Case 1. Roots of the Denominator of F(s) Are Real and Distinct

$$\mathcal{L}^{-1}\left[\frac{s-5}{s^2+s-6}\right]=\mathcal{L}^{-1}\left[\frac{s-5}{(s+3)(s-2)}\right]$$

$$f(s)=\frac{s-5}{(s+3)(s-2)}=\frac{A}{s+3}+\frac{B}{s-2}$$

$$(s+3)(s-2)\left[\frac{s-5}{(s+3)(s-2)}=\frac{A(s-2)+B(s+3)}{(s+3)(s-2)}\right]$$

$s=2$, $s=-3$

$$s-5=A(s-2)+B(s+3)$$

$s=2$, $s=-3$

$-3=5B$, $-8=-5A$

$-\frac{3}{5}=B$, $\frac{8}{5}=A$

$$\mathcal{L}^{-1}\left[\frac{8/5}{s+3}-\frac{3/5}{s-2}\right]=$$

$$\boxed{f(t)=\left(\frac{8}{5}e^{-3t}-\frac{3}{5}e^{2t}\right)u(t)}$$

$0 \stackrel{?}{=} 0$

$$s^3-3s^2-13s+15=0$$

$f(1)=1-3-13+15 \stackrel{?}{=} 0$

$0 \stackrel{\checkmark}{=} 0 \therefore (s-1)$

$f(2)=(8)-12-26+15=0$

$-15 \stackrel{?}{=} 0$

$f(-3)=-27-27+39+15=0$

$-54+54=0$

$0 \stackrel{\checkmark}{=} 0 \therefore (s+3)$

$f(5)=125-75-65+15=0$

$0 \stackrel{\checkmark}{=} 0 \therefore (s-5)$

$$\frac{A}{s-2}+\frac{B}{s+3}+\frac{C}{s-5}$$

Case 2. Roots of the Denominator of F(s) Are Real and Repeated

$$\mathcal{L}^{-1}\left[\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{(s+2)^2}\right]$$

$$\left[\frac{2}{(s+1)(s+2)^2}=\frac{A(s+2)^2+B(s+1)(s+2)+C(s+1)}{(s+1)(s+2)^2}\right]$$

$s=-2$, $s=-1$, $s=0$

$2=-C$, $2=A$, $2=2(4)+B(2)-2(1)$

$-2=C$

$-8+2+2=2B$

$4=2B$

$-2=B$

$$\mathcal{L}^{-1}[F(s)]=\mathcal{L}^{-1}\left[\frac{2}{(s+1)(s+2)^2}\right]=\mathcal{L}^{-1}\left[\frac{2}{s+1}-\frac{2}{s+2}-\frac{2}{(s+2)^2}\right]$$

$$2\mathcal{L}^{-1}\left[\frac{1}{(s+2)^2}\right]=e^{-2t}\cdot\frac{t^{2-1}}{(2-1)!}$$

$$\mathcal{L}^{-1}\left[\frac{n!}{(s+a)^{n+1}}\right]=e^{-at}\cdot\frac{t^{N-1}}{(N-1)!}u(t)$$

$$= 2e^{-2t}t$$

$$\mathcal{L}^{-1}\left[\frac{2}{s+1} - \frac{2}{s+2} - \frac{2}{(s+2)^2}\right]$$

$$f(t) = 2e^{-t} - 2e^{-2t} - 2e^{-2t}t$$

$$\boxed{f(t) = 2\left(e^{-t} - e^{-2t} - e^{-2t}t\right)u(t)}$$

CASE 3. ROOTS OF THE DENOMINATOR OF F (S) ARE COMPLEX OR IMAGINARY

$$\mathrm{L}^{-1}[F(s)] = \mathrm{L}^{-1}\left[\frac{3}{s(s^2+2s+5)}\right] = 3\mathcal{L}^{-1}\left[\frac{A}{s} + \frac{Bs+C}{s^2+2s+5}\right]$$

$$\cancel{D}\left[\frac{1}{\cancel{s(s^2+2s+5)}} = \frac{A(s^2+2s+5)+s(Bs+C)}{\cancel{s(s^2+2s+5)}}\right]\cancel{D}$$

$$1 = A(s^2+2s+5) + s(Bs+C)$$

$$s=0$$

$$1 = 5A$$

$$\frac{1}{5} = A$$

$$1 = \frac{1}{5}(s^2+2s+5) + Bs^2 + Cs$$

$$1 = \frac{s^2}{5} + \frac{2s}{5} + \frac{5}{5} + \frac{5Bs^2 + 5Cs}{5}$$

$$5 = s^2 + 2s + 5 + 5Bs^2 + 5Cs$$

$$0 = s^2(5B+1) + s(5C+2)$$

$$B = -\frac{1}{5}, \quad C = -\frac{2}{5}$$

$$3\mathcal{L}^{-1}\left[\frac{A}{s} + \frac{Bs+C}{s^2+2s+5}\right] = 3\mathcal{L}^{-1}\left[\frac{1/5}{s} - \frac{\frac{1}{5}s + \frac{2}{5}}{s^2+2s+5}\right]$$

$$= \frac{3}{5}\left\{\mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{s+2}{s^2+2s+5}\right]\right\}$$

$$\mathrm{L}^{-1}\left[\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right] = e^{-at}[\cos\omega t + \sin\omega t]\,u(t)$$

$$(s^2+2s)+5 \qquad (s+1)+2^2$$

$\to (s+1)+2^2$

$(s^2+2s)+5$

$(s+1)(s+1)$

s

$2s$

$s^2+2s+1+4$

$\boxed{(s+1)^2+2^2}$

$$\mathcal{L}^{-1}\left[\frac{s+2}{(s+1)^2+2^2}\right] = \frac{K_1(s+1)+K_2(2)}{(s+1)^2+2^2}$$

$s+2 = K_1(s+1) + 2K_2$

$s=-1, \quad s=0$

$1 = 2K_2 \qquad 2 = K_1+1$

$\frac{1}{2} = K_2 \qquad 1 = K_1$

$$\mathrm{L}^{-1}\left[\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right] = e^{-at}\left[cos\omega t + sin\omega t\right]u(t)$$

$$= \frac{3}{5}\left\{\mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{(s+1)+\frac{1}{2}(2)}{(s+1)^2+2^2}\right]\right\} =$$

$$\boxed{f(t) = \frac{3}{5}\left[1 - e^{-t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)\right]u(t)}$$