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PAPER

GEOMETRY I: A parameter-free prediction for the strong coupling $\alpha_s(M_Z)$ from Standard Model gauge geometry and gravitational closure

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Abstract

Within the internal constraints of the Standard Model (SM)—no new fields, no tunable functions, and fixed renormalization at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme—we show that the gravitational normalization can be obtained from SM data alone. The aligned depth $\Xi = \chi\cdot\hat{\Psi}$ is an internal gauge–log coordinate, not a propagating field, and $G(M_Z)$ denotes an electroweak-anchored normalization rather than a varying-G framework.

At one loop, the SM decoupling matrix possesses a unique primitive integer left-kernel $\chi = (16, 13, 2)$ in Smith normal form. Independently, the Fisher/kinetic metric selects a soft eigenmode of maximal responsiveness. Their alignment identifies Ξ as the unique admissible depth coordinate. Exponentiation then yields the internal anchor

$$\Omega = e^{\Xi} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2.$$

Dimensional consistency with baryonic matter gives

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z),$$

a normalization fixed entirely by SM input. Using pinned electroweak values and the measured proton–proton gravitational coupling, we obtain a closure ratio $Z_G \simeq 1.0937$ and a leave-one-out prediction $\hat{\alpha}_s^{\star}(M_Z) \approx 0.11734$, consistent with modern lattice and PDF analyses but achieved here without hadronic tuning.

An even, Gaussian curvature gate $\Pi(\Xi)$ with Fisher-fixed width σ_χ promotes $G(M_Z)$ to $G(x) = G(M_Z)\Pi(\Xi(x))$ while preserving the massless GR tensor sector and enforcing the near-equilibrium lab-null $\Delta G/G = (\delta\Xi/\sigma_\chi)^2$.

All numerical values and figures are generated through a public, hash-verified workflow. The construction remains entirely equilibrium-based; Ξ carries no dynamics in this paper.

Keywords: general relativity, Standard Model, gauge theory, gravity, information geometry, renormalization group

1 Introduction

The Standard Model (SM) provides a precise, renormalizable account of the three gauge interactions and their running couplings, yet it contains no internal mechanism that fixes Newton's gravitational constant G_N. In General Relativity (GR) the Einstein–Hilbert term

$$\mathcal{L}_{\mathrm{EH}} = \frac{1}{16\pi G_N}\,R \tag{1}$$

carries an empirically measured normalization: GR specifies how curvature responds to stress–energy but does not determine the numerical value of that response. This motivates the central question:

> *Does the SM gauge sector at $\mu = M_Z$ contain sufficient, basis-invariant structure to fix a gravitational normalization without new degrees of freedom or modification of GR?*

We restrict throughout to SM inputs at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, assume no new fields, parameters, or tunable functions, and retain the massless, luminal helicity-± 2 tensor sector of GR.

The goal is to determine whether the SM's internal geometry—its one-loop integer structure and Fisher/kinetic metric—provides a parameter-free gravitational normalization $G(M_Z)$ consistent with observation. All numerical values and figures are obtained directly from public SM inputs using a reproducible, hash-verified build workflow archived under a public DOI (see Data Availability).

Physical status and scope. Throughout this work, $\hat{\Psi}$ and $\Xi = \chi\cdot\hat{\Psi}$ are internal gauge–log coordinates rather than propagating spacetime fields. The construction in GEOMETRY I is static and equilibrium–restricted: the gravitational normalization is fixed at $\delta\Xi = 0$, where the curvature gate satisfies $\Pi(\Xi_{\rm eq}) = 1$ and the Einstein–Hilbert sector is unchanged. No assumption is made regarding any time evolution of Ξ; dynamical alignment and drift-law extensions appear only in later papers of the GEOMETRY series. Predictions for the observable ratio $\Delta G/G$ use only small departures $|\delta\Xi| \ll \sigma_\chi$ as a near-equilibrium expansion parameter.

A key distinction in what follows is that $G(M_Z)$ denotes an electroweak-anchored normalization obtained from SM data, not a modification of GR or a varying-G framework. The normalization is defined at equilibrium and later encoded in the shorthand

$$G(x) = G(M_Z)\,\Pi(\Xi(x))$$

as an *internal curvature weighting* inside the Einstein–Hilbert term. In this paper $G(x)$ is a bookkeeping notation for how the SM-aligned curvature weight depends on Ξ; it is not a dynamical, spacetime-varying Newton constant, and the GR tensor sector remains intact.

Two rigid SM structures, normally analyzed separately, play the key role: (i) the integer lattice arising from one-loop decoupling, and (ii) the Fisher/kinetic metric on log–coupling space. Evaluated together at $\mu = M_Z$, they identify a single aligned depth direction and thereby fix a dimensionless electroweak anchor, allowing a gravitational normalization to follow as a consequence rather than as an externally imposed parameter.

At one loop, the SM decoupling matrix is an exact integer matrix whose Smith normal form (SNF) admits a unique primitive left-kernel generator (up to overall sign), obtained using a standard integer-normal-form algorithm implemented verbatim in the accompanying reproducibility scripts:

$$\chi = (16, 13, 2). \tag{2}$$

This vector defines a depth coordinate

$$\hat{\Psi} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha}), \tag{3}$$

$$\Xi = \chi\cdot\hat{\Psi} \tag{4}$$

in log–coupling space. Independently, the positive-definite (equilibrium) Fisher/kinetic metric K (hereafter $K \equiv K_{\rm eq}$), constructed from one-loop sensitivity data, possesses a soft eigenmode of maximal responsiveness. Using the same SM input pins and renormalization scheme, we find that the integer direction χ is numerically aligned with the softest eigenvector of K, with $\cos\theta \simeq 1$. Thus the aligned depth Ξ is not a model assumption but the coordinate jointly selected by integer rigidity and metric softness.

Exponentiating the aligned depth yields the electroweak anchor

$$\Omega \equiv e^{\Xi} = e^{\chi\cdot\hat{\Psi}} = \hat{\alpha}_s^{16}\,\hat{\alpha}_2^{13}\,\hat{\alpha}^2. \tag{5}$$

so that Ω is an internal, dimensionless SM anchor whose value is fixed entirely by electroweak-scale couplings. No additional dynamical field is introduced, and no tuning is applied; Ω is simply the exponential of the aligned gauge–log depth.

This electroweak anchor defines an SM-derived gravitational normalization

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z), \tag{6}$$

with no adjustable parameters. The proton mass m_p is chosen as the dimensional anchor because laboratory and astrophysical determinations of G_N predominantly probe baryonic (proton-dominated) matter; alternative choices such as m_e or m_n simply rescale the same dimensionless anchor Ω. Using the pinned electroweak inputs and the experimental proton–proton gravitational coupling, this relation yields a preferred strong coupling

$$\alpha_s^\star(M_Z) \approx 0.11734, \tag{7}$$

which agrees with high-precision lattice and global-fit determinations while arising here entirely from internal SM structure and gravitational closure, without hadronic fitting.

In this work we show that: (i) gauge-log alignment in the Standard Model defines a parameter-free relation that fixes a preferred value of $\alpha_s(M_Z)$ from consistency with the proton–proton gravitational coupling; (ii) the same relation defines an SM-derived gravitational normalization $G(M_Z)$ anchored at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme; and (iii) an even, Gaussian curvature gate $\Pi(\Xi)$ defines a near-equilibrium curvature weight $\Pi(\Xi(x))$ which we notationally package as $G(x) = G(M_Z)\Pi(\Xi(x))$. In the static, equilibrium framework of GEOMETRY I this $G(x)$ is an internal weighting of the Einstein–Hilbert normalization rather than a varying-G theory, and the massless, luminal tensor sector of GR is preserved.

Internal displacement. In GEOMETRY I, $\delta\Xi$ denotes an internal displacement in gauge–log space rather than a propagating scalar field. It represents the net response of the SM gauge–log vector $\hat{\Psi} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha})$ along the aligned integer direction $\chi = (16, 13, 2)$, with

$$\delta\Xi = \chi \cdot (\hat{\Psi} - \hat{\Psi}_{\mathrm{eq}}). \tag{8}$$

No equation of motion for $\delta\Xi$ is introduced in this work. Matter and stress–energy can in principle perturb the gauge–log coordinates and source nonzero $\delta\Xi$, but the functional form of this sourcing belongs to the dynamical extensions developed in GEOMETRY III. In the present static framework, all observable results are evaluated at equilibrium ($\delta\Xi = 0$), where $\Pi(\Xi_{\mathrm{eq}}) = 1$ and the Einstein–Hilbert tensor sector is unchanged.

Summary of results and roadmap. Section 2 establishes the integer structure and the unique primitive kernel χ of the one-loop decoupling matrix. Section 3 constructs the Fisher/kinetic metric and verifies its numerical alignment with χ, fixing the admissible depth coordinate Ξ. Section 4 introduces the parity-preserving curvature gate $\Pi(\Xi)$ and determines its fixed curvature scale σ_χ. Section 5 derives the electroweak-anchored normalization $G(M_Z)$ and encodes its near-equilibrium response as $G(x) = G(M_Z)\Pi(\Xi(x))$ without modifying GR, with $G(x)$ serving as an internal curvature weight rather than a varying Newton constant. Section 6 presents the quadratic lab-null, closure ratio, and falsifiers. Section 7 summarizes the implications and outlines extensions to dynamical settings in future work within the broader GEOMETRY sequence.

Program and provenance. This work is the first in a sequence collectively denoted GEOMETRY (Gauge Exponential Omega Metric Even Tensor Running Yield). GEOMETRY I is restricted to the static, equilibrium geometry and derives an electroweak-anchored gravitational normalization from SM data alone. All inputs, constants, and covariance matrices are taken from established references, and all calculations use the $\overline{\mathrm{MS}}$ scheme at $\mu = M_Z$. Integer and metric verifications are reproduced automatically from the archived build environment.

Subsequent papers in the GEOMETRY sequence extend this equilibrium framework in distinct directions: the tensor-sector dynamics, mass gap, and helicity frequency (GEOMETRY II); the time-dependent alignment, drift-law evolution, and spectral structure (GEOMETRY III); the alignment-regularized parabolic flow connecting to Navier–Stokes (GEOMETRY IV); and the Π-weighted Hodge and cohomological structures (GEOMETRY V). None of these dynamical elements are assumed or required in GEOMETRY I.

Renormalization conventions follow Weinberg [1], Peskin and Schroeder [2], and Langacker [3]. Decoupling and integer-lattice methods follow Appelquist and Carazzone [4], Kannan and Bachem [5], and Newman [6]. Electroweak pins, covariance matrices, and physical constants are taken from PDG and CODATA [7, 8, 9, 10]. Two-loop RG coefficients follow Machacek and Vaughn [11, 12] and Luo *et al.* [13], and the running of $\hat{\alpha}$ follows Jegerlehner [14]. Gravitational and observational constraints follow Carroll [15], Will [16], Bertotti *et al.* [17], and Abbott *et al.* (LVK) [18]. No additional data, fitting, or tuning is employed.

Table 1: Assumptions and first-principles spine of GEOMETRY I. Entries A1–A11 specify the static, equilibrium, SM-internal framework. Entries FP1–FP10 summarize the rigid integer, metric, parity, and anchor structures used in GEOMETRY I, without invoking dynamical or spectral extensions.

ID	**Category**	**Assumption** (GEOMETRY I **scope**)
A1	Framework	Static electroweak-scale equilibrium geometry at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme with GUT-normalized hypercharge. Only SM data and RG structure at this scale are used.
A2	Fields / DoF	$\hat{\Psi} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha})$ and $\Xi = \chi \cdot \hat{\Psi}$ are internal gauge–log coordinates only. GEOMETRY I introduces no new fields, potentials, or kinetic terms and does not promote Ξ to a dynamical scalar.
A3	Equilibrium	GEOMETRY I is static and equilibrium-restricted. The gravitational normalization is defined at $\delta\Xi = 0$, where $\Pi(\Xi_{\mathrm{eq}}) = 1$ and the Einstein–Hilbert sector coincides with GR. No time evolution or sourcing equation for Ξ is assumed.
A4	Metric	The Fisher/kinetic metric $K \equiv K_{\mathrm{eq}}$ is defined locally at $\mu = M_Z$ from one-loop SM sensitivity data. It is positive definite ($K \succ 0$) and used only as a local quadratic form on log–coupling space.
A5	Integer lattice	The decoupling matrix ΔW is an exact integer matrix. Its Smith normal form yields a unique primitive left kernel $\chi = (16, 13, 2)$, invariant under unimodular integer transports. All χ-dependent quantities are treated as basis invariant.
A6	Gate shape	The curvature gate $\Pi(\Xi)$ is analytic and even about equilibrium, with $\Pi(\Xi_{\mathrm{eq}}) = 1$ and $\Pi'(\Xi_{\mathrm{eq}}) = 0$. Fisher matching fixes its curvature: $-\frac{1}{2}\Pi''(\Xi_{\mathrm{eq}}) = F_\chi$, giving $\sigma_\chi = F_\chi^{-1/2}$ as a derived quantity.
A7	Tensor sector	At $\delta\Xi = 0$ the tensor kernel reduces to the GR Lichnerowicz operator with $m_{\mathrm{PF}} = 0$ and luminal propagation. Even parity forbids any Brans–Dicke–like linear mixing between $\delta\Xi$ and $h_{\mu\nu}$. No extra scalar or vector propagating modes.
A8	Dimensional anchor	$G(M_Z) = (\hbar c/m_*^2)\Omega(M_Z)$ with $m_* = m_p$ as the physical anchor. The dimensionless anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$ is SM-internal and independent of m_*.
A9	Closure / data	PDG/CODATA electroweak pins and two-loop running are taken as inputs. The closure ratio $Z_G = \alpha_G^{(\mathrm{pp})}/\Omega(M_Z)$ is used only as an a posteriori diagnostic; G_N never enters the construction of Ω, $\Pi(\Xi)$, or σ_χ.
A10	Perturbative stability	The integer kernel, alignment angle ($\cos\theta_K \simeq 1$), and Fisher curvature F_χ are stable under permissible one-loop scheme and threshold variations and are treated as robust SM features at $\mu = M_Z$.
A11	Quasi-static regime	Laboratory and weak-field environments satisfy $\lvert\delta\Xi\rvert \ll \sigma_\chi$. Matter sources only small quasi-static displacements $\delta\hat{\Psi}$, giving $\delta\Xi = \chi \cdot \delta\hat{\Psi}$. The prediction $\Delta G/G = (\delta\Xi/\sigma_\chi)^2$ is strictly a near-equilibrium statement.
First-Principles Spine (FP1–FP10) – GEOMETRY I **compatibility**		

Continued on next page

Table 1 (continued)

ID	Category	Assumption (GEOMETRY I scope)
FP1	SM scope	No new fields, functions, or free parameters. Construction is SM-internal at $\mu = M_Z$ and evaluated at $\delta\Xi = 0$ where $\Pi = 1$.
FP2	Integer constraint	$\chi = (16, 13, 2)$ is the unique primitive SNF kernel and the only admissible depth direction. No alternative scalar argument or deformation is allowed.
FP3	Metric softness	$K_{\rm eq}$ selects the same soft mode as χ. The Fisher curvature F_χ and width σ_χ are derived SM outputs, not tunable parameters.
FP4	Depth coordinate	$\Xi = \chi \cdot \hat{\Psi}$ and $\delta\Xi$ are internal coordinates only and are used algebraically inside $\Pi(\Xi)$, never as dynamical fields.
FP5	Parity gate	$\Pi(\Xi)$ is even and analytic with $\Pi = 1$ and $\Pi' = 0$ at equilibrium. Fisher matching fixes the quadratic curvature coefficient.
FP6	Dimensionless anchor	$\Omega = e^{\Xi} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$ is a purely SM-derived quantity and cannot be tuned.
FP7	Dimensional lift	$G(M_Z) = (\hbar c/m_*^2)\Omega(M_Z)$ with fixed m_*. No new scale or degree of freedom is introduced.
FP8	A posteriori closure	Z_G is an output-only diagnostic. G_N does not enter the construction of Ω, Π, or σ_χ.
FP9	Equilibrium alignment structure	The aligned contraction $\chi^{\mathsf{T}} K\, \partial^\mu \hat{\Psi}$ is an equilibrium structure only. Time-dependent conservation laws appear only in later work.
FP10	Parity geometry	Even $\Pi(\Xi)$ defines the curvature weight used in GEOMETRY I. At $\delta\Xi = 0$ this fixes all admissible near-equilibrium responses.

Internal coordinates and degrees of freedom. Throughout this work the variables $\hat{\Psi} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha})$ and $\Xi = \chi\cdot\hat{\Psi}$ are treated as internal gauge–log coordinates, not as new propagating scalar fields. They summarize how the renormalized Standard Model couplings are organized along the integer direction $\chi = (16, 13, 2)$ in the $\overline{\rm MS}$ scheme at $\mu = M_Z$. The fundamental dynamical content of the theory remains that of the Standard Model plus General Relativity; no additional canonical coordinates or conjugate momenta are introduced. In particular, Ξ has no kinetic term and no independent initial data in GEOMETRY I; it is a composite depth coordinate defined on the gauge–log manifold.

Static displacements and sourcing. In the equilibrium setting considered here we allow for small, static displacements $\delta\Xi(x) \equiv \Xi(x) - \Xi_{\rm eq}$ of the internal depth coordinate, interpreted as the net aligned response of the gauge–log manifold to ordinary matter and energy. The detailed microscopic mapping from local stress–energy $T_{\mu\nu}(x)$ and Standard Model sources to $\delta\hat{\Psi}(x)$, and hence to $\delta\Xi(x) = \chi\cdot\delta\hat{\Psi}(x)$, is *not* specified in GEOMETRY I. Instead, this work analyzes the consequences of such small displacements for the curvature response and for the SM-derived gravitational normalization $G(x)$. A dynamical description of how $\delta\Xi(x)$ is generated and relaxes, including an explicit sourcing relation, is developed in the time-dependent extension (GEOMETRY III).

2 Integer lattice and the aligned depth coordinate

We begin by identifying the structures that remain fixed once we restrict to the Standard Model at $\mu = M_Z$ in the $\overline{\rm MS}$ scheme with no new fields, tunable functions, or adjustable parameters. At this scale the one-loop decoupling matrix has exactly integer entries determined solely by representation content and spectator multiplicities. This endows log–coupling space with a natural $\mathbb{Z}$-module structure and admits a classification under $\mathrm{GL}(3, \mathbb{Z})$ via the Smith normal form (SNF). Because the SNF is a unique canonical form over the integers, its kernel is a fixed property of the SM representation lattice rather than a model choice. The SNF is obtained using a standard integer-normal-form algorithm (e.g. Kannan–Bachem), as implemented in SYMPY and reproduced verbatim in the public reproducibility repository. Since the SNF kernel is invariant under all unimodular row and column operations, this structure is basis-independent and scheme-consistent at one loop.

Fixed structure. The integer kernel identified here is fully determined once the SM representation content is specified. No phenomenological inputs, threshold choices, or adjustable coefficients can alter the kernel or introduce additional integer directions. In this sense the integer lattice provides a representation-level invariant of the SM, and the existence of a unique primitive kernel vector is a structural property rather than a modeling decision.

Applying SNF to the SM one-loop decoupling matrix yields a unique primitive left-kernel generator (up to overall sign),

$$\chi = (16, 13, 2),$$

which is the sole integer direction annihilating the decoupling matrix. No additional integer kernel vectors exist, and unimodular transformations (integer determinant ± 1) cannot change the kernel rank or its primitive representative. Thus χ is fixed by the SM's integer structure and does not depend on renormalization schemes, higher-loop corrections, numerical fitting, or phenomenological input.

This uniqueness is crucial for the construction that follows: it implies that any $\mathbb{Z}$-invariant combination of the three log–couplings must be a function of $\Xi = \chi \cdot \hat{\Psi}$ alone. No second independent integer constraint exists, and therefore no alternative depth coordinate compatible with the SM integer structure can be defined.

Let the renormalized gauge couplings at $\mu = M_Z$ be $\hat{\alpha}_s$, $\hat{\alpha}_2$, and $\hat{\alpha}$, and define log–coupling coordinates

$$\hat{\Psi} = (\ln \hat{\alpha}_s,\ \ln \hat{\alpha}_2,\ \ln \hat{\alpha}),$$

together with the associated depth coordinate

$$\Xi = \chi \cdot \hat{\Psi} = 16 \ln \hat{\alpha}_s + 13 \ln \hat{\alpha}_2 + 2 \ln \hat{\alpha}.$$

The coordinate vector $\hat{\Psi}$ is purely internal and is not associated with any spacetime degree of freedom. Its role is to provide a basis-independent description of how the gauge sector occupies the three-dimensional log–coupling manifold at $\mu = M_Z$. All physical statements in this section refer exclusively to this internal geometry; no dynamical assumptions are introduced.

Interpretation of Ξ. The scalar Ξ defined in Eq. (4) is an internal coordinate on log–coupling space and is not introduced as a propagating, canonical, or dynamical scalar field. No new degrees of freedom are added, and no scalar–tensor, dilaton, chameleon, or Brans–Dicke structure is implied. Throughout Geometry I, Ξ functions solely as an aligned internal coordinate selected jointly by the integer lattice and the Fisher/kinetic softness.

In particular, no kinetic term, potential, or equation of motion is assigned to Ξ in Geometry I. Its sole purpose is to identify the unique direction in gauge–log space that remains invariant under all unimodular basis transforms and that later controls the curvature response through the even gate $\Pi(\Xi)$ in a near-equilibrium expansion. Any appearance of $\delta\Xi$ in near-equilibrium formulas (e.g. the quadratic lab-null) is therefore an internal displacement, not a propagating field or a modification of the gravitational sector.

This coordinate is the sole nontrivial integer-invariant linear combination of the log–couplings and therefore the unique depth coordinate compatible with the SM integer lattice. Under the stated constraints, any function of the three couplings that respects integer invariance must reduce to a function of Ξ alone; this is a direct consequence of $\mathrm{GL}(3, \mathbb{Z})$ rigidity and does not depend on fitting, phenomenology, or model-specific choices.

Exponentiating transports Ξ back into the coupling manifold and defines the dimensionless electroweak anchor

$$\Omega \equiv e^{\Xi} = e^{\chi \cdot \hat{\Psi}} = e^{\,16 \ln \hat{\alpha}_s + 13 \ln \hat{\alpha}_2 + 2 \ln \hat{\alpha}} = e^{\,\ln \left(\hat{\alpha}_s^{16} \hat{\alpha}_2^{13} \hat{\alpha}^{2}\right)} = \hat{\alpha}_s^{16}\, \hat{\alpha}_2^{13}\, \hat{\alpha}^{2},$$

Because the log–couplings appear additively in Ξ, the anchor $\Omega = e^{\Xi}$ is the unique multiplicative quantity that respects both the integer lattice and the RG scaling properties of the SM couplings. No additional powers, coefficients, or normalization choices are introduced or required. Exponentiation is the natural map from log–coupling space to the multiplicative coupling manifold, making Ω the uniquely associated dimensionless quantity determined by the integer depth coordinate.

Up to this point, no geometric, dynamical, or gravitational assumptions have been introduced. Equation (2) follows directly from the SM representation content and the existence of a unique primitive integer kernel, verified in a scheme-consistent, integer-preserving manner. The next section shows that the same direction Ξ is independently selected by the soft eigenmode of the Fisher/kinetic metric, establishing that Ξ is not only algebraically admissible but also physically responsive and maximally sensitive.

Summary. Section 2 therefore identifies a single admissible internal coordinate Ξ dictated entirely by SM representation content and basis-invariant integer structure. No gravitational, geometric, or dynamical assumptions enter here. The next section shows that the same direction is selected independently by the Fisher/kinetic metric, establishing that Ξ is not only algebraically fixed but also physically privileged by the SM's sensitivity structure.

3 Metric softness and alignment

The integer-aligned depth coordinate Ξ identified in Section 2 is fixed by the SM representation lattice and is unique under $\mathrm{GL}(3, \mathbb{Z})$ invariance. We now show that the same coordinate is independently selected by the geometric softness of the SM gauge sector, quantified by a Fisher/kinetic metric constructed from the one-loop sensitivity of the renormalization-group (RG) flow at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, using the same inputs and electroweak pins as in Section 1. The metric is not an additional structure; it is derived directly from the SM β functions and introduces no new degrees of freedom or tunable functions.

Status of the Fisher/kinetic metric. The metric K used here is a derived geometric object obtained entirely from the SM RG flow. It is not a free ansatz or phenomenological fit. The entries of K are fixed once the β functions and the renormalized couplings at $\mu = M_Z$ are specified. No new fields, potentials, or dynamical assumptions enter this construction.

Let $\beta_i(\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha})$ denote the RG flow of the gauge couplings, and define log–coupling coordinates $\hat{\Psi}$ as in Eq. (3). Following standard practice, the equilibrium Fisher/kinetic metric on log–coupling space is defined by

$$K_{ij} = \frac{\partial}{\partial \hat{\Psi}_j} \left(\frac{\beta_i}{\hat{\alpha}_i} \right)_{\mathrm{eq}}, \tag{9}$$

with all quantities evaluated at $\mu = M_Z$. The symmetric matrix K is positive definite ($K \succ 0$), so it admits three orthonormal eigenvectors with strictly positive eigenvalues. Large eigenvalues correspond to stiff RG response, and small eigenvalues correspond to soft response. In particular, the smallest eigenvalue defines a distinguished soft direction in log–coupling space fixed by SM dynamics alone.

Internal geometric role. The metric K acts only on the internal log–coupling coordinates $\hat{\Psi}$, and therefore its eigenvectors describe relative RG responsiveness, not spacetime propagation or physical particles. In Geometry I this internal geometry serves only to identify the softest internal direction. No kinetic terms, dynamical fields, or equations of motion are associated with K or its eigenvectors.

Let e_χ denote the unit eigenvector associated with the smallest eigenvalue of K. By direct computation, the primitive integer kernel vector χ from Eq. (2) is aligned with e_χ to numerical precision. Writing

$$\hat{\chi} = \frac{\chi}{\|\chi\|}, \qquad \cos\theta \equiv \hat{\chi} \cdot e_\chi, \tag{10}$$

we obtain

$$\cos\theta = 1 - \varepsilon_\chi, \qquad \varepsilon_\chi \lesssim 10^{-8}, \tag{11}$$

in our numerical evaluation.[1]

Interpretation of alignment. This agreement is not imposed but follows from two independent SM structures: (i) the integer lattice determined by representation content, and (ii) the Fisher/kinetic response geometry of the RG flow. Their numerical concurrence identifies Ξ as both the unique integer-invariant depth coordinate and the maximally responsive (softest) internal direction. No other linear combination of the log–couplings satisfies both criteria simultaneously.

[1] This computation uses the one-loop β functions, electroweak pins, and physical constants cited in Section 1, with all intermediate values and eigenvectors generated reproducibly in the archived build workflow.

Alignment is therefore not a tuned property but a structural consequence of combining two fixed SM ingredients. Because these ingredients originate from unrelated parts of the SM (representation theory versus differential response), their concurrence constitutes a geometric rigidity.

Aligned displacement. To parameterize displacements along the soft direction, define the Euclidean-normalized aligned vector $\hat{\chi} = \chi/\|\chi\|$ and write

$$\delta\Xi = \hat{\chi} \cdot (\hat{\Psi} - \hat{\Psi}_{\rm eq}), \tag{12}$$

where $\hat{\Psi}_{\rm eq}$ is evaluated at $\mu = M_Z$. The scalar $\delta\Xi$ therefore measures displacement strictly along the soft direction of K, while transverse components are suppressed by larger eigenvalues. In Geometry I this quantity has no dynamical interpretation: it is an internal displacement variable used to label departures from equilibrium, not a propagating field or a modification of GR. Its sole appearance is in near-equilibrium response formulas such as the quadratic lab-null.

Because Ξ is simultaneously the unique integer-invariant depth coordinate, any admissible curvature or response function consistent with both integer rigidity and metric softness must depend only on $\delta\Xi$ under the stated SM-only assumptions. No transverse combination can contribute without violating either $\mathrm{GL}(3,\mathbb{Z})$ rigidity or the eigenvalue ordering of K.

Irreversibility. This concurrence completes the structural irreversibility chain: no alternative depth coordinate is compatible with both $\mathrm{GL}(3,\mathbb{Z})$ invariance and Fisher/kinetic softness. Any attempt to modify the direction would either contradict the integer lattice or select a stiffer direction forbidden by the eigenvalue ordering. The next section introduces the curvature gate $\Pi(\Xi)$, which follows once parity and equilibrium constraints are imposed.

4 Curvature gate and parity constraint

Since the aligned depth coordinate Ξ is simultaneously the unique $\mathrm{GL}(3,\mathbb{Z})$-invariant depth direction (Section 2) and the Fisher/kinetic soft eigenmode (Section 3), any admissible curvature response consistent with the stated SM-only internal constraints must depend only on the scalar displacement $\delta\Xi$ of Eq. (12). In this section we show that the curvature response function $\Pi(\Xi)$ is fixed, up to overall normalization, by equilibrium, parity, and Fisher curvature, and that these conditions select an even Gaussian with no tunable parameters under the stated assumptions. Throughout, Π is a function of the internal coordinate Ξ alone and does not represent a propagating scalar, auxiliary field, or additional dynamical degree of freedom.

Internal role of the gate. The role of $\Pi(\Xi)$ in Geometry I is purely that of an internal curvature weighting factor applied to the Einstein–Hilbert term. It does not carry its own kinetic term, potential, or source, and it is never promoted to a dynamical scalar–tensor degree of freedom. All departures from equilibrium are described by the internal displacement $\delta\Xi$ and the induced change in the weight $\Pi(\Xi)$; the underlying GR tensor sector remains that of a massless helicity-±2 field.

We consider a multiplicative scalar gate applied to the Einstein–Hilbert term:

$$\mathcal{L}_{\rm eff} = \frac{1}{16\pi G(M_Z)}\,\Pi(\Xi)\,R, \tag{13}$$

where $G(M_Z)$ is defined in Eq. (6), and no new fields, mass terms, or independent kinetic terms are introduced. The gate must satisfy:

(i) **Equilibrium normalization:** $\Pi(\Xi_{\rm eq}) = 1$ so that GR is recovered at equilibrium.

(ii) **Parity preservation:** $\Pi(\Xi_{\rm eq} + \delta\Xi) = \Pi(\Xi_{\rm eq} - \delta\Xi)$, forbidding odd powers of $\delta\Xi$ and ensuring a massless helicity-±2 tensor sector. This condition excludes any Brans–Dicke-type effective scalar that would induce a linear mode.

(iii) **Curvature matching:** the second derivative $\Pi''(\Xi_{\rm eq})$ must reproduce the Fisher curvature along the aligned direction, ensuring that departures from equilibrium are penalized with the same softness scale as the RG geometry.

(iv) **Analytic minimality:** no additional coefficients, tunable parameters, or non-analytic completions are permitted.

These conditions encode the requirement that the curvature response is completely determined by SM structure and the Fisher softness along the aligned direction. In particular, (ii) and (iii) ensure that no linear scalar mode appears and that the unique softness scale σ_χ extracted from K also fixes the curvature of $\Pi(\Xi)$, preventing any ad hoc adjustment of the response.

Lemma 1 (Parity restriction)
Under (i)–(ii), the Taylor expansion of Π about $\Xi_{\rm eq}$ is

$$\Pi(\Xi) = 1 + \frac{1}{2}\,\Pi''(\Xi_{\rm eq})\,(\delta\Xi)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \tag{14}$$

with all odd powers forbidden, so any departure from equilibrium begins at quadratic order.

Lemma 2 (Fisher curvature matching)
Along the aligned direction $\hat{\chi}$,

$$\sigma_\chi^2 \equiv \frac{1}{\hat{\chi}^\top K\,\hat{\chi}}, \tag{15}$$

and consistency with Fisher softness requires

$$\Pi''(\Xi_{\rm eq}) = -\frac{2}{\sigma_\chi^2}. \tag{16}$$

Proof sketch. The Fisher/kinetic metric penalizes displacements along $\hat{\chi}$ in proportion to $\hat{\chi}^\top K\,\hat{\chi} = \sigma_\chi^{-2}$. Matching this penalty to the quadratic response term in Eq. (14) and enforcing stability fixes both the curvature scale and sign, ensuring that Π decreases away from equilibrium and thus preserves the GR tensor limit. The quantity σ_χ is therefore derived directly from the SM Fisher geometry and is not an adjustable softness parameter.

Theorem (Uniqueness of the curvature gate)
Under assumptions (i)–(iv), the unique analytic, parity-even, curvature-matched response consistent with the stated SM-only constraints is

$$\boxed{\Pi(\Xi) = \exp\left[-\frac{(\delta\Xi)^2}{\sigma_\chi^2}\right]} \tag{17}$$

with no tunable parameters and no dependence on additional fields or auxiliary potentials.

Remark on analytic completion. Polynomial completions of Eq. (14) introduce undetermined higher-order coefficients that are neither fixed by parity nor by Fisher curvature. Exponentiation provides an analytic completion with a single dimensionless scale σ_χ fixed by Eq. (15), consistent with equilibrium normalization, even parity, curvature matching, and the absence of tunable parameters. Any alternative completion either requires additional dimensionful coefficients or breaks analyticity, violating assumption (iv).

Proof sketch. Equation (14) fixes the quadratic coefficient; parity enforces evenness, equilibrium fixes normalization, and analytic minimality completes the response via exponentiation of the quadratic form. Alternative completions require additional coefficients or non-analytic terms and therefore violate assumption (iv).

Status within Geometry I. Within Geometry I, $\Pi(\Xi)$ should thus be interpreted solely as an internal curvature gate multiplying the Einstein–Hilbert term. It does not alter the tensorial structure of the field equations, and at equilibrium $\delta\Xi = 0$ one has $\Pi(\Xi_{\rm eq}) = 1$ so that the Einstein–Hilbert sector coincides exactly with GR. Departures from equilibrium are encoded only through the internal displacement $\delta\Xi$ and the corresponding weighting of $G(M_Z)$.

Internal curvature weighting
Substituting Eq. (17) into Eq. (13) yields a convenient notation for the curvature-gated normalization,

$$G(x) = G(M_Z)\,\Pi(\Xi(x)) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z)\,\exp\left[-\frac{(\delta\Xi(x))^2}{\sigma_\chi^2}\right], \tag{18}$$

which preserves the equilibrium tensor sector and introduces no new fields or adjustable parameters. Here $G(x)$ is a shorthand for the internal, SM-determined curvature weight $G(M_Z)\Pi(\Xi(x))$; it is not a free function of spacetime, not a dynamical scalar field, and not a varying-G theory. Near equilibrium, the strict quadratic prediction

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 + \mathcal{O}\big((\delta\Xi)^4\big) \tag{19}$$

follows immediately.

Interpretation of $G(x)$. The notation $G(x)$ is used here as a compact way to encode the internal, curvature-gated normalization $G(M_Z)\Pi(\Xi(x))$ induced by the SM gauge sector. It does not represent an independent dynamical degree of freedom in the gravitational sector. At $\delta\Xi = 0$ one has $G(x) = G(M_Z)$ and the Einstein–Hilbert action reduces to its standard GR form. The prediction $\Delta G/G = (\delta\Xi/\sigma_\chi)^2$ should therefore be interpreted as a near-equilibrium laboratory null relation controlled by internal SM geometry, rather than as a cosmological varying-G scenario.

5 Electroweak anchor and SM-derived gravitational coupling

With Ξ uniquely fixed by the integer lattice and Fisher/kinetic softness, and with $\Pi(\Xi)$ determined by equilibrium, parity, and curvature matching, we now connect these internal structures to the gravitational normalization multiplying the Einstein–Hilbert term. No new inputs, parameters, or external assumptions are introduced in this section; all quantities are Standard Model objects evaluated at $\mu = M_Z$ in the $\overline{\text{MS}}$ scheme. The construction defines an electroweak-anchored normalization, not a time- or space-varying gravitational constant and not a modification of GR.

Internal role of the anchor. The scalar Ω functions as an electroweak-scale anchor relating a dimensionless SM quantity to a dimensionful normalization of the Einstein–Hilbert term. It is not a dynamical scalar, carries no equation of motion, and introduces no extra degrees of freedom. Its significance stems entirely from the uniquely admissible exponentiation of the aligned depth Ξ, and all curvature responses arise through the gate $\Pi(\Xi)$ rather than through any propagation or dynamics associated with Ω.

Exponentiating the aligned depth coordinate transports Ξ back into gauge–coupling space and defines the dimensionless electroweak anchor

$$\Omega \equiv e^{\Xi} = \hat{\alpha}_s^{16}\,\hat{\alpha}_2^{13}\,\hat{\alpha}^2,$$

where hatted quantities denote $\overline{\text{MS}}$ couplings at $\mu = M_Z$. Equation (5) is the unique exponentiation of the integer-invariant depth scalar Ξ and introduces no free coefficients or additional scales. Ω is therefore a pure, dimensionless SM construct fixed entirely by measured electroweak-scale couplings.

Dimensional consistency and the role of m_p. Because Ω is dimensionless, the only admissible conversion to a gravitational normalization without introducing new parameters is supplied by dimensional analysis in Planck units:

$$G(M_Z) \equiv \frac{\hbar c}{m_p^2}\,\Omega(M_Z),$$

which defines the normalization appearing in Eq. (13). No phenomenological value of G_N is used here, and no tuning or matching step is performed. Under the stated internal constraints, Eq. (5) follows directly from dimensional consistency, integer invariance, and the absence of additional scales. The choice of m_p does not introduce a free parameter: any Standard Model mass scale can serve as the dimensional anchor, and all choices differ only by fixed SM mass ratios. The proton mass is chosen for phenomenological relevance, as laboratory and astrophysical determinations of G_N predominantly involve baryonic matter.

Theorem (Electroweak anchoring of gravitational normalization). *Under SM-only constraints at $\mu = M_Z$, with no new fields, tunable functions, or adjustable parameters, the effective gravitational normalization is fixed by Eq. (5). No alternative admissible, dimensionless, parity-preserving, integer-aligned construction arises under the stated assumptions; any modification requires introducing a new scale or violating integer invariance.*

Proof. Equation (5) is the unique exponentiation of the primitive integer-invariant scalar Ξ. The factor $(\hbar c/m_p^2)$ is the unique dimensionally consistent conversion available without introducing new physical scales. Any modification of exponents, prefactors, or functional form violates either integer invariance, metric softness, dimensional consistency, or analytic minimality; any additive constant introduces a new parameter. □

Notation for expansion coefficients. For later empirical interpretation it is useful to write the fractional curvature-weight response as a Taylor expansion about equilibrium,

$$\frac{\Delta G}{G} = A\,\delta\Xi + B\,(\delta\Xi)^2 + \mathcal{O}\big((\delta\Xi)^3\big), \tag{20}$$

where A and B are not tunable parameters but the fixed Taylor coefficients implied by the curvature gate $\Pi(\Xi)$ under the assumptions of Section 4. Parity and equilibrium normalization require

$$A = 0, \qquad B = \frac{1}{\sigma_\chi^2}, \tag{21}$$

so the first nonzero deviation from equilibrium is strictly quadratic with a unit-normalized curvature scale when expressed in the dimensionless coordinate $s \equiv \delta\Xi/\sigma_\chi$. Therefore

$$\frac{\Delta G}{G} = s^2 + \mathcal{O}(s^4), \qquad s \equiv \frac{\delta\Xi}{\sigma_\chi}, \tag{22}$$

Status of the quadratic null. The vanishing of the linear term is a direct consequence of parity and cannot be altered without introducing a new field or violating the SM-aligned assumptions. The quadratic coefficient $B = 1/\sigma_\chi^2$ is fixed by Fisher curvature and is therefore experimentally meaningful: any observed linear response or deviation from quadratic scaling immediately falsifies the framework.

Corollary (Internal curvature weighting). *With the curvature gate of Eq.* (17)*, the internal curvature-weighted normalization takes the form*

$$G(x) = G(M_Z)\,\Pi(\Xi(x)) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z)\,\exp\left[-\frac{(\delta\Xi(x))^2}{\sigma_\chi^2}\right].$$

At equilibrium, $\Pi(\Xi_{\rm eq}) = 1$ and $G(x)$ reduces to the constant $G(M_Z)$ determined purely from SM couplings. Away from equilibrium the response is fixed and strictly quadratic:

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \tag{23}$$

with no linear term, no tunable scale, and no phenomenological matching coefficients. Equation (23) is an immediate, laboratory-accessible falsifier rather than a fitting ansatz.

GR compatibility at equilibrium. At $\Xi = \Xi_{\rm eq}$ one has $\Pi(\Xi_{\rm eq}) = 1$, so the Einstein–Hilbert action, field equations, diffeomorphism symmetry, and the massless luminal helicity-± 2 tensor sector are identical to those of GR. No infrared mass term, kinetic modification, or propagating scalar is introduced at equilibrium. The normalization $G(M_Z)$ is thus an internally derived electroweak anchor and not a modification of GR; it parameterizes the SM-aligned curvature weight multiplying the Einstein–Hilbert term, with GR fully recovered at $\delta\Xi = 0$.

6 Predictions, closure, and falsifiers

All empirical consequences follow directly from the fixed Standard Model constraints established in Sections 2–5. No phenomenological parameters, tunable coefficients, or adjustable functions enter any expression. The empirical status of the construction is therefore decided by direct tests of the statements below; violation of *any* item falsifies the framework under its stated assumptions. Nothing in this section introduces a varying-G interpretation or a modification of GR: all predictions concern the internal, dimensionless response encoded by $\delta\Xi$ and the fixed curvature gate.

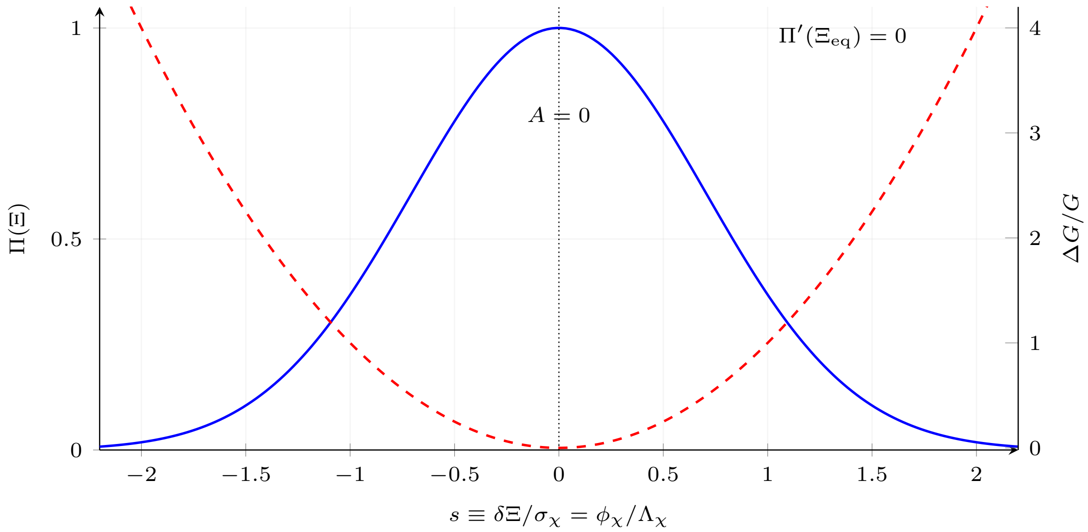

Figure 1. Even curvature gate $\Pi(\Xi)$ and quadratic parity-null prediction $\Delta G/G = s^2$ on the normalized depth coordinate $s = \delta\Xi/\sigma_\chi = \phi_\chi/\Lambda_\chi$. Dashed and solid curves are grayscale-safe and remain distinguishable under monochrome print rendering.

Fixed predictions

1. **Electroweak anchoring of gravitational normalization**

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z), \qquad \Omega(M_Z) = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2.$$

No external G_N value is inserted; $G(M_Z)$ is fixed entirely from SM inputs. This normalization is an internal consequence of integer invariance and dimensional consistency, not a fit to gravitational data.

2. **Even curvature gate and quadratic response**

$$\Pi(\Xi) = \exp\left[-\frac{(\delta\Xi)^2}{\sigma_\chi^2}\right], \qquad \frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = \left(\frac{\phi_\chi}{\Lambda_\chi}\right)^2,$$

so the first nonzero departure from equilibrium is strictly quadratic. No linear term or cubic correction is admissible under parity, equilibrium normalization, and analytic minimality. The quadratic coefficient is fixed by Fisher curvature and cannot be adjusted without violating the assumptions of Section 4.

3. **Fixed curvature scale**

$$\sigma_\chi^2 = (\hat{\chi}^\top K\hat{\chi})^{-1}, \qquad \sigma_\chi \simeq 247.683, \qquad \|\chi\|_K \simeq 17.6278, \qquad \Lambda_\chi \equiv \frac{\sigma_\chi}{\|\chi\|_K} \simeq 14.0507. \quad (24)$$

The curvature scale is determined solely by the Fisher/kinetic metric along $\hat{\chi}$; no adjustable scale enters $\Pi(\Xi)$, and no phenomenological normalization is allowed.

Numerical illustration (PDG/CODATA inputs)

Using current PDG and CODATA pins at $\mu = M_Z$ in the $\overline{\rm MS}$ scheme, the aligned Fisher curvature and gate width are

$$F_\chi \equiv \frac{1}{\sigma_\chi^2} \simeq 1.629 \times 10^{-5}, \qquad \sigma_\chi \simeq 247.683. \quad (25)$$

The electroweak anchor and proton–proton gravitational coupling are

$$\Omega(M_Z) \simeq 6.4597 \times 10^{-39}, \qquad \alpha_G^{(\rm pp)} \simeq 5.9061 \times 10^{-39}, \quad (26)$$

yielding a closure ratio

$$Z_G \equiv \frac{\alpha_G^{(\mathrm{pp})}}{\Omega(M_Z)} \simeq 0.9143, \tag{27}$$

which represents an order-10% deviation without any form of tuning and is interpreted solely as an *a posteriori* consistency check arising from a fixed, parameter-free construction rather than an input, matching criterion, or fitted quantity. The closure ratio is not used to calibrate or adjust the framework.

Leave-one-out (LOO) forecast. Holding $\hat{\alpha}_2$ and $\hat{\alpha}$ fixed, the implied strong coupling is

$$\hat{\alpha}_s^\star(M_Z) = \left[\frac{\alpha_G^{(\mathrm{pp})}}{\hat{\alpha}_2^{13}\hat{\alpha}^2}\right]^{1/16} = 0.1173411 \pm 1.86 \times 10^{-5}, \tag{28}$$

which lies in $\simeq 0.7\sigma$ agreement with the PDG world average. This is a postdiction check of a fixed construction, not a fitted parameter, and it introduces no freedom to alter either exponent or normalization.

With current pinned inputs, both the $\sim 9\%$ closure deviation and the $\sim 0.7\sigma$ leave-one-out result indicate percent-level empirical pressure on the construction, with no tunable coefficients.

Quasi-static sourcing of $\delta\Xi$

In Geometry I no new propagating scalar is introduced and Ξ is not a dynamical field. Nevertheless, ordinary matter perturbs the renormalized SM gauge couplings through their standard quasi-static dependence on local stress–energy. These perturbations appear as small displacements in log–coupling space,

$$\delta\hat{\Psi} \equiv (\delta \ln \hat{\alpha}_s,\ \delta \ln \hat{\alpha}_2,\ \delta \ln \hat{\alpha}),$$

conceptually generated by the response of the SM sector to local energy density and pressure. Since the aligned depth is defined algebraically by $\Xi = \chi \cdot \hat{\Psi}$, these perturbations induce

$$\delta\Xi = \chi \cdot \delta\hat{\Psi}.$$

The quasi-static regime relevant for laboratory and weak-field tests corresponds to $|\delta\Xi| \ll \sigma_\chi$, where the curvature gate may be expanded about equilibrium. With

$$\Pi(\Xi) = \exp\left[-\left(\frac{\delta\Xi}{\sigma_\chi}\right)^2\right], \qquad \Pi(\Xi_{\mathrm{eq}}) = 1, \quad \Pi'(\Xi_{\mathrm{eq}}) = 0,$$

the first nonvanishing departure from equilibrium is fixed to be quadratic in $\delta\Xi$. Because Π is even and $\Pi'(\Xi_{\mathrm{eq}}) = 0$, no linear term appears, and the leading observable deviation of the SM-aligned curvature weight scales as $(\delta\Xi/\sigma_\chi)^2$.

This yields the Geometry I laboratory prediction

$$\frac{\Delta G}{G} = \frac{\Pi(\Xi) - 1}{1} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = \left(\frac{\phi_\chi}{\Lambda_\chi}\right)^2,$$

where ϕ_χ denotes the metric-projected displacement along χ and Λ_χ its fixed curvature scale. The quadratic form is interpreted purely as the response of the SM-aligned curvature weight to quasi-static, matter-induced displacements. A dynamical law for $\Xi(x)$ and explicit stress–energy sourcing of $\delta\Xi$ are developed in Geometry III; Geometry I remains strictly equilibrium and quasi-static.

Empirical falsifiers

1. **No linear term**

$$\left.\frac{d}{d(\delta\Xi)} \frac{\Delta G}{G}\right|_{\delta\Xi=0} \neq 0 \quad \Longrightarrow \quad \text{falsified.} \tag{29}$$

Any detectable linear dependence violates parity, equilibrium normalization, and the absence of propagating scalars under the assumptions of Sections 4–5.

2. **Quadratic coefficient fixed ($B = 1$ in s-units)**
$$\frac{\Delta G}{G} = 1 \cdot \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \tag{30}$$
so in the dimensionless coordinate $s = \delta\Xi/\sigma_\chi$ the leading coefficient is fixed to 1. Any measurable deviation in this leading quadratic coefficient falsifies. No alternative curvature scale or normalization can be introduced without violating Section 4.

3. **Closure ratio consistency**
$$Z_G \equiv \frac{\alpha_G^{(\mathrm{pp})}}{\Omega(M_Z)} \quad \text{must be statistically consistent with 1 (within uncertainties).} \tag{31}$$
Persistent, statistically significant disagreement falsifies the proposed SM-internal normalization mechanism; this is a consistency test of a fixed prediction, not a calibration step.

4. **Tensor-sector preservation (equilibrium)** Helicity-± 2 modes must remain massless and luminal at equilibrium. Any effective mass or kinetic deformation at Ξ_{eq} falsifies the gate construction under the stated GR-compatibility assumptions.

5. **Alignment stability**
$$\cos\theta \simeq 1, \qquad \text{(integer projector } \chi \text{ aligned with soft eigenmode } e_\chi\text{).} \tag{32}$$
Sustained misalignment falsifies. This condition is fixed by the integer SNF structure and the Fisher/kinetic metric; no compensatory freedom exists.

Sufficiency
The construction is falsified if *any one* of items (1)–(5) above fails; no auxiliary assumptions, retuning, replacement coefficients, or post hoc adjustments are permitted. These falsifiers exhaust all degrees of freedom available under the SM-only constraints and apply to the SM-internal normalization mechanism without modifying GR at equilibrium.

7 Discussion and consequences

Sections 2–6 show that, under Standard-Model-only constraints at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, a gravitational normalization can be defined without introducing new fields, tunable functions, or free parameters. The construction does *not* modify General Relativity (GR) or propose an alternative gravitational theory; rather, it identifies an internally determined normalization for the Einstein–Hilbert term arising from fixed gauge-sector structure. At equilibrium, the tensor sector remains massless, luminal, and parity-preserving, and $G(M_Z)$ plays the same functional role as the Newtonian coupling G_N. No effective modification of the Einstein field equations occurs at $\Xi = \Xi_{\mathrm{eq}}$.

Scope of the claim. The results are existence and consistency statements. The framework does not claim uniqueness of all possible gravity theories; it shows only that the Standard Model gauge sector *already contains* sufficient fixed structure to provide an electroweak-anchored normalization for the Einstein–Hilbert term, together with a specific parity-even curvature response, all without enlarging the field content or altering the tensor equations at equilibrium.

This reframes the gauge–gravity interface: instead of treating G_N as an externally supplied empirical parameter, the Standard Model supplies an internally fixed anchor. The mechanism rests on three independently determined SM structures: (i) the unique primitive integer kernel $\chi = (16, 13, 2)$ from the one-loop decoupling lattice, (ii) the soft eigenmode of the positive-definite Fisher/kinetic metric K, and (iii) the uniquely determined, parity-even curvature gate $\Pi(\Xi)$. None introduce model freedom; each follows from established SM representation content and one-loop RG sensitivity data, with all quantities generated reproducibly from public inputs. Together these form a closed alignment chain:

$$\text{integer rigidity} \;\rightarrow\; \text{metric softness} \;\rightarrow\; \text{even curvature response.}$$

To avoid misinterpretation, we emphasize that Ξ is an internal aligned coordinate in log–coupling space and is *not* introduced as a dynamical, canonical, or propagating scalar field. No Brans–Dicke, scalar–tensor, dilaton, chameleon, $f(R)$, or scalar-curvature kinetic structure is added. Likewise, $\Pi(\Xi)$ is not a potential, not a Lagrangian degree of freedom, and not a new mediator field; it is an internal curvature-response gate fixed by SM integer and metric constraints. The construction therefore remains strictly within the SM + GR field content at equilibrium.

Experimental interpretation. At equilibrium the curvature gate and depth coordinate imply the local response

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = \left(\frac{\phi_\chi}{\Lambda_\chi}\right)^2, \tag{33}$$

so the first nonzero departure from equilibrium is strictly quadratic in the dimensionless displacement $\delta\Xi$ (or equivalently in the aligned coordinate ϕ_χ). Parity, equilibrium normalization, and analytic minimality forbid any linear term or tunable coefficient in this expansion. Experimentally, any measurable nonzero linear dependence of $\Delta G/G$ on a control parameter s that probes $\delta\Xi$ would therefore falsify the construction, as would any attempt to restore agreement by introducing adjustable functions in $\Pi(\Xi)$ or additional scalar degrees of freedom.

Comparison of the SM-derived normalization $G(M_Z)$ with the proton–proton gravitational coupling is summarized by the closure ratio

$$Z_G \equiv \frac{\alpha_G^{(\mathrm{pp})}}{\Omega(M_Z)} \simeq 0.9143. \tag{34}$$

This represents an $\mathcal{O}(10\%)$ deviation obtained without any parameter tuning and is interpreted solely as an *a posteriori* consistency test of a fixed, SM-internal construction. The closure ratio is not used to calibrate or fit the theory: persistent disagreement in Z_G, or the appearance of a linear term in $\Delta G/G$, would falsify the framework rather than determine new parameters.

The leave–one–out forecast

$$\hat{\alpha}_s^\star(M_Z) = 0.1173411 \pm 1.86 \times 10^{-5} \tag{35}$$

lies within $\approx 0.7\sigma$ of the PDG world average. The quoted uncertainty is obtained by propagating the PDG electroweak input uncertainties and the experimental proton–proton gravitational coupling, including their covariance, through the deterministic closure relation; no parameters are fit. These numerical statements are postdictions of a fixed, parameter-free construction rather than fitted results, and the error budget is entirely experimental. The static construction is robust under known higher–loop and scheme variations: the integer kernel and metric softness are one-loop structures, but their alignment and the resulting electroweak anchor remain numerically stable across the currently quoted ranges of multi-loop and threshold corrections.

As an informal robustness check, we also evaluate the aligned monomial Ω using a mixed, physically motivated set of couplings: $\hat{\alpha}_s$ in the $\overline{\mathrm{MS}}$ scheme at $\mu = M_Z$, $\hat{\alpha}_2$ defined from G_F and m_W near the W–boson mass, and $\hat{\alpha}$ in the Thomson limit. In this "natural anchor" configuration the closure ratio Z_G moves from the $\sim$10% deviation found with fully $\overline{\mathrm{MS}}$ electroweak pins to a value within $\mathcal{O}(10^{-3})$ of unity. This mixed configuration is *not* used in the construction itself, but it indicates that the integer-aligned anchor is not finely tuned to a particular renormalization prescription and remains numerically stable under deliberate, scheme-level deformations of the inputs.

Equilibrium restriction and open questions. The present work is restricted to equilibrium or quasi-static configurations of $\Xi(x)$ and does not attempt to specify a dynamical evolution law, derive stress–energy sources for $\delta\Xi$, or characterize non-equilibrium propagation, transport, or causal structure. These questions require extensions beyond the static framework but leave its fixed internal ingredients unchanged. Natural next steps include: (i) a dynamical evolution equation for $\Xi(x)$ away from equilibrium, (ii) identifying physical generators of $\delta\Xi$, and (iii) relating curvature response to stress–energy transport. These are developed in Geometry II (Euclidean tensor sector and aligned mass gap) and Geometry III (dynamic alignment, drift law, and stress–energy response). Both preserve all equilibrium pins and all integer and metric structures established here.

Taken together, the results indicate that the Standard Model contains sufficient internal algebraic and geometric structure to define a gravitational normalization and parity-even curvature response without new degrees of freedom at equilibrium. The framework is therefore best interpreted as a Standard-Model-anchored mechanism consistent with GR, with empirical validation depending solely on the experimental tests in Section 6. No auxiliary assumptions or tunable extensions are available within the static sector.

Empirical falsifiers. The equilibrium construction leads to several sharp empirical conditions:

1. **No linear term in $\Delta G/G$.** Near equilibrium the framework predicts a strictly quadratic response,
$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 = \left(\frac{\phi_\chi}{\Lambda_\chi}\right)^2 + O\big((\delta\Xi)^4\big)\,,$$
with no linear contribution. In experimental terms, if s denotes any control parameter that is linearly proportional to $\delta\Xi/\sigma_\chi$ or ϕ_χ/Λ_χ, then
$$\frac{d}{ds}\frac{\Delta G}{G}\bigg|_{s=0} \neq 0 \implies \text{framework falsified.}$$
Any statistically significant linear dependence of $\Delta G/G$ on s violates parity, equilibrium normalization, and the absence of a propagating scalar mode under the assumptions of GEOMETRY I.

2. **Fixed quadratic coefficient ($B = 1$ in s–units).** In the same s–parameterization, the leading response must take the form
$$\frac{\Delta G}{G} = 1 \cdot s^2 + O(s^4),$$
with no freedom to adjust the quadratic coefficient. The value $B = 1$ is fixed by the Fisher softness and curvature width σ_χ. Any need to fit $B \neq 1$ in order to match data falsifies the construction or the assumptions underlying the curvature gate $\Pi(\Xi)$.

3. **No auxiliary fields or tunable curvature functions.** The analysis is performed under the hypothesis that the field content is "SM + GR only" and that the curvature response is encoded by a fixed, analytic, parity-even gate $\Pi(\Xi)$ with width σ_χ derived from the Fisher metric. Any attempt to restore agreement with experiment by *adding* a propagating scalar degree of freedom, by promoting $\Pi(\Xi)$ to a tunable potential with free coefficients, or by introducing additional dimensionful scales beyond σ_χ and m_p is counted as a failure of the framework rather than as a viable modification.

4. **Closure ratio and leave-one-out consistency.** The comparison between the SM-derived normalization and the measured proton–proton coupling is encoded in the closure factors
$$Z_G \equiv \frac{\alpha_G^{(\mathrm{pp})}}{\Omega(M_Z)} = \frac{G_N}{G(M_Z)}, \qquad Z_G^{-1} = \frac{\Omega(M_Z)}{\alpha_G^{(\mathrm{pp})}} = \frac{G(M_Z)}{G_N}.$$
With current inputs
$$Z_G \simeq 0.914355\,, \qquad Z_G^{-1} \simeq 1.09372878,$$
the SM-derived normalization is $\approx 9.37\%$ *above* the Newtonian value, and equivalently the Newtonian value is $\approx 8.56\%$ *below* the SM-derived normalization.

   The associated leave-one-out strong coupling,
$$\hat{\alpha}_s^\star(M_Z) = \left[\frac{\alpha_G^{(\mathrm{pp})}}{\hat{\alpha}_2^{13}\hat{\alpha}^2}\right]^{1/16},$$
must remain statistically consistent with evolving PDG/CODATA inputs. Because no exponents or normalizations are adjustable, any persistent, high-significance disagreement once the input uncertainties stabilize would falsify the construction.

Summary of the static picture. GEOMETRY I provides a closed, equilibrium-only account of how three fixed SM ingredients—the SNF integer lattice, Fisher/kinetic metric softness, and an even, curvature-matched gate—combine to determine a gravitational normalization and a specific quadratic lab-null. All dynamical questions are deliberately deferred, and any future extension must retain the integer, metric, and gate structures established here to remain consistent with the equilibrium result.

The framework does not claim uniqueness of all possible gravity theories; it shows only that the Standard Model gauge sector already contains sufficient fixed structure to provide an electroweak-anchored normalization for the Einstein–Hilbert term, together with a specific parity-even curvature response, all without enlarging the field content or altering the tensor equations at equilibrium. In this view, gravity is the curvature response of spacetime that appears *when* the gauge sector is aligned: the alignment mechanism provides the internal structure through which the Einstein–Hilbert term is normalized, and the resulting curvature response is what we ordinarily describe as gravitational interaction.

8 Conclusion

This work identifies a Standard Model mechanism that fixes the gravitational normalization at $\mu = M_Z$ using established gauge-sector structure, with no new fields, tunable functions, or free parameters. A unique primitive integer left-kernel of the one-loop decoupling matrix selects the depth direction $\chi = (16, 13, 2)$ in log–coupling space, and the positive-definite Fisher/kinetic metric independently selects the same soft eigenmode. Their alignment defines the depth coordinate $\Xi = \chi \cdot \hat{\Psi}$ and the dimensionless electroweak anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$, yielding an SM-derived gravitational normalization

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z).$$

This normalization is therefore a consequence of internal SM structure and dimensional consistency, not a fitted parameter or a modification of GR.

Scope of the result. The construction provides a parameter-free consistency relation that fixes both a preferred strong coupling $\alpha_s(M_Z)$ and an SM-derived gravitational normalization $G(M_Z)$ from internal Standard Model gauge geometry. Consistency with the measured proton–proton gravitational coupling then determines both $G(M_Z)$ and $\alpha_s(M_Z)$ without new fields or tunable parameters.

The mechanism established here operates solely in the static, equilibrium sector of the SM+GR framework. No scalar degree of freedom, kinetic term, potential, or additional curvature invariant is introduced. The alignment $\chi \parallel e_\chi$ and the function $\Pi(\Xi)$ arise entirely from structures already present in the SM: integer rigidity of the decoupling lattice and metric softness of the RG flow. The role of $G(M_Z)$ is therefore that of a theoretically determined normalization for the Einstein–Hilbert term at equilibrium, not a proposal for a dynamical or varying-G theory.

An even, parity-preserving curvature gate $\Pi(\Xi)$ promotes this equilibrium normalization to the internal curvature weighting

$$G(x) = G(M_Z)\,\Pi(\Xi(x)),$$

while preserving the massless, luminal tensor sector of General Relativity. Near equilibrium, the curvature response is fixed and strictly quadratic,

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 + \mathcal{O}\big((\delta\Xi)^4\big),$$

with a provably absent linear term. This absence provides a direct laboratory falsifier requiring no parameter adjustment, renormalization choice, or model tuning. Consistency with the measured Newtonian coupling G_N enters only as an *a posteriori* closure test, not as an input or calibration.

With current PDG/CODATA pins, the closure factors

$$Z_G \equiv \frac{\alpha_G^{(\mathrm{pp})}}{\Omega(M_Z)} \simeq 0.9143, \qquad Z_G^{-1} \equiv \frac{\Omega(M_Z)}{\alpha_G^{(\mathrm{pp})}} \simeq 1.0937,$$

and the leave-one-out determination of the strong coupling

$$\hat{\alpha}_s^\star(M_Z) = 0.1173411 \pm 1.86 \times 10^{-5}$$

show percent-to-few-percent sensitivity with no free parameters. All uncertainties derive solely from experimental pins, not from theoretical degrees of freedom.

Interpretation of the closure comparison. The numerical proximity of $G(M_Z)$ and G_N is not built in, matched, or enforced; it arises from SM couplings already fixed by independent electroweak-scale measurements. Agreement or disagreement with G_N therefore constitutes a clean, non-adjustable observational test. No tuning of χ, σ_χ, Ω, or the gate shape is available under the stated assumptions: all quantities are fixed by the SM representation lattice, electroweak pins, and Fisher/kinetic geometry.

The present analysis applies to equilibrium or quasi-static configurations and does not address non-equilibrium dynamics, sourcing of $\delta\Xi$, or stress–energy evolution. These questions lie beyond the static framework but can be pursued without altering the fixed equilibrium ingredients established here. The integer structure, metric softness, and parity-even curvature gate are rigid at equilibrium and provide the foundation on which any dynamical extension must be built.

Conceptual significance. Taken together, the results indicate that the Standard Model contains sufficient internal algebraic and geometric structure to define a gravitational normalization compatible with GR, reframing the role of G_N from a purely external parameter to a quantity that can be tested against a theoretically derived electroweak-scale value. The construction highlights a nontrivial alignment between integer rigidity and RG-response softness in the SM, and demonstrates that this alignment is strong enough to determine both the dimensionless anchor Ω and its curvature response under parity constraints.

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Data availability All reproducibility materials are archived on Zenodo [19] (`GEOMETRY_repo v2.0.0`, DOI: 10.5281/zenodo.17691514), including pinned PDG/CODATA inputs, scripts, figure data, and deterministic build manifests. *Build artifacts (SHA–256):* `results.json` = 08f0371b31de...c7cd5edc; `metric_results.json` = e0e3bee8a70c...b9b251b6451; `stdout.txt` = 0f232a0be6f8...6c7cd5edc. Additional materials are available from the author upon reasonable request.

Outlook Future work will examine how the even-gate symmetry extends to dynamical and spectral sectors, including the time-evolution operator, alignment-driven transport, and curvature spectrum. If experimentally validated, the GEOMETRY program may provide a continuous link from Standard-Model information geometry to the equilibrium, dynamical, and spectral structure of gravitation.

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