Explanation of algorithms to calculate each curves

Calculating cubic spline

Theorem: $\forall n \in N, a = x_0 < x_1 < ... < x_n = b$ and function f exists exacly one cubic piecewise interpolating function s, satisfying the conditions $s''(a) = s''(b) = 0$

For each interval $[x_{k-1}, x_k], k = (1, 2, ..., n)$:

- $s(x) = h_k^{-1}\Bigg[\frac{1}{6}M_{k-1}(x_k - x)^3 + \frac{1}{6}M_k(x - x_k)^3 + (f(x_{k-1}) - \frac{1}{6}M_{k-1}h_k^2)$
  $(x_k - x) + (f(x_k) - \frac{1}{6}M_k h_k^2)(x - x_{k-1})\Bigg]$

where

- $M_k := s''(x_k), \lambda_k := h_k/(h_k + h_k + 1), h_k := x_k + x_{k-1}$

Values:

- $M_k := s''(x_k), (k = 0, 1, ..., n; M_0 = M_n = 0)$

satisfy the system of linear equations

- $\lambda_k M_{k-1} + 2M_k + (1 - \lambda_k)M_{k+1} = 6f[x_{k-1}, x_k, x_{k+1}], (k = 1, 2, ..., n - 1)$

Therefore, we can calculate the points using the following algorithm:
We calculate auxiliary quantities

- $p_1, p_2, ..., p_{n-1};$
  $q_0, q_1, ..., qn - 1;$
  $u_0, u_1, ..., un - 1$

in the following recurence relation:

- $q_0 := u_0 := 0,$
  $$\left.\begin{aligned} p_k &:= \lambda_k q_k + 2, \\ q_k &:= (\lambda_k - 1)/p_k, \\ u_k &:= (d_k - 1\lambda_k u_{k-1})/p_{k-1} \end{aligned}\right\} = (k = 1, 2, ..., n - 1)$$

where

- $d_k = 6f[x_{k-1}, x_k, xk + 1], \quad (k = 1, 2, ..., n - 1)$

Then

- $M_{n-1} = u_{n-1},$
  $M_k = u_k + q_k M_{k+1}, \quad (k = n - 2, n - 3, ..., 1)$

In the end, for each point $p_i = (x_i, y_i), \quad (i = 0, 1, ..., n),$

We create

- $[t_0, t_1, ..., t_n]; \quad t_j = \frac{j}{n}$

  and

  $[u_0, u_1, ..., u_m]; m =$(Rozmiar u); $u_j = \frac{j}{m}$

Next we evaluate

- $\left.\begin{matrix} s_x(t_k) = x_k \\ s_y(t_k) = y_k \end{matrix}\right\}(k = 0, 1, ..., n)$

In the end we return a sequence of points on the plot

- $\left[\left(s_x(u_0), s_y(u_0)\right), \dots, \left(s_x(u_m), s_y(u_m)\right)\right]$

Calculating Bézier curve

To calculate new point on out plot we will use **De Casteljau's** algorithm:

A Bézier curve B (of degree n, with constrol points $\beta_0, ..., \beta_n$) can be written in Berstein form as follows

- $B(t) = \sum_{i=0}^{n} \beta_i b_{i,n}(t),$

where b is a Bernstain basis polynomial

- $b_{i,n}(t) = \binom{n}{i}(1-t)^{n-i}t^i,$

The curve at point t_0 can be evaluated with the recurence relation

- $\beta_i^{(0)} := \beta_i, \quad (i = 0, 1, ..., n),$

  $\beta_i^{(j)} := \beta_i^{(j-1)}(1 - t_0) + \beta_{i+1}^{(j-1)} t_0, \quad (i = 0, 1, ..., n-j; \quad j = 1, 2, ..., n)$

The result $B(t_0)$ is given by

- $B(t_0) = \beta_0^{(n)}$

Our program creates a sequence of points on the plot using these calculations:

- $\left[\left(B_x(u_0), B_y(u_0)\right), \dots, \left(B_x(u_m), B_y(u_m)\right)\right]$