Estimating time-varying betas for a panel of stock returns using DCC-GARCH

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Theoretical Background

In a standard CAPM regression setting, a stock's excess return is modeled as $r_{i,t} = \alpha_i + \beta_i f_t + \varepsilon_{i,t},$, where the $r_{i,t}$ is the return of stock i at time t, f_t is the return of factors (e.g., market, SMB, or cyber-risk index in our case),and β_i is the exposure/sensitivity of stock i to factor f. Usually, β_i is assumed constant over the sample. But exposures shift: banks may become more sensitive to cyber-risk factors after a major incident, then less sensitive later, for example. So, we allow $\beta_{i,t}$ to change over time: $r_{i,t} = \alpha_i + \beta_{i,t} f_t + \varepsilon_{i,t}$, where $\beta_{i,t}$ is the time-varying beta.

There are two common approaches to estimate time-varying betas: DCC-GARCH and Kalman filter. In this short report, I will focus on DCC-GARCH (Dynamic Conditional Correlation - Generalized Autoregressive Conditional Heteroskedasticity) that models the time-varying covariance matrix of multiple return series. If you fit a multivariate DCC-GARCH on stock $r_{i,t}$ and factor f_t, you get conditional variance of factor: $\mathrm{Var}_t(f)$, and conditional covariance between stock and factor: $\mathrm{Cov}_t(r_i, f)$. Then $beta_{i,t} = \frac{\mathrm{Cov}_t(r_i,f)}{\mathrm{Var}_t(f)}$ So, the DCC model naturally produces dynamic betas.

Data

As for the factor, I use the market (MKT) factor from the Fama-French 3-Factor model. This is used as a proxy for the overall market return. Using a factor related to cyber-risk would be more relevant to the context of the project, but such data is not easy to obtain. Thus, I proceed with the market factor, which can be accessed through the Fama-French website. As for the panel of stock, given the context of the project, I consider six banking stocks from the S&P 500 index. The stock data is obtained using the WRDS database, specifically the CRSP dataset, as the university has a subscription to it. The data spans from June 30, 2015, to June 30, 2025, covering a period of roughly ten years. The stocks selected are: Citigroup (C), JPMorgan Chase (JPM), Bank of America (BAC), Wells Fargo (WFC), Goldman Sachs (GS). The factor used is: market factor $R_m - R_f$ (denoted as MKT).

Estimation

I start with defining the parameters. I work with an estimation window of 252 trading days (approximately one year), and confidence intervals are computed at the 95% level, as requested. To estimate the time-varying betas, I loop over each stock in the panel. For each stock, I extract its returns and the factor returns, and fit a GARCH(1,1) model using the `arch` library. The GARCH model captures the dynamic correlations between the stock and the factor over time. After fitting the model, I extract the conditional covariance between the stock and the factor, as well as the conditional variance of the factor. Using Equation 1, I compute the time-varying beta for each stock. I use the standard errors from the GARCH model to compute the 95% confidence intervals for the betas. For comparison, I also compute the OLS beta using the same returns using the `statsmodels` library. As a final step, I store the results in a dictionary for later use.

Results

The results of the analysis are presented in Table a.1. The table compares the constant ($\hat{\beta}_i$) and time-varying beta ($\hat{\beta}_{i,t}$) estimates across different stocks. The time-varying beta estimates show a wide range of values, indicating varying levels of systematic risk across stocks. The Figure a.1 provides a more nuanced view, capturing changes in risk exposure over time. Whether the beta estimates are statistically significant can be assessed using the confidence intervals.

Applying a new method (in my case DCC-GARCH) always comes with challenges. Understanding the underlying assumptions and limitations of the model is also crucial. In a longer-term project, I would consider comparing the DCC-GARCH results with those obtained from the Kalman filter approach. This comparison would provide insights into the robustness of the time-varying beta estimates and help identify any discrepancies between the two methods.

Appendix

Table a.1: Comparison of Beta Estimates across Firms

Stock	Constant Beta	Time-Varying Beta Mean (Std)	Time-Varying Beta Min – Max	Range
BAC	1.190	1.301 (0.441)	0.392 – 3.205	2.813
BK	0.997	1.110 (0.405)	0.408 – 3.541	3.133
C	1.290	1.345 (0.466)	0.412 – 3.616	3.204
GS	1.179	1.295 (0.390)	0.518 – 3.488	2.970
JPM	1.062	1.112 (0.382)	0.414 – 3.069	2.656
WFC	1.121	1.194 (0.455)	0.432 – 3.291	2.859

Figure a.1: Time-Varying Factor Betas for Banking Stocks (DCC-GARCH Approach)