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Notes:

- add a section about how to estimate the phase shift due to the thickness of the membrane, i.e. by calculating the tranfer matrix product(this can be found in previous work of the group). - add a section about how to estimate how parallel the two membranes are (this can be found in previous work of the group).

2 An Optical Cavity

An optical cavity is generally comprised of two highly reflective optical resonators. It is used for trapping any amount of light for an extended period of time, thus increasing the effective cross section of a given interaction between some object and the field inside the cavity.

Another attribute of the optical cavity is that it gives rise to so-called eigenstates related to the length of the cavity[1]. As the cavity does not expand infinitely, the allowed modes inside the cavity are limited to ones which fulfill the identity [2]

$$2d\cos\theta = m\lambda. \tag{1}$$

Here d is the length of the cavity, θ is the incidence angle of the light coupling into the cavity, λ is the wavelength of the light and $m = 1, 2, 3, ...$ is a positive integer describing the order of the mode. For the rest of this thesis it will be assumed that the laser couples into the cavity at normal incidence, meaning that $\theta = 0°$ and thus $\cos\theta = 1$.

2.1 The Fundamental Mode: A Gaussian Beam in the Large Waist Approximation

In order to describe the allowed modes within an optical cavity, it is first assumed that a single-mode field is linearly polarized. We then consider solutions to the wave equation

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}, \tag{2}$$

given as

$$\vec{E} = E_0(x, y, z)\vec{\epsilon}e^{ikz}. \tag{3}$$

Here $E_0(x, y, z)$ describes the electric field amplitude, $\vec{\epsilon}$ is denoted the polarization vector and $k = 2\pi/\lambda$ is the angular wave number of the field propagating along the z-axis. It is assumed that the electric field has a Gaussian transverse distribution[1].

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[1]When this is the case, the laser is said to operate in the lowest possible mode denoted TEM_{00}. This implies the assumption of ideal lasing conditions.

This is almost the simplest description of the propagating field, however, as the spacial dependence of the field amplitude still might cause problems, we consider the range in which this can be neglected.

It can be shown from the derivation of the Gaussian distribution that the waist of the beam $w(z)$, which depends on the spacial coordinate in the direction of propagation, can be described as [1]

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \tag{4}$$

where z is the distance from focus, w_0 is the beam waist at focus and z_R is the so-called *Rayleigh range*. The Rayleigh range describes the range in which the beam diverges slowly, whereas after this has been surpassed, the beam will begin to diverge more rapidly. By quick inspection of eq. (4) it is seen that

$$w(z) = \begin{cases} w_0, & \text{for } z = 0 \\ w_0\sqrt{2}, & \text{for } z = z_R, \end{cases} \tag{5}$$

which shows that the beam waist diverges no more than by a factor of $\sqrt{2}$ from the optimal case, for $0 \leq z \leq z_R$.

The Rayleigh range can be written as [2]

$$z_R = \frac{\pi w_0^2}{\lambda}, \tag{6}$$

which shows that a large beam waist will result in a long Rayleigh range. As an example, consider a beam of waist $w_0 = 200\mu m$ and wavelength $\lambda = 950nm$. This would result in a Rayleigh range of $z_R = 13.23cm$.

Finally we can conclude, that for any optical cavity significantly shorter than the Rayleigh range of the incident beam, the spacial dependence of the field amplitude inside the cavity is negligible, and the fundamental mode can be described simply by a linearly polarized plane wave

$$\vec{E} = E_0 \vec{\epsilon} e^{ikz}. \tag{7}$$

This is often referred to as the *large waist approximation* of a Gaussian beam, due to the dependence on w_0 of eq. (6).

2.2 The Fabry-Perot Interferometer

3 The Fano mirror: a sub wavelength grating

3.1 Geometric optical analysis

Considering an ideal grating with period d in the sub-wavelength regime, it can be shown that only a single mode of reflection/transmission is supported.

Any grating, of arbitrary dimensions, must comply with the very general *grating equation*[2] given as

$$\sin \theta_m = m \frac{\lambda}{d}, \tag{8}$$

for the special case of a linearly polarized plane wave incident on a grating placed normal to the direction of propagation. Now, inserting the sub-wavelength condition $d \ll \lambda$, it is easily seen that the right side of the equation blows up for any order of reflection $|m| > 0$, effectively showing that this is the aforementioned single supported mode in this regime. Furthermore, it can be equally easily seen that the propagation direction of the 0'th order mode is the same as the incident beam, i.e. normal to the grating.

3.2 Reflection/transmission spectra and line shape analysis

3.2.1 Lossless grating

We wish to analytically describe the wavelength-dependent spectra for the transmission and reflectivity of an infinite sub-wavelength grating. By first considering the case where absorption and thermal coupling effects are neglected, i.e. a lossless grating, we can assume conservation of energy and thereby the relations

$$|r_g|^2 + |t_g|^2 = 1 \quad \text{and} \quad |r_d|^2 + |t_d|^2 = 1, \tag{9}$$

where the subscripts g and d indicate the *grating* and *direct* transmissions and reflectivities, respectively. It is implied that the direct coefficients are constants and describe the transmission and reflectivity when the incident

wavelength is significantly detuned from any guided-mode resonance of the grating. Furthermore, it is also implied that the grating coefficients are functions of the incident wavelength.

We now assume a normal incident beam on the grating as a linearly polarized monochromatic plane wave, with a wavelength close to a guided-mode resonance of the grating. In order to describe the coefficients r_g and t_g we follow the formalism presented by Fan and Joannopoulos [3] and consider the likely paths of the incident light through the grating. It is quite intuitive to consider the case where the light is simply transmitted, and this shall be our first case hereafter denoted the *direct pathway*. Another case one might consider is the one where the incident light excites the guided-mode resonance in the grating, thus causing interference. This case is denoted the *indirect pathway* and decays more slowly than it's direct counterpart.

The interference caused when the guided mode is excited is often referred to as *Fano resonances*, due to its physical similarities to the description of interference between a discrete autoionized state and a bound continuum state first reported by Fano [4]. The cross section of inelastic scattering, when measured as a function of energy, showed characteristic asymmetric peaks. These were described as the aforementioned interference pattern between *direct* (the discrete state) and *indirect* (the continuum state) pathways.

By generalizing the model of Fan and Joannopoulos [3] we describe the transmission and recletivity coefficent amplitudes as

$$r_g = r_d + \frac{a}{k - k_1 + i\gamma} \quad \text{and} \quad t_g = t_d + \frac{b}{k - k_1 + i\gamma}, \qquad (10)$$

where $k = 2\pi/\lambda$ is the incident wave number, $k_1 = 2\pi/\lambda_1$ is the wave number according to the guided-mode resonance and γ is the HWHM (half width at half maximum) of the guided-mode resonance. Complex coefficients a and b describe the interference between the directly transmitted or reflected waves and the guided mode of the grating.

Note that in eq. (10) the right side of the expression for each coefficient corresponds to the continuum state i.e. the indirect pathway, while

the direct transmission and reflection coefficients take the role of the autoionized discrete state, i.e. the direct pathway[2] [4]

As we are dealing with an ideal, lossless, grating, we assume coefficients a and b to be equal, meaning that we specifically assume vertical symmetry throughout the grating. By considering eq. (9) this in turn leads to

$$a = b = -i\gamma(t_d + r_d), \tag{11}$$

which further yields an expression for the grating transmission amplitude coefficient on the form

$$t_g = t_d \frac{k - k_0}{k - k_1 + i\gamma}. \tag{12}$$

Here, the newly introduced $k_0 = 2\pi/\lambda_0$ is the zero-transmission/unity-reflectivity wave number.

To generalize eq. (12) to include non-unity reflectivity and non-zero transmission, we allow for $a \neq b$ meaning that the case of vertical asymmetry is included in the model. By assuming $r_d, t_d \in \mathbb{R}$, eq. (9) leads to the coupled differential equations

$$\begin{aligned} t_d x_a + r_d x_b &= 0, \quad \text{and} \\ x_a^2 + y_a^2 + x_b^2 + y_b^2 + 2t_d\gamma y_a + 2r_d\gamma y_b &= 0, \end{aligned} \tag{13}$$

where $\{x, y\}_{a,b}$ respectively denotes the real and imaginary parts of the coefficients a and b. Solving eqs. (13) leads to the correct complex reflectivity coefficients and the expression for the transmission coefficient amplitudes now reads

$$t_g = t_d \frac{k - k_0 + i\beta}{k - k_1 + i\gamma}, \tag{14}$$

where k_0 and β are defined from the expression for a found by solving eqs. (13), given as

$$a = t_d(k_1 - k_0 - i\gamma + i\beta). \tag{15}$$

Finally, this allows for non-zero transmission and non-unity reflectivity at wave number k_0.

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[2]The general eigenvector of a state comprised of a super-position between a discrete state and a continuum, i.e. a state vector corresponding to a Fano resonance, is given as $\Psi_E = a\phi + \int dE' b_{E'}\psi_{E'}$, given in eq. (2) in ref. [4], where a and $b_{E'}$ describes the probability of either pathway.

3.2.2 Lossy grating

In order to modify the above model such that losses, e.g. due to absorption or thermal coupling effects, are accounted for, we add a resonant loss term to the energy conservation relation in eq. (9). For this we introduce the resonant loss level L, which must be known in order to accurately calculate the complex reflectivity coefficients. The energy conservation relation is modified such that

$$|t_g|^2 + |r_g|^2 + \frac{c^2}{(k - k_1)^2 + \gamma^2} = 1, \tag{16}$$

where the coefficient $c^2 = L((k - k_1)^2 + \gamma^2)$ includes the resonant loss term L. A new set of coupled differential equations are found, using eq. (16), given as

$$\begin{aligned} t_d x_a + r_d x_b &= 0, \quad \text{and} \\ x_a^2 + y_a^2 + x_b^2 + y_b^2 + c^2 + 2t_d\gamma y_a + 2r_d\gamma y_b &= 0. \end{aligned} \tag{17}$$

It is easily identified that eq. (13) and eq. (17) differ only by the addition of coefficient c^2, and thereby the losses. Solving eq. (17) leads to the correct complex reflectivity coefficients, except that they now account for any losses associated with the grating.

In conclusion, the complete grating model consists of an expression for the transmission coefficients and a set of coupled differential equations for the reflection coefficients, shown in eq. (14) and eq. (17), respectively. The model on the form used for this project and subsequent thesis is derived in previous work by A. Darki et al. [5] and more recently T. Mitra et al. [6].

9 References

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