Project: Secrets of the Andorian tombs

Applied Boundary Value Problems Project 2018

Figure 1. Ancient Andorian tomb.
http://www.nationalgeographic.com.au/travel/dreamy-photographs-illuminate-life-in-dark-arctic-winters.aspx

The Andorian race has all but been annihilated. While searching for survivors, an array of ancient tombs has been discovered. A photo of one tomb is shown in Figures 1. Each tomb is a nearly hemispherical shell structure rumored to conceal vital Andorian secrets. Temperature activated locks safeguard artifacts stored within the walls of the tombs. Your team has been recruited to engineer a device to release the locking mechanism, with the hope of finding information needed to locate and save the remaining members of the Andorian race from certain doom. Your colleagues are developing controllers to maintain an axisymmetric temperature on the inner surface of a tomb. They are depending on you to determine an applied temperature profile that will open the lock. Beware, the entire structure will implode, killing anyone inside, if the walls are heated and the internal temperatures are not correct.

Important submission and grading information:

Students registered for ME201 or MTH281 can work in teams of 2. ***For team projects, each member is expected to contribute equally to the 1) analytical derivation, 2) simulation of results, and 3) writing of the report. At the end of your report, you are required to provide the % contribution of each member of your team to each of these 3 areas of the project.***

Students registered for ME400 or CHE400 must work alone.

The project report and annotated Mathematica .nb file are **due Mon Dec. 17, 2018 by noon**.

Extra 10 points if submitted by 11:40 AM on Wednesday Dec. 12.
No projects will be accepted after the due date.
As explained in the syllabus, each of the three regular exams count as 100, the homework will count collectively as 100 points, and this project will count as 25 points, for a total of 425 points.

A hardcopy of the report must be submitted for grading. This report must be in color if you use color to distinguish curves on plots. If in black/white, use dashed lines and gray-scale contours.

A .pdf file of the report and annotated .nb (or other simulation) file of the calculations must be submitted using blackboard. Please DO NOT email the files. If you work in a team, submit only one copy of each. File names should at least include last names of all authors, e.g. SmithandJones.pdf and SmithandJones.nb.

The report should be < ***10 pages in 12 point font***. Assume the reader knows nothing about the problem statement and include the following ***written in your own words***.

1. (5 points) Short abstract/summary
2. (10 points) Problem statement, with figure (can adapt figure from handout) and equations.
3. (15 points) Overview of your derivation, at least showing the separated equations with their solutions and the ***complete final solution for*** $\boldsymbol{u_{final}(\rho,\phi)}$ ***for arbitrary*** $\boldsymbol{T(\phi)}$.
4. Results, including
    a. (10 points) Solution $u_{trial}(\rho,\phi)$ for $T(\phi) = T_{trial}(\phi) = T_1$.
        i. Expression for $u_{trial}(\rho,\phi)$ in terms of T_1.
        ii. Calculation of T_1 so that $u_{trial}\left(\frac{a+b}{2},0\right) = T_{cr}$.
        iii. Plot of $u_{trial}(\rho,0)$ for $a < \rho < b$, showing $u_{trial}\left(\frac{a+b}{2},0\right) = T_{cr}$.
        iv. Contour plot of $u_{trial}(\rho,\phi)$ for $a < \rho < b$, $0 < \phi < \pi/2$, with color bar.
    b. (30 points) Solution $u_{final}(\rho,\phi)$ for your temperature profile $T(\phi) = T_{final}(\phi)$.
        i. Expression for and plot of your applied temperature profile $T_{final}(\phi)$.
        ii. Expression for $u_{final}(\rho,\phi)$ and values at the two critical points.
        iii. Plot of $u_{final}\big((a+b)/2,\phi\big)$ for $0 < \phi < \pi/2$, showing critical points.
        iv. Contour plot of $u_{final}(\rho,\phi)$ for $a < \rho < b$, $0 < \phi < \pi/2$, with color bar.
5. (10 points) Unique solution or extension to the project.
6. (10 points) Short discussion of your solution, plots, and sanity checks, including how you chose parameters in your final temperature profile to satisfy both constraints. Demonstrate your ***understanding***.

(10 points for report organization, clarity, writing style and code organization)

Problem Statement:

The tomb can be treated as a homogeneous spherical shell as shown in Figure 2. Additional tomb photos are given in Figure 3. Your task is to determine an appropriate temperature distribution $T(\phi)$ applied to the inner spherical surface of the hemispherical shell (at $\rho = a$) to maintain an internal temperature of a point and a ring within the hemispherical shell at a critical temperature ($u\left(\frac{a+b}{2}, 0\right) = 2\,u\left(\frac{a+b}{2}, \frac{\pi}{4}\right) = T_{cr}$). You can assume that the shell material is homogeneous.

<u>Specific parameters:</u>

The hemispherical shell has inner radius a=1 m and outer radius b=2 m.
The outside surface is at 0 ˚C.
The bottom surface is insulated.
You must chose an inside surface temperature profile
$u(a, \phi) = T(\phi)$ for $0 < \phi < \pi/2$ and solve for $u(\rho, \phi)$.

Your choice of $T(\phi)$ should have parameter you can vary to satisfy two additional constraints at critical points within the cross-section:

$$u\left(\frac{a+b}{2}, 0\right) = T_{cr} = 12\,^\circ C$$

$$u\left(\frac{a+b}{2}, \frac{\pi}{4}\right) = \frac{T_{cr}}{2} = 6\,^\circ C$$

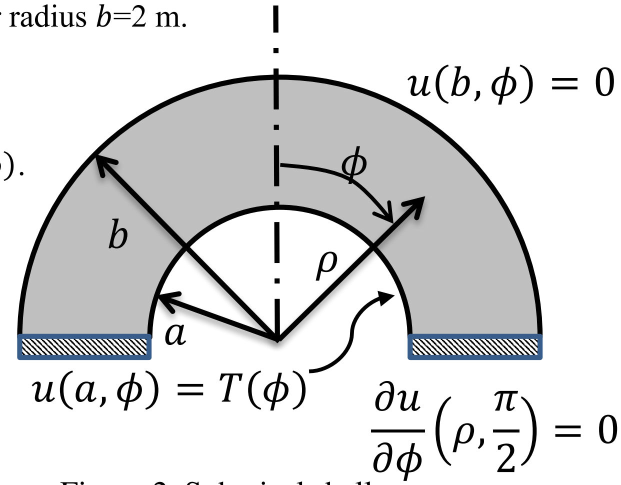

Figure 2. Spherical shell approximation for a tomb structure.

Tasks:

- State and solve the BVP for $u(\rho, \phi)$ for a general temperature distribution $T(\phi)$. Do not consider the two constraints yet.
- Use this general solution to determine and plot the solution when the applied temperature profile is $T(\phi) = T_{trial}(\phi) = T_1$. Note that there is only one parameter (T_1), so you will only be able to satisfy one of the two constraints. Choose T_1 to satisfy $u\left(\frac{a+b}{2}, 0\right) = T_{cr}$.
- Determine a temperature profile $T(\phi)$= $T_{final}(\phi)$ that will satisfy both constraints. Be creative. The goal is to show that you can do calculations and plots for an additional temperature profile and that you understand how the differences in the applied temperature profiles affect the temperature within the hemispherical shell.
- Define and present the solution to a related, but unique problem. You could simply modify or add a constraint or consider a unique temperature profile. You could solve a problem with different BCs at $\phi = \pi/2$ or at $\rho = b$, or even solve a problem with the same geometry in cylindrical coordinates.

Required derivations, results, and plots are listed on the previous page.

Extra credit will be given if you do additional work, such as show results for additional temperature profiles, present additional plots or sanity checks, or calculate and discuss energy flow $-K_0 \iint_S \nabla u \cdot \underline{n}\big|_{\rho=a} dS$ into the hemisphere. Extra credit will also be given for a creative story, but this will be limited to 5 points.

Report Tips:

Reports should be written in a professional manner and carefully checked. In particular, reports should be well organized, neatly **typed**, and proofread. Simulation codes and results should be in a separate file and annotated to be easy to follow. I should be able to run your code to reproduce your results. A sample report and annotated code can be found on the course website.

General suggestions for reports:

- Organize the report: use headings, write in paragraphs, put numbers in tables, number pages.
- Label all graphs, figures, and tables with a number and a caption.
- Describe each graph, figure, or table in text in the body of the report.
- Do NOT include any code in your report.
- Treat and punctuate equations as you would a phrase in a sentence (see your textbook).
- Be concise – shorter is typically better.
- Be precise, quantitative, and include units, do NOT include unrealistic significant figures.
- Don't break paragraphs with figures - put them on the bottom/top of a page (see textbook).
- Use proper grammar and a professional writing style.
- Feel free to be creative with the project story line.

ACADEMIC HONESTY STATEMENT FOR ME201/ME400/MTH281/CHE400:

- ***Students enrolled at the graduate level*** must do the project independently. ***Students enrolled at the undergraduate level*** are encouraged to work in teams of two. For team projects, each member is expected to contribute equally to the analytical derivation, the simulation of results, and the writing of the report. At the end of your report, you are required to provide a clear statement of the contributions of each member of your team to the project.
- **Do not share any part of your report or code, or copy any part of another report or code.** Writing style counts for only a few points of the project grade. Therefore, a correct solution and organized readable report are more important than writing style and grammar.

Figure 3. Photos of the Andorian guardian owl and additional Andorian tombs.
https://mu-peter.blogspot.com/2014/01/ookpik-snowy-owl.html
http://nationalpost.com/news/canada/theyre-not-human-how-19th-century-inuit-coped-with-a-real-life-invasion-of-the-walking-dead
http://www.stephengorman.com/where-men-and-bears-dance-inuit-traditions-in-a-changing-environment/