

Formule za drugi kolokvij iz Analize IIb

| $f(t)$ | $\mathcal{L}(f)(z)$ |
|--------------------------|-------------------------------|
| 1 | $\frac{1}{z}$ |
| t^n | $\frac{n!}{z^{n+1}}$ |
| e^{at} | $\frac{1}{z-a}$ |
| $\sin(at)$ | $\frac{a}{z^2+a^2}$ |
| $\cos(at)$ | $\frac{z}{z^2+a^2}$ |
| $t \sin(at)$ | $\frac{2az}{(z^2+a^2)^2}$ |
| $t \cos(at)$ | $\frac{z^2-a^2}{(z^2+a^2)^2}$ |
| $\operatorname{sh}(at)$ | $\frac{a}{z^2-a^2}$ |
| $\operatorname{ch}(at)$ | $\frac{z}{z^2-a^2}$ |
| $\sin(at) - at \cos(at)$ | $\frac{2a^3}{(z^2+a^2)^2}$ |

Definicija:

$$\mathcal{L}(f)(z) = \int_0^\infty f(t)e^{-zt}dt$$

Lastnosti:

$$\mathcal{L}(\alpha f + \beta g)(z) = (\alpha \mathcal{L}(f) + \beta \mathcal{L}(g))(z)$$

$$\mathcal{L}(f(t)e^{\alpha t})(z) = \mathcal{L}(f)(z - \alpha)$$

$$\mathcal{L}(f)'(z) = -\mathcal{L}(tf(t))(z)$$

$$\mathcal{L}(f)^{(n)}(z) = (-1)^n \mathcal{L}(t^n f(t))(z)$$

$$\mathcal{L}(f')(z) = z\mathcal{L}(f)(z) - f(0)$$

$$\mathcal{L}(f^{(n)})(z) = z^n \mathcal{L}(f)(z) - \sum_{i=1}^n z^{n-i} f^{(i-1)}(0)$$

Inverz:

$$\mathcal{L}^{-1}(F)(t) = \lim_{R \rightarrow \infty} \frac{1}{2\pi i} \int_{a-iR}^{a+iR} e^{zt} F(z) dz$$