Wireless System Performance - Miniproject

NDS 820

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Chapter 1

Patrick

1.1 MM1 - Narrowband multipath

1. Find an expression for the Doppler spread of the classical (bathtub) Doppler spectrum Hint : Shaum's Math. Handbook (14.239) and (14.237) .. i.e. integral solutions.
2. For same case as a) .. find the random-FM frequency shifts corresponding to the 10% and 1% probabilities.

1.1.1 Exercise 1

The Doppler spectrum (PSD) is given by:

$$S(f) = 2 \cdot \frac{\frac{1}{2\pi} \cdot \frac{1}{4\pi}}{\sqrt{(f_{d,max})^2 - (f)^2)}} \propto \frac{1}{\sqrt{(f_{d,max})^2 - f^2}} \tag{1.1}$$

The Doppler spread is defined as the standard deviation. The standard deviation is defines as:

$$\sigma = \sqrt{\int_f (f - \mu)^2 p(f) df} \tag{1.2}$$

where $p(x)$ is the PDF. and μ is the mean of the function. Since S is symmetric we know that $\mu = 0$.

The PDF can be estimated by S/P_t where P_t is the total power in the spectrum. Therefore we can say that:

$$P_t = \int_{-f_{d,max}}^{f_{d,max}} S(f) df \tag{1.3}$$

$$= \int_{-f_{d,max}}^{f_{d,max}} \frac{1}{\sqrt{(f_{d,max})^2 - f^2}} df \tag{1.4}$$

$$= \pi \tag{1.5}$$

We are using the 15.22 expression from the Mathematical Handbook of Formulas and Tables.

We can now say that the PDF is:

$$p(f) = \frac{S}{P_t} = \frac{1}{\sqrt{(f_{d,max})^2 - f^2} \cdot \pi} \tag{1.6}$$

Finally we can calculate σ by inserting into the definition of the standard deviation:

$$\sigma^2 = \int_{-f_d,max}^{f_d,max} x^2 \cdot \frac{S}{P_t} dt = \frac{1}{\pi} \int_{-f_d,max}^{f_d,max} \frac{x^2}{\sqrt{(f_{d,max})^2 - f^2}} \tag{1.7}$$

$$= \frac{1}{\pi} \cdot \frac{(f_{d,max})^2 \frac{\pi}{2}}{2} = \frac{(f_{d,max})^2}{4} \cdot 2 = \frac{(f_{d,max})^2}{2} \tag{1.8}$$

We are using the 15.24 expression from the Mathematical Handbook of Formulas and Tables.

Then σ (std(x)) will be:

$$\sigma = \sqrt{\frac{(f_{d,max})^2}{2}} \tag{1.9}$$

$$= \frac{f_{d,max}}{\sqrt{2}} = \frac{f_{d,max}}{1.4142} \tag{1.10}$$

1.1.2 Exercise 2

We use the general CDF for the Doppler spectrum:

$$Pr(\phi > x) = \frac{1}{2}(1 - \frac{x - f_d}{\sqrt{S_{fd}^2 + (x - f_d)^2}}) \tag{1.11}$$

Where $S_{fd}^2 = \frac{(f_{d,max})^2}{2}$

If we want to find the random-FM frequency shifts for 10% and 1% we will have to isolate x in $Pr(\phi)$

The reason this might be interesting is when we dimension the channel bandwidth. We will be interested in knowing how much of an impact random-FM shifts will have. By calculating the frequency at which the frequency shifts happens 1% of the time, we can verify that we have sufficient bandwidth to operate without frequency shifts in 99% of the cases.

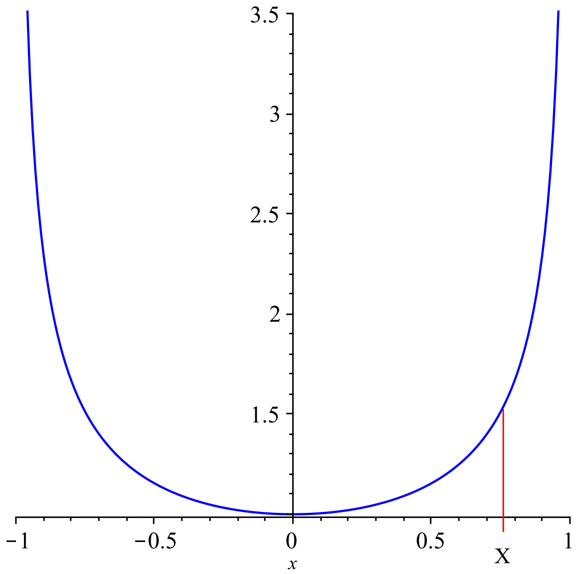


Figure 1.1: Caption

1.2 MM2 - Wideband multipath

1. Find an expression for the delay spread when the power delay profile is exponentially decaying $PDP(t) = A \cdot e^{(-a \cdot t)}$. Hint: it looks like the calculation we did last week!
2. What is the theoretical delay-spread for COST207 (GSM) 'urban' model (the continuous version)?
3. Find an analytical expression[1] for the frequency correlation function (FCF) for the exponential PDP.
4. What is the probability $p(tg > (delayspread + < t >))$, for the COST207 (GSM) 'urban' model.

[1] Hint: again, use what is available of standard solutions of integrals, transforms etc.. using Shaums outline, communication text books like Haykin etc. Then your main task reduces to establish the analytical expression in a form so they are recognizable as standard solutions and then apply them

1.2.1 Exercise 1

Average rms delay spread is given by:

$$S = \sqrt{\frac{\int_0^\infty \tau^2 \cdot P(\tau) d\tau}{P_m} - T_m^2} \tag{1.12}$$

We are given $E(|h(t)|^2)$ which is the PDP, the average power pr. delay. This is defined as:

$$P(\tau) = A \cdot exp(-\alpha \cdot \tau) \tag{1.13}$$

Total power (time integrated) is given by:

$$P_m = \int_0^\infty P(\tau) d\tau \tag{1.14}$$

Average mean delay:

$$T_m = \frac{\int_0^\infty \tau \cdot P(\tau) d\tau}{P_m} \tag{1.15}$$

We know that $PDP(\tau) = A \cdot e^{(-a \cdot \tau)}$

$$P_m = \int_0^\infty PDP(\tau) d\tau = \int_0^\infty A \exp\left(-\alpha\tau\right)\, d\tau \tag{1.16}$$

$$= A \int_0^\infty \exp\left(-\alpha\tau\right)\, d\tau = A \cdot \frac{\Gamma(n+1)}{\exp\left(n+1\right)} \tag{1.17}$$

$$= A \cdot \frac{\Gamma(0+1)}{\alpha} = \frac{A}{\alpha} \tag{1.18}$$

We are using the 15.76 expression from the Mathematical Handbook of Formulas and Tables.

$$T_m = \frac{\int_0^\infty \tau P_\tau d\tau}{P_m} = \frac{\int_0^\infty \tau \cdot A \cdot \exp(-\alpha\tau) d\tau}{\frac{A}{\alpha}} \tag{1.19}$$

$$= \frac{A \int_0^\infty \tau \cdot \exp(-\alpha\tau) d\tau}{\frac{A}{\alpha}} = \frac{A \cdot \Gamma(2) \cdot \alpha}{A \cdot \alpha^2} = \frac{1}{\alpha} \tag{1.20}$$

We are using the 15.76 expression from the Mathematical Handbook of Formulas and Tables.

$$S = \sqrt{\frac{\int_0^\infty \tau^2 \cdot P(\tau) d\tau}{P_m} - T_m^2} = \sqrt{\frac{\int_0^\infty \tau^2 \cdot A \cdot \exp\left(-\alpha\tau\right) d\tau}{\frac{A}{\alpha}} - \frac{1}{\alpha^2}} \tag{1.21}$$

$$= \sqrt{\frac{\frac{A \cdot \Gamma(3)}{\alpha^3}}{A \cdot \alpha} - \frac{1}{\alpha^2}} = \sqrt{\frac{2 \cdot \alpha \cdot A}{\alpha^3 \cdot A} - \frac{1}{\alpha^2}} \tag{1.22}$$

$$= \sqrt{\frac{1}{\alpha^2}} = \frac{1}{\alpha} \tag{1.23}$$

We are using the 15.76 expression from the Mathematical Handbook of Formulas and Tables.

1.2.2 Exercise 2

In this exercise, we must find the delay spread for the COST207 (GSM) model. As we've seen in the exercise before, it is equal to 1/a. So we must find a for this model.

We still have the same expression for PDP, $P(\tau) = A \cdot exp(-\alpha \cdot \tau)$, which we have to make an estimate of. We need to convert it to a linear scale which means we need to use $10 \cdot log_{10}$

Thanks to the curve given in the slides and on fig. 1.2,

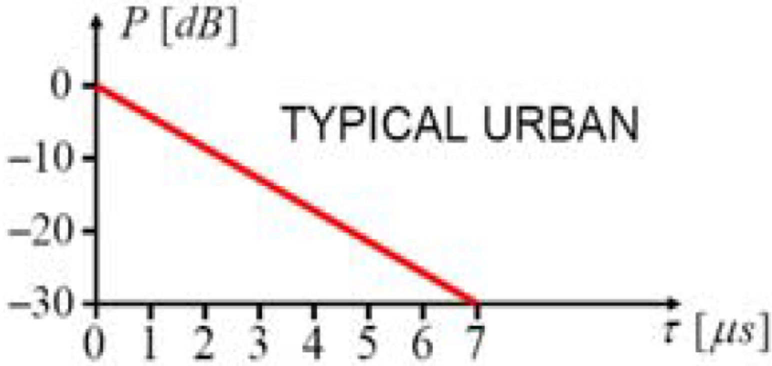


Figure 1.2

we know that

$$10log_{10}(P(0)) = 0 \tag{1.24}$$

$$10log_{10}(P(7)) = -30 \tag{1.25}$$

So this means, by replacing P with its expression in the first equation:

$$10log_{10}(A \cdot exp(-a \cdot 0)) = 0 \tag{1.26}$$

$$10log_{10}(A) = 0 \tag{1.27}$$

$$A = 1 \tag{1.28}$$

And then, by replacing P in the second expression, with A=1:

$$P_{log}(7) = 10log_{10}(exp(-a \cdot 7)) = -30 \tag{1.29}$$

$$log_{10}(exp(-a \cdot 7)) = -3 \tag{1.30}$$

$$exp(-a \cdot 7) = 10^{-3} \tag{1.31}$$

$$-a \cdot 7 = ln(10^{-3}) \tag{1.32}$$

$$a = -\frac{ln(10^{-3})}{7} = 0.9868 \tag{1.33}$$

At the end, we get that a is approximately equal to 0.9868 micro-hertz. Then we deduce the delay spread which is:

$$S = 1/\alpha = 1/0.9868 = 1.0134\mu s \tag{1.34}$$

1.2.3 Exercise 3

"Find an analytical expression[1] for the frequency correlation function (FCF) for the exponential PDP."

The frequency correlation function (FCF) is given as the inverse fourier transform of our power delay profile:

$$FCF = FFT^{-1}(PDP) \tag{1.35}$$

Where the PDP is the same from the earlier exercises:

$$P(\tau) = PDP(\tau) = A \cdot exp(-\alpha \cdot \tau) \tag{1.36}$$

And where the calculation for the frequency correlation function via inverse fourier transform is:

$$F^{-1}(P(\tau))(t) = \int_{-\infty}^{\infty} P(\tau) \cdot exp(2\pi j \cdot t \cdot \tau)\ d\tau \tag{1.37}$$

$$= \int_{0}^{\infty} A \cdot \exp\left(-\alpha\tau\right) \exp\left(2\pi j \cdot t \cdot \tau\right)\ d\tau \tag{1.38}$$

$$= \int_{0}^{\infty} A \cdot \exp\left(2\pi j \cdot t - \alpha\right) \cdot \tau\ d\tau \tag{1.39}$$

$$= \frac{A}{2\pi j \cdot t - \alpha} \tag{1.40}$$

1.2.4 Exercise 4

"What is the probability $p(\tau_g > (delayspread + < \tau >))$, for the COST207 (GSM) 'urban' model. Tg is the group delay (student t distribution)."

We use the CDF of the general doppler spectrum and insert our new conditions and reduce the expression.

$$Pr(\tau_g > delayspread + < \tau >) = \frac{1}{2}\left[1 - \frac{delayspread + < \tau > - \tau}{\sqrt{S_{fd}^2 + (delayspread + < \tau > - \tau))^2}}\right] \tag{1.41}$$

$$= \frac{1}{2}\left[1 - \frac{\sigma + < \tau > - < \tau >}{\sqrt{\sigma^2 + (\sigma + < \tau > - < \tau >)^2}}\right] \tag{1.42}$$

$$= \frac{1}{2}\left[1 - \frac{\sigma}{\sqrt{\sigma^2 + \sigma^2}}\right] \tag{1.43}$$

$$= \frac{1}{2}\left[1 - \frac{\sigma}{\sqrt{2} \cdot \sigma}\right] \tag{1.44}$$

$$= \frac{1}{2} - \frac{\sqrt{2}}{4} \tag{1.45}$$

1.3 MM3 - Diversity

1.3.1 Simulation

Make / generate two Rayleigh fading signals (Matlab has a generator , else you can make complex Guassian processes and use the amplitude form that)
1 : Plot two branch powers. Is there any thing to be gained using selection diversity? - or MRC W
For all cases assume identical noise floors on the branch signals (then MRC action is just a power consideration)
Find the 1% cumulative SNR level (hint: use the Matlab function 'sort.m'). Find the mean gain and diversity gain at 1% SNR level (hint : make a probability plot). Does Maximum Ratio Combining provide any noticeable extra gains?

The goal of this exercise is to compare the performance of three different diversity methods: Selection, Equal gain and Maximum ratio combining. This is done using Matlab simulation.

Two signals are generated and then the three diversity methods are applied to them:

```
N = 100000;
h1 = randn(N,1)+1j*randn(N,1);
p1 = abs(h1).^2;

h2 = randn(N,1)+1j*randn(N,1);
p2 = abs(h2).^2;
```

Figure 1.3: Generate data

Selection combination is applied like so:

```
% Selection combination
h3 = max(h1, h2);
p3 = abs(h3).^2;
cdf3 = sort(p3)./N;
semilogy(10*log10(cdf3), xax)
```

Figure 1.4: Selection combination

Here we simply select the samples from our two signals with the highest amplitude.

For Equal gain we divide the sum of samples with $\sqrt{2}$ (because we have 2 branches.) and then raise it to power 2.

```
% Equal gain combination
h4 = (sum([h1'; h2'])/sqrt(2)).^2;
p4 = h4;%abs(h4).^2;
cdf4 = sort(p4);
semilogy(10*log10(cdf4), xax)
```

Figure 1.5: Equal gain combination

The Maximum ratio combination is applied by summing up the power of in the signals.

```
% Maximum ratio combining
p5 = sum([p1'; p2']);
cdf5 = sort(p5)./N;
semilogy(10*log10(cdf5), xax)
```

Figure 1.6: Maximum ratio combination

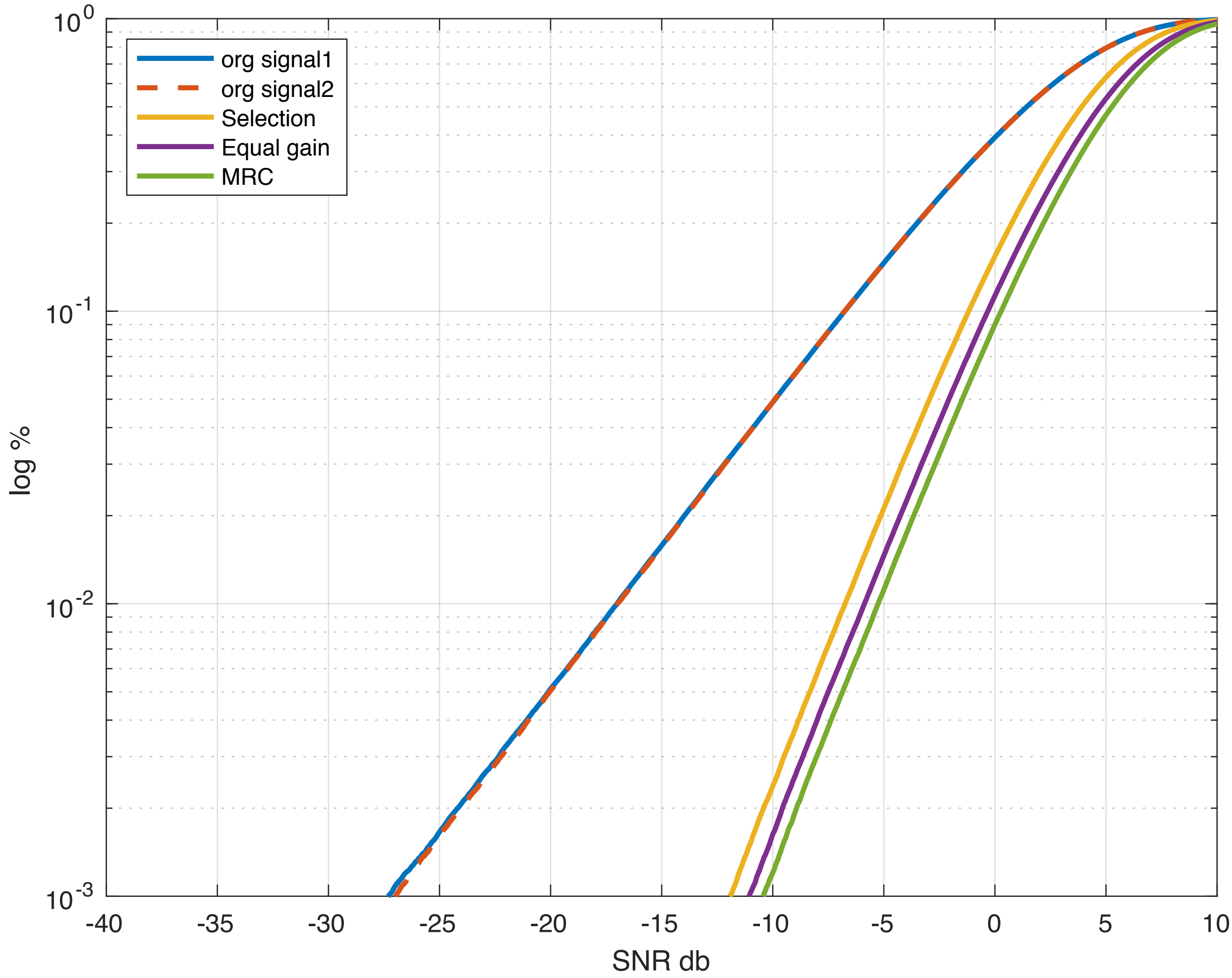


Figure 1.7: Diversity methods

Branch 1	Selection	Equal gain	MRC
-16.94 db	-6.8189 db	-5.9275 db	-5.2989

Table 1.1: 1% Results

Based on these results we can see that diversity helps. We can observe that MRC performs best, followed closely by Equal gain. Selection performs worst of the tree methods. This matches what is stated in the book "The Mobile Radio Propagation Channel".

From the probability plot on fig. 1.7 we can calculate the mean gain and the diversity gain. The mean gain is the shift in distribution and the diversity gain is the change in slope. In this example we compare to branch 1, org signal1.

The Mean gain (difference between the dBs at 1%) compared to branch 1:
Selective mean gain: 10.1211 dB
Equal mean gain: 11.0125 dB

MRC mean gain: 11.6411 dB

The Diversity gain (difference in slope) compared to branch 1 at 1% SNR:

$$G_{div} = \frac{P_{div}}{P_{branch}} \tag{1.46}$$

The diversity gain for selection is:

$$\frac{0.2108}{0.0200} = 10.5575 \tag{1.47}$$

Equal gain:

$$\frac{0.2573}{0.0200} = 12.8835 \tag{1.48}$$

MRC gain:

$$\frac{0.2969}{0.0200} = 14.8669 \tag{1.49}$$

So the MRC is better.

1.3.2 Theory

Narrow Band 1 : Assume a 'classical bathtub' Doppler spectrum at a location where a space diversity system is placed. What is the necessary antenna separation (in l) for an envelope correlation coefficient of 0.7?

2 : A Selection diversity combiner is subject to Rayleigh fading channels with equal mean power. How many diversity channels ('branches') are necessary to achieve a 14dB diversity gain at the 1% fading level? (assume all envelope signals are uncorrelated).

WB (more conceptual/'philosophical') 1 : Consider an exponentially decaying (power) impulse response with US. If an equalizer was NOT working properly phase-wise, what is the ratio of total collected power compared to an ideally working equalizer?

1.3.3 Theory - Exercise 1

In this exercise, we have a space diversity system and we want to now how much we should separate antenna to have an envelope correlation coefficient of 0.7 (which will have effects on the overall performance of the system).

We know that for a classical bathtub Doppler spectrum, the correlation coefficient is given by :

$$\rho_{env}(\frac{d}{\lambda}) = J_0^2(\frac{2*\pi*d}{\lambda}) \tag{1.50}$$

So in our case, we want:

$$0.7 = J_0^2(\frac{2*\pi*d}{\lambda}) \tag{1.51}$$

$$Lookinginatableofvaluesforzeroorderbesselfunctions, wethenknowthat : \tag{1.52}$$

$$\frac{2*\pi*d}{\lambda} = 1.1 \tag{1.53}$$

$$\frac{d}{\lambda} = \frac{1.1}{2*\pi} \tag{1.54}$$

$$\frac{d}{\lambda} = 0.175 \tag{1.55}$$

$$\tag{1.56}$$

So the distance that separate the antenna should be equal to 0.175 times the wavelength we are using.

1.3.4 Theory - Exercise 2

At first, we just have a simple emitting channel subject to Rayleigh fading. We want to know how many channels we should add in a space diversity system to gain 14db at the 1% fading level. The normal Rayleigh CDF is defined as follows:

$$P(snr) = 1 - exp\left(-\frac{SNR}{< SNR >}\right) \tag{1.57}$$

Let's start by computing the SNR at 1% fading level with just one channel :

$$0.01 = 1 - e^{-\frac{SNR}{<SNR>}} \tag{1.58}$$

$$0.99 = e^{-\frac{SNR}{<SNR>}} \tag{1.59}$$

$$\frac{SNR}{< SNR >} = -ln(0.99) \tag{1.60}$$

$$= 0.01 \tag{1.61}$$

$$\tag{1.62}$$

The Rayleigh with selection is as follows:

$$P_M(snr) = \left(1 - exp\left(-\frac{SNR}{< SNR >}\right)\right)^M \tag{1.63}$$

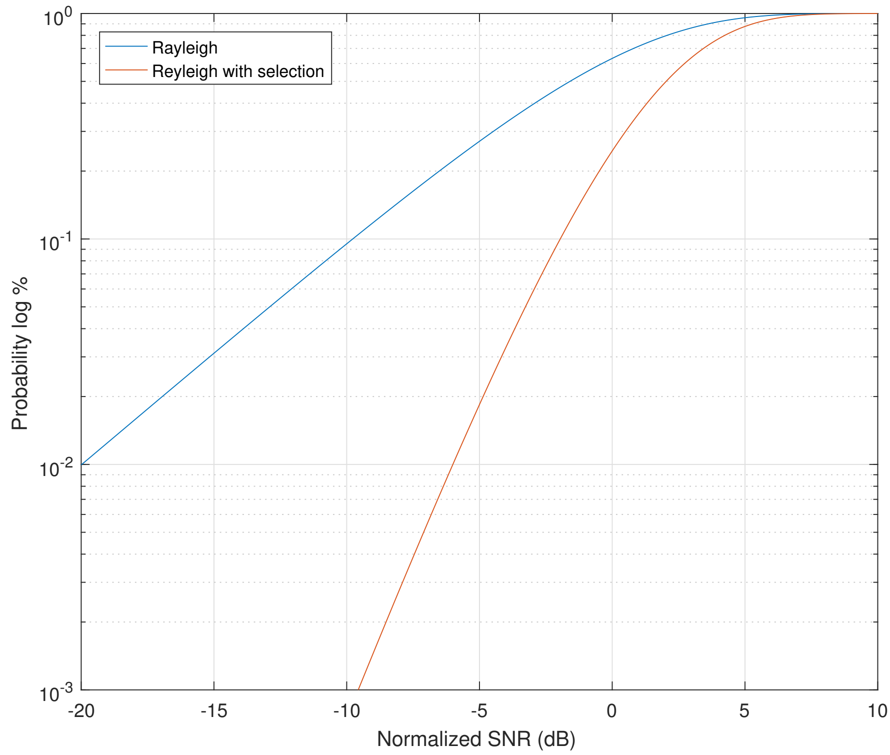
We want our normalized SNR to be 14db higher than 0.01 : it will have to be equal to 0.25.

$$(1 - e^{-0.25})^M = 0.01 \tag{1.64}$$

$$M = \frac{ln(0.01)}{ln(1 - e^{-0.25})} \tag{1.65}$$

$$M = 3.05 \tag{1.66}$$

So either we have no choice but to choose M=4, else maybe there is some other part of the system that can be improved to reach this 14db gain with 3 channels only.
On fig. 1.8 the difference between the standard Rayleigh and the modified Rayleigh can be seen.



Rayleigh
Reyleigh with selection
Probability log %
Normalized SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
-20
-15
-10
-5
0
5
10

Figure 1.8