Inverse Kinematics

Friday, 16 February 2024 8:11 am

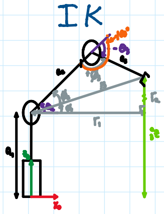
FK

$$\text{Joint Variables } \theta_n, d_n \rightarrow \begin{vmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & r_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & r_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{vmatrix} \rightarrow \text{Position Vector \& Orientation } {}^{M}_{E}V = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- For identifying the limits of your joint variables.
- For obtaining the trajectory solution.
- Easier to solve { Standardized. / Has only one real solution.

IK

$$\text{Position Vector } {}^{M}_{E}V = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow$$



$$\rightarrow \text{Joint Variables } \theta_n, d_n$$

- For mimicking the motion of a human arm
- For detailed positioning of end-effector

- Difficult to solve { Non-linear Solutions / Not standardized. / There can be no solution or there can be 1 or more solutions.

HTM_rad =

Position Vector

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[cos(t1)*cos(t2)*cos(t3) - cos(t1)*sin(t2)*sin(t3),    - cos(t1)*cos(t2)*sin(t3) - cos(t1)*cos(t3)*sin(t2),    sin(t1),   a2*cos(t1)*cos(t2) + a3*cos(t1)*cos(t2)*cos(t3) - a3*cos(t1)*sin(t2)*sin(t3)]
[cos(t2)*cos(t3)*sin(t1) - sin(t1)*sin(t2)*sin(t3),    - cos(t2)*sin(t1)*sin(t3) - cos(t3)*sin(t1)*sin(t2),   -cos(t1),   a2*cos(t2)*sin(t1) + a3*cos(t2)*cos(t3)*sin(t1) - a3*sin(t1)*sin(t2)*sin(t3)]
[              cos(t2)*sin(t3) + cos(t3)*sin(t2),                cos(t2)*cos(t3) - sin(t2)*sin(t3),          0,          a1 + a2*sin(t2) + a3*cos(t2)*sin(t3) + a3*cos(t3)*sin(t2)]
[                                0,                                   0,                                   0,                                   1]
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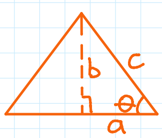
X
Y
Z

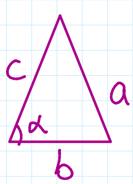
Graphical Method → for 3-DOF below

1) Pythagorean Theorem



$c^2 = a^2 + b^2$

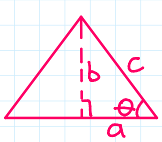
3) Law of Cosine



$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\alpha = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$

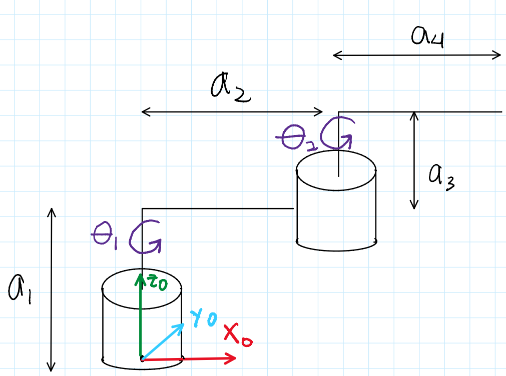
2) SOH CAH TOA



$\sin\theta = \frac{b}{c} = \frac{opp}{hyp}$

$\cos\theta = \frac{a}{c} = \frac{adj}{hyp}$

$\tan\theta = \frac{b}{a}$



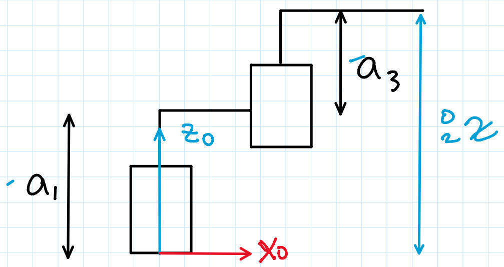
Given:

a_1 X
a_2 Y
a_3 Z
a_4

Hint: Given:

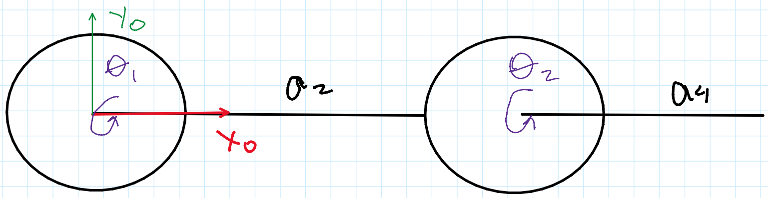
θ_1
θ_2

Front View

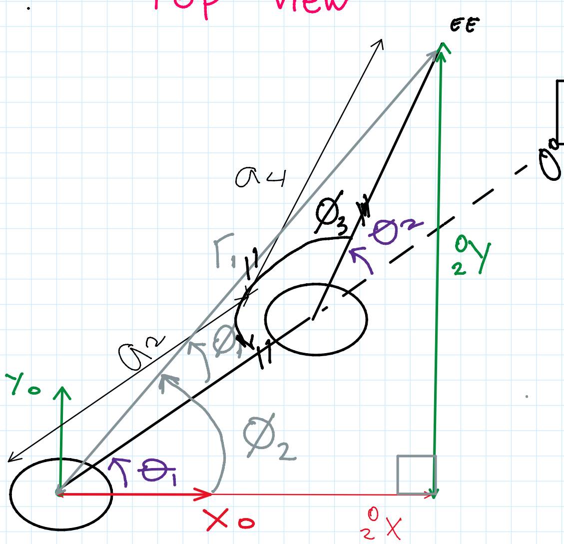


$a_1 \neq a_3$

${}^{0}_{2}z = \acute{a}_1 + \acute{a}_3$



Top View



$\phi_2 = \phi_1 + \theta_1$

$\boxed{\theta_1 = \phi_2 - \phi_1}$ (4)

$\boxed{\phi_2 = \tan^{-1}\left(\frac{{}^{0}_{2}Y}{{}^{0}_{2}X}\right)}$ (1)

$\boxed{r_1 = \sqrt{({}^{0}_{2}Y)^2 + ({}^{0}_{2}X)^2}}$ (2)

$\boxed{\phi_1 = \cos^{-1}\left(\frac{a_4^2 - a_2^2 - r_1^2}{-2a_2 r_1}\right)}$ (3)

$180^\circ = \phi_3 + \theta_2$

$\boxed{\theta_2 = 180^\circ - \phi_3}$ (6)

$$\boxed{\phi_3 = \cos^{-1}\left(\frac{r_1^2 - a_2^2 - a_4^2}{-2a_2 a_4}\right)}$$ (5)