Formula Summary

The SBN formula converts continuous input current into binary output through a **temperature-controlled stochastic threshold**.

The state of SBN is updated according to the equation:

$$\sigma = \text{sign}(\tanh(\beta I) - r)$$

where:

- I — input current
- β — the inverse temperature
- r — a uniform random number from [-1, 1]

Mathematical Connection

The uniform random number **implements** the probability:

$$P(\sigma_i = +1) = P(\tanh(\beta I_i) > r) = P(r < \tanh(\beta I_i))$$

Since $r \sim \text{Uniform}[-1, 1]$:

$$P(r < \tanh(\beta I_i)) = \frac{\tanh(\beta I_i) - (-1)}{1 - (-1)} = \frac{1 + \tanh(\beta I_i)}{2}$$

Sampling Implementation (How we achieve it)

$$\sigma_i = \text{sign}(\tanh(\beta I_i) - r)$$

Purpose: **Sampling mechanism** to achieve that probability

- Uniform random number r is the **tool** that converts probability → actual binary response
- Each time we run it, we get a concrete +1 or -1

Uniform random number

Maps from probability space to response space"

- **Probability space**: $P(\sigma_i = +1) \in [0, 1]$ (continuous)
- **Response space**: $\sigma_i \in \{-1, +1\}$ (discrete binary)

1. Stochastic SBN (Sampling Implementation)

$$\sigma_i = \text{sign}(\tanh(\beta I_i) - r)$$

2. Deterministic SBN (No randomness)

$$\sigma_i = \text{sign}(\tanh(\beta I_i))$$

Beta as "Smearing" Parameter

Large β (Sharp/Less Smeared)

- $\beta \to \infty$: $\tanh(\beta I_i) \to$ step function
- Nearly deterministic: $P(\sigma_i = +1) \approx 0$ or 1
- **Sharp transition** at $I_i = 0$

Small β (Smooth/More Smeared)

- $\beta \to 0$: $\tanh(\beta I_i) \to 0$ (flat)
- Maximum randomness: $P(\sigma_i = +1) \approx 0.5$ for all I_i
- **Gradual transition** - heavily smeared

Moderate β (Balanced)

- Sigmoid-like probability curve
- Smooth but responsive to input