

# Structural Equation Models 2019 / WEEK 5

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## Exercise 5.1

The subject of this exercise are the relations among the three dimensions of burnout and their invariance across elementary and secondary teachers. The data files consist of elementary (N=580) and secondary (N=692) teachers with 22 MBI items. The aim is to test the invariance of MBI across the samples.

Let us import the data file in R, prepare the data and build the model according to the input file.

```
elementary <- read.table("mbielm1.dat", header = FALSE)
secondary <- read.table("mbisec1.dat", header = FALSE)
names(elementary) <- names(secondary) <- paste("ITEM", 1:22, sep = "")

library(lavaan)

## This is lavaan 0.6-3
## lavaan is BETA software! Please report any bugs.
library(dplyr)

##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##   filter, lag
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union
library(knitr)
library(semPlot)

# Model for data "elementary" and also for data "secondary":
# F1 BY ITEM1 - ITEM3 ITEM6 ITEM8 ITEM13 ITEM14 ITEM16 ITEM20;
# F2 BY ITEM5 ITEM10 ITEM11 ITEM15 ITEM22;
# F3 BY ITEM4 ITEM7 ITEM9 ITEM12 ITEM17 - ITEM19 ITEM21;

model <- '
# F1
EE =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13 + ITEM14 + ITEM16 + ITEM20
# F2
DP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
# F3
PA =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21
'

fit_elem <- cfa(model, data = elementary, estimator="MLM")
fit_sec <- cfa(model, data = secondary, estimator="MLM")
```

```

# Model indices
group <- c("Elementary", "Secondary")
chisq = c(fitMeasures(fit_elem, "chisq.scaled"), fitMeasures(fit_sec, "chisq.scaled"))
df = c(fitMeasures(fit_elem, "df.scaled"), fitMeasures(fit_sec, "df.scaled"))
pvalue = c(fitMeasures(fit_elem, "pvalue.scaled"), fitMeasures(fit_sec, "pvalue.scaled"))
scf = c(fitMeasures(fit_elem, "chisq.scaling.factor"),
        fitMeasures(fit_sec, "chisq.scaling.factor")) %>% round(3)
cfi = c(fitMeasures(fit_elem, "cfi.robust"),
        fitMeasures(fit_sec, "cfi.robust")) %>% round(3)
tli = c(fitMeasures(fit_elem, "tli.robust"),
        fitMeasures(fit_sec, "tli.robust")) %>% round(3)
rmsea = c(fitMeasures(fit_elem, "rmsea.robust"),
          fitMeasures(fit_sec, "rmsea.robust")) %>% round(3)
srmr = c(fitMeasures(fit_elem, "srmr"),
          fitMeasures(fit_sec, "srmr")) %>% round(3)
tabledata <- data.frame("Group"=group, "Chi.square"=chisq, "df"=df,
                        "p-value"=pvalue, "Scaling correction factor"=scf,
                        "CFI"=cfi, "TLI"=tli, "RMSEA"=rmsea, "SRMR"=srmr)
kable(tabledata, caption = "Fit indices (Model 1)")

```

Table 1: Fit indices (Model 1)

Group	Chi.square	df	p.value	Scaling.correction.factor	CFI	TLI	RMSEA	SRMR
Elementary	802.7431	206	0	1.261	0.858	0.841	0.079	0.071
Secondary	971.0638	206	0	1.321	0.835	0.815	0.084	0.080

The fit indices of the initial models are shown in table 1. The scaling correction factor indices (1.261 and 1.321) support the use of MLM robust estimators. All fit indices imply poor fit of both models ( $p < 0.001$  [ $df=206$ ],  $CFI < 0.9$ ,  $TLI < 0.9$ ,  $RMSEA > 0.05$ ). Let us continue to model modification.

## Modification indices

```
modindices(fit_elem, minimum.value = 40, sort. = TRUE)
```

```

##      lhs op   rhs      mi    epc sepc.lv sepc.all sepc.nox
## 158 ITEM6 ~~ ITEM16 180.298 0.893 0.893 0.595 0.595
## 95  ITEM1 ~~ ITEM2 103.177 0.534 0.534 0.494 0.494
## 59   EE == ITEM12 81.319 -0.400 -0.515 -0.400 -0.400
## 260 ITEM10 ~~ ITEM11 67.743 0.688 0.688 0.832 0.832
## 323 ITEM18 ~~ ITEM19 43.669 0.279 0.279 0.340 0.340
## 298 ITEM4 ~~ ITEM7 42.833 0.184 0.184 0.294 0.294

```

```
modindices(fit_sec, minimum.value = 40, sort. = TRUE)
```

```

##      lhs op   rhs      mi    epc sepc.lv sepc.all sepc.nox
## 95  ITEM1 ~~ ITEM2 171.647 0.627 0.627 0.583 0.583
## 260 ITEM10 ~~ ITEM11 135.841 1.181 1.181 1.426 1.426
## 158 ITEM6 ~~ ITEM16 127.756 0.686 0.686 0.458 0.458
## 59   EE == ITEM12 118.156 -0.468 -0.561 -0.419 -0.419
## 250 ITEM5 ~~ ITEM15 77.216 0.580 0.580 0.355 0.355
## 271 ITEM11 ~~ ITEM15 60.947 -0.485 -0.485 -0.420 -0.420

```

```
## 122  ITEM2 ~~ ITEM20  53.024 -0.324 -0.324 -0.316 -0.316
## 249  ITEM5 ~~ ITEM11  48.297 -0.446 -0.446 -0.369 -0.369
## 314  ITEM9 ~~ ITEM19  46.617  0.360  0.360  0.358  0.358
## 52    EE  == ITEM10  45.623 -0.394 -0.473 -0.302 -0.302
```

The results (modification indices MI and expected parameter change EPC) suggest that we should add  $\text{ITEM6} \sim \text{ITEM16}$ ,  $\text{ITEM1} \sim \text{ITEM2}$  and  $\text{ITEM10} \sim \text{ITEM11}$ . Also a cross-loading of item 12 on factor F1 (EE) is implied. These results are shared in both groups. Let us modify the model:

## Model 2

```
model2 <- '
EE =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13
    + ITEM14 + ITEM16 + ITEM20 + ITEM12
DP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
PA =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

ITEM6 ~~ ITEM16
ITEM1 ~~ ITEM2
ITEM10 ~~ ITEM11
'

fit_elem2 <- cfa(model2, data = elementary, estimator="MLM")
fit_sec2 <- cfa(model2, data = secondary, estimator="MLM")

# Model indices
group <- c("Elementary", "Secondary")
chisq = c(fitMeasures(fit_elem2, "chisq.scaled"),
          fitMeasures(fit_sec2, "chisq.scaled"))
df = c(fitMeasures(fit_elem2, "df.scaled"),
        fitMeasures(fit_sec2, "df.scaled"))
pvalue = c(fitMeasures(fit_elem2, "pvalue.scaled"),
            fitMeasures(fit_sec2, "pvalue.scaled"))
scf = c(fitMeasures(fit_elem2, "chisq.scaling.factor"),
        fitMeasures(fit_sec2, "chisq.scaling.factor")) %>% round(3)
cfi = c(fitMeasures(fit_elem2, "cfi.robust"),
        fitMeasures(fit_sec2, "cfi.robust")) %>% round(3)
tli = c(fitMeasures(fit_elem2, "tli.robust"),
        fitMeasures(fit_sec2, "tli.robust")) %>% round(3)
rmsea = c(fitMeasures(fit_elem2, "rmsea.robust"),
          fitMeasures(fit_sec2, "rmsea.robust")) %>% round(3)
srmr = c(fitMeasures(fit_elem2, "srmr"),
          fitMeasures(fit_sec2, "srmr")) %>% round(3)
tabledata <- data.frame("Group"=group, "Chi.square"=chisq, "df"=df,
                        "p-value"=pvalue, "Scaling correction factor"=scf,
                        "CFI"=cfi, "TLI"=tli, "RMSEA"=rmsea, "SRMR"=srmr)
kable(tabledata, caption = "Fit indices (Model 2)")
```

Table 2: Fit indices (Model 2)

Group	Chi.square	df	p.value	Scaling.correction.factor	CFI	TLI	RMSEA	SRMR
Elementary	470.3767	202	0	1.243	0.937	0.928	0.053	0.053

Group	Chi.square	df	p.value	Scaling.correction.factor	CFI	TLI	RMSEA	SRMR
Secondary	572.5523	202	0	1.311	0.921	0.909	0.059	0.058

The indices improved quite a lot for both models, but we need to continue modifying the model:

### Modification indices

```
modindices(fit_elem2, minimum.value = 30, sort. = TRUE)
```

```
##      lhs op   rhs      mi   epc sepc.lv sepc.all sepc.nox
## 305  ITEM4 ~~ ITEM7 38.931 0.174   0.174   0.284   0.284
## 323 ITEM18 ~~ ITEM19 38.744 0.266   0.266   0.333   0.333
```

```
modindices(fit_sec2, minimum.value = 30, sort. = TRUE)
```

```
##      lhs op   rhs      mi   epc sepc.lv sepc.all sepc.nox
## 57    EE =~ ITEM11 67.177 0.472   0.532   0.339   0.339
## 318 ITEM9 ~~ ITEM19 43.690 0.355   0.355   0.357   0.357
## 263 ITEM5 ~~ ITEM15 35.576 0.416   0.416   0.310   0.310
```

In the group of elementary teachers, items 4 and 7 have the highest values for the residual covariance. The group of secondary teachers has the highest MI value for the cross-loading of item 11 on F1. We must specify model 3 separately for these groups according to these findings.

### Model 3

```
model3elem <- '
EE =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13
    + ITEM14 + ITEM16 + ITEM20 + ITEM12
DP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
PA =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

ITEM6 ~~ ITEM16
ITEM1  ~~ ITEM2
ITEM10 ~~ ITEM11
ITEM7  ~~ ITEM4
'

model3sec <- '
EE =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13
    + ITEM14 + ITEM16 + ITEM20 + ITEM12 + ITEM11
DP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
PA =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

ITEM6 ~~ ITEM16
ITEM1  ~~ ITEM2
ITEM10 ~~ ITEM11
'

fit_elem3 <- cfa(model3elem, data = elementary, estimator="MLM")
fit_sec3  <- cfa(model3sec, data = secondary, estimator="MLM")
```

```

# Model indices
group <- c("Elementary", "Secondary")
chisq = c(fitMeasures(fit_elem3, "chisq.scaled"),
          fitMeasures(fit_sec3, "chisq.scaled"))
df = c(fitMeasures(fit_elem3, "df.scaled"),
        fitMeasures(fit_sec3, "df.scaled"))
pvalue = c(fitMeasures(fit_elem3, "pvalue.scaled"),
            fitMeasures(fit_sec3, "pvalue.scaled"))
scf = c(fitMeasures(fit_elem3, "chisq.scaling.factor"),
         fitMeasures(fit_sec3, "chisq.scaling.factor")) %>% round(3)
cfi = c(fitMeasures(fit_elem3, "cfi.robust"),
         fitMeasures(fit_sec3, "cfi.robust")) %>% round(3)
tli = c(fitMeasures(fit_elem3, "tli.robust"),
         fitMeasures(fit_sec3, "tli.robust")) %>% round(3)
rmsea = c(fitMeasures(fit_elem3, "rmsea.robust"),
           fitMeasures(fit_sec3, "rmsea.robust")) %>% round(3)
srmr = c(fitMeasures(fit_elem3, "srmr"),
          fitMeasures(fit_sec3, "srmr")) %>% round(3)
tabledata <- data.frame("Group"=group, "Chi.square"=chisq, "df"=df,
                        "p-value"=pvalue, "Scaling correction factor"=scf,
                        "CFI"=cfi, "TLI"=tli, "RMSEA"=rmsea, "SRMR"=srmr)
kable(tabledata, caption = "Fit indices (Model 3)")

```

Table 3: Fit indices (Model 3)

Group	Chi.square	df	p.value	Scaling.correction.factor	CFI	TLI	RMSEA	SRMR
Elementary	445.8005	201	0	1.224	0.944	0.935	0.051	0.051
Secondary	522.9823	201	0	1.307	0.931	0.921	0.055	0.055

## Modification indices

```

modindices(fit_elem3, minimum.value = 30, sort. = TRUE)

##      lhs op   rhs    mi   epc sepc.lv sepc.all sepc.nox
## 323 ITEM18 ~~ ITEM19 32.503 0.247  0.247    0.319    0.319

modindices(fit_sec3, minimum.value = 30, sort. = TRUE)

##      lhs op   rhs    mi   epc sepc.lv sepc.all sepc.nox
## 318 ITEM9  ~~ ITEM19 42.687 0.351  0.351    0.355    0.355

```

The MI and EPC values imply that continuing modification is not needed for the elementary teachers group. According to MI and EPC values the model for secondary teachers group can be fine tuned further by allowing residual covariance of items 9 and 19. Model 4 will be done separately for secondary teachers group only.

## Model 4

```

model4sec <- '
EE =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13
    + ITEM14 + ITEM16 + ITEM20 + ITEM12 + ITEM11
DP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
PA =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

```

```

ITEM6 ~~ ITEM16
ITEM1  ~~ ITEM2
ITEM10 ~~ ITEM11
ITEM9  ~~ ITEM19
'

fit_sec4 <- cfa(model4sec, data = secondary, estimator="MLM")

# Model indices
group <- c("Secondary")

chisq = fitMeasures(fit_sec4, "chisq.scaled")
df = fitMeasures(fit_sec4, "df.scaled")
pvalue = fitMeasures(fit_sec4, "pvalue.scaled")
scf = fitMeasures(fit_sec4, "chisq.scaling.factor") %>% round(3)
cfi = fitMeasures(fit_sec4, "cfi.robust") %>% round(3)
tli = fitMeasures(fit_sec4, "tli.robust") %>% round(3)
rmsea = fitMeasures(fit_sec4, "rmsea.robust") %>% round(3)
srmr = fitMeasures(fit_sec4, "srmr") %>% round(3)
tabledata <- data.frame("Group"=group, "Chi.square"=chisq, "df"=df,
                        "p-value"=pvalue, "Scaling correction factor"=scf,
                        "CFI"=cfi, "TLI"=tli, "RMSEA"=rmsea, "SRMR"=srmr)
kable(tabledata, caption = "Fit indices (Model 4)")

```

Table 4: Fit indices (Model 4)

	Group	Chi.square	df	p.value	Scaling.correction.factor	CFI	TLI	RMSEA	SRMR
chisq.scaled	Secondary	492.2819	200	0	1.308	0.938	0.928	0.053	0.054

Now we have achieved baseline models for both teacher groups. Let us print their fit indices:

```

group <- c("Elementary", "Secondary")
chisq = c(fitMeasures(fit_elem3, "chisq.scaled"),
          fitMeasures(fit_sec4, "chisq.scaled"))
df = c(fitMeasures(fit_elem3, "df.scaled"),
        fitMeasures(fit_sec4, "df.scaled"))
pvalue = c(fitMeasures(fit_elem3, "pvalue.scaled"),
            fitMeasures(fit_sec4, "pvalue.scaled"))
scf = c(fitMeasures(fit_elem3, "chisq.scaling.factor"),
         fitMeasures(fit_sec4, "chisq.scaling.factor")) %>% round(3)
cfi = c(fitMeasures(fit_elem3, "cfi.robust"),
         fitMeasures(fit_sec4, "cfi.robust")) %>% round(3)
tli = c(fitMeasures(fit_elem3, "tli.robust"),
         fitMeasures(fit_sec4, "tli.robust")) %>% round(3)
rmsea = c(fitMeasures(fit_elem3, "rmsea.robust"),
           fitMeasures(fit_sec4, "rmsea.robust")) %>% round(3)
srmr = c(fitMeasures(fit_elem3, "srmr"),
          fitMeasures(fit_sec4, "srmr")) %>% round(3)
tabledata <- data.frame("Group"=group, "Chi.square"=chisq, "df"=df,
                        "p-value"=pvalue, "Scaling correction factor"=scf,
                        "CFI"=cfi, "TLI"=tli, "RMSEA"=rmsea, "SRMR"=srmr)

```

```
kable(tabledata, caption = "Fit indices (Baseline models)")
```

Table 5: Fit indices (Baseline models)

Group	Chi.square	df	p.value	Scaling.correction.factor	CFI	TLI	RMSEA	SRMR
Elementary	445.8005	201	0	1.224	0.944	0.935	0.051	0.051
Secondary	492.2819	200	0	1.308	0.938	0.928	0.053	0.054

## Exercise 5.2

Let us combine the groups and specify the common baseline (configural) model and draw the graphs.

*# Combine (merge) the data sets into one:*

```
elmsec <- merge(data.frame(elementary, group = "elementary"),
               data.frame(secondary, group = "secondary"),
               all = TRUE, sort = FALSE)
```

*# Configural model (common baseline model for both groups simultaneously):*

```
INV1model <- '
# EE: EmotionalExhaustion (including ITEM12!)
# EP: Depersonalization
# PA: PersonalAccomplishment
EE =~ ITEM1 + ITEM2 + ITEM3 + ITEM6 + ITEM8 + ITEM13
    + ITEM14 + ITEM16 + ITEM20 + ITEM12
DP =~ ITEM5 + ITEM10 + ITEM11 + ITEM15 + ITEM22
PA =~ ITEM4 + ITEM7 + ITEM9 + ITEM12 + ITEM17 + ITEM18 + ITEM19 + ITEM21

# Residual covariances (in both groups)
ITEM1 ~~ ITEM2
ITEM6 ~~ ITEM16
ITEM10 ~~ ITEM11

# Group-specific model parameters
ITEM4 ~~ c(NA,0)*ITEM7 # only for elm
EE =~ c(0,NA)*ITEM11 # only for sec
ITEM9 ~~ c(0,NA)*ITEM19 # only for sec
'
```

```
fit_measures <- c("chisq.scaled", "df.scaled", "pvalue.scaled",
                 "chisq.scaling.factor", "cfi.robust", "tli.robust",
                 "rmsea.robust", "srmr_bentler")
```

```
fitconfig <- cfa(INV1model, data=elmsec, estimator="MLM", group = "group",
                mimic = "Mplus")
fitMeasures(fitconfig, fit_measures)
```

```
##          chisq.scaled          df.scaled          pvalue.scaled
##          958.341          401.000          0.000
## chisq.scaling.factor          cfi.robust          tli.robust
##          1.242          0.940          0.930
##          rmsea.robust
```

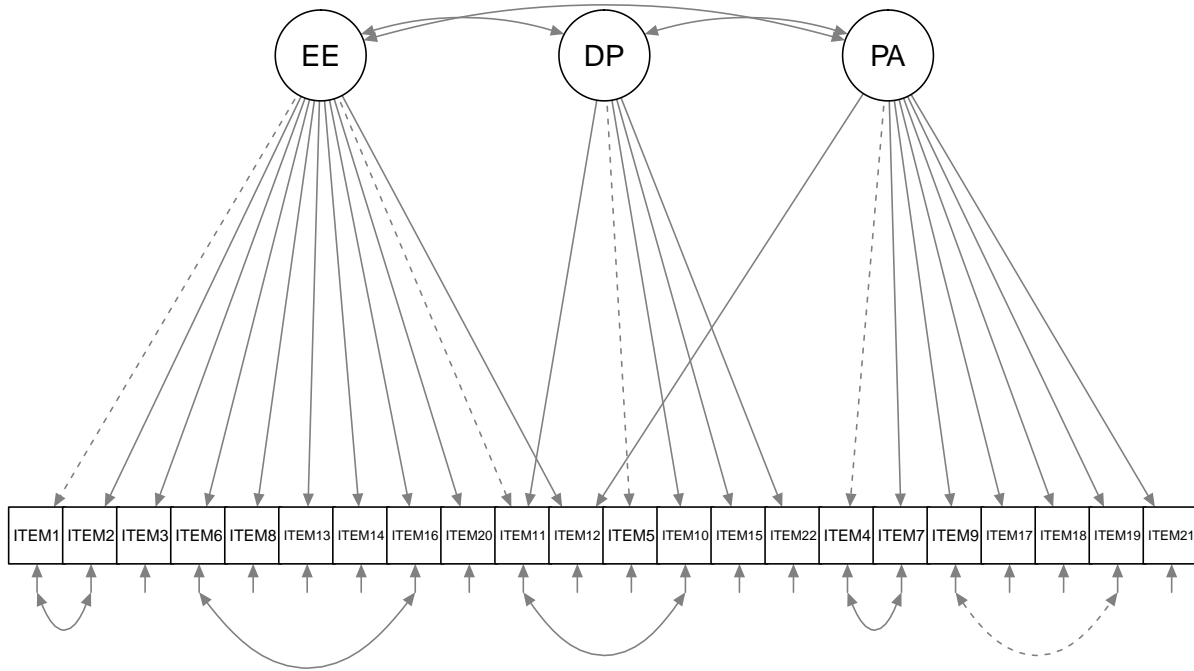
##

0.052

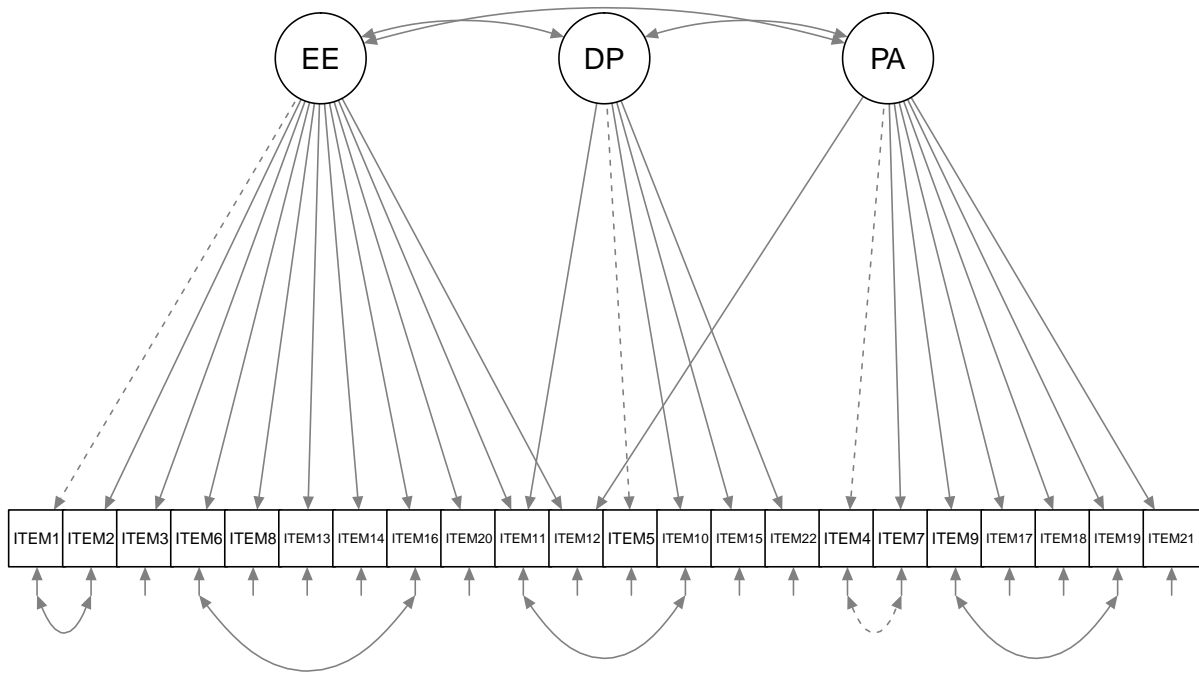
Graphs of the common baseline model (configural model) are presented below. The first graph is for the elementary school teachers and the second graph is for the secondary school teachers.

```
semPaths(fitconfig, style="lisrel", intercepts=FALSE, mar=c(8,1,8,1))
```

1







Unfortunately I did not have enough time to complete the rest of these tasks properly - I will have to check into these thoroughly while doing next weeks tasks!