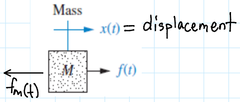
Translational Mechanical System - 4205

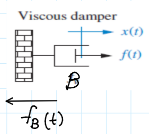
Tuesday, 5 March 2024 2:59 pm



$f_m(t) = ma \rightarrow \frac{d^2x(t)}{dt^2}$

$\boxed{f_m(t) = M\frac{d^2x(t)}{dt^2}}$ (1)

$\boxed{F_m(s) = Ms^2X(s)}$ (2)

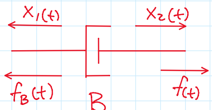


$f_B(t) = Bv \rightarrow \frac{dx(t)}{dt}$

$\boxed{f_B(t) = B\frac{dx(t)}{dt}}$ (3)

$\boxed{F_B(s) = BsX(s)}$ (4)

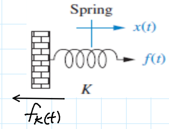
fixed damper



$\boxed{f_B(t) = B\left[\dot{x}_1(t) - \dot{x}_2(t)\right]}$ (5)

$\boxed{F_B(s) = Bs\left[X_1(s) - X_2(s)\right]}$ (6)

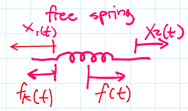
free damper



$\boxed{f_K(t) = Kx(t)}$ (7)

$\boxed{F_K(s) = KX(s)}$ (8)

fixed spring

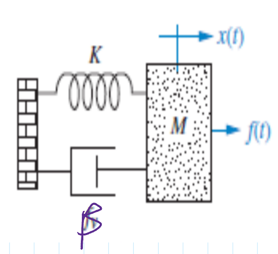


$\boxed{f_K(t) = K\left[x_1(t) - x_2(t)\right]}$ (9)

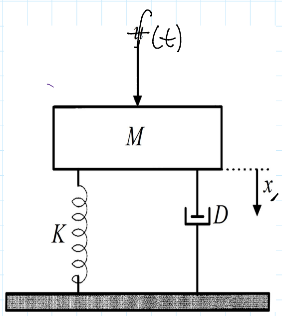
$\boxed{F_K(s) = K\left[X_1(s) - X_2(s)\right]}$ (10)

free spring

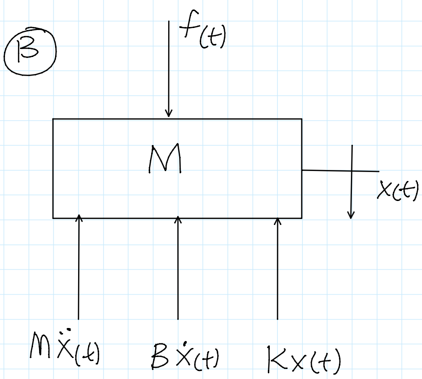
A)



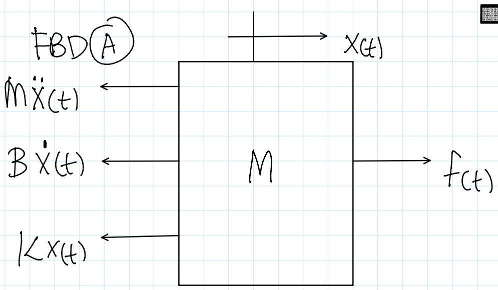
B)



FBD (B)



FBD (A)



$\frac{X(s)}{F(s)}$, $F_m(s)$, $F_B(s)$, $F_K(s)$

$M = B = K = 1$

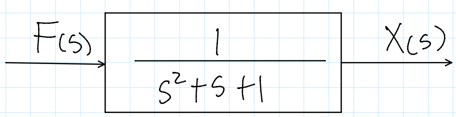
$$f(t) = M\ddot{x}(t) + B\dot{x}(t) + Kx(t)$$

$$F(s) = Ms^2X(s) + BsX(s) + KX(s)$$

$$F(s) = s^2X(s) + sX(s) + X(s)$$

$$F(s) = X(s)\left[s^2 + s + 1\right]$$

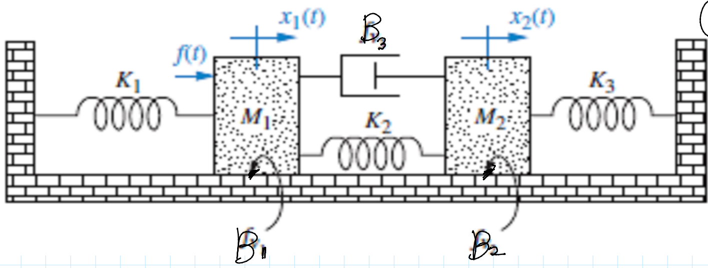
$$\frac{X(s)}{F(s)} = \frac{1}{s^2+s+1}$$



Degrees of Freedom (DOF)

Defined as the minimum number of independent parameters/ variables/ coordinates needed to describe a system completely.

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure



2 independent variables

→ 2 - DOF

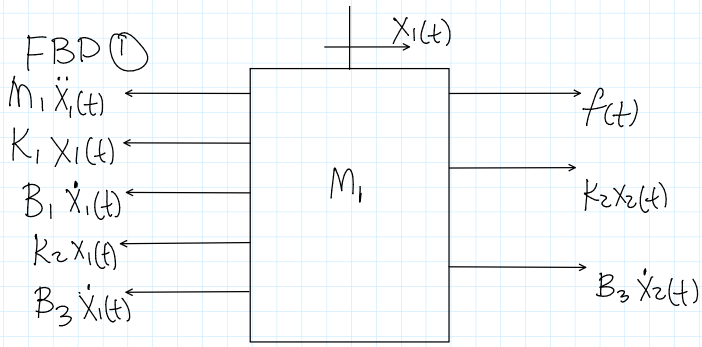
→ 2 independent equations of motion

$M_1 = M_2 = 1$

$B_1 = B_2 = B_3 = 2$

$K_1 = K_2 = K_3 = 3$

FBD 1

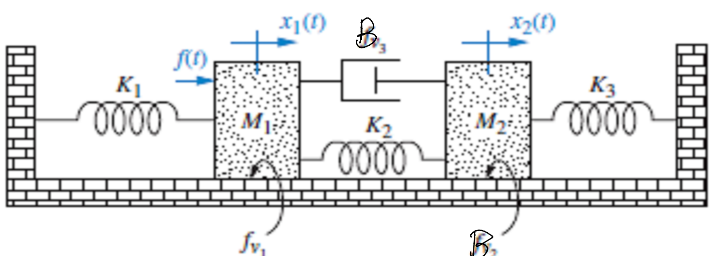


$$f(t) = M_1\ddot{X}_1(t) + (B_1 + B_3)\dot{X}_1(t) + (K_1 + K_2)X_1(t) - B_3\dot{X}_2(t) - K_2X_2(t)$$

$$F(s) = s^2X_1(s) + 4sX_1(s) + 6X_1(s) - 2sX_2(s) - 3X_2(s)$$

$$F(s) = X_1(s)[s^2 + 4s + 6] - X_2(s)[2s + 3] \quad (1)$$

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure

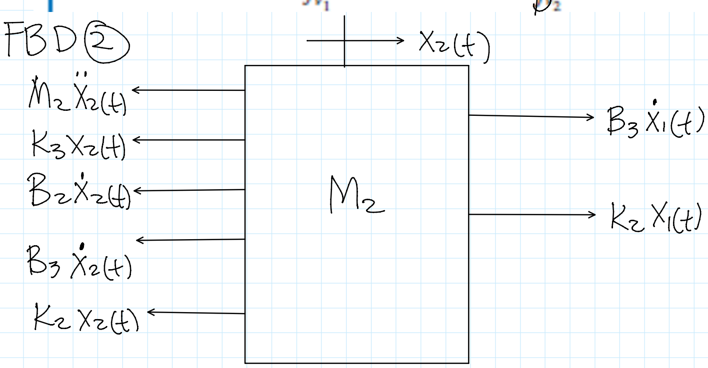


$M_1 = M_2 = 1$

$B_1 = B_2 = B_3 = 2$

$K_1 = K_2 = K_3 = 3$

FBD 2



$$0 = M_2\ddot{X}_2(t) + (B_2 + B_3)\dot{X}_2(t) + (K_2 + K_3)X_2(t) - B_3\dot{X}_1(t) - K_2X_1(t)$$

$$0 = s^2X_2(s) + 4sX_2(s) + 6X_2(s) - 2sX_1(s) - 3X_1(s)$$

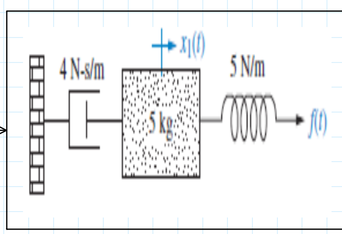
$$0 = X_2(s)[s^2 + 4s + 6] - X_1(s)[2s + 3] \quad (2)$$

$$X_2(s) = \frac{\begin{array}{cc} A & C \\ \end{array} \begin{vmatrix} s^2+4s+6 & F(s) \\ -(2s+3) & 0 \end{vmatrix}}{\begin{vmatrix} s^2+4s+6 & -(2s+3) \\ -(2s+3) & s^2+4s+6 \end{vmatrix} \begin{array}{cc} A & B \end{array}} = \frac{0 - [-(2s+3)F(s)]}{\begin{array}{r} s^4+4s^3+6s^2 \\ 4s^3+16s^2+24s \\ 6s^2+24s+36 \\ \hline s^4+8s^3+28s^2+48s+36 - (4s^2+6s+6s+9) \\ s^4+8s^3+24s^2+36s+27 \end{array}}$$

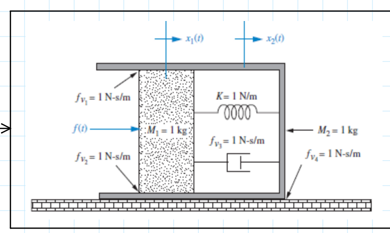
$$X_2(s) = \frac{(2s+3)F(s)}{s^4+8s^3+24s^2+36s+27}$$

$$F(s) \longrightarrow \boxed{\frac{2s+3}{s^4+8s^3+24s^2+36s+27}} \longrightarrow X_2(s)$$

B. Whole G(s) Examples



$F_K(s)$

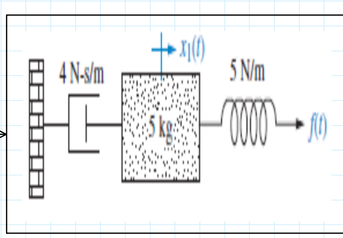


$F_{M2}(s)$

$$F(s) \longrightarrow \boxed{G_1} \xrightarrow{F_K(s)} \boxed{G_2} \longrightarrow F_{M_2}(s)$$

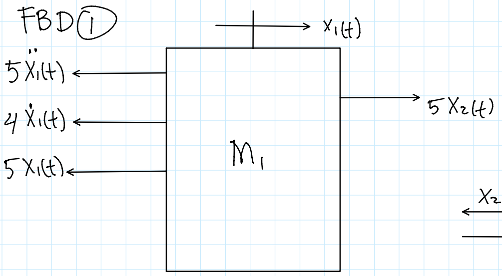
$$F(s) \longrightarrow \boxed{G_1 G_2} \longrightarrow F_{M_2}(s)$$

F(s)



$F_K(s)$

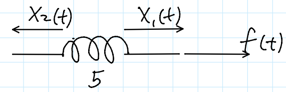
FBD ①

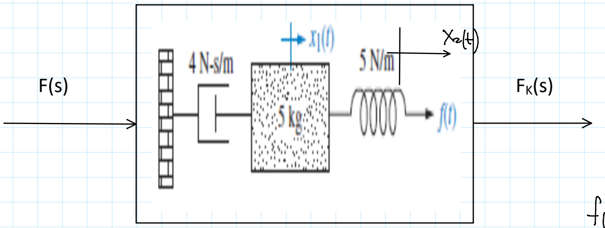


$$0 = 5\ddot{x}_1(t) + 4\dot{x}_1(t) + 5x_1(t) - 5x_2(t)$$

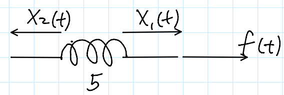
$$0 = 5s^2X_1(s) + 4sX_1(s) + 5X_1(s) - 5X_2(s)$$

$$0 = X_1(s)[5s^2+4s+5] - 5X_2(s) \quad (1)$$





FBD ②



$$f(t) = 5x_2(t) - 5x_1(t)$$

$$F(s) = 5X_2(s) - 5X_1(s) \quad (2)$$

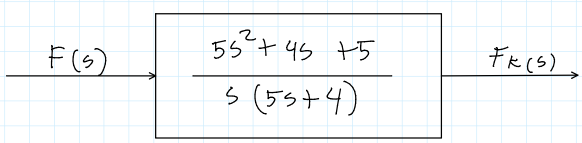
$$X_2(s) = \frac{\begin{array}{cc} A & C \\ \left|\begin{matrix} 5s^2+4s+5 & 0 \\ -5 & F(s) \end{matrix}\right| \end{array}}{\begin{array}{cc} \left|\begin{matrix} 5s^2+4s+5 & -5 \\ -5 & 5 \end{matrix}\right| \\ A & B \end{array}} = \frac{F(s)[5s^2+4s+5] - 0}{25s^2+20s+\cancel{25}-\cancel{25}}$$

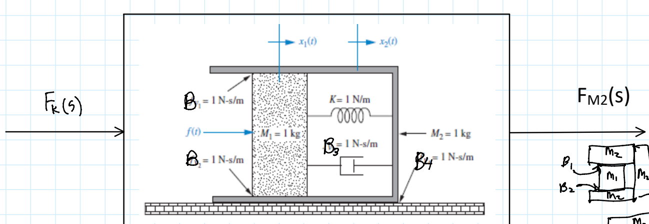
$$\frac{X_2(s)}{F(s)} = \frac{5s^2+4s+5}{5s(5s+4)}$$

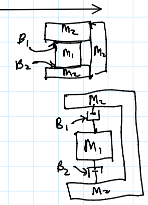
$$F_K(s) = K X_2(s)$$

$$\frac{F_K(s)}{K} = X_2(s)$$

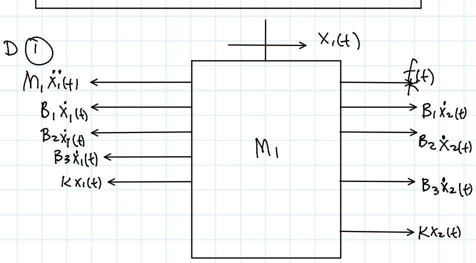
$$5\left[\frac{F_K(s)}{5F(s)} = \frac{5s^2+4s+5}{5s(5s+4)}\right]5$$

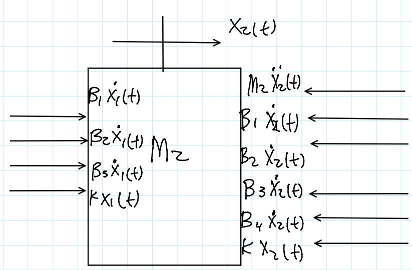






FBD ①

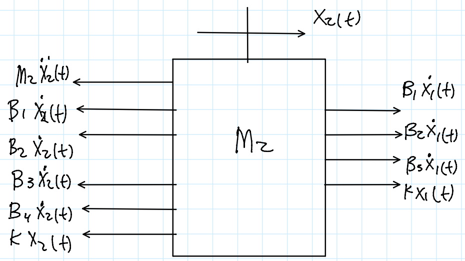




$f_k(t) = M_1\ddot{x}_1(t) + (B_1+B_2+B_3)\dot{x}_1(t) + Kx_1(t) - (B_1+B_2+B_3)\dot{x}_2(t) - Kx_2(t)$

$F_k(s) = s^2X_1(s) + 3sX_1(s) + X_1(s) - 3sX_2(s) - X_2(s)$

$F_k(s) = X_1(s)\left[s^2+3s+1\right] - X_2(s)\left[3s+1\right]$



$0 = M_2\ddot{x}_2(t) + (B_1+B_2+B_3+B_4)\dot{x}_2(t) + Kx_2(t)$
$\quad - (B_1+B_2+B_3)\dot{x}_1(t) - Kx_1(t)$

$0 = s^2X_2(s) + 4sX_2(s) + X_2(s) - 3sX_1(s) - X_1(s)$

$0 = X_2(s)\left[s^2+4s+1\right] - X_1(s)\left[3s+1\right]$

$F_k(s) = X_1(s)\left[s^2+3s+1\right] - X_2(s)\left[3s+1\right]$

$0 = X_2(s)\left[s^2+4s+1\right] - X_1(s)\left[3s+1\right]$

$$X_2(s) = \frac{\begin{array}{c}A \qquad\qquad C\\ \begin{vmatrix} s^2+3s+1 & F_k(s) \\ -(3s+1) & 0 \end{vmatrix}\end{array}}{\begin{array}{c}\begin{vmatrix} s^2+3s+1 & -(3s+1) \\ -(3s+1) & s^2+4s+1 \end{vmatrix}\\ A \qquad\qquad B\end{array}} = \frac{0 - \left[-(3s+1)F_k(s)\right]}{\begin{array}{r} s^4+4s^3+s^2 \\ 3s^3+12s^2+3s \\ s^2+4s+1 \\ \hline s^4+7s^3+14s^2+7s+1 - (9s^2+3s+3s+1) \end{array}}$$

$$= s^4+7s^3+5s^2+s$$

$$\frac{X_2(s)}{F_k(s)} = \frac{3s+1}{s(s^3+7s^2+5s+1)}$$

$F_{M_2}(s) = M_2s^2X_2(s)$

$\frac{F_{M_2}(s)}{M_2s^2} = X_2(s)$

$$\cancel{s^2}\left[\frac{F_{M_2}(s)}{\cancel{s^2}F_k(s)} = \frac{3s+1}{\cancel{s}(s^3+7s^2+5s+1)}\right]s^{\cancel{2}}$$