# Odvodi

| $f(x)$ | $f'(x)$ |
|--------|---------|
| $x^n$ | $nx^{n-1}$ |
| $a^x$ | $a^x \ln(x)$ |
| $e^x$ | $e^x$ |
| $\ln x$ | $\frac{1}{x}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\frac{1}{\cos^2 x}$ |
| $\cot x$ | $-\frac{1}{\sin^2 x}$ |
| $\arcsin x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\arccos x$ | $-\frac{1}{\sqrt{1-x^2}}$ |
| $\arctan x$ | $\frac{1}{1+x^2}$ |
| $\operatorname{sh} x$ | $\operatorname{ch} x$ |
| $\operatorname{ch} x$ | $-\operatorname{sh} x$ |
| $\operatorname{th} x$ | $\frac{1}{\operatorname{ch}^2 x}$ |
| $\operatorname{cth} x$ | $-\frac{1}{\operatorname{sh}^2 x}$ |
| $\operatorname{arsh} x$ | $\frac{1}{\sqrt{x^2+1}}$ |
| $\operatorname{arth} x$ | $\frac{1}{1-x^2}$ |

# Integrali

| $f(x)$ | $\int f(x)dx$ |
|--------|---------------|
| $x^n$ | $\frac{x^{n+1}}{n+1}(n \neq 1)$ |
| $\frac{1}{x}$ | $\ln|x|$ |
| $e^x$ | $e^x$ |
| $\sin x$ | $-\cos x$ |
| $\cos x$ | $\sin x$ |
| $\frac{1}{\cos^2 x}$ | $\tan x$ |
| $\frac{1}{\sin^2 x}$ | $-\cot x$ |
| $\frac{1}{\sqrt{1-x^2}}$ | $\arcsin x$ |
| $\frac{1}{\sqrt{1+x^2}}$ | $\operatorname{arsh} x = \ln|x + \sqrt{x^2+1}|$ |
| $\frac{1}{1+x^2}$ | $\arctan x$ |

## Per Partes

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

$$\int u dv = uv - \int v du$$

## Racionalne funkcije

$\int \frac{p(x)}{q(x)}dx,\qquad p(x),\, q(x)$ sta polinoma

1. Če je $st(p(x)) <= st(p(x))$ polinoma delimo
2. $q(x)$ razdelimo na linearne in kvadratne faktorje
3. Izraz pod integralom razcepimo na parcialne ulomke
   $\frac{p(x)}{q(x)} = \left[\frac{A_1}{x-a_1} + \cdots + \frac{A_{n_1}}{(x-a_1)^{n_1}}\right] + \cdots +$
   $\left[\frac{Z_1}{x-a_k} + \cdots + \frac{Z_{n_k}}{(x-a_k)^{n_k}}\right] + \left[\frac{\alpha_1 x + \beta_1}{x^2+b_1 x+c_1} + \cdots + \frac{\alpha_{m_1} x+\beta_{m_1}}{(x^2+b_1 x+c_1)^{n_1}}\right] +$
   $\cdots + \left[\frac{\varphi_1 x+\omega_1}{x^2+b_l x+c_l} + \cdots + \frac{\varphi_{m_l} x+\omega_{m_l}}{(x^2+b_l x+c_l)^{m_l}}\right]$
4. Integriramo vsakega zase

$$k \geq 2 \qquad st(p(x)) \leq 2k-1$$
$$st(q(x)) \leq 2k-3 \qquad (ax^2+bx+c) \qquad \text{nerazcepen v } \mathbb{R}$$
$$I = \int \frac{p(x)}{(ax^2+bx+c)^k} = \int \frac{Ax+B}{ax^2+bx+c} + \frac{q(x)}{(ax^2+bx+c)^{k-1}}$$

A,B, $q(x)$ poiščemo tako da enačbo odvajamo.

## Korenske funkcije

1. $\int f(\sqrt{ax+b})dx \qquad t = \sqrt{ax+b}$

2. $\int f(\sqrt{ax^2+bx+c})dx$

   a $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ ga prevedemo na oblike:
   - Če je $a < 0 : \int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$
   - Če je $a > 0 : \int \frac{dx}{\sqrt{x^2+c}} = \ln|x + \sqrt{x^2+c}|$

   b $\int \frac{p(x)}{\sqrt{ax^2+bx+c}} = q(x)\sqrt{ax^2+bx+c} + A\int \frac{dx}{\sqrt{ax^2+bx+c}}$
   $st(p(x)) - 1 = st(q(x))$ A, q(x) poiščemo z odvanjanjem

## Kotne funkcije

1.
$$\int \sin(ax)\sin(bx)dx = \int -\frac{1}{2}\left[\cos(a+b)x - \cos(a-b)x\right]dx =$$
$$= -\frac{1}{2}\left[\frac{sin(a-b)x}{(a-b)} - \frac{sin(a+b)x}{(a+b)}\right]$$
$$\int \cos(ax)\cos(bx)dx = \int \frac{1}{2}\left[\cos(a+b)x + \cos(a-b)x\right]dx \ldots$$
$$\int \sin(ax)\cos(bx)dx = \int \frac{1}{2}\left[\sin(a+b)x + \sin(a-b)x\right]dx \ldots$$

2. $\int \cos^m x \sin^n x\, dx$

   (a) Eno od stevil $m,n$ je liho (npr. $m = 2k+1$)
   $$\int \cos^{2k} x \cos x \sin^n x\, dx = \int t^n (1-t^2)^k dt$$
   $$t = \sin x \quad dt = \cos x\, dx$$
   $$\cos^{2k} x = (\cos^2 x)^k = (1-t^2)^k$$

   (b) $m, n$ sta oba soda, $m = 2m_1, n = 2n_1$
   $$\int \cos^{2m_1} x \sin^{2n_1} x\, dx = \int (\cos^2 x)^{m_1}(\sin^2 x)^{n_1} dx =$$
   $$= \int \left(\frac{1+\cos 2x}{2}\right)^{m_1}\left(\frac{1-\cos 2x}{2}\right)^{n_1} =$$
   $$= \text{vsota integralov oblike } \int \cos^k 2x\, dx$$

   kjer je $k \leq m_1 + n_1 = \frac{1}{2}(m+n) < m+1$
   Ce je $k$ lih gremo po 1 tocki
   Ce je $k$ sod ponovimo postopek

   (c) $\int R(\cos x, \sin x)dx$ ($R \ldots$ racinonalni izraz)
   $$t = \tan \frac{x}{2} \quad \cos x = \frac{1-t^2}{t^2+1}$$
   $$\sin x = \frac{2t}{t^2+1} \quad dx = \frac{2}{t^2+1}dt$$
   $$t = \tan x \quad \cos x = \frac{1}{\sqrt{t^2+1}}$$
   $$\sin x = \frac{t}{\sqrt{t^2+1}} \quad dx = \frac{dt}{t^2+1}$$

## Uporaba integralov

1. Ploscina ravnisnkih likov

2. Dolzina krivulj

$$s = \int_b^a \sqrt{1 + f'(x)^2}\,dx$$

3. Prostornina vrtenine

$$V = \pi \int_b^a f(x)^2\,dx$$

4. Povrsina vrtenine

$$S = 2\pi \int_b^a f(x)\sqrt{1 + f(x)^2}\,dx$$

# Kotne funkcije

## Adicijski izreki

$$\sin x \pm y = \sin x \cos y \pm \sin y \cos x$$
$$\cos x \mp y = \cos x \cos y \mp \sin x \sin y$$
$$\tan x \pm y = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

## Faktorizacija

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$
$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\cos x - \cos y = 2 \sin \frac{x-y}{2} \sin \frac{x+y}{2}$$

## Razclenevanje

$$\sin x \sin y = -\frac{1}{2}\left(\cos\left(x+y\right) - \cos\left(x-y\right)\right)$$
$$\cos x \cos y = \frac{1}{2}\left(\cos\left(x+y\right) + \cos\left(x-y\right)\right)$$
$$\sin x \cos y = \frac{1}{2}\left(\sin\left(x+y\right) + \sin\left(x-y\right)\right)$$

$$\cos^2 x = \frac{\cos 2x + 1}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$