

Tarea 1: Operaciones con Números Complejos

Sean $z_1 = 5 + 2i$ $z_2 = 8 + 4i$ $z_3 = -1 + \frac{5}{3}i$

$z_4 = -1 + 3i$ $z_5 = \frac{2}{3} - \frac{1}{2}i$

Realizar las siguientes operaciones, utilizando la representación más adecuada.

1) $z_3 + z_5 =$

$$\left(-1 - \frac{5}{3}i\right) + \left(\frac{2}{3} - \frac{1}{2}i\right) = \left(-1 - \frac{5}{3}i + \frac{2}{3} - \frac{1}{2}i\right)$$

$$= \left(-1 + \frac{2}{3}\right) + \left(-\frac{5}{3}i - \frac{1}{2}i\right) = \boxed{-\frac{1}{3} - \frac{13}{6}i}$$

2) $\frac{1}{2}z_2 + z_4 =$

$$\frac{1}{2}(8 + 4i) + (-1 + 3i)$$

$$4 + 2i - 1 + 3i = \boxed{3 + 5i}$$

3) $z_4 \cdot z_1 = (-1 + 3i)(5 + 2i) = -5 - 2i + 15i + (i^2)(-1)$

$$= -5 - 2i + 15i - 1 = \boxed{-6 + 13i}$$

4) $z_5(z_1 + z_4) = \left(\frac{2}{3} - \frac{1}{2}i\right)(5 + 2i) + (-1 + 3i)$

$$= \left(\frac{2}{3} - \frac{1}{2}i\right)(4 + 5i)$$

$$= \frac{8}{3} - \frac{4}{2}i + \frac{10}{3}i - \frac{5}{2}i^2$$

$$= \frac{8}{3} - 2i + \frac{10}{3}i + \frac{5}{2}$$

$$= \boxed{\frac{31}{6} + \frac{4}{3}i}$$

$$5) \frac{z_1}{z_4} = \frac{5+2i}{-1+3i} \cdot \frac{(-1+3i)}{(-1+3i)} = \frac{-5-15i-2i-6(-1)}{(1-9i^2)(1+9)} \\ = \frac{1-17i}{10} = \frac{1}{10} - \frac{17i}{10}$$

$$6) \frac{z_4 \cdot z_1}{z_3 + z_5} = \frac{(-1+3i)(5+2i)}{(-1+\frac{5}{3}i) + (\frac{2}{3}-\frac{1}{2}i)} = \frac{-5+13i}{-\frac{1}{3}-\frac{13i}{6}} \\ = \frac{(-\frac{1}{3} + \frac{13i}{6})}{(-\frac{1}{3} + \frac{13i}{6})} = \frac{\frac{11}{3} - \frac{143}{6}i + \frac{13i}{3} + \frac{169}{6}}{(-\frac{1}{3})^2 + (\frac{13i}{6})^2(-1)}$$

$$= \frac{-\frac{49}{2} - \frac{169}{6}i}{\frac{1}{9} + \frac{13}{6}} = \frac{-\frac{49}{2} - \frac{169}{6}i}{\frac{41}{18}}$$

$$= \frac{\frac{49}{2}}{\frac{41}{18}} - \frac{\frac{169}{6}}{\frac{41}{18}}i = \frac{882}{82} - \frac{3042}{246}i \\ = \frac{441}{41} - \frac{507}{41}i$$