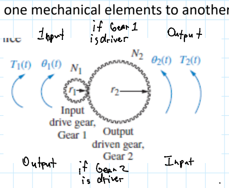
Gear System

Thursday, 14 March 2024 8:05 am

Gears are mechanical elements w/ teeth, used to transmit power from one mechanical elements to another.



N = number of teeth

$T(t)$ = Torque

$\theta(t)$ = angular displacement

if G1 is driver

$\theta_1(t) \rightarrow \boxed{G(s)} \rightarrow \theta_2(t)$ $\quad$ $T_1(t) \rightarrow \boxed{G(s)} \rightarrow T_2(t)$

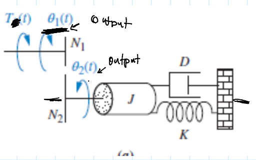
$\theta_1(t) \propto \theta_2(t)$ $\quad$ $T_1(t) \propto T_2(t)$

$\theta_1(t) = k * \theta_2(t)$ $\quad$ $T_1(t) = k * T_2(t)$

1 - DOF

1 - independent equation

$$\text{Gear Ratio} = \frac{N_{Driven}}{N_{Driver}} = \frac{N^{Destination}}{N^{Source}}$$



$$G(s) = \frac{\theta_1(s)}{T(s)} \quad or \quad \frac{\theta_2(s)}{T(s)}$$

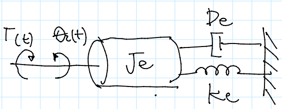
$N_1 = 5$

$N_2 = 10$

$J = D = k = 1$

if Gear 1 is driver : $\frac{N^D}{N^S} = \frac{N_2}{N_1} = \frac{10}{5} = 2$

if Gear 2 is driver : $\frac{N^D}{N^S} = \frac{N_1}{N_2} = \frac{5}{10} = \frac{1}{2}$



$$\frac{\theta_1(s)}{T(s)}$$

$$J_e = \Sigma J * \left(\frac{N^D}{N^S}\right)^2$$

$$D_e = \Sigma D * \left(\frac{N^D}{N^S}\right)^2$$

$$K_e = \Sigma K * \left(\frac{N^D}{N^S}\right)^2$$

$$\frac{N^D}{N^S} = \frac{N_1}{N_1} = \left(\frac{5}{5}\right) = 1$$

$$J_e = 1 * \left(\frac{N_1}{N_2} = \frac{5}{10}\right)^2 = \frac{1}{4}$$

$$D_e = 1 * \left(\frac{5}{10}\right)^2 = \frac{1}{4}$$

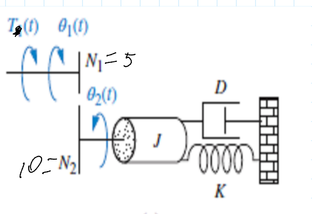
$$K_e = 1 * \left(\frac{5}{10}\right)^2 = \frac{1}{4}$$

$$T(t) = J_e \ddot{\theta}_1(t) + D_e \dot{\theta}_1(t) + K_e \theta_1(t)$$

$$T(t) = \frac{1}{4}\ddot{\theta}(t) + \frac{1}{4}\dot{\theta}(t) + \frac{1}{4}\theta(t)$$

$$T(s) = \frac{1}{4}\theta_1(s)\left[s^2 + s + 1\right]$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{4}{s^2 + s + 1}}$$



$$\frac{\theta_2(s)}{T(s)}$$

$$J_e = 1 * \left(\frac{N_2}{N_2} = \frac{10}{10}\right)^2 = 1$$

$$D_e = 1 * \left(\frac{10}{10}\right)^2 = 1$$

$$K_e = 1 * \left(\frac{10}{10}\right)^2 = 1$$

$$\frac{N^D}{N^S} = \frac{10}{5} = 2$$

$$2\,T(t) = J_e \ddot{\theta}_2(t) + D_e \dot{\theta}_2(t) + K_e \theta_2(t)$$

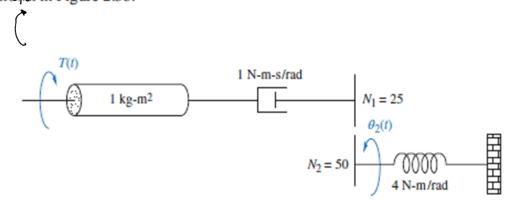
$$2\,T(t) = \ddot{\theta}_2(t) + \dot{\theta}_2(t) + \theta_2(t)$$

$$2\,T(s) = s^2\theta_2(s) + s\,\theta_2(s) + \theta_2(s)$$

$$\left\{2\,T(s) = \theta_2(s)\left[s^2 + s + 1\right]\right\} \frac{1}{2T(s)} \cdot 2 \cdot \frac{1}{s^2+s+1}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{2}{s^2 + s + 1}}$$

PROBLEM: Find the transfer function, $G(s) = \theta_2(s)/T(s)$, for the rotational mechanical system with gears shown in Figure 2.33.



$\frac{\theta_1(s)}{T(s)}$ and $\frac{\theta_2(s)}{T(s)}$

$J_e = 1 * \left(\frac{50}{25}\right)^2 = 4$

$D_e = 1 * \left(\frac{50}{25}\right)^2 = 4$

$K_e = 4 * \left(\frac{50}{50}\right)^2 = 4$

$\frac{\theta_2(s)}{T(s)}$ $\qquad \frac{N^D}{N^S} = \frac{N_2}{N_1} = \frac{50}{25} = 2$

$2T(t) = J_e\ddot{\theta}_2(t) + D_e\dot{\theta}_2(t) + K_e\theta_2(t)$

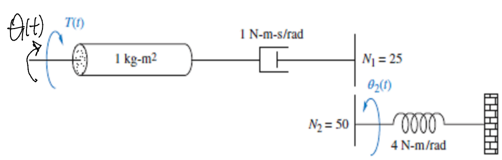
$\cancel{2}T(s) = 4s^2\theta_2(s) + 4s\theta_2(s) + 4\theta_2(s)$

$T(s) = 2s^2\theta_2(s) + 2s\theta_2(s) + 2\theta_2(s)$

$T(s) = 2\theta_2(s)\left[s^2 + s + 1\right]$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{1/2}{s^2+s+1}}$$

PROBLEM: Find the transfer function, $G(s) = \theta_2(s)/T(s)$, for the rotational mechanical system with gears shown in Figure 2.33.



$\frac{\theta_1(s)}{T(s)}$

$J_e = 1 * \left(\frac{25}{25}\right)^2 = 1$

$D_e = 1 * \left(\frac{25}{25}\right)^2 = 1$

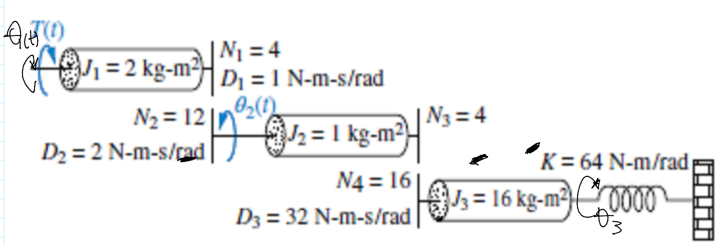
$K_e = 4\left(\frac{N_1}{N_2} = \frac{25}{50}\right)^2 = 1$

$\frac{N^D}{N^S} = \frac{N_1}{N_1} = \frac{25}{25} = 1$

$T(t) = J_e\ddot{\theta}_1(t) + D_e\dot{\theta}_1(t) + K_e\theta_1(t)$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{1}{s^2+s+1}}$$

36. For the rotational system shown in Figure P2.21, find the transfer function, $G(s) = \theta_2(s)/T(s)$. [Section: 2.7]



$J_e = 2 \cdot \left(\frac{12}{4}\right)^2 + 1 * \left(\frac{12}{12} \cdot \frac{4}{4}\right)^2 + 16 * \left(\frac{4}{16}\right)^2 = 20$

$D_e = 1 * \left(\frac{12}{4}\right)^2 + 2\left(\frac{12}{12} \cdot \frac{4}{4}\right)^2$

$$+\;2\left(\frac{12}{12}\cdot\frac{4}{4}\right)^2 + 32\left(\frac{4}{16}\right)^2 = 13$$

$$K_e = 64 * \left(\frac{4}{16}\right)^2 = 4$$

$$\frac{\theta_2(s)}{T(s)} \qquad \frac{N^D}{N^S} = \frac{N_2}{N_1} = \frac{12}{4} = 3$$

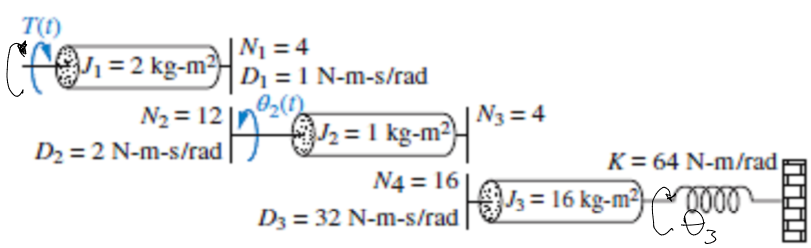
$$3T(t) = J_e\ddot{\theta}_2(t) + D_e\dot{\theta}_2(t) + K_e\theta_2(t)$$

$$3T(s) = 20s^2\theta_2(s) + 13s\,\theta_2(s) + 4\theta_2(s)$$

$$\left\{3T(s) = \theta_2(s)\left[20s^2+13s+4\right]\right\} \quad \frac{1}{3T(s)},\ 3,\ \frac{1}{20s^2+13s+4}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{3}{20s^2+13s+4}}$$

36. For the rotational system shown in Figure P2.21, find the transfer function, $G(s) = \theta_2(s)/T(s)$. [Section: 2.7]



$$\boxed{\frac{\theta_3(s)}{T(s)}}$$

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$$J_e = 2*\left(\frac{12}{4}*\frac{16}{4}\right)^2 + 1*\left(\frac{16}{4}\right)^2 + 16*\left(\frac{16}{16}\right)^2 = 320$$

$$D_e = 1*\left(\frac{12}{4}*\frac{16}{4}\right)^2 + 2*\left(\frac{16}{4}\right)^2 + 32*\left(\frac{16}{16}\right)^2 = 208$$

$$K_e = 64\left(\frac{16}{16}\right)^2 = 64$$

$$\frac{\theta_3(s)}{T(s)} \qquad \frac{N^D}{N^S} = \frac{12}{4}*\frac{16}{4} = 12$$

$$12T(t) = J_e\ddot{\theta}_3(t) + D_e\dot{\theta}_3(t) + K_e\theta_3(t)$$

$$12T(s) = 320s^2\theta_3(s) + 208s\,\theta_3(s) + 64\theta_3(s)$$

$$12T(s) = 16\theta_3(s)\left[20s^2 + 13s + 4\right]$$

$$\frac{12}{16} = \frac{\theta_3(s)\left[20s^2+13s+4\right]}{T(s)}$$

$$\frac{3}{4} = \frac{\theta_3(s)\left[20s^2+13s+4\right]}{T(s)}$$

$$\boxed{\frac{\theta_3(s)}{T(s)} = \frac{3/4}{20s^2+13s+4}}$$