Gauge-Aligned Gravity Emergence (GAGE): SM-derived gravitational coupling $G(Q)$

Michael DeMasi, DNP[1]

[1] *Independent Researcher, Milford, CT 06460, USA*

(Dated: November 5, 2025)

Within the SM at $\mu = M_Z$ ($\overline{\text{MS}}$), a unique primitive projector $\boldsymbol{\chi} = (16, 13, 2)$ defines the gauge–log depth $\Xi = \boldsymbol{\chi}\cdot\hat{\boldsymbol{\Psi}}$. An even curvature gate $\Pi(\Xi) = \exp[-(\Delta\Xi)^2/\sigma_\chi^2]$ with $\Pi'(\Xi_{\text{eq}}) = 0$ yields a GR-normalized, massless tensor sector ($m_{\text{PF}} = 0$) and $G(x) = G(M_Z)\Pi(\Xi(x))$, where $G(M_Z) = \hat{\Omega}(M_Z)\hbar c/m_p^2$ and $\hat{\Omega} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$ (hats: $\overline{\text{MS}}$ at $\mu = M_Z$). Two direct tests: (i) quadratic lab-null $\Delta G/G \simeq (\Delta\Xi/\sigma_\chi)^2$; (ii) closure/LOO with $\hat{\Omega}(M_Z)/\alpha_G^{(\text{pp})} = 1.09373393$ and $\hat{\alpha}_s^\star(M_Z) = 0.1173411 \pm 1.86\times10^{-5}$. Inputs are SM-pinned; metrology is used only *a posteriori*.

Motivation. Despite its precision, the Standard Model has not yielded a self-contained derivation of Newton's constant G_N or a massless spin-2 graviton within its gauge structure. We propose an *alignment principle*: near the electroweak scale, the gauge system aligns with the soft eigenmode of the equilibrium kinetic metric, enforcing parity-even curvature at the lab point. At $\mu = M_Z$ in $\overline{\text{MS}}$, the SNF-certified projector $\boldsymbol{\chi} = (16, 13, 2)$ fixes the gauge–log depth $\Xi = \boldsymbol{\chi}\cdot\hat{\boldsymbol{\Psi}}$; an even gate $\Pi(\Xi)$ with $\Pi'(\Xi_{\text{eq}}) = 0$ yields a GR-normalized, massless spin-2 sector with no new fields or tunable parameters and defines the emergent coupling G. The framework is parameter-free and falsifiable: with $s_\Xi \equiv \Delta\Xi/\sigma_\chi$, $\phi_\chi \equiv \Delta\Xi/\,\|\boldsymbol{\chi}\|_{\mathbf{K}_{\text{eq}}}$, and $\Lambda_\chi \equiv \sigma_\chi/\,\|\boldsymbol{\chi}\|_{\mathbf{K}_{\text{eq}}}$, one finds $\Delta G/G = s_\Xi^2 = (\phi_\chi/\Lambda_\chi)^2$; closure is tested *a posteriori* against metrology via $\alpha_G^{(\text{pp})} = G_N m_p^2/(\hbar c)$.

Premise. Work in logarithmic coupling space $\hat{\boldsymbol{\Psi}} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha})$, where multiplicative renormalization becomes additive and basis transports are linear [1–3]. We seek a *single*, basis-invariant scalar depth in this space that governs the curvature response and defines the emergent coupling $G(x)$. Its existence is not assumed but must arise from SM structure and be protected by a symmetry enforcing an even equilibrium response (*i.e.*, a null first derivative).

Alignment principle (physical motivation). Let $\mathbf{K}_{\text{eq}} \succ 0$ denote the equilibrium kinetic metric on $\hat{\boldsymbol{\Psi}}$. Small variations organize along its *soft eigenmode*—the direction of minimal kinetic curvature. We find numerically, using SM pins [4, 5], that the gauge system aligns its response along this mode. Alignment has two immediate consequences: (i) **Parity protection.** Near equilibrium, the response is even, so the leading deviation is quadratic in the depth displacement. The resulting *parity-null* is directly testable—any observed linear term falsifies the mechanism. (ii) **Tensor-sector normalization.** With even parity at the laboratory point, the linearized tensor dynamics coincide with GR (luminal helicity-2, no Pauli–Fierz mass). The graviton sector thus *emerges* as the parity-even curvature response of the aligned gauge system [6, 7].

This alignment-first view provides the physical intuition before the algebra: the next sections identify the unique direction (certificate), define the depth Ξ, specify the even gate $\Pi(\Xi)$, derive the β-function for G, and establish closure and falsifiers.

In Fisher-metric terms, alignment corresponds to motion along the soft eigenvector of $\mathbf{K}_{\text{eq}}$—the direction of least informational curvature. Equivalently, systems minimize Fisher resistance by cohering along the soft mode, the information-geometry analog of least action.

Integer certificate and depth. The alignment principle requires a single scalar coordinate in coupling space that remains invariant under renormalization–basis changes. From the one-loop decoupling lattice of the SM, the Smith normal form (SNF) isolates a unique primitive left-kernel generator [8–10],

$$\boldsymbol{\chi} = (16, 13, 2),$$

certifying that only one integer combination of gauge couplings remains invariant under one-loop decoupling transformations (Appelquist–Carazzone regime). This integer projector defines the gauge–log depth

$$\Xi = \boldsymbol{\chi}\cdot\hat{\boldsymbol{\Psi}}, \qquad \hat{\boldsymbol{\Psi}} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha}),$$

with local fluctuation $\Delta\Xi = \Xi - \Xi_{\text{eq}}$. Log coordinates render multiplicative renormalization additive and basis transports linear [1, 2]: $\hat{\boldsymbol{\Psi}}' = A\hat{\boldsymbol{\Psi}}$, $\boldsymbol{\chi}' = A^{-\top}\boldsymbol{\chi}$, so $\Xi = \boldsymbol{\chi}\cdot\hat{\boldsymbol{\Psi}}$ is basis-invariant. Under metric transport $K' = A^{-\top}KA^{-1}$,

$$\|\boldsymbol{\chi}\|_K^2 = \boldsymbol{\chi}^\top K\,\boldsymbol{\chi} = \boldsymbol{\chi}'^\top K'\boldsymbol{\chi}',$$

so both $\|\boldsymbol{\chi}\|_K$ and the gate scale $\Lambda_\chi = \sigma_\chi/\|\boldsymbol{\chi}\|_K$ remain invariant under basis choice. The integer certificate, scalar depth, and their transport properties complete the algebraic foundation for the curvature gate introduced next.

SNF certificate (sketch). $\Delta W_{\text{EM}} = \begin{bmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{bmatrix}$ has integer rank 2, giving $\ker_{\mathbb{Z}}(\Delta W_{\text{EM}}) = \text{span}\{\pm\chi_{\text{EM}}\}$ with $\chi_{\text{EM}} = (-10, -18, 1)$ and $\Delta W_{\text{EM}}\chi_{\text{EM}} = 0$. Unimodular transport $M^\top\chi_{\text{EM}} = \boldsymbol{\chi} = (16, 13, 2)$ fixes the primitive generator, unique up to sign (full proof in SM Sec. S1).

Even gate and definition of $G(x)$. Having identified the invariant scalar depth $\Xi = \boldsymbol{\chi}\cdot\hat{\boldsymbol{\Psi}}$, we introduce the curvature-response function (the *gate*) that modulates the emergent gravitational coupling. The gate must satisfy: (i) even parity about equilibrium $[\Pi'(\Xi_{\rm eq}) = 0]$; (ii) analyticity and positivity for all Ξ; (iii) normalization $\Pi(\Xi_{\rm eq}) = 1$ (recovering GR at equilibrium); and (iv) minimal parameter freedom.

A minimal analytic, even, and positive form satisfying these is the Gaussian gate

$$\Pi(\Xi) = \exp\Big[-\frac{(\Delta\Xi)^2}{\sigma_\chi^2}\Big], \qquad \Pi'(\Xi_{\rm eq}) = 0, \quad \Pi(\Xi_{\rm eq}) = 1,$$

which enforces parity-even curvature modulation and smooth suppression beyond the Planck-thin envelope ($|\Delta\Xi| \sim \sigma_\chi$). With $s_\Xi = \Delta\Xi/\sigma_\chi$, $\phi_\chi = \Delta\Xi/\|\boldsymbol{\chi}\|_{\mathbf{K}_{\rm eq}}$, and $\Lambda_\chi = \sigma_\chi/\|\boldsymbol{\chi}\|_{\mathbf{K}_{\rm eq}}$, the laboratory null becomes $\Delta G/G = s_\Xi^2 = (\phi_\chi/\Lambda_\chi)^2$. The width σ_χ is fixed by Fisher curvature from SM covariance at $\mu = M_Z$ [4, 5, 11], leaving no tunable parameters.

Gate parity. If Π is even and C^2 near $\Xi_{\rm eq}$, then $\Pi'(\Xi_{\rm eq}) = 0$ and $\Pi(\Xi_{\rm eq}+\Delta\Xi) = 1-(\Delta\Xi)^2/\sigma_\chi^2+\mathcal{O}((\Delta\Xi)^4)$; any $\mathcal{O}((\Delta\Xi))$ term falsifies parity or alignment. Writing the laboratory template as

$$\frac{\Delta G}{G} = A\,s + B\,s^2 + \mathcal{O}(s^3), \qquad s \equiv \frac{\Delta\Xi}{\sigma_\chi},$$

an even gate predicts $A = 0$, $B = 1$. Equivalently, in the ϕ_χ normalization,

$$\frac{\Delta G}{G} = A'\phi_\chi + B'\phi_\chi^2 + \dots, \qquad A' = 0, \quad B' = 1/\Lambda_\chi^2.$$

Any reproducible odd term ($A \neq 0$ or $A' \neq 0$) falsifies alignment parity.

Definition (natural units). In natural units ($\hbar = c = 1$), the SM fixes the gravitational coupling via the dimensionless product

$$G \equiv \frac{\Omega}{m_p^2}, \qquad \Omega \equiv \alpha_s^{16}\alpha_2^{13}\alpha^2, \qquad G(x) = G\,\Pi(\Xi(x)). \tag{1}$$

For SI units, restore $\hbar c$:

$$G(M_Z) = \frac{\hat{\Omega}(M_Z)\,\hbar c}{m_p^2}, \qquad \hat{\Omega} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2. \tag{2}$$

$$\boxed{\Omega \;=\; \hat{\alpha}_s^{16}\,\hat{\alpha}_2^{13}\,\hat{\alpha}^2} \qquad \boxed{G \;=\; \frac{\Omega}{m_p^2}}$$

$$\boxed{\frac{G_*\,\Pi\big(\hat{\Xi}\big)}{G_*} \;=\; \Pi\big(\hat{\Xi}\big) \;=\; \exp\Big[-\frac{(\hat{\Xi}-\hat{\Xi}^{(\rm eq)})^2}{\sigma_\chi^2}\Big]}$$

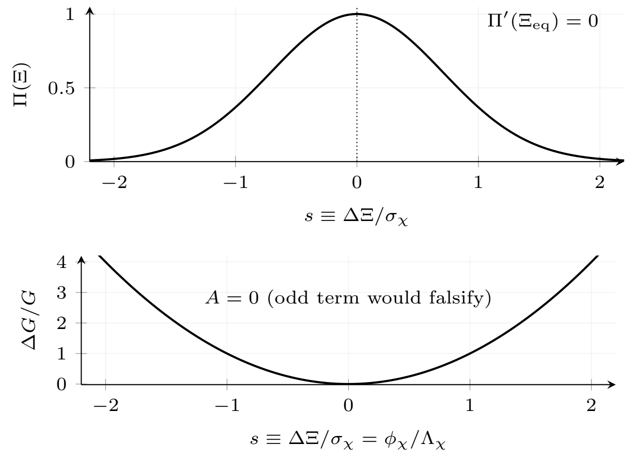

FIG. 1. Even gate $\Pi(\Xi) = \exp[-(\Delta\Xi)^2/\sigma_\chi^2]$ vs $s = \Delta\Xi/\sigma_\chi$ with parity condition $\Pi'(\Xi_{\rm eq}) = 0$, enforcing the quadratic response $\Delta G/G \simeq (\Delta\Xi)^2/\sigma_\chi^2 = \phi_\chi^2/\Lambda_\chi^2$. Any odd term As would falsify alignment.

Parity-even lab null and tensor sector. Even parity of the gate enforces a quadratic curvature response near equilibrium. With $\mathbf{K}_{\rm eq}\succ 0$ and alignment of $\hat{\chi} = \boldsymbol{\chi}/\,\|\boldsymbol{\chi}\|_{\mathbf{K}_{\rm eq}}$ to the soft eigenvector of $\mathbf{K}_{\rm eq}$,

$$\frac{\Delta G}{G} \simeq \frac{(\Delta\Xi)^2}{\sigma_\chi^2} = \frac{\phi_\chi^2}{\Lambda_\chi^2}, \qquad \phi_\chi = \frac{\boldsymbol{\chi}^\top(\hat{\boldsymbol{\Psi}} - \langle\hat{\boldsymbol{\Psi}}\rangle)}{\|\boldsymbol{\chi}\|_{\mathbf{K}_{\rm eq}}}, \tag{3}$$

$$\Lambda_\chi = \frac{\sigma_\chi}{\|\boldsymbol{\chi}\|_{\mathbf{K}_{\rm eq}}}. \tag{4}$$

This constitutes the *quadratic lab null*: any linear term in $\Delta G/G$ falsifies parity or alignment.

No h–ϕ mixing at linear order. $S \supset \frac{M_{\rm P}^2}{2}\Pi(\Xi)R$. Since $\Pi'(\Xi_{\rm eq}) = 0$, the variation $\delta\Omega = M_{\rm P}^2\Pi'(\Xi_{\rm eq})\,\delta\Xi$ vanishes, removing the $(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\delta\Omega$ term at $\mathcal{O}(h)$. The remaining equation $\mathcal{E}_{\mu\nu}^{\alpha\beta}h_{\alpha\beta} = 0$ is the Lichnerowicz operator with $m_{\rm PF} = 0$.

Because $\Pi'(\Xi_{\rm eq}) = 0$, the linearized equation reduces to

$$\Delta_L h_{\mu\nu} \equiv -\Box h_{\mu\nu} = 0, \tag{5}$$

so the tensor mode is GR-normalized, massless, and luminal (helicity-2) [6, 7, 12, 13].

Pinned scales. With $K_{\rm eq}\succ 0$, the gate scale is

$$\Lambda_\chi = \frac{\sigma_\chi}{\|\chi\|_{K_{\rm eq}}}. \tag{6}$$

From SM pins at $\mu = M_Z$ one finds $\sigma_\chi = 247.683$ and $\|\chi\|_{K_{\rm eq}} = 17.6278$, giving $\Lambda_\chi = 14.0507$ (derivation in the Supplement).

TABLE I. Equilibrium kinetic metric $K_{\rm eq}$: eigenvalues and alignment diagnostics.

Eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$	$\|\chi\|_{K_{\rm eq}}$	$\cos\theta_K$	ε_χ
(0.7243, 2.0156, 3.2599)	17.6278	0.9999999998	$< 10^{-8}$

Running of G and Ward-flatness. Because G is composed of SM couplings, its renormalization follows theirs. Differentiating $\Xi = \boldsymbol{\chi}\cdot\hat{\boldsymbol{\Psi}}$ gives

$$\beta_\Xi \equiv \frac{d\Xi}{d\ln Q} = 16\frac{\beta_{\alpha_s}}{\alpha_s} + 13\frac{\beta_{\alpha_2}}{\alpha_2} + 2\frac{\beta_\alpha}{\alpha} = 0$$

(1 loop; Ward-flat).

Thus Ξ and G are stationary at one loop, avoiding basis artifacts [1, 2]. Preregistered flatness windows and masks are evaluated in the SM; Fig. 2 shows the monitor $F(Q)$ within bounds.

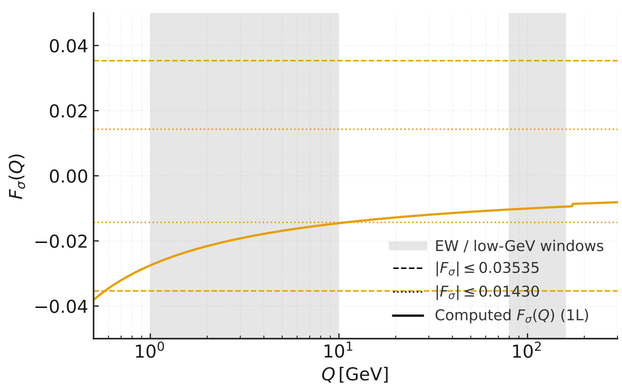

FIG. 2. Projected flatness monitor $F(Q)$ within preregistered electroweak and low-GeV windows. One-loop flatness corresponds to $\beta_\Xi = 0$; mild threshold kinks appear outside the EW window.

Beyond equilibrium, slow variation of $K_{\rm eq}$ produces adiabatic tracking of the soft mode (dynamic alignment) and a small two-loop drift (quantified in the Supplement).

The running of G follows directly:

$$\beta_G \equiv \frac{d\ln G}{d\ln Q} = 16\frac{\beta_{\alpha_s}}{\alpha_s} + 13\frac{\beta_{\alpha_2}}{\alpha_2} + 2\frac{\beta_\alpha}{\alpha} = 0 + O(\hat{\alpha}_i^2), \quad (7)$$

so G is flat at one loop with bounded higher-order drift [14]. Physically,

$$G(x) = G\,\Pi(\Xi(x)), \qquad G = \frac{\Omega}{m_p^2}. \quad (8)$$

At $\mu = M_Z$ ($\overline{\rm MS}$), comparison is made *a posteriori* to the metrological coupling $\alpha_G^{(\rm pp)}$ [15, 16].

Closure and prediction. The *closure test* links the SM invariant to metrology:

$$\hat{\Omega}(M_Z) = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2 \overset{?}{=} \alpha_G^{(\rm pp)} \equiv \frac{G_N m_p^2}{\hbar c}. \quad (9)$$

If equality holds within uncertainties,

$$G(\Xi_{\rm eq}) = \frac{\hat{\Omega}(M_Z)\hbar c}{m_p^2} = G(M_Z). \quad (10)$$

Inverting gives the leave-one-out (LOO) forecast,

$$\hat{\alpha}_s^\star(M_Z) = \left[\frac{\alpha_G^{(\rm pp)}}{\hat{\alpha}_2^{13}\hat{\alpha}^2}\right]^{1/16} = 0.1173411 \pm 1.86\times10^{-5}, \quad (11)$$

consistent with the PDG mean (-0.73σ) [4, 5]. The closure ratio

$$\frac{\hat{\Omega}(M_Z)}{\alpha_G^{(\rm pp)}} = 1.09373393 \quad (+9.37\%) \quad (12)$$

serves as the empirical benchmark [4, 11, 15, 16].

Matching (UV→IR). Define the IR normalization via a dimensionless matching factor Z_G:

$$G(M_Z) = \frac{\hat{\Omega}(M_Z)\,\hbar c}{m_p^2}, \qquad G_N \equiv Z_G\,G(M_Z), \quad (13)$$

$$Z_G = \frac{\alpha_G^{(\rm pp)}}{\hat{\Omega}(M_Z)} = 0.91430\ (-8.57\%). \quad (14)$$

This Z_G absorbs scheme, threshold, and higher-order effects; the full budget is detailed in the Supplement.

Helicity scale. At equilibrium, soft-mode alignment fixes

$$\Lambda_\chi = \frac{\sigma_\chi}{\|\chi\|_{K_{\rm eq}}} = 14.0507\ (\approx 14.05), \quad (15)$$

$$\omega_{\rm hel} = \Lambda_\chi^{-1}, \quad (16)$$

$$T_{\rm hel} = 2\pi\,\Lambda_\chi \simeq 88\,t_P. \quad (17)$$

This *Planck-thin curvature envelope* sets the coherence length of the emergent tensor mode and bounds the quadratic response $\Delta G/G \simeq \phi_\chi^2/\Lambda_\chi^2$ (see SM Sec. S8).

Falsifiers (any suffices). The construction is parameter-free; any single failure falsifies it:

1. **Non-unique integer certificate.** SNF must yield a unique primitive left-kernel generator $\boldsymbol{\chi} = (16, 13, 2)$; any alternate integer solution of comparable norm breaks the projection symmetry [8–10].

2. **Odd (linear) curvature response.** With $\Pi'(\Xi_{\rm eq}) = 0$, a measured linear term As in $\Delta G/G = As + Bs^2 + \ldots$ (where $s = \Delta\Xi/\sigma_\chi$) contradicts the quadratic lab-null in Eq. (4). *Example (illustrative):* taking $s = 9$ from SM pins gives $\Delta G/G \simeq s^2 = (\phi_\chi/\Lambda_\chi)^2 = 1.32\times10^{-3}$, accessible to symmetric $\pm s$ clock/torsion tests; fits must be consistent with $A = 0$.

3. **Tensor-sector anomaly.** Any Pauli–Fierz mass or non-luminal dispersion violates GR-normalized propagation (Eq. (5)) [7, 12, 13].

4. **Metric instability or misalignment.** $K_{\rm eq} \succ 0$ with $\cos\theta_K \simeq 1$ must hold; a negative eigenvalue or deviation beyond ε_χ signals ghost/locking failure (Table 1).

5. **Ward-flatness violation.** The projected flow must satisfy $\beta_\Xi = 0$ within preregistered EW and low-GeV masks (Eq. ()); significant drift implies RG-scheme dependence or breakdown of aligned projection [1, 2].

6. **Closure or LOO failure.** Deviation of $\hat{\Omega}/\alpha_G^{(\rm pp)}$ from unity beyond pinned uncertainties (Eq. (12)), or $\hat{\alpha}_s^\star(M_Z)$ outside PDG bounds (Eq. (11)), falsifies identification of G as SM-derived [4, 5, 15, 16].

Implications. Gravity emerges as the parity-even curvature response of the Standard Model gauge sector, with

$$G(x) = G(M_Z)\,\Pi(\Xi(x))$$

fixed entirely by $\{\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha}\}$ at $\mu = M_Z$ [4–6, 11]. Two direct signatures make the framework falsifiable: (i) the quadratic lab-null $\Delta G/G \simeq (\Delta\Xi/\sigma_\chi)^2$, and (ii) the closure ratio $\hat{\Omega}/\alpha_G^{(\rm pp)} = 1.09373393$. Any reproducible odd-parity term or closure mismatch beyond pinned uncertainties would refute the mechanism [7, 12, 13].

The spurion-even gate and χ-projection imply a conserved alignment current,

$$\begin{aligned} J^\mu_\chi &= \Pi(\Xi)\,\chi^\top K_{\rm eq}\,\partial^\mu\hat{\Psi}, \\ \partial_\mu J^\mu_\chi &= 0 + \mathcal{O}\big((\Delta\Xi)^3, 2\text{L drift}, \varepsilon_\chi\big), \end{aligned} \tag{18}$$

the Noether current associated with rigid depth shifts of Ξ at equilibrium. This conservation law is the gauge-depth analog of energy–momentum conservation under spacetime translations: alignment preserves curvature "energy" within the gauge sector. Operationally, any reproducible odd (linear) response [$A \neq 0$ in $\Delta G/G = As + Bs^2 + \cdots$] corresponds to $\partial_\mu J^\mu_\chi \neq 0$ and thus falsifies alignment.

Scope note. This Letter derives the gravitational coupling G from SM couplings and demonstrates empirical closure. Dynamic tensor-sector extensions—including the helicity frequency, Planck-thin envelope, drift law, and the conserved alignment current—are detailed in the Supplemental Material. All analytic derivations and pinned numerical checks are reproducible from the accompanying `GAGE_repo`. A companion study will extend the alignment–conservation framework to a fully dynamical formulation, developing the stress-energy, effective Lagrangian, geometric-fiber, informational, and temporal extensions that complete the GAGE description of gravity.

This work was conducted independently with no external funding. The author thanks the Particle Data Group (PDG), CODATA, Overleaf, and Python for open-access tools and data. An AI-assisted writing tool (OpenAI ChatGPT) was used for language editing and organizational assistance; all scientific content and responsibility rest with the author. The author declares no competing interests.

Data availability All pins, scripts, and figure data are archived at Zenodo (`GAGE_repo v1.0.0`, DOI 10.5281/zenodo.17537647). The Letter and Supplemental Material contain all equations, definitions, and figure descriptions. Additional materials are available from the author upon reasonable request.

[1] S. Weinberg, *The Quantum Theory of Fields, Vol. 2: Modern Applications* (Cambridge University Press, Cambridge, UK, 1996).
[2] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, MA, 1995).
[3] P. Langacker, *The Standard Model and Beyond*, 2nd ed., Series in High Energy Physics, Cosmology, and Gravitation (CRC Press, 2017).
[4] S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024), and 2025 update.
[5] T. Dorigo and M. Tanabashi, in *Review of Particle Physics*, edited by Particle Data Group (Oxford University Press, 2025) p. 083C01, published in Prog. Theor. Exp. Phys. 2025 (8), 083C01.
[6] S. M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity* (Addison-Wesley, 2004).
[7] C. M. Will, Living Rev. Relativ. **17**, 4 (2014).
[8] T. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975).
[9] R. Kannan and A. Bachem, SIAM J. Comput. **8**, 499 (1979).
[10] M. Newman, Linear Algebra Appl. **254**, 367 (1997).
[11] J. Erler *et al.*, in *Review of Particle Physics* (Particle Data Group, 2024).
[12] B. Bertotti, L. Iess, and P. Tortora, Nature **425**, 374 (2003).
[13] R. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration and KAGRA Collaboration), Phys. Rev. D (2021), combined bound $m_g \leq 1.27 \times 10^{-23}\,\mathrm{eV}/c^2$ (90% C.L.), arXiv:2112.06861 [gr-qc].
[14] F. Jegerlehner, Nucl. Part. Phys. Proc. **303–305**, 1 (2018), see also arXiv:1705.00263.
[15] P. J. Mohr, D. B. Newell, B. N. Taylor, and E. Tiesinga, Rev. Mod. Phys. **97**, 025002 (2025).
[16] D. Robinson and P. A. Zyla, in *Review of Particle Physics* (Particle Data Group, 2024) topical review, revised 2024.