Vrije Universiteit Amsterdam - School of Business and Economics
Quantitative Investing

# Portfolio Selection Assignment

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The portfolio selection assignment consists of two parts. For the first part, I was asked to construct a portfolio based on my pre-knowledge, which will serve as a benchmark. The second part, which is the main body of the assignment, asked me to revise my portfolio each course week based on the material of that week. I needed to motivate my decisions, which will show whether I was able to apply the material that was discussed in class.

## Part I.

### 1.1 Task

For this assignment I was asked to construct a portfolio that consists of stocks from the S&P500. I received a hypothetical amount of $100,000 for which I can buy stocks or obtain a risk-free interest rate of 1% (annual). Any stock I decided to buy can only be bought in a positive integer number. The risk-free interest rate I can think of as depositing money in a bank account such that any amount can be chosen as a deposit. I was asked to hand in the Excel sheet with the number of stocks held per stock, the amount invested at the risk-free interest rate, and all relative weights in the portfolio. I need to make sure it adds up to $100,000; I cannot be short or leave money on the table. Any investment at the risk-free interest rate should also be explicitly written.

### 1.2 Introduction and Background

The way I construct my first portfolio is up to me and what I know so far about finance. As for my educational background, I graduated with a Bachelor of Business Administration, which consisted of rather few financial courses. To compensate for this shortcoming, I wrote several research papers on financial markets, using different methodologies. Thanks to these papers, I was hired to work as an equity analyst at a Hungarian investment bank – this is where I came from. During my work there, I had the opportunity to gain insight into different aspects of financial analyses, from reporting on individual public companies and recommending investment opportunities, to developing econometric and machine learning-based models to predict foreign exchange rates and stock prices.

### 1.3 First Portfolio Construction *(Week 1)*

Even though I have been developing more sophisticated models for portfolio/stock selection, I would stick to a relatively simple strategy for the first part of the assignment. One reason for this is that I want it to be a baseline model for the following weeks' solutions, while the other reason is the given constraints – e.g., working with only yearly prices.

My solution begins with obtaining the previous year's close prices, then calculating a one-year percentage return, without considering dividends. I omit stocks without price data, i.e., that were not listed on the stock exchange in the previous year. Stocks with non-positive returns are also filtered out and receive no allocation. Then, I calculate the weight for each stock by dividing the stock's return by the sum of all returns. Multiplying the weights by the allocated $100,000 will give the dollar amount I want to invest in each stock. To get how many shares to buy, I divide the allocated amount by the latest available close price. Since I can buy only an integer number of shares, I round this value. Rounding has a slight chance to result in an allocation of more money than the specified amount; therefore, I am supposed to round these values down. However, I tested it, and it does not exceed this amount, so I decided to simply round it. After rounding and allocating the total amount of money, I invest the leftover cash into the risk-free asset. With this, I implemented a classic positive momentum strategy.

# Part II.

### 2.1 Task

For the second part of this assignment I am asked to update the initial portfolio that I handed in at the first hand-in date. In the course, we discussed topics that could help me strengthen my personal investment portfolio. It is up to me to determine and motivate whether material that we discussed in class is appropriate to incorporate in my portfolio. I could argue it is the case or it is not, but I need to write a well-founded motivation for my decision to update my portfolio. I can also have the opportunity not to update my portfolio, but in this case, I need to explain why I do so based on the class material, this can also be supported by an analysis.

### 2.2 Tangency Portfolio using Mean-Variance Analysis *(Week 2)*

In the second part of the assignment, I decided to switch from my initial momentum-based portfolio to a mean–variance based tangency portfolio. In mean-variance portfolio theory, investors evaluate portfolios solely by their expected return $\mu$ and variance $\sigma^2$. With quadratic utility (or normally distributed returns), these two parameters are sufficient statistics for choice. Preferences are monotone: higher $\mu$ is preferred, while higher $\sigma$ is disliked. The goal is therefore to identify efficient portfolios, which yield the highest expected return for a given level of risk.

Let $\mu \in \mathbb{R}^N$ denote expected returns, $\Sigma \in \mathbb{R}^{N\times N}$ the positive-definite covariance matrix of asset returns, and $1$ the vector of ones. For a target return $\hat{\mu}$, the minimum-variance portfolio solves

$$\min_{w} \ \tfrac{1}{2} w'\Sigma w \quad \text{s.t.} \quad w'\mu = \hat{\mu}, \ \ 1'w = 1.$$

The closed-form solution is

$$w_{\hat{\mu}} = \frac{C\hat{\mu} - A}{D}\,\Sigma^{-1}\mu + \frac{B - A\hat{\mu}}{D}\,\Sigma^{-1}1,$$

with

$$A = 1'\Sigma^{-1}\mu, \quad B = \mu'\Sigma^{-1}\mu, \quad C = 1'\Sigma^{-1}1, \quad D = BC - A^2.$$

Varying $\hat{\mu}$ traces out the minimum-variance frontier. The upper branch of this curve is the efficient frontier, representing all portfolios with the best attainable return at each variance level.

Among all efficient portfolios, one stands out: the portfolio that offers the highest slope in mean-variance space – i.e., the greatest increase in expected return per additional unit of risk. This is the tangency portfolio.

Formally, it solves

$$\max_{w} \frac{w'\mu}{\sqrt{w'\Sigma w}} \quad \text{s.t.} \quad 1'w = 1,$$

which is equivalent to maximizing the portfolio Sharpe ratio when no risk-free asset is included. The solution is proportional to $t \propto \Sigma^{-1}\mu$. Scaling is determined by the normalization $1't = 1$. Thus,

$$\boxed{t = \frac{\Sigma^{-1}\mu}{1'\Sigma^{-1}\mu}}$$

This portfolio represents the unique combination of risky assets that achieves the steepest risk-return trade-off along the efficient frontier. All other efficient portfolios are linear combinations of this tangency portfolio and the global minimum-variance portfolio. Including a risk-free asset would alter the efficient frontier, making it a straight line from the risk-free rate to the tangency portfolio. However, in this assignment, I handle the risky assets first, and then allocate any leftover cash to the risk-free asset.

In constructing the tangency portfolio, the unconstrained mean-variance solution gave weights that included negative values (short positions) and values greater than one (leveraged longs). Since I do not want to allow short selling in this assignment, I decided on a practical modification: I set all negative weights to zero and then renormalized the positive weights so that they sum to one. This way, my portfolio remains long-only while preserving as much of the original tangency solution as possible. Although this adjustment is not strictly mean-variance optimal under the no-shorting constraint, it is a reasonable compromise that avoids excessive complexity in the optimization while ensuring realistic, fully invested allocations.

As for the allocation, I used the same approach as in the first part of the assignment, only now with the new tangency weights.

### 2.3 Placeholder *(Week 3)*

### 2.4 Placeholder *(Week 4)*

### 2.5 Placeholder *(Week 5)*

### 2.6 Placeholder *(Week 6)*