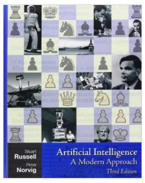
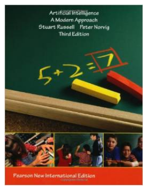
Propositional and Predicate Logics

Purpose: Gain experience with propositional and prepositional (first-order) logic by solving many small problems.

Note: The textbook, *Artificial Intelligence: A Modern Approach, 3rd edition*, exists in two versions:



"Blue version"



"Green version"

All references to chapters and figures in the textbook are given as 1.2.3 / 2.3.4, for the blue and the green version respectively.

1 Models and Entailment in Propositional Logic

1. For each statement below, determine whether the statement is true or false by building the complete model table.

   (a) $A \land \neg B \models A \lor B$

   (b) $A \lor B \models A \land \neg B$

   (c) $A \Leftrightarrow B \models A \Rightarrow B$

   (d) $(A \Leftrightarrow B) \Leftrightarrow C \models A \lor \neg B \lor \neg C$

   (e) $(\neg A \land B) \land (A \Rightarrow B)$ is satisfiable

   (f) $(\neg A \land B) \land (A \Leftrightarrow B)$ is satisfiable

2. Consider a logical knowledge base with 100 variables, $A_1, A_2, \ldots, A_{100}$. This will have $Q = 2^{100}$ possible models. For each logical sentence below, give the number of models that satisfy it.

   Feel free to express your answer as a fraction of Q (without writing out the whole number $1267650600228229401496703205376 = 2^{100}$) or to use other symbols to represent large numbers.

   Example: The logical sentence A_1 will be satisfied by $\frac{1}{2}Q = \frac{1}{2}2^{100} = 2^{99}$ models.

   (a) $A_{31} \land \neg A_{76}$

   (b) $A_{44} \land A_{49} \land A_{78}$

   (c) $A_{44} \lor A_{49} \lor A_{78}$

   (d) $A_{70} \Rightarrow \neg A_{92}$

   (e) $(A_7 \Leftrightarrow A_{72}) \land (A_{83} \Leftrightarrow A_{84})$

   (f) $\neg A_9 \land \neg A_{19} \land A_{37} \land A_{50} \land A_{68} \land A_{73} \land A_{79} \land A_{81}$

3. Consider the Wumpus world in Figure 7.2 / 6.2 . Suppose the agent starts in room in the *bottom right corner*, i.e. $[4, 1]$. The agent then moves north/upward twice, visiting the rooms $[4, 2]$ and $[4, 3]$. The agent perceives a breeze in $[4, 1]$ and $[4, 3]$, but not in $[4, 2]$. From these percepts, we are interested in determining the possible configuration of pits in the adjacent rooms, i.e. $[3, 1]$, $[3, 2]$, $[3, 3]$ and $[4, 4]$.

   Build the full model table of all possible worlds by constructing a truth table with the variables $P_{3,1}$, $P_{3,2}$, $P_{3,3}$ and $P_{4,4}$, each of which specifies whether there is a pit in a particular room. Mark the worlds in which the knowledge base is true, i.e. the pit configurations that are consistent with the perceived breezes. Additionally, mark the worlds in which each of the following sentences is true:

   α_1 = "There is a pit in $[3, 1]$".
   α_2 = "There is a pit in $[3, 3]$".
   α_3 = "There is a pit in $[3, 3]$ or $[4, 4]$".

   Based on the truth table, which of the sentences α_1, α_2 and α_3 can you conclude are entailed by the knowledge base? I.e., specify which of the following are true: $KB \models \alpha_1$, $KB \models \alpha_2$ and $KB \models \alpha_3$.

2 Resolution in Propositional Logic

1. Convert each of the following sentences to Conjunctive Normal Form (CNF).
   (a) $\neg A \vee (B \wedge C)$
   (b) $\neg(A \Rightarrow B) \wedge \neg(C \Rightarrow D)$
   (c) $\neg(A \Rightarrow B) \vee \neg(C \Rightarrow D)$
   (d) $(A \Rightarrow B) \Leftrightarrow C$
2. Consider the following Knowledge Base (KB):
   - $(X \wedge \neg Y) \Rightarrow \neg B$
   - $\neg X \Rightarrow C$
   - $B \wedge \neg Y$
   - $A \Rightarrow \neg C$

   Use resolution to show that $KB \models \neg A$
3. Do exercise 7.17 / 6.18 from the textboox ("Consider the following sentence. . . "), but with the following the sentence instead of the one in the textbook:

   $((Food \vee Drinks) \Rightarrow Party) \Rightarrow (\neg Party \Rightarrow \neg Food)$

3 Representations in First-Order Logic

1. Consider a first-order logical knowledge base that describes worlds containing movies, actors, directors and characters. The vocabulary contains the following symbols:
   - PlayedInMovie(a,m): predicate. Actor/person a played in the movie m
   - PlayedCharacter(a,c): predicate. Actor/person a played character c
   - CharacterInMovie(c,m): predicate. Character c is in the movie m.
   - Directed(p,m): person p directed movie m.

- Constants related to the name of the movie, person or character with obvious meaning (to simplify you may use the surname or abbreviation).

Express the following statements in First-Order Logic:

(a) The character "Batman" was played by Christian Bale, George Clooney and Val Kilmer.

(b) Heath Ledger and Christian Bale did not play the same characters.

(c) Christian Bale played in all "Batman" movies directed by Christopher Nolan (*tip*: note that in this case Batman is a character of the movie, not the name of the movie).

(d) "The Joker" and "Batman" are characters that appear together in some movies.

(e) Kevin Costner directed and starred in the same movie.

(f) George Clooney and Tarantino never played in the same movie and Tarantino never directed a film that George Clooney played.

(g) Uma Thurman played a character in *some* movies directed by Tarantino.

2. Arithmetic assertions can be written using FOL. Use the predicates ($<, \leq, \neq, =$), usual arithmetic operations as function symbol (+,-,x,/), biconditionals to create new predicates, and integer number constants to express the following statements in FOL:

   (a) An integer number x is divisible by y if there is some integer z less than x such that $x = z \times y$ (in other words, define the predicate $Divisible(x, y)$).

   (b) A number is even if and only if it is divisible by 2 (define the predicate $Even(x)$).

   (c) An odd number is not divisible by 2 (define the predicate $Odd(x)$).

   (d) The result of summing an even number with 1 is an odd number (define the predicate $Odd(x)$).

   (e) A prime number is divisible only by itself.

3. Translate into first-order logic the sentence "Everyone's DNA is unique and is derived from their parents' DNA." You must specify the precise intended meaning of your vocabulary terms. (*Hint:* Do not use the predicate $Unique(x)$, since uniqueness is not really a propery of an object in itself!)

4 Resolution in First-Order Logic

1. Find the unifier (θ) – if possible – for each pair of atomic sentences. $Philosopher(x)$, $StudentOf(y, x)$, $Write(x, z)$, $Read(y, z)$ and $Book(z)$ are predicates, while $TeacherOf(y)$ is a function that maps a philosophy student to its teacher name and $Author(z)$ maps a book to its author.

   (a) Philosopher(x) ... Philosopher(Plato)   Answer: $\theta = \{x/Plato\}$

   (b) Write(Plato, TheRepublic) ... Write(Plato, y)

   (c) Read(x, Metaphysics) ... Read(Peter, y)

   (d) Write(x, Fear And Trembling) ... Write(Kierkegaard, x)

   (e) Write(Kant,CritiqueOfPureReason) ... Write(Author(y),y)

2. Using the same predicates of the previous question perform skolemization with the following expressions:

   (a) ∃x∃y: Philosopher(x) ∧ StudentOf(y,x)

(b) $\forall$ y,x: Philosopher(x) $\wedge$ StudentOf(y,x) $\rightarrow$ [$\exists$z: Book(z) $\wedge$ Write(x,z) $\wedge$ Read(y,z)]

3. Use resolution to prove SuperActor(Tarantino) given the information below. You must first convert each sentence into CNF. Feel free to show only the applications of the resolution rule that lead to the desired conclusion. For each application of the resolution rule, show the unification bindings, θ. We are using in this case the same predicates of Exercise 3.1 (movies, actors, etc).

   - $\forall$x: SuperActor(x) $\leftrightarrow$ [$\exists$ m: PlayedInMovie(x,m) $\wedge$ Directed(x,m)]

   - $\forall$ m: Directed(Tarantino,m) $\leftrightarrow$ PlayedInMovie(UmaThurman,m)

   - $\exists$ m: PlayedInMovie(UmaThurman,m) $\wedge$ PlayedInMovie(Tarantino,m)

   (a) Show all the steps in the proof (or the diagram).

   (b) Translate the information given in FOL into English (or Norwegian) and describe in high level the reasoning you could apply in English to have the same result (in other words, describe a proof of the result in natural language).