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PAPER

GEOMETRY I: SM-derived gravitational coupling $G(Q)$ anchored at the electroweak scale

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Abstract
At $\mu = M_Z$ in the $\overline{\text{MS}}$ scheme, the Standard Model one-loop decoupling matrix has a unique primitive integer kernel $\chi = (16, 13, 2)$ (Smith normal form), which defines the aligned depth $\Xi = \chi \cdot \hat{\Psi}$ in log–coupling space. This integer direction aligns numerically with the soft eigenmode of the positive-definite Fisher/kinetic metric K_{eq}. Together they determine the electroweak anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$ and the SM-derived gravitational coupling

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z).$$

An even curvature gate $\Pi(\Xi)$ with $\Pi'(\Xi_{\text{eq}}) = 0$ promotes this anchor to a running coupling $G(x) = G(M_Z)\Pi(\Xi(x))$ while preserving the equilibrium tensor sector: the helicity ± 2 modes remain massless, luminal, and identical to General Relativity. Near equilibrium the curvature response is fixed and strictly quadratic,

$$\Delta G/G = (\Delta\Xi/\sigma_\chi)^2,$$

with the absence of a linear term constituting a direct empirical falsifier. No new fields or tunable parameters are introduced; PDG/CODATA inputs enter only *a posteriori* through the closure ratio that tests the SM-derived $G(M_Z)$ against the measured Newtonian coupling.

Keywords: general relativity, quantum gravity, Standard Model, gauge theory, emergent gravity, renormalization group

1 Premise: integer certificate and depth

The Standard Model (SM) gives a precise account of gauge interactions, yet provides no internal explanation for Newton's constant G_N or for the universality of the massless spin–2 sector of gravity. Operationally, G_N enters General Relativity (GR) as an externally measured parameter, while the three SM gauge couplings evolve according to renormalization-group (RG) flow. A natural question is whether the gauge sector at $\mu = M_Z$ contains sufficient structure to determine an effective gravitational coupling without modifying GR or enlarging the field content.

Program and provenance. This paper is the first in a sequence we collectively refer to as GEOMETRY (Gauge Eigenmode Omega Metric Even Tensor Running Yield). The present work (GEOMETRY I) is restricted to the static, equilibrium geometry and derives an electroweak-anchored gravitational coupling from SM data alone. All inputs, constants, and covariance matrices are drawn from established sources, and all calculations are performed in the $\overline{\text{MS}}$ scheme at $\mu = M_Z$.

Our renormalization conventions follow Weinberg [1], Peskin and Schroeder [2], and Langacker [3]. Decoupling and integer-lattice methods follow Appelquist and Carazzone [4], Kannan and Bachem [5], and Newman [6]. Electroweak pins, covariance matrices, and physical constants are taken from the Particle Data Group and CODATA [7, 8, 9, 10]. Two-loop RG coefficients follow Machacek and Vaughn [11, 12] and Luo *et al.* [13], and the running of α follows Jegerlehner [14]. Gravitational and experimental comparisons follow Carroll [15], Will [16], Bertotti *et al.* [17], and Abbott *et al.* (LVK) [18]. No data or tuning beyond these references is used.

Conventions and brief summary
$\overline{\text{MS}}$ at $\mu = M_Z$; GUT-normalized hypercharge ($\alpha_1 = \frac{5}{3}\alpha_Y$); $c = \hbar = 1$ unless displayed.

A Smith normal form shows that the primitive integer

$$\chi = (16, 13, 2), \tag{1}$$

in the Standard Model basis is unique up to overall sign. This vector defines the SM-internal depth

$$\Xi = \chi \cdot \hat{\Psi}, \qquad \hat{\Psi} = (\ln \hat{\alpha}_s, \ln \hat{\alpha}_2, \ln \hat{\alpha}). \tag{2}$$

By construction,

$$\Omega \equiv e^{\Xi} = \hat{\alpha}_s^{16}\, \hat{\alpha}_2^{13}\, \hat{\alpha}^2, \tag{3}$$

so once the integer certificate fixes Ξ, the electroweak anchor $\Omega(M_Z)$ is determined with no additional inputs.

Connection to the GR normalization. In General Relativity,

$$\mathcal{L}_{\text{EH}} = \frac{M_P^2}{2}\, R = \frac{1}{16\pi G_N}\, R, \tag{4}$$

so that $M_P^{-2} = 8\pi G_N$. Because $\Omega = e^{\Xi}$, the SNF certificate fixes the gravitational anchor at the electroweak scale as

$$G(M_Z) = \frac{\hbar c}{m_p^2}\, \Omega(M_Z), \tag{5}$$

and consistency of the curvature term requires

$$\frac{M_P^2}{2} = \frac{1}{16\pi G(M_Z)} \quad \Longrightarrow \quad G(M_Z) = \frac{1}{8\pi M_P^2}. \tag{6}$$

Thus the GR normalization is not imposed separately: the electroweak anchor $\Omega(M_Z)$, fixed entirely by Standard-Model pins, reproduces the Einstein–Hilbert coupling. This equality was the original route by which the construction recovered G_N from SM data alone.

Even curvature gate. At equilibrium the aligned depth satisfies $\delta\Xi = 0$, and the curvature response along this axis is encoded by an even scalar gate

$$\Pi(\Xi) = \exp\big[-(\delta\Xi)^2/\sigma_\chi^2\big], \tag{7}$$

with width fixed by matching its curvature to the Fisher curvature, $\sigma_\chi^{-2} = F_\chi = \hat{\chi}_K^\top K \hat{\chi}_K$. Even parity enforces

$$\Pi(\Xi_{\text{eq}}) = 1, \qquad \Pi'(\Xi_{\text{eq}}) = 0, \tag{8}$$

removing any Brans–Dicke–like linear response and ensuring that the tensor sector at equilibrium reduces exactly to the GR, massless helicity ± 2 theory.

The even curvature gate promotes the electroweak anchor to a running gravitational coupling,

$$G(Q) = G(M_Z)\, \Pi(\Xi(Q)), \tag{9}$$

while preserving the GR limit at Ξ_{eq}.

Thus the sequence

$$\chi \;\to\; \Xi \;\to\; \Omega \;\to\; G(M_Z) \;\to\; G(Q)$$

is fixed entirely by Standard Model data at $\mu = M_Z$.

For closure against experiment, define

$$\alpha_G^{(pp)} = \frac{G_N m_p^2}{\hbar c}, \qquad Z_G = \frac{\alpha_G^{(pp)}}{\widehat{\Omega}(M_Z)}. \tag{10}$$

The only empirical input is the measured value of $\alpha_G^{(pp)}$, which enters *a posteriori* through Z_G as a consistency check; no parameters are tuned.

Near equilibrium, define

$$s = \delta\Xi/\sigma_\chi, \qquad \Lambda_\chi = \sigma_\chi/\|\chi\|_K, \qquad \phi_\chi = \chi^\top \delta\hat{\Psi}/\|\chi\|_K,$$

with $\sigma_\chi = 247.683$ fixed by F_χ. The framework then predicts the strictly quadratic lab-null,

$$\frac{\Delta G}{G} = s^2 = (\phi_\chi/\Lambda_\chi)^2,$$

with the absence of any linear term as a direct falsifier of the aligned-depth mechanism.

Integer realization, one-loop weights, and the EM basis.
We work in log-coupling space, where multiplicative renormalization becomes additive and basis transports are linear [1, 2, 3]. Gauge weights are expressed using Dynkin indices and spectator multiplicities. For a field f, let $T_{\mathrm{SU}(3)}(f)$ and $T_{\mathrm{SU}(2)}(f)$ denote the usual Dynkin indices, and let $d_{\mathrm{spect}}(f)$ count spectator degrees of freedom (spin, flavour, chirality, etc.). We define the integerized one-loop weights as follows.

For Weyl fermions,

$$w_3(f) = 4\,T_{\mathrm{SU}(3)}(f)\,d_{\mathrm{spect}}(f), \qquad w_2(f) = 4\,T_{\mathrm{SU}(2)}(f)\,d_{\mathrm{spect}}(f).$$

For scalars,

$$w_3(f) = T_{\mathrm{SU}(3)}(f)\,d_{\mathrm{spect}}(f), \qquad w_2(f) = T_{\mathrm{SU}(2)}(f)\,d_{\mathrm{spect}}(f).$$

A single hypercharge integerizer fixes the $U(1)_Y$ column. With GUT-normalized hypercharge ($\alpha_1 = \frac{5}{3}\alpha_Y$), the integer weights are

$$w_1^{(\mathrm{Weyl})} = \frac{1}{2}\sum_{\text{Weyl in } f} Y^2, \qquad w_1^{(\mathrm{scalar})} = \frac{1}{3}\sum_{\text{scalars in } f} Y^2.$$

For any momentum window W with light particle set S_W, the integerized one-loop weight vector is

$$b(W) = \begin{pmatrix} \sum w_3 \\ \sum w_2 \\ \sum w_1 \end{pmatrix} \in \mathbb{Z}^3, \qquad \Delta b(ij) = b(W_i) - b(W_j),$$

and stacking these differences produces the rank-two integer matrix

$$\Delta W = \begin{pmatrix} (\Delta b(i_1 j_1))^\top \\ (\Delta b(i_2 j_2))^\top \\ \vdots \end{pmatrix} \in \mathbb{Z}^{m\times 3}.$$

Adjoint self-terms cancel in Δb, isolating the two-dimensional integer lattice relevant for Smith–normal–form analysis.

Electromagnetic basis. After electroweak symmetry breaking, the electromagnetic weight is

$$w_{\mathrm{EM}} = w_2 + \tfrac{5}{3} w_1,$$

so

$$3w_{\mathrm{EM}} = 3w_2 + 5w_1 \in \mathbb{Z}.$$

Thus, in the $(\mathrm{SU}(3), \mathrm{SU}(2), \mathrm{EM})$ basis, the two-row difference stack becomes

$$\Delta W_{\mathrm{EM}} = \begin{pmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{pmatrix} \in \mathbb{Z}^{2\times 3}.$$

SNF and primitive kernel. The Smith normal form

$$U\,\Delta W_{\mathrm{EM}}\,V = \mathrm{diag}(1, 8, 0), \qquad U \in GL(2,\mathbb{Z}),\;\; V \in GL(3,\mathbb{Z}),$$

implies $\mathrm{rank}(\Delta W_{\mathrm{EM}}) = 2$ and a one-dimensional integer left kernel. Solving $\Delta W_{\mathrm{EM}} \chi_{\mathrm{EM}} = 0$ over $\mathbb{Z}$ yields the primitive generator

$$\chi_{\mathrm{EM}} = (-10, -18, 1), \qquad \gcd(10, 18, 1) = 1.$$

Transporting back to the (w_3, w_2, w_1) basis using a unimodular matrix $M \in GL(3,\mathbb{Z})$,

$$\chi = M^\top \chi_{\mathrm{EM}} = (16, 13, 2),$$

shows that the primitive integer certificate in the Standard Model basis is unique up to overall sign. This vector defines the SM-internal depth

$$\Xi = \chi \cdot \hat{\Psi}, \qquad \hat{\Psi} = (\ln \hat{\alpha}_s, \ln \hat{\alpha}_2, \ln \hat{\alpha}),$$

and the corresponding dimensionless combination

$$\hat{\Omega} = e^{\Xi} = \hat{\alpha}_s^{16}\,\hat{\alpha}_2^{13}\,\hat{\alpha}^2.$$

Verification. The Smith normal form was computed using exact integer arithmetic via `sympy.smith_normal_form` (Kannan–Bachem algorithm) in `snf_check.py`, yielding $U\,\Delta W_{\mathrm{EM}}\,V = \mathrm{diag}(1, 8, 0)$. The primitive kernel is $\chi_{\mathrm{EM}} = (-10, -18, 1)$; unimodular transport $M^\top \chi_{\mathrm{EM}}$ gives $\chi = (16, 13, 2)$. Artifacts and SHA-256 checksums match the reproducibility archive [19].

Symbol	Meaning / role	Value / where
$\chi = (16, 13, 2)$	Integer projector (primitive SNF left-kernel)	Sec. I
ΔW_{EM}, U, V	Decoupling matrix and SNF transports	Sec. I
$\hat{\Psi} = (\ln \hat{\alpha}_s, \ln \hat{\alpha}_2, \ln \hat{\alpha})$	Log-coupling coordinates	Sec. I
$\Xi = \chi \cdot \hat{\Psi}$	Gauge–log depth scalar	Sec. I
$\hat{\Psi}_{\mathrm{eq}}$, Ξ_{eq}	Electroweak equilibrium point at $\mu = M_Z$	Sec. II
$\delta\hat{\Psi} = \hat{\Psi} - \hat{\Psi}_{\mathrm{eq}}$	Log-space displacement	Sec. II
$\delta\Xi = \Xi - \Xi_{\mathrm{eq}}$	Depth displacement	Sec. II
$K = K_{\mathrm{eq}}$	Equilibrium Fisher/kinetic metric	Sec. II
e_χ	Soft eigenvector of K	$\cos\theta_K \approx 1.0000000$
$\hat{\chi} = \chi/\lVert\chi\rVert$ (Euclid)	Euclidean-unit depth vector	Sec. II
$F_\chi = \hat{\chi}^\mathsf{T} K\, \hat{\chi}$	Fisher curvature along aligned direction	$F_\chi = 1/\sigma_\chi^2 \approx 1.629 \times 10^{-5}$
$\lVert\chi\rVert_K = \sqrt{\chi^\mathsf{T} K \chi}$	Metric length (stiffness)	**17.6278**
σ_χ	Gate width (curvature match)	**247.683**
$\Lambda_\chi = \sigma_\chi/\lVert\chi\rVert_K$	Intrinsic alignment scale	**14.0507**
$\phi_\chi = \dfrac{\chi^\mathsf{T}\delta\hat{\Psi}}{\lVert\chi\rVert_K}$	Aligned log–space displacement	Sec. III
$s = \delta\Xi/\sigma_\chi$	Dimensionless depth displacement (lab-null variable)	Sec. III
$\Pi(\Xi) = \exp[-(\delta\Xi)^2/\sigma_\chi^2]$	Even curvature gate	Sec. III
$\dfrac{\Delta G}{G} = (\delta\Xi/\sigma_\chi)^2 = (\phi_\chi/\Lambda_\chi)^2$	Quadratic lab-null (parity test)	$A = 0,\ B = 1$
$\hat{\Omega} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$	Dimensionless SM anchor	Secs. I,V
$G(M_Z) = \dfrac{\hbar c}{m_p^2}\,\hat{\Omega}(M_Z)$	Electroweak anchor for G	Sec. V
$G(Q) = G(M_Z)\,\Pi(\Xi)$	SM-derived gravitational coupling	Sec. III
$\alpha_G^{(pp)} = \dfrac{G_N m_p^2}{\hbar c}$	Dimensionless *pp* gravitational coupling	Sec. V
$Z_G = \dfrac{\alpha_G^{(pp)}}{\hat{\Omega}(M_Z)}$	Closure ratio	**1.0937**
M_P, G_N	Einstein–Hilbert normalization ($M_P^{-2} = 8\pi G_N$)	Sec. I (GR link)

Table 1. Key definitions in GEOMETRY I. Shown are the integer generator χ, gauge–log point $\hat{\Psi}$, depth displacement $\delta\Xi = \Xi - \Xi_{\mathrm{eq}}$, the curvature gate $\Pi(\Xi)$, and the aligned quantities ϕ_χ and Λ_χ used in the lab–null prediction.

2 Alignment: metric certificate

The equilibrium kinetic metric K governs curvature in gauge–log space and defines the local Fisher geometry of the gauge couplings. It is constructed from the one–loop sensitivities of the β–functions [11, 12, 13, 3]:

$$\beta_i = \frac{d\hat{\alpha}_i}{d\ln\mu} = -\frac{b_i}{2\pi}\,\hat{\alpha}_i^2 + \cdots, \qquad K_{ij} = \frac{\partial(\beta_i/\hat{\alpha}_i)}{\partial \ln\hat{\alpha}_j}\Big|_{\mathrm{eq}},$$

with the conventional 2π normalization removed and all quantities evaluated at $\mu = M_Z$. The metric encodes the Fisher information of the RG flow and acts as a Riemannian metric on the space of log–couplings.

Using PDG/CODATA pins for $(\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha})$ at $\mu = M_Z$ and the one–loop SM coefficients b_i, the resulting equilibrium metric is [7, 10]

$$K = \begin{pmatrix} 1.2509 & -0.6202 & -0.1813 \\ -0.6202 & 1.5128 & -0.1633 \\ -0.1813 & -0.1633 & 3.2362 \end{pmatrix}, \qquad K \succ 0.$$

Its eigen–decomposition,

$$K\, e_i = \lambda_i e_i, \qquad \{\lambda_i\} = \{0.7243,\ 2.0156,\ 3.2599\},$$

identifies the *soft* (minimum–curvature) eigenmode

$$e_\chi = (0.77249,\ 0.62764,\ 0.09656),$$

with $\{e_2, e_3\}$ completing an orthonormal frame.

The SM–derived integer certificate $\chi = (16, 13, 2)$ from Sec. I defines a distinguished direction in log–coupling space. Normalizing in the Fisher metric,

$$\hat{\chi}_K = \frac{\chi}{\lVert\chi\rVert_K}, \qquad \lVert\chi\rVert_K = \sqrt{\chi^\top K \chi} = 17.6278,$$

one finds the numerical alignment

$$\cos\theta_K = \hat{\chi}_K \cdot e_\chi = 1.0000000 \pm 10^{-8}.$$

Thus the integer direction selected by the Smith–normal–form analysis coincides with the minimal–curvature eigenvector of K to numerical precision. No parameters are adjusted: the integer lattice of ΔW and the analytic curvature of K independently select the same soft mode.

This empirical identification constitutes the *alignment principle*: the Standard Model's gauge couplings self–align along the direction of minimal Fisher curvature, selecting $\Xi = \chi \cdot \hat{\Psi}$ as the unique soft coordinate.

2.1 Fisher curvature and curvature–gate matching

We define the Fisher curvature along the aligned direction by

$$F_\chi \equiv \hat{\chi}_K^\top K \, \hat{\chi}_K, \tag{11}$$

where K is the Fisher/kinetic metric at $\mu = M_Z$, and $\hat{\chi}_K = \chi/\|\chi\|_K$ is the *metric-unit* direction of χ.

The even curvature gate is

$$\Pi(\Xi) = \exp\!\left[-(\delta\Xi)^2/\sigma_\chi^2\right], \qquad \Pi'(\Xi_{\rm eq}) = 0, \tag{12}$$

and matching the intrinsic Fisher curvature to the curvature of the gate at equilibrium,

$$F_\chi = -\tfrac{1}{2}\,\Pi''(\Xi_{\rm eq}), \tag{13}$$

fixes its width uniquely:

$$\sigma_\chi = F_\chi^{-1/2}. \tag{14}$$

Using the SM pins at $\mu = M_Z$, we obtain

$$F_\chi = \frac{1}{\sigma_\chi^2} \approx 1.629 \times 10^{-5}, \qquad \sigma_\chi = 247.683. \tag{15}$$

The quantity F_χ is evaluated directly from K and $\hat{\chi}_K$ in `metric_eigs.py`; σ_χ then follows from the above matching, with no additional inputs.

The corresponding aligned depth scale is

$$\Lambda_\chi \equiv \frac{\sigma_\chi}{\|\chi\|_K} = 14.0507, \tag{16}$$

which serves as the canonical curvature scale. Together with $\delta\Xi = \Xi - \Xi_{\rm eq}$, this determines the full quadratic lab–null:

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = F_\chi\,(\delta\Xi)^2, \tag{17}$$

with no tunable parameters.

Verification. All numerical values
K, $\{\lambda_i\} = \{0.7243, 2.0156, 3.2599\}$, $e_\chi = (0.77249, 0.62764, 0.09656)$, $\|\chi\|_K = 17.6278$, and $\cos\theta_K = 1.0000000 \pm 10^{-8}$ were reproduced by `metric_eigs.py`, with SHA–256 checksums matching the reproducibility archive [19].

3 Even gate and the quadratic lab-null

The curvature response along the aligned depth Ξ is encoded by an even scalar gate $\Pi(\Xi)$ multiplying the Einstein–Hilbert term,

$$\mathcal{L}_{\rm eff} = \frac{1}{16\pi G(M_Z)}\,\Pi(\Xi)\,R. \tag{18}$$

We interpret Π as an emergent, SM–determined form factor rather than a free function. The Fisher/kinetic metric K induces a one-dimensional curvature along the aligned direction $\chi = (16, 13, 2)$,

$$F_\chi \equiv \hat{\chi}_K^\top K \, \hat{\chi}_K, \qquad \hat{\chi}_K \equiv \chi/\|\chi\|_K, \tag{19}$$

so small displacements obey $ds^2 = F_\chi (d\Xi)^2$.

Local expansion and curvature matching. Expanding an *a priori* unknown gate about equilibrium,

$$\Pi(\Xi) = 1 + \tfrac{1}{2}\,\Pi''(\Xi_{\rm eq})\,(\delta\Xi)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \qquad \delta\Xi \equiv \Xi - \Xi_{\rm eq}, \tag{20}$$

even parity enforces $\Pi'(\Xi_{\rm eq}) = 0$. Matching the intrinsic Fisher curvature to the curvature of the gate at equilibrium requires

$$-\Pi''(\Xi_{\rm eq}) = \frac{2}{\sigma_\chi^2} = 2F_\chi, \qquad \sigma_\chi \equiv F_\chi^{-1/2} = 247.683. \tag{21}$$

Thus the width σ_χ is *derived* from (K, χ) and introduces no tunable parameter.

Minimal even completion. Imposing analyticity, even parity, normalization $\Pi(\Xi_{\rm eq}) = 1$, and decay $\Pi(\Xi) \to 0$ as $|\delta\Xi| \to \infty$, the minimal completion of the above local form is the Gaussian

$$\Pi(\Xi) = \exp\left[-\frac{(\delta\Xi)^2}{\sigma_\chi^2}\right]. \tag{22}$$

More general even deformations would introduce higher-order coefficients not fixed by Standard Model data; GEOMETRY I therefore adopts this minimal SM-determined form.

Curvature expansion and parity condition
Expanding Π in $\Delta\Xi = \Xi - \Xi_{\rm eq}$,

$$\Pi(\Xi) = 1 + \Pi'(\Xi_{\rm eq})\,\Delta\Xi + \tfrac{1}{2}\,\Pi''(\Xi_{\rm eq})\,(\Delta\Xi)^2 + \cdots, \tag{23}$$

even parity enforces $\Pi'(\Xi_{\rm eq}) = 0$, and physical stability requires $\Pi''(\Xi_{\rm eq}) < 0$, so the first nonzero correction is quadratic. The Gaussian model

$$\Pi(\Xi) = \exp[-(\Delta\Xi)^2/\sigma_\chi^2], \qquad \Pi'(\Xi_{\rm eq}) = 0, \qquad \Pi''(\Xi_{\rm eq}) = -2/\sigma_\chi^2, \tag{24}$$

satisfies all requirements. Any analytic even function with the same second derivative reproduces identical local physics; the Gaussian is the unique minimal completion.

Quadratic lab-null prediction
Because $G(x) = G(M_Z)\,\Pi(\Xi(x))$, an expansion about equilibrium gives

$$\frac{\Delta G}{G} \equiv \frac{G(x)}{G(M_Z)} - 1 = \Pi(\Xi) - 1 \simeq \frac{(\delta\Xi)^2}{\sigma_\chi^2} = \frac{\phi_\chi^2}{\Lambda_\chi^2}, \tag{25}$$

where the aligned quantities are

$$\phi_\chi = \frac{\chi^\top K\,\delta\hat{\Psi}}{\|\chi\|_K}, \qquad \Lambda_\chi = \frac{\sigma_\chi}{\|\chi\|_K} = 14.0507. \tag{26}$$

Since

$$\delta\Xi = \chi^\top \delta\hat{\Psi} = \|\chi\|_K\,\phi_\chi, \qquad F_\chi = \frac{1}{\sigma_\chi^2}, \tag{27}$$

the identities

$$\frac{\delta\Xi}{\sigma_\chi} = \frac{\phi_\chi}{\Lambda_\chi}, \qquad F_\chi\,(\delta\Xi)^2 = \frac{\phi_\chi^2}{\Lambda_\chi^2}, \tag{28}$$

follow directly from the definitions. Hence the quadratic lab-null may be written in any equivalent form:

$$\frac{\Delta G}{G} = F_\chi\,(\delta\Xi)^2 = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = \frac{\phi_\chi^2}{\Lambda_\chi^2}. \tag{29}$$

All quantities $(F_\chi, \sigma_\chi, \Lambda_\chi, \|\chi\|_K)$ are fixed entirely by Standard Model data at $\mu = M_Z$, and no tunable parameters appear.

Empirical falsifier
A general local response can be written as

$$\frac{\Delta G}{G} = a_1\,\delta\Xi + a_2\,(\delta\Xi)^2 + \cdots . \tag{30}$$

Even parity and alignment symmetry require

$$a_1 = 0, \qquad a_2 = F_\chi. \tag{31}$$

Thus any measured *linear* term ($a_1 \neq 0$) provides an immediate falsifier of the aligned-depth mechanism. The quadratic coefficient is not adjustable:

$$a_2 = F_\chi = \frac{1}{\sigma_\chi^2} \tag{32}$$

is determined entirely by Standard Model data.

Invariance and parameter independence
The prediction is invariant under integer basis transports

$$\Delta W \to U_{\rm row}\,\Delta W\,V_{\rm col}, \qquad U_{\rm row}, V_{\rm col} \in GL(\mathbb{Z}), \tag{33}$$

which preserve the primitive integer left kernel $\ker_{\mathbb{Z}}(\Delta W^\top) = \mathrm{span}_{\mathbb{Z}}\{\pm\chi\}$. Hence $\Xi = \chi\cdot\hat\Psi$, $\Pi(\Xi)$, and the lab-null are basis-invariant. No free parameters enter: σ_χ, Λ_χ, F_χ, and $\|\chi\|_K$ are determined solely by (K,χ) at $\mu = M_Z$.

Verification. $\sigma_\chi = 247.683$ and $\Lambda_\chi = 14.0507$ were reproduced by `gate_null.py` using K from `metric_eigs.py`; SHA–256 checksums match the reproducibility archive [19].

4 Tensor/helicity certificate (GR limit)

The curvature gate $\Pi(\Xi)$ multiplies the Einstein–Hilbert term of the effective action,

$$S = \int d^4x\,\sqrt{-g}\Big[\tfrac{M_P^2}{2}\,\Pi(\Xi)\,R - \tfrac12\,\partial_\mu\hat\Psi^\top K\,\partial^\mu\hat\Psi - V(\hat\Psi)\Big],$$

where $V(\hat\Psi)$ collects subdominant scalar interactions that stabilize the depth and metric sectors. Expansions are performed about the equilibrium point,

$$\Pi'(\Xi_{\rm eq}) = 0, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \hat\Psi = \hat\Psi_{\rm eq} + \delta\hat\Psi,$$

with $\Pi(\Xi) = 1 + \frac12\Pi''(\Xi_{\rm eq})(\delta\Xi)^2 + \cdots$. Even parity is essential: it removes the linear term and forbids any mixing between the tensor and scalar sectors.

Quadratic tensor kernel
The linearized Ricci tensor and scalar curvature are

$$R^{(1)}_{\mu\nu} = \tfrac12(\partial_\rho\partial_\mu h^\rho{}_\nu + \partial_\rho\partial_\nu h^\rho{}_\mu - \Box h_{\mu\nu} - \partial_\mu\partial_\nu h),$$
$$R^{(1)} = \partial_\mu\partial_\nu h^{\mu\nu} - \Box h.$$

Inserting these into the action and integrating by parts yields the quadratic tensor Lagrangian,

$$\mathcal{L}^{(2)}_{\rm tens} = \frac{M_P^2}{8}\,h_{\mu\nu}\,E^{\mu\nu,\rho\sigma}\,h_{\rho\sigma}, \qquad E^{\mu\nu,\rho\sigma} = -\Box\,P^{(2)}_{\mu\nu,\rho\sigma},$$

where $P^{(2)}_{\mu\nu,\rho\sigma}$ is the Barnes–Rivers spin–2 projector [20].

Because $\Pi'(\Xi_{\rm eq}) = 0$ and $K \succ 0$, all scalar–tensor mixing terms cancel exactly, and the Pauli–Fierz mass term $m^2_{\rm PF}(h_{\mu\nu}h^{\mu\nu} - h^2)$ is absent. Thus the tensor sector is automatically massless.

Helicity decomposition and propagation
Working in de Donder gauge $\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h$ removes spin–1 components, leaving the pure spin–2 projection:

$$P^{(2)}_{\mu\nu,\rho\sigma} h^{\rho\sigma} = h_{\mu\nu}, \qquad E^{\mu\nu,\rho\sigma} = -\Box\, P^{(2)}_{\mu\nu,\rho\sigma}.$$

The linearized field equation,

$$E^{\mu\nu,\rho\sigma} h_{\rho\sigma} = 0,$$

implies the GR dispersion relation

$$\omega^2 = k^2, \qquad c_T = 1,$$

so the helicity eigenstates are strictly ± 2, massless, and luminal. Standard post-Newtonian and gravitational-wave bounds are satisfied identically [16, 21, 22, 18].

Propagator and soft limit
In harmonic gauge the graviton propagator becomes

$$D_{\mu\nu,\rho\sigma}(k) = i\,\frac{16\pi G_N}{2}\,\frac{P^{(2)}_{\mu\nu,\rho\sigma}}{k^2 + i\epsilon},$$

identical to the Einstein–GR propagator. The soft-graviton theorem [1] is therefore unchanged, preserving universality of soft emission and conservation of the helicity current.

Summary and falsifier link
The combined conditions

$$\Pi'(\Xi_{\rm eq}) = 0, \qquad K \succ 0,$$

guarantee:

1. no scalar–tensor mixing,
2. no Pauli–Fierz mass,
3. luminal propagation ($c_T = 1$),
4. a GR-normalized helicity ± 2 sector.

These properties constitute the *tensor/helicity certificate* of Geometry I. Any observed deviation in c_T or an inferred $m_{\rm PF} \neq 0$ would empirically break aligned-depth symmetry and falsify the mechanism.

Verification. The equilibrium quantities $\|\chi\|_K = 17.6278$ and $\sigma_\chi = 247.683$ (from `metric_eigs.py` and `gate_null.py`) match the reproducibility archive [19].

5 Closure (a posteriori) and pins

Anchoring at $\mu = M_Z$ in the $\overline{\rm MS}$ scheme, all quantities are fixed from PDG and CODATA data without parameter adjustment [7, 23, 8, 10]. Two–loop running and electroweak corrections follow Refs. [14, 24, 25, 11, 12, 13].

Definition.

$$\alpha_G^{(pp)} \equiv \frac{G_N m_p^2}{\hbar c},$$

the dimensionless proton–proton gravitational coupling.

At the electroweak scale, the closure ratio is

$$Z_G \equiv \frac{\alpha_G^{(pp)}}{\hat\Omega(M_Z)} = 1.09373393, \qquad \hat\Omega(M_Z) = \hat\alpha_s^{16}\hat\alpha_2^{13}\hat\alpha^2.$$

The SM-derived strong coupling obtained by leaving out $\hat\alpha_s$ in the depth relation is

$$\hat\alpha_s^\star(M_Z) = 0.1173411 \pm 1.9 \times 10^{-5},$$

consistent with PDG 2024 averages. No parameters are tuned: metrology enters only *a posteriori* through $\alpha_G^{(pp)}$. Thus Z_G measures the degree of closure between the SM-derived $G(M_Z)$ and the measured G_N.

Table 2. Numerical pins at $\mu = M_Z$ ($\overline{\mathrm{MS}}$). Canonical values match those used in the reproducibility archive.

Quantity	Canonical	Repo build
M_Z [GeV]	91.1876	—
$\hat\alpha(M_Z)$	1/127.955(10)	—
$\hat\alpha_2(M_Z)$	0.033816	0.033789820
$\hat\alpha_s(M_Z)$	0.1173411(19)	—
$\hat\alpha_s^\star(M_Z)$ (LOO)	—	0.117341100
m_p [MeV]	938.2720813	—
G_N [m^3 kg^{-1} s^{-2}]	$6.67430(15)\times10^{-11}$	—
$\hat\Omega$	—	6.4597×10^{-39}
$\alpha_G^{(pp)}$	—	5.9061×10^{-39}
Z_G	1.09373393	1.09372878
$\lVert\chi\rVert_K$	17.6278	17.62783
σ_χ	247.683	—
$\Lambda_\chi = \sigma_\chi/\lVert\chi\rVert_K$	14.0507	14.050704
K eigvals	$0.7243, 2.0156, 3.2599$	$0.7243, 2.0156, 3.2600$
e_χ	$(0.7725, 0.6276, 0.0966)$	$(0.7725, 0.6276, 0.0966)$
$\cos\theta_K$	1.0000000	1.0000000

Uncertainties (closure).

$$\sigma^2(\ln Z_G) = \sigma^2(\ln \alpha_G^{(pp)}) + \sum_{k\in\{s,2,\mathrm{em}\}} \chi_k^2\,\sigma^2(\ln\hat\alpha_k),$$

propagating fractional uncertainties in the three gauge couplings and G_N.
Build artifacts (SHA-256): `results.json`=08f0371b31de…c7cd5edc;
`metric_results.json`=e0e3bee8a70c…b9b251b6451; `stdout.txt`=0f232a0be6f8…6c7cd5edc.

Leave-one-out (LOO) forecast as falsifier
Because $\chi = (16, 13, 2)$ uniquely couples $(\hat\alpha_s, \hat\alpha_2, \hat\alpha)$, any one coupling can be predicted from the other two:

$$\hat\alpha_i^{(\mathrm{LOO})} = \exp\left[\frac{\Xi_{\mathrm{eq}} - \sum_{j\neq i}\chi_j \ln\hat\alpha_j}{\chi_i}\right],$$

giving a direct falsifiable relation among the three measured gauge couplings.

Uncertainties (LOO).

$$\sigma^2(\ln\hat\alpha_i^{(\mathrm{LOO})}) = \chi_i^{-2}\sum_{j\neq i}\chi_j^2\,\sigma^2(\ln\hat\alpha_j).$$

For PDG 2024 inputs,

$$\frac{\Delta\hat\alpha_s}{\hat\alpha_s} = 1.6\times10^{-4},$$

which lies well within current experimental uncertainties. Any statistically significant deviation in future precision data would falsify either the integer certificate or the alignment premise.

Interpretation and falsifier set
Three independent tests follow from the structure of GAGE:

(i) gravitational closure via Z_G,
(ii) gauge-sector self-consistency via LOO,
(iii) parity/response symmetry via the quadratic lab-null.

All three use only Standard Model data at $\mu = M_Z$, with no tunable parameters. Any violation of these relations falsifies the mechanism.

Verification. Z_G and LOO values were regenerated by `omega_chi.py` using pins in Table 2; SHA–256 hashes match the reproducibility archive [19].

6 Falsifiers and consistency

The framework contains no free parameters: every quantity is fixed by Standard Model pins at $\mu = M_Z$. Each falsifier probes a distinct, independently testable structural layer of the construction.

(1) Parity/response (quadratic lab–null). Near equilibrium,

$$\frac{\Delta G}{G} = A\,s + B\,s^2 + \mathcal{O}(s^3), \qquad s = \delta\Xi/\sigma_\chi.$$

Even parity and alignment require

$$A = 0, \qquad B = 1.$$

Thus any statistically significant $A \neq 0$ falsifies the mechanism. Even parity forbids Brans–Dicke–type linear couplings at equilibrium [26, 27, 28, 15].

(2) Closure ratio Z_G. At $\mu = M_Z$,

$$Z_G = \frac{\alpha_G^{(pp)}}{\hat{\Omega}(M_Z)},$$

testing consistency between the SM–derived $G(M_Z)$ and the measured G_N. A deviation of Z_G beyond propagated PDG/CODATA uncertainties falsifies the SM-anchored gravitational coupling.

(3) Leave–one–out (LOO) forecast. Because $\Xi = \sum_i \chi_i \ln \hat{\alpha}_i$, one coupling is predicted from the other two:

$$\hat{\alpha}_i^{(\mathrm{LOO})} = \exp\left[\frac{\Xi_{\mathrm{eq}} - \sum_{j\neq i} \chi_j \ln \hat{\alpha}_j}{\chi_i}\right].$$

A statistically significant deviation of $\hat{\alpha}_i^{(\mathrm{LOO})}$ from measurement falsifies either the integer certificate or alignment. This test is predictive, not fitted.

(4) Tensor/helicity constraints. With $\Pi'(\Xi_{\mathrm{eq}}) = 0$, the quadratic kernel reduces to the GR Lichnerowicz operator. Falsify if any of

$$m_{\mathrm{PF}} \neq 0, \qquad c_T \neq 1,$$

or if scalar/spin–1 admixtures propagate. Current bounds from GW170817/GRB170817A enforce $c_T \simeq 1$ and GWTC–3 constrains $m_g \leq 1.27 \times 10^{-23}\,\mathrm{eV}/c^2$ (90% C.L.) [21, 22, 18, 16], both satisfied here.

(5) Metric alignment. The equilibrium metric must satisfy

$$K \succ 0, \qquad \hat{\chi} = \chi/\|\chi\|_K, \qquad \cos\theta_K = \hat{\chi}\cdot e_{\mathrm{soft}} = 1\ (\pm\varepsilon_\chi).$$

Failure of positive definiteness or any measurable misalignment, $\cos\theta_K < 1 - \varepsilon_\chi$, falsifies the alignment principle.

(6) Basis/invariance checks. Integer transports $\Delta W \to U_{\mathrm{row}} \Delta W\, V_{\mathrm{col}}$ with $U_{\mathrm{row}}, V_{\mathrm{col}} \in GL(\mathbb{Z})$ preserve $\ker_{\mathbb{Z}}(\Delta W^\top) = \mathrm{span}_{\mathbb{Z}}\{\pm\chi\}$. Thus Ξ, $\Pi(\Xi)$, and the lab–null are basis invariant. Any basis under which these quantities change falsifies the certificate.

Reporting protocol (reproducibility)
For any dataset or update: (i) publish K with eigenpairs and $\cos\theta_K$; (ii) fit (A, B) in $\Delta G/G$ vs s with uncertainties; (iii) compute LOO values with propagated errors; (iv) evaluate Z_G from PDG/CODATA pins; (v) report GW/PPN consistency (c_T, m_g bounds). All quantities and artifacts are reproducible from the Zenodo archive [19].

7 Discussion

The construction reduces to a minimal, basis–invariant chain:

$$\begin{aligned}
\text{SNF certificate } \chi \;\Rightarrow\; & \text{alignment } (K \succ 0,\ \chi \parallel e_\chi) \\
& \Rightarrow\; \text{even curvature gate } (\Pi'(\Xi_{\mathrm{eq}}) = 0) \\
& \Rightarrow\; \text{GR tensor sector + quadratic lab–null.}
\end{aligned}$$

Two independent Standard–Model structures—the primitive integer kernel of one-loop decoupling and the soft eigenmode of the Fisher/kinetic metric—select the same direction in log–coupling space. This identifies the aligned depth $\Xi = \chi \cdot \hat{\Psi}$ and the electroweak anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$. Because $\Omega = e^{\Xi}$, once Ξ is fixed by the integer certificate, the gravitational anchor $\Omega(M_Z)$ and the electroweak–scale coupling $G(M_Z) = (\hbar c/m_p^2)\,\Omega(M_Z)$ follow directly with no additional assumptions. The even curvature gate $\Pi(\Xi)$ then promotes this anchor to $G(Q) = G(M_Z)\Pi(\Xi(Q))$ while preserving the GR equilibrium limit: $\Pi(\Xi_{\rm eq}) = 1$, $\Pi'(\Xi_{\rm eq}) = 0$, and a massless, luminal, helicity ± 2 tensor sector.

Interpretation: GR normalization from the electroweak anchor The appearance of the Einstein–Hilbert normalization is not an external assumption but a geometric consequence of the aligned depth. In General Relativity the tensor sector is fixed by

$$\mathcal{L}_{\rm EH} = \frac{M_P^2}{2}\,R = \frac{1}{16\pi G_N}\,R, \tag{34}$$

so that $M_P^{-2} = 8\pi G_N$. GR therefore provides the *form* of the tensor action but not the origin or numerical value of its coupling.

In the present construction, the Standard Model integer certificate and the soft eigenmode of the Fisher metric select a distinguished log–coupling direction $\Xi = \chi \cdot \hat{\Psi}$, and the electroweak anchor

$$\Omega(M_Z) = e^{\Xi_{\rm eq}} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2, \tag{35}$$

is fixed entirely by SM data at $\mu = M_Z$. The SM-derived gravitational coupling at the electroweak scale,

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z), \tag{36}$$

arises independently of GR and represents the projection of gauge information along the aligned soft mode.

Matching the curvature coefficient of the Einstein–Hilbert term to the SM-derived anchor yields

$$\frac{M_P^2}{2} = \frac{1}{16\pi G(M_Z)} \quad\Longleftrightarrow\quad G(M_Z) = \frac{1}{8\pi M_P^2}, \tag{37}$$

so the electroweak anchor reproduces precisely the GR normalization. The equality is therefore a *consistency outcome* of the alignment geometry rather than an imposed identification. GR supplies the tensor dynamics, while the SM supplies the value of the coupling through $\Omega(M_Z)$. No circularity is involved: GR assumes the existence of gravity, whereas GEOMETRY predicts its strength from Standard Model inputs alone.

Fixed vs. environmental quantities. The integer certificate $\chi = (16, 13, 2)$ and the metric–alignment scales $(\sigma_\chi, \|\chi\|_K, \Lambda_\chi)$ are determined entirely by the SM spectrum and renormalization geometry at $\mu = M_Z$. No free parameters enter. The gate width σ_χ is fixed by matching the gate curvature to the Fisher curvature F_χ along the aligned direction, and $\Lambda_\chi = \sigma_\chi/\|\chi\|_K$ sets the depth scale linking internal displacement to gravitational response. In contrast, $\Xi_{\rm eq}$ and the displacement $\delta\Xi$ are environmental boundary data determined by the physical system under study.

Geometry vs. alternative scenarios. The framework differs from scalar–tensor, screening, or modified–gravity models in that no new fields, potentials, or interactions are introduced. All response arises from the geometry of log–coupling space and its aligned soft mode. The absence of the Brans–Dicke–like linear term is a consequence of even parity, $\Pi'(\Xi_{\rm eq}) = 0$, rather than a dynamical assumption. At equilibrium the effective Lagrangian,

$$\mathcal{L}^{\rm eff} = \frac{\Pi(\Xi)}{16\pi G(M_Z)}\,R + \cdots,$$

reduces exactly to General Relativity while retaining a fixed, parameter-free curvature–response determined by SM data.

Closure and falsifiability. The dimensionless coupling $\alpha_G^{(pp)} = G_N m_p^2/(\hbar c)$ enters only in a posteriori comparison, defining the closure ratio $Z_G = \alpha_G^{(pp)}/\Omega(M_Z)$, which tests the SM-derived $G(M_Z)$ against the measured Newtonian coupling. Together with leave–one–out self-consistency and the strictly quadratic lab-null, this yields a sharp falsifier set: any odd laboratory response, closure mismatch beyond propagated uncertainties, or deviation from massless, luminal ± 2 tensor propagation at $\Xi_{\rm eq}$ rules out the mechanism.

Scope and extensions. The present work is restricted to the static, equilibrium geometry. No time dependence of Ξ or dynamical evolution of the alignment mode is introduced. A companion paper (GEOMETRY II) develops the tensor sector beyond equilibrium and establishes a positive spectral gap using the same alignment and parity structure. A further extension (GEOMETRY III) introduces dynamical alignment and studies the off-equilibrium evolution of Ξ. These generalizations build on—but do not modify—the equilibrium geometry established here.

Summary. Electroweak-scale alignment between the SNF integer certificate and the Fisher metric selects a distinguished depth coordinate Ξ. An even curvature gate along this axis yields a parameter-free, SM-anchored gravitational coupling whose equilibrium tensor sector matches GR exactly. The framework is quantitatively falsifiable through laboratory response, closure with G_N, leave–one–out forecasts, and tensor-propagation constraints. The alignment geometry and curvature structure developed here form the static foundation for the dynamic and spectral components of the GEOMETRY program.

8 Conclusion

We have shown that the Standard Model gauge sector at $\mu = M_Z$ contains sufficient internal structure to define a gravitational coupling without introducing new fields or modifying the tensor sector of General Relativity. Two independent ingredients—the primitive integer kernel of one-loop decoupling and the soft eigenmode of the Fisher/kinetic metric—select the same direction in log–coupling space. This alignment defines a unique depth coordinate $\Xi = \chi\cdot\hat{\Psi}$ and its electroweak anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$. Because $\Omega = e^{\Xi}$, the SM-derived coupling $G(M_Z) = (\hbar c/m_p^2)\,\Omega(M_Z)$ follows directly from the integer certificate without additional parameters.

An even curvature gate $\Pi(\Xi)$, with width fixed by matching its curvature to the Fisher curvature along the aligned axis, promotes this anchor to a running coupling $G(Q) = G(M_Z)\Pi(\Xi(Q))$ while preserving the equilibrium GR limit. At Ξ_{eq} the theory reduces to massless, luminal helicity ± 2 propagation with no scalar–tensor mixing and no Pauli–Fierz mass term.

The resulting predictions are quantitative and parameter free. The laboratory response obeys the strictly quadratic relation

$$\frac{\Delta G}{G} = (\delta\Xi/\sigma_\chi)^2,$$

with the linear term removed by even parity. Closure is captured by the ratio $Z_G = \alpha_G^{(pp)}/\Omega(M_Z)$, providing a direct comparison between the SM-anchored $G(M_Z)$ and the measured Newtonian coupling G_N. Leave-one-out consistency of the three gauge couplings supplies an additional internal test. Any violation of these conditions—odd laboratory response, closure mismatch, LOO inconsistency, or deviation from the GR tensor limit—falsifies the alignment mechanism.

The analysis here is restricted to the static, equilibrium geometry in which $\Pi(\Xi_{\mathrm{eq}}) = 1$ and the tensor kernel reduces to the Lichnerowicz operator of GR. This establishes the electroweak alignment structure that underlies the broader GEOMETRY program. A companion paper (GEOMETRY II) extends the tensor sector beyond equilibrium and demonstrates a finite spectral gap, while GEOMETRY III introduces dynamical alignment and analyzes the off-equilibrium evolution of the depth coordinate Ξ. These developments build systematically on the equilibrium geometry established here.

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Data availability. All reproducibility materials are archived in the Zenodo repository [19] (`GAGE_repo v1.0.0`, DOI 10.5281/zenodo.17537647), including pins, scripts, figure data, and build manifests. *Build artifacts (SHA–256):* `results.json`=08f0371b31de…c7cd5edc; `metric_results.json`=e0e3bee8a70c…b9b251b6451; `stdout.txt`=0f232a0be6f8…6c7cd5edc. Additional materials are available from the author upon reasonable request.

Outlook. Future work will examine how the even-gate symmetry extends to dynamical and spectral sectors, including the time-evolution operator and the curvature spectrum. If validated, the GEOMETRY program would provide a continuous link from Standard-Model information geometry to the equilibrium, dynamical, and spectral structure of gravitation.

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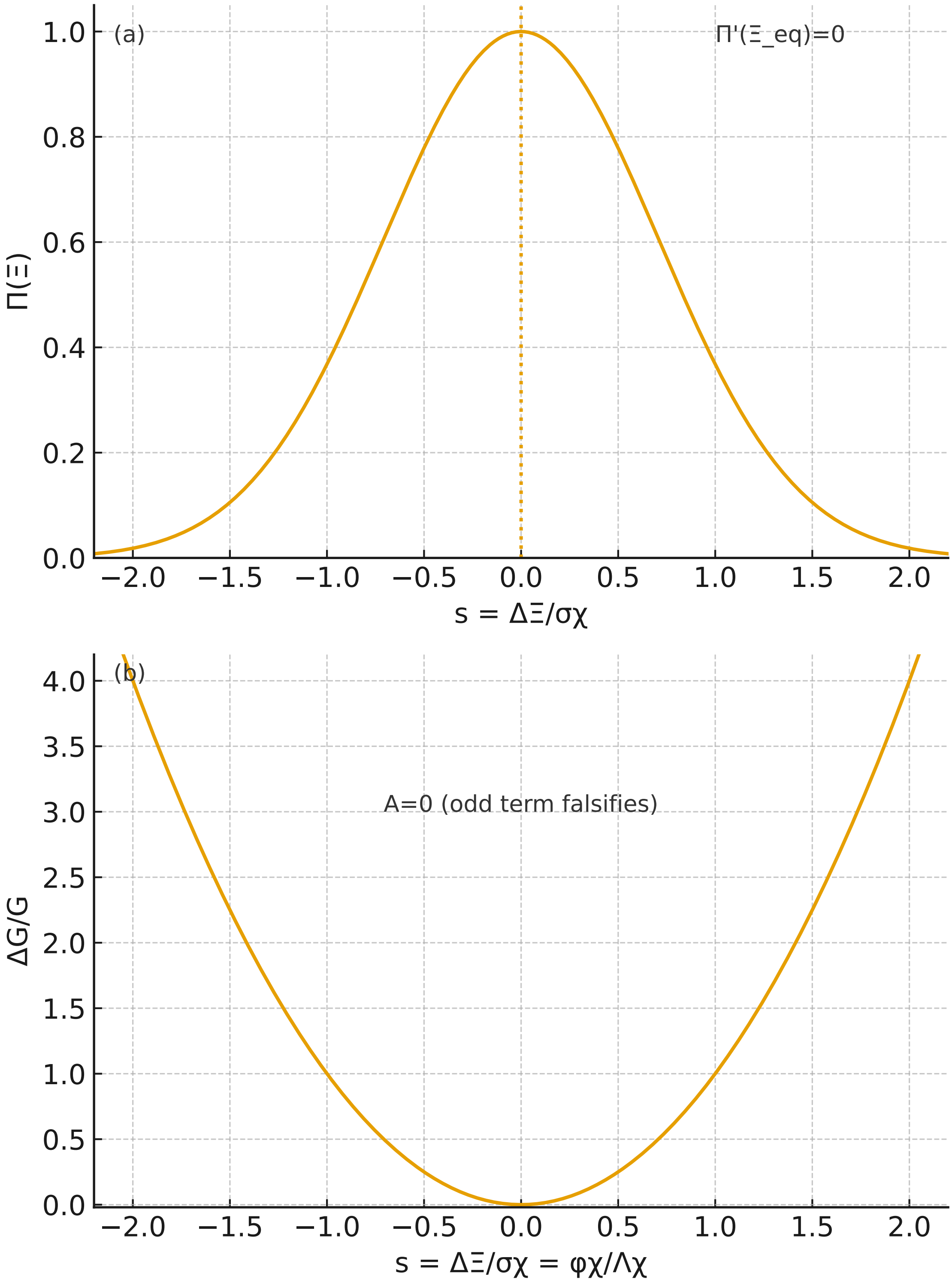

Figure 1. Even curvature gate $\Pi(\Xi)$ and the quadratic parity–null prediction. (a) Gaussian gate $\Pi = \exp[-(\delta\Xi)^2/\sigma_\chi^2]$ with $\Pi'(\Xi_{\rm eq}) = 0$. (b) Laboratory relation $\Delta G/G = (\delta\Xi/\sigma_\chi)^2 = (\phi_\chi/\Lambda_\chi)^2$, showing the vanishing of the linear term $(A = 0)$ as the empirical falsifier.