VRIJE UNIVERSITEIT AMSTERDAM - SCHOOL OF BUSINESS AND ECONOMICS
CREDIT, COMPLEXITY AND SYSTEMIC RISK

Migration & Default Risk

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In this report, I analyze one-period credit migration and default risk for bond portfolios. The objective is to study how portfolio concentration, issuer-level diversification, and asset correlation affect the distribution of portfolio losses. I consider two stylized portfolios with different credit quality profiles and evaluate their risk using Monte Carlo simulation under a one-factor Merton model. By comparing concentrated and diversified portfolio structures across varying correlation levels, the analysis highlights the roles of idiosyncratic and systematic risk in shaping credit portfolio tail losses.

1 CONCENTRATED PORTFOLIO

1.1 Model setup and assumptions

In this section, I analyze a concentrated credit portfolio following a CreditMetrics-style one-period migration and default framework. The total portfolio market value is fixed at 1500 € million and allocated pro rata across rating classes. In the concentrated setting, each rating class is represented by a single issuer, meaning that the entire capital allocated to a given rating is exposed to one obligor only.

Rating migrations are driven by a one-factor Merton model, where issuer asset returns depend on a common systematic factor and an idiosyncratic shock. I consider asset correlations $\rho \in \{0, 33\%, 66\%, 100\%\}$. Portfolio losses arise from rating-driven changes in bond market values at the end of the period.

Table 1.1 summarizes the resulting issuer setup, including invested market values and implied bond units per issuer.

Table 1.1: Issuer Setup for Concentrated Portfolios

Portfolio	Rating	Weight	MV_0	V_0	Units
Investment Grade	AAA	0.60	900.0	99.40	9.054326
Investment Grade	AA	0.30	450.0	98.39	4.573636
Investment Grade	BBB	0.10	150.0	92.79	1.616554
Junk	BB	0.60	900.0	90.11	9.987793
Junk	B	0.35	525.0	86.60	6.062356
Junk	CCC	0.05	75.0	77.16	0.972006

1.2 Calibration checks

1.2.1 Default threshold validation

Before running the simulation, I verify that the migration thresholds implied by the transition matrix are implemented correctly. For a BBB-rated issuer, the one-period default probability is $P(\text{BBB} \to \text{D}) = 0.001680$. The analytically implied default threshold is therefore

$$z = \Phi^{-1}(0.001680) = -2.9327.$$

The threshold used in the simulation matches this value exactly, with zero numerical discrepancy, confirming correct calibration of the migration bins.

1.2.2 Monte Carlo convergence

To assess simulation stability, I repeat the Monte Carlo experiment for the Junk portfolio at $\rho = 33\%$ using three different random seeds with $N = 200{,}000$ scenarios. The resulting 99.5% VaR values vary within a range of 7.32 € million, which I consider acceptable given the size of tail losses. I therefore retain $N = 200{,}000$ as the baseline simulation size.

1.3 Simulation methodology

For each simulation scenario, I draw one realization of the common systematic factor and independent idiosyncratic shocks for each issuer. Asset returns are mapped to rating outcomes using the calibrated migration thresholds. End-of-period bond values are assigned based on the realized rating, and portfolio value is obtained by aggregating issuer-level values. Portfolio losses are defined as the difference between the initial portfolio value and the simulated end-of-period value. From the simulated loss distribution, I compute expected portfolio value, Value-at-Risk, and Expected Shortfall.

1.4 Results

Table 1.2 reports the results for both portfolios across different correlation levels. Expected portfolio values remain close to the initial value, indicating correct calibration of expected losses. In contrast, tail risk measures increase substantially with higher correlation, particularly for the Junk portfolio.

For the Investment Grade portfolio, the 99.5% VaR rises from approximately 27 € million at $\rho = 0$ to nearly 50 € million under perfect correlation. For the Junk portfolio, tail risk is significantly larger: the 99.5% VaR exceeds 300 € million at moderate correlation levels and reaches almost 480 € million when $\rho = 100\%$. Expected Shortfall follows a similar pattern and highlights the severe loss potential under joint adverse migration scenarios.

Table 1.2: Concentrated Portfolio Results

Portfolio	Rho (ρ)	Expected Value	$\text{VaR}_{0.90}$	$\text{VaR}_{0.995}$	$\text{ES}_{0.90}$	$\text{ES}_{0.995}$
Investment Grade	0.00	1499.9459	5.88	27.13	11.70	56.37
Investment Grade	0.33	1499.9754	6.76	33.79	12.73	56.42
Investment Grade	0.66	1499.9611	5.88	37.50	14.45	72.01
Investment Grade	1.00	1499.9825	-1.42	49.97	0.49	85.17
Junk	0.00	1499.5412	48.47	290.03	109.12	305.46
Junk	0.33	1499.7680	48.47	307.41	112.32	377.01
Junk	0.66	1500.1156	48.47	370.28	122.89	445.93
Junk	1.00	1499.6101	48.47	478.67	147.20	478.67

1.5 Interpretation

The results clearly illustrate the impact of concentration and correlation on credit portfolio risk. With only one issuer per rating class, idiosyncratic diversification is absent, and losses are driven by large discrete migration events. Increasing asset correlation amplifies the likelihood of joint downgrades and defaults, leading to highly fat-tailed loss distributions. This effect is particularly pronounced for the Junk portfolio, where lower initial credit quality makes the portfolio highly sensitive to systematic shocks.

2 Diversified portfolio

Having analyzed the concentrated portfolio, I now extend the framework to a diversified setting in order to isolate the effect of issuer-level diversification on credit migration risk. All model assumptions, transition probabilities, bond valuations, and simulation parameters remain unchanged. The only difference relative to the previous section is the portfolio granularity.

2.1 Model setup and assumptions

In the diversified portfolio, each rating class is represented by 100 distinct issuers instead of a single obligor. The total market value allocated to each rating class is unchanged, but it is divided equally across issuers. As a result, each issuer carries a small exposure and migrates independently conditional on the common systematic factor. This setup introduces diversification purely through portfolio structure, without altering expected migration behavior.

2.2 Simulation methodology

The simulation methodology follows the same Monte Carlo procedure as in the concentrated case. For each scenario, I draw one realization of the common systematic factor and independent idiosyncratic shocks for each issuer. Asset returns are mapped to rating outcomes using the calibrated migration thresholds, and end-of-period bond values are assigned accordingly. Portfolio losses are computed as the difference between the initial portfolio value and the simulated end-of-period value. Expected portfolio values, Value-at-Risk, and Expected Shortfall are then estimated from the simulated loss distribution.

2.3 Results

Table 2.1 reports the results for the diversified portfolios across different correlation levels. Expected portfolio values remain close to the initial market value, indicating that diversification does not materially affect expected losses.

Risk measures, however, are substantially reduced relative to the concentrated case at low and moderate correlation levels. For the Investment Grade portfolio, the 99.5% VaR is close to zero when $\rho = 0$ and remains limited at intermediate correlation levels. A similar pattern is observed for the Junk portfolio, where diversification significantly reduces both VaR and Expected Shortfall when idiosyncratic risk dominates.

As correlation increases, tail risk rises for both portfolios, and at $\rho = 100\%$ the risk measures converge to the same levels observed in the concentrated setting. In this extreme case, diversification no longer provides protection, as all issuers migrate simultaneously in response to the common factor.

Table 2.1: Diversified Portfolio Results

Portfolio	Rho (ρ)	Expected Value	$\text{VaR}_{0.90}$	$\text{VaR}_{0.995}$	$\text{ES}_{0.90}$	$\text{ES}_{0.995}$
Investment Grade	0.00	1499.9617	0.80	2.19	1.28	2.59
Investment Grade	0.33	1499.9566	3.22	16.97	7.35	24.56
Investment Grade	0.66	1499.9561	3.58	31.35	11.49	52.72
Investment Grade	1.00	1500.0037	-1.42	49.97	0.47	84.10
Junk	0.00	1499.9270	6.52	14.30	9.27	16.32
Junk	0.33	1499.9191	32.64	137.51	65.55	172.64
Junk	0.66	1499.9201	38.67	265.98	108.16	325.43
Junk	1.00	1500.0983	48.47	478.67	144.71	478.67

2.4 Interpretation

The diversified results demonstrate that issuer-level diversification is highly effective in reducing credit migration risk when asset correlation is low or moderate. By splitting exposures across many obligors, idiosyncratic migration shocks largely average out, leading to thinner loss tails and substantially lower risk measures. However, diversification does not eliminate systematic risk, and under perfect correlation the portfolio behaves as if it were fully concentrated. These findings highlight the central role of correlation in determining the effectiveness of diversification in credit portfolios.

3 Analysis

3.1 Comparison between Portfolio I and Portfolio II

Table 3.1 compares the concentrated and diversified portfolio results at the 99.5% confidence level. The most striking difference between Portfolio I (Investment Grade) and Portfolio II (Junk) is the overall magnitude of tail risk. Across all correlation levels, the Junk portfolio exhibits substantially higher Value-at-Risk and Expected Shortfall, reflecting its lower initial credit quality and higher sensitivity to downgrades and default.

Diversification has a strong mitigating effect in both portfolios, but the absolute reduction in tail risk is far larger for the Junk portfolio. At low correlation, diversification reduces the 99.5% VaR of the Junk portfolio by more than an order of magnitude, whereas the Investment Grade portfolio already exhibits relatively limited tail risk even in the concentrated case. This highlights that diversification is most valuable when baseline credit risk is high.

Table 3.1: Comparison of Concentrated vs Diversified Portfolio Results

		$\text{VaR}_{0.995}$		$\text{ES}_{0.995}$	
Portfolio	Rho (ρ)	Concentrated	Diversified	Concentrated	Diversified
Investment Grade	0.00	27.13	2.19	56.37	2.59
Investment Grade	0.33	33.79	16.97	56.42	24.56
Investment Grade	0.66	37.50	31.35	72.01	52.72
Investment Grade	1.00	49.97	49.97	85.17	84.10
Junk	0.00	290.03	14.30	305.46	16.32
Junk	0.33	307.41	137.51	377.01	172.64
Junk	0.66	370.28	265.98	445.93	325.43
Junk	1.00	478.67	478.67	478.67	478.67

3.2 Impact of correlation

The effect of diversification depends critically on the level of asset correlation. At $\rho = 0$, diversification is highly effective: idiosyncratic migration shocks largely cancel out, leading to very low tail risk in the diversified portfolios. As correlation increases, the benefit of diversification diminishes steadily, since a growing fraction of portfolio risk is driven by the common systematic factor.

At $\rho = 100\%$, diversification provides no risk reduction at all. In this extreme case, all issuers migrate identically, and the diversified portfolios collapse to the same loss distributions as their concentrated counterparts. This explains why both the 99.5% VaR and Expected Shortfall coincide exactly across the concentrated and diversified portfolios when correlation is perfect.

3.3 Tail fatness and loss distribution shape

To assess tail fatness, I examine the ratio of the 99.5% Expected Shortfall to the 99.5% Value-at-Risk. For a standard normal distribution, this ratio is approximately 1.125. In the credit portfolios considered here, this ratio is consistently higher, particularly for the Junk portfolio and at elevated correlation levels.

This indicates that credit loss distributions are substantially more fat-tailed than the underlying asset return distribution. The presence of discrete rating migrations and default events generates large jump losses, which inflate Expected Shortfall relative to VaR. The effect is strongest when systematic risk dominates, confirming that credit portfolios exhibit pronounced tail risk even when driven by normally distributed asset returns.

References

[1] Vrije Universiteit Amsterdam (2026). Credit, Complexity and Systemic Risk - Lecture Slides.

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