Electrical System

Tuesday, 27 February 2024 7:52 am

R 1Ω L 1H C 1mf

$i(t)$ $i(t)$ $i(t)$

R, L, C } constant

$E = IR$

$V = IR$

$E_{emf} = IR$

$$\mathcal{L}\left\{ v_R(t) = R\, i(t) \right. \quad ①$$

small

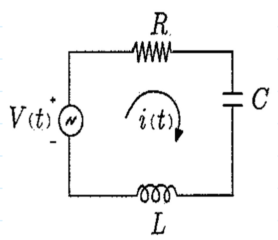
$$v_L(t) = L \frac{di(t)}{dt} \quad ②$$

instantaneous

$$\int i(t) = C \int \frac{dv_c(t)}{dt}$$

$$\frac{1}{C}\int i(t)\, d(t) = v_c(t) \quad ③$$

$$V_R(s) = R I(s) \quad ④ \qquad V_L(s) = L s I(s) \quad ⑤ \qquad V_C(s) = \frac{I(s)}{Cs} \quad ⑥$$



$$G(s) = \frac{V_o(s)}{V_i(s)} \to V_R(s),\ V_C(s),\ V_L(s),\ I(s)$$

$$V_i(s) \to \boxed{G(s)} \to$$

$$G(s) = \frac{I(s)}{V_i(s)}$$

① KVL / mesh / Nodal
② Laplace Transform
③ Linear Algebra (sub, elim, Cramer's)
④ Solve for $G(s)$

$$G(s) = \frac{I(s)}{V_i(s)}$$

① $V_i(t) = V_R(t) + V_c(t) + V_L(t)$

$$V_i(t) = R\, i(t) + \frac{1}{C}\int i(t)\, dt + L \frac{di(t)}{dt}$$

② $V_i(s) = R I(s) + \frac{1}{Cs} I(s) + L s I(s)$

③ $V_i(s) = I_s \left(R + \frac{1}{Cs} + Ls \right)$

④ $\left[V_i(s) = I_s \left(\frac{RCs + 1 + LCs^2}{Cs} \right) \right] \frac{1}{V_i(s)} \cdot \frac{Cs}{RCs + 1 + LCs^2}$

$$\frac{I(s)}{V_i(s)} = \frac{Cs}{LCs^2 + RCs + 1}$$

if $C = 4$
$L = 3$
$R = 2$

$$= \frac{4s}{12s^2 + 8s + 1}$$

$$V_i(s) \to \boxed{\frac{4s}{12s^2 + 8s + 1}} \to I(s)$$

$$V_i(s) \to \boxed{\frac{4s}{12s^2 + 8s + 1}} \to I(s)$$

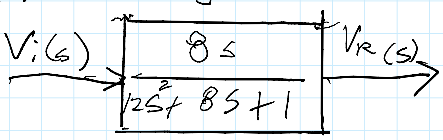
$$\frac{V_R(s)}{V_i(s)} = ?, \quad \frac{V_L(s)}{V_i(s)} = ?, \quad \frac{V_C(s)}{V_i(s)} = ?$$

$$V_R(s) = R I(s) \quad ④ \qquad V_L(s) = L s I(s) \quad ⑤ \qquad V_C(s) = \frac{I(s)}{Cs} \quad ⑥$$

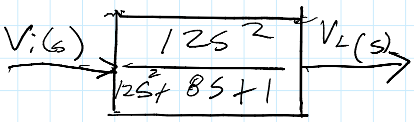
$$I(s) = \frac{V_R(s)}{R} \qquad I(s) = \frac{V_L(s)}{Ls} \qquad I(s) = Cs\, V_c(s)$$

$$\frac{I(s)}{V_i(s)} = \frac{4s}{12s^2+8s+1}$$

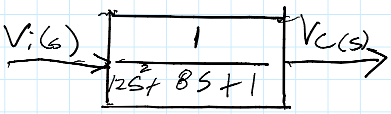
$$R\left[\frac{V_R(s)}{\cancel{R}V_i(s)} = \frac{4s}{12s^2+8s+1}\right] R=2 = \frac{8s}{12s^2+8s+1}$$



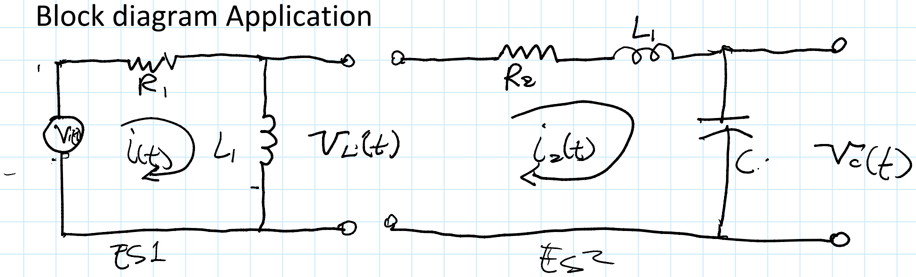
$$Ls\left\{\frac{V_L(s)}{\cancel{Ls}V_i(s)} = \frac{4s}{12s^2+8s+1}\right\} Ls \rightarrow 3 = 12s^2$$

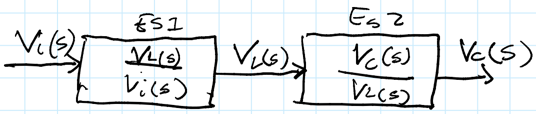


$$\frac{1}{Cs}\left\{\frac{Cs\,V_C(s)}{V_i(s)} = \frac{\cancel{4s}}{12s^2+8s+1}\right\} \frac{1}{\cancel{Cs}} \rightarrow 4$$

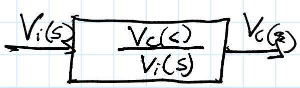


Block diagram Application





$$G_0(s) = \frac{V_C(s)}{V_i(s)}$$



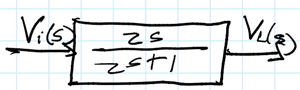
$R_1 = R_2 = 1,\quad L_1 = L_2 = 2,\quad C = 3$

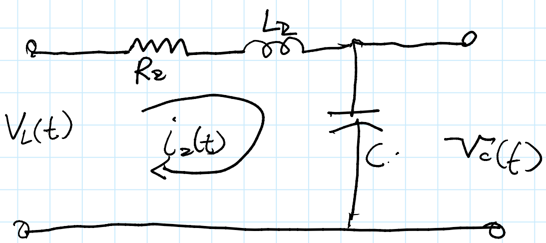
$$G_1(s) = \frac{V_L(s)}{V_i(s)}$$

① $V_i(t) = R\,i(t) + L\frac{di(t)}{dt}$ ② $V_L(t) = L\frac{di(t)}{dt}$

② $V_i(s) = R\,I(s) + Ls\,I(s)$ ② $V_L(s) = Ls\,I(s)$

$$\frac{V_L(s)}{V_i(s)} = \frac{Ls\,\cancel{I(s)}}{\cancel{I(s)}[Ls+R]} = \frac{2s}{2s+1}$$

$G_1(s)$ 



$$G_2(s) = \frac{V_c(s)}{V_L(s)}$$

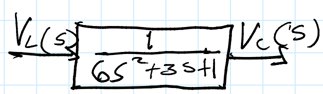
① $V_L(t) = R_2 i_2(t) + L_2 \frac{di_2(t)}{dt} + \frac{1}{C}\int i_2(t)\,dt$ ② $V_c(t) = \frac{1}{C}\int i_2(t)\,dt$

① $V_L(s) = R_2 I_2(s) + L_2 s I_2(s) + \frac{1}{Cs} I_2(s)$ ② $V_c(s) = \frac{1}{Cs} I_2(s)$

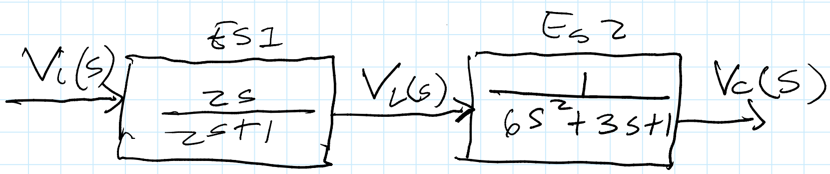
$$\frac{V_c(s)}{V_L(s)} = \frac{\cancel{I_2(s)}/Cs}{\cancel{I_2(s)}\left[L_2 s + R_2 + \frac{1}{Cs}\right]}$$

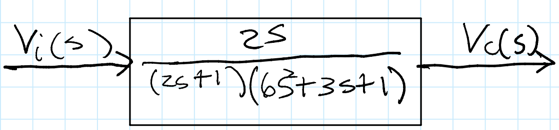
$$= \frac{1}{L_2 C s^2 + R_2 C s + 1} = \frac{1}{6s^2 + 3s + 1}$$

$G_2(s)$

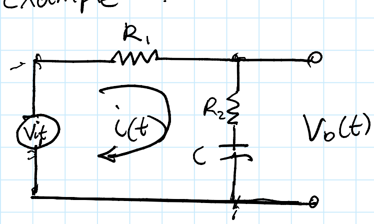


$$G_0(s) = \frac{V_c(s)}{V_i(s)}$$





Example 3:



$$G(s) = \frac{V_o(s)}{V_i(s)}$$

$V_i(t) = V_{R_1}(t) + V_{R_2}(t) + V_c(t)$ ② $V_o(t) = V_{R_2}(t) + V_c(t)$

$V_{R_2}(t) = V_o(t) - V_c(t)$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 I(s) + \frac{I(s)}{Cs}}{R_1 I(s) + R_2 I(s) + \frac{I(s)}{Cs}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2Cs+1}{\cancel{Cs}}}{\frac{R_1Cs+R_2Cs+1}{\cancel{Cs}}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2Cs+1}{R_1Cs+R_2Cs+1}$$

$$V_i(s) \longrightarrow \boxed{\frac{R_2Cs+1}{R_1Cs+R_2Cs+1}} \longrightarrow V_o(s)$$

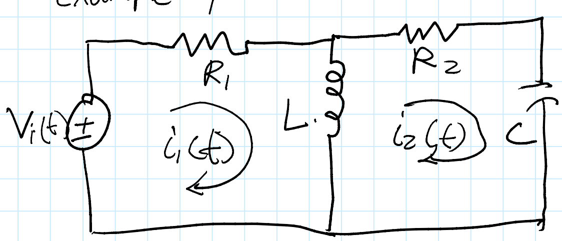
$$\frac{V_{R_2}(s)}{V_i(s)} = \;?$$

$$\frac{V_{R_2}(s) + V_C(s)}{V_i(s)} = \frac{R_2Cs+1}{R_1Cs+R_2Cs+1}$$

$$\frac{V_{R_2}(s)}{V_i(s)} + \frac{V_C(s)}{\cancel{V_i(s)}} = \frac{R_2Cs+1}{R_1Cs+R_2Cs+1} - \frac{\frac{1}{\cancel{Cs}}\cancel{I(s)}}{\cancel{I(s)}\left[\frac{R_1Cs+R_2Cs+1}{\cancel{Cs}}\right]}$$

$$V_i(s) \longrightarrow \boxed{\frac{R_2Cs}{R_1Cs+R_2Cs+1}} \longrightarrow V_C(s)$$

Example 4:



$$G(s) = \frac{I_2(s)}{V(s)}$$

if: $R_1 = R_2 = 1$

$L = 2$

$C = 3$

①

$$V_i(t) = R_1 i_1(t) + L\frac{di_1(t)}{dt} - L\frac{d\,i_2(t)}{dt}$$

$$V_i(s) = I_1(s)\left[Ls + R_1\right] - Ls\,I_2(s)$$

②

$$0 = Li_2(t) + R_2 i_2(t) + Ci_2(t) - Li_1(t)$$

$$0 = I_2(s)\left[Ls + R_2 + \frac{1}{Cs}\right] - Ls\,I_1(s)$$

$$I_2(s) = \frac{\begin{vmatrix} Ls+R_{\cancel{1}} & V_i(s) \\ -Ls & 0 \end{vmatrix}}{\begin{vmatrix} Ls+R_1 & -Ls \\ -Ls & \frac{LCs^2+R_2Cs+1}{Cs} \end{vmatrix}} = \frac{0 - (-Ls\,V_i(s))}{\left(\frac{L^2Cs^3+R_2CLs^2+Ls+R_1LCs^2+R_1R_2Cs+R_1}{Cs}\right)}$$

$$I_2(s) = \frac{LCs^2\,V_i(s)}{L^2Cs^3-R_2CLs^2+Ls+R_1LCs^2+R_1R_2Cs+R_1}$$

$$\frac{I_2(s)}{V_i(s)} = \frac{LCs^2}{L^2Cs^3-R_2CLs^2+Ls+R_1LCs^2+R_1R_2Cs+R_1}$$

$$= \frac{(2)(3)s^2}{(2)^2(3)s^3-(1)(3)(2)s^2+2s+(1)(2)(3)s^2+(1)(1)(3)s+1}$$

$$= \frac{6s^2}{12s^3-6s^2+2s+6s^2+3s+1}$$

$$= \frac{6s^2}{12s^3+5s+1}$$

$$V_i(s) \longrightarrow \boxed{\frac{6s^2}{12s^3+5s+1}} \longrightarrow I_2(s)$$