Translational Mechanical System - ME 4210

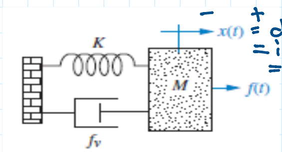
Saturday, 8 February 2025 4:52 pm

Translation - motion along straight line

Degrees of Freedom (DOF)

Defined as the minimum number of independent parameters/ variables/ coordinates needed to describe a system completely.

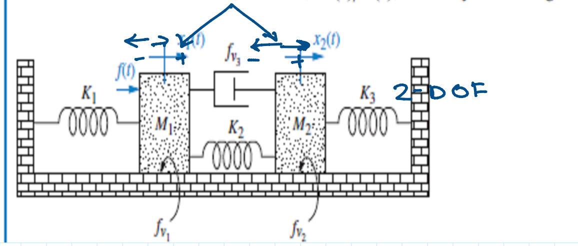
1-DOF



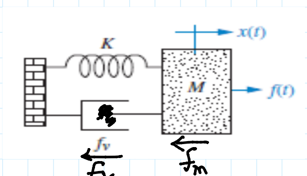
$x(t)$ = displacement = independent variable = used to describe the system

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure

independent



2-DOF



$f(t)$ = input force

$f_v = B = D$ = damper = friction

M = Mass

$f_m = Ma$

$f_m = M \frac{d^2 x(t)}{dt^2}$

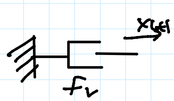
$\boxed{① f_m = M\ddot{x}}$

$f_v = BV$

$f_v = B \frac{dx(t)}{dt}$

$\boxed{② f_v = B\dot{x}}$

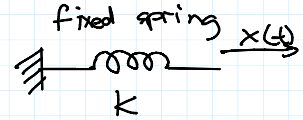
fixed damper



Spring

$\boxed{④ f_K = K\,x(t)}$

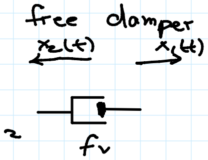
fixed spring



$\boxed{③ f_v = B(\dot{x}_1 - \dot{x}_2)}$

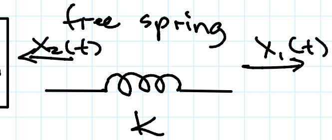
$f(t) = B\dot{x}_1 - B\dot{x}_2$

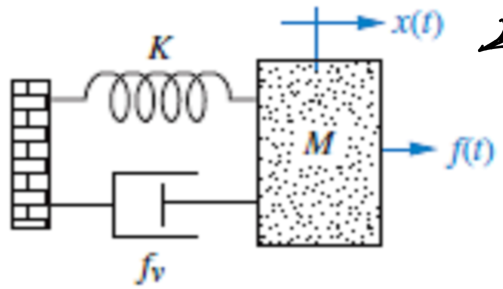
free damper



$\boxed{⑤ f_K = K[x_1(t) - x_2(t)]}$

free spring





MATLAB

1) Physical Model - Physical Simulink Model

2) Time Domain Model → D. E.

↳ Block Diagram Simulink Model

3) Frequency Domain Model

↳ Laplace Transform

↳ Transfer Function

↳ B. D. simulink model

↳ Programming Language

4) Time Domain State Space Model

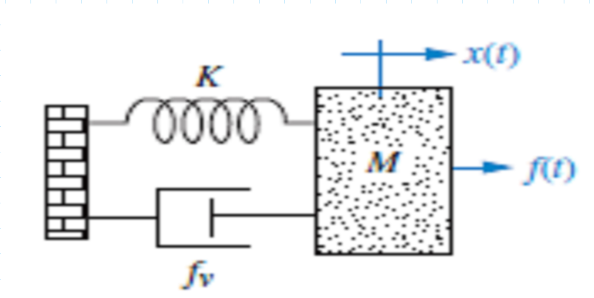
↳ D. E.

↳ Matrix & Vectors

↳ Programming Language

↳ Modern Modelling

Time Domain Model



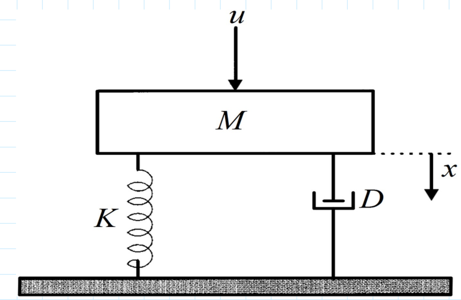
① $f_m = M\ddot{x}$

② $f_v = B\dot{x}$

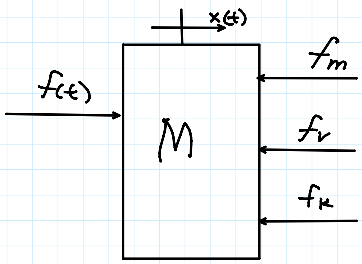
③ $f_v = B(\dot{x}_1 - \dot{x}_2)$

④ $f_K = K\,x(t)$

⑤ $f_K = K[x_1(t) - x_2(t)]$



FBD



$$f(t) = f_m + f_B + f_K$$

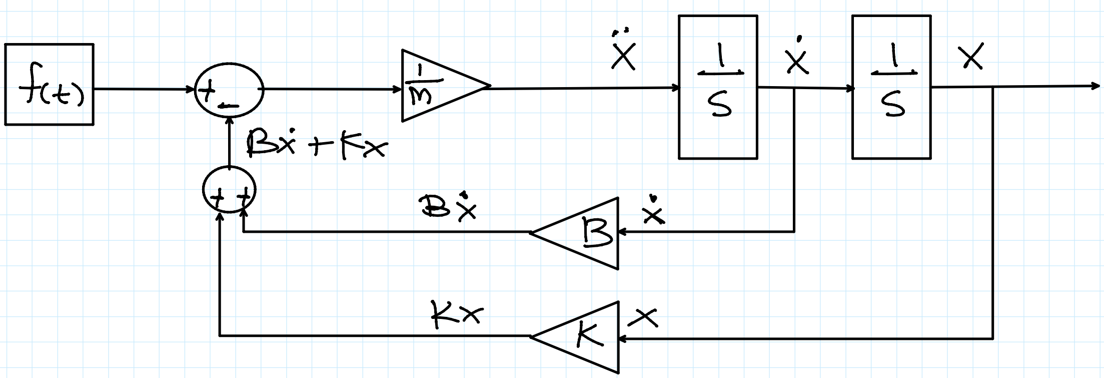
$$f(t) = M\ddot{x} + B\dot{x} + Kx$$

$$\boxed{M\ddot{x} = f(t) - B\dot{x} - Kx}$$

$$\boxed{\ddot{x} = \frac{1}{M}\left[f(t) - B\dot{x} - Kx\right]}$$

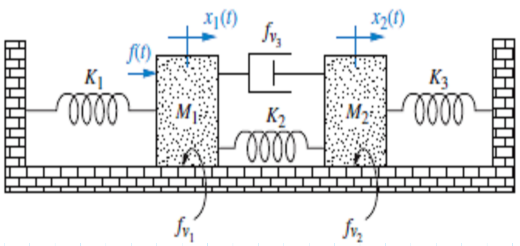
$$\ddot{x} \longrightarrow \int \longrightarrow \dot{x} \longrightarrow \int x$$

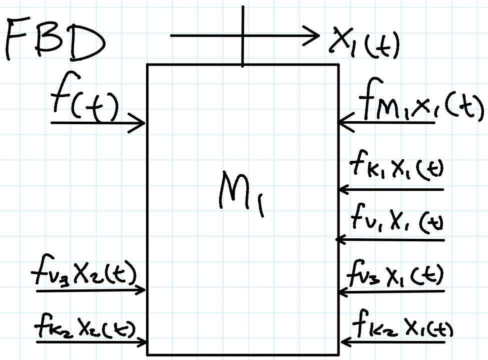
integrator



$$\boxed{\frac{1}{M}\left[f(t) - B\dot{x} - Kx\right] = \ddot{x}}$$

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure





$$\boxed{① \ f_m = m\ddot{x}}$$

$$\boxed{② \ f_v = B\dot{x}}$$

$$\boxed{③ \ f_v = B(\dot{x}_1 - \dot{x}_2)}$$

$$\boxed{④ \ f_K = K\,x(t)}$$

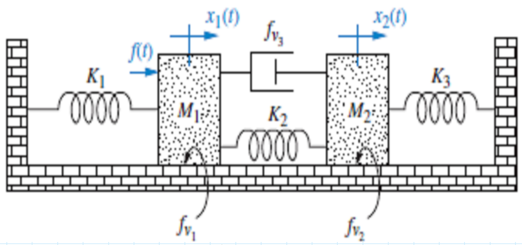
$$\boxed{⑤ \ f_K = K[x_1(t) - x_2(t)]}$$

$$f(t) = f_{M_1 x_1(t)} + f_{K_1 x_1(t)} + f_{v_1 x_1(t)} + f_{v_3 x_1(t)} + f_{K_2 x_1(t)} - f_{v_3 x_2(t)} - f_{K_2 x_2(t)}$$

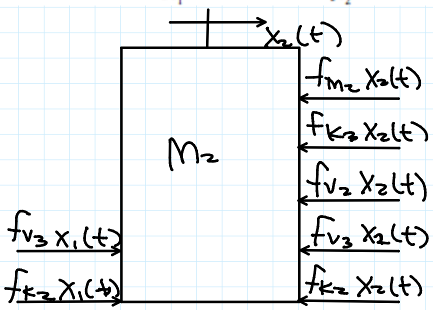
$$f(t) = M_1\ddot{x}_1 + B_3(\dot{x}_1 - \dot{x}_2) + K_2(x_1 - x_2) + B_1\dot{x}_1 + K_1 x_1$$

$$\boxed{\ddot{x}_1 = \frac{1}{M_1}\left[f(t) - B_3(\dot{x}_1 - \dot{x}_2) - K_2(x_1 - x_2) - B_1\dot{x}_1 - K_1 x_1\right]}$$

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure



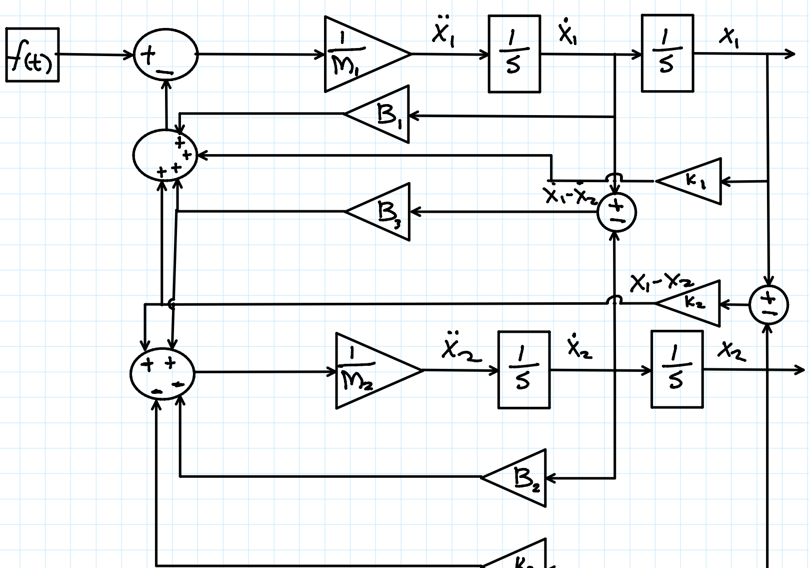
$$0 = f_{M_2}x_2(t) + f_{K_3}x_2(t) + f_{v_2}x_2(t) + f_{v_3}x_2(t) + f_{K_2}x_2(t) - f_{v_3}x_1(t) - f_{K_2}x_1(t)$$



$$0 = M_2\ddot{x}_2 + B_3(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1) + B_2\dot{x}_2 + K_3x_2$$

$$\boxed{\ddot{x}_2 = \frac{1}{M_2}\left[-B_3(\dot{x}_2 - \dot{x}_1) - K_2(x_2 - x_1) - B_2\dot{x}_2 - K_3x_2\right]}$$

$$\boxed{\ddot{x}_1 = \frac{1}{M_1}\left[f(t) - B_3(\dot{x}_1 - \dot{x}_2) - K_2(x_1 - x_2) - B_1\dot{x}_1 - K_1x_1\right]}$$



$$\boxed{\ddot{x}_2 = \frac{1}{M_2}\left[+B_3(-\dot{x}_2 + \dot{x}_1) + K_2(-x_2 + x_1) - B_2\dot{x}_2 - K_3x_2\right]}$$