Zadania Studium Talent 1

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December 2021

1 Definicja kresu dolnego

$$\text{L} = \inf \text{B} \Leftrightarrow \begin{array}{c} 1.\forall\ b \in B : b \geq L \\ 2.\forall\ \varepsilon > 0\ \exists b_\varepsilon \in B < L - \varepsilon \end{array}$$

2 Ciagi

Niech m = Cn
Jeśli n$\to \infty$, to m$\to \infty$
$\lim_{m\to\infty}(1 + \frac{1}{m})^m = e$

Jeśli Cn$\to -\infty$, to $(1 + \frac{1}{Cn})^{Cn}_{n\to\infty} \to e$
$\lim_{n\to\infty}(1 + \frac{1}{Cn})^{Cn} = L$
Niech m = -Cn$\longrightarrow$jeżeli n$\to \infty$, to m$\to \infty$
$L = \lim_{m\to\infty}(1-\frac{1}{m})^{-m} = \lim_{m\to\infty}(\frac{1}{\frac{m-1}{m}})^m = \lim_{m\to\infty}(\frac{m}{m-1})^m$
Niech a = m-1
$L = \lim_{a\to\infty}(\frac{a+1}{a})^{a+1} = \lim_{a\to\infty}[(\frac{a+1}{a})(1 + \frac{1}{a})^a] = e$
$\lim_{n\to\infty}(\frac{a+1}{a}) = 1 \wedge \lim_{n\to\infty}(1 + \frac{1}{a})^a = e$

3 Limesy

$\lim_{n\to\infty}(\frac{5n-1}{5n+3})^{2n-1}$

$(\frac{5n-1}{5n+3})^{2n-1} = [(\frac{5n+3}{5n-1})^{2n-1}]^{-1} = [(\frac{5n-1}{5n-1} + \frac{4}{5n-1})^{2n-1}]^{-1} =$

$= [(1 + \frac{1}{\frac{5n-1}{4}})^{\frac{5n-1}{4} * \frac{8}{5} - \frac{3}{5}}]^{-1} = [e^{\frac{8}{5}} : 1]^{-1} = e^{-\frac{8}{5}}$

$\lim_{n\to\infty}(\frac{5n-1}{6n+2})^n = 0$, ponieważ $\lim_{n\to\infty}(\frac{5n-1}{6n+2}) = 0$

$\lim_{n\to\infty}(\frac{5n-1}{3n+2})^n = \infty$, ponieważ $\lim_{n\to\infty}(\frac{5n-1}{3n+2}) = \infty$