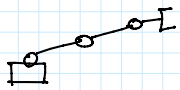
Jacobian Matrix Part 3: Singularity, Inverse Velocity and Force-Torque Analysis

Monday, 15 April 2024 8:53 am

Singularities

↳ Is a point where the robot loses one or more DOF. Here, it is impossible to move the end effector (tool) in a particular direction, regardless of the joint rates.



1) Wrist Singularity
2) Shoulder Singularity
3) Elbow singularity

Singularity = D(J)

→ if D(J) = 0, singularity

→ if D(J) ≠ 0, not singularity

$$Det(J) = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$Det(J) = a_1 \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1 \begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} - c_1 \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

Inverse Velocity

$$\dot{E} = J\dot{q}$$

$$J^{-1}\dot{E} = \dot{q}$$

$$\overline{J} \quad \overline{J}$$

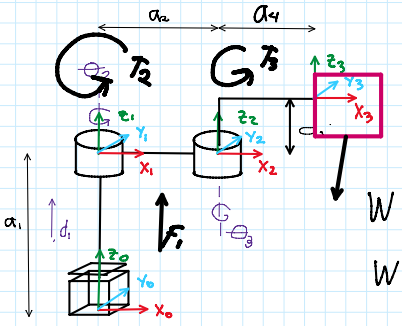
$$\begin{bmatrix} 0 & 0 & 1 \\ -1.592\times10^{-2} & 9.19\times10^{-3} & 0 \\ 3.09\times10^{-2} & 1.35\times10^{-3} & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{d}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$\dot{z} = \dot{d}_1$$

$$-1.592\times10^{-2}\,\dot{x} + 9.19\times10^{-3}\,\dot{y} = \dot{\theta}_2$$

$$3.09\times10^{-2}\,\dot{x} + 1.35\times10^{-3}\,\dot{y} = \dot{\theta}_3$$

Torques & Forces Analysis



W = due to gravity

W = mg

Notes:

→ We ignore the weight of the manipulator

→ We ignore the Force & Torque eq't. of manipulator

Prismatic → F

Revolute → T

Joints Torque-Force vector → $J = (J)^T F$ ← EE Torque force vector

$$\begin{vmatrix} F_1 \\ T_2 \\ T_3 \end{vmatrix} = (J)^T \begin{bmatrix} F_x \\ F_y \\ F_z \\ N_x \\ N_y \\ N_z \end{bmatrix}$$

$$M = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \qquad (M)^T = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$\begin{vmatrix} F_1 \\ T_2 \\ T_3 \end{vmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -119.09 & 10.42 & 0 \\ -59.9 & 10.42 & 0 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ N_x \\ N_y \\ N_z \end{bmatrix}$$

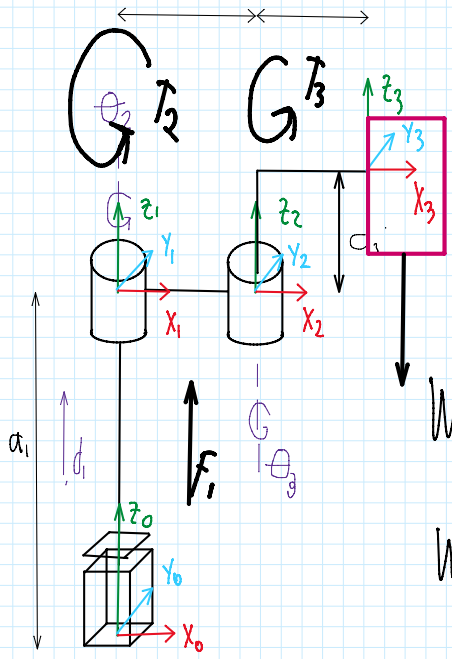
?N ?N

$F_1 = F_z$

$T_2 = -119.09 F_x + 10.42 F_y$

$T_3 = -59.9 F_x + 10.42 F_y$

$F_1 = ma = m_1 \frac{d^2 d_1}{dt^2}$ $\quad T_2 = I_2 \frac{d^2\theta_2}{dt^2}$ $\quad T_3 = I_3 \frac{d^2\theta_3}{dt^2}$



W = due to gravity

$W = mg$