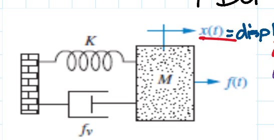
Degrees of Freedom (DOF)

Defined as the minimum number of independent parameters/ variables/ coordinates needed to describe a system completely.

1-DOF

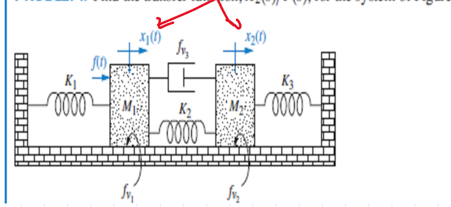


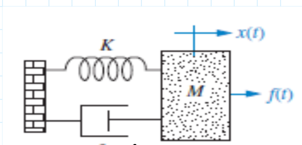
$x(t)$ = displacement
independent variable
used to describe the system

2-DOF

used to describe the system
independent

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure





$f_v = B$ $\quad f_m(t)$

friction = $f_B(t)$

$f(t)$ = input force

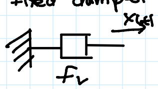
$f_m(t) = Ma$

$$\boxed{① \; f_m(t) = M\frac{d^2x(t)}{dt^2}}$$

$f_B(t) = f_v V$

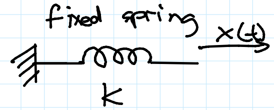
$$\boxed{② \; f_B(t) = f_v \frac{dx(t)}{dt}}$$

fixed damper



$$\boxed{④ \; f_k(t) = K x(t)}$$

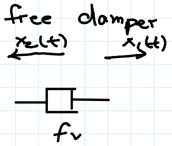
fixed spring



$f_B(t) = f_v(V_1 - V_2)$

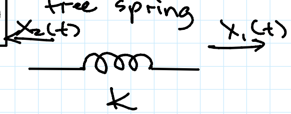
$$\boxed{③ \; f_B(t) = f_v[x_1(t) - x_2(t)]}$$

free damper

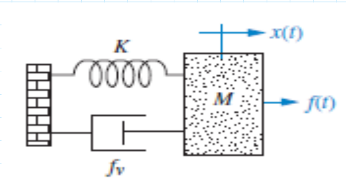


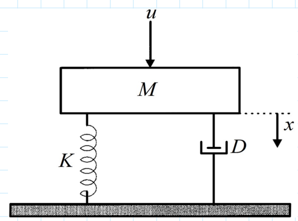
$$\boxed{⑤ \; f_k(t) = K[x_1(t) - x_2(t)]}$$

free spring



A. Physical Model

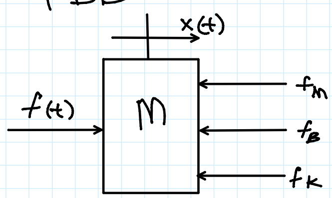


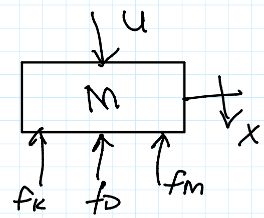


B. Time Domain

- DE of the physical model

FBD





$$f(t) = f_m + f_B + f_K$$

$$f(t) = M\ddot{x} + f_v\dot{x} + Kx$$

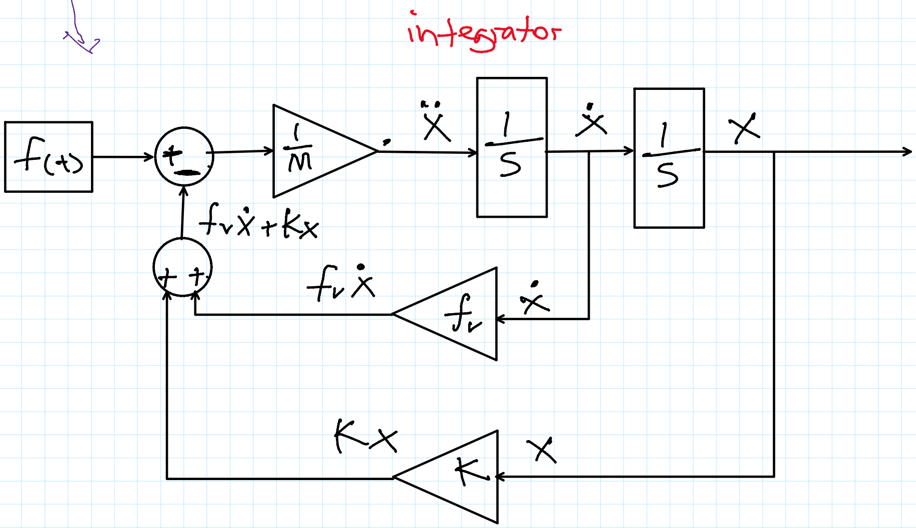
$$M\ddot{x} = f(t) - f_v\dot{x} - Kx$$

$$u = f_m + f_D + f_K$$

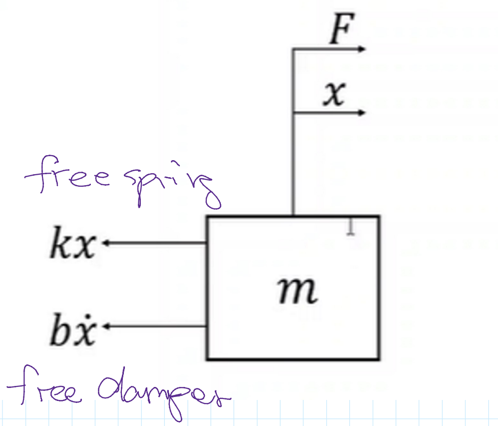
$$u = M\ddot{x} + f_v\dot{x} + Kx$$

$$\boxed{\ddot{x} = \frac{1}{M}\left[f(t) - f_v\dot{x} - Kx\right]}$$

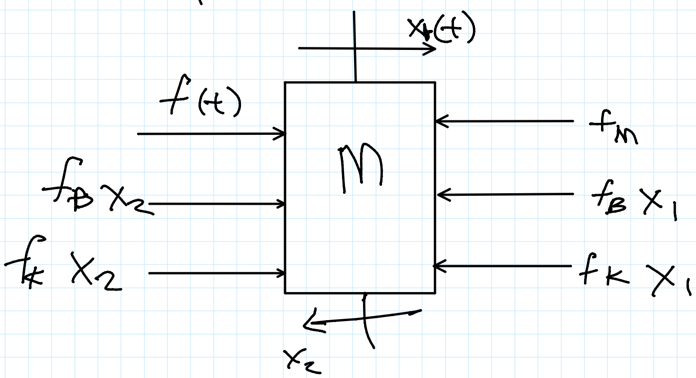
$$\ddot{x} \to \int \to \dot{x} \to \int x$$



$$\frac{1}{M}\left[f(t) - (f_v\dot{x} + Kx)\right] = \ddot{x}$$



FBD

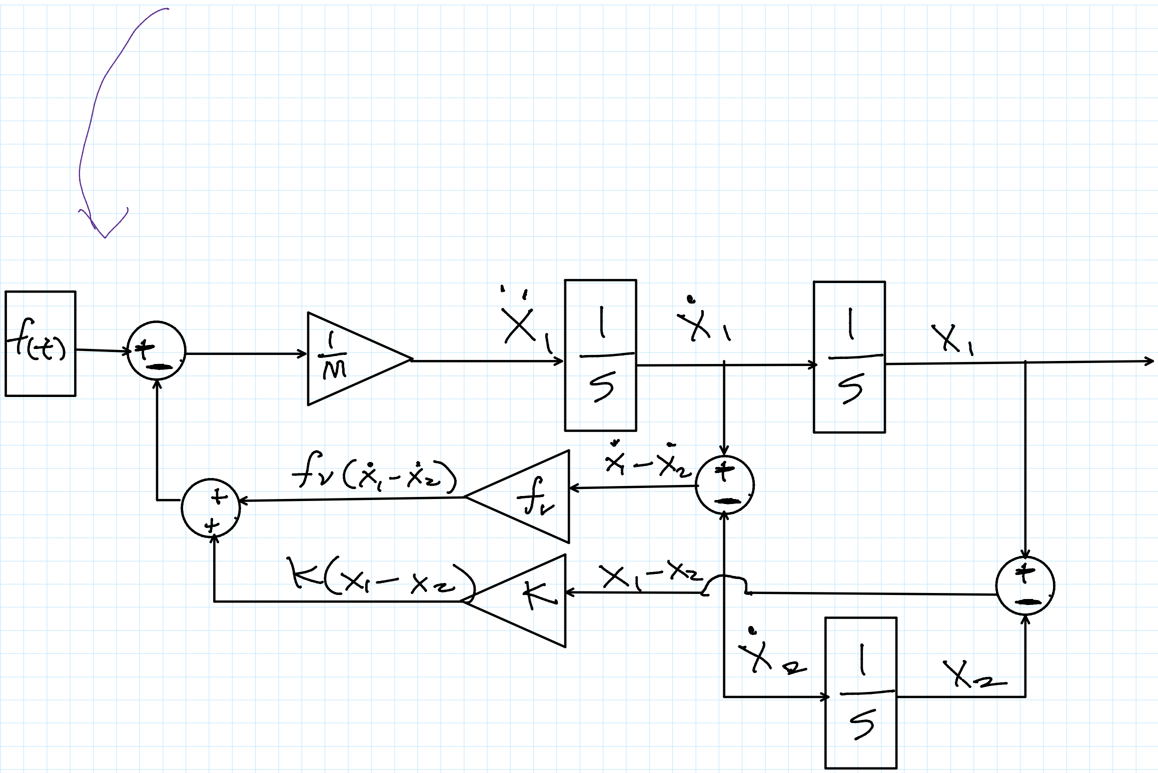


$$f(t) = f_M + f_B x_1 + f_K x_1 - f_B x_2 - f_K x_2$$

$$f(t) = M\ddot{x}_1 + f_v \dot{x}_1 + K x_1 - f_v \dot{x}_2 - K x_2$$

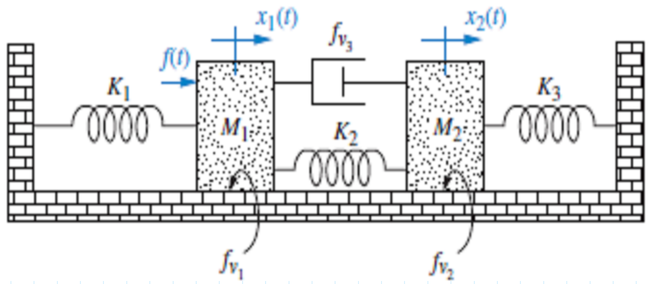
$$f(t) = M\ddot{x}_1 + f_v(\dot{x}_1 - \dot{x}_2) + K(x_1 - x_2)$$

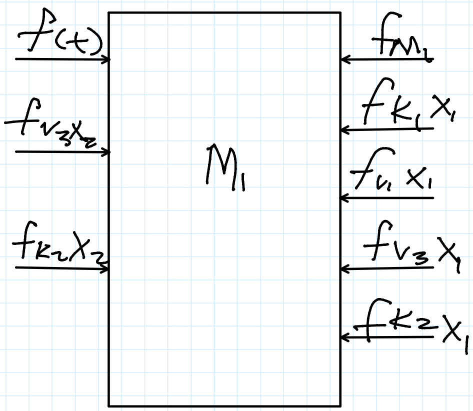
$$\boxed{\ddot{x}_1 = \frac{1}{M}\left[f(t) - f_v(\dot{x}_1 - \dot{x}_2) - K(x_1 - x_2)\right]}$$

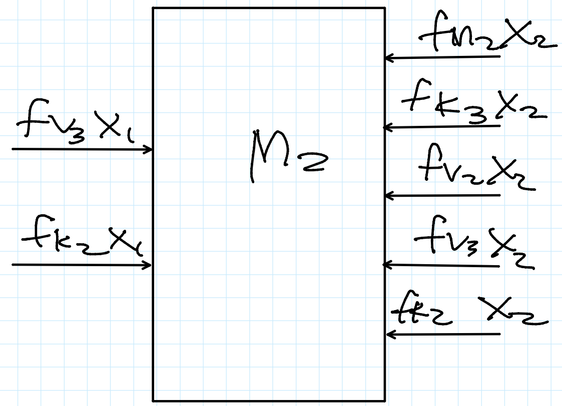


f(t)
+
−
1/M
$\ddot{x}_1$
1/s
$\dot{x}_1$
1/s
x_1
$f_v(\dot{x}_1-\dot{x}_2)$
f_v
$\dot{x}-\dot{x}_2$
+
−
+
+
$K(x_1-x_2)$
K
x_1-x_2
+
−
$\dot{x}_2$
1/s
x_2

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure







$$f(t) = M_1\ddot{x}_1 + f_{v_1}\dot{x}_1 + f_{v_3}\dot{x}_1 + K_1x_1 + K_2x_1 - f_{v_3}\dot{x}_2 - K_2x_2$$

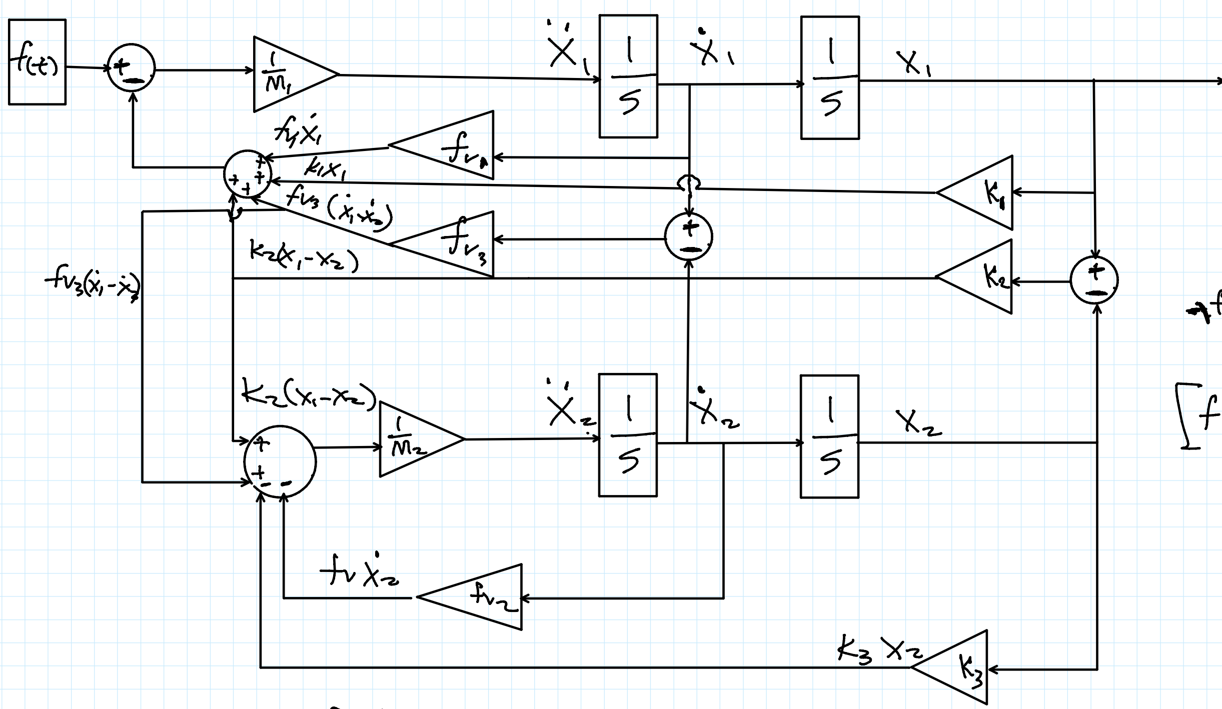
$$f(t) = M_1\ddot{x}_1 + f_{v_1}\dot{x}_1 + f_{v_3}(\dot{x}_1 - \dot{x}_2) + K_1x_1 + K_2(x_1 - x_2) \quad (1)$$

$$0 = M_2\ddot{x}_2 + f_{v_2}\dot{x}_2 + f_{v_3}\dot{x}_2 + f_{K_2}x_2 + f_{K_3}x_2 - f_{v_3}\dot{x}_1 - f_{K_2}x_1$$

$$0 = M_2\ddot{x}_2 + f_{v_2}\dot{x}_2 + f_{v_3}(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1) + K_3x_2 \quad (2)$$

$$\ddot{x}_1 = \frac{1}{M_1}\left[f(t) - f_{v_1}\dot{x}_1 - f_{v_3}(\dot{x}_1 - \dot{x}_2) - K_1x_1 - K_2(x_1 - x_2)\right] \quad (3)$$

$$\ddot{x}_2 = \frac{1}{M_2}\left[-f_{v2}\dot{x}_2 - f_{v3}(\dot{x}_2-\dot{x}_1) - K_2(x_2-x_1) - K_3 x_2\right] \quad (2)$$



$$-f_{v3}(\dot{x}_1 - x_2) \cancel{=} f_{v3}(\dot{x}_1-\dot{x}_2)$$

$$-f_{v3}(-x_2+\dot{x}_1) + K_2(-x_2+x_1)$$

$$- \cdot - = +$$

$$\left[f_{v3}(\dot{x}_1-\dot{x}_2) + k_2(x_1-x_2) - f_{v2}\dot{x}_2 - k_3 x_2\right]\frac{1}{m} = \ddot{x}_2$$

$$\ddot{x}_2 = \frac{1}{M_2}\left[-f_{v2}\dot{x}_2 + f_{v3}(-\dot{x}_2+\dot{x}_1) + K_2(-x_2+x_1) - K_3 x_2\right]$$

$$+ = - \cdot -$$