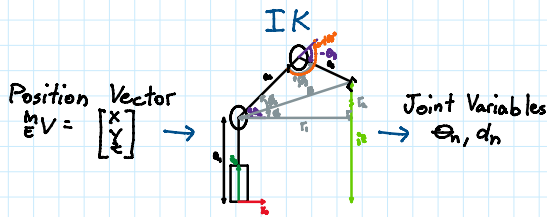


- For identifying the limits of your joint variables.

- For obtaining the trajectory solution.

- Easier to solve if standardized, has only one real solution.



- For mimicking the motion of a human arm

- For detailed positioning of end-effector

- Difficult to solve

- Non-linear Solutions
- Not standardized.
- There can be no solution or there can be 1 or more solutions.

HTM\_rad =

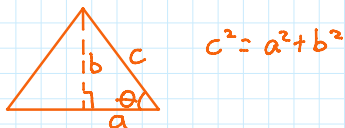
$$\begin{bmatrix} \cos(t_1)\cos(t_2)\cos(t_3) - \cos(t_1)\sin(t_2)\sin(t_3), & -\cos(t_1)\cos(t_2)\sin(t_3) - \cos(t_1)\cos(t_3)\sin(t_2), \\ \cos(t_2)\cos(t_3)\sin(t_1) - \sin(t_1)\sin(t_2)\sin(t_3), & -\cos(t_2)\sin(t_1)\sin(t_3) - \cos(t_3)\sin(t_1)\sin(t_2), \\ \cos(t_2)\sin(t_3) + \cos(t_3)\sin(t_2), & \cos(t_2)\cos(t_3) - \sin(t_2)\sin(t_3), \\ 0, & 0, \end{bmatrix}$$

Position Vector

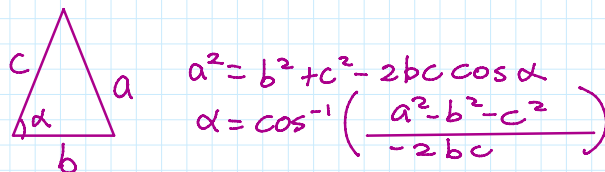
$$\begin{bmatrix} \sin(t_1), & a_2\cos(t_1)\cos(t_2) + a_3\cos(t_1)\cos(t_2)\cos(t_3) - a_3\cos(t_1)\sin(t_2)\sin(t_3) \\ -\cos(t_1), & a_2\cos(t_2)\sin(t_1) + a_3\cos(t_2)\cos(t_3)\sin(t_1) - a_3\sin(t_1)\sin(t_2)\sin(t_3) \\ 0, & a_1 + a_2\sin(t_2) + a_3\cos(t_2)\sin(t_3) + a_3\cos(t_3)\sin(t_2) \\ 0, & 1 \end{bmatrix}$$

Graphical Method → for 3-DOF below

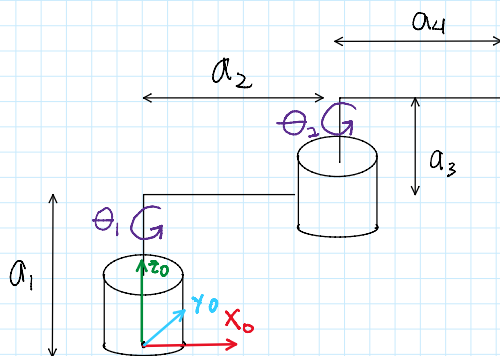
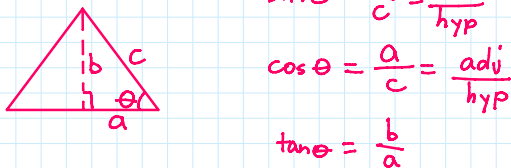
1) Pythagorean Theorem



3) Law of Cosine



2) SOH CAHTOA



Given:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

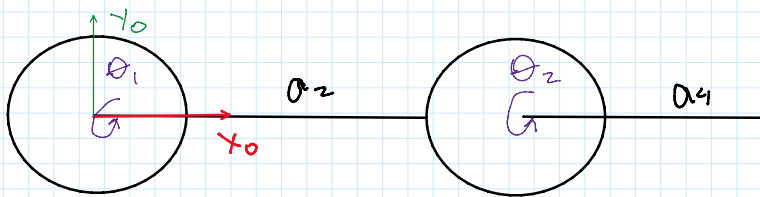
Hindi: Given:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

A diagram of a 3-link planar robot arm. The base joint is at the origin of a coordinate system with a red arrow pointing right labeled  $x_0$ . The first link is vertical, with height  $a_1$  and joint angle  $z_0$ . The second link is horizontal, with length  $a_2$  and joint angle  $z_1$ . The third link is vertical, with height  $a_3$  and joint angle  $z_2$ . The end-effector is at the top of the third link.

$$a_1 \neq a_3$$

$${}_2^0 z = \bar{a}_1 + \bar{a}_3$$



A diagram of a three-link planar robot arm. The base joint is at the origin of a coordinate system with axes  $x_0$  (red) and  $y_0$  (green). The first link has length  $a_1$  and makes an angle  $\theta_1$  with the  $x_0$  axis. The second link has length  $a_2$  and makes an angle  $\theta_2$  with the extension of the first link. The third link has length  $a_3$  and makes an angle  $\theta_3$  with the extension of the second link. The end-effector is at coordinates  $(x_0, y_0)$ . A dashed line indicates the angle  $\phi_2$  between the second and third links. A right-angle symbol is shown at the origin.

$$\phi_2 = \phi_1 + \theta_1$$

$$\ominus_1 = \ominus_2 - \ominus_1 \quad (4)$$

$$\phi_2 = \tan^{-1} \left( \frac{{}^0Y_2}{{}^0X_2} \right) \quad (1)$$

$$r_1 = \sqrt{(2y)^2 + (2x)^2} \quad (2)$$

$$\phi_1 = \cos^{-1} \left( \frac{a_4^2 - a_2^2 - r_1^2}{-2a_2r_1} \right) \quad (3)$$

$$180^\circ = \phi_3 + \theta_2$$

$$\theta_2 = 180^\circ - \phi_3$$

$$\phi_3 = \cos^{-1} \left( \frac{r_1^2 - a_2^2 - a_4^2}{-2a_2a_4} \right) \quad (5)$$