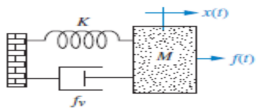
Translational Mechanical System: Frequency Domain

Thursday, 27 February 2025 6:55 pm

Find the transfer function $G_{(s)} = X_{(s)}/F_{(s)}$.



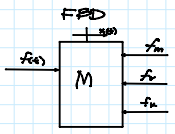
① $f_m = M\ddot{x}$

② $f_k = Kx(t)$

③ $f_v = B\dot{x}$

④ $f_k = K[x_1(t) - x_2(t)]$

⑤ $f_v = B(\dot{x}_1 - \dot{x}_2)$



$f(t) = f_m + f_v + f_k$

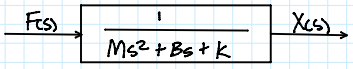
$f(t) = M\ddot{x} + B\dot{x} + Kx$

$\mathcal{L}\{f(t) = M\ddot{x} + B\dot{x} + Kx\}$

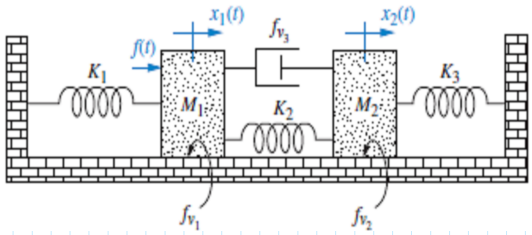
$F(s) = Ms^2X(s) + BsX(s) + kX(s)$

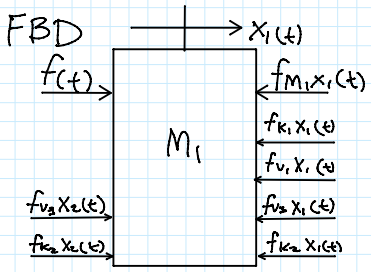
$F(s) = X(s)[Ms^2 + Bs + k]$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$



PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure





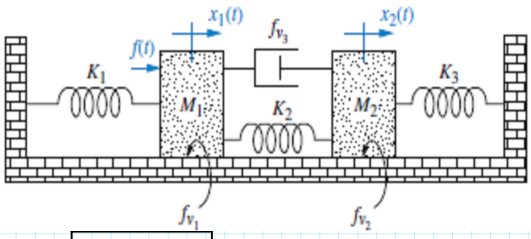
$f(t) = f_{M_1}x_1(t) + f_{K_1}x_1(t) + f_{v_1}x_1(t) + f_{v_3}x_1(t) + f_{K_2}x_1(t) - f_{v_3}x_2(t) - f_{K_2}x_2(t)$

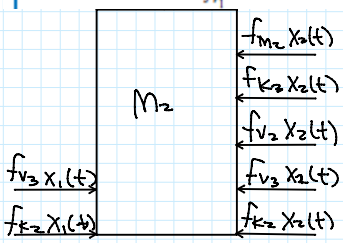
$f(t) = M_1\ddot{x}_1 + B_1\dot{x}_1 + B_3\dot{x}_1 + k_1x_1 + k_2x_1 - B_3\dot{x}_2 - k_2x_2$

$F(s) = M_1s^2X_1(s) + B_1sX_1(s) + B_3sX_1(s) + K_1X_1(s) + k_2X_1(s) - B_3sX_2(s) - k_2X_2(s)$

$F(s) = X_1(s)[M_1s^2 + B_1s + B_3s + k_1 + k_2] - X_2(s)[B_3s + k_2]$

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure





$$0 = f_{M_2}x_2(t) + f_{K_3}x_2(t) + f_{v_2}x_2(t) + f_{v_3}x_2(t) + f_{K_2}x_2(t) - f_{v_3}x_1(t) - f_{K_2}x_1(t)$$

$$0 = M_2\ddot{x}_2 + B_2\dot{x}_2 + B_3\dot{x}_2 + k_2x_2 + k_3x_2 - B_3\dot{x}_1 - k_2x_1$$

$$0 = M_2s^2X_2(s) + B_2sX_2(s) + B_3sX_2(s) + k_2X_2(s) + k_3X_2(s) - B_3sX_1(s) - k_2X_1(s)$$

$$0 = X_2(s)\left[M_2s^2 + B_2s + B_3s + k_2 + k_3\right] - X_1(s)\left[B_2s + k_2\right]$$

$$F(s) = X_1(s)\left[M_1s^2 + B_1s + B_3s + k_1 + k_2\right] - X_2(s)\left[B_3s + k_2\right] \quad (1)$$

$$0 = X_2(s)\left[M_2s^2 + B_2s + B_3s + k_2 + k_3\right] - X_1(s)\left[B_2s + k_2\right] \quad (2)$$

$$\frac{X_1(s)}{F(s)} = ?$$

$$X_2(s) = \frac{X_1(s)\left[B_2s + k_2\right]}{\left[M_2s^2 + B_2s + B_3s + k_2 + k_3\right]}$$

$$F(s) = X_1(s)\left[M_1s^2 + B_1s + B_3s + k_1 + k_2\right] - \left\{\frac{X_1(s)\left[B_2s + k_2\right]}{\left[M_2s^2 + B_2s + B_3s + k_2 + k_3\right]}\right\}\left[B_3s + k_2\right]$$

$$\rightarrow -\left[B_2B_3s^2 + B_2k_2s + B_3k_2s + k_2^2\right]$$

$$F(s) = X_1(s)\left\{\frac{\left[M_1s^2 + B_1s + B_3s + k_1 + k_2\right]\left[M_2s^2 + B_2s + B_3s + k_2 + k_3\right] - X_1(s)\left[B_2s + k_2\right]\left[B_3s + k_2\right]}{\left[M_2s^2 + B_2s + B_3s + k_2 + k_3\right]}\right\}$$

$$M_1M_2s^4 + M_1B_2s^3 + M_1B_3s^3 + M_1k_2s^2 + M_1k_3s^2 + M_2B_1s^3 + B_1B_2s^2 + B_1B_3s^2 + B_1k_2s + B_1k_3s + M_2B_3s^3 + B_2B_3s^2 + B_3s^2 + B_3k_2s + B_3k_3s + M_2k_1s^2 + B_2k_1s + B_3k_1s + K_1K_2 + K_1K_2 + M_2k_2s^2 + B_2k_2s + B_3k_2s + k_2^2 + k_2k_3$$

$$F(s)=X_1(s)\left[\frac{M_1M_2s^4+(M_1B_3+M_1B_3+M_2B_1+M_2B_3)s^3+(M_1K_2+M_1K_3+B_1B_2+B_1B_3+B_3+M_2k_1+M_2k_2)s^2+(B_1k_2+B_1k_3+B_3k_2+B_3k_3+B_2k_1+B_3k_1)s+k_1k_2+k_1k_3+k_2k_3}{M_2s^2+B_2s+B_3s+k_2+k_3}\right]$$

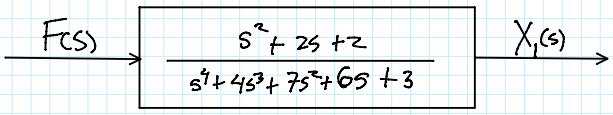
$$\frac{M_2s^2+B_2s+B_3s+k_2+k_3}{M_1M_2s^4+(M_1B_3+M_1B_3+M_2B_1+M_2B_3)s^3+(M_1K_2+M_1K_3+B_1B_2+B_1B_3+B_3+M_2k_1+M_2k_2)s^2+(B_1k_2+B_1k_3+B_3k_2+B_3k_3+B_2k_1+B_3k_1)s+k_1k_2+k_1k_3+k_2k_3}=\frac{X_1(s)}{F(s)}$$

if $M_1=M_2=1$

$B_1=B_2=B_3=1$

$K_1=k_2=k_3=1$

$$\frac{s^2+2s+2}{s^4+4s^3+7s^2+6s+3}=\frac{X_1(s)}{F(s)}$$



$$F(s)=X_1(s)\left[M_1s^2+B_1s+B_3s+k_1+k_2\right]-X_2(s)\left[B_3s+k_2\right] \quad (1)$$

$$0=X_2(s)\left[M_2s^2+B_2s+B_3s+k_2+k_3\right]-X_1(s)\left[B_2s+k_2\right] \quad (2)$$

$$\frac{X_2(s)}{F(s)}=?$$

$$X_2(s)\left[M_2s^2+B_2s+B_3s+k_2+k_3\right]=X_1(s)\left[B_2s+k_2\right]$$

$$\frac{X_2(s)\left[M_2s^2+B_2s+B_3s+k_2+k_3\right]}{B_2s+k_2}=X_1(s)$$

$$F(s)=\frac{X_2(s)\left[M_2s^2+B_2s+B_3s+k_2+k_3\right]\left[M_1s^2+B_1s+B_3s+k_1+k_2\right]-X_2(s)\left[B_3s+k_2\right]}{B_2s+k_2}$$

$$F(s)=X_2(s)\left\{\frac{\left[M_2s^2+B_2s+B_3s+k_2+k_3\right]\left[M_1s^2+B_1s+B_3s+k_1+k_2\right]-\left[B_3s+k_2\right]\left[B_2s+k_2\right]}{B_2s+k_2}\right\}$$

if $M_1=M_2=1$

$B_1=B_2=B_3=1$

$K_1=k_2=k_3=1$

$$F(s) = X_2(s) \left\{ \frac{[s^2+2s+2][s^2+2s+2] - [s+1][s+1]}{s+1} \right\}$$

$$F(s) = X_2(s) \left\{ \frac{s^4+2s^3+2s^2+2s^3+4s^2+4s+2s^2+4s+4 - [s^2+s+s+1]}{s+1} \right\}$$

$$F(s) = X_2(s) \left\{ \frac{s^4+4s^3+8s^2+8s+4 \quad - [s^2+s+s+1]}{s+1} \right\}$$

$$F(s) = X_2(s) \left\{ \frac{s^4+4s^3+7s^2+6s+3}{s+1} \right\}$$

$$\frac{X_2(s)}{F(s)} = \frac{s+1}{s^4+4s^3+7s^2+6s+3}$$