APPENDIX A

PROOF OF EQUATION (27)

Proof. We aim to derive an upper bound on the delay violation probability in a multi-hop satellite network. Let $W(n)$ denote the total end-to-end delay across H hops, and $W_i(n)$ the queuing delay at the i-th hop. The probability that the total delay exceeds threshold k can be written as:

$$\mathbb{P}(W(n) \geq k) = \mathbb{P}\left(\sum_{i=1}^{H} W_i(n) \geq k\right). \tag{1}$$

Using the Boole's inequality, this can be upper-bounded by:

$$\mathbb{P}(W(n) \geq k) \leq \inf_{\sum_{i=1}^{H} k_i = k} \sum_{i=1}^{H} \mathbb{P}(W_i(n) \geq k_i). \tag{2}$$

From martingale-based delay analysis, the delay violation probability at each hop satisfies:

$$\mathbb{P}(W_i(n) \geq k_i) \leq EH^i e^{-\theta_i^* K_i^s k_i}. \tag{3}$$

Substituting into the upper bound, we obtain an optimization problem for minimizing the sum of individual bounds:

$$\min_{k_1, \ldots, k_H} \quad \sum_{i=1}^{H} EH^i e^{-\theta_i^* K_i^s k_i}$$
$$\text{s.t.} \quad \sum_{i=1}^{H} k_i = k, \quad k_i \geq 0 \quad \forall i. \tag{4}$$

The objective seeks the optimal delay split $\{k_i\}$ across H hops that minimizes the total upper bound on the end-to-end violation probability. To solve this constrained optimization, we define the Lagrangian:

$$\mathcal{L}(k_1, \ldots, k_H; \lambda) = \sum_{i=1}^{H} EH^i e^{-\theta_i^* K_i^s k_i} + \lambda\left(\sum_{i=1}^{H} k_i - k\right). \tag{5}$$

Taking the derivative with respect to k_i and setting it to zero yields the optimality condition:

$$\frac{\partial \mathcal{L}}{\partial k_i} = -EH^i \theta_i^* K_i^s e^{-\theta_i^* K_i^s k_i} + \lambda = 0. \tag{6}$$

Solving for k_i, we obtain:

$$k_i = -\frac{1}{\theta_i^* K_i^s} \ln \left(\frac{\lambda}{EH^i \theta_i^* K_i^s} \right). \tag{7}$$

Substitute this into the constraint $\sum k_i = k$, we get:

$$\sum_{i=1}^{H} -\frac{1}{\theta_i^* K_i^s} \ln \left(\frac{\lambda}{EH^i \theta_i^* K_i^s} \right) = k. \tag{8}$$

Define $\omega = \sum_{i=1}^{H} \frac{1}{\theta_i^* K_i^s}$ and $p_i = \frac{1}{\theta_i^* K_i^s \omega}$, where $\sum p_i = 1$, then solving the above yields:

$$\lambda = \prod_{i=1}^{H} \left(EH^i \theta_i^* K_i^s \right)^{-p_i} e^{\frac{k}{\omega}}. \tag{9}$$

Substituting λ back into the objective gives:

$$\mathbb{P}(W(n) \geq k) \leq \prod_{i=1}^{H} \left(EH^i \theta_i^* K_i^s \omega \right)^{p_i} e^{-\frac{k}{\omega}}. \tag{10}$$

Let the multiplicative coefficient in the delay bound be denoted by

$$D = \prod_{i=1}^{H} (EH^i \theta_i^* K_s^i \omega)^{\frac{1}{\theta_i^* K_s^i \omega}}. \tag{11}$$

Taking the natural logarithm of both sides yields:

$$\ln D = \sum_{i=1}^{H} p_i \ln(EH^i \theta_i^* K_i^s \omega). \tag{12}$$

Since $\ln(\cdot)$ is concave and p_i is a convex combination ($\sum p_i = 1$), we apply the Jensen's

inequality:

$$\ln D \leq \sum_{i=1}^{H} p_i \ln(EH^i). \tag{13}$$

Exponentiating both sides leads to:

$$D = \prod_{i=1}^{H} (EH^i \theta_i^* K_i^s \omega)^{p_i} \leq \sum_{i=1}^{H} EH^i. \tag{14}$$

This expression indicates that the proportional coefficient of the multi-hop delay distribution can be approximated by the sum of the coefficients from each hop. Substituting D into the delay bound expression gives the final approximation:

$$\mathbb{P}(W(n) \geq k) \leq \sum_{i=1}^{H} EH^i e^{-\frac{k}{\omega}}. \tag{15}$$

Equation (27) has been proofed.