

Forward Kinematics - MEXE-3201

Monday, 15 November 2021 9:01 pm

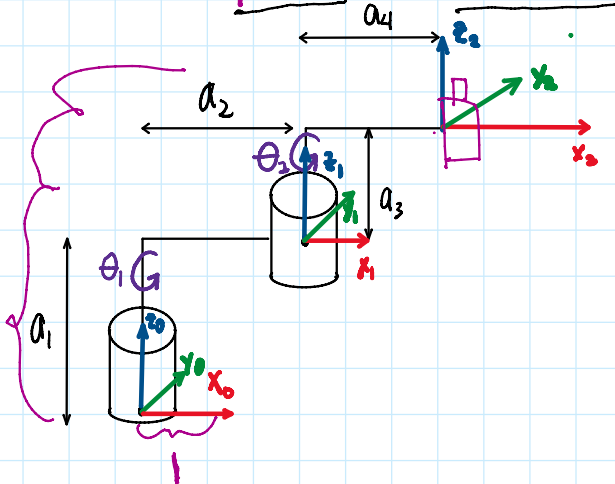
Kinematics is the Science of motion which treats motion without considering the forces which causes the motion.

Forward Kinematics is the geometrical problem of computing the position vector and orientation of the end-effector given the joint variables.

$${}^{n-1}T_n = {}^{n-1}H_n : \begin{matrix} \text{RM} & \rightarrow & \text{PV} \\ 3 \times 3 & & 3 \times 1 \end{matrix} \left\{ \begin{array}{cccc} c\theta_1 & -s\theta_1 c\theta_2 & s\theta_1 c\theta_2 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 c\theta_2 & -c\theta_1 c\theta_2 & l_1 s\theta_1 \\ 0 & s\theta_2 & c\theta_2 & d_1 \\ 0 & 0 & 0 & 1 \end{array} \right\}$$

FK
 ↳ Solving for PV
 ↳ Given any joint variables
 neg Rotation Matrix

Find the position & direction of the spray at the end-effector.



$$\begin{aligned} \theta_1 &= 90^\circ & a_1 &= 3 \\ \theta_2 &= -90^\circ & a_2 &= 3 \\ & & a_3 &= 3 \\ & & a_4 &= 3 \end{aligned}$$

Process 1:

- Rotation matrix
- Position Vector
- Concatenate to HTM
- Multiplication

Process 2:

- DH Notation
- Parametric Table
- HTM formula
- Multiplication

$H_{0,2} =$

$$\begin{matrix} x_2 & y_2 & z_2 & PV \\ \begin{matrix} x_0 \\ y_0 \\ z_0 \end{matrix} & \begin{bmatrix} \cos(t_1)\cos(t_2) - \sin(t_1)\sin(t_2) & -\cos(t_1)\sin(t_2) - \sin(t_1)\cos(t_2) & 0 \\ \cos(t_1)\sin(t_2) + \sin(t_1)\cos(t_2) & \cos(t_1)\cos(t_2) - \sin(t_1)\sin(t_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} & \begin{bmatrix} a_2\cos(t_1) + a_4\cos(t_1)\cos(t_2) - a_4\sin(t_1)\sin(t_2) \\ a_2\sin(t_1) + a_4\cos(t_1)\sin(t_2) + a_4\cos(t_2)\sin(t_1) \\ a_1 + a_3 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} x_2 & y_2 & z_2 \\ x_0 & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & x_0 & \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & z_0 & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{matrix}$$

$${}^0P_2 = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} \begin{matrix} x \\ y \\ z \end{matrix}$$

Forward Kinematics

Graphical form of Forward Kinematics

