

In[1]:= $a = 1; b = 2; \rho = \frac{a+b}{2}$

Out[1]= $\frac{3}{2}$

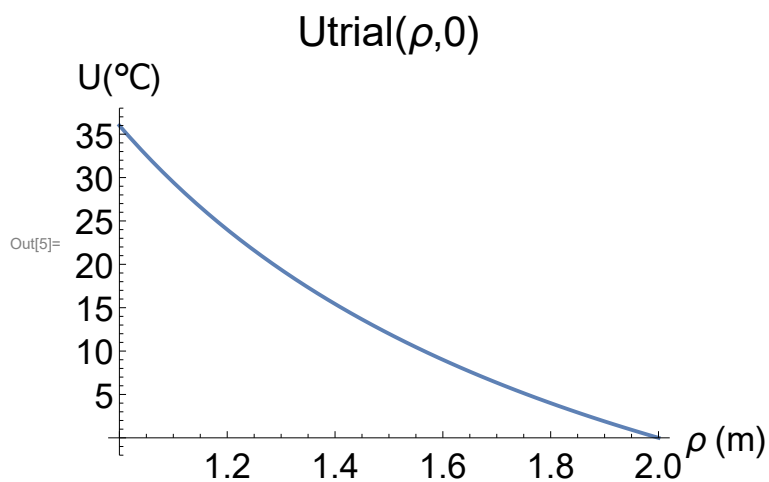
In[3]:= $U[\rho_-, \varphi_-, nMax_] := \sum_{n=0}^{nMax} An[n] * (-b^{4n+1} * \rho^{-1-2n} + \rho^{2n}) * LegendreP[2 * n, Cos[\varphi]]$

In[2]:= $An[n_] := \frac{(4n+1)}{a^{2n} - b^{4n+1} * a^{-1-2n}} \int_0^\pi T * LegendreP[2 * n, Cos[\varphi]] * Sin[\varphi] d\varphi$

1. (*Trial*)

In[4]:= $Utrial[\rho_-, \varphi_] := -36 (-b * \rho^{-1} + \rho^0)$

In[5]:= $Plot[Utrial[\rho, 0], \{\rho, a, b\}, PlotLabel \rightarrow "Utrial(\rho, 0)", PlotRange \rightarrow All, AxesLabel \rightarrow \{\rho \text{ (m)}, "U(^{\circ}C)" \}, BaseStyle \rightarrow \{FontSize \rightarrow 18\}, PlotStyle \rightarrow Thick]$



(*Critical point:*)

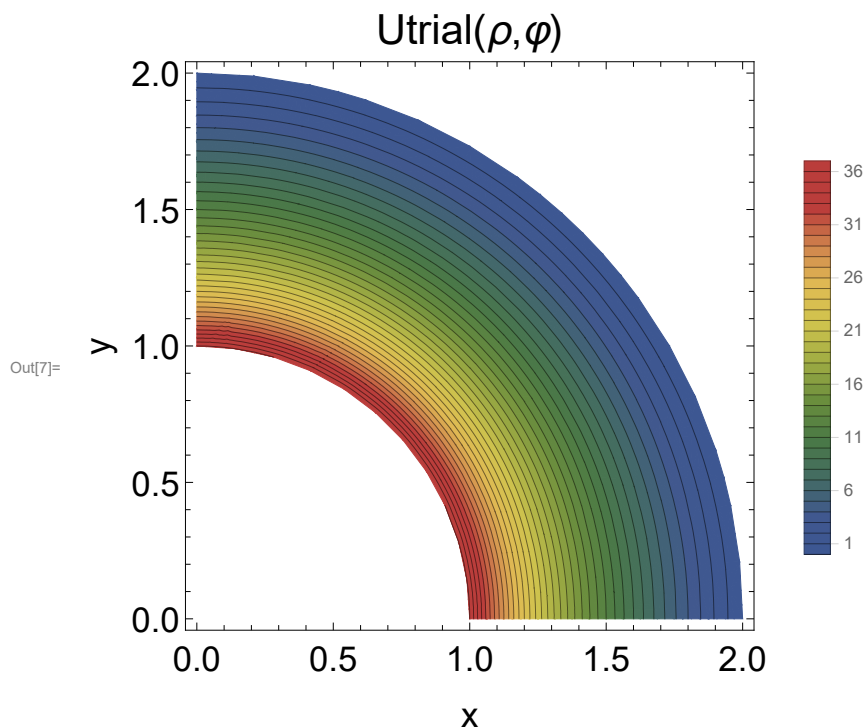
In[6]:= $Utrial[\frac{a+b}{2}, 0]$

Out[6]= 12

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In[7]:= ContourPlot[Utrial[Sqrt[x^2 + y^2], ArcTan[y, x]],
  {x, y} ∈ Annulus[{0, 0}, {1, 2}, {0,  $\frac{\pi}{2}$ }], PlotLabel → "Utrial(ρ,φ)",
  BaseStyle → {FontSize → 18}, FrameLabel → {"x", "y"}, ColorFunction -> "DarkRainbow",
  ContourLabels → False, Contours → Range[-20, 100, 1], PlotLegends → Automatic]

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2. (*Final*)

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In[8]:= Solve[A0 * (ρ^0 - b^1 * ρ^-1) + A1 * LegendreP[2, Cos[θ]] * (-b^5 * ρ^-3 + ρ^2) == 12 &&
  A0 * (ρ^0 - b^1 * ρ^-1) + A1 * LegendreP[2, Cos[ $\frac{\pi}{4}$ ]] * (-b^5 * ρ^-3 + ρ^2) == 6, {A0, A1}]

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Out[8]= $\left\{ \left\{ A_0 \rightarrow -12, A_1 \rightarrow -\frac{864}{781} \right\} \right\}$

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In[9]:= Anfimal[n_] :=
  
$$\frac{(4n+1)}{a^{2n} - b^{4n+1} * a^{-1-2n}} \int_0^{\frac{\pi}{2}} (c * (\cos[\varphi])^2 + d) * \text{LegendreP}[2 * n, \cos[\varphi]] * \sin[\varphi] \, d\varphi$$


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In[10]:= Anfimal[0]

Out[10]= $-\frac{c}{3} - d$

In[11]:= Anfimal[1]

Out[11]= $-\frac{2c}{93}$

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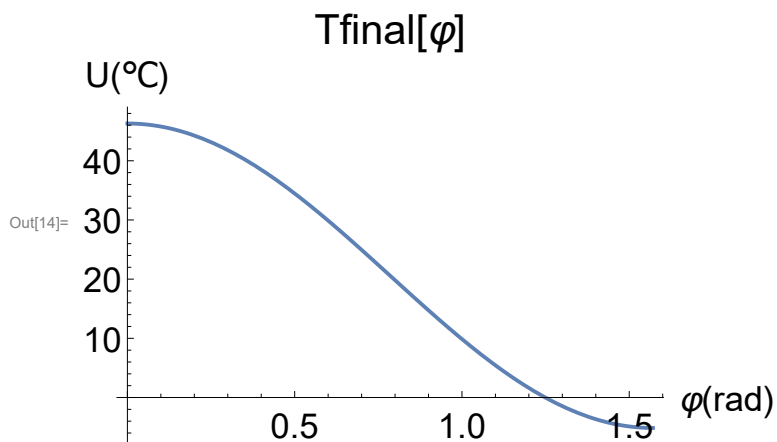
In[12]:= Solve[- $\frac{c}{3} - d == -12$  && - $\frac{2c}{93} == -\frac{864}{781}$ , {c, d}]

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Out[12]= $\left\{ \left\{ c \rightarrow \frac{40176}{781}, d \rightarrow -\frac{4020}{781} \right\} \right\}$

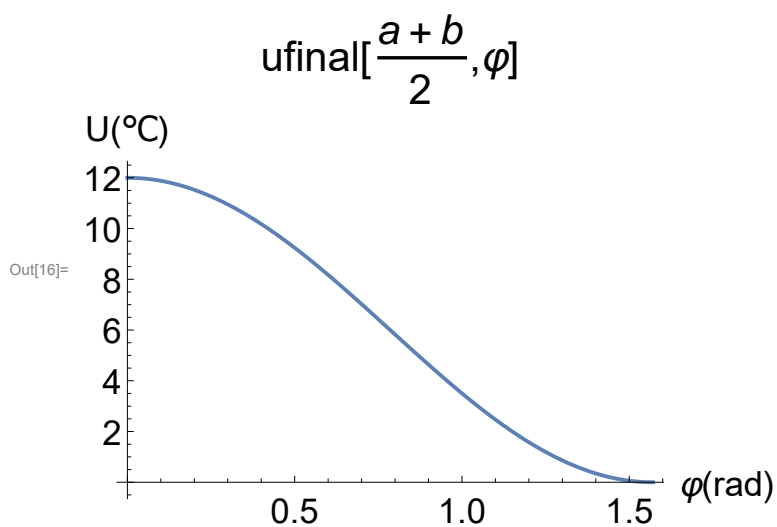
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In[13]:= Tfinal[φ_] :=  $\frac{40176}{781} * (\text{Cos}[\varphi])^2 - \frac{4020}{781}$ 
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In[14]:= Plot[Tfinal[φ], {φ, 0,  $\frac{\pi}{2}$ }, PlotLabel → "Tfinal[φ]", PlotRange → All,
  AxesLabel → {"φ(rad)", "U(°C)"}, BaseStyle → {FontSize → 18}, PlotStyle → Thick]
```



```
In[15]:= ufinal[ρ_, φ_] :=  $-12 * (\rho^0 - b^1 * \rho^{-1}) - \frac{864}{781} * \text{LegendreP}[2, \text{Cos}[\varphi]] * (-b^5 * \rho^{-3} + \rho^2)$ 
```

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In[16]:= Plot[ufinal[ $\frac{a+b}{2}$ , φ], {φ, 0,  $\frac{\pi}{2}$ }, PlotLabel → "ufinal[ $\frac{a+b}{2}$ , φ]", PlotRange → All,
  AxesLabel → {"φ(rad)", "U(°C)"}, BaseStyle → {FontSize → 18}, PlotStyle → Thick]
```



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In[19]:= (*Critical point:*)
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ufinal[ $\frac{a+b}{2}$ , 0]
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ufinal[ $\frac{a+b}{2}$ ,  $\frac{\pi}{4}$ ]
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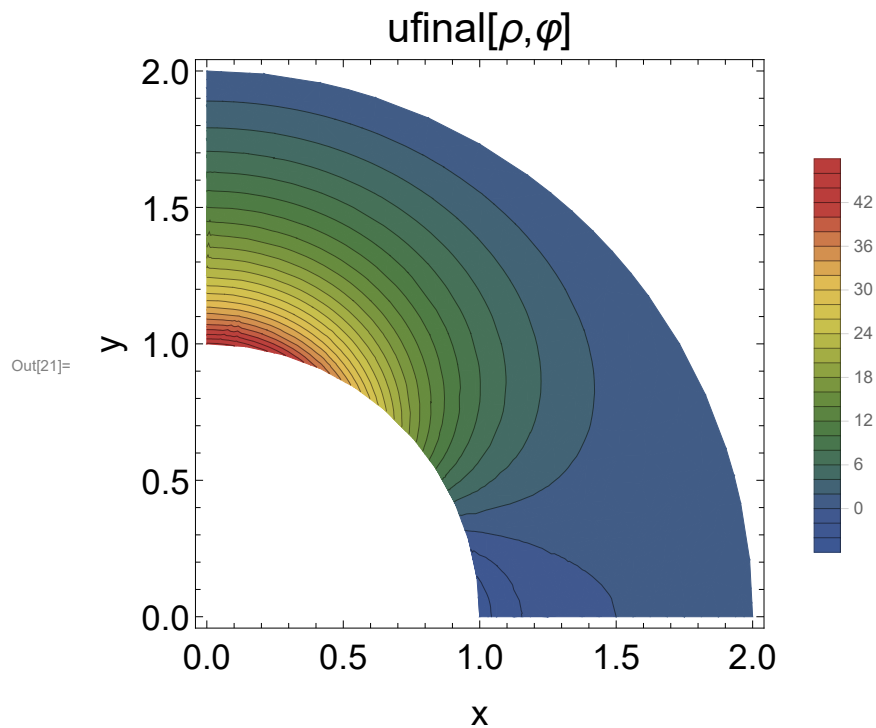
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Out[19]= 12
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Out[20]= 6
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In[21]:= ContourPlot[ufinal[Sqrt[x^2 + y^2], ArcTan[y, x]], {x, y} ∈ Annulus[{0, 0}, {1, 2}],
PlotLabel → "ufinal[ρ, φ]", BaseStyle → {FontSize → 18},
FrameLabel → {"x", "y"}, ColorFunction → "DarkRainbow", ContourLabels → False,
Contours → Range[-20, 100, 2], PlotRange → All, PlotLegends → Automatic]

```



3. (*extension*)

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In[22]:= Solve[B0 * (ρ^0 - b^1 * ρ^-1) + B1 * LegendreP[2, Cos[θ]] * (-b^5 * ρ^-3 + ρ^2) +
B2 * LegendreP[4, Cos[θ]] * (-b^17 * ρ^-9 + ρ^8) == 12 &&
B0 * (ρ^0 - b^1 * ρ^-1) + B1 * LegendreP[2, Cos[π/4]] * (-b^5 * ρ^-3 + ρ^2) +
B2 * LegendreP[4, Cos[π/4]] * (-b^17 * ρ^-9 + ρ^8) == 6 &&
B0 * (ρ^0 - b^1 * ρ^-1) + B1 * LegendreP[2, Cos[π/8]] * (-b^5 * ρ^-3 + ρ^2) +
B2 * LegendreP[4, Cos[π/8]] * (-b^17 * ρ^-9 + ρ^8) == 9, {B0, B1, B2}]

```

Out[22]=

$$\left\{ \left\{ B0 \rightarrow -\frac{6 \left(19 - 42 \cos\left[\frac{\pi}{8}\right]^2 + 20 \cos\left[\frac{\pi}{8}\right]^4 \right)}{5 \left(1 - 3 \cos\left[\frac{\pi}{8}\right]^2 + 2 \cos\left[\frac{\pi}{8}\right]^4 \right)}, \right. \right.$$

$$\left. B1 \rightarrow -\frac{216 \left(1 - 48 \cos\left[\frac{\pi}{8}\right]^2 + 56 \cos\left[\frac{\pi}{8}\right]^4 \right)}{5467 \left(1 - 3 \cos\left[\frac{\pi}{8}\right]^2 + 2 \cos\left[\frac{\pi}{8}\right]^4 \right)}, B2 \rightarrow \frac{241864704 \left(-3 + 4 \cos\left[\frac{\pi}{8}\right]^2 \right)}{596775515735 \left(1 - 3 \cos\left[\frac{\pi}{8}\right]^2 + 2 \cos\left[\frac{\pi}{8}\right]^4 \right)} \right\} \right\}$$

$$\begin{aligned} \text{In[23]:= } \text{uextension}[\rho_-, \varphi_-] := & -\frac{6 \left(19 - 42 \cos\left[\frac{\pi}{8}\right]^2 + 20 \cos\left[\frac{\pi}{8}\right]^4\right)}{5 \left(1 - 3 \cos\left[\frac{\pi}{8}\right]^2 + 2 \cos\left[\frac{\pi}{8}\right]^4\right)} * (\rho^0 - b^1 * \rho^{-1}) - \\ & \frac{216 \left(1 - 48 \cos\left[\frac{\pi}{8}\right]^2 + 56 \cos\left[\frac{\pi}{8}\right]^4\right)}{5467 \left(1 - 3 \cos\left[\frac{\pi}{8}\right]^2 + 2 \cos\left[\frac{\pi}{8}\right]^4\right)} * \text{LegendreP}[2, \cos[\varphi]] * (-b^5 * \rho^{-3} + \rho^2) + \\ & \frac{241864704 \left(-3 + 4 \cos\left[\frac{\pi}{8}\right]^2\right)}{596775515735 \left(1 - 3 \cos\left[\frac{\pi}{8}\right]^2 + 2 \cos\left[\frac{\pi}{8}\right]^4\right)} (-b^{17} * \rho^{-9} + \rho^8) * \text{LegendreP}[4, \cos[\varphi]] \end{aligned}$$

$$\text{In[24]:= } \text{Anextension}[n_-] := \frac{(4n+1)}{a^{2n} - b^{4n+1} * a^{-1-2n}} \int_0^{\frac{\pi}{2}} (\text{uextension}[a, \varphi]) * \text{LegendreP}[2 * n, \cos[\varphi]] * \sin[\varphi] \, d\varphi$$

In[25]:= **Anextension[0]**
Anextension[1]
Anextension[2]
Anextension[3]

$$\text{Out[25]= } -\frac{132}{5}$$

$$\text{Out[26]= } \frac{1728}{5467}$$

$$\text{Out[27]= } -\frac{126805794471936}{304952288540585}$$

$$\text{Out[28]= } 0$$

$$\text{In[34]:= } \text{Ann}[n_-] := \frac{(4n+1)}{a^{2n} - b^{4n+1} * a^{-1-2n}}$$

$$\int_0^{\frac{\pi}{2}} (p * (\cos[\varphi])^4 + q * (\cos[\varphi])^2 + s) * \text{LegendreP}[2 * n, \cos[\varphi]] * \sin[\varphi] \, d\varphi$$

Ann[0]
Ann[1]
Ann[2]
Ann[3]

$$\text{Out[35]= } -\frac{p}{5} - \frac{q}{3} - s$$

$$\text{Out[36]= } -\frac{2}{651} (6p + 7q)$$

$$\text{Out[37]= } -\frac{8p}{17885}$$

$$\text{Out[38]= } 0$$

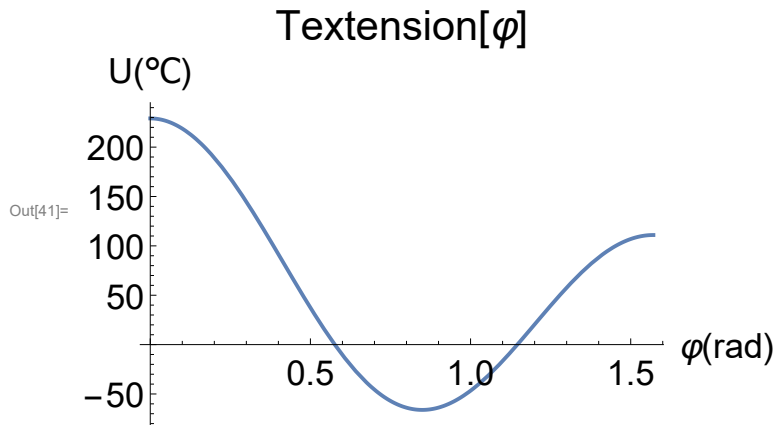
$$\text{In[39]:= } \text{Solve}\left[-\frac{p}{5} - \frac{q}{3} - s == -\frac{132}{5} \&\& -\frac{2}{651} (6p + 7q) == \frac{1728}{5467} \&\& -\frac{8p}{17885} == -\frac{126805794471936}{304952288540585}, \{p, q, s\}\right]$$

$$\text{Out[39]= } \left\{\left\{p \rightarrow \frac{15850724308992}{17050729021}, q \rightarrow -\frac{10806650612890272}{13316619365401}, s \rightarrow \frac{1477892485145460}{13316619365401}\right\}\right\}$$

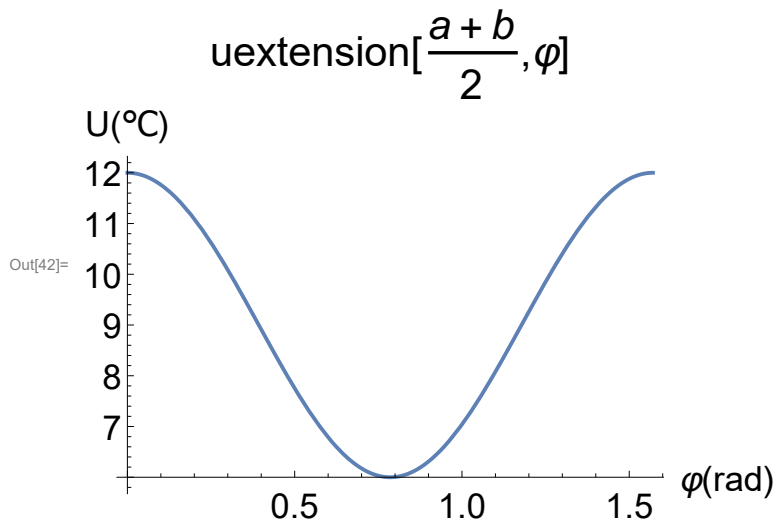
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In[40]:= Textension[φ_] :=
  
$$\frac{15\,850\,724\,308\,992}{17\,050\,729\,021} * (\text{Cos}[\varphi])^4 + -\frac{10\,806\,650\,612\,890\,272}{13\,316\,619\,365\,401} * (\text{Cos}[\varphi])^2 + \frac{1\,477\,892\,485\,145\,460}{13\,316\,619\,365\,401}$$

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In[41]:= Plot[Textension[φ], {φ, 0,  $\frac{\pi}{2}$ }, PlotLabel → "Textension[φ]", PlotRange → All,
  AxesLabel → {"φ(rad)", "U(°C)"}, BaseStyle → {FontSize → 18}, PlotStyle → Thick]
```



```
In[42]:= Plot[uextension[ $\frac{a+b}{2}$ , φ], {φ, 0,  $\frac{\pi}{2}$ }, PlotLabel → "uextension[ $\frac{a+b}{2}$ , φ]", PlotRange → All,
  AxesLabel → {"φ(rad)", "U(°C)"}, BaseStyle → {FontSize → 18}, PlotStyle → Thick]
```



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In[43]:= ContourPlot[uextension[ $\sqrt{x^2 + y^2}$ , ArcTan[y, x]],
  {x, y}  $\in$  Annulus[{0, 0}, {1, 2}, {0,  $\frac{\pi}{2}}$ ], PlotLabel  $\rightarrow$  "uextension[ $\rho, \phi$ ",
  BaseStyle  $\rightarrow$  {FontSize  $\rightarrow$  18}, FrameLabel  $\rightarrow$  {"x", "y"},
  ColorFunction  $\rightarrow$  "DarkRainbow", ContourLabels  $\rightarrow$  False,
  Contours  $\rightarrow$  Range[-20, 560, 3], PlotRange  $\rightarrow$  All, PlotLegends  $\rightarrow$  Automatic]

```

