28) $\sum_{n=0}^{+\infty} \frac{2+n}{2}$ serie $= 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$

ragione $= \frac{1}{2}$ $a_1 = 1$

$$S_n = n \cdot \left(\frac{a_1 + a_n}{2}\right)$$
$$= n\left(\frac{1}{2} + \frac{2+n}{4}\right)$$
$$= n\left(1 + \frac{n}{4}\right)$$
$$= \frac{1}{4}n^2 + n$$

$\lim_{n \to +\infty} \left(\frac{1}{4}n^2 + n\right) = +\infty$ divergente positivamente

29) $\sum_{n=1}^{+\infty} \left(\frac{5n^2+n}{n^2} - \frac{1}{n}\right)$ serie $= 5, 5, 5, \ldots$

$q = 0$ $a_1 = 5$

$$a_n = \frac{5n^2+n}{n^2} - \frac{1}{n}$$
$$= \frac{\cancel{n}(5n+1)}{n^{\cancel{2}}} - \frac{1}{n}$$
$$= \frac{5n}{n} = 5$$

$$S_n = n\left(\frac{5+5}{2}\right)$$
$$= 5n$$

$\lim_{n \to +\infty} S_n = +\infty$ divergente positivamente

39) $\sum_{n=0}^{+\infty} \left(5n - \frac{3}{2}\pi\right)$ $\quad$ serie $= -\frac{3}{2}\pi,\ 5-\frac{3}{2}\pi,\ 10-\frac{3}{2}\pi, \ldots$

$$q = 5 \qquad a_1 = -\frac{3}{2}\pi$$

$$S_n = n\left(-\frac{3}{4}\pi + \left(\frac{5}{2}n - \frac{3}{4}\pi\right)\right)$$

$$= n\left(\frac{5}{2}n - \frac{3}{2}\pi\right)$$

$$= \frac{5}{2}n^2 - \frac{3}{2}n\pi \qquad \lim_{n\to+\infty}\left(\frac{5}{2}n^2 - \frac{3}{2}n\pi\right) = +\infty$$

divergente positivamente

40) $\sum_{n=1}^{+\infty} \left(\frac{4}{7}\right)^n$ $\quad$ serie $= \frac{4}{7}, \frac{16}{49}, \ldots$

$$q = \frac{4}{7} \qquad a_1 = \frac{4}{7}$$

$$S_n = \frac{4}{7} \cdot \frac{1-\left(\frac{4}{7}\right)^n}{1-\frac{4}{7}} =$$

$$= \frac{4}{7} \cdot \frac{7}{3} \cdot \left(1 - \left(\frac{4}{7}\right)^n\right)$$

$$= \frac{4}{3}\left(1 - \left(\frac{4}{7}\right)^n\right) \qquad \lim_{n\to+\infty}\left(\frac{4}{3}\left(1-\left(\frac{4}{7}\right)^n\right)\right) = \frac{4}{3}$$

convergente

42) $$\sum_{n=1}^{+\infty} (-1)^n \cos(2n^2)$$

serie = $-1, 1, -1, \ldots$

indeterminata

47) $$\sum_{n=0}^{+\infty} \left(\frac{2}{7}\right)^{2n}$$ $q = \frac{4}{49}$ $\quad S_n = \frac{1}{1-\frac{4}{49}} = \frac{49}{45}$

$-1 < q < 1$ convergente

50) $$\sum_{n=0}^{+\infty} \left(\frac{1}{2^n}\right)^4$$ $q = \left(\frac{1}{2}\right)^4$ $\quad S_n = \frac{1}{1-\frac{1}{16}} = \frac{16}{15}$

$-1 < q < 1$ convergente

52) $$\sum_{n=0}^{+\infty} (4-\sqrt{8})^n$$ $q = 4 - \sqrt{8}$ $\quad q > 1$ divergente

61) $$\sum_{n=1}^{+\infty} (5x-9)^n$$ $q = 5x - 9$

$$\begin{cases} 5x-9 > -1 \\ 5x-9 < 1 \end{cases} \quad \begin{cases} x > \frac{8}{5} \\ x < 2 \end{cases} \quad \frac{8}{5} < x < 2 \text{ convergente}$$

65) $$\sum_{n=0}^{+\infty} (x^2-x+4)^n$$ $q = x^2 - x + 4$

$$\begin{cases} x^2-x+4 > -1 \\ x^2-x+4 < 1 \end{cases}$$

$x^2 - x + 5 = 0 \quad \forall x \in \mathbb{R}$

$x^2 - x + 5 = 0 \quad \nexists x \in \mathbb{R}$

57) $\displaystyle\sum_{n=0}^{+\infty} e^{\frac{2n}{x}}$ $\qquad q = e^{\frac{2}{x}}$

$$\begin{cases} e^{\frac{2}{x}} > -1 \\ e^{\frac{2}{x}} < 1 \end{cases} \qquad \begin{cases} \forall x \in \mathbb{R} \\ \frac{2}{x} < 0 \end{cases} \qquad \underline{x < 0} \text{ convergente}$$

81) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{4n^2 + 4n}$ $\qquad a_n = \frac{1}{4n(n+1)}$

$$\frac{1}{4n(n+1)} = \frac{A}{4n} + \frac{B}{n+1}$$

$$\frac{A(\cancel{4}n+1) + B(4n)}{4n(n+1)} = \frac{An + A + 4Bn}{4n(n+1)}$$

$$\begin{cases} A = 1 \\ 4B = -A \quad B = -\frac{1}{4} \end{cases}$$

$$\sum_{n=1}^{+\infty} \left(\frac{1}{4n} - \frac{1}{4(n+1)}\right) \qquad \text{serie} = \left(\frac{1}{4} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{12}\right) \dots$$

$$\lim_{n \to +\infty} \left(\frac{1}{4} - \frac{1}{4(n+1)}\right) = \frac{1}{4} \quad \text{convergente}$$