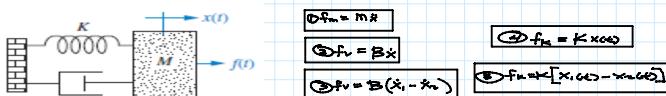
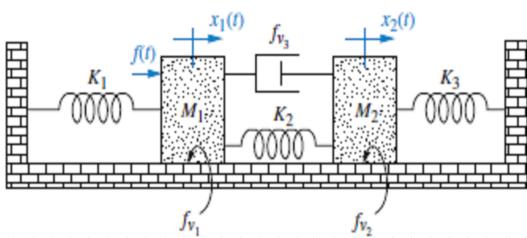


Find the transfer function $G(s) = X_1(s)/F(s)$.

$\sum f_{\omega} = M\ddot{x} + B\dot{x} + Kx$
 $F(s) = M s^2 X(s) + B s X(s) + K X(s)$
 $F(s) = X(s) [M s^2 + B s + K]$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{M s^2 + B s + K}$$

$$\begin{array}{c} F(s) \\ \longrightarrow \end{array} \boxed{\frac{1}{M s^2 + B s + K}} \quad \begin{array}{c} X(s) \\ \longrightarrow \end{array}$$

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure

FBD

$f(t) \rightarrow$

M_1

$f_{n1} x_1(t)$
 $f_{d1} x_1(t)$
 $f_{e1} x_1(t)$
 $f_{n2} x_2(t)$
 $f_{d2} x_2(t)$
 $f_{e2} x_2(t)$

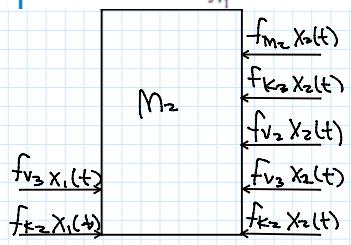
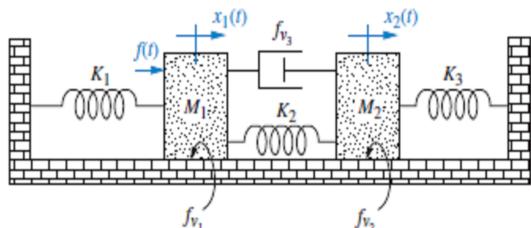
$$\begin{aligned} f(t) &= f_{m1} x_1(t) + f_{k1} x_1(t) + f_{v1} x_1(t) \\ &\quad + f_{n2} x_1(t) + f_{k2} x_1(t) - f_{v3} x_2(t) \\ &\quad - f_{k2} x_2(t) \end{aligned}$$

$$f(t) = M_1 \ddot{x}_1 + B_1 \dot{x}_1 + B_3 \dot{x}_1 + K_1 x_1 + K_2 x_1 - B_3 \dot{x}_2 - K_2 x_2$$

$$F(s) = M_1 s^2 X_1(s) + B_1 s X_1(s) + B_3 s X_1(s) + K_1 X_1(s) + K_2 X_1(s) - B_3 s X_2(s) - K_2 X_2(s)$$

$$F(s) = X_1(s) [M_1 s^2 + B_1 s + B_3 s + K_1 + K_2] - X_2(s) [B_3 s + K_2]$$

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure



$$0 = f_{m_2}x_2(t) + f_{k_3}x_2(t) + f_{v_2}x_2(t) \\ + f_{v_3}x_2(t) + f_{k_2}x_2(t) - f_{v_3}x_1(t) \\ - f_{k_2}x_1(t)$$

$$0 = M_2 \ddot{x}_2 + B_2 \dot{x}_2 + B_3 \dot{x}_2 + k_2 x_2 + k_3 x_2 - B_3 \dot{x}_1 - k_2 x_1$$

$$0 = M_2 s^2 X_2(s) + B_2 s X_2(s) + B_3 s X_2(s) + k_2 X_2(s) + k_3 X_2(s) - B_3 s X_1(s) - k_2 X_1(s)$$

$$0 = X_2(s) [M_2 s^2 + B_2 s + B_3 s + k_2 + k_3] - X_1(s) [B_3 s + k_2]$$

$$F(s) = X_1(s) [M_1 s^2 + B_1 s + B_3 s + k_1 + k_2] - X_2(s) [B_3 s + k_2] \quad (1)$$

$$0 = X_2(s) [M_2 s^2 + B_2 s + B_3 s + k_2 + k_3] - X_1(s) [B_3 s + k_2] \quad (2)$$

$$\frac{X_1(s)}{F(s)} = ?$$

$$X_2(s) = \frac{X_1(s) [B_3 s + k_2]}{[M_2 s^2 + B_2 s + B_3 s + k_2 + k_3]}$$

$$F(s) = X_1(s) [M_1 s^2 + B_1 s + B_3 s + k_1 + k_2] - \left\{ \frac{X_1(s) [B_3 s + k_2]}{[M_2 s^2 + B_2 s + B_3 s + k_2 + k_3]} \right\} [B_3 s + k_2] \rightarrow [B_2 B_3 s^2 + B_2 k_2 s + B_3 k_2 s + k_2^2]$$

$$F(s) = X_1(s) \left\{ \frac{[M_1 s^2 + B_1 s + B_3 s + k_1 + k_2] [M_2 s^2 + B_2 s + B_3 s + k_2 + k_3] - X_1(s) [B_3 s + k_2] [B_3 s + k_2]}{[M_2 s^2 + B_2 s + B_3 s + k_2 + k_3]} \right\}$$

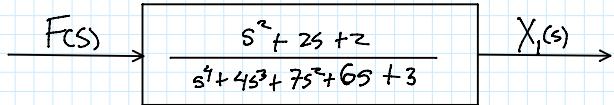
$$M_1 M_2 s^4 + M_1 B_2 s^3 + M_1 B_3 s^3 + M_1 k_2 s^2 + M_1 k_3 s^2 + M_2 B_1 s^3 + B_1 B_2 s^2 + B_1 B_3 s^2 + B_1 k_2 s + B_1 k_3 s + M_2 B_3 s^3 + B_2 B_3 s^2 + B_3 s^2 + B_3 k_2 s + B_3 k_3 s + M_2 k_1 s^2 + B_2 k_1 s + B_3 k_1 s + k_1 k_2 + k_1 k_3 + M_2 k_2 s^2 + B_2 k_2 s + B_3 k_2 s + k_2^2 + k_2 k_3$$

$$F(s) = X_1(s) \left[\frac{M_1 M_2 s^2 + (M_1 B_3 + M_2 B_3 + M_3 B_1 + M_2 B_2) s^3 + (M_1 K_2 + M_1 K_3 + B_1 B_2 + B_1 B_3 + B_2 + M_2 K_1 + M_3 K_2) s^2 + (B_1 K_2 + B_1 K_3 + B_3 K_2 + B_3 K_3 + B_2 K_1 + B_3 K_1) s + K_1 K_2 + K_1 K_3 + K_2 K_3}{M_2 s^2 + B_2 s + B_3 s + K_2 + K_3} \right]$$

$$\frac{M_2 s^2 + B_2 s + B_3 s + K_2 + K_3}{M_2 s^2 + (M_1 B_3 + M_2 B_3 + M_3 B_1 + M_2 B_2) s^3 + (M_1 K_2 + M_1 K_3 + B_1 B_2 + B_1 B_3 + B_2 + M_2 K_1 + M_3 K_2) s^2 + (B_1 K_2 + B_1 K_3 + B_3 K_2 + B_3 K_3 + B_2 K_1 + B_3 K_1) s + K_1 K_2 + K_1 K_3 + K_2 K_3} = \frac{X_1(s)}{F(s)}$$

if $M_1 = M_2 = 1$
 $B_1 = B_2 = B_3 = 1$
 $K_1 = K_2 = K_3 = 1$

$$\frac{s^2 + 2s + 2}{s^4 + 4s^3 + 7s^2 + 6s + 3} = \frac{X_1(s)}{F(s)}$$



$$F(s) = X_1(s) [M_1 s^2 + B_1 s + B_3 s + K_1 + K_2] - X_2(s) [B_3 s + K_2] \quad (1)$$

$$0 = X_2(s) [M_2 s^2 + B_2 s + B_3 s + K_2 + K_3] - X_1(s) [B_2 s + K_2] \quad (2)$$

$$\frac{X_2(s)}{F(s)} = ?$$

$$X_2(s) [M_2 s^2 + B_2 s + B_3 s + K_2 + K_3] = X_1(s) [B_2 s + K_2]$$

$$\frac{X_2(s) [M_2 s^2 + B_2 s + B_3 s + K_2 + K_3]}{B_2 s + K_2} = X_1(s)$$

$$F(s) = \frac{X_2(s) [M_2 s^2 + B_2 s + B_3 s + K_2 + K_3] [M_1 s^2 + B_1 s + B_3 s + K_1 + K_2] - X_1(s) [B_3 s + K_2]}{B_2 s + K_2}$$

$$F(s) = X_2(s) \left\{ \frac{[M_2 s^2 + B_2 s + B_3 s + K_2 + K_3] [M_1 s^2 + B_1 s + B_3 s + K_1 + K_2] - [B_3 s + K_2] [B_2 s + K_2]}{B_2 s + K_2} \right\}$$

if $M_1 = M_2 = 1$
 $B_1 = B_2 = B_3 = 1$
 $K_1 = K_2 = K_3 = 1$

$$F(s) = X_2(s) \left\{ \frac{[s^2 + 2s + 2][s^2 + 2s + 2] - [s+1][s+1]}{s+1} \right\}$$

$$F(s) = X_2(s) \left\{ \frac{s^4 + 2s^3 + 2s^2 + 2s^3 + 4s^2 + 4s + 2s^2 + 4s + 4 - [s^2 + s + s + 1]}{s+1} \right\}$$

$$F(s) = X_2(s) \left\{ \frac{s^4 + 4s^3 + 8s^2 + 8s + 4 - [s^2 + s + s + 1]}{s+1} \right\}$$

$$F(s) = X_2(s) \left\{ \frac{s^4 + 4s^3 + 7s^2 + 6s + 3}{s+1} \right\}$$

$$\frac{X_2(s)}{F(s)} = \frac{s+1}{s^4 + 4s^3 + 7s^2 + 6s + 3}$$

