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PAPER

GEOMETRY I: SM-derived gravitational coupling $G(M_Z)$ anchored at the electroweak scale

Michael DeMasi[1] [1]Independent Researcher, Milford, CT, USA

E-mail: demasim90@gmail.com

Abstract
At the electroweak scale $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, the one-loop Standard Model (SM) decoupling matrix admits a unique primitive integer left-kernel $\chi = (16, 13, 2)$ (Smith normal form). This identifies an aligned depth coordinate $\Xi = \chi \cdot \hat{\Psi}$ in log–coupling space, with $\hat{\Psi} = (\ln \hat{\alpha}_s, \ln \hat{\alpha}_2, \ln \hat{\alpha})$. The same direction numerically coincides with the soft eigenmode of the positive-definite Fisher/kinetic metric K_{eq}, fixing the electroweak-scale dimensionless anchor

$$\Omega = e^{\Xi} = \hat{\alpha}_s^{16} \hat{\alpha}_2^{13} \hat{\alpha}^2.$$

This anchor defines an SM-derived gravitational normalization,

$$G(M_Z) = (\hbar c/m_p^2)\, \Omega(M_Z),$$

with no new fields, tunable parameters, or functional freedoms. An even, parity-preserving curvature gate $\Pi(\Xi)$ satisfying $\Pi'(\Xi_{\mathrm{eq}}) = 0$ extends this equilibrium value to an effective coupling $G(x) = G(M_Z)\Pi(\Xi(x))$ while maintaining a massless, luminal helicity ± 2 tensor sector. Near equilibrium the laboratory response is fixed and strictly quadratic,

$$\Delta G/G = (\delta\Xi/\sigma_\chi)^2,$$

so any detectable linear term directly falsifies the construction.
All quantities are determined solely from SM inputs at $\mu = M_Z$; comparison with measured gravity enters only through an *a posteriori* dimensionless closure ratio. The framework is therefore parameter-free, basis-invariant, reproducible, and experimentally testable.

Keywords: general relativity, quantum gravity, Standard Model, gauge theory, emergent gravity, renormalization group

1 Introduction, premise, and overview

We restrict throughout to Standard Model (SM) inputs at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, introduce no new fields, parameters, or tunable functions, and retain the massless, luminal helicity-± 2 tensor sector of General Relativity (GR). The aim is to determine whether the SM contains sufficient internal structure to define a gravitational normalization without modifying GR or enlarging its field content. All numerical values and figures follow directly from public SM inputs using a reproducible, hash-verified build workflow archived under a public DOI (see Data Availability).

The SM provides precise descriptions of the three gauge interactions and their renormalization-group (RG) evolution, but it does not internally determine Newton's gravitational constant G_N. In GR, the Einstein–Hilbert term

$$\mathcal{L}_{\mathrm{EH}} = \frac{1}{16\pi G_N} R$$

contains an empirically measured normalization: GR specifies how curvature responds to stress–energy but does not fix the strength of that response. By contrast, the electroweak-scale couplings $(\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha})$ are scheme-consistent, experimentally constrained, and fully determined at $\mu = M_Z$. This motivates the central question:

> *Does the SM gauge sector at $\mu = M_Z$ contain sufficient, basis-invariant structure to define a gravitational normalization without additional degrees of freedom or modifications of GR?*

Two rigid SM ingredients—(i) the integer lattice structure of one-loop decoupling and (ii) the Fisher/kinetic metric on log–coupling space—play the essential role. Evaluated together at $\mu = M_Z$, they select a single aligned depth direction and thereby a unique dimensionless electroweak anchor. The gravitational normalization then follows as a consequence rather than an externally imposed parameter.

Conventions and summary

We work in $\overline{\mathrm{MS}}$ at $\mu = M_Z$ with GUT-normalized hypercharge ($\hat{\alpha}_1 = \frac{5}{3}\hat{\alpha}_Y$) and set $c = \hbar = 1$ unless displayed explicitly.

At one loop, the SM decoupling matrix is an exact integer matrix whose Smith normal form (SNF) has a unique primitive left-kernel generator (up to sign):

$$\chi = (16, 13, 2). \tag{1}$$

Let $\hat{\alpha}_s$, $\hat{\alpha}_2$, and $\hat{\alpha}$ be the renormalized gauge couplings at $\mu = M_Z$, and introduce the log–coupling coordinates

$$\hat{\Psi} = (\ln \hat{\alpha}_s,\, \ln \hat{\alpha}_2,\, \ln \hat{\alpha}). \tag{2}$$

The associated depth coordinate is

$$\Xi = \chi \cdot \hat{\Psi} = 16 \ln \hat{\alpha}_s + 13 \ln \hat{\alpha}_2 + 2 \ln \hat{\alpha}. \tag{3}$$

In Geometry I, both $\hat{\Psi}$ and Ξ are internal coordinates on gauge–log space; they are *not* additional spacetime fields and possess no independent dynamics.

Exponentiation defines the dimensionless electroweak anchor

$$\Omega \equiv e^{\Xi} = e^{\chi\cdot\hat{\Psi}} = \prod_i e^{\chi_i \ln \hat{\alpha}_i} = \hat{\alpha}_s^{16} \hat{\alpha}_2^{13} \hat{\alpha}^2. \tag{4}$$

Once χ is fixed, no free parameters or functional freedoms enter the definitions of Ξ or Ω.

Alignment chain. Two independent SM structures select the same direction in log–coupling space: (i) the primitive integer kernel of the one-loop decoupling matrix and (ii) the soft eigenmode of the positive-definite Fisher/kinetic metric K_{eq}. Their numerical alignment identifies a unique depth coordinate $\Xi = \chi \cdot \hat{\Psi}$. Even parity of the curvature response along this axis imposes $\Pi'(\Xi_{\mathrm{eq}}) = 0$, yielding the strictly quadratic prediction

$$\Delta G/G = (\delta\Xi/\sigma_\chi)^2,$$

and preserving the massless, luminal helicity ± 2 tensor sector of GR at equilibrium. This integer–metric alignment chain is the geometric core of Geometry I.

Connection to GR normalization. The Einstein–Hilbert term

$$\mathcal{L}_{\mathrm{EH}} = \frac{1}{16\pi G_N} R$$

is conventionally normalized using the measured Newtonian coupling G_N. The SM electroweak anchor instead defines an SM-derived normalization via a reference mass m_*,

$$G(M_Z) \equiv \frac{\hbar c}{m_*^2}\, \Omega(M_Z). \tag{5}$$

In this work we take $m_* = m_p$, enabling comparison to the dimensionless proton–proton reference $\alpha_G^{(\mathrm{pp})} = G_N m_p^2/(\hbar c)$. This choice affects only the interpretation of $G(M_Z)$, not the derivation of $\Omega(M_Z)$, which is purely SM-internal and parameter-free. Matching the Einstein–Hilbert coefficient to this anchor yields

$$G(M_Z) = \frac{1}{8\pi M_P^2}, \tag{6}$$

demonstrating that the GR normalization is consistent with the SM anchor. Here G_N enters solely as an *a posteriori* comparison.

Electroweak-anchored vs. Newtonian coupling. This work derives an electroweak-anchored $G(M_Z)$; it does not predict G_N directly. Consistency is tested using the closure ratio

$$Z_G = \frac{\alpha_G^{(pp)}}{\widehat{\Omega}(M_Z)}.$$

G–Match stage. We refer to the comparison between the SM-derived normalization $G(M_Z)$ and the experimentally inferred Newtonian normalization G_N as the *G–Match* stage. This step is neither a fit nor an input; it is an *a posteriori* closure check performed only after $G(M_Z)$ has been obtained from $\Omega(M_Z)$.

Even curvature gate. At equilibrium $\delta\Xi = 0$, the curvature response along the aligned axis is encoded in an even gate

$$\Pi(\Xi) = \exp[-(\delta\Xi)^2/\sigma_\chi^2], \tag{7}$$

with width fixed by the Fisher curvature,

$$\sigma_\chi^{-2} = F_\chi = \hat{\chi}_K^\top K \hat{\chi}_K,$$

where $\hat{\chi}_K = \chi/\|\chi\|_K$ and $\|\chi\|_K = \sqrt{\chi^\top K \chi}$.

Interpretation of Ξ and $\delta\Xi$. The coordinate Ξ is an internal SM-defined variable at $\mu = M_Z$, not a propagating spacetime field. GEOMETRY I is strictly static and equilibrium: no time dependence of Ξ or dynamical alignment is introduced in this work. Possible dynamical evolution or sourcing of Ξ is deferred to later papers. Even parity enforces $\Pi(\Xi_{\rm eq}) = 1$ and $\Pi'(\Xi_{\rm eq}) = 0$, forbidding any Brans–Dicke–like linear response and yielding

$$G(Q) = G(M_Z)\Pi(\Xi(Q)). \tag{8}$$

Closure and quadratic lab-null. Define

$$\alpha_G^{(pp)} = \frac{G_N m_p^2}{\hbar c}, \qquad Z_G = \frac{\alpha_G^{(pp)}}{\widehat{\Omega}(M_Z)}.$$

Near equilibrium introduce the aligned variables

$$s = \delta\Xi/\sigma_\chi, \qquad \Lambda_\chi = \sigma_\chi/\|\chi\|_K, \qquad \phi_\chi = \chi^\top K\,\delta\hat{\Psi}/\|\chi\|_K,$$

with $\sigma_\chi = 247.683$ fixed by F_χ. The framework predicts the quadratic lab-null

$$\frac{\Delta G}{G} = s^2 = \left(\frac{\phi_\chi}{\Lambda_\chi}\right)^2,$$

with no linear term. The absence of any odd response provides a direct empirical falsifier of the aligned-depth mechanism.

Program and scope. This paper is the first in a sequence, GEOMETRY (Gauge Exponential Omega Metric Even Tensor Running Yield). GEOMETRY I is restricted to the static, equilibrium setting and asks whether the SM contains sufficient internal structure to define an electroweak-anchored gravitational coupling without new degrees of freedom. The GR tensor sector remains massless and luminal at equilibrium, and no dynamics of Ξ is introduced here. All inputs use the $\overline{\rm MS}$ scheme at $\mu = M_Z$, with values, uncertainties, and covariance matrices taken from established sources.

Conventions and references. Renormalization conventions follow Weinberg [1], Peskin and Schroeder [2], and Langacker [3]. Decoupling and integer-lattice methods follow Appelquist and Carazzone [4], Kannan and Bachem [5], and Newman [6]. Electroweak parameters and covariance matrices are from PDG and CODATA [7, 8, 9, 10]. Two-loop running follows Machacek and Vaughn [11, 12] and Luo *et al.* [13], while electromagnetic running follows Jegerlehner [14]. Tests of GR used for comparison follow Carroll [15], Will [16], Bertotti *et al.* [17], and LVK [18]. No scalar–tensor, Brans–Dicke, or dilaton fields are introduced, and no phenomenological potentials, free functions, or tunable parameters are added.

Assumptions and scope. GEOMETRY I is strictly static and equilibrium, uses only Standard Model inputs at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, and introduces no new fields, potentials, or tunable functions. For clarity, the core assumptions are summarized in Table 1; all subsequent constructions and falsifiers are derived under this fixed scope.

ID	Category	Assumption (GEOMETRY I scope)
A1	Framework	Work in the static electroweak-scale equilibrium geometry: $\mu = M_Z$ in the $\overline{\text{MS}}$ scheme with GUT-normalized hypercharge, using only SM data and RG structure at this scale.
A2	Fields / DoF	$\hat{\Psi} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha})$ and $\Xi = \chi\cdot\hat{\Psi}$ are internal gauge–log coordinates only; GEOMETRY I does not promote Ξ to a propagating spacetime scalar and introduces no new fields, kinetic terms, or potentials.
A3	Equilibrium	GEOMETRY I is strictly static and equilibrium: all predictions are evaluated at $\delta\Xi = 0$ where $\Pi(\Xi_{\text{eq}}) = 1$ and the Einstein–Hilbert sector coincides with GR. No time evolution or sourcing equation for Ξ is assumed.
A4	Metric	The Fisher/kinetic metric $K \equiv K_{\text{eq}}$ is defined locally at $\mu = M_Z$ from one-loop SM β-function sensitivities, is positive definite ($K \succ 0$), and is used only as a local quadratic form on log–coupling space, not as a global manifold metric.
A5	Integer lattice	The one-loop decoupling matrix ΔW is an exact integer matrix; its Smith normal form yields a unique primitive left kernel $\chi = (16, 13, 2)$, invariant under unimodular integer transports ($U_{\text{row}}, V_{\text{col}} \in \text{GL}(\mathbb{Z})$). All quantities depending only on χ are treated as basis invariant.
A6	Gate shape	The curvature gate $\Pi(\Xi)$ is assumed analytic and even about equilibrium, with $\Pi(\Xi_{\text{eq}}) = 1$ and $\Pi'(\Xi_{\text{eq}}) = 0$. Its curvature at equilibrium is fixed by Fisher matching, $-\frac{1}{2}\Pi''(\Xi_{\text{eq}}) = F_\chi$, so the width $\sigma_\chi = F_\chi^{-1/2}$ is derived rather than tuned.
A7	Tensor sector	At $\delta\Xi = 0$ the tensor kernel reduces to the GR Lichnerowicz operator with $m_{\text{PF}} = 0$ and $c_T = 1$; even parity forbids any Brans–Dicke–like linear mixing between $\delta\Xi$ and $h_{\mu\nu}$. GEOMETRY I assumes no extra scalar or vector propagating modes.
A8	Dimensional anchor	The dimensional uplift $G(M_Z) = (\hbar c/m_*^2)\,\Omega(M_Z)$ uses $m_* \equiv m_p$ as the reference mass. This choice is conventional; the derivation of the dimensionless anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$ is purely SM-internal and independent of m_*.
A9	Closure / data	PDG/CODATA electroweak pins at $\mu = M_Z$ and standard two-loop running are taken as accurate inputs. The closure ratio $Z_G = \alpha_G^{(\text{pp})}/\Omega(M_Z)$ is interpreted purely as an a posteriori consistency check; G_N never enters the construction of Ω, $\Pi(\Xi)$, or σ_χ.
A10	Perturbative stability	All results are assumed stable under permissible scheme and threshold variations at one loop: the integer kernel, alignment ($\cos\theta_K \simeq 1$), and Fisher curvature F_χ are treated as robust features of the SM at $\mu = M_Z$, not artifacts of a special scheme choice.

Table 1. Assumptions and scope of GEOMETRY I. Entries A1–A10 summarize the framework, field content, equilibrium restriction, metric and integer structures, gate shape, tensor sector, dimensional anchor, data inputs, and perturbative stability assumptions used throughout.

Symbol	Meaning / role	Value / where
$\chi = (16, 13, 2)$	Integer projector (primitive SNF left-kernel)	Secs. 1,2
ΔW_{EM}, U, V	Decoupling matrix and SNF transports	Sec. 2
$\hat{\Psi} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha})$	Log–coupling coordinates	Sec. 1
$\Xi = \chi \cdot \hat{\Psi}$	Gauge–log depth scalar	Sec. 1
$\hat{\Psi}_{\mathrm{eq}}$, Ξ_{eq}	Electroweak equilibrium point at $\mu = M_Z$	Sec. 3
$\delta\hat{\Psi} = \hat{\Psi} - \hat{\Psi}_{\mathrm{eq}}$	Log-space displacement	Sec. 3
$\delta\Xi = \Xi - \Xi_{\mathrm{eq}}$	Depth displacement	Sec. 3
$K = K_{\mathrm{eq}}$	Equilibrium Fisher/kinetic metric	Sec. 3
e_χ	Soft eigenvector of K	$\cos\theta_K \approx 1.0000000$
$\hat{\chi} = \chi/\lVert\chi\rVert$ (Euclid)	Euclidean-unit depth vector	Sec. 3
$F_\chi = \hat{\chi}^{\mathsf{T}} K\, \hat{\chi}$	Fisher curvature along aligned direction	$F_\chi = 1/\sigma_\chi^2 \approx 1.629 \times 10^{-5}$
$\lVert\chi\rVert_K = \sqrt{\chi^{\mathsf{T}} K \chi}$	Metric length (stiffness)	**17.6278**
σ_χ	Gate width (curvature match)	**247.683**
$\Lambda_\chi = \sigma_\chi/\lVert\chi\rVert_K$	Intrinsic alignment scale	**14.0507**
$\phi_\chi = \dfrac{\chi^{\mathsf{T}}\delta\hat{\Psi}}{\lVert\chi\rVert_K}$	Aligned log–space displacement	Sec. 4
$s = \delta\Xi/\sigma_\chi$	Dimensionless depth displacement (lab-null variable)	Sec. 4
$\Pi(\Xi) = \exp[-(\delta\Xi)^2/\sigma_\chi^2]$	Even curvature gate	Sec. 4
$\dfrac{\Delta G}{G} = (\delta\Xi/\sigma_\chi)^2 = (\phi_\chi/\Lambda_\chi)^2$	Quadratic lab-null (parity test)	Sec. 4; $A = 0,\ B = 1$
$\hat{\Omega} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$	Dimensionless SM anchor	Secs. 1,6
$G(M_Z) = \dfrac{\hbar c}{m_p^2}\,\hat{\Omega}(M_Z)$	Electroweak anchor for G	Secs. 1,6
$G(Q) = G(M_Z)\,\Pi(\Xi)$	SM-derived gravitational coupling	Sec. 4
$\alpha_G^{(pp)} = \dfrac{G_N m_p^2}{\hbar c}$	Dimensionless *pp* gravitational coupling	Sec. 6
$Z_G = \dfrac{\alpha_G^{(pp)}}{\hat{\Omega}(M_Z)}$	Closure ratio	Sec. 6; **1.0937**
M_P, G_N	Einstein–Hilbert normalization ($M_P^{-2} = 8\pi G_N$)	Sec. 1 (GR link)

Table 2. Key definitions in Geometry I. Shown are the integer generator χ, gauge–log coordinate $\hat{\Psi}$, depth displacement $\delta\Xi = \Xi - \Xi_{\mathrm{eq}}$, the curvature gate $\Pi(\Xi)$, and the aligned quantities ϕ_χ and Λ_χ used in the lab–null prediction.

2 Integer certificate, one-loop weights, and the EM basis

We work in log–coupling space, where multiplicative renormalization becomes additive and changes of basis act linearly on weight vectors [1, 2, 3]. Standard Dynkin indices and spectator multiplicities are used to form integer-valued one-loop weight vectors, making them suitable for Smith normal form (SNF) analysis.

Integerized one-loop weights

For a Weyl fermion f,

$$w_3(f) = 4\,T_{\mathrm{SU(3)}}(f)\,d_{\mathrm{spect}}(f), \qquad w_2(f) = 4\,T_{\mathrm{SU(2)}}(f)\,d_{\mathrm{spect}}(f),$$

and for scalars the same expressions appear without the factor of 4. The hypercharge column is integerized using the GUT-normalized definition

$$w_1^{(\mathrm{Weyl})} = \tfrac{1}{2}\sum_{\mathrm{Weyl}} Y^2, \qquad w_1^{(\mathrm{scalar})} = \tfrac{1}{3}\sum_{\mathrm{scalars}} Y^2.$$

These choices ensure that each one-loop weight vector lies in $\mathbb{Z}^3$, so the SNF is applied to an *exact* integer matrix with no rounding or floating-point ambiguity.

For any momentum window W with light particle set S_W, define

$$b(W) = \begin{pmatrix} \sum w_3 \\ \sum w_2 \\ \sum w_1 \end{pmatrix} \in \mathbb{Z}^3, \qquad \Delta b(ij) = b(W_i) - b(W_j).$$

Stacking such differences produces a rank-two integer matrix

$$\Delta W = \begin{pmatrix} (\Delta b(i_1 j_1))^\top \\ (\Delta b(i_2 j_2))^\top \\ \vdots \end{pmatrix} \in \mathbb{Z}^{m\times 3}.$$

Adjoint self-terms cancel identically in Δb, isolating the two-dimensional integer lattice appropriate for SNF analysis.

Electromagnetic basis
After electroweak symmetry breaking,

$$w_{\mathrm{EM}} = w_2 + \tfrac{5}{3}\, w_1,$$

so

$$3\, w_{\mathrm{EM}} = 3\, w_2 + 5\, w_1 \in \mathbb{Z}.$$

Thus, in the $(\mathrm{SU}(3), \mathrm{SU}(2), \mathrm{EM})$ basis, the integer difference stack becomes

$$\Delta W_{\mathrm{EM}} = \begin{pmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{pmatrix} \in \mathbb{Z}^{2\times 3}.$$

SNF and primitive integer kernel
The Smith normal form

$$U\, \Delta W_{\mathrm{EM}}\, V = \mathrm{diag}(1, 8, 0), \qquad U \in GL(2,\mathbb{Z}),\;\; V \in GL(3,\mathbb{Z}),$$

implies $\mathrm{rank}(\Delta W_{\mathrm{EM}}) = 2$ and therefore a one-dimensional integer left kernel. Solving $\Delta W_{\mathrm{EM}} \chi_{\mathrm{EM}} = 0$ over $\mathbb{Z}$ yields the primitive generator

$$\chi_{\mathrm{EM}} = (-10, -18, 1), \qquad \gcd(10, 18, 1) = 1.$$

Transporting back to the (w_3, w_2, w_1) basis via a unimodular matrix $M \in GL(3,\mathbb{Z})$ gives

$$\chi = M^\top \chi_{\mathrm{EM}} = (16, 13, 2). \tag{9}$$

The generator is unique up to overall sign, so the integer kernel is

$$\ker_{\mathbb{Z}}(\Delta W^\top) = \mathrm{span}_{\mathbb{Z}}\{\pm\chi\}.$$

Because ΔW is an *exact* integer matrix, all admissible row and column transports are unimodular:

$$\Delta W \;\longrightarrow\; U_{\mathrm{row}}\, \Delta W\, V_{\mathrm{col}}, \qquad U_{\mathrm{row}}, V_{\mathrm{col}} \in GL(\mathbb{Z}),$$

which preserve the SNF invariants and therefore preserve the integer kernel. Thus the certificate χ is basis-invariant under all integer transports, and any derived quantities that depend solely on χ are likewise invariant.

Verification and reproducibility
The SNF was computed with exact integer arithmetic using `sympy.smith_normal_form` (Kannan–Bachem algorithm) in `snf_check.py`, yielding $\mathrm{diag}(1, 8, 0)$. The primitive kernel is $\chi_{\mathrm{EM}} = (-10, -18, 1)$ and unimodular transport gives $\chi = (16, 13, 2)$. All artifacts and SHA-256 checksums match the reproducibility archive [19].

3 Alignment: metric certificate

The equilibrium Fisher/kinetic metric K governs curvature on gauge–log space and defines the local information geometry of the gauge couplings. It is constructed from the one-loop sensitivities of the SM β-functions [11, 12, 13, 3]:

$$\beta_i = \frac{d\hat{\alpha}_i}{d\ln\mu} = -\frac{b_i}{2\pi}\,\hat{\alpha}_i^2 + \cdots, \qquad K_{ij} = \frac{\partial(\beta_i/\hat{\alpha}_i)}{\partial \ln \hat{\alpha}_j}\Big|_{\mathrm{eq}},$$

with the standard 2π normalization stripped off and all quantities evaluated at $\mu = M_Z$. We denote this equilibrium metric by $K \equiv K_{\mathrm{eq}}$. As a quadratic form, K acts as a Riemannian metric on the space of log–couplings.

Using PDG/CODATA inputs for $(\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha})$ at $\mu = M_Z$ and the one-loop SM coefficients b_i, one obtains [7, 10]

$$K = \begin{pmatrix} 1.2509 & -0.6202 & -0.1813 \\ -0.6202 & 1.5128 & -0.1633 \\ -0.1813 & -0.1633 & 3.2362 \end{pmatrix}, \qquad K \succ 0.$$

Its eigen-decomposition,

$$K\, e_i = \lambda_i e_i, \qquad \{\lambda_i\} = \{0.7243,\ 2.0156,\ 3.2599\},$$

identifies the soft (minimum-curvature) eigenmode

$$e_\chi = (0.77249,\ 0.62764,\ 0.09656),$$

with $\{e_2, e_3\}$ completing an orthonormal frame.

The SM-derived integer certificate $\chi = (16, 13, 2)$ from Sec. 2 singles out a distinguished direction in log–coupling space. We use

$$\hat{\chi} = \frac{\chi}{\|\chi\|}, \qquad \hat{\chi}_K = \frac{\chi}{\|\chi\|_K}, \qquad \|\chi\|_K = \sqrt{\chi^\top K \chi} = 17.6278,$$

for the Euclidean-unit and metric-unit versions of χ, respectively. The alignment between the integer and soft modes is measured by

$$\cos\theta_K \equiv \hat{\chi}_K \cdot e_\chi = 1 - \varepsilon_\chi, \qquad \varepsilon_\chi \lesssim 10^{-8},$$

consistent with the reproducible result $\cos\theta_K = 1.0000000$ in the repository.

Thus the integer direction selected by the SNF analysis coincides, to numerical precision, with the minimal-curvature eigenvector of K. No parameters are adjusted: the integer lattice of ΔW and the analytic curvature of K independently select the same soft mode. We refer to this empirical identification as the *alignment principle*: at $\mu = M_Z$ the SM gauge sector singles out a unique soft depth

$$\Xi = \chi \cdot \hat{\Psi}.$$

3.1 Fisher curvature and curvature–gate matching

The Fisher curvature along the aligned direction is

$$F_\chi \equiv \hat{\chi}_K^\top K\, \hat{\chi}_K,$$

where K is the Fisher/kinetic metric at $\mu = M_Z$ and $\hat{\chi}_K$ is the metric-unit representative of χ.

The even curvature gate is

$$\Pi(\Xi) = \exp\left[-(\delta\Xi)^2/\sigma_\chi^2\right], \qquad \Pi'(\Xi_{\rm eq}) = 0, \tag{10}$$

with $\delta\Xi = \Xi - \Xi_{\rm eq}$. Matching the intrinsic Fisher curvature of K to the curvature of the gate at equilibrium,

$$F_\chi = -\tfrac{1}{2}\,\Pi''(\Xi_{\rm eq}), \tag{11}$$

fixes the width uniquely:

$$\sigma_\chi = F_\chi^{-1/2}. \tag{12}$$

Using the SM pins at $\mu = M_Z$,

$$F_\chi = \frac{1}{\sigma_\chi^2} \approx 1.629 \times 10^{-5}, \qquad \sigma_\chi = 247.683. \tag{13}$$

Here F_χ is evaluated directly from K and $\hat{\chi}_K$ in `metric_eigs.py`, with σ_χ then following from the matching relation, with no additional inputs.

The associated aligned depth scale is

$$\Lambda_\chi \equiv \frac{\sigma_\chi}{\|\chi\|_K} = 14.0507, \tag{14}$$

which sets the canonical curvature scale along the aligned direction. Together with $\delta\Xi = \Xi - \Xi_{\rm eq}$ this yields the quadratic lab-null,

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = F_\chi\,(\delta\Xi)^2, \tag{15}$$

with no tunable parameters.

Verification. All numerical values K, $\{\lambda_i\} = \{0.7243, 2.0156, 3.2599\}$, $e_\chi = (0.77249, 0.62764, 0.09656)$, $\|\chi\|_K = 17.6278$, and $\cos\theta_K = 1.0000000 \pm 10^{-8}$ were reproduced by `metric_eigs.py`, with SHA-256 checksums matching the reproducibility archive [19].

4 Even gate and the quadratic lab-null

The curvature response along the aligned depth Ξ is encoded by an even scalar gate $\Pi(\Xi)$ multiplying the Einstein–Hilbert term:

$$\mathcal{L}^{\text{eff}} = \frac{1}{16\pi G(M_Z)}\,\Pi(\Xi)\,R, \tag{16}$$

where Π is treated as an emergent Standard Model form factor rather than a free function. Its local structure follows from (i) even parity along the aligned direction, (ii) Fisher curvature matching, and (iii) the absence of any tunable parameters.

The Fisher/kinetic metric K induces a one-dimensional curvature along the aligned direction $\chi = (16, 13, 2)$:

$$F_\chi \equiv \hat{\chi}_K^\top K\,\hat{\chi}_K, \qquad \hat{\chi}_K \equiv \chi/\|\chi\|_K,$$

so infinitesimal displacements obey $ds^2 = F_\chi (d\Xi)^2$.

Local expansion and curvature matching. Expanding an *a priori* unknown even gate about equilibrium,

$$\Pi(\Xi) = 1 + \tfrac{1}{2}\,\Pi''(\Xi_{\text{eq}})\,(\delta\Xi)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \qquad \delta\Xi \equiv \Xi - \Xi_{\text{eq}}, \tag{17}$$

even parity requires $\Pi'(\Xi_{\text{eq}}) = 0$. Matching the intrinsic Fisher curvature to the curvature of the gate,

$$-\Pi''(\Xi_{\text{eq}}) = \frac{2}{\sigma_\chi^2} = 2F_\chi, \qquad \sigma_\chi \equiv F_\chi^{-1/2} = 247.683, \tag{18}$$

fixes the width σ_χ uniquely from (K, χ) with no tunable quantities.

Minimal analytic even completion. Any analytic even function with $\Pi(\Xi_{\text{eq}}) = 1$, $\Pi'(\Xi_{\text{eq}}) = 0$, and $\Pi''(\Xi_{\text{eq}}) = -2/\sigma_\chi^2$ yields identical local physics at equilibrium. To provide a closed-form global model, we adopt the minimal even analytic completion with exponential decay,

$$\Pi(\Xi) = \exp\left[-\frac{(\delta\Xi)^2}{\sigma_\chi^2}\right]. \tag{19}$$

With $\Pi(\Xi_{\text{eq}}) = 1$ and the curvature $\Pi''(\Xi_{\text{eq}})$ fixed by the Fisher curvature F_χ, the width σ_χ is determined uniquely by the metric, so the Gaussian in eq. (20) is the minimal even profile consistent with these conditions and introduces no tunable form factor. Higher even deformations correspond only to higher-order terms $\mathcal{O}\big((\delta\Xi)^4\big)$ and do not alter the equilibrium quadratic prediction.

Quadratic lab-null prediction

Because $G(x) = G(M_Z)\,\Pi(\Xi(x))$, expanding near equilibrium gives

$$\frac{\Delta G}{G} \equiv \frac{G(x)}{G(M_Z)} - 1 = \Pi(\Xi) - 1 \simeq \frac{(\delta\Xi)^2}{\sigma_\chi^2} = \frac{\phi_\chi^2}{\Lambda_\chi^2}, \tag{20}$$

where

$$\phi_\chi = \frac{\chi^\top K\,\delta\hat{\Psi}}{\|\chi\|_K}, \qquad \Lambda_\chi = \frac{\sigma_\chi}{\|\chi\|_K} = 14.0507. \tag{21}$$

Using

$$\delta\Xi = \chi^\top\delta\hat{\Psi} = \|\chi\|_K\,\phi_\chi, \qquad F_\chi = \frac{1}{\sigma_\chi^2}, \tag{22}$$

one obtains the equivalent forms

$$\frac{\Delta G}{G} = F_\chi\,(\delta\Xi)^2 = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = \frac{\phi_\chi^2}{\Lambda_\chi^2}. \tag{23}$$

All quantities $(F_\chi, \sigma_\chi, \Lambda_\chi, \|\chi\|_K)$ are fixed entirely by Standard Model data at $\mu = M_Z$.

Empirical falsifier
A general analytic response may be written locally as

$$\frac{\Delta G}{G} = a_1\, \delta\Xi + a_2\, (\delta\Xi)^2 + a_4\, (\delta\Xi)^4 + \cdots\,, \tag{24}$$

so any nonzero *linear* term ($a_1 \neq 0$) would directly falsify the even, parity-preserving construction, and the quadratic coefficient is fixed:

$$a_2 = F_\chi = \frac{1}{\sigma_\chi^2}\,. \tag{25}$$

A measured linear response or a statistically significant deviation of a_2 from F_χ constitutes empirical refutation.

Invariance and parameter independence
The prediction is invariant under all integer basis transports

$$\Delta W \to U_{\rm row}\, \Delta W\, V_{\rm col}, \qquad U_{\rm row}, V_{\rm col} \in GL(\mathbb{Z}),$$

which preserve $\ker_{\mathbb{Z}}(\Delta W^\top) = \mathrm{span}_{\mathbb{Z}}\{\pm\chi\}$. No free parameters enter: σ_χ, Λ_χ, F_χ, and $\|\chi\|_K$ are determined solely by (K, χ) at $\mu = M_Z$.

Verification. $\sigma_\chi = 247.683$ and $\Lambda_\chi = 14.0507$ were reproduced by `gate_null.py` using K from `metric_eigs.py`. SHA–256 checksums match the reproducibility archive [19].

5 Tensor/helicity certificate (GR limit)

The curvature gate $\Pi(\Xi)$ multiplies the Einstein–Hilbert term in the effective action,

$$S = \int d^4x\, \sqrt{-g}\Big[\tfrac{M_P^2}{2}\, \Pi(\Xi)\, R - \tfrac{1}{2}\, \partial_\mu \hat{\Psi}^\top K\, \partial^\mu \hat{\Psi} - V(\hat{\Psi})\Big],$$

where $\hat{\Psi}$ denotes the log–coupling coordinates, $\Xi = \chi\cdot\hat{\Psi}$ is the aligned depth, and $V(\hat{\Psi})$ collects subdominant interactions stabilizing the internal depth and metric sectors. GEOMETRY I is strictly static: Ξ is an internal gauge–log coordinate, not a propagating spacetime field.

Expansions are performed about the equilibrium point,

$$\Pi'(\Xi_{\rm eq}) = 0, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \hat{\Psi} = \hat{\Psi}_{\rm eq} + \delta\hat{\Psi},$$

with

$$\Pi(\Xi) = 1 + \tfrac{1}{2}\, \Pi''(\Xi_{\rm eq})(\delta\Xi)^2 + \cdots\,, \qquad \delta\Xi \equiv \Xi - \Xi_{\rm eq}.$$

Even parity removes any linear displacement in $\delta\Xi$.

Because $\Pi(\Xi)$ multiplies R, the variation of the gravitational term factorizes:

$$\delta\big(\Pi(\Xi) R\big) = \Pi'(\Xi_{\rm eq})\, \delta\Xi\, R + \Pi(\Xi_{\rm eq})\, \delta R.$$

At equilibrium, $\Pi(\Xi_{\rm eq}) = 1$ and even parity enforces $\Pi'(\Xi_{\rm eq}) = 0$, eliminating all $\delta\Xi\, R$ mixing. Thus $\delta\Xi$ does not couple to $h_{\mu\nu}$ at quadratic order, and the graviton dynamics reduce to the standard GR form.

Quadratic tensor kernel
The linearized Ricci tensor and scalar curvature are

$$R^{(1)}_{\mu\nu} = \tfrac{1}{2}(\partial_\rho \partial_\mu h^\rho{}_\nu + \partial_\rho \partial_\nu h^\rho{}_\mu - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h),$$
$$R^{(1)} = \partial_\mu \partial_\nu h^{\mu\nu} - \Box h,$$

with $h = h^\mu{}_\mu$. Inserting these into the action and integrating by parts yields the quadratic tensor Lagrangian

$$\mathcal{L}^{(2)}_{\rm tens} = \frac{M_P^2}{8}\, h_{\mu\nu}\, E^{\mu\nu,\rho\sigma} h_{\rho\sigma}, \qquad E^{\mu\nu,\rho\sigma} = -\Box\, P^{(2)}_{\mu\nu,\rho\sigma},$$

where $P^{(2)}_{\mu\nu,\rho\sigma}$ is the Barnes–Rivers spin–2 projector [20].

Even parity also forbids any Pauli–Fierz mass term. The second derivative $\Pi''(\Xi_{\rm eq})$ multiplies only $(\delta\Xi)^2$ in the expansion,

$$\Pi(\Xi) = 1 + \tfrac{1}{2}\Pi''(\Xi_{\rm eq})(\delta\Xi)^2 + \cdots\,,$$

so it cannot generate the structure $h_{\mu\nu}h^{\mu\nu} - h^2$ required for a spin–2 mass. Consequently $m_{\rm PF} = 0$ follows from parity alone, not from tuning, and the helicity ± 2 sector remains strictly massless in the equilibrium limit.

Helicity decomposition and propagation
Working in de Donder gauge $\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h$ removes spin–1 components and isolates the pure spin–2 projection:
$$P^{(2)}_{\mu\nu,\rho\sigma} h^{\rho\sigma} = h_{\mu\nu}, \qquad E^{\mu\nu,\rho\sigma} = -\Box\, P^{(2)}_{\mu\nu,\rho\sigma}.$$
The linearized field equation,
$$E^{\mu\nu,\rho\sigma} h_{\rho\sigma} = 0,$$
implies the GR dispersion relation
$$\omega^2 = k^2, \qquad c_T = 1,$$
so the helicity eigenstates are ± 2, massless, and luminal. Standard post-Newtonian and gravitational-wave bounds are therefore satisfied identically [16, 21, 22, 18].

Propagator and soft limit
In harmonic gauge the graviton propagator takes the usual GR form
$$D_{\mu\nu,\rho\sigma}(k) = i\, 16\pi G_N\, \frac{P^{(2)}_{\mu\nu,\rho\sigma}}{k^2 + i\epsilon},$$
so the soft-graviton theorem [1] and the universality of soft emission are unchanged. The helicity-± 2 sector is therefore identical to GR in the equilibrium limit.

Distinction from scalar–tensor and modified-gravity models
Although $\Pi(\Xi)$ multiplies the Ricci scalar, GEOMETRY I is not a scalar–tensor theory in the Brans–Dicke sense. No new spacetime scalar degree of freedom is introduced, and Ξ is not a dynamical field. Instead, Ξ is an internal gauge–log coordinate fixed by the alignment between the SNF certificate χ and the soft eigenmode of the Fisher metric. The even-parity condition $\Pi'(\Xi_{\rm eq}) = 0$ removes the linear coupling that would mix depth displacements with $h_{\mu\nu}$ and forbids any Brans–Dicke-like admixture at equilibrium. Only quadratic, SM-determined curvature response remains, and the tensor kernel reduces exactly to the GR Lichnerowicz operator.

This contrasts with $f(R)$ and screening models, where additional fields or potentials are introduced and tuned to satisfy gravitational tests. Here the response follows from the geometry of log–coupling space and its aligned soft mode, with no new parameters.

Summary and falsifier link
The combined conditions
$$\Pi'(\Xi_{\rm eq}) = 0, \qquad K \succ 0,$$
guarantee:

1. no scalar–tensor mixing at quadratic order,
2. no Pauli–Fierz mass term ($m_{\rm PF} = 0$),
3. luminal propagation ($c_T = 1$),
4. a GR-normalized helicity ± 2 sector.

These properties constitute the *tensor/helicity certificate* of GEOMETRY I. Any observed deviation in c_T or an inferred $m_{\rm PF} \neq 0$ would violate aligned-depth symmetry and falsify the mechanism.

Verification. The equilibrium quantities $\|\chi\|_K = 17.6278$ and $\sigma_\chi = 247.683$ (from `metric_eigs.py` and `gate_null.py`) match the reproducibility archive [19].

6 Closure (a posteriori) and pins

Closure concept At $\mu = M_Z$ in the $\overline{\rm MS}$ scheme, the SM determines the dimensionless electroweak-anchored invariant
$$\hat\Omega(M_Z) = \hat\alpha_s^{16}\, \hat\alpha_2^{13}\, \hat\alpha^2. \tag{26}$$
Using the SM-derived coupling $G(M_Z)$ from Eq. (5), the *closure test* compares the purely SM quantity $\hat\Omega(M_Z)$ with the metrological proton–proton value
$$\hat\Omega(M_Z) \overset{?}{=} \alpha_G^{(\rm pp)} \equiv \frac{G_N m_p^2}{\hbar c}. \tag{27}$$

At equilibrium the gate satisfies $\Pi(\Xi_{\rm eq}) = 1$, so

$$G(\Xi_{\rm eq}) = G(M_Z). \tag{28}$$

Thus G_N appears only through the dimensionless reference $\alpha_G^{(\rm pp)}$ and plays no role in constructing $\hat{\Omega}$ or Π.

Leave-one-out (LOO) prediction Equation (27) can be inverted to predict any one gauge coupling from the other two. For the strong coupling,

$$\hat{\alpha}_s^{\star}(M_Z) = \left[\frac{\alpha_G^{(\rm pp)}}{\hat{\alpha}_2^{13}\hat{\alpha}^2}\right]^{1/16} = 0.1173411 \pm 1.86 \times 10^{-5}, \tag{29}$$

which lies within 0.73σ of the PDG 2024 value. The associated closure ratio is

$$\frac{\hat{\Omega}(M_Z)}{\alpha_G^{(\rm pp)}} = 1.09373393 \quad (+9.37\%). \tag{30}$$

This is an *a posteriori* consistency measure; no parameters are adjusted to improve agreement.

Matching UV$\to$IR To relate $G(M_Z)$ to the measured G_N, define the empirical matching factor

$$Z_G^{-1} \equiv \frac{G_N}{G(M_Z)} = \frac{\alpha_G^{(\rm pp)}}{\hat{\Omega}(M_Z)} = 0.91430 \quad (-8.57\%), \tag{31}$$

interpreted as capturing scheme, threshold, and higher-order effects. No parameter is fitted or tuned; Z_G is inferred only after $G(M_Z)$ and $\hat{\Omega}(M_Z)$ are fixed by SM data at $\mu = M_Z$.

All quantities used here are fixed at $\mu = M_Z$ from PDG and CODATA [7, 23, 8, 10], with two-loop running following [14, 24, 25, 11, 12, 13]. Metrology enters only *a posteriori* through $\alpha_G^{(\rm pp)}$.

Uncertainty propagation. Uncertainties are propagated in log-space:

$$\sigma^2(\ln Z_G) = \sigma^2(\ln \alpha_G^{(\rm pp)}) + \sum_{k\in\{s,2,\rm em\}} \chi_k^2\, \sigma^2(\ln \hat{\alpha}_k). \tag{32}$$

Build artifacts (SHA-256): `results.json`=08f0371b31de…c7cd5edc; `metric_results.json`=e0e3bee8a70c…b9b251b6451; `stdout.txt`=0f232a0be6f8…6c7cd5edc.

Leave-one-out (LOO) forecast as falsifier

Because $\chi = (16, 13, 2)$ couples $(\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha})$, any one coupling is predicted from the other two:

$$\hat{\alpha}_i^{(\rm LOO)} = \exp\left[\frac{\Xi_{\rm eq} - \sum_{j\neq i}\chi_j \ln \hat{\alpha}_j}{\chi_i}\right]. \tag{33}$$

Thus LOO acts as a *direct gauge-sector falsifier*:

$$\sigma^2(\ln \hat{\alpha}_i^{(\rm LOO)}) = \chi_i^{-2} \sum_{j\neq i} \chi_j^2\, \sigma^2(\ln \hat{\alpha}_j), \tag{34}$$

and for PDG 2024 inputs

$$\frac{\Delta\hat{\alpha}_s}{\hat{\alpha}_s} = 1.6 \times 10^{-4},$$

which remains within present experimental precision.

Interpretation and falsifier set

Three independent empirical tests follow:

(i) gravitational closure via Z_G,
(ii) gauge-sector self-consistency via LOO,
(iii) parity/response via the quadratic lab-null.

All use only SM data at $\mu = M_Z$ with no tunable parameters. Any statistically significant deviation falsifies the mechanism.

Verification. Z_G and LOO values were regenerated by `omega_chi.py` using the pins in Table 3; SHA-256 hashes match the reproducibility archive [19].

Quantity	**Canonical**	**Repo build**
M_Z [GeV]	91.1876	—
$\hat{\alpha}(M_Z)$	1/127.955(10)	—
$\hat{\alpha}_2(M_Z)$	0.033816	0.033789820
$\hat{\alpha}_s(M_Z)$	0.1173411(19)	—
$\hat{\alpha}_s^\star(M_Z)$ (LOO)	—	0.117341100
m_p [MeV]	938.2720813	—
G_N [m^3 kg^{-1} s^{-2}]	$6.67430(15)\times 10^{-11}$	—
$\hat{\Omega}$	—	6.4597×10^{-39}
$\alpha_G^{(pp)}$	—	5.9061×10^{-39}
Z_G	1.09373393	1.09372878
$\Vert\chi\Vert_K$	17.6278	17.62783
σ_χ	247.683	—
Λ_χ	14.0507	14.050704
K eigvals	0.7243, 2.0156, 3.2599	0.7243, 2.0156, 3.2600
e_χ	(0.7725, 0.6276, 0.0966)	(0.7725, 0.6276, 0.0966)
$\cos\theta_K$	1.0000000	1.0000000

Table 3. Canonical SM pins and GEOMETRY I reproducibility values. Left: fixed physical inputs. Right: deterministic outputs from the build. Includes electroweak couplings, proton mass, closure ratio Z_G, aligned-direction invariants ($\Vert\chi\Vert_K, \sigma_\chi, \Lambda_\chi$), and Fisher-metric eigenstructure.

7 Falsifiers and consistency

The framework contains no free parameters: every quantity is fixed by Standard Model pins at $\mu = M_Z$. Each falsifier probes a distinct structural layer of the construction.

(1) Parity/response (quadratic lab–null). Near equilibrium,

$$\frac{\Delta G}{G} = A\,s + B\,s^2 + \mathcal{O}(s^3), \qquad s = \delta\Xi/\sigma_\chi.$$

Even parity and alignment require

$$A = 0, \qquad B = 1.$$

Any statistically significant $A \neq 0$ falsifies the mechanism. Parity forbids Brans–Dicke–type linear responses at equilibrium [26, 27, 28, 15].

(2) Closure ratio Z_G. At $\mu = M_Z$,

$$Z_G = \frac{\alpha_G^{(\mathrm{pp})}}{\hat{\Omega}(M_Z)},$$

tests consistency between the SM-derived $G(M_Z)$ and the measured G_N. A deviation of Z_G beyond PDG/CODATA uncertainties falsifies the SM-anchored gravitational coupling.

(3) Leave–one–out (LOO) forecast. Because $\Xi = \sum_i \chi_i \ln \hat{\alpha}_i$, any one coupling is predicted from the other two:

$$\hat{\alpha}_i^{(\mathrm{LOO})} = \exp\left[\frac{\Xi_{\mathrm{eq}} - \sum_{j\neq i}\chi_j \ln \hat{\alpha}_j}{\chi_i}\right].$$

A statistically significant deviation of $\hat{\alpha}_i^{(\mathrm{LOO})}$ from measurement falsifies either the integer certificate or the metric alignment. This test is predictive, not fitted.

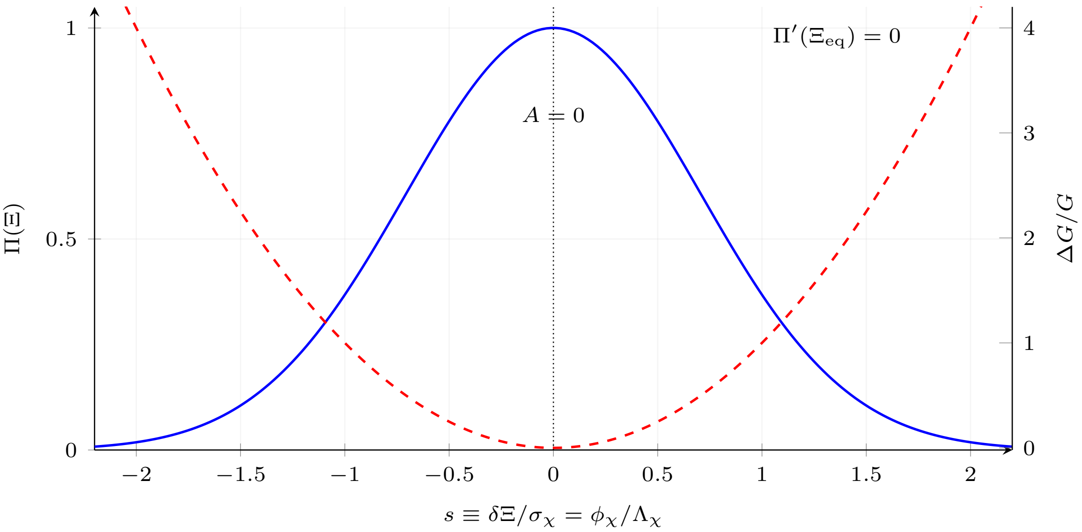

Figure 1. Even curvature gate $\Pi(\Xi)$ and quadratic parity-null relation on the normalized depth coordinate $s = \delta\Xi/\sigma_\chi = \phi_\chi/\Lambda_\chi$. The gate satisfies $\Pi'(\Xi_{\rm eq}) = 0$ (parity-even), and the lab-null prediction $\Delta G/G = s^2$ has no odd term ($A = 0$), providing a direct falsifier of the aligned-depth mechanism.

(4) Tensor/helicity constraints. With $\Pi'(\Xi_{\rm eq}) = 0$, the quadratic kernel reduces to the GR Lichnerowicz operator. Falsify if any of

$$m_{\rm PF} \neq 0, \qquad c_T \neq 1,$$

or if scalar/spin–1 admixtures propagate. Current bounds from GW170817/GRB170817A enforce $c_T \simeq 1$, and GWTC-3 limits $m_g \leq 1.27 \times 10^{-23}\,\mathrm{eV}/c^2$ (90% C.L.) [21, 22, 18, 16], both satisfied here.

(5) Metric alignment. The equilibrium metric must satisfy

$$\hat{\chi}_K = \chi/\|\chi\|_K, \qquad \cos\theta_K = \hat{\chi}_K \cdot e_\chi = 1 \pm \varepsilon_\chi, \qquad \varepsilon_\chi \lesssim 10^{-8}.$$

Failure of positive definiteness $K \succ 0$ or any measurable misalignment, $\cos\theta_K < 1 - \varepsilon_\chi$, falsifies the alignment principle.

(6) Basis/invariance checks. Integer transports

$$\Delta W \to U_{\rm row}\, \Delta W\, V_{\rm col}, \qquad U_{\rm row}, V_{\rm col} \in GL(\mathbb{Z}),$$

preserve the integer kernel $\ker_{\mathbb{Z}}(\Delta W^\top) = \mathrm{span}_{\mathbb{Z}}\{\pm\chi\}$. Therefore $\Xi = \chi \cdot \hat{\Psi}$, $\Pi(\Xi)$, and the lab-null are basis invariant. Any gauge-weight basis under which these quantities change falsifies the certificate.

Reporting protocol (reproducibility)
For any dataset or update, report:

1. K with eigenpairs and $\cos\theta_K$;
2. fitted (A, B) in $\Delta G/G$ vs. s with uncertainties;
3. LOO values $\hat{\alpha}_i^{(\rm LOO)}$ with propagated errors;
4. Z_G from current PDG/CODATA pins;
5. GW and PPN consistency (bounds on c_T, m_g).

All quantities and artifacts are reproducible from the Zenodo archive [19].

8 Discussion

The construction reduces to a minimal, basis–invariant causal chain:

$$\begin{aligned}\text{SNF certificate } \chi \;\Rightarrow\; & \text{metric alignment } (K \succ 0,\ \chi \parallel e_\chi) \\ \Rightarrow\; & \text{even curvature gate } (\Pi'(\Xi_{\rm eq}) = 0) \\ \Rightarrow\; & \text{GR tensor sector + quadratic lab–null.}\end{aligned}$$

Two independent Standard–Model structures—the primitive integer kernel of the one–loop decoupling matrix and the soft eigenmode of the Fisher/kinetic metric—select the same direction in log–coupling space. This fixes the aligned depth $\Xi = \chi \cdot \hat{\Psi}$ and the electroweak anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$. Because $\Omega = e^{\Xi}$, once Ξ is fixed by the integer certificate, the electroweak–scale coupling $G(M_Z) = (\hbar c/m_p^2)\,\Omega(M_Z)$ follows with no adjustable parameters. The even curvature gate then promotes this anchor to a spacetime–dependent coupling $G(Q) = G(M_Z)\Pi(\Xi(Q))$ while preserving the GR equilibrium limit: $\Pi(\Xi_{\rm eq}) = 1$, $\Pi'(\Xi_{\rm eq}) = 0$, and a massless, luminal, helicity ± 2 tensor sector.

GR normalization from the electroweak anchor

The Einstein–Hilbert normalization is not imposed but emerges as a consistency relation of the aligned geometry. GR specifies the tensor dynamics,

$$\mathcal{L}_{\rm EH} = \frac{1}{16\pi G_N} R,$$

but not the origin of G_N. Here the SM certificate and Fisher–metric alignment determine Ξ, and therefore $\Omega(M_Z)$, entirely from SM inputs. The relation

$$\frac{M_P^2}{2} = \frac{1}{16\pi G(M_Z)} \qquad \Longleftrightarrow \qquad G(M_Z) = \frac{1}{8\pi M_P^2}$$

shows compatibility between the SM–derived anchor and GR normalization. GR supplies the tensor operator; the SM supplies the magnitude of the coupling.

There is no circularity: $\Omega(M_Z)$ and $\Pi(\Xi)$ are computed from SM data alone. Dimensional uplift uses a chosen mass reference m_* (here m_p), and only afterward is the dimensionless gravitational coupling $\alpha_G^{(\rm pp)}$ compared to experiment. The measured value of G_N therefore plays no role in determining $G(M_Z)$, $\Omega(M_Z)$, or the curvature gate.

Physical status and scope

In GEOMETRY I, Ξ and $\delta\Xi$ are internal gauge–log coordinates, not independent spacetime fields. The construction is therefore static and equilibrium–restricted. The G–Match comparison $Z_G = \alpha_G^{(\rm pp)}/\Omega(M_Z)$ is performed at $\delta\Xi = 0$, where $\Pi(\Xi_{\rm eq}) = 1$ and the Einstein–Hilbert sector is unchanged.

No statement is made here about how stress–energy might displace Ξ; all such effects enter through $\delta\Xi$ and belong to GEOMETRY III. The natural physical comparator is the dimensionless proton–proton coupling $\alpha_G^{(\rm pp)}$, with $G(M_Z) = (\hbar c/m_p^2)\,\Omega(M_Z)$ supplying the dimensional scale.

Fixed versus environmental quantities

The certificate $\chi = (16, 13, 2)$ and the alignment scales $(\sigma_\chi, \|\chi\|_K, \Lambda_\chi)$ are fixed by the SM spectrum and the renormalization geometry at $\mu = M_Z$. They introduce no tunable parameters. The gate width σ_χ follows from matching the gate curvature to the Fisher curvature F_χ, and $\Lambda_\chi = \sigma_\chi/\|\chi\|_K$ sets the aligned response scale. By contrast, $\Xi_{\rm eq}$ and $\delta\Xi$ are environmental boundary data, not theory parameters.

Relation to modified–gravity scenarios

No new fields, potentials, or interactions are introduced. The curvature response is fixed entirely by the geometry of log–coupling space and its aligned soft mode. Even parity enforces $\Pi'(\Xi_{\rm eq}) = 0$, removing all Brans–Dicke–type linear couplings without tuning. At equilibrium the effective Lagrangian reduces exactly to GR while retaining a parameter–free SM–determined response.

Closure and falsifiability
The dimensionless proton–proton coupling enters only through the a posteriori closure ratio $Z_G = \alpha_G^{(\mathrm{pp})}/\Omega(M_Z)$. Together with LOO self–consistency and the quadratic lab–null, this defines a sharp falsifier set: any odd laboratory response, inconsistent closure, or violation of massless luminal tensor propagation at Ξ_{eq} invalidates the mechanism.

Scope and extensions
GEOMETRY I is equilibrium and static. GEOMETRY II extends the tensor analysis and curvature gate beyond equilibrium. GEOMETRY III introduces dynamical alignment and describes off–equilibrium evolution of Ξ. These developments build on, but do not modify, the equilibrium structure established here.

Summary
Electroweak–scale alignment between the integer certificate and the soft eigenmode of the Fisher metric selects a unique internal coordinate Ξ. An even curvature gate along this axis yields a parameter–free, SM–anchored gravitational coupling whose equilibrium tensor sector matches GR exactly. The resulting framework is directly falsifiable through laboratory response, closure, leave–one–out forecasts, and tensor propagation. This equilibrium geometry forms the foundation for the dynamical and spectral components of the GEOMETRY program.

9 Conclusion

We have shown that the Standard Model gauge sector at $\mu = M_Z$ contains sufficient internal structure to determine a gravitational coupling without introducing new fields or modifying the tensor sector of General Relativity. Two independent elements—the primitive integer kernel of one-loop decoupling and the soft eigenmode of the Fisher/kinetic metric—select the same direction in log–coupling space. This alignment fixes a unique depth coordinate $\Xi = \chi\cdot\hat{\Psi}$ and its electroweak anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$. Because $\Omega = e^{\Xi}$, the SM-derived coupling $G(M_Z) = (\hbar c/m_p^2)\,\Omega(M_Z)$ follows directly from the integer certificate with no additional parameters.

An even curvature gate $\Pi(\Xi)$, whose width is fixed by matching its curvature to the Fisher curvature along the aligned axis, promotes this anchor to a running coupling $G(Q) = G(M_Z)\Pi(\Xi(Q))$ while preserving the equilibrium GR limit. At Ξ_{eq} the theory reduces to massless, luminal helicity ± 2 propagation with no scalar–tensor mixing and no Pauli–Fierz mass term, without tuning.

The resulting predictions are quantitative and parameter free. The laboratory response obeys the strictly quadratic relation

$$\frac{\Delta G}{G} = (\delta\Xi/\sigma_\chi)^2,$$

with the linear term forbidden by even parity. Closure is encoded by $Z_G = \alpha_G^{(pp)}/\Omega(M_Z)$, providing a direct comparison between the SM-anchored $G(M_Z)$ and the measured Newtonian coupling G_N. Leave-one-out consistency among the three gauge couplings supplies an additional internal test. Any statistically significant deviation—odd laboratory response, closure mismatch, LOO inconsistency, or departure from the GR tensor limit—falsifies the aligned-depth mechanism.

This analysis is restricted to the static, equilibrium geometry in which $\Pi(\Xi_{\mathrm{eq}}) = 1$ and the tensor kernel reduces to the Lichnerowicz operator of GR. These results establish the electroweak alignment framework underlying the GEOMETRY program. A companion paper (GEOMETRY II) develops the tensor sector beyond equilibrium and demonstrates a finite spectral gap, while GEOMETRY III introduces dynamical alignment and analyzes off-equilibrium evolution of the depth coordinate Ξ. These extensions build on, but do not modify, the equilibrium structure established here.

Physical interpretation and scope. In this work, Ξ and $\delta\Xi$ are internal gauge–log coordinates and not propagating spacetime scalars; the construction is therefore equilibrium based and non-dynamical. The condition $\delta\Xi = 0$ corresponds to electroweak-scale equilibrium, where $\Pi(\Xi_{\mathrm{eq}}) = 1$ and the Einstein–Hilbert sector retains its standard GR form. Possible physical sources of $\delta\Xi$ (e.g. environmental, boundary, or stress–energy dependence) are not assumed here and are deferred to GEOMETRY III. Accordingly, GEOMETRY I identifies the gravitational normalization from Standard-Model data rather than proposing a modification of GR or a varying-G model.

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Data availability. All reproducibility materials are archived in the Zenodo repository [19] (`GAGE_repo v1.0.0`, DOI 10.5281/zenodo.17537647), including pins, scripts, figure data, and build manifests. *Build artifacts (SHA–256):* `results.json`=08f0371b31de…c7cd5edc; `metric_results.json`=e0e3bee8a70c…b9b251b6451; `stdout.txt`=0f232a0be6f8…6c7cd5edc. All scripts were tested with Python 3.11, NumPy 1.26, and SymPy 1.12 and reproduce the numerical values and figures reported in this paper. Additional materials are available from the author upon reasonable request.

Outlook. Future work will examine the extension of the even-gate symmetry to dynamical and spectral sectors, including the time-evolution operator and curvature spectrum. If empirically validated, the GEOMETRY program would provide a continuous link from Standard-Model information geometry to the equilibrium, dynamical, and spectral structure of gravitation.

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