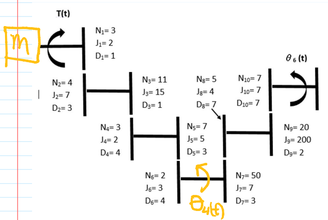
G(s) for Electromechanical Systems

Wednesday, October 28, 2020 5:02 PM



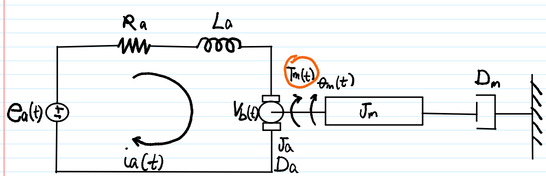
$$\frac{\theta_4(s)}{T(s)}$$

$$\frac{N^D}{N^S} = \frac{N_2}{N_1} * \frac{N_4}{N_3} * \frac{N_6}{N_5} = \frac{4}{3} * \frac{3}{11} * \frac{2}{7} = \frac{8}{77}$$

$$\frac{8}{77} T(t) = J_e \ddot{\theta}_4(t) + D_e \dot{\theta}_4(t)$$

$$J_e = \sum J \left(\frac{N_D}{N_S}\right)^2$$

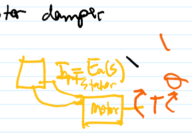
$$J_e = 2\left(\frac{4}{3} * \frac{3}{11} * \frac{2}{7}\right)^2 + 7\left(\frac{3}{11} * \frac{2}{7}\right)^2 + 15\left(\frac{3}{11} * \frac{2}{7}\right)^2 + 2\left(\frac{2}{7}\right)^2 + 5\left(\frac{2}{7}\right)^2 + 3\left(\frac{2}{2}\right)^2$$
$$+ 7\left(\frac{50}{50}\right)^2 + 4\left(\frac{50}{5}\right)^2 + 200\left(\frac{50}{5}\right)^2 + 7\left(\frac{20}{7} * \frac{50}{5}\right)^2$$
$$= 26{,}125.012$$



ARMATURE-CONTROLLED DC SERVOMOTOR

$e_a(t)$ = input voltage
R_a = armature resistance
L_a = armature inductance
i_a = armature current
$V_b(t)$ = back electromotive force
$T_m(t)$ = torque applied to the load by the motor
J_m = Moment of Inertia (Load side)
D_m = Damper (Load side)
$\theta_m(t)$ = output angular displacement

J_a = motor inertia
D_a = motor damper



$$G(s) = \frac{T_J, T_D, T_K, \theta}{E_a(s)}$$

KVL @ Electrical Side:

$$\mathcal{L}\left\{e_a(t) - R_a i_a(t) - L_a \frac{d\, i_a(t)}{dt} - V_b(t) = 0\right\}$$

$$E_a(s) - R_a I_a(s) - L_a s I_a(s) - V_b(s) = 0$$

$V_b \propto \omega_m(t) \rightarrow$ angular velocity of motor

$\hookrightarrow \frac{d\theta_m(t)}{dt}$

$$V_b(t) = K_b \frac{d\theta_m(t)}{dt}$$

$\hookrightarrow$ back emf constant

$$V_b(s) = K_b s \theta_m(s)$$

Substituting $V_b(s)$:

$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + K_b s \theta_m(s)$$

$$E_a(s) = [R_a + L_a s]\, I_a(s) + K_b s \theta_m(s)$$

$T_m(t) \propto i_a(t)$

K_t = motor torque constant

$$T_m(s) = K_t I_a(s)$$

$$I_a(s) = \frac{T_m(s)}{K_t}$$

Substituting $I_a(s)$:

$$E_a(s) = \left(\frac{T_m(s)}{K_t}\right)(R_a + L_a s) + K_b s \theta_m(s) \quad (1)$$

Mechanical Side:

$$T_m(t) = J_m \ddot{\theta}_m(t) + D_m \dot{\theta}_m(t)$$

$$T_m(s) = J_m s^2 \theta_m(s) + D_m s \theta_m(s)$$
$$T_m(s) = [J_m s^2 + D_m s]\, \theta_m(s) \quad (2)$$

$$E_a(s) = \left(\frac{[J_m s^{\cancel{2}} + D_m \cancel{s}]\, s\theta_m(s)}{K_t}\right)(R_a + \cancel{L_a s}) + K_b s\theta_m(s)$$

∴ assume L_a is very small compared to R_a w/ch is usual for DC motors.

$$E_a(s) = s\theta_m(s)\left[\frac{R_a}{K_t}(J_m s + D_m) + K_b\right]$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s\left[\frac{R_a}{K_t}(J_m s + D_m) + K_b\right]}$$

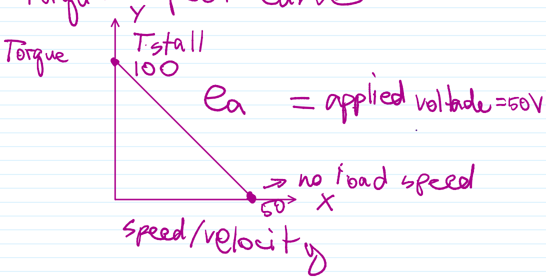
$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t}{s[R_a J_m s + R_a D_m + K_b K_t]} \cdot \frac{\frac{1}{R_a J_m}}{\frac{1}{R_a J_m}}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t / R_a J_m}{s\left[s + \frac{D_m}{J_m} + \frac{K_b K_t}{R_a J_m}\right]}$$

I. $$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t / R_a J_m}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}}$$ ✓

II. $$\boxed{J_m = J_a + J_L\left(\frac{N^p}{N^s}\right)^2;\quad D_m = D_a + D_L\left(\frac{N^p}{N^s}\right)^2;\quad K_m = K_L\left(\frac{N^p}{N^s}\right)^2}$$

Torque Speed Curve



$e_a = 50$

$T_{stall} = 100$

$\omega_{no\ load} = 50$

III. $$\boxed{\frac{K_t}{R_a} = \frac{T_{stall}}{e_a};\quad K_b = \frac{e_a}{\omega_{no\ load}}}$$

if given is like this: $T_m = -8\omega_m + 200$

IV. $$T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a$$

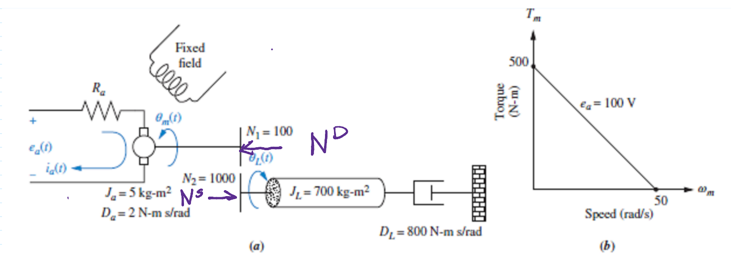
e_a is always given.

V. $$\boxed{\theta_m(s) = \theta_L(s)\left(\frac{N^p}{N^s}\right)}$$

$$\left\{ \left(\frac{N^D}{N^S}\right) \frac{\theta_L(s)}{E_a(s)} = \frac{1}{s[s+1]} \right\} \frac{N^S}{N^D}$$

$$\frac{\theta_L(s)}{E_a(s)} = \frac{N^S/N^D}{s[s+1]}$$

PROBLEM: Given the system and torque-speed curve of Figure 2.39(a) and (b), find the transfer function, $\theta_L(s)/E_a(s)$.



$$E_a(s) \longrightarrow \boxed{\frac{0.0417}{s(s+1.667)}} \longrightarrow \theta_L(s)$$

$e_a = 100$ V
$T_{stall} = 500$ N-m
$\omega_{nload} = 50$ rad/s

III. $\boxed{\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} \quad ; \quad K_b = \frac{e_a}{\omega_{no\,load}}}$

$\frac{K_t}{R_a} = \frac{500}{100} = 5 \quad ; \quad K_b = \frac{100}{50} = 2$

II. $\boxed{J_m = J_a + J_L\left(\frac{N^D}{N^S}\right)^2 ; \; D_m = D_a + D_L\left(\frac{N^D}{N^S}\right)^2 ; \; K_m = K_L\left(\frac{N^D}{N^S}\right)^2}$

$J_m = 5 + 700 \cdot \left(\frac{100}{1000}\right)^2 = 12 \quad ; \quad D_m = 2 + 800\left(\frac{100}{1000}\right)^2$

$D_m = 10$

I. $\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/R_a J_m}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}} = \frac{(5)\left(\frac{1}{12}\right)}{s\left[s + \frac{1}{12}(10+10)\right]}$

$$= \frac{\frac{5}{\cancel{12}}}{s\left[\frac{12s + 20}{\cancel{12}}\right]}$$

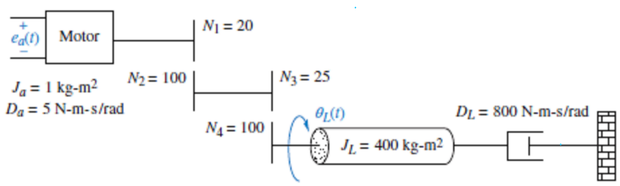
$$E_a(s) \longrightarrow \boxed{\frac{5}{4s[3s+5]}} \longrightarrow \theta_m(s)$$

V. $\boxed{\theta_m(s) = \theta_L(s)\left(\frac{N^P}{N^S}\right)}$ $\theta_m(s) = \theta_L(s)\left(\frac{1000}{100}\right)$

$$\frac{10\,\theta_L(s)}{E_a(s)} = \frac{5}{4s[3s+5]}$$

$$E_a(s) \longrightarrow \boxed{\frac{1}{8s[3s+5]}} \longrightarrow \theta_L(s)$$ ✓

PROBLEM: Find the transfer function, $G(s) = \theta_L(s)/E_a(s)$, for the motor and load shown in Figure 2.40. The torque-speed curve is given by $T_m = -8\omega_m + 200$ when the input voltage is 100 volts.



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