

# Structural Equation Models 2019

## Week 2

### Exercise 2.1

The null hypothesis is that self-concept (SC) is a multidimensional construct composed of four factors:

- General SC (GSC)
- Academic SC (ASC)
- English SC (ESC)
- Mathematics SC (MSC)

Alternative hypothesis is that self-concept is not a multidimensional construct composed of four factors.

Let us bring the data in R and prepare it for analysis:

```
library(lavaan)

## This is lavaan 0.6-3
## lavaan is BETA software! Please report any bugs.

library(semPlot)

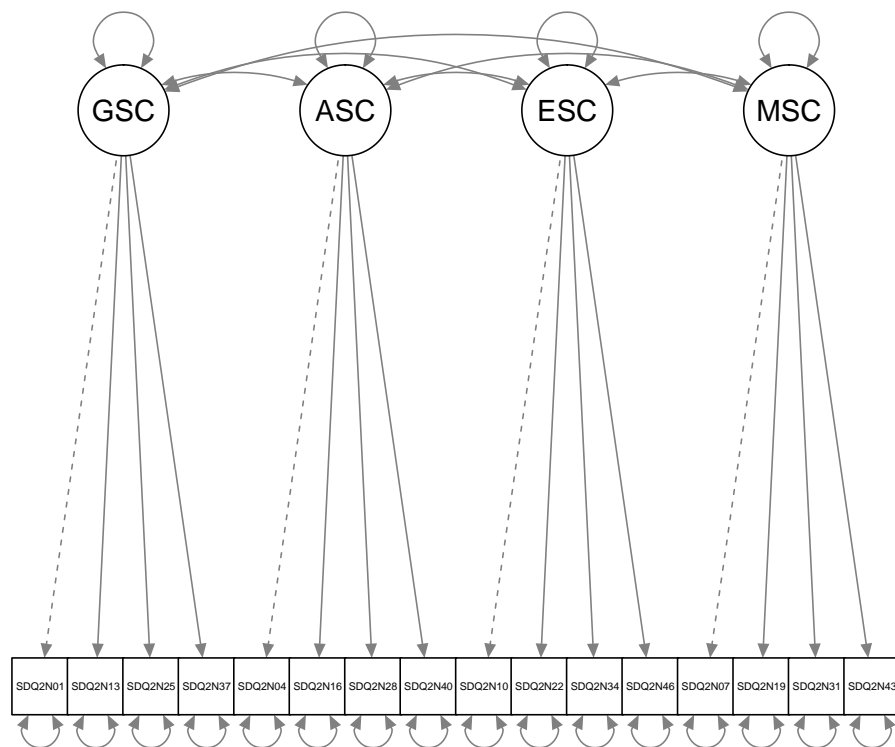
ex2.1 <- read.fortran(file = "ASC7INDM.DAT", c("40F1.0", "X", "6F2.0"))
colnames(ex2.1) <- c("SPPCN08", "SPPCN18", "SPPCN28", "SPPCN38", "SPPCN48",
  "SPPCN58", "SPPCN01", "SPPCN11", "SPPCN21", "SPPCN31",
  "SPPCN41", "SPPCN51", "SPPCN06", "SPPCN16", "SPPCN26",
  "SPPCN36", "SPPCN46", "SPPCN56", "SPPCN03", "SPPCN13",
  "SPPCN23", "SPPCN33", "SPPCN43", "SPPCN53", "SDQ2N01",
  "SDQ2N13", "SDQ2N25", "SDQ2N37", "SDQ2N04", "SDQ2N16",
  "SDQ2N28", "SDQ2N40", "SDQ2N10", "SDQ2N22", "SDQ2N34",
  "SDQ2N46", "SDQ2N07", "SDQ2N19", "SDQ2N31", "SDQ2N43",
  "MASTENG1", "MASTMAT1", "TENG1", "TMAT1", "SENG1", "SMAT1")
```

Then let us specify the model, visualize the model structure:

```
modelF4 <- "
GSC =~ SDQ2N01 + SDQ2N13 + SDQ2N25 + SDQ2N37
ASC =~ SDQ2N04 + SDQ2N16 + SDQ2N28 + SDQ2N40
ESC =~ SDQ2N10 + SDQ2N22 + SDQ2N34 + SDQ2N46
MSC =~ SDQ2N07 + SDQ2N19 + SDQ2N31 + SDQ2N43
"

fitF4 <- cfa(modelF4, data = ex2.1)

semPaths(fitF4, layout='tree2')
```



```
summary(fitF4, fit.measures = T)
```

```
## lavaan 0.6-3 ended normally after 49 iterations
##
## Optimization method          NLMINB
## Number of free parameters    38
##
## Number of observations       265
##
## Estimator                    ML
## Model Fit Test Statistic     159.112
## Degrees of freedom           98
## P-value (Chi-square)         0.000
##
## Model test baseline model:
##
## Minimum Function Test Statistic 1703.155
## Degrees of freedom             120
## P-value                        0.000
##
## User model versus baseline model:
##
## Comparative Fit Index (CFI)    0.961
## Tucker-Lewis Index (TLI)      0.953
##
## Loglikelihood and Information Criteria:
##
## Loglikelihood user model (H0)  -6562.678
## Loglikelihood unrestricted model (H1) -6483.122
##
```

```

##      Number of free parameters                      38
##      Akaike (AIC)                                13201.356
##      Bayesian (BIC)                              13337.386
##      Sample-size adjusted Bayesian (BIC)          13216.905
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                                          0.049
##      90 Percent Confidence Interval                0.034  0.062
##      P-value RMSEA <= 0.05                        0.556
##
## Standardized Root Mean Square Residual:
##
##      SRMR                                          0.048
##
## Parameter Estimates:
##
##      Information                                Expected
##      Information saturated (h1) model            Structured
##      Standard Errors                            Standard
##
## Latent Variables:
##      Estimate   Std.Err   z-value   P(>|z|)
##      GSC =~
##      SDQ2N01      1.000
##      SDQ2N13      1.083    0.154    7.044    0.000
##      SDQ2N25      0.851    0.132    6.455    0.000
##      SDQ2N37      0.934    0.131    7.131    0.000
##      ASC =~
##      SDQ2N04      1.000
##      SDQ2N16      1.279    0.150    8.520    0.000
##      SDQ2N28      1.247    0.154    8.097    0.000
##      SDQ2N40      1.259    0.156    8.048    0.000
##      ESC =~
##      SDQ2N10      1.000
##      SDQ2N22      0.889    0.103    8.658    0.000
##      SDQ2N34      0.670    0.148    4.539    0.000
##      SDQ2N46      0.843    0.117    7.225    0.000
##      MSC =~
##      SDQ2N07      1.000
##      SDQ2N19      0.841    0.058   14.495    0.000
##      SDQ2N31      0.952    0.049   19.516    0.000
##      SDQ2N43      0.655    0.049   13.298    0.000
##
## Covariances:
##      Estimate   Std.Err   z-value   P(>|z|)
##      GSC ~~
##      ASC          0.415    0.078    5.292    0.000
##      ESC          0.355    0.072    4.947    0.000
##      MSC          0.635    0.118    5.387    0.000
##      ASC ~~
##      ESC          0.464    0.078    5.921    0.000
##      MSC          0.873    0.134    6.519    0.000
##      ESC ~~

```

```
##      MSC                0.331    0.100    3.309    0.001
##
## Variances:
##      Estimate Std.Err z-value P(>|z|)
##      .SDQ2N01    1.198    0.126    9.537    0.000
##      .SDQ2N13    1.119    0.124    9.019    0.000
##      .SDQ2N25    1.056    0.107    9.897    0.000
##      .SDQ2N37    0.771    0.087    8.821    0.000
##      .SDQ2N04    1.394    0.128   10.900    0.000
##      .SDQ2N16    0.616    0.068    9.020    0.000
##      .SDQ2N28    0.896    0.090    9.959    0.000
##      .SDQ2N40    0.952    0.095   10.029    0.000
##      .SDQ2N10    0.653    0.082    7.941    0.000
##      .SDQ2N22    0.657    0.075    8.735    0.000
##      .SDQ2N34    2.590    0.233   11.128    0.000
##      .SDQ2N46    1.201    0.118   10.183    0.000
##      .SDQ2N07    0.854    0.100    8.551    0.000
##      .SDQ2N19    1.228    0.121   10.153    0.000
##      .SDQ2N31    0.365    0.065    5.649    0.000
##      .SDQ2N43    0.964    0.092   10.473    0.000
##      GSC         0.613    0.137    4.464    0.000
##      ASC         0.561    0.126    4.453    0.000
##      ESC         0.668    0.116    5.749    0.000
##      MSC         2.307    0.273    8.460    0.000
```

The hypothesis that SC has four factors is not supported by the results (chi square statistic = 159.112,  $p = 0.000$ ), which suggest that the fit of the data to the model is not adequate and null hypothesis should be rejected. However, the indices CFI (0.961), TLI (0.953) and RMSEA (0.049) support the null hypothesis.

## Exercise 2.2

### a) Hypothesis 2:

The null hypothesis is that SC has two factors:

- General SC (GSC)
- Academic SC (ASC)

Alternative hypothesis is that SC does not have two factors. Let us form the model:

```
modelF2 <- "
GSC =~ SDQ2N01 + SDQ2N13 + SDQ2N25 + SDQ2N37
OSC =~ SDQ2N04 + SDQ2N16 + SDQ2N28 + SDQ2N40
+ SDQ2N10 + SDQ2N22 + SDQ2N34 + SDQ2N46
+ SDQ2N07 + SDQ2N19 + SDQ2N31 + SDQ2N43
"

fitF2 <- cfa(modelF2, data = ex2.1)
summary(fitF2, fit.measures = T)

## lavaan 0.6-3 ended normally after 38 iterations
##
## Optimization method          NLMINB
## Number of free parameters    33
##
```

```

##      Number of observations                265
##
##      Estimator                            ML
##      Model Fit Test Statistic             457.653
##      Degrees of freedom                   103
##      P-value (Chi-square)                 0.000
##
## Model test baseline model:
##
##      Minimum Function Test Statistic      1703.155
##      Degrees of freedom                    120
##      P-value                              0.000
##
## User model versus baseline model:
##
##      Comparative Fit Index (CFI)          0.776
##      Tucker-Lewis Index (TLI)            0.739
##
## Loglikelihood and Information Criteria:
##
##      Loglikelihood user model (H0)        -6711.949
##      Loglikelihood unrestricted model (H1) -6483.122
##
##      Number of free parameters            33
##      Akaike (AIC)                         13489.897
##      Bayesian (BIC)                       13608.028
##      Sample-size adjusted Bayesian (BIC)  13503.401
##
## Root Mean Square Error of Approximation:
##
##      RMSEA                                0.114
##      90 Percent Confidence Interval        0.103  0.125
##      P-value RMSEA <= 0.05                0.000
##
## Standardized Root Mean Square Residual:
##
##      SRMR                                0.101
##
## Parameter Estimates:
##
##      Information                          Expected
##      Information saturated (h1) model      Structured
##      Standard Errors                      Standard
##
## Latent Variables:
##
##      Estimate  Std.Err  z-value  P(>|z|)
##      GSC =~
##      SDQ2N01    1.000
##      SDQ2N13    1.048    0.151    6.930    0.000
##      SDQ2N25    0.860    0.131    6.542    0.000
##      SDQ2N37    0.890    0.128    6.957    0.000
##      OSC =~
##      SDQ2N04    1.000
##      SDQ2N16    1.263    0.170    7.440    0.000

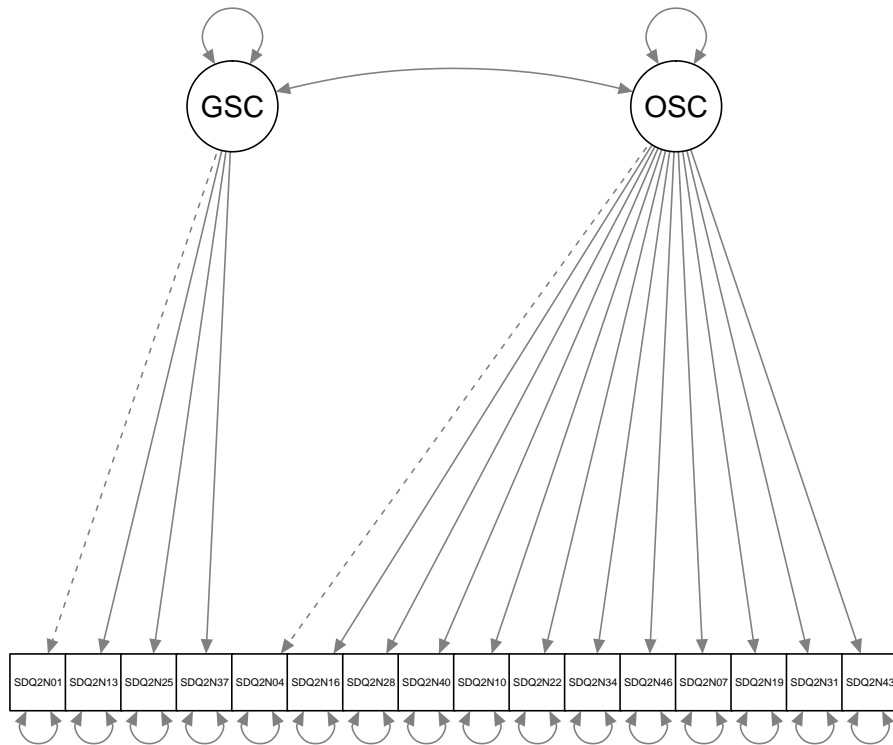
```

```
##      SDQ2N28      1.276    0.177    7.221    0.000
##      SDQ2N40      1.235    0.176    7.026    0.000
##      SDQ2N10      0.581    0.123    4.736    0.000
##      SDQ2N22      0.558    0.117    4.786    0.000
##      SDQ2N34      0.065    0.161    0.406    0.685
##      SDQ2N46      0.514    0.132    3.885    0.000
##      SDQ2N07      2.069    0.262    7.885    0.000
##      SDQ2N19      1.871    0.242    7.721    0.000
##      SDQ2N31      2.021    0.247    8.192    0.000
##      SDQ2N43      1.442    0.193    7.481    0.000
##
## Covariances:
##      Estimate Std.Err z-value P(>|z|)
##      GSC ~~
##      OSC      0.340    0.068    4.975    0.000
##
## Variances:
##      Estimate Std.Err z-value P(>|z|)
##      .SDQ2N01      1.170    0.127    9.216    0.000
##      .SDQ2N13      1.134    0.127    8.906    0.000
##      .SDQ2N25      1.026    0.107    9.582    0.000
##      .SDQ2N37      0.799    0.090    8.842    0.000
##      .SDQ2N04      1.495    0.134   11.171    0.000
##      .SDQ2N16      0.799    0.076   10.490    0.000
##      .SDQ2N28      1.018    0.095   10.695    0.000
##      .SDQ2N40      1.138    0.105   10.828    0.000
##      .SDQ2N10      1.166    0.103   11.364    0.000
##      .SDQ2N22      1.043    0.092   11.360    0.000
##      .SDQ2N34      2.888    0.251   11.510    0.000
##      .SDQ2N46      1.554    0.136   11.425    0.000
##      .SDQ2N07      1.191    0.123    9.654    0.000
##      .SDQ2N19      1.247    0.124   10.067    0.000
##      .SDQ2N31      0.575    0.073    7.852    0.000
##      .SDQ2N43      0.996    0.095   10.442    0.000
##      GSC      0.641    0.142    4.508    0.000
##      OSC      0.461    0.114    4.034    0.000
```

The hypothesis that SC has two factors is not supported by the results (chi square statistic = 457.653,  $p = 0.000$ ), which suggest that the fit of the data to the model is not adequate and null hypothesis should be rejected. In addition the indices CFI (0.776), TLI (0.739) and RMSEA (0.144) are not supporting the null hypothesis.

Let us visualize the model structure:

```
library(semPlot)
semPaths(fitF2, layout='tree2')
```



### b) Hypothesis 3:

The null hypothesis is that SC is unidimensional (only one SC factor).

Alternative hypothesis is that SC is not unidimensional.

```
modelF1 <- "
SC =~ SDQ2N01 + SDQ2N13 + SDQ2N25 + SDQ2N37
+ SDQ2N04 + SDQ2N16 + SDQ2N28 + SDQ2N40
+ SDQ2N10 + SDQ2N22 + SDQ2N34 + SDQ2N46
+ SDQ2N07 + SDQ2N19 + SDQ2N31 + SDQ2N43
"
```

```
fitF1 <- cfa(modelF1, data = ex2.1)
summary(fitF1, fit.measures = T)
```

```
## lavaan 0.6-3 ended normally after 43 iterations
##
## Optimization method          NLMINB
## Number of free parameters    32
##
## Number of observations       265
##
## Estimator                    ML
## Model Fit Test Statistic     531.918
## Degrees of freedom          104
## P-value (Chi-square)         0.000
##
## Model test baseline model:
##
## Minimum Function Test Statistic 1703.155
```

```

## Degrees of freedom                120
## P-value                          0.000
##
## User model versus baseline model:
##
## Comparative Fit Index (CFI)        0.730
## Tucker-Lewis Index (TLI)          0.688
##
## Loglikelihood and Information Criteria:
##
## Loglikelihood user model (H0)      -6749.081
## Loglikelihood unrestricted model (H1) -6483.122
##
## Number of free parameters          32
## Akaike (AIC)                      13562.162
## Bayesian (BIC)                    13676.713
## Sample-size adjusted Bayesian (BIC) 13575.256
##
## Root Mean Square Error of Approximation:
##
## RMSEA                            0.125
## 90 Percent Confidence Interval      0.114 0.135
## P-value RMSEA <= 0.05              0.000
##
## Standardized Root Mean Square Residual:
##
## SRMR                             0.104
##
## Parameter Estimates:
##
## Information                      Expected
## Information saturated (h1) model  Structured
## Standard Errors                  Standard
##
## Latent Variables:
##      Estimate  Std.Err  z-value  P(>|z|)
## SC =~
## SDQ2N01      1.000
## SDQ2N13      1.158    0.247    4.690    0.000
## SDQ2N25      0.903    0.209    4.330    0.000
## SDQ2N37      1.126    0.224    5.018    0.000
## SDQ2N04      1.407    0.278    5.063    0.000
## SDQ2N16      1.772    0.310    5.716    0.000
## SDQ2N28      1.775    0.317    5.605    0.000
## SDQ2N40      1.744    0.315    5.541    0.000
## SDQ2N10      0.859    0.197    4.362    0.000
## SDQ2N22      0.816    0.187    4.371    0.000
## SDQ2N34      0.181    0.222    0.815    0.415
## SDQ2N46      0.756    0.202    3.732    0.000
## SDQ2N07      2.743    0.471    5.826    0.000
## SDQ2N19      2.505    0.434    5.768    0.000
## SDQ2N31      2.711    0.454    5.970    0.000
## SDQ2N43      1.929    0.341    5.659    0.000
##

```



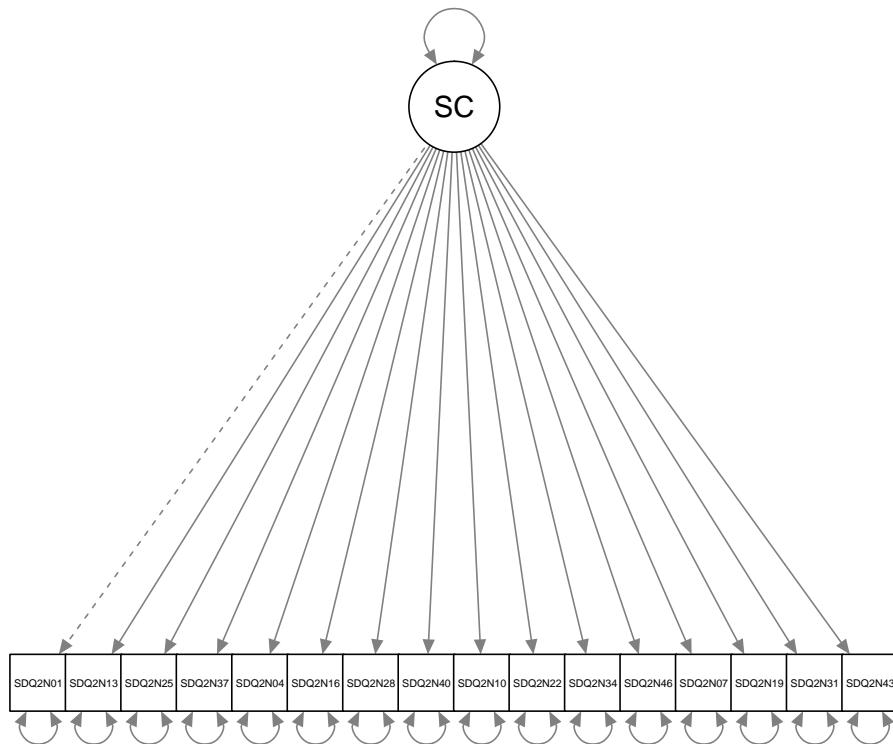
```
## Variances:
##
```

	Estimate	Std.Err	z-value	P(> z )
## .SDQ2N01	1.565	0.138	11.335	0.000
## .SDQ2N13	1.508	0.134	11.266	0.000
## .SDQ2N25	1.299	0.115	11.338	0.000
## .SDQ2N37	0.994	0.089	11.160	0.000
## .SDQ2N04	1.469	0.132	11.140	0.000
## .SDQ2N16	0.762	0.073	10.368	0.000
## .SDQ2N28	0.994	0.093	10.633	0.000
## .SDQ2N40	1.093	0.102	10.742	0.000
## .SDQ2N10	1.140	0.101	11.333	0.000
## .SDQ2N22	1.022	0.090	11.332	0.000
## .SDQ2N34	2.882	0.250	11.508	0.000
## .SDQ2N46	1.535	0.135	11.409	0.000
## .SDQ2N07	1.311	0.132	9.913	0.000
## .SDQ2N19	1.316	0.129	10.186	0.000
## .SDQ2N31	0.650	0.078	8.367	0.000
## .SDQ2N43	1.040	0.099	10.520	0.000
## SC	0.246	0.083	2.972	0.003

The hypothesis that SC is unidimensional is not supported by the results (chi square statistic = 531.918,  $p = 0.000$ ), which suggest that the fit of the data to the model is not adequate and null hypothesis should be rejected. In addition the indices CFI (0.730), TLI (0.688) and RMSEA (0.125) are not supporting the null hypothesis.

Let us visualize the model structure:

```
library(semPlot)
semPaths(fitF1, layout='tree2')
```



Overall the best fit of these three models is the first one with four factors since it is the only model that had at least some support for the model fit.