3101: Rotation Matrix

Monday, 3 October 2022 7:12 am

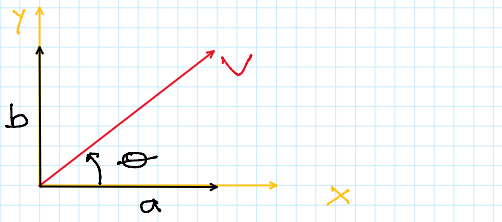
Ideal Manipulator

3 → Translation ⇒ Position Vectors

3 → O/R → Rotation Matrix

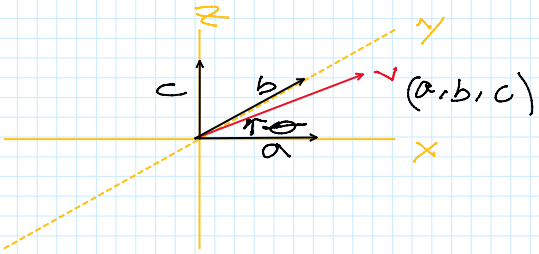
6 DOF

Projection

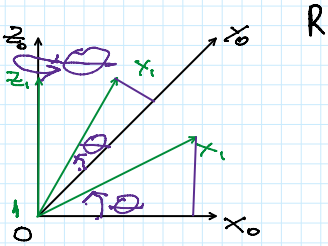


$\tan\theta = \frac{b}{a}$

(a, b)

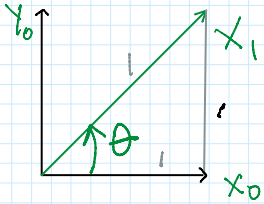


RM: How the rotation of Fn is projected on Fn-1.



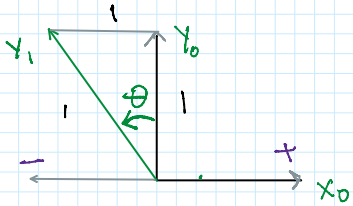
SOH CAH TOA

$$\begin{array}{c} \\ X_{n-1} \\ Y_{n-1} \\ Z_{n-1} \end{array} \begin{array}{c} \begin{array}{ccc} X_n & Y_n & Z_n \end{array} \\ \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \end{array}$$



$\cos\theta = \frac{X_0}{X_1}$

$\sin\theta = \frac{Y_0}{X_1}$



$\frac{X_0}{Y_1} = \sin\theta$

$\frac{Y_0}{Y_1} = \cos\theta$

$$\begin{array}{c} \\ X_0 \\ Y_0 \\ Z_0 \end{array} \begin{array}{c} \begin{array}{ccc} X_1 & Y_1 & Z_1 \end{array} \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

"Z Rotation Matrix"

$$\begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c|ccc} & X_1 & Y_1 & Z_1 \\ X_0 & 1 & 0 & 0 \\ Y_0 & 0 & c\theta & -s\theta \\ Z_0 & 0 & s\theta & c\theta \end{array}$$

"X Rotation Matrix"

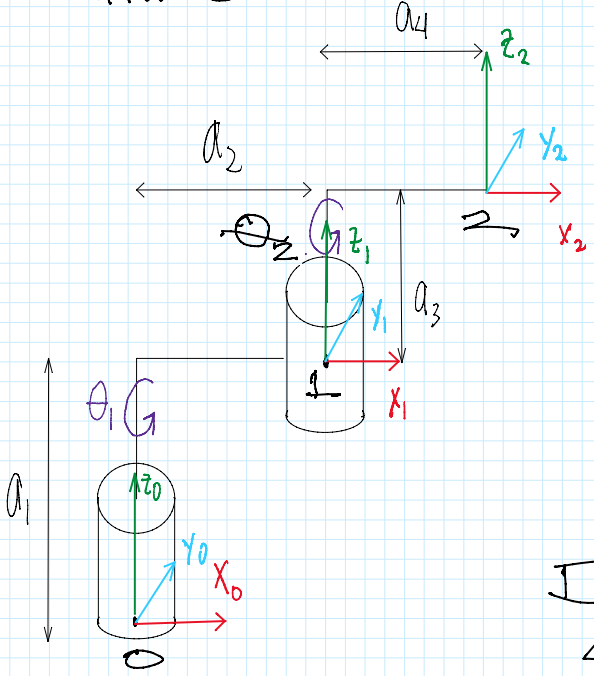
$$\begin{array}{c|ccc} & X_1 & Y_1 & Z_1 \\ X_0 & c\theta & 0 & s\theta \\ Y_0 & 0 & 1 & 0 \\ Z_0 & -s\theta & 0 & c\theta \end{array}$$

"Y Rotation Matrix"

$$\begin{array}{c|ccc} & X_1 & Y_1 & Z_1 \\ X_0 & c\theta & -s\theta & 0 \\ Y_0 & s\theta & c\theta & 0 \\ Z_0 & 0 & 0 & 1 \end{array}$$

"Z Rotation Matrix"

ref. frame ← ${}^{n-1}_{n}R$ → projected frame



$${}^{0}_{2}R = {}^{0}_{1}R \; {}^{1}_{2}R$$

$$[3\times3] \quad [3\times3]$$

$${}^{0}_{1}R = \begin{array}{c|ccc} & X_1 & Y_1 & Z_1 \\ X_0 & c\theta_1 & -s\theta_1 & 0 \\ Y_0 & s\theta_1 & c\theta_1 & 0 \\ Z_0 & 0 & 0 & 1 \end{array}$$

← stationary

"Z Rotation" ← moving Matrix

$$\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity Matrix ⇔ 1 ⇔ No Rotation

$${}^{1}_{2}R = \begin{array}{c|ccc} & X_1 & Y_1 & Z_1 \\ X_0 & 1 & 0 & 0 \\ Y_0 & 0 & 1 & 0 \\ Z_0 & 0 & 0 & 1 \end{array}$$

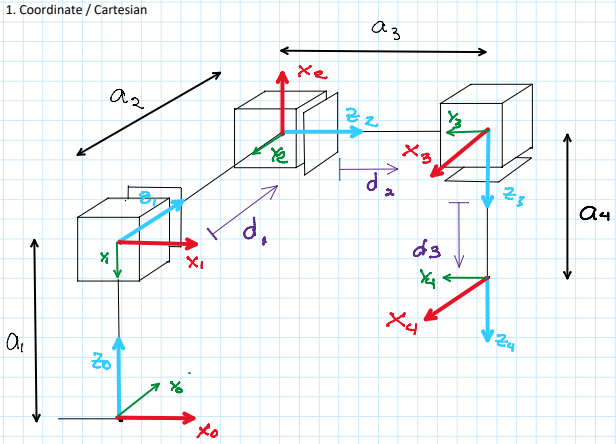
"Z Rotation"

$$\begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{2}R = \begin{pmatrix} \cos(t1)\cos(t2) - \sin(t1)\sin(t2), & -\cos(t1)\sin(t2) - \cos(t2)\sin(t1), & 0 \\ \cos(t1)\sin(t2) + \cos(t2)\sin(t1), & \cos(t1)\cos(t2) - \sin(t1)\sin(t2), & 0 \end{pmatrix}$$

$$
{}^{0}_{2}R = \begin{pmatrix} \cos(t1)\cos(t2) - \sin(t1)\sin(t2), & -\cos(t1)\sin(t2) - \cos(t2)\sin(t1), & 0 \\ \cos(t1)\sin(t2) + \cos(t2)\sin(t1), & \cos(t1)\cos(t2) - \sin(t1)\sin(t2), & 0 \\ 0, & 0, & 1 \end{pmatrix}
$$

1. Coordinate / Cartesian



$$
{}^{0}_{4}R = {}^{0}_{1}R\,{}^{1}_{2}R\,{}^{2}_{3}R\,{}^{3}_{4}R
$$

$$
{}^{0}_{1}R = \begin{array}{c} \\ x_0 \\ y_0 \\ z_0 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & y_1 & z_1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \end{array} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

Stationary

Moving matrix