# Introduction to Linear Algebra

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## Lec01 The Geometry of Linear Equations

n linear equations, n unknowns;

* Row picture
* Column picture \*
* Matrix form

[Row Picture](https://www.geogebra.org/calculator/barfggqy)

[Column Picture](https://www.geogebra.org/calculator/twzn2dwj)

Find b linear combination of columns.

What's the combination?

It's will fill all plane.

Linear Combination

Can I solve every AX = b for every b?

Do the combinations of the columns fill all the 3D space?

9 equations, 9 unknowns.

When the column is not independent: col9 = col7 + col8

We can get 8D space.

matrix multiply by the vector

AX is a comb. of column of A.

Dot product

### Recitation

Solve , and find out

its "row picture" and "column picture"

## Rec 1 | MIT 18.085 Computational Science and Engineering I, fall 2008

Vectors / Matrices / Subspaces

  u     v      w

[ 1   [ 0    [ 0

 -1     1      0

  0 ]  -1 ]    1 ]

## Lect02 Matrix Elimination

Elimination

 x + 2y + z = 2

3x + 8y + z = 12

   4y + z = 2

[ |1| 2  1      [ |1| 2  1      [ |1|  2   1

   3  8  1   ->    0 |2| -2  ->    0  |2| -2

   0  4  1 ]       0  4  1 ]       0   0  |5| ]

pivot

Elimination failure

When pivot $\neq$ 0, row exchange.

But in the end, the pivot = 0 means the elimination failed

Back substitution

[ 1 2 1 |2     [ 1 2 1  |2    [ 1 2 1  |2

  3 8 1 |12 ->   0 2 -2 |6 ->   0 2 -2 |6

  0 4 1 |2 ]     0 4 1  |2 ]    0 0 5  |-10]

augmented matrix $Ux = c$

x + 2y + z = 2

    2y -2z = 6

        5z = -10

[ - - -     [ 3

  - - -   \*   4  = 3col1 + 4col2 + 5col3

  - - - ]     5]

matrix \* column = column

[ 1     [ - - -

  2   \*   - - -   = 1row1 + 2row2 + 7row3

  7 ]     - - - ]

row \* matrix = row

Matrix

[ 1 0 0     [ 1 2 1     [ 1 2 1

 -3 1 0   \*   3 8 1   =   0 2 -2

  0 0 1 ]     0 4 1 ]     0 4 1 ]

[  1row1 + 0row2 + 0row3

  -3row1 + 1row2 + 0row3

   0row1 + 0row2 + 1row3 ]

Elementary Matrix

$E\_{21}$ element at (2, 1) -> 0

$E\_{32}(E\_{21}A) = U$

$(E\_{32}E\_{21})A = U$

$Permutation$

Exchange row1 and row2

[ 0 1   \* [ a b   = [ c d

  1 0 ]     c d ]     a b ]

$PA = U$

Exchange column1 and column2

[ a b    [ 0 1    = [ b a

  c d ]    1 0 ]      d c ]

**## Inverse matrix**

[ 1 0 0  [ 1 0 0   [ 1 0 0

  3 1 0   -3 1 0  =  0 1 0

  0 0 1 ]  0 0 1 ]   0 0 1 ]

$E^{-1}E = I$

\* Recitation

Solve, using method of elimination:

## Lect03 Multiplication and Inverse Matrices

[ - - -   [ - - -     [ - - -

  - - -     - - -   =   - - -

  - - - ]   - - - ]     - - - ]

$AB = C$

C\_{34}  = row3 of A \* column4 of B

        = $a\_{31}\cdot b\_{14} + a\_{32}\cdot b\_{24} + \cdots$

        = $\sum^{n}\_{k = 1}{a\_{3k}b\_{k4}}$

$m\times n \cdot n \times p = m \times p$

column1 of B give the combination of the A column = column1 of C

row1 of A give the combination of the B row = row1 of C

2 7

3 8

4 9

1 6

0 0

=

2

3

4

1 6

+

7

8

9

0 0

C is the sum of (column of A) $\cdot$ (row of B)

$\sum^n\_{k}{colA\_{k}\times rowB\_{k}}$

**## Block Multiplication**

A\_{1} A\_{2}

A\_{3} A\_{4}

B\_{1} B\_{2}

B\_{3} B\_{4}

=

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