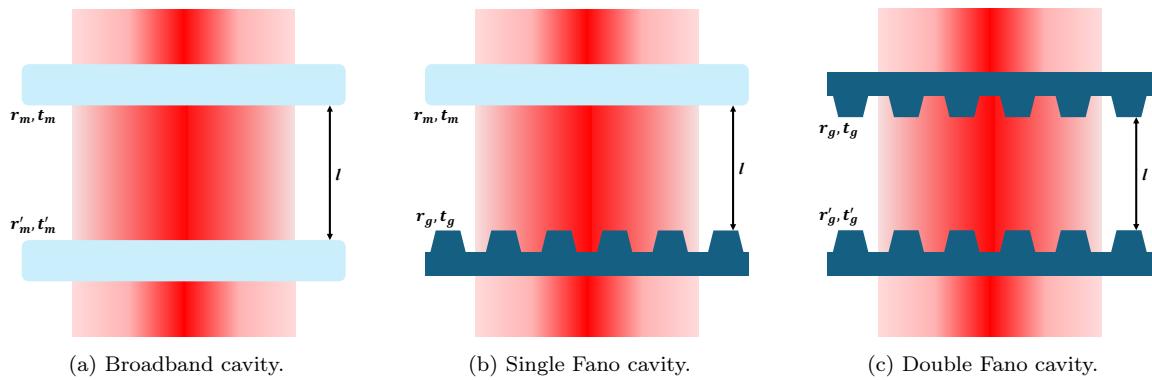


Contents

1	Introduction	1
2	Theory	2
2.1	The Fabry-Perot interferometer	2
2.1.1	Transmission	2
2.1.2	Finesse	4
2.1.3	Cavity length	6
2.1.4	The Fundamental Mode: A Gaussian Beam in the Large Waist Approximation	6
2.2	The Fano mirror: a sub wavelength grating	9
2.2.1	Geometric optical analysis	9
2.2.2	Reflection/transmission spectra and line shape analysis	9
2.2.3	Lossless grating	9
2.2.4	Lossy grating	12
2.3	The Fano cavity	13
2.3.1	The single Fano cavity model	13
2.3.2	Transmission linewidth	14
2.4	The double Fano cavity	18
2.4.1	The double fano transmission model	18
2.4.2	Transmission linewidth	19
2.4.3	Single and double Fano cavity comparison	20
2.4.4	Additional cavity losses	22
2.4.5	Spectral detuning (lossless)	24
2.4.6	Spacial detuning (lossless)	27
3	Method	34
3.1	The experimental setup	34
3.1.1	Tunable diode laser	34
3.1.2	$\lambda/2$ - waveplate	35
3.1.3	Optical telescope	36
3.1.4	Transmission, reflection and incident photo-detectors	36
3.1.5	The double Fano cavity measurement setup	37
3.2	Characterization of sub-wavelength grating	39

3.2.1	The alignment procedure	40
3.2.2	Adjusting and measuring the beam waist	42
3.2.3	Obtaining normalized transmission/reflection spectra	44
3.3	Cavity measurements	45
3.3.1	Determining the cavity length from the FSR	45
3.3.2	Normalization	45
3.3.3	Single fano cavity characterization	45
3.3.4	Aligning the cavity	45
3.3.5	Double fano cavity characterization	45
3.3.6	Off-resonance Fabry-Perot cavity (alignment technique)	45
3.3.7	Centering of the top grating (pinhole method)	45
3.4	Parallelism study (deviation from normal incident)	45
4	Simulations	46
4.1	The single Fano cavity	46
4.2	The double Fano cavity	46
5	Experimental results	47
5.1	The single Fano cavity	47
5.2	The double Fano cavity	47
5.2.1	Realizing the double fano model	47
5.2.2	Double fano off-resonance Fabry-Perot cavity	47
5.2.3	The double fano linewidth	47
6	Discussion	48
6.1	Optimal configuration for double fano cavity - spacial limitation for the cavity length	48
6.2	Spacial drift of the piezo ring	48
6.3	Noise reduction - coupled/uncoupled mechanical/acoustic vibration (the plexi-glass box)	48
6.4	Broadening sources (especially for long cavities)	48
6.5	Improvements to the setup	48
7	Appendix	49
8	References	50



1 Introduction

2 Theory

2.1 The Fabry-Perot interferometer

The Fabry-Perot interferometer, also known as an optical cavity, is generally comprised of two reflective optical resonators. It might make use of the configuration sketched in figure 2.

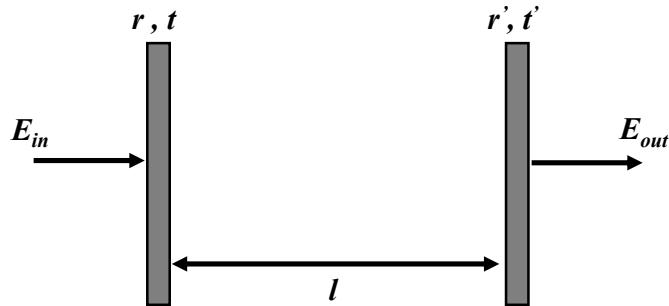


Figure 2: Sketch of a planar Fabry-Perot cavity.

Two planar slabs described by each their respective reflectivity r and transmission t are placed parallel and at a distance l from eachother. The transmitted field E_{out} is then given as a super position of all beams coupling into the cavity by E_{in} . In this way the Fabry-Perot cavity gives rise to so-called eigenstates[1] related to it's length l , derived from considering the path difference between successive parallel beams through the cavity. The allowed modes inside the cavity are generally limited to ones which fulfill the identity[2]

$$2l \cos \theta = m\lambda. \quad (1)$$

Here θ is the incidence angle of the light coupling into the cavity, λ is the wavelength of the light and $m = 1, 2, 3, \dots$ is a positive integer describing the order of the allowed mode. For the rest of this thesis it will be assumed that the laser couples into the cavity at normal incidence, meaning that $\theta = 0^\circ$ and thus $\cos \theta = 1$, as depicted in figure 2.

2.1.1 Transmission

In order to determine the transmission through the Fabry-Perot interferometer, we once again consider the configuration in figure 2. It is further,

initially, assumed that resonators are both lossless, such that

$$|r|^2 + |t|^2 = |r'|^2 + |t'|^2 = 1. \quad (2)$$

This means that all losses, e.g. absorption, associated with the interaction between the field and the resonators are neglected.

In order to formulate the transmitted field E_{out} in terms of the ingoing one E_{in} , we first assume that the ingoing field can be described well by a plane-wave propagating in the z-direction

$$E_{in} = E_{0,in}e^{ikz}, \quad (3)$$

where $k = 2\pi/\lambda$ is the wave number and $E_{0,in}$ is the amplitude of the field. We then consider E_{out} to be comprised of contributions for each roundtrip inside the cavity. This can be written as an infinite geometrical series given as

$$\begin{aligned} E_{out} &= tt'E_{0,in}e^{ikz} + tt'E_{0,in}e^{ikz}rr'e^{i\delta} \\ &\quad + tt'E_{0,in}e^{ikz}(rr'e^{i\delta})^2 + tt'E_{0,in}e^{ikz}(rr'e^{i\delta})^3 + \dots \\ &= tt'E_{0,in}e^{ikz} \sum_{m=0}^{\infty} (rr'e^{i\delta})^m \end{aligned} \quad (4)$$

where $\delta = 2kl$. The first term of the series corresponds to a direct transmission through the cavity, and each term thereafter corresponds to the respective contribution to the transmission after the $m'th$ round trip.

By evaluating the series it is seen that it indeed converges, and setting $z = l$ the final expression for the transmission through a planar Fabry-Perot cavity is found as

$$E_{out} = E_{0,in} \frac{tt'e^{i\delta/2}}{1 - rr'e^{i\delta}}. \quad (5)$$

Figure 3 shows an example of the normalized transmission $|E_{out}|^2/|E_{0,in}|^2$ of an arbitrary Fabry-Perot cavity of length $l \approx 50\mu m$ with reflectivity

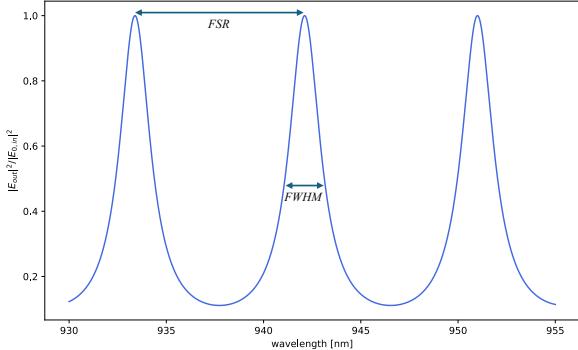


Figure 3: Example of a Fabry-Pérot cavity transmission spectrum for cavity length $l \approx 50\mu m$ and reflectivity $|r|^2 = |r'|^2 = 0.5$. The *Free Spectral Range* and *Full Width at Half Maximum*, respectively.

$|r|^2 = |r'|^2 = 0.5$. The two defining features marked on the figure are the *Free Spectral Range (FSR)* and *Full Width at Half Maximum (FWHM)*. As clearly depicted on the figure, the FSR is defined as the, in this case, spectral distance between two peaks¹, while the FWHM is simply the linewidth of each peak.

2.1.2 Finesse

Another important characteristic of an optical cavity is the so-called *finesse* \mathcal{F} , which generally describes the quality of the cavity by the number of round trips the light makes on average before being transmitted. It can thus be said that the finesse corresponds to the order m in the geometrical series describing the transmitted field E_{out} in eq. (4), making the series, in any practical sense, finite.

There is a variety of ways one can obtain the finesse of a cavity. By examination of the spectrum one can find the finesse in terms of the FSR and FWHM, as seen in figure 3, given as

$$\mathcal{F} = \frac{FSR}{FWHM}, \quad (6)$$

¹The definition of the FSR is not limited to the distance between peaks in wavelength. As seen in eq. (1) the cavity length and the resonant wavelengths are linearly dependent, and the length can thus just as well be varied resulting in a similar spectrum for the cavity transmission.

which indicates the resulting peaks of a cavity of higher finesse are narrower for the same FSR.

The finesse can also be expressed in terms of the reflectivity/transmission of the resonators of the cavity, as the defining physical action is that of the light either being transmitted from the cavity or not. The finesse in terms of r and t , for both resonators, are given as

$$\mathcal{F} = \frac{2\pi}{1 - |r|^2 + 1 - |r'|^2} = \frac{2\pi}{|t|^2 + |t'|^2}. \quad (7)$$

Figure 4 shows an example of the transmission spectra of a high finesse cavity of reflectivity $|r|^2 = 0.9$ together with a cavity of relatively low finesse with reflectivity $|r|^2 = 0.3$. It is clearly seen that the linewidth of the high finesse cavity is much narrower than the low finesse case, which follows from eq. (6).

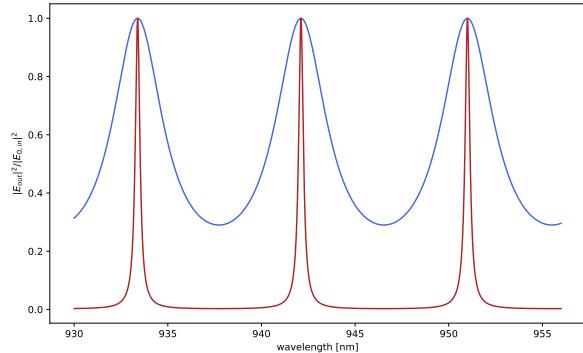


Figure 4: The red line shows the transmission spectrum of a *high* finesse Fabry-Perot cavity of reflectivity $|r|^2 = 0.90$, while the blue line shows the transmission spectrum of a *low* finesse cavity with reflectivity $|r|^2 = 0.30$.

Eq. (7) however is only true for the case of a lossless cavity, i.e. when eq. (2) is fulfilled. In practice, any cavity will have some amount of losses, which would have to be taken into account when calculating the finesse. When losses are present eq. (2) instead generally reads

$$|r|^2 + |t|^2 + L + L' = 1, \quad (8)$$

where L and L' indicates the fractional losses of each resonator.

In this case the finesse would be given as

$$\mathcal{F} = \frac{2\pi}{1 - |r|^2 + 1 - |r'|^2} = \frac{2\pi}{|t|^2 + |t'|^2 + L_{total}}, \quad (9)$$

where $L_{total} = L + L'$ are the total cavity losses.

The effect on the transmission spectrum of a cavity with losses is that the level of the normalized transmission will not reach 1, as some light is lost to i.g. absorption effects for each round trip of the cavity.

2.1.3 Cavity length

So far we have only considered effects that are independent of the length of the Fabry-Perot cavity, and by examining figure 4 it is seen that the only previously discussed feature that does not vary when changing the reflectivity and transmission (and by extension also the finesse) of the cavity, is the FSR. The FSR is defined as the distance between adjacent peaks of the transmission spectrum, and is given as

$$FSR = \frac{c}{2nl}, \quad (10)$$

where c is the speed of light in vacuum, n is the refractive index of the medium inside the cavity and l is the length of the cavity. The inverse proportionality of the cavity length l and the FSR is easily seen, and since the finesse is constant when varying the cavity length, the same must apply for the FWHM. Figure 5 shows a comparison of the transmission through two Fabry-Perot cavities of length $100\mu m$ and $150\mu m$, respectively. It is evident that the FSR depends on the cavity length, as $FSR_{100\mu m} > FSR_{150\mu m}$. It is also seen, that while the FSR becomes smaller with increasing cavity lengths, the linewidth does too, this is a consequence of the finesse \mathcal{F} being constant in terms of the cavity length.

2.1.4 The Fundamental Mode: A Gaussian Beam in the Large Waist Approximation

In order to describe the allowed modes within an optical cavity, it is first assumed that a single-mode field is linearly polarized. We then consider

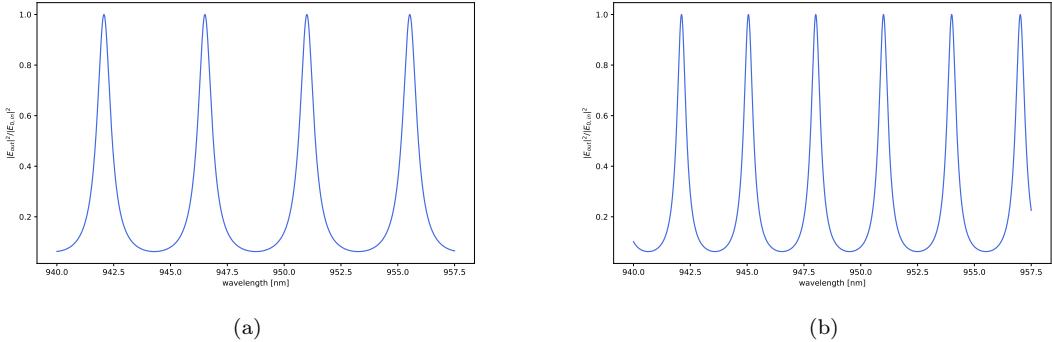


Figure 5: (a) shows the transmission spectrum of a Fabry-Perot cavity of length $l \approx 100\mu m$, while (b) shows the transmission spectrum of a cavity of length $l \approx 150\mu m$.

solutions to the wave equation

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}, \quad (11)$$

given as

$$\vec{E} = E_0(x, y, z) \vec{\epsilon} e^{ikz}. \quad (12)$$

Here $E_0(x, y, z)$ describes the electric field amplitude, $\vec{\epsilon}$ is denoted the polarization vector and $k = 2\pi/\lambda$ is the angular wave number of the field propagating along the z-axis. It is assumed that the electric field has a Gaussian transverse distribution².

This is almost the simplest description of the propagating field, however, as the spacial dependence of the field amplitude still might complicate things, we consider the range in which this can be neglected.

It can be shown from the derivation of the Gaussian distribution that the waist of the beam $w(z)$, which depends on the spacial coordinate in the direction of propagation, can be described as [1]

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \quad (13)$$

where z is the distance from focus, w_0 is the beam waist at focus and z_R is the so-called *Rayleigh range*. The Rayleigh range describes the range in

²When this is the case, the laser is said to operate in the lowest possible mode denoted TEM_{00} . This implies the assumption of ideal lasing conditions.

which the beam diverges slowly, whereas after this has been surpassed, the beam will begin to diverge more rapidly. By quick inspection of eq. (13) it is seen that

$$w(z) = \begin{cases} w_0, & \text{for } z = 0 \\ w_0\sqrt{2}, & \text{for } z = z_R, \end{cases} \quad (14)$$

which shows that the beam waist diverges no more than by a factor of $\sqrt{2}$ from the optimal case, for $0 \leq z \leq z_R$. Considering the case where $z \neq 0$ but however much smaller than the Rayleigh range z_R , we can further inspect eq. (13) and find that this leads to negligible changes in the waist of the beam. Specifically, it can fairly easily be seen that

$$\left(\frac{z}{z_R}\right)^2 \approx 0, \quad \text{for } z \ll z_R \quad (15)$$

which in turn leads to

$$w(z) \approx w_0. \quad (16)$$

The Rayleigh range can be written as [2]

$$z_R = \frac{\pi w_0^2}{\lambda}, \quad (17)$$

which, through the exponential dependence on w_0 , shows that a large beam waist will result in a long Rayleigh range. As an example, consider a beam of waist $w_0 = 200\mu m$ and wavelength $\lambda = 950nm$. This would result in a Rayleigh range of $z_R = 13.23cm$.

Finally we can conclude, that any optical cavity, for which the total distance travelled inside the cavity³, is significantly shorter than the Rayleigh range of the incident beam, the spacial dependence of the field amplitude inside the cavity is negligible, and the fundamental mode can be described simply by a linearly polarized plane wave

$$\vec{E} = E_0 \vec{e} e^{ikz}. \quad (18)$$

³For any optical cavity the incident light will travel a distance inside the cavity according to, not only the length of the cavity, but also the amount of round trips the light makes when confined inside the cavity.

This is often referred to as the *large waist approximation* of a Gaussian beam, due to the dependence on w_0 of eq. (17).

2.2 The Fano mirror: a sub wavelength grating

2.2.1 Geometric optical analysis

Considering an ideal grating with period d in the sub-wavelength regime, it can be shown that only a single mode of reflection/transmission is supported.

Any grating, of arbitrary dimensions, must comply with the very general *grating equation*[2] given as

$$\sin \theta_m = m \frac{\lambda}{d}, \quad (19)$$

for the special case of a linearly polarized plane wave incident on a grating placed normal to the direction of propagation. Now, inserting the sub-wavelength condition $d \ll \lambda$, it is easily seen that the right side of the equation blows up for any order of reflection $|m| > 0$, effectively showing that this is the aforementioned single supported mode in this regime. Furthermore, it can be equally easily seen that the propagation direction of the 0'th order mode is the same as the incident beam, i.e. normal to the grating.

2.2.2 Reflection/transmission spectra and line shape analysis

2.2.3 Lossless grating

We wish to analytically describe the wavelength-dependent spectra for the transmission and reflectivity of an infinite sub-wavelength grating. By first considering the case where absorption and thermal coupling effects are neglected, i.e. a lossless grating, we can assume conservation of energy and thereby the relations

$$|r_g|^2 + |t_g|^2 = 1 \quad \text{and} \quad |r_d|^2 + |t_d|^2 = 1, \quad (20)$$

where the subscripts g and d indicate the *grating* and *direct* transmissions and reflectivities, respectively. It is implied that the direct coefficients are

constants and describe the transmission and reflectivity when the incident wavelength is significantly detuned from any guided-mode resonance of the grating. Furthermore, it is also implied that the grating coefficients are functions of the incident wavelength.

We now assume a normal incident beam on the grating as a linearly polarized monochromatic plane wave, with a wavelength close to a guided-mode resonance of the grating. In order to describe the coefficients r_g and t_g we follow the formalism presented by Fan and Joannopoulos [3] and consider the likely paths of the incident light through the grating. It is quite intuitive to consider the case where the light is simply transmitted, and this shall be our first case hereafter denoted the *direct pathway*. Another case one might consider is the one where the incident light excites the guided-mode resonance in the grating. This case is denoted the *indirect pathway* and decays more slowly than its direct counterpart.

The interference caused when the guided mode is excited is often referred to as *Fano resonances*, due to its physical similarities to the description of interference between a discrete autoionized state and a bound continuum state first reported by Fano [4]. The cross section of inelastic scattering, when measured as a function of energy, showed characteristic asymmetric peaks. These were described as the aforementioned interference pattern between *direct* (the discrete state) and *indirect* (the continuum state) pathways.

By generalizing the model of Fan and Joannopoulos [3] we describe the transmission and reflectivity coefficient amplitudes as

$$r_g = r_d + \frac{a}{k - k_1 + i\gamma} \quad \text{and} \quad t_g = t_d + \frac{b}{k - k_1 + i\gamma}, \quad (21)$$

where $k = 2\pi/\lambda$ is the incident wave number, $k_1 = 2\pi/\lambda_1$ is the wave number according to the guided-mode resonance and γ is the HWHM (half width at half maximum) of the guided-mode resonance. Complex coefficients a and b describe the interference between the directly transmitted or reflected waves and the guided mode of the grating.

Note that in eq. (21) the right side of the expression for each coefficient corresponds to the continuum state i.e. the indirect pathway, while

the direct transmission and reflection coefficients take the role of the autoionized discrete state, i.e. the direct pathway⁴ [4]

As we are dealing with an ideal, lossless, grating, we assume coefficients a and b to be equal, meaning that we specifically assume vertical symmetry throughout the grating. By considering eq. (20) this in turn leads to

$$a = b = -i\gamma(t_d + r_d), \quad (22)$$

which further yields an expression for the grating transmission amplitude coefficient on the form

$$t_g = t_d \frac{k - k_0}{k - k_1 + i\gamma}. \quad (23)$$

Here, the newly introduced $k_0 = 2\pi/\lambda_0 = k_1 - i\gamma(r_d/t_d)$ is the zero-transmission/unity-reflectivity wave number.

To generalize eq. (23) to include non-unity reflectivity and non-zero transmission, we allow for $a \neq b$ [5][6][7], meaning that the case of vertical asymmetry is included in the model[8]. By assuming $r_d, t_d \in \mathbb{R}$, eq. (20) leads to the coupled differential equations

$$\begin{aligned} t_d x_a + r_d x_b &= 0, \quad \text{and} \\ x_a^2 + y_a^2 + x_b^2 + y_b^2 + 2t_d\gamma y_a + 2r_d\gamma y_b &= 0, \end{aligned} \quad (24)$$

where $\{x, y\}_{a,b}$ respectively denotes the real and imaginary parts of the coefficients a and b . Solving eqs. (24) leads to the correct complex reflectivity coefficients and the expression for the transmission coefficient amplitudes now reads

$$t_g = t_d \frac{k - k_0 + i\beta}{k - k_1 + i\gamma}, \quad (25)$$

where k_0 and β are defined from the expression for a found by solving eqs. (24), given as

$$a = t_d(k_1 - k_0 - i\gamma + i\beta). \quad (26)$$

Finally, this allows for non-zero transmission and non-unity reflectivity at wave number k_0 .

⁴The general eigenvector of a state comprised of a super-position between a discrete state and a continuum, i.e. a state vector corresponding to a Fano resonance, is given as $\Psi_E = a\phi + \int dE' b_{E'} \psi_{E'}$, given in eq. (2) in ref. [4], where a and $b_{E'}$ describes the probability of either pathway.

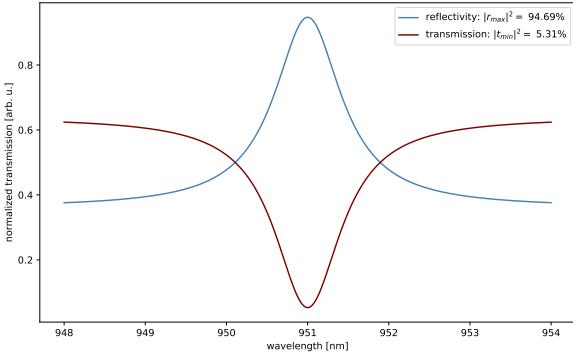


Figure 6: Simulated reflectivity and transmission spectra of an arbitrary lossless grating.

2.2.4 Lossy grating

In order to modify the above model such that losses, e.g. due to absorption or thermal coupling effects, are accounted for, we add a resonant loss term to the energy conservation relation in eq. (20). For this we introduce the resonant loss level L , which must be known in order to accurately calculate the complex reflectivity coefficients. The energy conservation relation is modified such that

$$|t_g|^2 + |r_g|^2 + \frac{c^2}{(k - k_1)^2 + \gamma^2} = 1, \quad (27)$$

where the coefficient $c^2 = L((k - k_1)^2 + \gamma^2)$ includes the resonant loss term L . A new set of coupled differential equations are found, using eq. (27), given as

$$\begin{aligned} t_d x_a + r_d x_b &= 0, \quad \text{and} \\ x_a^2 + y_a^2 + x_b^2 + y_b^2 + c^2 + 2t_d\gamma y_a + 2r_d\gamma y_b &= 0. \end{aligned} \quad (28)$$

It is easily identified that eq. (24) and eq. (28) differ only by the addition of coefficient c^2 , and thereby the losses. Solving eq. (28) leads to the correct complex reflectivity coefficients, except that they now account for any losses associated with the grating.

In conclusion, the complete grating model consists of an expression for the transmission coefficients and a set of coupled differential equations for the reflection coefficients, shown in eq. (25) and eq. (28), respectively. The

model on the form used for this project and subsequent thesis is derived in previous work by A. Darki et al. [9] and more recently T. Mitra et al. [10].

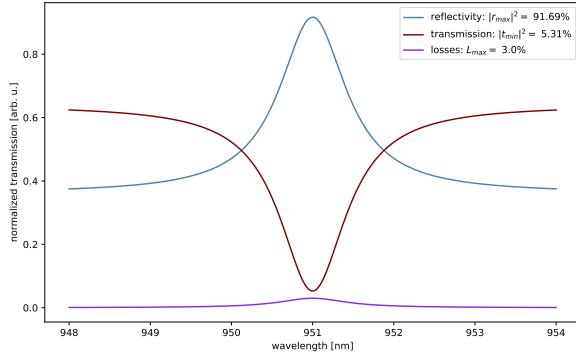


Figure 7: Simulated reflectiviy and transmission spectra of an arbitrary lossy grating.

2.3 The Fano cavity

2.3.1 The single Fano cavity model

The single Fano cavity consists of a planer broadband mirror, and a sub-wavelength grating, i.e. a Fano mirror, as described in section 2.2 and seen in figure 8 where schematics of the single Fano and broadband cavity configurations are shown. While the broadband mirror has fixed optical properties, the Fano mirror has transmission and reflection coefficients dependent on the incident wavelength, according to solutions to the coupled differential equations of eq. (28).

In order to model the single Fano cavity transmission spectra, we therefore consider the transmission function of a normal incident and planer Fabry-Perot cavity in eq. (5) with $r, t \rightarrow r_m, t_m$ and $r', t' \rightarrow r_g(\lambda), t_g(\lambda)$ [11]. Here the subscript m indicates the broadband *mirror* coefficients, and g is for *grating*, which indicates the coefficients of the Fano mirror. Rewriting eq. (5) such that it describes the normalized transmission amplitude, through the single Fano cavity, $T_{cav} = |E_{out}|^2/|E_{0,in}|^2$ we get

$$T_{cav} = \left| \frac{t_m t_g(\lambda) e^{i\phi}}{1 - r_m r_g e^{2i\phi}} \right|^2, \quad (29)$$

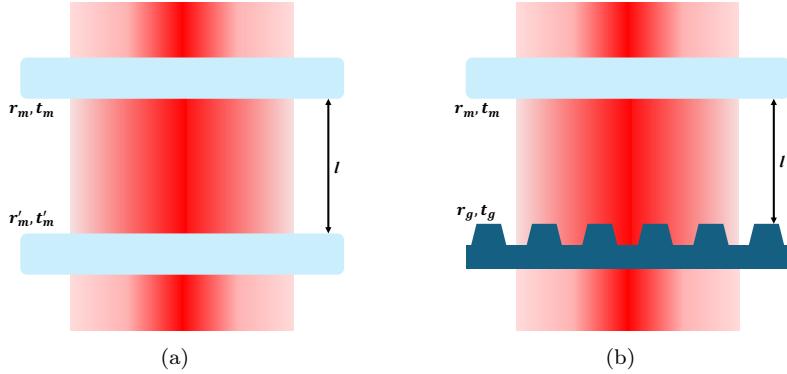


Figure 8: In (a) is seen the schematics of a cavity consisting of two broadband mirrors with transmission and reflectivity coefficients t_m , r_m , t'_m , and r'_m . (b) shows the schematics of a single Fano cavity consisting of one broadband mirror with coefficients t_m , r_m and one Fano mirror with wavelength dependent coefficients t_g , r_g . The reflectors in either cavity are separated by a cavity length l .

where $\phi = 2\delta = kl$, $k = 2\pi/\lambda$ and l is the cavity length, as is consistent the general case described in section 2.2.

2.3.2 Transmission linewidth

In order to analytically describe how the transmission spectrum at, or close to, the overall resonance⁵ behaves as a function of the incident wavelength, we attempt to generalize the Fano model for this specific scenario. Considering the case where the cavity resonance closely resembles the guided-mode resonance of the Fano mirror (the zero-transmission wavelength), eq. (29) can be approximated well by

$$T_{cav} \approx \frac{A}{1 + \left(\frac{\Delta}{1-\nu\Delta}\right)^2}, \quad (30)$$

where $\Delta = (\lambda - \lambda_c)/\delta\lambda$ is the detuning from the cavity resonance normalized by the HWHM $\delta\lambda$, and ν is a constant describing the asymmetry of the single Fano transmission spectrum. [10][9]

From eq. (30) it can be shown that the HWHM of the Fano transmission profile around the overall resonance wavelength, i.e. when $\lambda_c \approx \lambda_0$, is

⁵This terminology is used for the scenario where the guided-mode, cavity mode and input laser mode all meet the resonance condition $\lambda_g \approx \lambda_c \approx \lambda_l$, where g, c, l stands for grating/guided-mode, cavity and laser, respectively.

approximately given as

$$\delta\lambda \approx \frac{1}{\frac{1}{\delta\lambda_c} + \frac{1}{\delta\lambda_g}}, \quad (31)$$

where

$$\delta\lambda_c = \frac{\lambda_0^2}{8\pi l}(|t_g(\lambda_0)|^2 + |t_m|^2 + L) \quad (32)$$

is the HWHM of a broadband cavity and

$$\delta\lambda_g = \frac{\gamma\lambda}{2(1 - r_d)}(|t_g(\lambda_0)|^2 + |t_m|^2 + L) \quad (33)$$

is the HWHM of the Fano cavity in the so-called Fano regime.[10][9] In eqs. (31)-(33) λ_0 is the Fano cavity resonance wavelength, l is the cavity length, $L = (1 - |r_g(\lambda_0)|^2 - |t_g(\lambda_0)|^2)$ is the total additional losses of the cavity when on resonance, $\gamma\lambda$ is the width of the guided-mode resonance of the Fano mirror and r_d is the off-resonance, or *direct*, reflectivity of the Fano mirror.

The *Fano regime* and its counterpart the so-called *standard regime* are defined for a given single Fano cavity, by its length l . By inspection of eqs. (32) and (33) it is seen that for $l \rightarrow \infty$ the linewidth in eq. (31) is dominated by the broadband cavity term, while for the opposite case, $l \rightarrow 0$, the linewidth is predominantly given by the Fano cavity term.

Generally the Fano regime describes the cavity lengths for which eq. (31) shows a significant divergence from the broadband linewidth in eq. (32). Oppositely, when in the standard regime the broadband and Fano cavity produces resonance transmission peaks of comparable, if not equal, linewidths.

Figures 9a and 9b depicts two examples, in each their regime, of single Fano cavity transmission spectra together with their respective complimentary broadband cavity transmission profiles, and the reflectiviy amplitude of the Fano mirror used to model both profiles. As shown in section 2.2 any Fano mirror, or grating, is described by five characterizing parameters. For the one used to model the transmission spectra in figures 9a and 9b

these are

$$\begin{aligned} \lambda_0 &= 951\text{nm}, \quad \lambda_1 = 951\text{nm}, \quad t_d = 80\%, \\ \gamma\lambda &= 0.5\text{nm} \quad \text{and} \quad \beta = 0, \end{aligned} \quad (34)$$

where we remember that $\lambda_{0,1}$ are the resonant wavelengths for, respectively, the cavity- and guided-modes, t_d is the direct transmission, $\gamma\lambda$ is the width of the guided-mode resonance and β is a constant describing the losses of the grating, which here are set to 0, and thus neglected. The broadband mirror parameters are chosen such that they perfectly resemble the transmission and reflectivity of the Fano mirror on resonance, which is arbitrarily chosen such that $|r_m|^2 = 95\%$ and $|t_m|^2 = 5\%$.

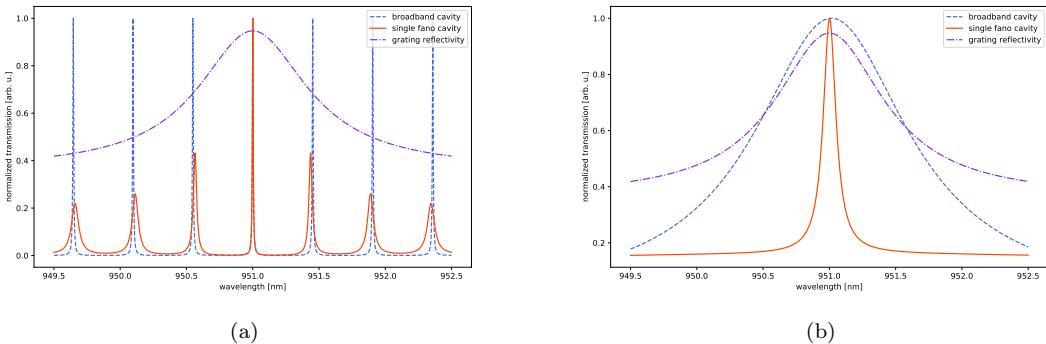


Figure 9: In (a) is seen the comparison of broadband and single Fano cavity transmission spectra for a cavity length of $l \approx 1000\mu\text{m}$, i.e. in the *standard* regime. (b) shows the same comparison, but for a cavity length of $l \approx 5\mu\text{m}$, i.e. in the so-called *Fano* regime.

Figure 9a shows the transmission spectra of the two cavities for a length of $l \approx 1000\mu\text{m}$, i.e. in the standard regime. It is clear from inspection of the figure that the resonance transmission profile of the standard broadband cavity is not wavelength dependent, in the sense that all fringes appear to have the same high finesse \mathcal{F} , i.e. ratio between the FSR and HWHM. This is not the case for the Fano cavity which is due to the wavelength dependence of the optical properties of the Fano mirror, as this causes the transmission and reflectivity to *only* match those of the broadband mirror when on resonance. Furthermore, no significant difference in linewidth is seen for the transmission of the two cavities on resonance, as predicted by eq. (31).

In figure 9b, the transmission spectra of both cavities are shown for a length of $l \approx 5\mu\text{m}$, i.e. in the Fano regime, where it is clearly seen

that while the standard cavity experiences broadening for shorter cavity lengths⁶, this is not the case for the Fano cavity transmission peak.

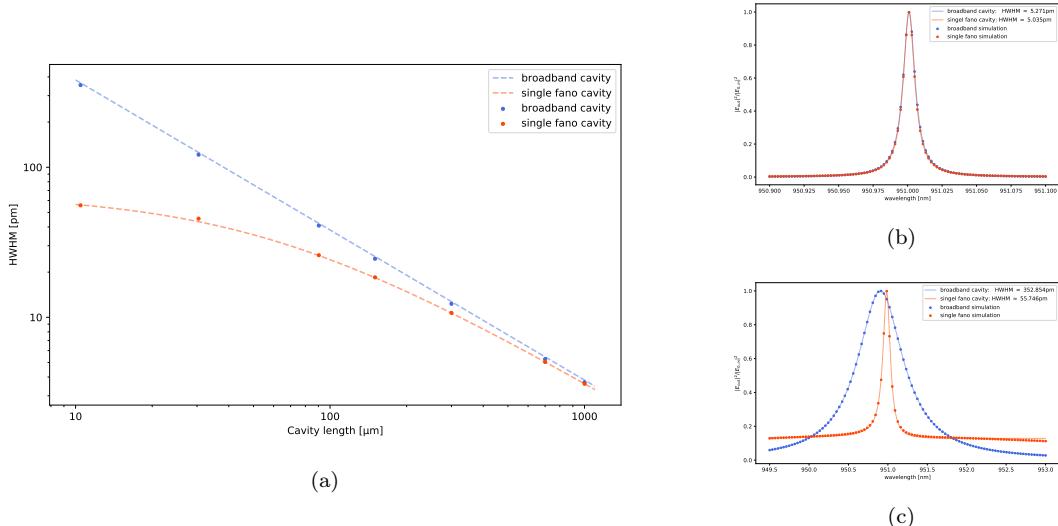


Figure 10: (a) shows the approximate analytical resonance linewidths (eq. (31)) as a function of cavity length for the broadband and single Fano cavities together with linewidths of transmission profiles simulated using eq. (29) and eq. (5), found as parameters of least squares fits, for comparison. In (b) and (c) is seen transmission spectra of broadband and single Fano cavities of lengths $\sim 700\mu\text{m}$ and $\sim 10\mu\text{m}$, respectively. The spectra shown indicate each their respective linewidths, and are examples of the values plotted in (a).

Figure 10a models the behavior of the linewidth of the single Fano cavity compared with the one for a broadband cavity of similar optical properties, as a function of wavelength. Here it is easily seen where the linewidth of the single Fano transmission begins to saturate, and hence deviate from the one of the broadband cavity. The plotted line in the figure is calculated using eq. (31) while the points depict linewidths found as a fitting parameter from a least squares fit of the general Fano model in eq. (30) to transmission spectra simulated by the Fabry-Perot (eq. (5)) and single Fano (eq. (29)) transmission functions. Finally, it can be concluded that the approximate analytical expression for the linewidth of the broadband and single fano cavities in eq. (31) correlates very well with the values found from the simulated spectra.

⁶This is a natural consequence of the shorter cavity as this causes the lifetime inside the cavity to fall and hence the transmission HWHM to rise. The HWHM is inversely proportional to the lifetime and goes as $\delta\lambda \propto 1/\tau$.

2.4 The double Fano cavity

2.4.1 The double fano transmission model

While the single Fano cavity is usually known in the appropriate litterature as simply a *Fano cavity*, I have insisted on including the fact that it contains only one Fano mirror, and hence denoted it the *single* Fano cavity. This addition is justified by the contents of this section, and namely that we now move on to the *double* Fano cavity, which as the name suggests consists of two Fano mirrors. The schematics of this configuration is shown together with the one for the single Fano cavity in figure 11.

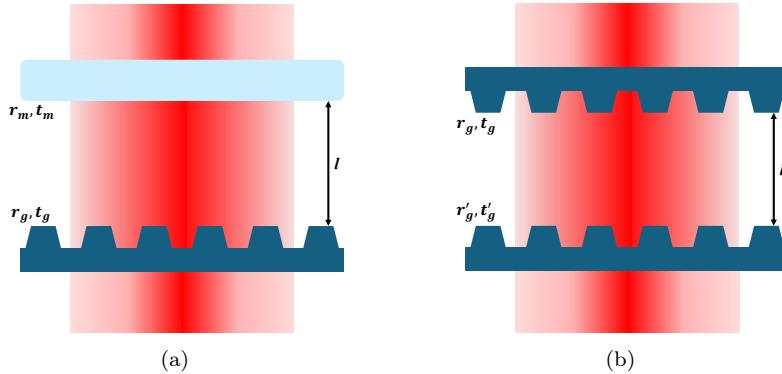


Figure 11: (a) shows schematics of the single Fano cavity consisting of a broadband mirror with transmission and reflectivity coefficients t_m, r_m and a Fano mirror with coefficients t_g, r_g . (b) shows the schematics of the double Fano cavity consisting of two Fano mirrors with coefficients t_g, r_g, t'_g, r'_g . Both cavities are separated by a cavity length l .

Here it is evident that instead of having one set of reflectivity and transmission coefficients that depend on the incident wavelength, we now have two. In order to model the transmission of the double Fano cavity, we once again consider the transmission function for the normal incident and planar Fabry-Perot cavity in eq. (5), this time with $r, t \rightarrow r_g(\lambda), t_g(\lambda)$ and $r', t' \rightarrow r'_g(\lambda), t'_g(\lambda)$ [11]. We rewrite the Fabry-Perot transmission function with the addressed substitutions of the optical coefficients and such that it describes the normalized transmission amplitudes $T_{cav} = |E_{out}|^2/|E_{0,in}|^2$ and get

$$T_{cav} = \frac{t_g(\lambda)t'_g(\lambda)e^{i\phi}}{1 - r_g(\lambda)r'_g(\lambda)e^{2i\phi}}. \quad (35)$$

The subscript g indicates a grating transmission or reflectivity. Figure 12 shows an example of the normalized transmission spectrum of an $\sim 30\mu\text{m}$

double Fano cavity on- and off-resonance, found using eq. (35).

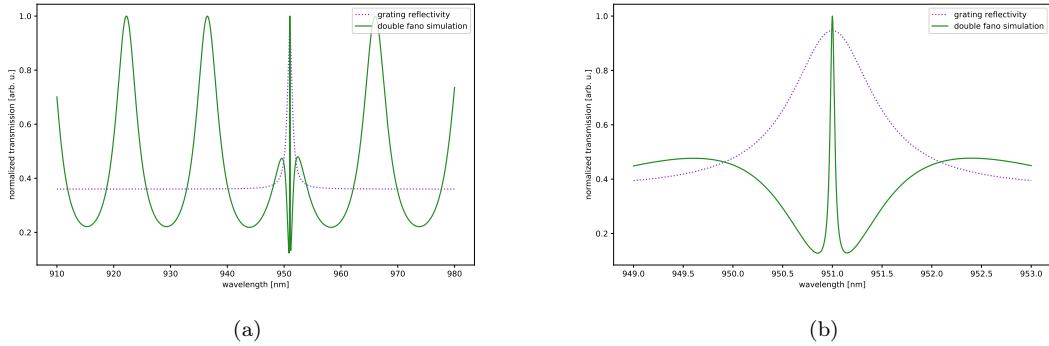


Figure 12: An example of a normalized double Fano transmission spectrum of an $\sim 30\mu\text{m}$ cavity. (a) shows a long-range wavelength scan, depicting both the on- and off-resonance behaviour of the double Fano cavity transmission, while (b) shows the transmission focussed specifically around the resonance position. Both examples are shown together with the reflectivity of the Fano mirror used to model them.

2.4.2 Transmission linewidth

In order to describe the analytical linewidth of the transmission profile of the double Fano cavity, we take a similar approach as in the single Fano case outlined in section 2.3.2. It can be shown from eq. (31) when including the wavelength dependence of the optical coefficients of both Fano mirrors, that the HWHM $\delta\lambda$ of the double Fano cavity transmission profile is approximately given as

$$\delta\lambda^{double} \approx \frac{1}{\frac{1}{\delta\lambda_c} + \frac{1}{\delta\lambda_g^{double}}}, \quad (36)$$

where

$$\delta\lambda_c = \frac{\lambda_0^2}{8\pi l} (|t_g(\lambda_0)|^2 + |t'_g(\lambda_0)|^2 + L) \quad (37)$$

is still the HWHM of a broadband cavity and

$$\delta\lambda_g^{double} = \frac{\gamma\lambda}{4(1 - r_d)} (|t_g(\lambda_0)|^2 + |t'_g(\lambda_0)|^2 + L). \quad (38)$$

is the HWHM of the double Fano cavity in the Fano regime. Note that $\delta\lambda^{double} = \delta\lambda^{single}/2$ for $l \rightarrow 0$ when eq. (36) is predominantly given by

eq. (38). In this brief evaluation of the estimated analytical linewidth of the double Fano transmission profile it is assumed that all defining parameters of the two Fano mirrors are identical, except for the cavity and guided-mode resonance wavelengths $\lambda_{0,1}$. Namely the following relevant parameters are assumed identical,

$$r_d = r'_d \text{ and } \gamma\lambda = \gamma\lambda'. \quad (39)$$

In this way any spectral detuning of the two Fano mirrors used to make a cavity is included in the analytical expression. The spectral detuning, and the effect hereof, will be further described in section 2.4.5.

2.4.3 Single and double Fano cavity comparison

Using the analytical expression for the double Fano cavity transmission in eq. (36) we are now in a position to compare the single and double Fano cavities. Note that we at this point only consider the ideal case of the double Fano cavity where additional cavity losses are neglected and the two Fano mirrors used are identical, i.e. the cavity is said to be *symmetrical*. The additional cavity losses are explicitly set to be given as

$$L = 1 - |r_g|^2 - |t_g|^2 = 0. \quad (40)$$

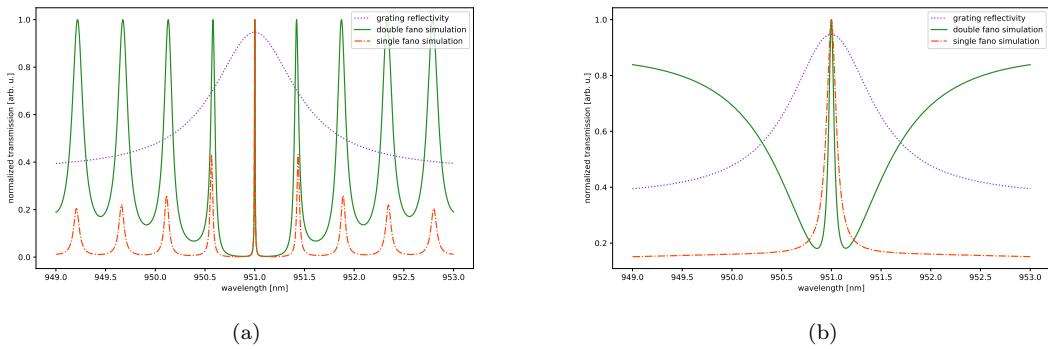


Figure 13: Comparison of the single and double Fano transmission spectra in the *standard* and *Fano* regimes. (a) shows the spectral comparison for an $\sim 1000\mu\text{m}$ cavity, i.e. in the *standard* regime, while (b) shows the same for an $\sim 5\mu\text{m}$ cavity, i.e. in the *Fano* regime. The Fano mirror reflectivity is depicted in both (a) and (b).

Figure 13 shows the transmission of the ideal double Fano cavity and the corresponding single Fano cavity for comparison.

Figure 13a shows the transmission for a cavity length of $\sim 1000\mu m$ which is well-inside the standard regime outlined in section 2.3.2 where the standard broadband and single Fano cavities produce transmission spectra of roughly identical linewidths. In this regime, due to the $1/l$ proportionality of the FSR, the off-resonance behavior of the double Fano cavity transmission is visible in the range plottet. It is seen, contrary to the single Fano cavity, that the transmission at each cavity resonance reaches a normalized transmission of $|E_{out}|^2/|E_{0,in}|^2 = 1$. This is due to the fact that the two gratings, while they have wavelength dependent transmission and reflectivity coefficients, always have identical ones for the ideal case, which maximizes the Fabry-Perot transmission function and ensures unity transmission at any cavity resonance. The minimum level of the cavity changes according to the reflectivity of the Fano mirrors and by that the HWHM also changes as we move further from the guided-mode resonance. Both the minimum transmission level and the HWHM converge when moving away from the resonance wavelength, when the reflectivity becomes constant.

Figure 13b shows the transmission of a double Fano cavity of length $\sim 5\mu m$ which, contrary to the one in figure 13a, is well within the Fano regime. It is seen that the double Fano cavity transmission produces a resonance peak with a HWHM narrower than the one for the single Fano cavity, as is predicted in eq. (38). Furthermore, the immediate off-resonance behavior of the double Fano cavity transmission in the Fano regime, is drastically different than for the single Fano cavity. This is due to the collective higher transmission in this region compared with the single Fano case where the broadband mirror has a constant, and often high, reflectivity and hence a correspondingly low transmission.

Figure 14a shows the analytical linewidth calculated and compared for the broadband, single Fano, and double Fano cavities, calculated using eqs. (32), (31), (36). These are compared with linewidths found as fitting parameters from a least squares fit of the general Fano transmission formula given in eq. (30) to transmission spectra calculated using eqs. (5), (29), and (35). According to eq. (36) the linewidth of the double Fano cavity transmission should converge to exactly half that of the single Fano

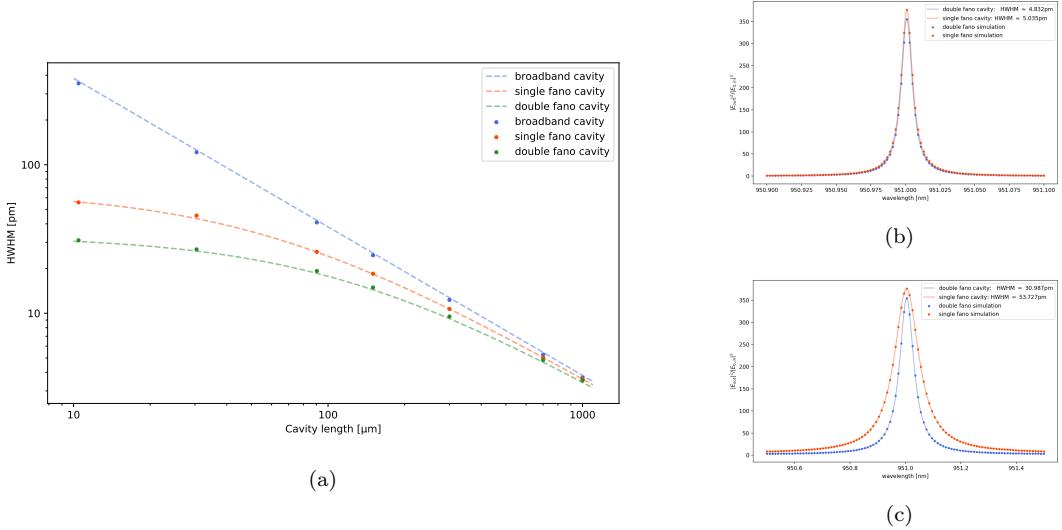


Figure 14: (a) shows the analytical resonance linewidths (eqs. (31), (36), and (32)) as a function of the cavity length for the broadband, single and double Fano cavities together with linewidths of corresponding profiles simulated using eqs. (29), (35), and (5) for comparison. In (b) and (c) is seen intracavity spectra of single and double Fano cavities of lengths $\sim 700\mu m$, and $\sim 10\mu m$, respectively. The spectra shown indicate each their respective linewidths, and are examples of the values plotted in (a). Note however, that the broadband cavity peak has been left out of (b) and (c).

cavity, meaning that

$$\delta\lambda_{double} = \frac{\delta\lambda_{single}}{2}, \quad \text{for } l \rightarrow 0. \quad (41)$$

This is supported well by the simulation depicted in figure 14a.

Figures 14b and 14c contain examples of intracavity⁷ spectra of single and double Fano cavities and corresponding least squares fits to the general Fano model in order to determine their linewidths.

2.4.4 Additional cavity losses

Thus far we have only examined a lossless double Fano cavity where

$$|r_g|^2 + |t_g|^2 = 1 \quad (42)$$

is fulfilled for both Fano mirrors used to construct the cavity.

⁷The reason these are shown as intracavity spectra instead of transmission spectra is due to the immediate off-resonance behavior of the double Fano cavity transmission. The shape makes accurate fitting a challenge, and while the intracavity spectra cannot be measured in the lab, the linewidth when simulated is identical to the resulting transmission peaks.

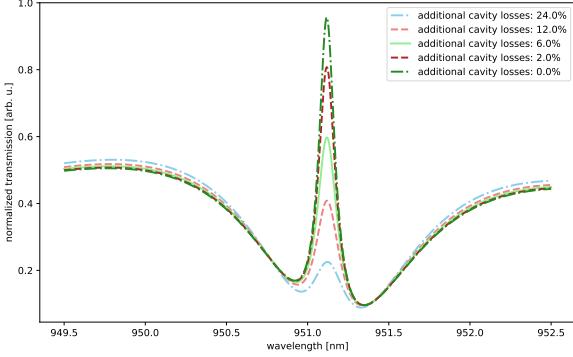


Figure 15: Resonant transmission spectra for different values of the additional loss term L , of a symmetric double Fano cavity of length $\sim 30\mu\text{m}$.

In this section we will investigate what happens when we introduce additional cavity losses⁸

$$L = 1 - |r_g|^2 - |t_g|^2, \quad (43)$$

leading to the modified condition

$$|r_g|^2 + |t_g|^2 + L = 1. \quad (44)$$

Figure 15 shows double Fano transmission spectra for a symmetric cavity with varying values for the additional cavity losses. It is readily seen that the maximum value reached in each spectrum is rapidly reduced with the introduction of these losses.

Since the linewidth is defined as the HWHM of the transmission profile, this will naturally vary as a function of additional cavity losses. This is depicted in figure 16a where the HWHM is shown for different values of L . Examples of intracavity spectra taken for different values of L are shown in figures 16b and 16c, along with their respective least squares fits to the general Fano model in eq. (30) and linewidths found as fitting parameters hereof.

Figure 16a shows the simulated linewidths of intracavity spectra as a function of total cavity losses L , and is compared with the analytical

⁸Not to be confused with the often used definition of cavity losses given as $L = 1 - |r|^2$ where anything not being reflected back into the cavity is considered to have a contribution to the losses.

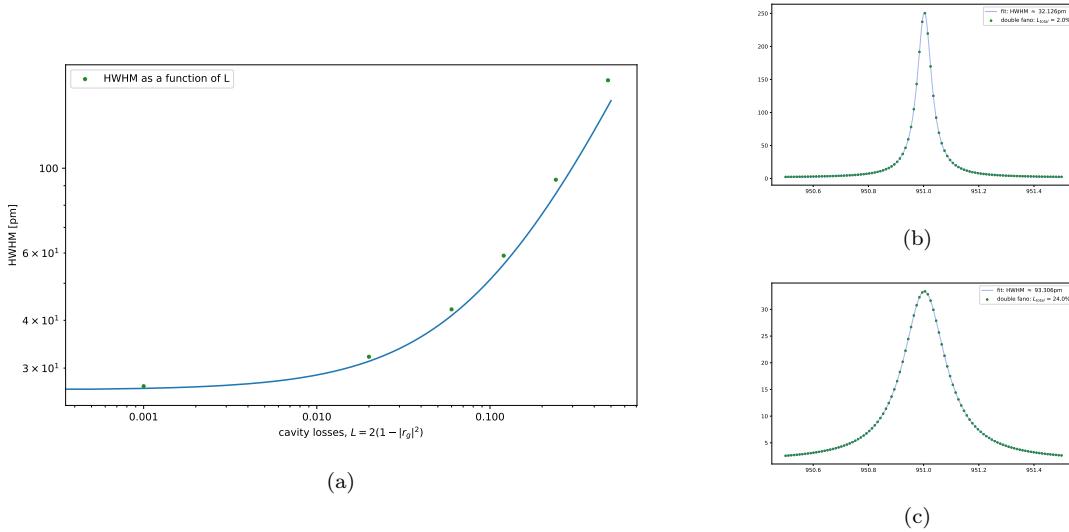


Figure 16: (a) shows the linewidth (HWHM) of a symmetric double Fano cavity of length $\sim 30\mu\text{m}$ as a function of additional cavity losses $L = 2(|r_g|^2 - |t_g|^2)$. Each point is found as a fitting parameter of a least squares fit of the intracavity double Fano spectrum for a certain value of L to the general Fano model. The plotted line indicates the analytical value of the linewidth (eq. 31) as a function of L , for comparison. In (b) and (c) are seen examples of intracavity double Fano spectra, with their respective linewidths, for cavities of $L = 2\%$ and $L = 24\%$, respectively.

formula for the double Fano linewidth in eq. (36). It is seen that the analytical model proves, once again, to be a powerful predicting technique for the linewidth, and especially for cavities with relatively low losses.

2.4.5 Spectral detuning (lossless)

Up until this point, it has been assumed that the Fano mirrors making up the double Fano cavity has been identical, namely the cavity has been *symmetric*. However, in practice this is very unlikely to be the case, as any Fano mirror constructed is bound to have some uncertainties attached to the physical parameters describing it. For that reason we investigate the effect of an *assymmetric* cavity on the resulting transmission profile. Here we remember that each Fano mirror is described by a set of parameters, λ_0 , λ_1 , t_d , $\gamma\lambda$ and β , which respectively describe the cavity resonance wavelength, guided-mode resonance wavelength, direct transmission, guided-mode resonance linewidth and additional losses of each grating. In order to model only a spectral detuning, we therefore simply change the parameters regarding the cavity and guided-mode resonance wavelengths, λ_0 and λ_1

by an amount corresponding to the detuning we wish to study. For this section the parameters will be given as

$$\begin{aligned} \lambda_0 &= 951\text{nm}, \quad \lambda_1 = 951\text{nm}, \quad t_d = 80\%, \\ \gamma\lambda &= 0.5\text{nm} \quad \text{and} \quad \beta = 10^{-6} \end{aligned} \quad (45)$$

for the unchanged Fano mirror, and

$$\begin{aligned} \lambda'_0 &= \lambda_0 + \Delta, \quad \lambda'_1 = \lambda_1 + \Delta, \quad t'_d = t_d, \\ \gamma\lambda' &= \gamma\lambda \quad \text{and} \quad \beta' = \beta \end{aligned} \quad (46)$$

for the *detuned* Fano mirror. Figure 17 shows the normalized reflectivity and transmission spectra of the Fano mirrors described by the parameters in eqs. (45) and (46).

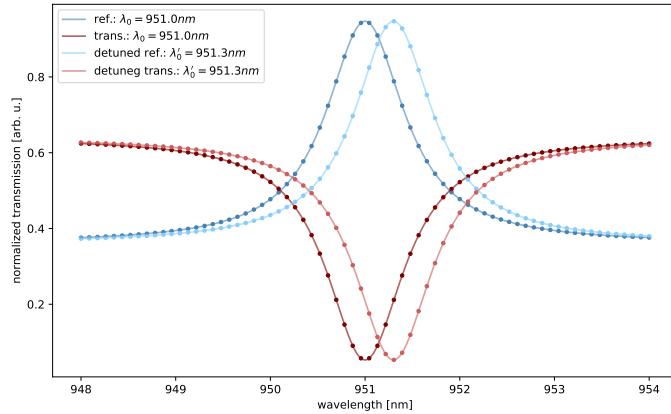


Figure 17: The normalized reflectivity and transmission spectra of the Fano mirrors described in eqs. (45) and (46) for detuning $\Delta = 0.3\text{nm}$.

As has been observed in previous sections, the Fano resonance transmission peak is present at the point where the grating reflectivity $r_g(\lambda)$ is maximized and transmission $t_g(\lambda)$ minimized. However, when $\lambda_0 \neq \lambda'_0$ and $\lambda_1 \neq \lambda'_1$, this is no longer a trivial conclusion to draw. The question of whether the cavity resonance should be tuned to match the guided-mode resonance wavelength of one grating or the other, or maybe somewhere in between does not have an obvious answer. This will be further expanded upon later in section 2.4.6, but in order to move forward with the investigation of the spectral detuning we, for now, accept that the optimal cavity

length, is the one corresponding to a cavity resonance λ_t given, exactly between the two guided-mode resonance wavelengths, as

$$\lambda_t = \frac{\lambda_{0,1} + \lambda'_{0,1}}{2}. \quad (47)$$

The subscript $0,1$ indicate that the two are interchangable, as the two are assumed identical at this time, and t is for *transmission* as this is, later on, to be used experimentally as the *transmission wavelength*.

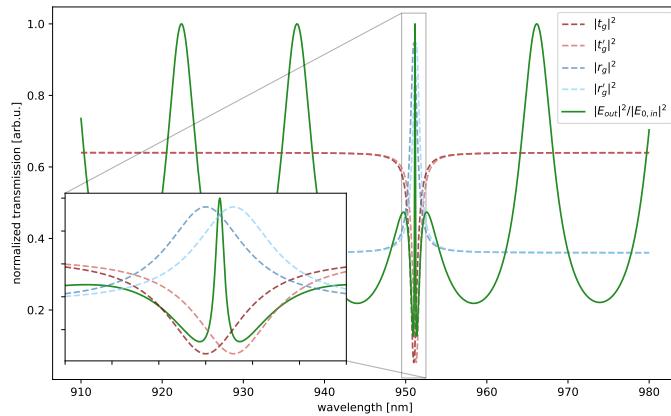


Figure 18: The double Fano transmission spectra of a cavity detuned by $\Delta = 0.3\text{nm}$, as seen in figure 17, together with the reflectivity and transmission spectra of the Fano mirrors used for the simulation.

Figure 18 shows the transmission spectrum of a detuned double Fano cavity with parameters corresponding to the transmission and reflectivity spectra in figure 17, meaning that $\Delta = 0.3\text{nm}$. The transmission wavelength λ_t is chosen such that eq. (47) is fulfilled, and correspondingly it is seen that the transmission peak is placed exactly between the maximum (minimum) reflectivites (transmissions) of the two grating, i.e. between the two guided-mode resonance wavelengths. Furthermore, it can be concluded that the detuning is chosen such that the overlap between the guided-mode resonances is still substantiel enough for them to couple and hence for the the overall Fano resonance to be excited.

While knowing that a detuning of 0.3nm is acceptable in terms of exciting the Fano resonance in the lossless case is a nice result, it does not provide

much in terms of estimating the acceptable detuning for any experimental purposes. In figure 19a the double Fano transmission is shown for increasing detuning Δ and transmission wavelength $\lambda_t = (\lambda_{0,1} + \lambda'_{0,1})/2$.

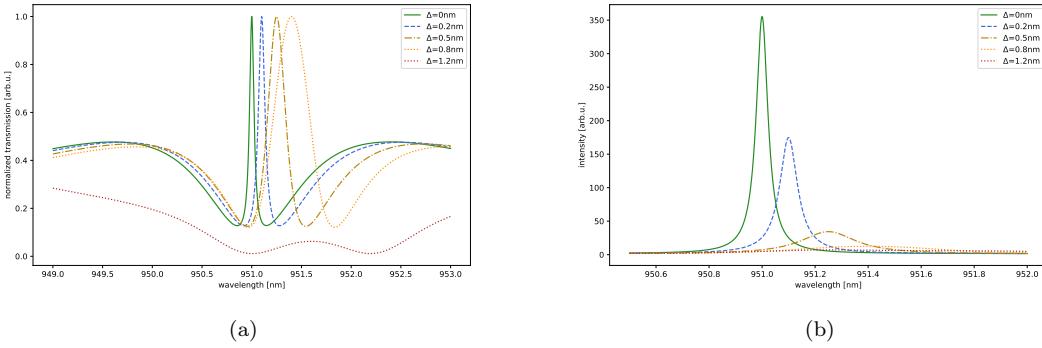


Figure 19: In (a) is seen lossless double Fano transmission spectra for increasing value of the detuning $\Delta = |\lambda_{0,1} - \lambda'_{0,1}|$. It is readily seen that the transmission wavelength $\lambda_t = (\lambda_{0,1} + \lambda'_{0,1})/2$ goes up, and that the linewidth likewise becomes larger with increasing detuning. The spectrum for $\Delta = 1.2\text{nm}$ shows the effect of the detuning becoming too large for the guided-modes of the two Fano mirrors to have substantial overlap and thus to sustain the Fano resonance. (b) shows the intracavity spectra corresponding to the transmission spectra in (a).

It is readily seen that with increasing positive detuning, relative to the resonant wavelength of the unchanged Fano mirror, the peak shifts to higher wavelengths, which in itself is trivial from eq. (47). The linewidth is also seen to increase with the detuning, and the peak eventually collapses when the overlap between the guided-mode resonance profiles of the two Fano mirrors becomes too small to sustain the Fano resonance. Figure 19b shows the intracavity spectra corresponding to the transmission spectra in figure 19a, and provides valuable insight into the mode density inside the cavity for the different values of Δ . It is clearly demonstrated by the two figures that the spectral overlap is a very crucial parameter of the double Fano cavity, and is paramount in describing the cavity's ability to sustain the Fano resonance modes.

2.4.6 Spacial detuning (lossless)

As mentioned above in section 2.4.5, any spectral detuning gives rise to the potential of a spacial detuning as $2l = m\lambda$ (eq. (1)) must be fulfilled for any sustained plane-wave mode inside a normal incident optical cavity. We denote the length corresponding to the resonance wavelength of the

unchanged Fano mirror as simply l , while the one for the detuned Fano mirror will be denoted as l' . Previously we assumed that the optimal length of a detuned double Fano cavity is the one where the so-called transmission wavelength is given as

$$\lambda_t = \frac{\lambda_{0,1} + \lambda'_{0,1}}{2}. \quad (48)$$

And while this does turn out to be the case, we must investigate the resonant length-dependence of the linewidth of the Fano resonance profile in order to arrive at this conclusion indefinitely.

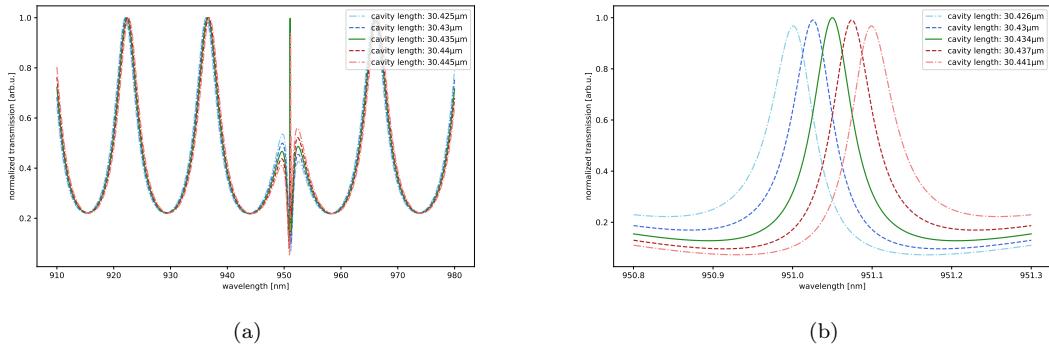


Figure 20: (a) shows lossless long-range transmission spectra of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu m$ with $\Delta = 0.1nm$. (b) shows the same spectra as seen in (a), zoomed in on the range around the transmission wavelength λ_t .

Figure 20 shows double Fano resonance transmission profiles plotted for varying cavity length $l \rightarrow l'$ with detuning $\Delta = 0.1nm$. Looking at figure 20a we see the long range scan of the transmission profiles where there still appears to be a substantial overlap between the guided-modes of the two Fano mirrors. The main differences are seen as changes in the intensity in the immediate off-resonance regime. This is due to the dramatic wavelength (and thus length) dependence in the areas close to the resonance position, which makes even very small changes visible on the spectra.

Figure 20b shows the same spectra, but enhanced around the resonance wavelength, and this too shows that the substantial guided-mode overlap results in little visible change, except for shifting the peak position, when scanning between the resonant cavity lengths.

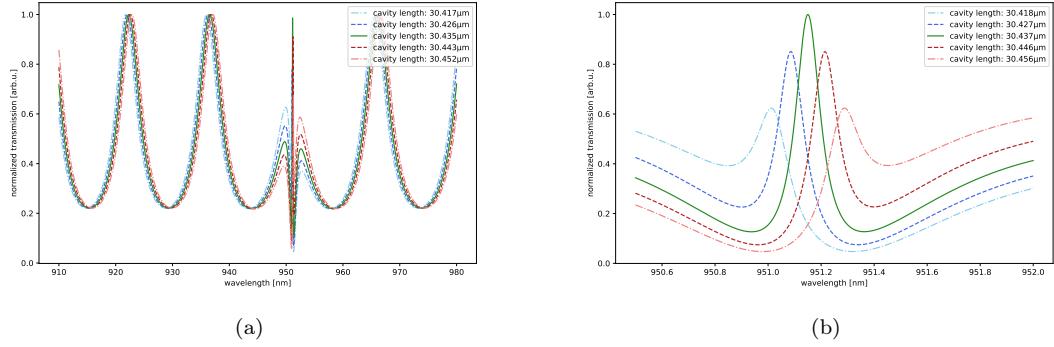


Figure 21: (a) shows lossless long-range transmission spectra of double Fano cavities of lengths $l \rightarrow l' \approx 30\mu m$ with $\Delta = 0.3nm$. (b) shows the same spectra as seen in (a), zoomed in on the range around the transmission wavelength λ_t .

Figure 21 shows similar transmission profiles as in figure 20, only with an increased detuning of $\Delta = 0.3nm$. Looking at the long range scan in figure 21a and comparing with the one in figure 20a, it is seen that the displacement of the off-resonance Fabry-Perot like fringes have increased with the slightly higher detuning. This is not surprising as the Fabry-Perot cavity modes, like any other interference pattern inside a cavity must fulfill the brightness identity $2l = m\lambda$. Furthermore, the immediate off-resonance regime shows a slight increase in the intensity displacement when compared with the less detuned example.

Figure 21b, which shows the spectra enhanced around the resonance position, provides a more detailed image of what happens to the transmission profiles as a function of the cavity length. Here it is seen that the peaks at the edges of the length interval have varied in both position and height, much more dramatically than the ones in figure 20b. In short, the increased detuning has also increased the fractional length-dependence of the double Fano cavity⁹, as the resonance transmission profile seem to change *more* when varying the length of the cavity between the guided-mode resonance lengths, in this case.

Figure 22 shows the effect of a relatively large detuning of $\Delta = 0.8nm$. This is simply included for a visual representation of a detuning which

⁹It must be noted here that the range in which the cavity length is scanned is increased with the detuning. So the length-dependence is fractional in the sense that the step size is defined by the size of the interval.

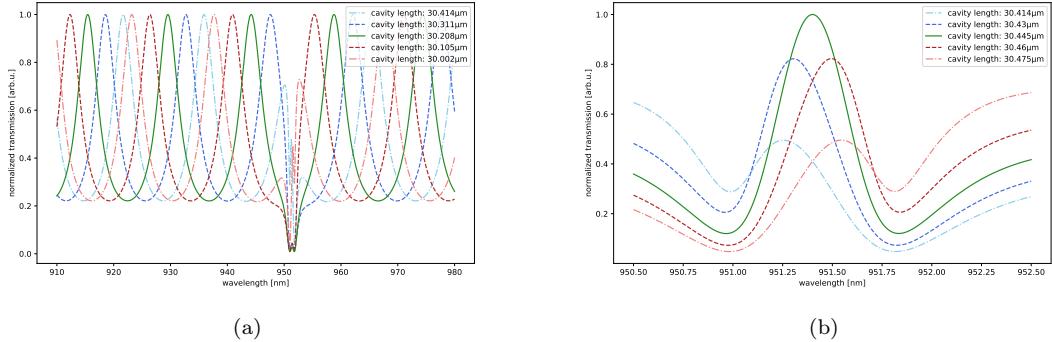


Figure 22: (a) shows lossless long-range transmission spectra of double Fano cavities of lengths $l \rightarrow l' \approx 30 \mu\text{m}$ with $\Delta = 0.8 \text{ nm}$. (b) shows the same spectra as seen in (a), zoomed in on the range around the transmission wavelength λ_t .

is considered "too large" for any practical use. The long range scan in figure 22a shows that the Fabry-Perot like fringes are now too displaced to be compared in terms of their positions, and figure 22b showing the spectra enhanced around the resonance wavelength shows roughly the same trend as in figure 21b, only that this example is greatly broadened in comparison. It is though noted, that while broadened, the Fano resonance mode is sustained even for the relatively large detuning.

In order to get a clear and qualitative picture of the double Fano cavity transmission profile as a function of both cavity length and wavelength, we visualize the varying of both in a heat map and let the color indicate the transmission intensity. This is shown in figure 23a which depicts a lossless cavity of length $l \approx 20 \mu\text{m}$ and detuning $\Delta = 0.3 \text{ nm}$. Here the movement of the resonance peak as a function of the cavity length is clearly seen as a slope of the "line" representing the high intensity region due to the Fano resonance.

Considering only the heat map, it is not easily visible which cavity length is the optimal one, however, the *magenta*, *cyan*, and *limegreen* lines across the heat map indicate slices which are depicted separately in figure 23b. It is seen by analysing the transmission profiles of these cavities of specific lengths, how they vary in both position and linewidth. The resonance wavelength positions are given relative to the cavity lengths chosen. The

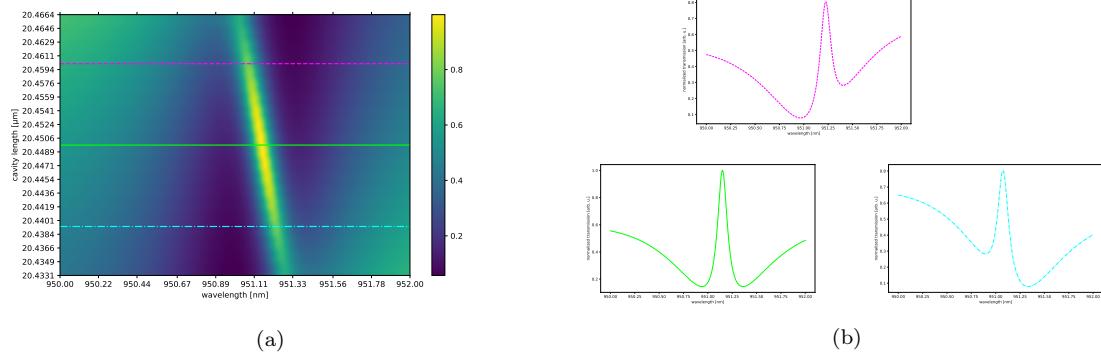


Figure 23: (a) shows a heat map of the lossless double Fano cavity transmission as a function of wavelength and cavity lengths ranging $l \rightarrow l' \approx 20\mu m$ for $\Delta = 0.3nm$. (b) show slices of the heat map for comparing different cavity lengths, and are indicated by their respective color and linetype in (a). The respecitve cavity lengths and linewidths are given as: $l_{magenta} = (0.2l + 0.8l')$, $HWHM_{magenta} = 71.3pm$, $l_{cyan} = (0.8l + 0.2l')$, $HWHM_{cyan} = 71.3pm$ and $l_{lime} = (l + l')/2$, $HWHM_{lime} = 58.6pm$.

lengths and corresponding linewidths are given as

$$\begin{aligned} l_{magenta} &= 0.2l + 0.8l' \rightarrow HWHM_{magenta} = 71.3pm \\ l_{cyan} &= 0.8l + 0.2l' \rightarrow HWHM_{cyan} = 71.3pm \\ l_{lime} &= (l + l')/2 \rightarrow HWHM_{lime} = 58.6pm. \end{aligned} \quad (49)$$

It turns out that the linewidths of the cyan and magenta transmission profiles are identical¹⁰, while the limegreen transmission profile seem to have a narrower profile at resonance.

This trend is further examined in figure 24 where the linewidths of intracavity spectra are shown as a function of the cavity length. The parameters used are the same as in figure 23 and clearly indicates that the optimal cavity length of a detuned double Fano cavity is the one fulfilling

$$\lambda_t = \frac{\lambda_{0,1} + \lambda'_{0,1}}{2}, \quad (50)$$

for the transmission wavelength λ_t , and thus corresponding cavity length l_t .

¹⁰This is only exactly correct because the only varied parameter of the two Fano mirrors are the cavity- and guided-mode-resonant wavelengths λ_0 and λ_1 . In a more realistic case where several parameters vary, the attributes of the transmission profiles are unlikely to act fully symmetrically when varying the length $l \rightarrow l'$.

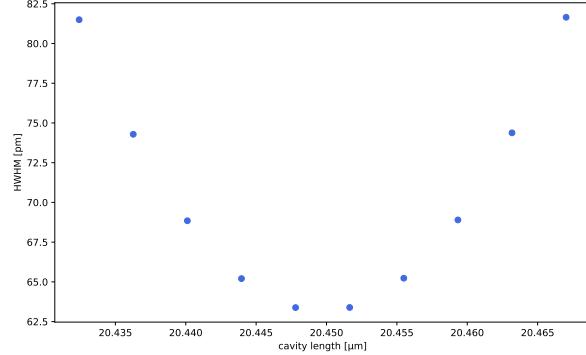


Figure 24: Linewidth as a function of cavity length $l \rightarrow l'$. Each point is found as a fitting parameter from a least squares fit of the intracavity lossless double Fano spectrum for each respective cavity length, to the generalized Fano model in eq. (30).

As a visual and qualitative representation of the effect of increasing the detuning Δ , figure 25 shows heat maps similar to the one in figure 23, but for a range of values for the detuning $\Delta = 0.1\text{nm} \rightarrow \Delta = 0.9\text{nm}$. It is readily seen that the aforementioned slope of the high intensity region, indicating the resonance peak, increases with the detuning. This is a representation of the peak moving to higher wavelengths, both for the optimal transmission wavelength and for the one closer to the detuned Fano mirror guided-mode resonance. The broadening of the peak is also displayed in a way that is, while only qualitative, convincing.

The last image in figure 25, displays the case of $\Delta = 0.9\text{nm}$. Here it is seen that the spectral overlap has become too small to sustain the Fano resonance inside the cavity, and the intensity at the, previously, optimal transmission wavelength is therefore at a minimum. If however, we look at the areas of the heat map corresponding to cavity lengths of exactly l or l' and wavelengths $\lambda_{0,1}$ or $\lambda'_{0,1}$, we see that there is a slight increase in the transmission intensity. This is due to the fact that the cavity, in this position, corresponds to a single Fano cavity with a low finesse, as one of the Fano mirrors are exactly on resonance while the other is so far away that it acts as simply a poor "broadband" mirror.

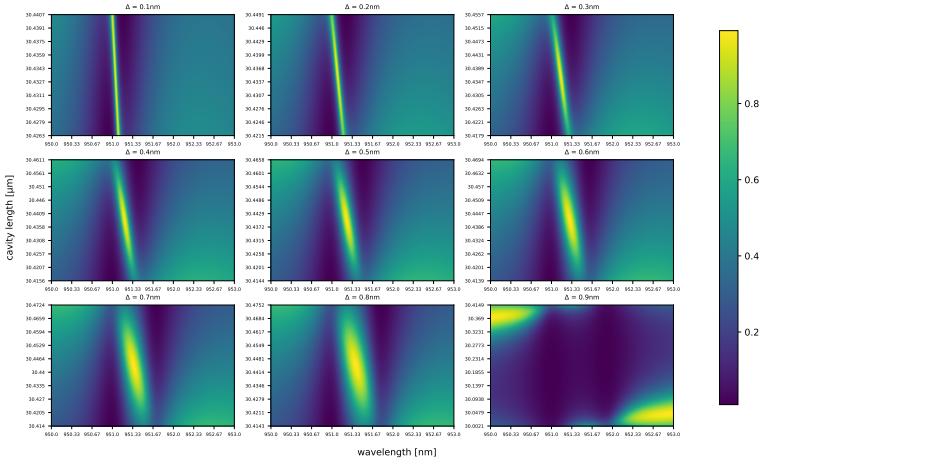


Figure 25: Series of heat maps showing lossless double Fano transmission spectra as a function of the cavity length, for increasing detuning Δ . The movement of the resonant wavelength is shown as the increasing slope of the high intensity region representing the resonance, for increasing detuning Δ . The broadening as a function of detuning is easily seen as it is qualitatively displayed in the series of figures until the spectral overlap makes the cavity unable of sustaining the Fano resonance.

3 Method

3.1 The experimental setup

The experimental setup used to optically characterize the Fano mirrors, single- and double Fano cavities is illustrated in figure 26 where the whole setup is sketched in figure 26a and the specific part of the setup surrounding the cavity, outlined by the dashed line in figure 26a, is subsequently shown in figure 26b.

In order to effectively conduct the experiments in this project, it is imperative to be able to control certain parameters. Each element in the experimental setup is thoroughly considered for each their purpose in this regard, these will be outlined in this section.

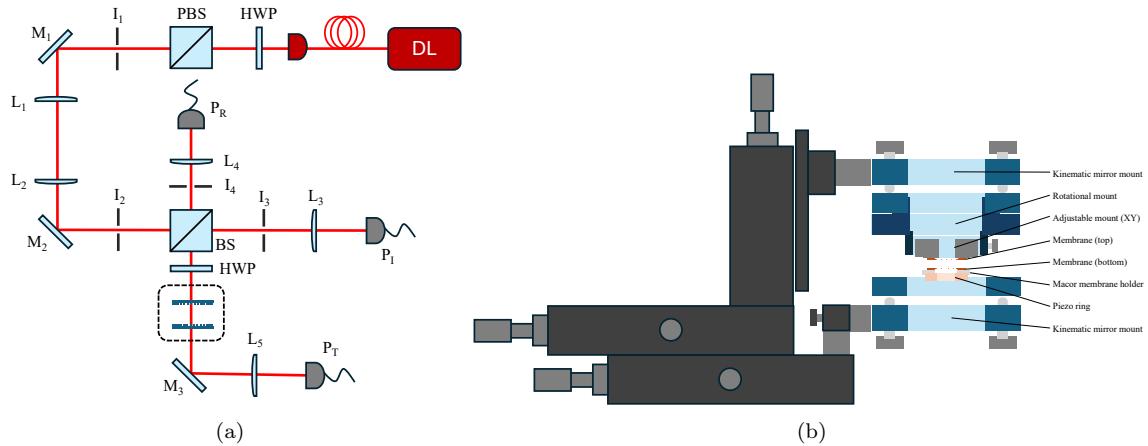


Figure 26: (a) shows schematics of the experimental setup for measuring cavity transmission- and Fano mirrors transmission/reflectivity spectra. The dashed line indicate the location of the cavity for measurements, and the setup is seen in (b). Note, the setup is shown with an optical cavity present, and is thus modified accordingly when prepared for Fano mirror measurements. The components not outlined in this section are regular mirrors labeled M_{1-3} apertures used for alignment labeled I_{1-4} .

3.1.1 Tunable diode laser

As shown in figure 26 the laser source used for the optical characterizations is coupled into the setup through an optical fiber. The laser used is a *Toptica DLC Pro* tunable CW diode laser with a range for the transmission wavelength of $910 - 980\text{nm}$. The optical fiber is a *P3-780PM-FC-10* fiber

from Thorlabs which is a single mode¹¹ polarization-maintaining optical fiber with an effective range of $770 - 1100\text{nm}$. Between the Toptica laser and the incoupling end of the fiber, a $\lambda/2$ -plate (HWP) and *polarizing beam splitter* (PBS) is placed in order to be able to control the incident power of the laser and to only couple linearly polarized light into the setup.

The light being emitted from the optical fiber is sent through another HWP and PBS in order to be able to control the resulting polarization in the setup even more precisely, should there be any discrepancies of the light coupled into the fiber.

3.1.2 $\lambda/2$ - waveplate

A $\lambda/2$ -waveplate, or HWP, is constructed of a so-called bi-refringent material (most commonly crystalline quartz), which means that it has slightly different refractive indices for incident light of different polarization axis'. Generally a HWP will have a *fast-* and *slow axis*, where it is understood that light polarized along the fast axis experiences a lower refractive index (and hence moves faster), than that along the slow axis. In this way the HWP separates the components of unpolarized light that has perpendicular and parallel polarizations with respect to the fast axis.

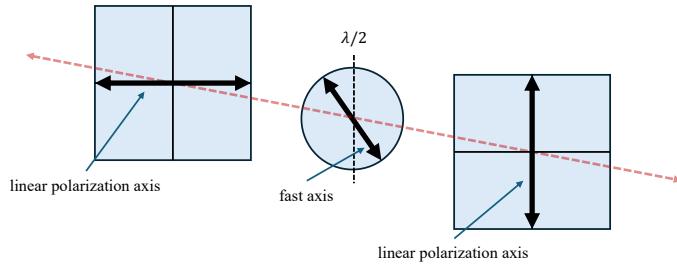


Figure 27: A simple sketch of the effect of a HWP on linearly polarized light.

The effect of the HWP on linearly polarized light, is an effective rotation of the polarization, this is sketched simply in figure 27. It can be shown that the polarization axis is rotated according to the angle between the fast axis of the HWP and the incident polarization axis. A relative angle

¹¹A single mode fiber can only sustain the TEM00 mode, which means that the output is known to be perfectly Gaussian.

θ will thus result in a rotation of 2θ , e.g. if $\theta = 45^\circ$, this will constitute a rotation from completely p-polarized light to completely s-polarized light. This is the specific scenario sketched in figure 27[12]. In this way a rotating HWP can allow one to alter an incident linearly polarized beam to be polarized along any axis, and is thus a necessary component for this particular setup.

3.1.3 Optical telescope

The linearly polarized light transmitted through the PBS passes through plano-convex lenses of positive focal lengths f and f' , L_1 and L_2 , which together makes up an optical telescope used to manipulate the beam waist w_0 incident on the cavity or Fano mirror.

Figure 28 shows schematics of the general way an optical telescope is utilized to manipulate the beam waist of a laser beam.

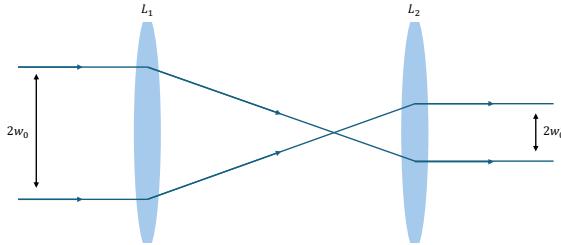


Figure 28

When the incident beam hits L_1 it is focused according to its focal length f , and by inserting another lens L_2 of a relatively longer focal length one can *catch* the beam at the desired beam waist. If the focal length f' of L_2 is sufficiently long, compared to the path of the beam after the lenses, the beam will be approximately collimated and remain at the waist obtained when incident on L_2 .

3.1.4 Transmission, reflection and incident photo-detectors

After passing the optical telescope the beam reaches a simple 50/50 beam splitter (BS) which transmits 50% of the light while reflecting another 50%. The reflected light is then incident on the cavity or Fano mirror (target).

The transmitted light passes through a lens L_3 , which is focused on a photo-detector P_I used for reference measurements and later normalization. Since the tunable laser in nature varies in power with the wavelength, it is necessary to keep track of these fluctuations and correct for them in data analysis.

The transmitted light is sent through, yet another, HWP which in this case is used only to alter the polarization of the light incident on the target. After the beam, or a portion of it, has passed the target it is sent through a lens L_5 focused onto the transmission detector P_T .

The part of the light incident on the target that is *not* transmitted, is reflected back onto the BS which then transmits 50% once again, and thus reflects the other 50%. The transmitted part here is focused through the lens L_4 onto the reflection detector P_R .

3.1.5 The double Fano cavity measurement setup

The cavity measurement setup shown in figure 26b is the one used to measure the transmission spectrum of a double Fano cavity consisting of two Fano mirrors. This part of the setup consists of two sets of standard $PT1 \mu\text{m}$ -stages from Thorlabs combined to provide precise movement of each Fano mirror in the xy-plane.

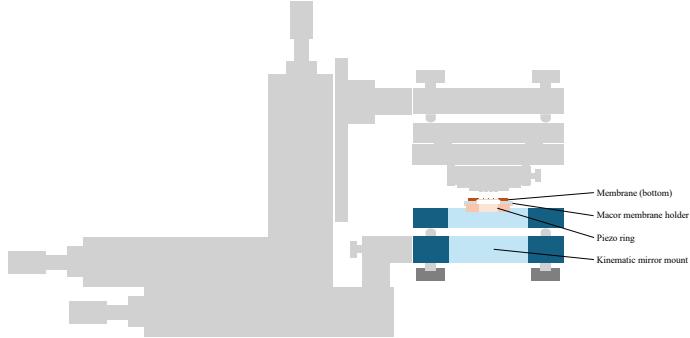


Figure 29: Schematics of the *bottom* part of the double Fano cavity setup.

Examining the structure from the bottom (as it is built), a kinematic mirror mount is attached to the lower set of xy-stages, this is used to control the angular degrees of freedom of the Fano mirror which makes up the bottom of the cavity. On the mirror mount, a piezo ring is firmly attached and connected to a piezo driver and that to a function generator

in order to control the expansion of the piezo ring both by application alternating and fixed current. Lastly, in order to be able to place the Fano mirror or simply membrane on the piezo ring, a ceramic Macor membrane holder is used. The bottom part of the cavity setup is highlighted in figure 29.

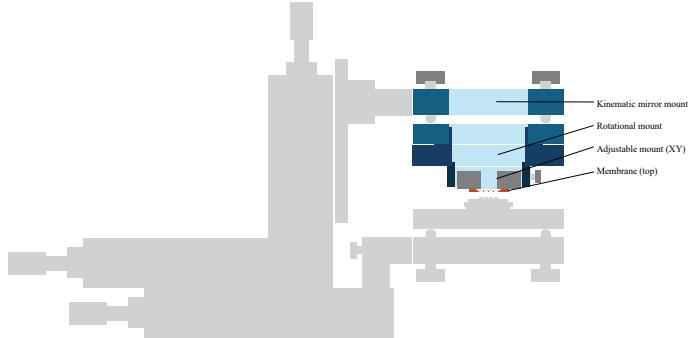


Figure 30: Schematics of the *top* part of the double Fano cavity setup.

The part of the setup built to control the Fano mirror making up the top of the cavity is slightly more complicated, as this is the Fano mirror that is, for practical reasons, aligned last. This means that additional degrees of freedom must be controlled by movement of the Fano mirror itself. The alignment procedure will be explained in detail in sections 3.2 and 3.3.

The top part of the cavity setup is attached to the second set of xy-stages and additionally to an *NFL5DP20* stage from Thorlabs in the z-direction in order to be able to change the length of the cavity. As for the bottom part of the cavity setup, a kinematic mirror mount acts as the base of the construction. This is, once again, to control the angular degrees of freedom of the corresponding Fano mirror. On the mirror mount, a standard rotational mount with a 1 inch inner winding is attached in order to be able to control the rotational degree of freedom of the Fano mirror. An additional xy-adjustable mount is then used in order to effectively place the Fano mirror in the center of the rotational mount to ensure the rotational axis is in the center of the membrane. Finally, the Fano mirror is taped to a costume mount created to fit into the xy-adjustable mount. The top part of the cavity setup is highlighted in figure 30.

What has been outlined here is the setup utilized to optically characterize the double Fano cavity. If one wishes to do so for the single Fano cavity

instead, the setup would be modified such that the top part of the setup, highlighted in figure 30, would only consist of the kinematic mirror mount. Inside the mirror mount would then be placed a broadband mirror. The rotational- and xy-adjustable mounts would here be redundant due to the rotational symmetry of any standard broadband mirror.

3.2 Characterization of sub-wavelength grating

The sub-wavelength gratings, or Fano mirrors, used are commercially available high-quality silicone nitride *SiN* membranes suspended on a *Si* frame, which have been patterned into a grating. Figure 31a shows a packaged membrane from Norcada, which is the company who has provided all gratings/membranes used in this project. Figure 31b is a close-up picture of a bare membrane which have not yet been patterned, and includes a scale to provide perspective regarding the physical dimensions.

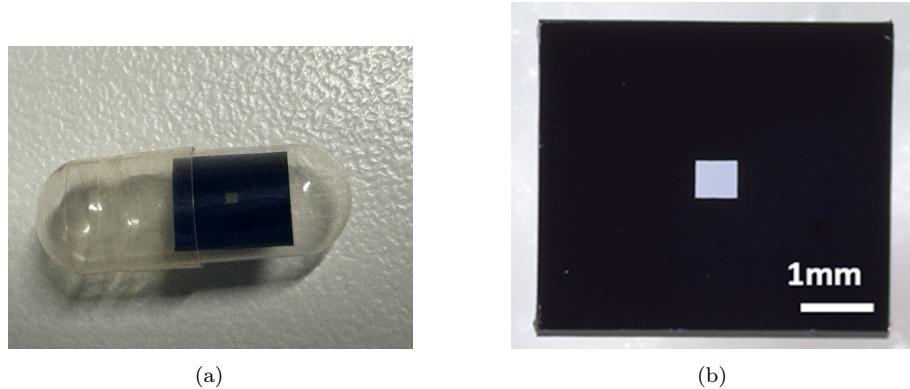


Figure 31: (a) shows an image of a packaged bare membrane, which has not yet been patterned, from Norcada. (b) is a close-up image of a bare membrane like the one in figure 31a.

The patterned area of the Fano mirrors used throughout this project is $400 \times 400\mu m$, and this along with detailed dimensions of the whole membrane is sketched in figure 32.

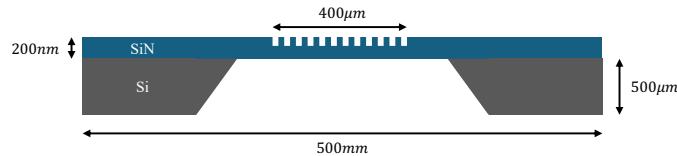


Figure 32: A simple sketch of the profile of a patterned membrane, i.e. a Fano mirror. It is seen that the width of the patterned area is $400\mu m$.

3.2.1 The alignment procedure

In order to align the Fano mirror for its optical characterization only, the setup utilized is a simplified one, compared with the double Fano cavity setup shown in figure 26b. This setup corresponds to one where only the *bottom* part is included, i.e. corresponding to the one highlighted in figure 29. It must here be noted, that if the purpose of the Fano mirror alignment is to create a cavity, the top part shown in figure 30 must of course be utilized as well.

The goal of the alignment procedure is to ensure that the coupling between the guided-mode and the mode of the laser is maximized, this is outlined in section 2.2. The transmission of the Fano mirror is measured during alignment with a *PM100D* digital handheld optical power meter from Thorlabs, and progression of the alignment procedure is understood as minimizing the transmission. The behavior of the Fano mirror transmission, when perfectly aligned, can be seen in figure 7, where it has been simulated for a Fano mirror of arbitrary parameters. When aligned, the guided-mode resonance wavelength can thus be found. This is practically done by scanning the expected wavelength range using the Toptica laser and recording the wavelength at which the transmission is minimized.

The three degrees of freedom, in which the alignment of the Fano mirror is optimized, are the spacial coordinates (x, y), the rotational degree of freedom, and the two angular degrees of freedom. It is assumed at this point that the waist of the beam incident on the Fano mirror is optimal, this will be expanded upon in section 3.2.2.

The general order in which the alignment is done is given as:

1. xy-plane alignment.
2. Alignment of the angular degrees of freedom (adjusting the kinematic mirror mount).
3. Rotational alignment (adjusting the HWP placed prior to the cavity setup).

xy-plane alignment

The spacial alignment of the Fano mirror in the xy-plane is simply done by movement of the xy-stages (linear μm -stages from Thorlabs) shown in figure 26b. Here it is crucial to know how the transmission level behaves qualitatively as a function of the xy-positions when the wavelength of the laser is off-resonance with respect to the guided-mode of the Fano mirror. Due to the effective thickness of the membrane in patterned and un-patterned areas being different, the transmission properties will vary slightly. This can be utilized in order to align these parameters effectively. Assuming one of the coordinates, x or y , is optimal, and one scans across the other as indicated in figure 33, it will be evident that the transmission is constant when the entire beam is incident on the bare membrane, while it will experience a slight decrease in the patterned area, simply due to the lower effective thickness of the structure in this area. For this reason the optimal alignment in this direction is found as the minimum transmission as a function hereof.

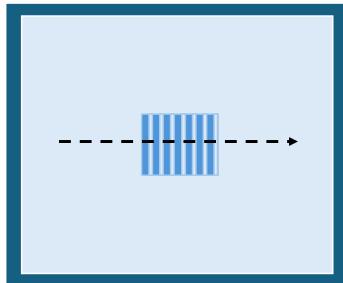


Figure 33: Simple sketch of a patterned membrane, i.e. a Fano mirror. The darker area surrounding the *SiN* membrane is the *Si* frame and the dark blue lines in the center indicate the $400 \times 400 \mu m$ patterned area. The black dotted arrow is indicative of scanning across the membrane, by moving it relative to the beam, using one of the linear stages.

Angular alignment

Adjusting the angular alignment, which is done by adjusting the screws on the kinematic mirror mount holding the piezo ring and the Macor membrane holder, is crucial for the evaluation of the guided-mode resonance wavelength. As reported by Parthenopoulos et al. in [7] the resonance wavelength of the guided-mode depends strongly on the angle of incidence.

In figure 26a the components I_{1-4} indicate apertures (iris') used for the

angular alignment. Specifically I_1 and I_2 are used to track the back-reflection of the beam incident on the Fano mirror to ensure a high degree of overlap as possible. A complete overlap indicates that the incident beam is normal to the Fano mirror surface.

Rotational alignment

The Fano mirrors provided by Norcada are of so-called TM polarization, meaning that the magnetic field is parallel to the grating lines, and is thus perpendicular to the electric field. Since the polarization is defined along the axis of the electric field, this means that the polarization axis must also be perpendicular to the grating lines, as this ensures the interaction is maximized[13].

For this reason the rotational alignment is vital to achieving the correct minimum transmission wavelength of the Fano mirror.

Since no rotational degree of freedom is appointed the Fano mirror itself, and that this would greatly complicate the alignment procedure, the rotational alignment is achieved by changing the polarization of the incident light. This done by adjusting the rotating HWP placed immediately before the light hits the Fano mirror surface and is possible due to its effect on linearly polarized light illustrated in figure 27.

3.2.2 Adjusting and measuring the beam waist

In section 3.2.1 we assumed an optimal beam waist as the alignment procedure was outlined. This was due to the fact that the beam waist optimization is unnecessary unless the entire setup is rebuilt or if the size of the Fano mirror is changed. Since the patterned area of the Fano mirrors characterized in this project was only $400 \times 400\mu m$, the beam waist optimization was only done once.

As is evident from figure 28 how an optical telescope was utilized to tune the waist size w_0 of the collimated laser beam. In order to determine the optimal value of w_0 the second lens L_2 was moved back and forth without diverging from the position where the optical axis passed through the center of the lens. The minimum transmission was recorded for each

iteration, and the transmission would converge towards its minimum value (or at least the lowest achievable value). When the minimum transmission as a function of w_0 was found, this was measured using the *razor blade method*. The razor blade method is a simple tool to, fairly accurately, measure the beam waist of a Gaussian laser beam by *cutting* the beam gradually with a blade. The distance the blade moves is recorded along with the transmitted intensity, and the two are thus directly correlated[14].

An arbitrary Gaussian distribution, corresponding to a laser beam in the TEM00 mode, is shown on the left in figure 34, while the right plot shows the integral of the distribution as a function of the razor blade position, which corresponds to an error function.

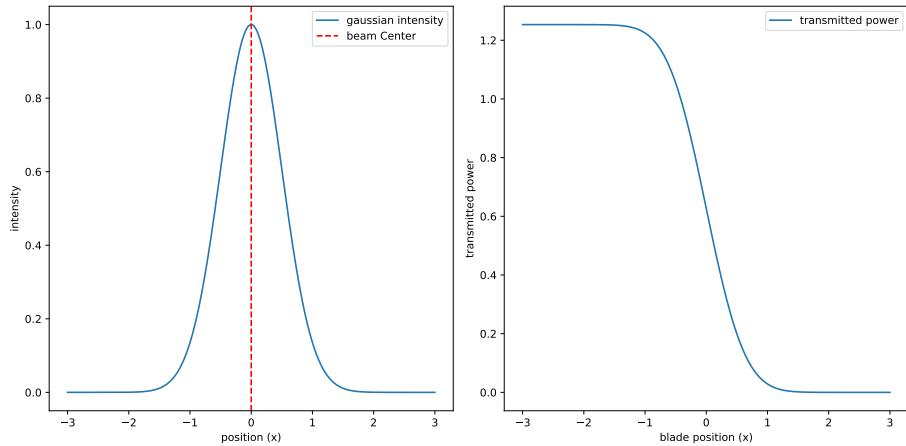


Figure 34: The left plot shows an arbitrary Gaussian distribution, and the red dashed line indicate the placement of the maximum value. The right plot shows the transmitted intensity as a function of the position of the razor blade, as the razor blade method is simulated, i.e. the beam is gradually covered with the blade.

In practice the blade position was recorded by counting the turns made on a linear μm -stage with the razor blade attached, and the transmitted intensity was recorded live by a power meter from Thorlabs. The power of the laser beam without being *cut* by the blade was tuned to $\sim 1\text{mW}$ such that the percentile change was intuitively seen. The linewidth of the Gaussian beam is then approximately given as the distance between the two cut-off points where the transmitted intensity is given as 16% and

84%¹². The optimal value for the linewidth was found to be

$$\delta\lambda_{400\mu m} \approx 160\mu m. \quad (51)$$

3.2.3 Obtaining normalized transmission/reflection spectra

When the Fano mirror and the surrounding setup is completely aligned, it is time for the optical characterization. In order to do so, a series of measurements is necessary to obtain normalized transmission and reflection spectra which rightfully represent the optical properties of the Fano mirror. The structure of such measurements is given as follows:

1. The transmission and reflection intensities as a function of wavelength are measured using the reflection detector P_R and the transmission detector P_T , while the light incident on the Fano mirror is measured using detector P_I . The positions of all detectors are shown in figure 26a.
2. The Fano mirror is removed and identical measurements are done for an "empty" setup. The purpose of this measurement is to measure the signal when *all* the incident light is transmitted, and *no* light is reflected. This data is used to normalize the transmission intensities obtained in the first measurement.
3. On the position of the Fano mirror, a highly reflective (HR) mirror is now placed, and the measurement is repeated. This is corresponding to *all* the incident light being reflected, and *no* light being transmitted. This data is used to normalize the reflection intensities obtained in the first measurement.
4. Lastly, a measurement of the background signal is taken, by simply blocking the laser before it enters the setup and repeating the measurements.

The normalized transmission and reflectivity values are now given as

$$blahblah \quad (52)$$

¹²In reality this interval corresponds to the width according to two standard deviations 2σ , which is only approximately equal to the linewidth.

3.3 Cavity measurements

3.3.1 Determining the cavity length from the FSR

Figures:

- Linewidth as a function of "time" - to see the reduction of the linewidth as the piezo reaches an equilibrium where any time-dependent drift is reduced.

3.3.2 Normalization

3.3.3 Single fano cavity characterization

3.3.4 Aligning the cavity

Incluce:

- Scan for optimal cavity length
- Measure the spectral linewidth
-

3.3.5 Double fano cavity characterization

3.3.6 Off-resonance Fabry-Perot cavity (alignment technique)

3.3.7 Centering of the top grating (pinhole method)

3.4 Parallelism study (deviation from normal incident)

4 Simulations

4.1 The single Fano cavity

4.2 The double Fano cavity

Figures:

- Simulated spectra of M3 and M5.
- Simulated length scans of M3 and M5.
- M3/M5 cavity trans. spectra (on resonance + full range)
 - for lengths: $l_{M3} \rightarrow l_{M5}$
 - for length: $l = 1/2 \cdot (l_{M3} + l_{M5})$
- Optimal result comparison with single fano/broadband cavities of similar losses.
- Optimal result comparison with the ideal case from prev. section.
- Simulated linewidth as a function of cavity length (include the same for broadband and single fano cavities).

5 Experimental results

5.1 The single Fano cavity

Figures:

- Single fano cavity transmission as a function of wavelength.
- Short scan of the single fano cavity transmission, with found linewidth.
- Long scan Fabry-Perot fringes for determining FSR -> cavity length.
- linewidth as a function of cavity length (compare with broadband cavity).

5.2 The double Fano cavity

5.2.1 Realizing the double fano model

Figures:

- Fit of the double fano model (long + short cavity)

5.2.2 Double fano off-resonance Fabry-Perot cavity

Figures:

- Off-resonance double fano transmission as a function of wavelength (show that the off resonance transmission goes close to 100 percent for a well-aligned cavity).

5.2.3 The double fano linewidth

Figures:

- "Semi-short" scan data, fit to the double fano transmission model.
- Short scan data, fit to the Fano function (for measuring linewidth).
- Linewidth as a function of cavity length (compare double fano, single fano and broadband cavitites).

6 Discussion

- 6.1 Optimal configuration for double fano cavity - spacial limitation for the cavity length
- 6.2 Spacial drift of the piezo ring
- 6.3 Noise reduction - coupled/uncoupled mechanical/acoustic vibration (the plexi-glass box)
- 6.4 Broadening sources (especially for long cavities)
- 6.5 Improvements to the setup

7 Appendix

- All peaks and fits used for figure 10a.
- All peaks and fits used for figure 14a.
- The derivation of equation 30
- The derivation of equation 31
- Same derivations for the double fano cavity.

8 References

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