# Introduction to Linear Algebra

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## Lec01 The Geometry of Linear Equations

n linear equations, n unknowns;

* Row picture
* Column picture \*
* Matrix form

[Row Picture](https://www.geogebra.org/calculator/barfggqy)

[Column Picture](https://www.geogebra.org/calculator/twzn2dwj)

Find b linear combination of columns.

What's the combination?

It's will fill all plane.

Linear Combination

Can I solve every AX = b for every b?

Do the combinations of the columns fill all the 3D space?

9 equations, 9 unknowns.

When the column is not independent: col9 = col7 + col8

We can get 8D space.

matrix multiply by the vector

AX is a comb. of column of A.

Dot product

### Recitation

Solve , and find out

its "row picture" and "column picture"

## Rec 1 | MIT 18.085 Computational Science and Engineering I, fall 2008

Vectors / Matrices / Subspaces

Let w = u + v;

=>c1+c2+c3=0, is a plane.

All results of CX, all combinations of (u, v, w\*) give only a plane.

### Recitation

Suppose A is a matrix s.t. the complete solution to

What can we say about columns of A?

A(Xp + CXs) = b

If c = 0, AXp = b; c = 1, AXp + AXs = b

* AXs = 0
* C2 + C3 =b
* 2C2 + C3 = 0
* C2=-b, C3=2b

, Ax = 0

* Dim(N(A)) = 1, rank(A) = 3-1 = 2
* C1 is not a multiple of b.

## Lect02 Matrix Elimination

Elimination

Pivot

Elimination failure

When pivot 0, row exchange.

But in the end, the pivot = 0 means the elimination failed

Back substitution

augmented matrix

Matrix multiplication details

matrix \* column = column

row \* matrix = row

Matrix

**Elementary Matrix**

**Permutation**

Exchange row1 and row2

Exchange column1 and column2

**Inverse matrix**

### Recitation

Solve, using method of elimination:

## Lect03 Multiplication and Inverse Matrices

#1

M\*n \* n\*p = m \* p

#2 column1 of B give the combination of the A column = column1 of C

#3 row1 of A give the combination of the B row = row1 of C

column of A \* row of B

#4

**Block Multiplication**

**Inverse** (square matrices)

Invertible, nonsingular.

Singular case. No inverse.

It can’t get [1, 0], get the pivot.

Singular: if I can find a Vector X(X ≠ 0) with

AX = 0.

Gauss-Jordan (Solve 3 equations at once)

### Recitation

Find the conditions on a and b that make the matrix A invertible, and find when it exists

A is not invertible if a = 0 or a = b

## Lect04 Factorization into A = LU

Transpose

Because