E 1

P3 E1

System's equation $m\ddot{x} + k_1 x + k_3 x^3 = u$

We introduce de err $e = x - x_n$

$$\dot{e} = \dot{x} - \dot{x}_n$$

$$\ddot{e} = \ddot{x} - \ddot{x}_n = \frac{u}{m} - \frac{k_1}{m}x - \frac{k_3}{m}x^3 - \ddot{x}_n$$

$$\ddot{e} = \frac{u}{m} - \frac{k_1}{m}(e+x_n) - \frac{k_3}{m}(e+x_n)^3 - \ddot{x}_n$$

We propo L function candidate:

$$V = \cancel{\tfrac{1}{2}e}\ \frac{1}{2}m\dot{e}^2 + \frac{1}{2}\cancel{e}\,k_1 e^2$$

$$\dot{V} = m\dot{e}\ddot{e} + k_1\dot{e}e$$

$$\dot{V} = \dot{e}\left[u - k_1(e+x_n) - k_3(e+x_n)^3\right] + k_1\dot{e}e$$

To ensure is negative definite we set

$$u - k_1(e+x_n) - k_3(e+x_n)^3 + k_1 e - \ddot{x}_n = -\underset{>0}{k_d}\,\dot{e}$$

Which provides control function

$$u = k_1(e+x_n) + k_3(e+x_n)^3 + \ddot{x}_n - k_1 e - k_d\dot{e}$$

$e = x - x_n$

$$u = k_1 x + k_3 x^3 + \ddot{x}_n - k_1(x - x_n) - k_d(\dot{x} - \dot{x}_n)$$

$$u = k_1 x_n + k_3 x^3 + \ddot{x}_n - k_d(\dot{x} - \dot{x}_n)$$

Figure 1: p3e1_eq

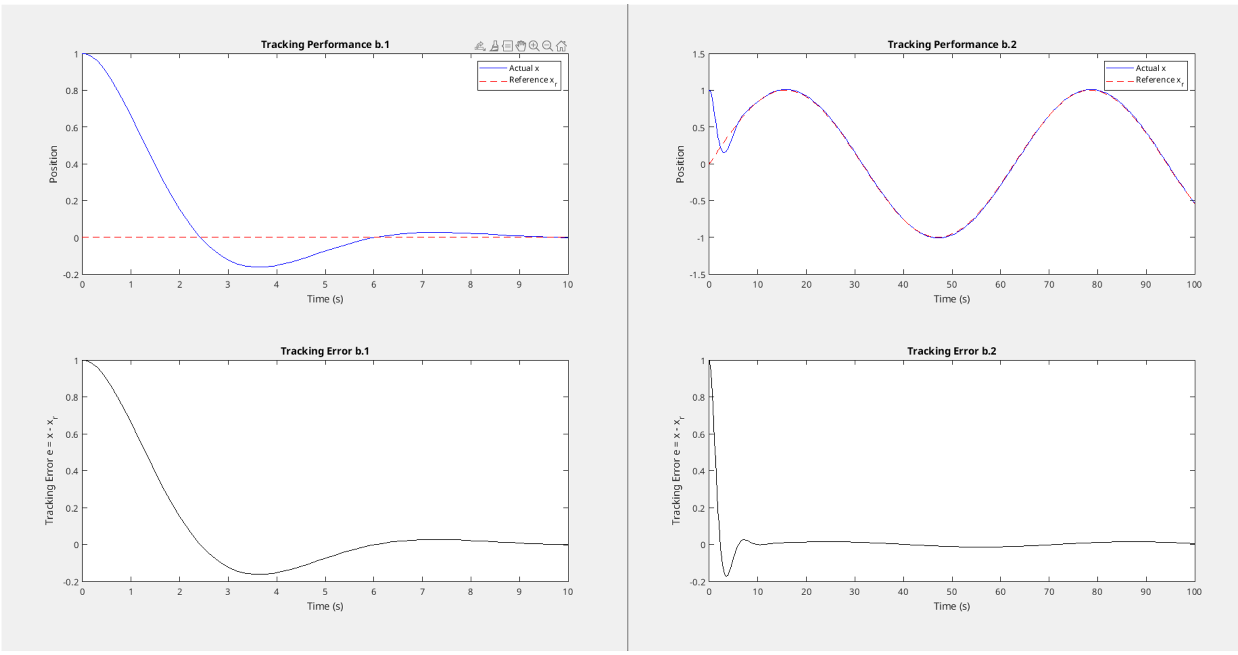

Figure 2: p3e1_plots

E 2

We start with the code provided in `EulerEqsExample` and `Euler`, and add the control law. To get the gain values, I defined a desired decay rate T and got from gain values for a critically damped system:

$$P_i = \frac{2I_i}{T}; \qquad K_i = \frac{P^2}{I_i}$$

We can validate the control law by plotting the error ε in a semi-log plot and comparing it with the expected decay rate given the desired decay rate.

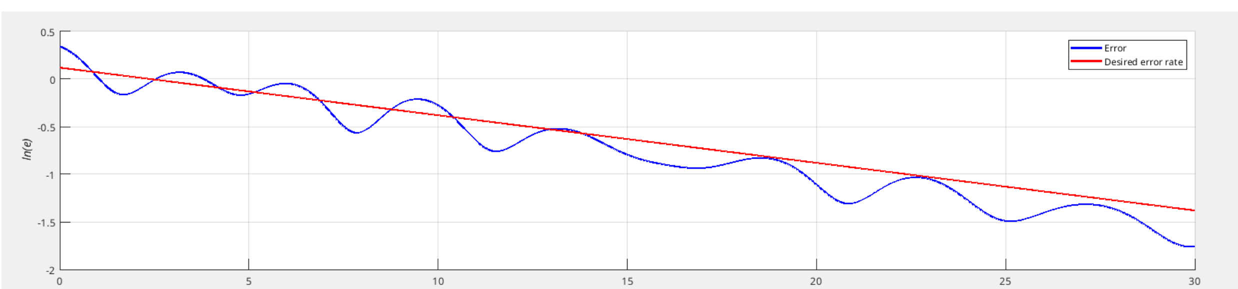

Figure 3: p3e2_plots

C 1

We use the same control law that is provided by the teachers in the `SpacecraftControlMRP` function:

$$\boldsymbol{u} = -[K]\boldsymbol{\sigma} - [P]\boldsymbol{\omega} - \boldsymbol{L}$$

And after the calculation, I added a filter so that each of the components is not bigger than the specified u_{limit}.

For the values of the K_i and P_i, I used the same strategy used in E2. Then, for part b, I used a bigger decay time so any value of u_i has an absolute value greater than u_{limit}.

The following plot shows the difference in response between the two cases.

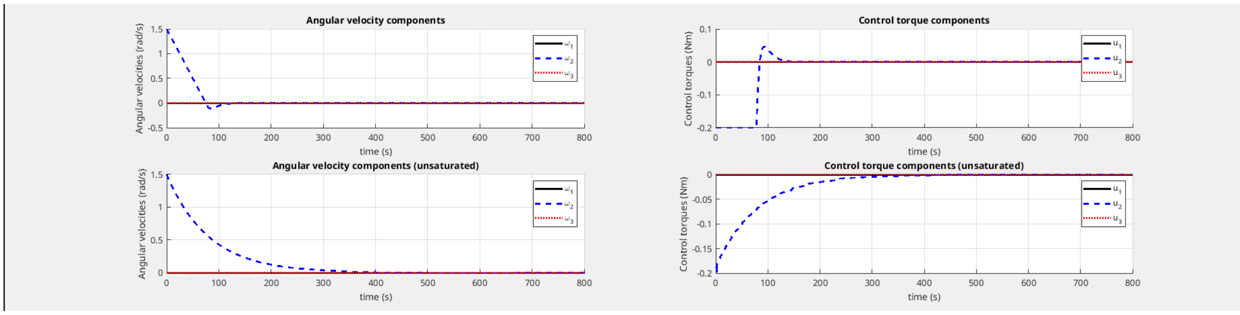

Figure 4: p3c1_plots

c

In my case, the difference to get to a steady state was in the order of **350 seconds**.

C 2

a

$$H_0 = I\omega_0 \; ; \qquad \tau_{avg} \approx m_{max} B_{avg} \; ; \qquad t \approx \frac{||H_0||}{\tau_{avg}}$$

```
t =
    2.2782 h
```

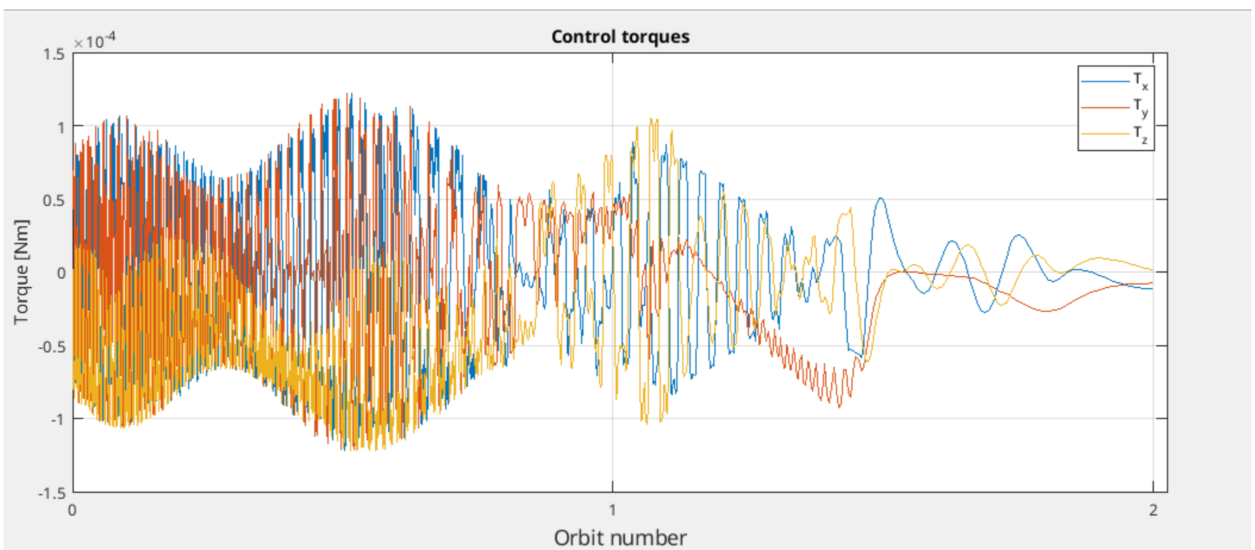

Figure 5: p3c2_plots

On the plots, we can see that the systems gets closer to stability after 1.5 orbits, which means that it takes around 2.5-3h. First, we need to understand that the controller is not able to generate a maximum torque durig the entire detumble process as the magnetic field is not always going to be perfectly oriented for the magnetorque to work at its maximum efficiency. However, the system alreaddy gets at a relatively low angular speed in the 2.3h that we intially estimated.

b

The limiting is mainly the maximum allowed torque, that allows the satellite to avoid saturation and once the magnetorque is smaller than the one required required, the satellite can reach a steady state faster.

In my case, providing the maximum allowed torque of **4 Am^2**, we can reach a relatively steady state in less than an orbit.

c

During my exploration, if we created undersired torque that is 10% greater than the $m_{max} B_{avg}$, it would start creating a bigger rotational speed than the desired one. However, in this case, I would say that any percentage over the expected one would make it tumble as the control law doesn't have any integral part to account for the cumulative error.

A 1

a

Position

$$\boldsymbol{\sigma}_{er} = \frac{(1 - \boldsymbol{\sigma}_r^T \boldsymbol{\sigma})\boldsymbol{\sigma} - (1 - \boldsymbol{\sigma}^T \boldsymbol{\sigma})\boldsymbol{\sigma}_r + 2\boldsymbol{\sigma} \times \boldsymbol{\sigma}_r}{1 + \boldsymbol{\sigma}^T \boldsymbol{\sigma}_r + \boldsymbol{\sigma}^T \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}_r^T \boldsymbol{\sigma}_r}$$

$$\dot{\boldsymbol{\sigma}} = \frac{1}{4} B(\boldsymbol{\sigma})\omega$$

$$B(\boldsymbol{\sigma}) = (1 - \boldsymbol{\sigma}^T \boldsymbol{\sigma}) I_{3\times3} + 2[\boldsymbol{\sigma}\times] + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T$$

Angular velocity

$$\omega = 4B^{-1}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}}$$

$$\omega_{er} = \omega - \omega_r$$

Rigid body equation

$$\mathbf{I}_C \dot{\omega} + \omega \times \mathbf{I}_C \omega = \boldsymbol{u}$$

I'll use a PD-like control structure with feedforward terms:

$$\boldsymbol{u} = \omega \times \mathbf{I}_C \omega - \omega_r \times \mathbf{I}_C \omega_r - P\boldsymbol{\sigma}_{er} - K\omega_{er}$$

where: - $P = \text{diag}(P_1, P_2, P_3)$ are the position feedback gains - $K = \text{diag}(K_1, K_2, K_3)$ are the velocity feedback gains

Gain Selection

We define a single decay time T. Assuming a critically damped system:

$$P_i = \frac{2I_i}{T}$$

$$K_i = \frac{4I_i}{T^2}$$

As we can see from the analytical performance, when T is smaller than the natural period of the reference trajectory, the performance is very good:

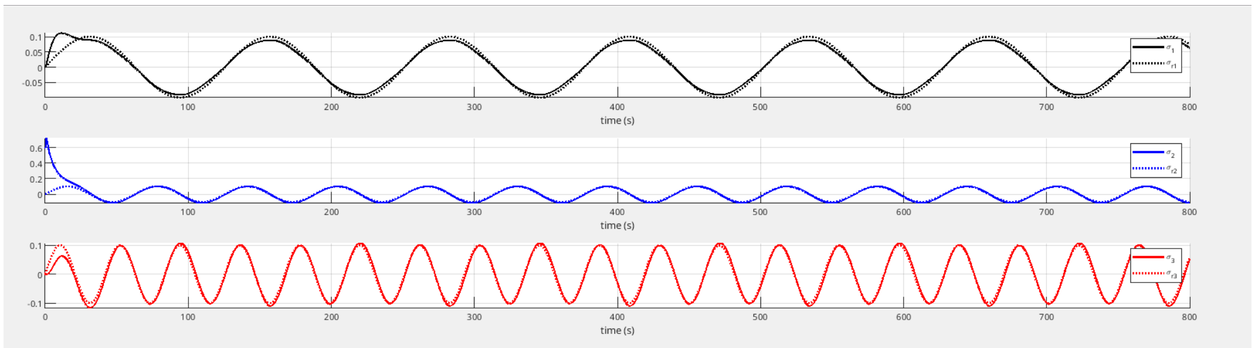

Figure 6: p3a1_plots1

Under higher amplitudes, the system was not as reliable and the control law would not be able to keep up with the reference trajectory.

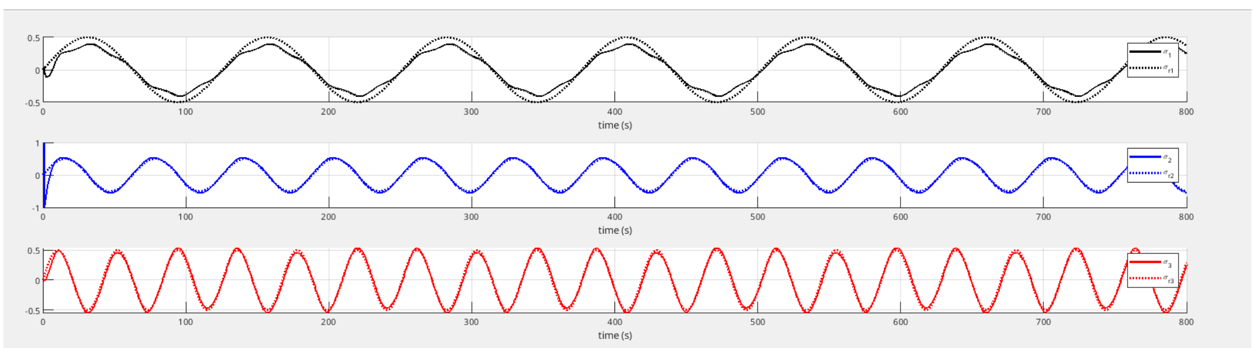

Figure 7: p3a1_plots2

A 2

Similar to the previous control law, without the feedforward terms, we can provide a simpler PI-D control structure:

$$\boldsymbol{u} = -K_p\beta - K_d\beta - z; \quad z = K_i\int_0^t \beta$$

After modifying the code to provide that control law and using the dynamics learned from the previous problem sheets, the following result shows the system was able to reach the desired steady state.

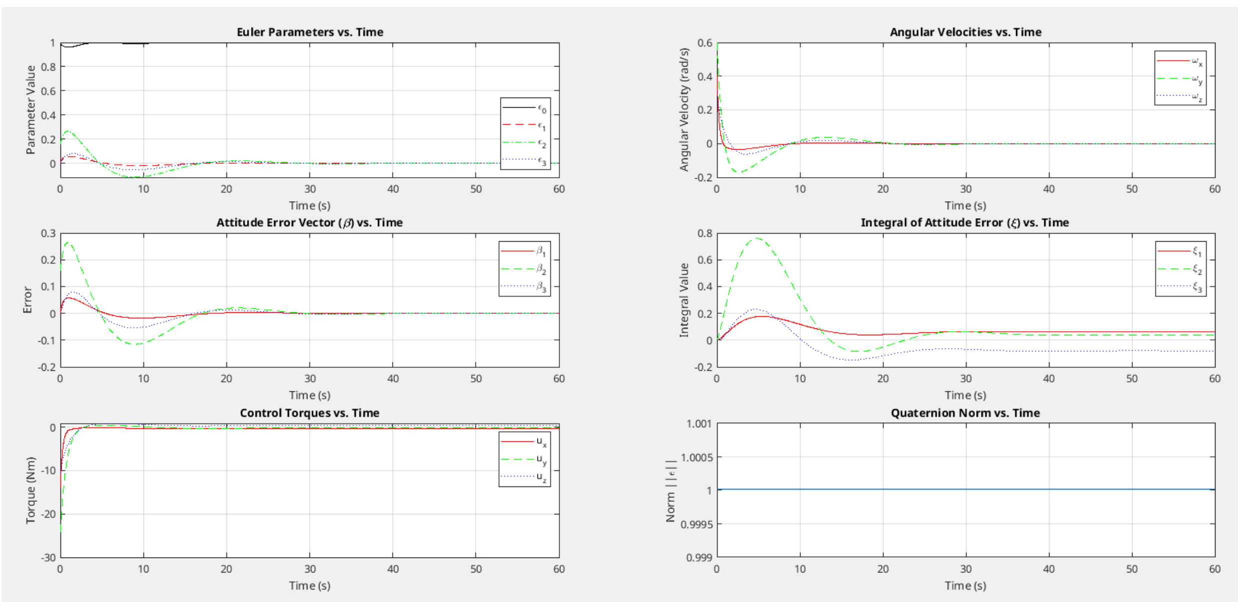

Figure 8: p3a2_plots