}{1 - r_m r_g e^{2i\phi}} \right|^2, \quad (29)$$

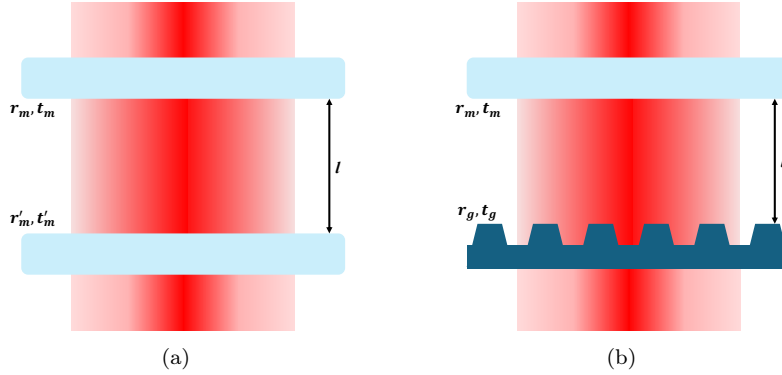


Figure 8: In (a) is seen the schematics of a cavity consisting of two broadband mirrors with transmission and reflectivity coefficients t_m , r_m , t'_m , and r'_m . (b) shows the schematics of a single Fano cavity consisting of one broadband mirror with coefficients t_m , r_m and one Fano mirror with wavelength dependent coefficients t_g , r_g . The reflectors in either cavity are separated by a cavity length l .

where $\phi = 2\delta = kl$, $k = 2\pi/\lambda$ and l is the cavity length, as is consistent the general case described in section 2.2.

2.3.2 Transmission linewidth

In order to analytically describe how the transmission spectrum at, or close to, the overall resonance⁵ behaves as a function of the incident wavelength, we attempt to generalize the Fano model for this specific scenario. Considering the case where the cavity resonance closely resembles the guided-mode resonance of the Fano mirror (the zero-transmission wavelength), eq. (29) can be approximated well by

$$T_{cav} \approx \frac{A}{1 + \left(\frac{\Delta}{1-\nu\Delta}\right)^2}, \quad (30)$$

where $\Delta = (\lambda - \lambda_c)/\delta\lambda$ is the detuning from the cavity resonance normalized by the HWHM $\delta\lambda$, and ν is a constant describing the asymmetry of the single Fano transmission spectrum.

From eq. (30) it can be shown that the HWHM of the Fano transmission

⁵This terminology is used for the scenario where the guided-mode, cavity mode and input laser mode all meet the resonance condition $\lambda_g \approx \lambda_c \approx \lambda_l$, where g, c, l stands for grating/guided-mode, cavity and laser, respectively.

profile around the resonance wavelength $\delta\lambda$ is approximately given as

$$\delta\lambda \approx \frac{1}{\frac{1}{\delta\lambda_c} + \frac{1}{\delta\lambda_g}}, \quad (31)$$

where $\delta\lambda_c$, $\delta\lambda_g$ are the contributions to the linewidth from the broadband cavity and Fano cavity, respectively.

It can be shown that there are two regimes of interest when examining the linewidth of the Fano cavity transmission profile. The first of these is the standard cavity regime, where the linewidth $\delta\lambda$ is predominantly described by the contribution from the standard cavity $\delta\lambda_c$. In this regime the broadband and Fano cavity produces resonance transmission peaks of comparable, if not equal, linewidths. The other is the Fano regime where a significant reduction in linewidth is found when comparing the Fano and broadband cavity transmission profiles, as the linewidth is described mostly by the contribution of $\delta\lambda_g$.

In order to determine where the transition between the standard and the Fano cavity regimes occur, we look at the expressions for $\delta\lambda_c$ and $\delta\lambda_g$ in eq. (31).

The standard cavity linewidth is given as

$$\delta\lambda_c = \frac{\lambda_0^2}{8\pi l} (|t_g(\lambda_0)|^2 + |t_m|^2 + L), \quad (32)$$

while the Fano cavity linewidth in the Fano regime is given as

$$\delta\lambda_g = \frac{\gamma\lambda}{2(1 - r_d)} (|t_g(\lambda_0)|^2 + |t_m|^2 + L). \quad (33)$$

Here λ_0 is the cavity resonance wavelength, l is the cavity length, $L = 1 - |r_g(\lambda_0)|^2$ is the total losses of the cavity when on resonance, $\gamma\lambda$ is the width of the guided-mode resonance of the Fano mirror and r_d is the off-resonance, or *direct*, reflectivity of the Fano mirror.

Figures 9a and 9b depicts two examples of single Fano cavity transmission spectra together with their respective complimentary broadband cavity transmission profiles, and the reflectivity amplitude of the Fano mirror

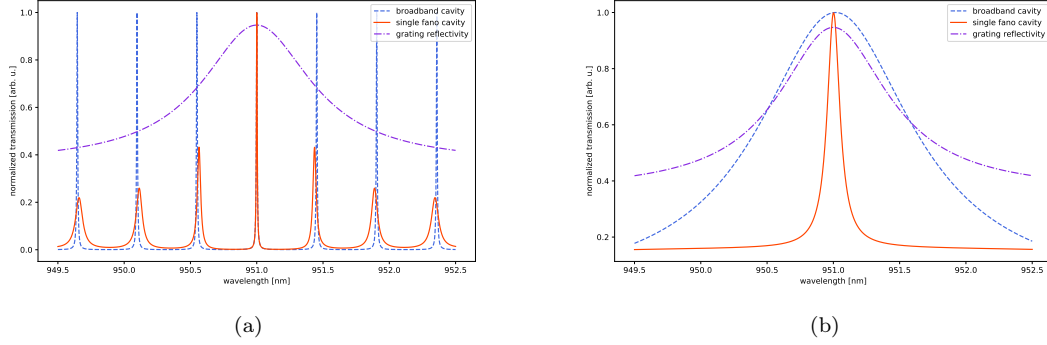


Figure 9: In (a) is seen the comparison of broadband and single Fano cavity transmission spectra for a cavity length of $l \approx 1000\mu m$, i.e. in the *standard* regime. (b) shows the same comparison, but for a cavity length of $l \approx 5\mu m$, i.e. in the so-called *Fano* regime.

used to model both profiles. As shown in section 2.2 any Fano mirror, or grating, is described by five characterizing parameters. For the one used to model the transmission spectra in figures 9a and 9b these are

$$\begin{aligned} \lambda_0 = 951nm, \quad \lambda_1 = 951nm, \quad t_d = 80\%, \\ \gamma\lambda = 0.5nm \quad \text{and} \quad \beta = 10^{-6}, \end{aligned} \quad (34)$$

where we remember that $\lambda_{0,1}$ are the resonant wavelengths for, respectively, the cavity- and guided-mode, t_d is the direct transmission, $\gamma\lambda$ is the width of the guided-mode resonance and β is a constant describing the losses of the grating, which here is neglected when solving the coupled differential equations of eq. (28). The broadband mirror parameters are chosen such that it perfectly resembles the transmission and reflectivity of the grating on resonance, which is arbitrarily chosen such that $|r_m|^2 = 95\%$ and $|t_m|^2 = 5\%$.

It is clear from inspection of figure 9a that the resonance transmission profile of the standard broadband cavity is not wavelength dependent, in the sense that all fringes, when cavity losses are neglected, appear to have the same high finesse \mathcal{F} , i.e. ratio between the FSR and HWHM. This is not the case for the Fano cavity which is, naturally, due to the wavelength dependence of the optical properties of the Fano mirror, as this causes the transmission and reflectivity to *only* match those of the broadband mirror when on resonance.

In figure 9b, the transmission spectra of both cavities are shown *in* the

Fano regime, where it is clearly seen that while the standard cavity experiences broadening for shorter cavity lengths⁶, this is not the case for the Fano cavity transmission peak.

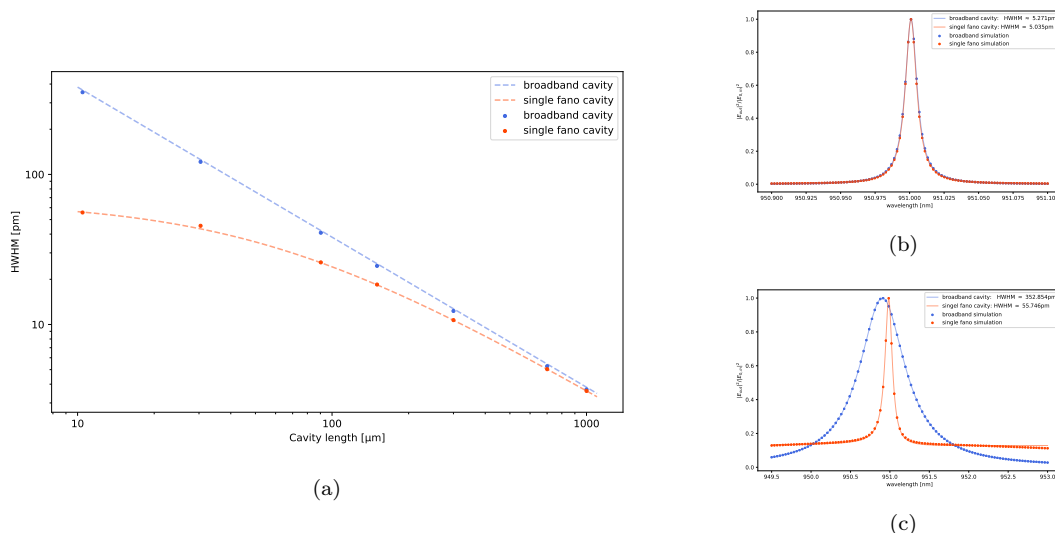


Figure 10: (a) shows the approximate analytical resonance linewidths (eq. (31)) as a function of cavity length for the broadband and single Fano cavities together with linewidths of transmission profiles simulated using eq. (29) and eq. (5), found as parameters of least squares fits, for comparison. In (b) and (c) is seen transmission spectra of broadband and single Fano cavities of lengths $\sim 700\mu\text{m}$ and $\sim 10\mu\text{m}$, respectively. The spectra shown indicate each their respective linewidths, and are examples of the values plotted in (a).

Figure 10a models the behavior of the linewidth of the single Fano cavity compared with the one for a broadband cavity of similar optical properties, as a function of wavelength. Here it is easily seen where the linewidth of the single Fano begins to saturate, and hence deviate from the one of the broadband cavity. The plotted line in the figure is calculated using eq. (31) while the points depict linewidths found as a fitting parameter from a least squares fit of the general Fano model in eq. (30) to transmission spectra simulated by the Fabry-Perot (eq. (5)) and single Fano (eq. (29)) transmission functions. Finally, it can be concluded that the approximate analytical expression for the linewidth of the broadband and single fano cavities in eq. (31) correlates very well with the values found from the simulated spectra.

⁶This is a natural consequence of the shorter cavity as this causes the lifetime inside the cavity to fall and hence the transmission HWHM to rise. The HWHM is inversely proportional to the lifetime and goes as $\delta\lambda \propto 1/\tau$.

2.4 The double Fano cavity

2.4.1 The double fano transmission model

While the single Fano cavity is usually known in the appropriate literature as simply a *Fano cavity*, I have insisted on including the fact that it contains only one Fano mirror, and hence denoted it the *single* Fano cavity. This addition is justified by the contents of this section, and namely that we now move on to the *double* Fano cavity, which as the name suggests consists of two Fano mirrors. The schematics of this configuration is shown together with the one for the single Fano cavity in figure 11.

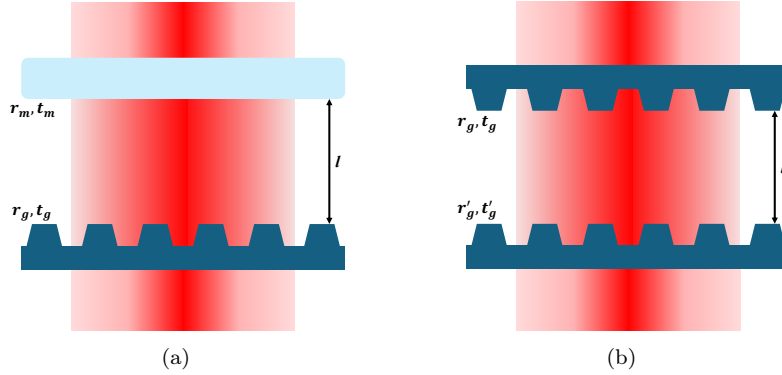


Figure 11: (a) shows schematics of the single Fano cavity consisting of a broadband mirror with transmission and reflectivity coefficients t_m, r_m and a Fano mirror with coefficients t_g, r_g . (b) shows the schematics of the double Fano cavity consisting of two Fano mirrors with coefficients t_g, r_g, t'_g, r'_g . Both cavities are separated by a cavity length l .

Here it is evident that instead of having one set of reflectivity and transmission coefficients that depend on the incident wavelength, we now have two. In order to model the transmission of the double Fano cavity, we once again consider the transmission function for the normal incident and planar Fabry-Perot cavity in eq. (5), this time with $r, t \rightarrow r_g(\lambda), t_g(\lambda)$ and $r', t' \rightarrow r'_g(\lambda), t'_g(\lambda)$. We rewrite the Fabry-Perot transmission function with the addressed substitutions of the optical coefficients and such that it describes the normalized transmission amplitudes $T_{cav} = |E_{out}|^2/|E_{0,in}|^2$ and get

$$T_{cav} = \frac{t_g(\lambda)t'_g(\lambda)e^{i\phi}}{1 - r_g(\lambda)r'_g(\lambda)e^{2i\phi}}. \quad (35)$$

The subscript g indicates a grating transmission or reflectivity. Figure 12 shows an example of the normalized transmission spectrum of an $\sim 30\mu m$

double Fano cavity on- and off-resonance, found using eq. (35).

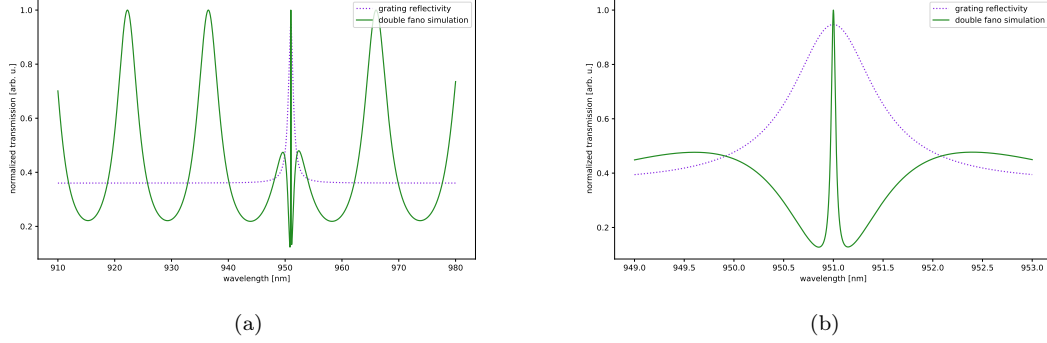


Figure 12: An example of a normalized double Fano transmission spectrum of an $\sim 30\mu\text{m}$ cavity. (a) shows a long-range wavelength scan, depicting both the on- and off-resonance behaviour of the double Fano cavity transmission, while (b) shows the transmission focussed specifically around the resonance position. Both examples are shown together with the reflectivity of the Fano mirror used to model them.

2.4.2 Transmission linewidth

$$\delta\lambda \approx \frac{1}{\frac{1}{\delta\lambda_c} + \frac{1}{\delta\lambda_g^{\text{double}}}}, \quad (36)$$

$$\delta\lambda_c = \frac{\lambda_0^2}{8\pi l} (|t_g(\lambda_0)|^2 + |t_m|^2 + L), \quad (37)$$

$$\delta\lambda_g^{\text{double}} = \frac{\gamma\lambda}{4(1 - r_d)} (|t_g(\lambda_0)|^2 + |t_m|^2 + L). \quad (38)$$

2.4.3 Single and double Fano cavity comparison

Using the analytical expression for the double Fano cavity transmission in eq. (36) we are now in a position to compare the single and double Fano cavities. Note that we at this point only consider the ideal case of the double Fano cavity where additional cavity losses are neglected and the two Fano mirrors used are identical, i.e. the cavity is said to be *symmetrical*. The additional cavity losses are explicitly set to be given as

$$L = 1 - |r_g|^2 - |t_g|^2 = 0. \quad (39)$$

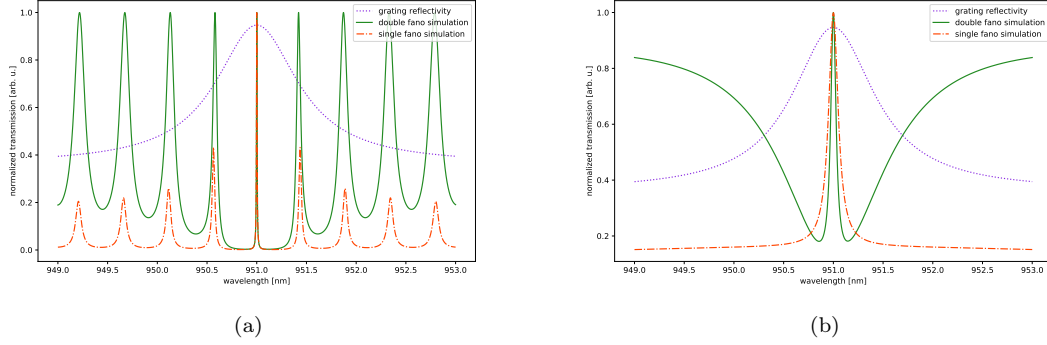


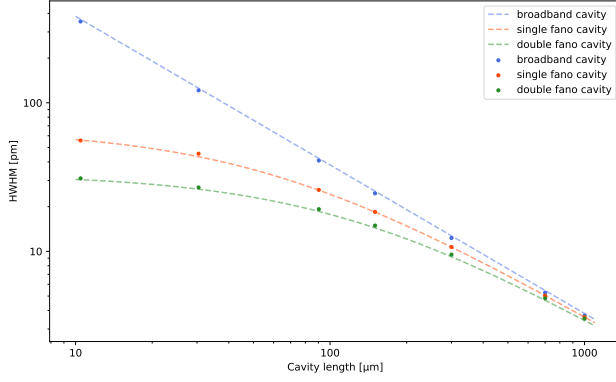
Figure 13: Comparison of the single and double Fano transmission spectra in the *standard* and *Fano* regimes. (a) shows the spectral comparison for an $\sim 1000\mu\text{m}$ cavity, i.e. in the *standard* regime, while (b) shows the same for an $\sim 5\mu\text{m}$ cavity, i.e. in the *Fano* regime. The Fano mirror reflectivity is depicted in both (a) and (b).

Figure 13 shows the transmission of the ideal double Fano cavity and the corresponding single Fano cavity for comparison.

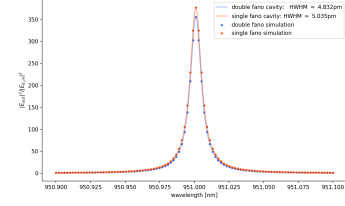
Figure 13a shows the transmission for a cavity length of $\sim 1000\mu\text{m}$ which is well-inside the standard regime outlined in section 2.3.2 where the standard broadband and single Fano cavities produce transmission spectra of roughly identical linewidths. In this regime, due to the $1/l$ proportionality of the FSR, the off-resonance behavior of the double Fano cavity transmission is visible in the range plotted. It is seen, contrary to the single Fano cavity, that the transmission at each cavity resonance reaches a normalized transmission of $|E_{out}|^2/|E_{0,in}|^2 = 1$. This is due to the fact that the two gratings, while they have wavelength dependent transmission and reflectivity coefficients, always have identical ones for the ideal case, which maximizes the Fabry-Perot transmission function and ensures unity transmission at any cavity resonance. The minimum level of the cavity changes according to the reflectivity of the Fano mirrors and by that the HWHM also changes as we move further from the guided-mode resonance. Both the minimum transmission level and the HWHM converge when moving away from the resonance wavelength, when the reflectivity becomes constant.

Figure 13b shows the transmission of a double Fano cavity of length $\sim 5\mu\text{m}$ which, contrary to the one in figure 13a, is well within the Fano regime. It is seen that the double Fano cavity transmission produces

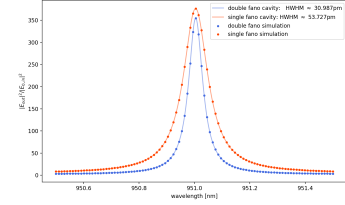
a resonance peak with a HWHM narrower than the one for the single Fano cavity, as is predicted in eq. (38). Furthermore, the immediate off-resonance behavior of the double Fano cavity transmission in the Fano regime, is drastically different than for the single Fano cavity. This is due to the collective higher transmission in this region compared with the single Fano case where the broadband mirror has a constant, and often high, reflectivity and hence a correspondingly low transmission.



(a) The analytical resonance linewidths (eqs. (31), (36), and (32)) as a function of the cavity length for the broadband, single and double Fano cavities are shown together with linewidths of corresponding transmission profiles simulated using eqs. (29), (35), and (5) for comparison.



(b) Intracavity transmission spectra of single and double Fano cavities of length $\sim 700\mu m$.



(c) Intracavity transmission spectra of single and double Fano cavities of length $\sim 10\mu m$.

Figure 14a shows the analytical linewidth calculated and compared for the broadband, single Fano, and double Fano cavities, calculated using eqs. (32), (31), (36). These are compared with linewidths found as fitting parameters from a least squares fit of the general Fano transmission formula given in eq. (30) to transmission spectra calculated using eqs. (5), (29), and (35). According to eq. (36) the linewidth of the double Fano cavity transmission should converge to exactly half that of the single Fano cavity, meaning that

$$\delta\lambda_{double} = \frac{\delta\lambda_{single}}{2}, \quad \text{for } l \rightarrow 0. \quad (40)$$

This is supported well by the simulation depicted in figure 14a.

Figures 14b and 14c contain examples of intracavity⁷ spectra of single and double Fano cavities and corresponding least squares fits to the general Fano model in order to determine their linewidths.

2.4.4 Additional cavity losses

Thus far we have only examined a lossless double Fano cavity where

$$|r_g|^2 + |t_g|^2 = 1 \quad (41)$$

is fulfilled for both Fano mirrors used to construct the cavity.

In this section we will investigate what happens when we introduce additional cavity losses⁸

$$L = 1 - |r_g|^2 - |t_g|^2, \quad (42)$$

leading to the modified condition

$$|r_g|^2 + |t_g|^2 + L = 1. \quad (43)$$

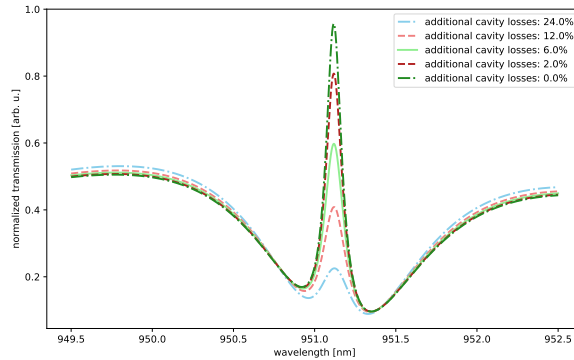


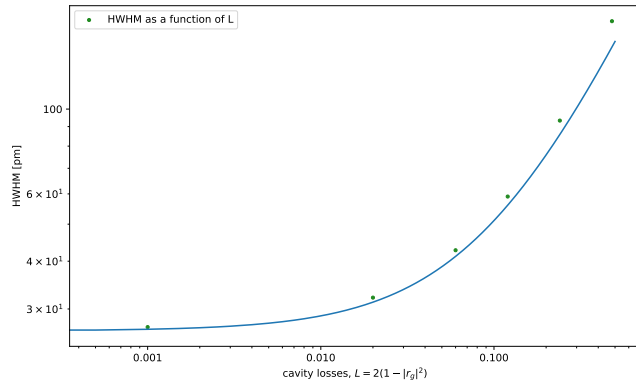
Figure 15: length $\sim 30\mu m$

⁷The reason these are shown as intracavity spectra instead of transmission spectra is due to the immediate off-resonance behavior of the double Fano cavity transmission. The shape makes accurate fitting a challenge, and while the intracavity spectra cannot be measured in the lab, the linewidth when simulated is identical to the resulting transmission peaks.

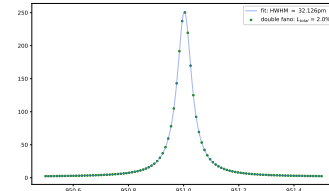
⁸Not to be confused with the often used definition of cavity losses given as $L = 1 - |r|^2$ where anything not being reflected back into the cavity is considered to have a contribution to the losses.

Figure 15 shows double Fano transmissison spectra as a function of wavelength for a symmetric cavity with varying values for the additional cavity losses. It is readily seen that the maximum value reached in each spectrum is rapidly reduced with the introduction of these losses.

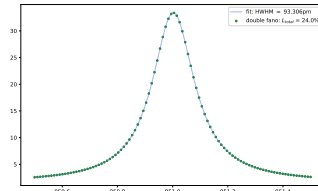
Since the linewidth is defined as the HWHM of the transmission profile, this will naturally vary as a function of additional cavity losses. This is depicted in figure 16a where the HWHM is shown for different values of L . Examples of intracavity spectra taken for different values of L are shown in figures 16b and 16c, along with their respective least squares fits to the general Fano model in eq. (30) and linewidths found as fitting parameters hereof.



(a) HWHM of a symmetric $\sim 30\mu m$ double Fano cavity as a function of additional cavity losses $L = 2(|r_g|^2 - |t_g|^2)$. Improve this figure by adding the analytical prediction of the linewidth as a function of losses.



(b) Double Fano transmission spectra with cavity length $\sim 30\mu m$ and $L = 2\%$.



(c) Double Fano transmission spectra with cavity length $\sim 30\mu m$ and $L = 24\%$.

Figure 16

Figure 16a shows the simulated linewidths of intracavity spectra as a function of total cavity losses L , and is compared with the analytical formula for the double Fano linewidth in eq. (36). It is seen that the analytical model proves, once again, to be a powerful predicting technique for the linewidth, and especially for cavities with relatively low losses.

2.4.5 Spectral detuning (lossless)

Up until this point, it has been assumed that the Fano mirrors making up the double Fano cavity has been identical, namely the cavity has been *symmetric*. However, in practice this is very unlikely to be the case, as any Fano mirror constructed is bound to have some uncertainties attached to the physical parameters describing it. For that reason we investigate the effect of an *asymmetric* cavity on the resulting transmission profile. Here we remember that each Fano mirror is describes by a set of parameters, λ_0 , λ_1 , t_d , $\gamma\lambda$ and β , which respectively describe the cavity resonance wavelength, guided-mode resonance wavelength, direct transmission, guided-mode resonance linewidth and additional losses of each grating. In order to model only a spectral detuning, we therefore simply change the parameters regarding the cavity and guided-mode resonance wavelengths, λ_0 and λ_1 by an amount corresponding to the detuning we wish to study. For this section the parameters will be given as

$$\begin{aligned}\lambda_0 &= 951nm, \quad \lambda_1 = 951nm, \quad t_d = 80\%, \\ \gamma\lambda &= 0.5nm \quad \text{and} \quad \beta = 10^{-6}\end{aligned}\tag{44}$$

for the unchanged Fano mirror, and

$$\begin{aligned}\lambda'_0 &= \lambda_0 + \Delta, \quad \lambda'_1 = \lambda_1 + \Delta, \quad t'_d = t_d, \\ \gamma\lambda' &= \gamma\lambda \quad \text{and} \quad \beta' = \beta\end{aligned}\tag{45}$$

for the *detuned* Fano mirror. Figure 17 shows the normalized reflectivity and transmission spectra of the Fano mirrors described by the parameters in eqs. (44) and (45).

As has been observed in previous sections, the Fano resonance transmission peak is present at the point where the grating reflectivity $r_g(\lambda)$ is maximized and transmission $t_g(\lambda)$ minimized. However, when $\lambda_0 \neq \lambda'_0$ and $\lambda_1 \neq \lambda'_1$, this is no longer a trivial conclusion to draw. The question of whether the cavity resonance should be tuned to match the guided-mode resonance wavelength of one grating or the other, or maybe somewhere in between does not have an obvious answer. This will be further expanded upon later in section 2.4.6, but in order to move forward with the investigation of the spectral detuning we, for now, accept that the optimal cavity

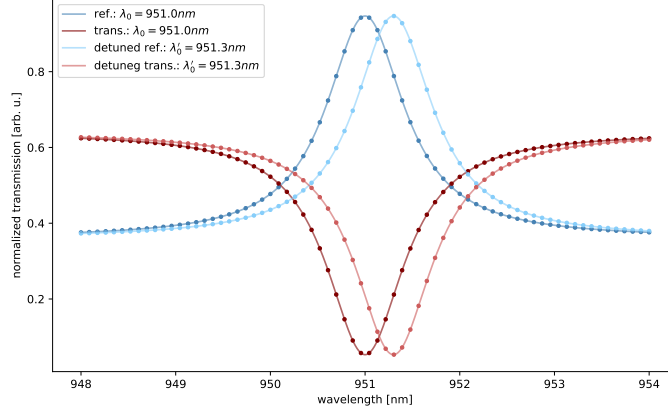


Figure 17: The normalized reflectivity and transmission spectra of the Fano mirrors described in eqs. (44) and (45) for detuning $\Delta = 0.3nm$.

length, is the one corresponding to a cavity resonance λ_t given, exactly between the two guided-mode resonance wavelengths, as

$$\lambda_t = \frac{\lambda_{0,1} + \lambda'_{0,1}}{2}. \quad (46)$$

The subscript $0,1$ indicate that the two are interchangeable, as the two are assumed identical at this time, and t is for *transmission* as this is, later on, to be used experimentally as the *transmission wavelength*.

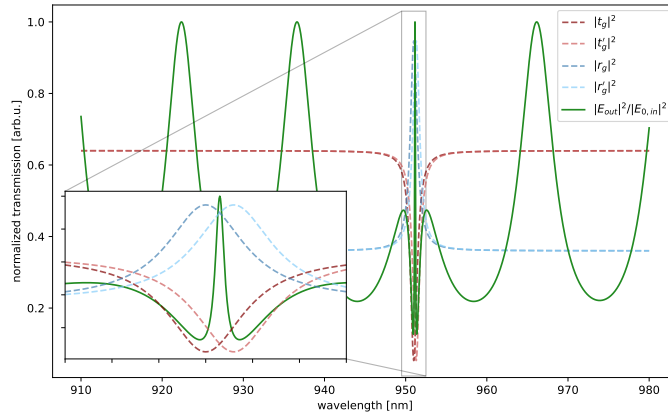
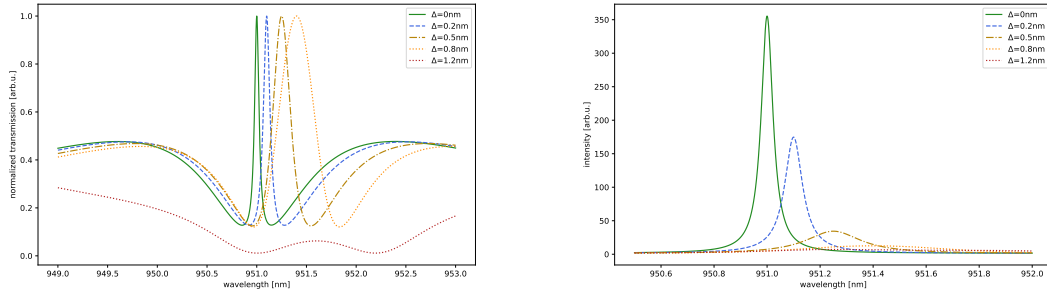


Figure 18: The double Fano transmission spectra of a cavity detuned by $\Delta = 0.3nm$, as seen in figure 17, together with the reflectivity and transmission spectra of the Fano mirrors used for the simulation.

Figure 18 shows the transmission spectrum of a detuned double Fano cavity with parameters corresponding to the transmission and reflectivity spectra in figure 17, meaning that $\Delta = 0.3nm$. The transmission wavelength λ_t is chosen such that eq. (46) is fulfilled, and correspondingly it is seen that the transmission peak is placed exactly between the maximum (minimum) reflectivities (transmissions) of the two grating, i.e. between the two guided-mode resonance wavelengths. Furthermore, it can be concluded that the detuning is chosen such that the overlap between the guided-mode resonances is still substantial enough for them to couple and hence for the overall Fano resonance to be excited.

While knowing that a detuning of $0.3nm$ is acceptable in terms of exciting the Fano resonance in the lossless case is a nice result, it does not provide much in terms of estimating the acceptable detuning for any experimental purposes. In figure 19a the double Fano transmission is shown for increasing detuning Δ and transmission wavelength $\lambda_t = (\lambda_{0,1} + \lambda'_{0,1})/2$.

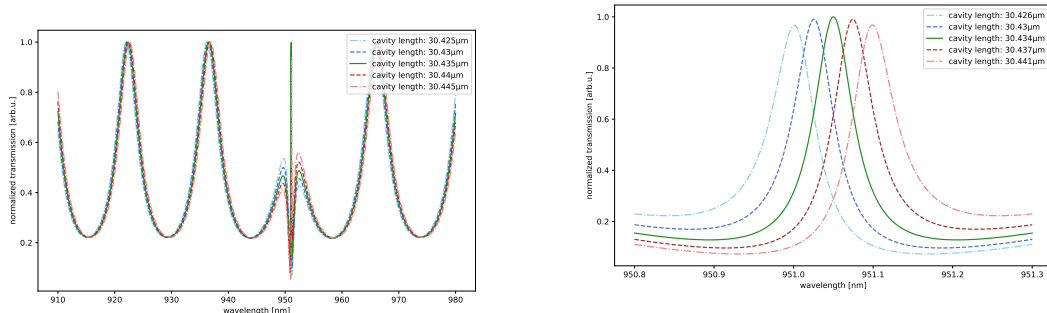


(a) Double Fano cavity transmission spectra for different values of the spectral detuning Δ . (b) Double Fano cavity intracavity spectra for different values of the spectral detuning Δ .

Figure 19

It is readily seen that with increasing positive detuning, relative to the resonant wavelength of the unchanged Fano mirror, the peak shifts to higher wavelengths, which in itself is trivial from eq. (46). The linewidth is also seen to increase with the detuning, and the peak eventually collapses when the overlap between the guided-mode resonance profiles of the two Fano mirrors becomes too small to sustain the Fano resonance. Figure 19b shows the intracavity spectra corresponding to the transmission spectra in figure 19a, and provides valuable insight into the mode density inside the cavity for the different values of Δ . It is clearly demonstrated by the

two figures that the spectral overlap is a very crucial parameter of the double Fano cavity, and is paramount in describing the cavity's ability to sustain the Fano resonance modes.



(a) Full-range transmission spectra of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu\text{m}$ with $\Delta = 0.1\text{nm}$
(b) Transmission spectra zoomed in on a range around the resonance wavelength of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu\text{m}$ with $\Delta = 0.1\text{nm}$

Figure 20

2.4.6 Spacial detuning (lossless)

As mentioned above in section 2.4.5, any spectral detuning gives rise to the potential of a spacial detuning as $2l = m\lambda$ (eq. (1)) must be fulfilled for any sustained plane-wave mode inside a normal incident optical cavity. We denote the length corresponding to the resonance wavelength of the unchanged Fano mirror as simply l , while the one for the detuned Fano mirror will be denoted as l' . Previously we assumed that the optimal length of a detuned double Fano cavity is the one where the so-called transmission wavelength is given as

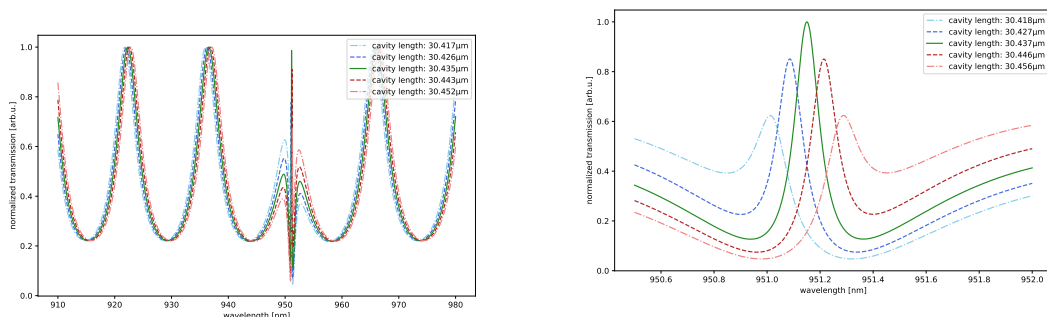
$$\lambda_t = \frac{\lambda_{0,1} + \lambda'_{0,1}}{2}. \quad (47)$$

And while this does turn out to be the case, we must investigate the resonant length-dependence of the linewidth of the Fano resonance profile in order to arrive at this conclusion indefinitely.

Figure 20 shows double Fano resonance transmission profiles plotted for varying cavity length $l \rightarrow l'$ with detuning $\Delta = 0.1\text{nm}$. Looking at figure 20a we see the long range scan of the transmission profiles where there still appears to be a substantial overlap between the guided-modes

of the two Fano mirrors. The main differences are seen as changes in the intensity in the immediate off-resonance regime. This is due to the dramatic wavelength (and thus length) dependence in the areas close to the resonance position, which makes even very small changes visible on the spectra.

Figure 20b shows the same spectra, but enhanced around the resonance wavelength, and this too shows that the substantial guided-mode overlap results in little visible change, except for shifting the peak position, when scanning between the resonant cavity lengths.



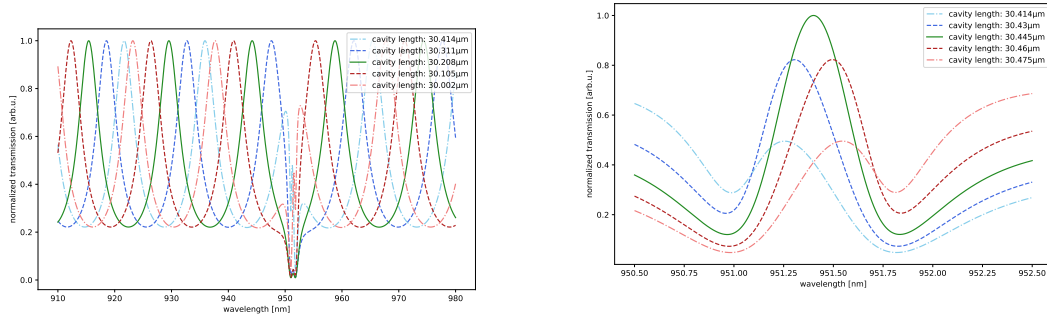
(a) Full-range transmission spectra of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu\text{m}$ with $\Delta = 0.3\text{nm}$
(b) Transmission spectra zoomed in on a range around the resonance wavelength of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu\text{m}$ with $\Delta = 0.3\text{nm}$

Figure 21

Figure 21 shows similar transmission profiles as in figure 20, only with an increased detuning of $\Delta = 0.3\text{nm}$. Looking at the long range scan in figure 21a and comparing with the one in figure 20a, it is seen that the displacement of the off-resonance Fabry-Perot like fringes have increased with the slightly higher detuning. This is not surprising as the Fabry-Perot cavity modes, like any other interference pattern inside a cavity must fulfill the brightness identity $2l = m\lambda$. Furthermore, the immediate off-resonance regime shows a slight increase in the intensity displacement when compared with the less detuned example.

Figure 21b, which shows the spectra enhanced around the resonance position, provides a more detailed image of what happens to the transmission profiles as a function of the cavity length. Here it is seen that the peaks at the edges of the length interval have varied in both position and height, much more dramatically than the once in figure 20b. In short, the in-

creased detuning has also increased the fractional length-dependence of the double Fano cavity⁹, as the resonance transmission profile seem to change *more* when varying the length of the cavity between the guided-mode resonance lengths, in this case.



(a) Full-range transmission spectra of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu m$ with $\Delta = 0.8nm$ (b) Transmission spectra zoomed in on a range around the resonance wavelength of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu m$ with $\Delta = 0.8nm$

Figure 22

Figure 22 shows the effect of a relatively large detuning of $\Delta = 0.8nm$. This is simply included for a visual representation of a detuning which is considered "too large" for any practical use. The long range scan in figure 22a shows that the Fabry-Perot like fringes are now too displaced to be compared in terms of their positions, and figure 22b showing the spectra enhanced around the resonance wavelength shows roughly the same trend as in figure 21b, only that this example is greatly broadened in comparison. It is though noted, that while broadened, the Fano resonance mode is sustained even for the relatively large detuning.

In order to get a clear and qualitative picture of the double Fano cavity transmission profile as a function of both cavity length and wavelength, we visualize the varying of both in a heat map and let the color indicate the transmission intensity. This is shown in figure 23a which depicts a lossless cavity of length $l \approx 20\mu m$ and detuning $\Delta = 0.3nm$. Here the movement of the resonance peak as a function of the cavity length is clearly seen as a slope of the "line" representing the high intensity region due to the Fano resonance.

⁹It must be noted here that the range in which the cavity length is scanned is increased with the detuning. So the length-dependence is fractional in the sense that the step size is defined by the size of the interval.

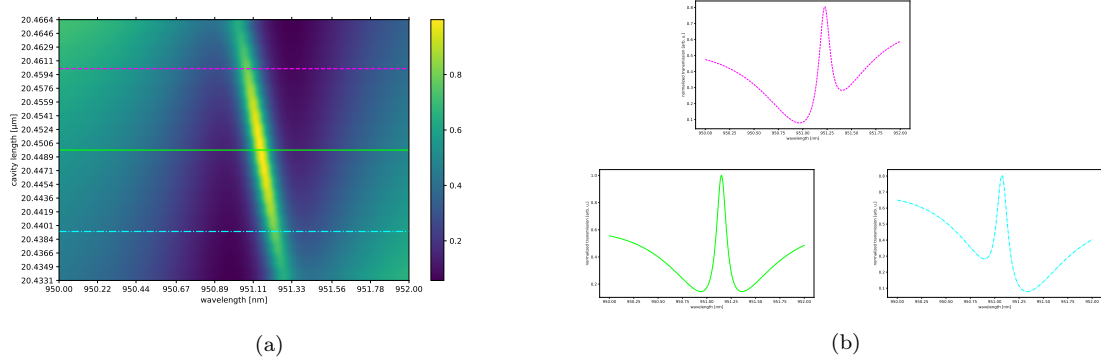


Figure 23: (a) shows a heat map of the double Fano cavity transmission as a function of wavelength and cavity lengths ranging $l \rightarrow l' \approx 20\mu\text{m}$ for $\Delta = 0.3\text{nm}$. (b) show slices of the heat map for comparing different cavity lengths, and are indicated by their respective color and linetype in (a). The respective cavity lengths and linewidths are given as: $l_{\text{magenta}} = (0.2l + 0.8l')$, $WHM_{\text{magenta}} = 71.3\text{pm}$, $l_{\text{cyan}} = (0.8l + 0.2l')$, $WHM_{\text{cyan}} = 71.3\text{pm}$ and $l_{\text{lime}} = (l + l')/2$, $WHM_{\text{lime}} = 58.6\text{pm}$.

Considering only the heat map, it is not easily visible which cavity length is the optimal one, however, the *magenta*, *cyan*, and *limegreen* lines across the heat map indicate slices which are depicted separately in figure 23b. It is seen by analysing the transmission profiles of these cavities of specific lengths, how they vary in both position and linewidth. The resonance wavelength positions are given relative to the cavity lengths chosen. The lengths and corresponding linewidths are given as

$$\begin{aligned}
 l_{\text{magenta}} &= 0.2l + 0.8l' \rightarrow WHM_{\text{magenta}} = 71.3\text{pm} \\
 l_{\text{cyan}} &= 0.8l + 0.2l' \rightarrow WHM_{\text{cyan}} = 71.3\text{pm} \\
 l_{\text{lime}} &= (l + l')/2 \rightarrow WHM_{\text{lime}} = 58.6\text{pm}.
 \end{aligned} \tag{48}$$

It turns out that the linewidths of the cyan and magenta transmission profiles are identical¹⁰, while the limegreen transmission profile seem to have a narrower profile at resonance.

This trend is further examined in figure 24 where the linewidths of intracavity spectra are shown as a function of the cavity length. The parameters used are the same as in figure 23 and clearly indicates that the optimal

¹⁰This is only exactly correct because the only varied parameter of the two Fano mirrors are the cavity- and guided-mode-resonant wavelengths λ_0 and λ_1 . In a more realistic case where several parameters vary, the attributes of the transmission profiles are unlikely to act fully symmetrically when varying the length $l \rightarrow l'$.

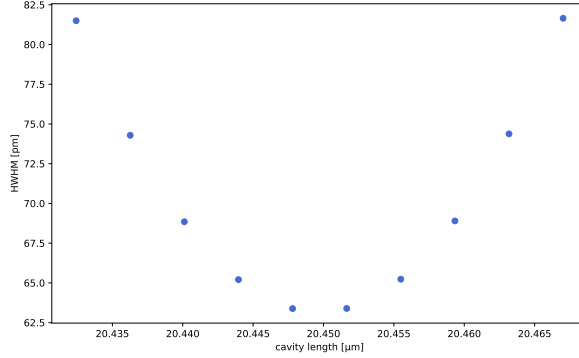


Figure 24: Linewidths, found as fitting parameters of a least squares fit to the general Fano model, as a function of cavity length $l \rightarrow l'$.

cavity length of a detuned double Fano cavity is the one fulfilling

$$\lambda_t = \frac{\lambda_{0,1} + \lambda'_{0,1}}{2}, \quad (49)$$

for the transmission wavelength λ_t , and thus corresponding cavity length l_t .

As a visual and qualitative representation of the effect of increasing the detuning Δ , figure 25 shows heat maps similar to the one in figure 23, but for a range of values for the detuning $\Delta = 0.1nm \rightarrow \Delta = 0.9nm$. It is readily seen that the aforementioned slope of the high intensity region, indicating the resonance peak, increases with the detuning. This is a representation of the peak moving to higher wavelengths, both for the optimal transmission wavelength and for the one closer to the detuned Fano mirror guided-mode resonance. The broadening of the peak is also displayed in a way that is, while only qualitative, convincing.

The last image in figure 25, displays the case of $\Delta = 0.9nm$. Here it is seen that the spectral overlap has become too small to sustain the Fano resonance inside the cavity, and the intensity at the, previously, optimal transmission wavelength is therefore at a minimum. If however, we look at the areas of the heat map corresponding to cavity lengths of exactly l or l' and wavelengths $\lambda_{0,1}$ or $\lambda'_{0,1}$, we see that there is a slight increase in the transmission intensity. This is due to the fact that the cavity, in this position, corresponds to a single Fano cavity with a low finesse, as one of

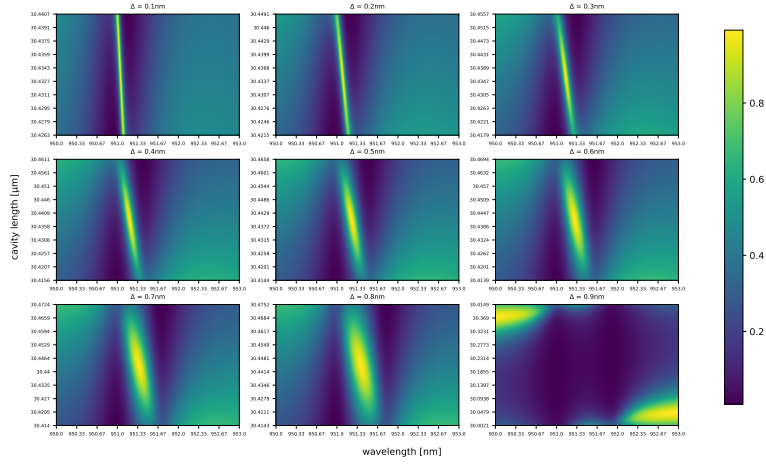
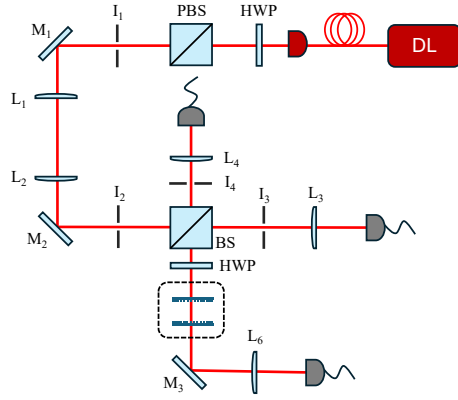
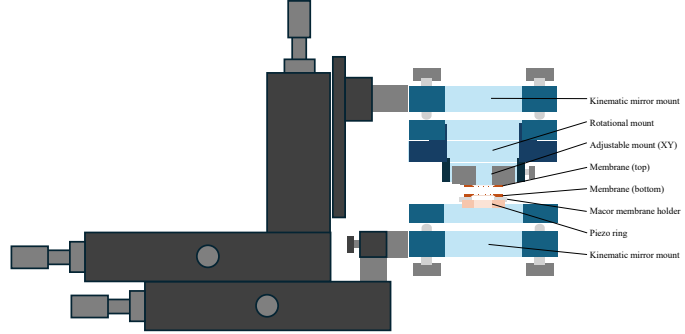


Figure 25: Series of heat maps showing double Fano transmission profiles for varying cavity lengths and wavelengths for increasing detuning Δ . The movement of the resonant wavelength is shown as the increasing slope of the high intensity region representing the resonance, for increasing detuning Δ . The broadening as a function of detuning is easily seen as it is qualitatively displayed in the series of figures until the spectral overlap makes the cavity unable of sustaining the Fano resonance.

the Fano mirrors are exactly on resonance while the other is so far away that it acts as simply a poor "broadband" mirror.



(a) Schematics of the experimental setup for measuring the transmission through a double Fano cavity. The cavity setup shown in (b) is



(b) Sketch of the part of the experimental setup containing the optical cavity.

Method

3 Method

3.1 The experimental setup

3.2 Characterization of sub-wavelength grating

Figures:

- Pictures of the membrane before patterning.
- AFM profile of the grating after patterning. (talk about profilometry and the physical parameters of the grating and their meaning for project).
- MAYBE: MIST simulation showing that the model accurately predicts the simulated spectra.

3.2.1 Obtaining normalized transmission/reflection spectra

3.2.2 Adjusting the beam waist - the optical telescope

3.2.3 The alignment procedure

3.2.4 Normalization

3.3 Cavity measurements

3.3.1 Determining the cavity length from the FSR

Figures:

- Linewidth as a function of "time" - to see the reduction of the linewidth as the piezo reaches an equilibrium where any time-dependent drift is reduced.

3.3.2 Normalization

3.3.3 Single fano cavity characterization

3.3.4 Aligning the cavity

Include:

- Scan for optimal cavity length
- Measure the spectral linewidth
-

3.3.5 Double fano cavity characterization

3.3.6 Off-resonance Fabry-Perot cavity (alignment technique)

3.3.7 Centering of the top grating (pinhole method)

3.4 Parallelism study (deviation from normal incident)

Results

4 Simulations

4.1 The single Fano cavity

4.2 The double Fano cavity

Figures:

- Simulated spectra of M3 and M5.
- Simulated length scans of M3 and M5.
- M3/M5 cavity trans. spectra (on resonance + full range)
for lengths: $l_{M3} \rightarrow l_{M5}$
for length: $l = 1/2 \cdot (l_{M3} + l_{M5})$
- Optimal result comparison with single fano/broadband cavities of similar losses.
- Optimal result comparison with the ideal case from prev. section.
- Simulated linewidth as a function of cavity length (include the same for broadband and single fano cavities).

5 Experimental results

5.1 The single Fano cavity

Figures:

- Single fano cavity transmission as a function of wavelength.
- Short scan of the single fano cavity transmission, with found linewidth.
- Long scan Fabry-Perot fringes for determining FSR \rightarrow cavity length.
- linewidth as a function of cavity length (compare with broadband cavity).

5.2 The double Fano cavity

5.2.1 Realizing the double fano model

Figures:

- Fit of the double fano model (long + short cavity)

5.2.2 Double fano off-resonance Fabry-Perot cavity

Figures:

- Off-resonance double fano transmission as a function of wavelength (show that the off resonance transmission goes close to 100 percent for a well-aligned cavity).

5.2.3 The double fano linewidth

Figures:

- "Semi-short" scan data, fit to the double fano transmission model.
- Short scan data, fit to the Fano function (for measuring linewidth).
- Linewidth as a function of cavity length (compare double fano, single fano and broadband cavities).

6 Discussion

- 6.1 Optimal configuration for double fano cavity - spacial limitation for the cavity length
- 6.2 Spacial drift of the piezo ring
- 6.3 Noise reduction - coupled/uncoupled mechanical/acoustic vibration (the plexi-glass box)
- 6.4 Broadening sources (especially for long cavities)
- 6.5 Improvements to the setup

7 Appendix

- All peaks and fits used for figure 10a.
- All peaks and fits used for figure 14a.
- The derivation of equation 30
- The derivation of equation 31
- Same derivations for the double fano cavity.

8 References

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