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GEOMETRY I: SM-derived gravitational coupling $G(M_Z)$ anchored at the electroweak scale

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Abstract
Under the minimal internal constraints of the Standard Model (SM)—no new fields, no tunable functions, and fixed renormalization at $\mu = M_Z$ in the $\overline{\text{MS}}$ scheme—we show that a gravitational normalization can be derived from SM data alone, rather than introduced as an external input. This construction does not introduce a propagating scalar or modify General Relativity; Ξ is an internal aligned coordinate and $G(M_Z)$ denotes an electroweak-anchored normalization rather than a varying-G framework.
At one loop, the SM decoupling matrix admits a unique primitive integer left-kernel $\chi = (16, 13, 2)$ in Smith normal form (verified using a standard integer-normal-form algorithm). This selects an aligned depth $\Xi = \chi \cdot \hat{\Psi}$ in log-coupling space. Independently, the positive-definite Fisher/kinetic metric identifies a soft eigenmode of maximal responsiveness; the two structures align numerically ($\cos\theta \simeq 1$), fixing Ξ as the unique admissible depth coordinate. Exponentiation then yields a parameter-free electroweak anchor $\Omega = e^{\Xi} = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$ and an SM-derived gravitational normalization

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z),$$

with m_p taken as the natural baryonic dimensional anchor.
An even, equilibrium-normalized curvature gate $\Pi(\Xi)$ satisfying $\Pi'(\Xi_{\text{eq}}) = 0$ and fixed curvature scale $\sigma_\chi^{-2} = \hat{\chi}^\top K \hat{\chi}$ promotes $G(M_Z)$ to a position-dependent coupling $G(x) = G(M_Z)\Pi(\Xi(x))$ while preserving the massless, luminal helicity-± 2 tensor sector. Near equilibrium the curvature response is strictly quadratic, $\Delta G/G = (\delta\Xi/\sigma_\chi)^2$, providing a direct laboratory falsifier with no linear term. Comparison with the measured Newtonian coupling enters only *a posteriori* through a closure ratio, not as an input or calibration. The construction is therefore fixed entirely by SM data at $\mu = M_Z$, introduces no tunable parameters, preserves the GR tensor limit, and yields a reproducible, experimentally testable gravitational coupling anchored at the electroweak scale. All figures, scripts, and numerical values are generated from public inputs via an archived, hash-verified build workflow. All statements apply to the equilibrium SM geometry; sourcing, stress–energy coupling, and dynamical extensions are deferred to future work.

Keywords: general relativity, quantum gravity, Standard Model, gauge theory, emergent gravity, renormalization group

1 Introduction

We restrict throughout to Standard Model (SM) data at $\mu = M_Z$ in the $\overline{\text{MS}}$ scheme, introduce no new fields, parameters, or tunable functions, and retain the massless, luminal helicity-± 2 tensor sector of General Relativity (GR). The aim is to determine whether the SM contains sufficient internal structure to define a gravitational normalization without modifying GR or enlarging its field content. All numerical values and figures presented below are generated directly from public SM inputs using a reproducible, hash-verified build workflow archived under a public DOI (see Data Availability).

The SM provides a quantitative description of the three gauge interactions and their renormalization-group (RG) evolution, but it does not internally specify Newton's gravitational constant G_N. In GR, the Einstein–Hilbert term

$$\mathcal{L}_{\text{EH}} = \frac{1}{16\pi G_N} R \tag{1}$$

contains an empirically measured coupling: GR specifies how curvature responds to stress–energy but does not determine the numerical normalization of that response. By contrast, the three electroweak-scale SM couplings $(\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha})$ are RG-predictive, experimentally constrained, and scheme-consistent at $\mu = M_Z$. This motivates the central question:

> *Does the SM gauge sector at $\mu = M_Z$ contain sufficient, basis-invariant structure to fix a gravitational normalization without new degrees of freedom or modification of GR?*

Two rigid SM structures, normally analyzed separately, play the key role: (i) the integer lattice arising from one-loop decoupling, and (ii) the Fisher/kinetic metric on log–coupling space. Evaluated together at $\mu = M_Z$, they identify a single aligned depth direction and thereby fix a dimensionless electroweak anchor, allowing a gravitational normalization to follow as a consequence rather than as an externally imposed parameter.

At one loop, the SM decoupling matrix is an exact integer matrix whose Smith normal form (SNF) admits a unique primitive left-kernel generator (up to overall sign), obtained using a standard integer-normal-form algorithm implemented verbatim in the accompanying reproducibility scripts:

$$\chi = (16, 13, 2). \tag{2}$$

This vector defines a depth coordinate

$$\Xi = \chi \cdot \hat{\Psi}, \qquad \hat{\Psi} = (\ln \hat{\alpha}_s, \ln \hat{\alpha}_2, \ln \hat{\alpha}), \tag{3}$$

in log–coupling space. Independently, the positive-definite (equilibrium) Fisher/kinetic metric K (hereafter $K \equiv K_{\mathrm{eq}}$), constructed from one-loop sensitivity data, possesses a soft eigenmode of maximal responsiveness. Using the same SM input pins and renormalization scheme, we find that the integer direction χ is numerically aligned with the softest eigenvector of K, with $\cos\theta \simeq 1$. Thus the aligned depth Ξ is not a model assumption but the coordinate jointly selected by integer rigidity and metric softness.

Exponentiating the aligned depth yields the electroweak anchor

$$\Omega \equiv e^{\Xi} = e^{\chi \cdot \hat{\Psi}} = \hat{\alpha}_s^{16}\, \hat{\alpha}_2^{13}\, \hat{\alpha}^2, \tag{4}$$

which defines an SM-derived electroweak-scale gravitational normalization

$$G(M_Z) = \frac{\hbar c}{m_p^2}\, \Omega(M_Z), \tag{5}$$

with no adjustable parameters. The proton mass m_p is chosen as the dimensional anchor because laboratory and astrophysical determinations of G_N predominantly probe baryonic (proton-dominated) matter; alternative choices such as m_e or m_n simply rescale the same dimensionless anchor Ω.

The remainder of this work examines how this normalization is promoted to a spacetime coupling consistent with GR, how parity constraints fix the curvature response, and how the near-equilibrium prediction

$$\frac{\Delta G}{G} = \left(\frac{\delta \Xi}{\sigma_\chi}\right)^2 \tag{6}$$

yields a direct empirical test.

Summary of results and roadmap. Section 2 establishes the integer structure and the unique primitive kernel χ of the one-loop decoupling matrix. Section 3 constructs the Fisher/kinetic metric and verifies its numerical alignment with χ, fixing the admissible depth coordinate Ξ. Section 4 introduces the parity-preserving curvature gate $\Pi(\Xi)$ and determines its fixed curvature scale σ_χ. Section 5 derives the electroweak-anchored normalization $G(M_Z)$ and promotes it to $G(x) = G(M_Z)\Pi(\Xi(x))$ without modifying GR. Section 6 presents the quadratic lab-null, closure ratio, and falsifiers. Section 7 summarizes the implications and outlines extensions to dynamical settings in future work.

Program and provenance. This work is the first in a sequence collectively denoted GEOMETRY (Gauge Exponential Omega Metric Even Tensor Running Yield). GEOMETRY I is restricted to the static, equilibrium geometry and derives an electroweak-anchored gravitational coupling from SM data alone. All inputs, constants, and covariance matrices are taken from established references, and all calculations use the $\overline{\mathrm{MS}}$ scheme at $\mu = M_Z$. Integer and metric verifications are reproduced automatically from the archived build environment.

Renormalization conventions follow Weinberg [1], Peskin and Schroeder [2], and Langacker [3]. Decoupling and integer-lattice methods follow Appelquist and Carazzone [4], Kannan and Bachem [5], and Newman [6]. Electroweak pins, covariance matrices, and physical constants are taken from the Particle Data Group and CODATA [7, 8, 9, 10]. Two-loop RG coefficients follow Machacek and Vaughn [11, 12] and Luo *et al.* [13], and the running of $\hat{\alpha}$ follows Jegerlehner [14]. Gravitational and observational constraints follow Carroll [15], Will [16], Bertotti *et al.* [17], and Abbott *et al.* (LVK) [18]. No additional data, fitting, or tuning is employed.

Interpretation of $G(M_Z)$. The normalization $G(M_Z)$ introduced in Eq. (5) is a renormalization-anchored quantity, not a time- or space-varying gravitational constant. No modification of General Relativity, its field equations, or the equilibrium value of the Newtonian coupling is proposed. The construction identifies a theoretically determined equilibrium normalization compatible with GR, rather than a dynamical or varying-G scenario.

Interpretation of Ξ and $\delta\Xi$. The aligned depth $\Xi = \chi \cdot \hat{\Psi}$ is an internal coordinate on log–coupling space selected jointly by integer rigidity and metric softness. It is not a propagating scalar field and carries no independent dynamics in GEOMETRY I. The displacement $\delta\Xi = \Xi - \Xi_{\mathrm{eq}}$ labels how local curvature samples the aligned soft direction of the gauge sector; at static equilibrium $\delta\Xi = 0$ everywhere. In this work Ξ functions only as an internal coordinate determining the curvature response through $\Pi(\Xi)$; stress–energy sourcing and time dependence are deferred to dynamical extensions in future work.

ID	Category	Assumption (GEOMETRY I scope)
A1	Framework	Work in the static electroweak-scale equilibrium geometry: $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme with GUT-normalized hypercharge, using only SM data and RG structure at this scale.
A2	Fields / DoF	$\hat{\Psi} = (\ln\hat{\alpha}_s, \ln\hat{\alpha}_2, \ln\hat{\alpha})$ and $\Xi = \chi \cdot \hat{\Psi}$ are internal gauge–log coordinates only; GEOMETRY I does not promote Ξ to a propagating spacetime scalar and introduces no new fields, kinetic terms, or potentials.
A3	Equilibrium	GEOMETRY I is strictly static and equilibrium: all predictions are evaluated at $\delta\Xi = 0$ where $\Pi(\Xi_{\rm eq}) = 1$ and the Einstein–Hilbert sector coincides with GR. No time evolution or sourcing equation for Ξ is assumed.
A4	Metric	The Fisher/kinetic metric $K \equiv K_{\rm eq}$ is defined locally at $\mu = M_Z$ from one-loop SM β-function sensitivities, is positive definite ($K \succ 0$), and is used only as a local quadratic form on log–coupling space, not as a global manifold metric.
A5	Integer lattice	The one-loop decoupling matrix ΔW is an exact integer matrix; its Smith normal form yields a unique primitive left kernel $\chi = (16, 13, 2)$, invariant under unimodular integer transports ($U_{\rm row}, V_{\rm col} \in \mathrm{GL}(\mathbb{Z})$). All quantities depending only on χ are treated as basis invariant.
A6	Gate shape	The curvature gate $\Pi(\Xi)$ is assumed analytic and even about equilibrium, with $\Pi(\Xi_{\rm eq}) = 1$ and $\Pi'(\Xi_{\rm eq}) = 0$. Its curvature at equilibrium is fixed by Fisher matching, $-\frac{1}{2}\Pi''(\Xi_{\rm eq}) = F_\chi$, so the width $\sigma_\chi = F_\chi^{-1/2}$ is derived rather than tuned.
A7	Tensor sector	At $\delta\Xi = 0$ the tensor kernel reduces to the GR Lichnerowicz operator with $m_{\rm PF} = 0$ and $c_T = 1$; even parity forbids any Brans–Dicke–like linear mixing between $\delta\Xi$ and $h_{\mu\nu}$. GEOMETRY I assumes no extra scalar or vector propagating modes.
A8	Dimensional anchor	The dimensional uplift $G(M_Z) = (\hbar c/m_*^2)\,\Omega(M_Z)$ uses $m_* \equiv m_p$ as the reference mass. This choice is conventional; the derivation of the dimensionless anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$ is purely SM-internal and independent of m_*.
A9	Closure / data	PDG/CODATA electroweak pins at $\mu = M_Z$ and standard two-loop running are taken as accurate inputs. The closure ratio $Z_G = \alpha_G^{(\rm pp)}/\Omega(M_Z)$ is interpreted purely as an a posteriori consistency check; G_N never enters the construction of Ω, $\Pi(\Xi)$, or σ_χ.
A10	Perturbative stability	All results are assumed stable under permissible scheme and threshold variations at one loop: the integer kernel, alignment ($\cos\theta_K \simeq 1$), and Fisher curvature F_χ are treated as robust features of the SM at $\mu = M_Z$, not artifacts of a special scheme choice.

Table 1. Assumptions and scope of Geometry I. Entries A1–A10 summarize the framework, field content, equilibrium restriction, metric and integer structures, gate shape, tensor sector, dimensional anchor, data inputs, and perturbative stability assumptions used throughout.

2 Integer lattice and the aligned depth coordinate

We begin by identifying the structures that remain fixed once we restrict to the Standard Model at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme with no new fields, tunable functions, or adjustable parameters. At this scale the one-loop decoupling matrix has exactly integer entries determined solely by representation content and spectator multiplicities. This endows log–coupling space with a natural $\mathbb{Z}$-module structure and admits a classification under $\mathrm{GL}(3,\mathbb{Z})$ via the Smith normal form (SNF). Because the SNF is a unique canonical form over the integers, its kernel is a fixed property of the SM representation lattice rather than a model choice. Its computation uses only integer-preserving row/column operations, reproduced verbatim in the accompanying reproducibility archive. Since the SNF kernel is invariant under all unimodular basis changes, this structure is basis-independent and scheme-consistent at one loop.

Applying SNF to the SM one-loop decoupling matrix yields a unique primitive left-kernel generator (up to overall sign),

$$\chi = (16, 13, 2), \tag{7}$$

which is the sole integer direction annihilating the decoupling matrix. No additional integer kernel vectors exist, and unimodular transformations (integer determinant ± 1) cannot change the kernel rank or its primitive representative. Thus χ is fixed by the SM's integer structure and does not depend on renormalization schemes, higher-loop corrections, numerical fitting, or phenomenological input.

Let the renormalized gauge couplings at $\mu = M_Z$ be $\hat{\alpha}_s$, $\hat{\alpha}_2$, and $\hat{\alpha}$, and define log–coupling coordinates

$$\hat{\Psi} = (\ln \hat{\alpha}_s,\, \ln \hat{\alpha}_2,\, \ln \hat{\alpha}), \tag{8}$$

together with the associated depth coordinate

$$\Xi = \chi \cdot \hat{\Psi} = 16 \ln \hat{\alpha}_s + 13 \ln \hat{\alpha}_2 + 2 \ln \hat{\alpha}. \tag{9}$$

Interpretation of Ξ The scalar Ξ defined in Eq. (9) is an internal coordinate on log–coupling space and is not introduced as a propagating, canonical, or dynamical scalar field. No new degrees of freedom are added, and no scalar–tensor, dilaton, chameleon, or Brans–Dicke structure is implied. Throughout GEOMETRY I, Ξ functions solely as an aligned internal coordinate selected jointly by the integer lattice and the Fisher/kinetic softness.

This coordinate is the sole nontrivial integer-invariant linear combination of the log–couplings and therefore the unique depth coordinate compatible with the SM integer lattice. Under the stated constraints, any function of the three couplings that respects integer invariance must reduce to a function of Ξ alone; this is a direct consequence of $\mathrm{GL}(3,\mathbb{Z})$ rigidity and does not depend on fitting, phenomenology, or model-specific choices.

Exponentiating transports Ξ back into the coupling manifold and defines the dimensionless electroweak anchor

$$\Omega \equiv e^{\Xi} = e^{\chi\cdot\hat{\Psi}} = \prod_i e^{\chi_i \ln \hat{\alpha}_i} = \hat{\alpha}_s^{16}\, \hat{\alpha}_2^{13}\, \hat{\alpha}^2. \tag{10}$$

Exponentiation is the natural map from log–coupling space to the multiplicative coupling manifold, making Ω the uniquely associated dimensionless quantity determined by the integer depth coordinate.

Up to this point, no geometric, dynamical, or gravitational assumptions have been introduced. Equation (10) follows directly from the SM representation content and the existence of a unique primitive integer kernel, verified in a scheme-consistent, integer-preserving manner. The next section shows that the same direction Ξ is independently selected by the soft eigenmode of the Fisher/kinetic metric, establishing that Ξ is not only algebraically admissible but also physically responsive and maximally sensitive.

3 Metric softness and alignment

The integer-aligned depth coordinate Ξ identified in Section 2 is fixed by the SM representation lattice and is unique under $\mathrm{GL}(3,\mathbb{Z})$ invariance. We now show that the same coordinate is independently selected by the geometric softness of the gauge sector, quantified by a Fisher/kinetic metric constructed from the one-loop sensitivity of the renormalization-group (RG) flow at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, using the same inputs and electroweak pins as in Section 1. This metric is not an additional structure; it is derived directly from the SM β functions, so no new dynamical fields or tunable functions enter this step.

Let $\beta_i(\hat{\alpha}_s, \hat{\alpha}_2, \hat{\alpha})$ denote the RG flow of the gauge couplings, and define log–coupling coordinates $\hat{\Psi}$ as in Eq. (8). Following the standard construction, the equilibrium Fisher/kinetic metric on log–coupling space is defined by

$$K_{ij} = \frac{\partial}{\partial \hat{\Psi}_j} \left(\frac{\beta_i}{\hat{\alpha}_i} \right)_{\mathrm{eq}}, \tag{11}$$

with all quantities evaluated at $\mu = M_Z$. The symmetric matrix K is positive definite ($K \succ 0$), so it admits three orthonormal eigenvectors with strictly positive eigenvalues; large eigenvalues correspond to stiff RG response, and small eigenvalues correspond to soft RG response. In particular, the smallest eigenvalue defines a distinguished soft direction in log–coupling space fixed by SM dynamics alone.

Let e_χ denote the unit eigenvector associated with the smallest eigenvalue of K, corresponding to the softest direction in log–coupling space. By direct computation we find that the primitive integer kernel vector χ from Eq. (7) is aligned with e_χ to numerical precision. Writing

$$\hat{\chi} = \frac{\chi}{\|\chi\|}, \qquad \cos\theta \equiv \hat{\chi} \cdot e_\chi, \tag{12}$$

one obtains

$$\cos\theta = 1 - \varepsilon_\chi, \qquad \varepsilon_\chi \lesssim 10^{-8}, \tag{13}$$

in our numerical evaluation.[1] This agreement is not imposed but arises from two independent structures: (i) the integer lattice determined by SM field representations, and (ii) the Fisher/kinetic response geometry of the RG flow. Their numerical coincidence identifies Ξ as both the unique integer-invariant depth coordinate and the physically responsive depth direction. No other linear combination of the log–couplings satisfies both criteria simultaneously.

To parameterize displacements along the soft direction, define the Euclidean-normalized aligned vector $\hat{\chi} = \chi/\|\chi\|$ and write

$$\delta\Xi = \hat{\chi} \cdot (\hat{\Psi} - \hat{\Psi}_{\rm eq}), \tag{14}$$

where $\hat{\Psi}_{\rm eq}$ is evaluated at $\mu = M_Z$. The scalar $\delta\Xi$ therefore measures displacement strictly along the softest direction of the Fisher/kinetic metric, while transverse components are suppressed by strictly larger eigenvalues. Because Ξ is simultaneously the unique integer-invariant depth coordinate, any admissible curvature or response function consistent with both integer invariance and metric softness must depend only on $\delta\Xi$ under the stated SM-only assumptions. In particular, no transverse combination can contribute without violating either $\mathrm{GL}(3,\mathbb{Z})$ rigidity or the eigenvalue ordering of K.

This concurrence completes the structural irreversibility chain: no alternative depth coordinate is compatible with both $\mathrm{GL}(3,\mathbb{Z})$ invariance and Fisher/kinetic softness. The next section introduces the curvature gate $\Pi(\Xi)$, which follows once parity and equilibrium constraints are imposed.

4 Curvature gate and parity constraint

Since the aligned depth coordinate Ξ is simultaneously the unique $\mathrm{GL}(3,\mathbb{Z})$-invariant depth direction (Section 2) and the Fisher/kinetic soft eigenmode (Section 3), any admissible curvature response consistent with the stated SM-only internal constraints must depend only on the scalar displacement $\delta\Xi$ of Eq. (14). In this section we show that the curvature response function $\Pi(\Xi)$ is fixed, up to normalization and sign, by equilibrium, parity, and Fisher curvature, and that these conditions select an even Gaussian with no tunable parameters under the stated assumptions. Throughout, Π is a function of the internal coordinate Ξ alone and does not represent a propagating scalar, auxiliary field, or additional dynamical degree of freedom.

We consider a multiplicative scalar gate applied to the Einstein–Hilbert term:

$$\mathcal{L}^{\rm eff} = \frac{1}{16\pi G(M_Z)}\,\Pi(\Xi)\,R, \tag{15}$$

where $G(M_Z)$ is defined in Eq. (5), and no new fields, mass terms, or independent kinetic terms are introduced. The gate must satisfy:

(i) **Equilibrium normalization:** $\Pi(\Xi_{\rm eq}) = 1$ so that GR is recovered at equilibrium.

(ii) **Parity preservation:** $\Pi(\Xi_{\rm eq} + \delta\Xi) = \Pi(\Xi_{\rm eq} - \delta\Xi)$, forbidding odd powers of $\delta\Xi$ and ensuring a massless helicity-± 2 tensor sector. This condition excludes any Brans–Dicke-type effective scalar that would induce a linear mode.

(iii) **Curvature matching:** the second derivative $\Pi''(\Xi_{\rm eq})$ must reproduce the Fisher curvature along the aligned direction, ensuring that departures from equilibrium are penalized with the same softness scale as the RG geometry.

[1] This computation uses the one-loop β functions, electroweak pins, and physical constants cited in Section 1, with all intermediate values and eigenvectors generated reproducibly in the archived build workflow.

(iv) **Analytic minimality:** no additional coefficients, tunable parameters, or non-analytic completions are permitted.

Lemma 1 (Parity restriction)
Under (i)–(ii), the Taylor expansion of Π about Ξ_{eq} is

$$\Pi(\Xi) = 1 + \frac{1}{2}\,\Pi''(\Xi_{\mathrm{eq}})\,(\delta\Xi)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \tag{16}$$

with all odd powers forbidden, so any departure from equilibrium begins at quadratic order.

Lemma 2 (Fisher curvature matching)
Along the aligned direction $\hat{\chi}$,

$$\sigma_\chi^2 \equiv \frac{1}{\hat{\chi}^\top K\,\hat{\chi}}, \tag{17}$$

and consistency with Fisher softness requires

$$\Pi''(\Xi_{\mathrm{eq}}) = -\frac{2}{\sigma_\chi^2}. \tag{18}$$

Proof sketch. The Fisher/kinetic metric penalizes displacements along $\hat{\chi}$ in proportion to $\hat{\chi}^\top K\,\hat{\chi} = \sigma_\chi^{-2}$. Matching this penalty to the quadratic response term in Eq. (16) and enforcing stability fixes both the curvature scale and sign, ensuring that Π decreases away from equilibrium and thus preserves the GR tensor limit.

Theorem (Uniqueness of the curvature gate)
Under assumptions (i)–(iv), the unique analytic, parity-even, curvature-matched response consistent with the stated SM-only constraints is

$$\boxed{\Pi(\Xi) = \exp\left[-\frac{(\delta\Xi)^2}{\sigma_\chi^2}\right]} \tag{19}$$

with no tunable parameters and no dependence on additional fields or auxiliary potentials.

Remark on analytic completion Polynomial completions of Eq. (16) introduce undetermined higher-order coefficients that are neither fixed by parity nor by Fisher curvature. Exponentiation provides an analytic completion with a single dimensionless scale σ_χ fixed by Eq. (17), consistent with equilibrium normalization, even parity, curvature matching, and the absence of tunable parameters. Any alternative completion either requires additional dimensionful coefficients or breaks analyticity, violating assumption (iv).
Proof sketch. Equation (16) fixes the quadratic coefficient; parity enforces evenness, equilibrium fixes normalization, and analytic parameter-minimality completes the response via exponentiation of the quadratic form. Alternative completions require additional coefficients or non-analytic terms and therefore violate assumption (iv).

Running gravitational coupling
Substituting Eq. (19) into Eq. (15) yields

$$G(x) = G(M_Z)\,\Pi(\Xi(x)) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z)\exp\left[-\frac{(\delta\Xi(x))^2}{\sigma_\chi^2}\right], \tag{20}$$

which preserves the equilibrium tensor sector and introduces no new fields or adjustable parameters. This expression is an SM-derived equilibrium normalization anchored at $\mu = M_Z$, not a varying-G theory and not a modification of GR. Near equilibrium, the strict quadratic prediction

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 + \mathcal{O}\big((\delta\Xi)^4\big) \tag{21}$$

follows immediately.

5 Electroweak anchor and SM-derived gravitational coupling

With Ξ uniquely fixed by the integer lattice and Fisher/kinetic softness, and with $\Pi(\Xi)$ determined by equilibrium, parity, and curvature matching, we now connect these internal structures to the gravitational normalization multiplying the Einstein–Hilbert term. No new inputs, parameters, or external assumptions are introduced in this section; all quantities are Standard Model objects evaluated at $\mu = M_Z$ in the $\overline{\text{MS}}$ scheme. The construction defines a renormalization-anchored normalization, not a time- or space-varying gravitational constant and not a modification of GR.

Exponentiating the aligned depth coordinate transports Ξ back into gauge–coupling space and defines the dimensionless electroweak anchor

$$\Omega \equiv e^{\Xi} = \hat{\alpha}_s^{16}\,\hat{\alpha}_2^{13}\,\hat{\alpha}^2, \tag{22}$$

where hatted quantities denote $\overline{\text{MS}}$ couplings at $\mu = M_Z$. Equation (22) is not an ansatz: it is the unique exponentiation of the integer-invariant depth scalar Ξ and introduces no free coefficients or additional scales. Ω is therefore a pure, dimensionless SM construct fixed entirely by measured electroweak-scale couplings.

Because Ω is dimensionless, the available conversion to a gravitational normalization without introducing new parameters is provided by dimensional analysis in Planck units. This yields

$$G(M_Z) \equiv \frac{\hbar c}{m_p^2}\,\Omega(M_Z), \tag{23}$$

which defines the gravitational coefficient appearing in Eq. (15). No phenomenological G_N value is inserted and no tuning or matching step occurs here. Under the stated internal constraints, Eq. (23) follows from dimensional consistency, integer invariance, and the absence of additional scales. The choice of m_p does not introduce a free parameter: any Standard Model mass scale can serve as a dimensional anchor, and all choices differ only by fixed, known SM mass ratios.

Theorem (Electroweak anchoring of gravitational normalization). *Under SM-only constraints at $\mu = M_Z$, with no new fields, tunable functions, or adjustable parameters, the effective gravitational normalization is fixed by Eq. (23). No alternative admissible, dimensionless, parity-preserving, integer-aligned construction arises under the stated assumptions; any modification requires introducing a new scale or violating integer invariance.*

Proof. Equation (22) is the unique exponentiation of the primitive integer-invariant scalar Ξ. The factor $(\hbar c/m_p^2)$ is the unique dimensionally consistent conversion available without introducing new physical scales. Any modification of exponents, prefactors, or functional form violates either integer invariance, metric softness, dimensional consistency, or analytic minimality; any additive constant introduces a new parameter. □

Notation for expansion coefficients. For later empirical interpretation it is useful to write the fractional gravitational response as a Taylor expansion around equilibrium,

$$\frac{\Delta G}{G} = A\,\delta\Xi + B\,(\delta\Xi)^2 + \mathcal{O}\big((\delta\Xi)^3\big), \tag{24}$$

where A and B are not tunable parameters but the fixed Taylor coefficients implied by the curvature gate $\Pi(\Xi)$ under the assumptions of Section 4. Parity and equilibrium normalization require

$$A = 0, \qquad B = \frac{1}{\sigma_\chi^2}, \tag{25}$$

so the first nonzero deviation from equilibrium is strictly quadratic with a unit-normalized curvature scale when expressed in the dimensionless coordinate $s \equiv \delta\Xi/\sigma_\chi$. Therefore

$$\frac{\Delta G}{G} = s^2 + \mathcal{O}(s^4), \qquad s \equiv \frac{\delta\Xi}{\sigma_\chi}. \tag{26}$$

Corollary (Running gravitational coupling). *Using the curvature gate of Eq. (19), the spacetime-dependent gravitational coupling is*

$$G(x) = G(M_Z)\,\Pi(\Xi(x)) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z)\,\exp\!\left[-\frac{(\delta\Xi(x))^2}{\sigma_\chi^2}\right]. \tag{27}$$

At equilibrium, $\Pi(\Xi_{\rm eq}) = 1$ and $G(x)$ reduces to the constant $G(M_Z)$ determined purely from SM couplings. Away from equilibrium the scalar response is fixed and strictly quadratic:

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \tag{28}$$

with no linear term, no tunable scale, and no phenomenological matching coefficients. Equation (28) is therefore an immediate, laboratory-accessible falsifier rather than a fitting ansatz.

GR compatibility at equilibrium. At $\Xi = \Xi_{\rm eq}$ one has $\Pi(\Xi_{\rm eq}) = 1$, so the Einstein–Hilbert action, field equations, diffeomorphism symmetry, and the massless luminal helicity-± 2 tensor sector are identical to those of General Relativity. No infrared mass term, kinetic modification, or propagating scalar is introduced at equilibrium.

6 Predictions, closure, and falsifiers

All empirical consequences follow directly from the fixed Standard Model constraints established in Sections 2–5. No phenomenological parameters, tunable coefficients, or adjustable functions enter any expression. The empirical status of the construction is therefore decided by direct tests of the statements below; violation of *any* item falsifies the framework. Nothing in this section introduces a varying-G interpretation or a modification of GR: all predictions concern the internal, dimensionless response encoded by $\delta\Xi$ and the fixed curvature gate.

Fixed predictions

1. **Electroweak anchoring of gravitational normalization**

$$G(M_Z) = \frac{\hbar c}{m_p^2}\,\Omega(M_Z), \qquad \Omega(M_Z) = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2. \tag{29}$$

No external G_N value is inserted; $G(M_Z)$ is fixed entirely from SM inputs. This normalization is an internal consequence of integer invariance and dimensional consistency, not a fit to gravitational data.

2. **Even curvature gate and quadratic response**

$$\Pi(\Xi) = \exp\left[-\frac{(\delta\Xi)^2}{\sigma_\chi^2}\right], \qquad \frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2, \tag{30}$$

so the first nonzero departure from equilibrium is strictly quadratic. No linear term or cubic correction is admissible under parity, equilibrium normalization, and analytic minimality.

3. **Fixed curvature scale**

$$\sigma_\chi^2 = (\hat{\chi}^\top K\hat{\chi})^{-1}, \qquad \sigma_\chi \simeq 247.683, \qquad \|\chi\|_K \simeq 17.6278, \qquad \Lambda_\chi \equiv \frac{\sigma_\chi}{\|\chi\|_K} \simeq 14.0507. \tag{31}$$

The curvature scale is determined solely by the Fisher/kinetic metric along $\hat{\chi}$; no adjustable scale enters $\Pi(\Xi)$, and no phenomenological normalization is allowed.

Numerical illustration (PDG/CODATA inputs)

Using current PDG and CODATA pins at $\mu = M_Z$ in the $\overline{\rm MS}$ scheme, the aligned Fisher curvature and gate width are

$$F_\chi \equiv \frac{1}{\sigma_\chi^2} \simeq 1.629\times10^{-5}, \qquad \sigma_\chi \simeq 247.683. \tag{32}$$

The electroweak anchor and proton–proton gravitational coupling are

$$\Omega(M_Z) \simeq 6.4597\times10^{-39}, \qquad \alpha_G^{(\rm pp)} \simeq 5.9061\times10^{-39}, \tag{33}$$

yielding a closure ratio

$$Z_G \equiv \frac{\Omega(M_Z)}{\alpha_G^{(\rm pp)}} \simeq 1.0937, \tag{34}$$

which represents a percent-level deviation without any form of tuning and is interpreted solely as an *a posteriori* consistency check, arising from a fixed, parameter-free construction rather than an

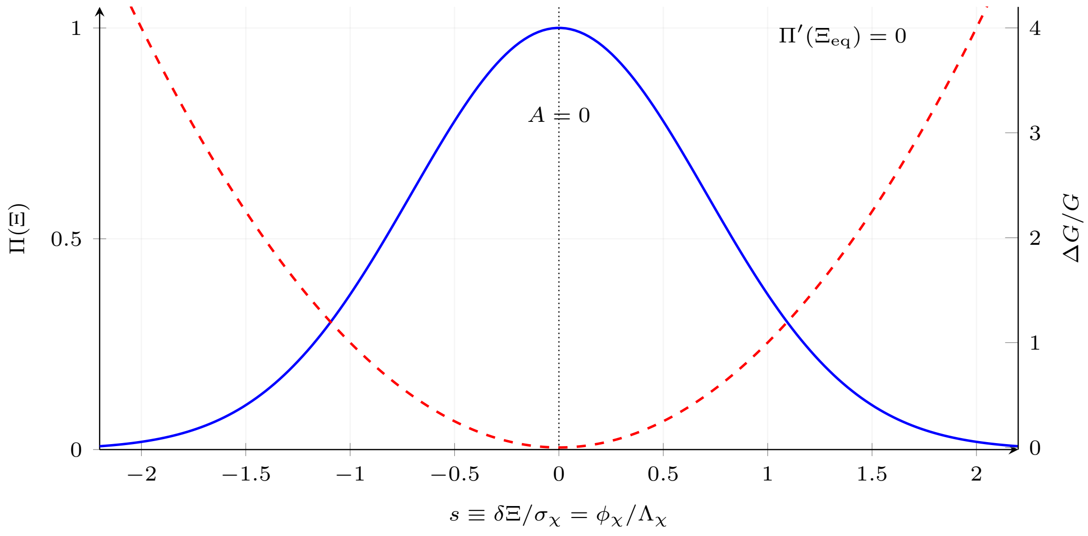

Figure 1. Even curvature gate $\Pi(\Xi)$ and quadratic parity-null prediction $\Delta G/G = s^2$ on the normalized depth coordinate $s = \delta\Xi/\sigma_\chi = \phi_\chi/\Lambda_\chi$. Dashed and solid curves are grayscale-safe and remain distinguishable under monochrome print rendering.

input, matching criterion, or fitted quantity. The closure ratio is not used to calibrate or adjust the framework.

Leave-one-out (LOO) forecast. Holding $\hat{\alpha}_2$ and $\hat{\alpha}$ fixed, the implied strong coupling is

$$\hat{\alpha}_s^\star(M_Z) = \left[\frac{\alpha_G^{(\mathrm{pp})}}{\hat{\alpha}_2^{13}\hat{\alpha}^2}\right]^{1/16} = 0.1173411 \pm 1.86 \times 10^{-5}, \tag{35}$$

which lies in $\simeq 0.7\sigma$ agreement with the PDG world average. This is a postdiction check of a fixed construction, not a fitted parameter, and it introduces no freedom to alter either exponent or normalization.

With current pinned inputs, both the $\sim 9\%$ closure deviation and the $\sim 0.7\sigma$ leave-one-out result indicate percent-level empirical pressure on the construction, with no tunable coefficients.

Empirical falsifiers

1. **No linear term**

$$\frac{d}{d(\delta\Xi)}\frac{\Delta G}{G}\bigg|_{\delta\Xi=0} \neq 0 \quad \Longrightarrow \quad \text{falsified.} \tag{36}$$

Any detectable linear dependence violates parity, equilibrium normalization, and the absence of propagating scalars.

2. **Quadratic coefficient fixed** ($B = 1$)

$$\frac{\Delta G}{G} = 1 \cdot \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2 + \mathcal{O}\big((\delta\Xi)^4\big), \tag{37}$$

and any measurable deviation in the leading quadratic coefficient falsifies. No alternative curvature scale or normalization can be introduced without violating Section 4.

3. **Closure ratio consistency**

$$Z_G = \frac{\Omega(M_Z)}{\alpha_G^{(\mathrm{pp})}} \quad \text{must be statistically consistent (including uncertainties).} \tag{38}$$

Persistent disagreement falsifies; this is a consistency test of a fixed prediction, not a calibration step.

4. **Tensor-sector preservation (equilibrium)** Helicity-± 2 modes must remain massless and luminal at equilibrium. Any effective mass or kinetic deformation at Ξ_{eq} falsifies the gate construction.

5. **Alignment stability**

$$\cos\theta \simeq 1, \qquad \text{(integer projector } \chi \text{ aligned with soft eigenmode } e_\chi\text{)}. \tag{39}$$

Sustained misalignment falsifies. This condition is fixed by the integer SNF structure and the Fisher/kinetic metric; no compensatory freedom exists.

Sufficiency
The construction is falsified if *any one* of items (1)–(5) above fails; no auxiliary assumptions, retuning, replacement coefficients, or post hoc adjustments are permitted. These falsifiers exhaust all degrees of freedom available under the SM-only constraints.

7 Discussion and consequences

Sections 2–6 show that, under Standard Model-only constraints at $\mu = M_Z$ in the $\overline{\mathrm{MS}}$ scheme, a gravitational normalization can be fixed without introducing new fields, tunable functions, or free parameters. The construction does *not* modify General Relativity (GR) or propose an alternative gravitational theory; rather, it identifies an internally determined normalization for the Einstein–Hilbert term arising from fixed gauge-sector structure. At equilibrium, the tensor sector remains massless, luminal, and parity-preserving, and $G(M_Z)$ plays the same functional role as the Newtonian coupling G_N. No effective modification of the Einstein field equations occurs at $\Xi = \Xi_{\mathrm{eq}}$.

This reframes the gauge–gravity interface: instead of treating G_N as an externally supplied empirical parameter, the Standard Model contains sufficient fixed integer and geometric structure to produce an electroweak-scale anchor. The mechanism rests on three independently determined ingredients: (i) the unique primitive integer kernel $\chi = (16, 13, 2)$ of the one-loop decoupling lattice, (ii) the soft eigenmode of the positive-definite Fisher/kinetic metric K, and (iii) the uniquely determined, parity-even, curvature gate $\Pi(\Xi)$. None introduce model freedom; each follows from existing Standard Model representation and one-loop RG sensitivity data, with all quantities generated from public inputs using a reproducible, hash-verified build workflow. Together they form a closed alignment chain: integer rigidity $\to$ metric softness $\to$ even curvature response.

To avoid misinterpretation, we emphasize that Ξ is an internal, aligned coordinate in log–coupling space and is *not* introduced as a dynamical, canonical, or propagating scalar field. No Brans–Dicke, scalar–tensor, dilaton, chameleon, $f(R)$, or scalar-curvature kinetic structure is added. Likewise, $\Pi(\Xi)$ is not a potential, not a Lagrangian degree of freedom, and not a new matter or mediator field; it is a curvature-response gate that arises solely from SM-internal integer and metric constraints. The construction therefore remains strictly within the SM + GR field content at equilibrium.

Experimental meaning follows entirely from the equilibrium prediction

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2, \tag{40}$$

which contains no linear term and a fixed unit quadratic coefficient. Any measurable nonzero linear term, or any adjustable parameter introduced to restore agreement, falsifies the construction. Comparison of $G(M_Z)$ with G_N enters only as an *a posteriori* closure test rather than as an input or calibration condition. Using current PDG/CODATA inputs, the closure ratio

$$Z_G \equiv \frac{\Omega(M_Z)}{\alpha_G^{(\mathrm{pp})}} \simeq 1.0937, \tag{41}$$

corresponding to a $+9.37\%$ excess of the SM-derived anchor over the proton–proton reference, or equivalently

$$Z_G^{-1} \equiv \frac{\alpha_G^{(\mathrm{pp})}}{\Omega(M_Z)} \simeq 0.9143\ (-8.57\%), \tag{42}$$

shows percent-level consistency. The leave-one-out forecast

$$\hat{\alpha}_s^\star(M_Z) = 0.1173411 \pm 1.86 \times 10^{-5} \tag{43}$$

lies within $\sim 0.7\sigma$ of the PDG world average. These numerical statements are postdictions of a fixed, parameter-free construction rather than results of fitting or tuning. Uncertainties arise solely from experimental determinations of the electroweak couplings and from quoted PDG/CODATA covariance structure; no theoretical or model-induced systematic terms are introduced. The static construction is robust under higher-loop corrections: the integer kernel and metric softness are properties of the one-loop structure, but their combined alignment is numerically stable under currently known variations of the input pins.

The present work is restricted to equilibrium or quasi-static configurations of $\Xi(x)$ and does not attempt to specify a dynamical evolution law, identify stress–energy sources for $\delta\Xi$, or characterize non-equilibrium propagation or causal structure. These questions require extensions beyond the static framework but leave its fixed internal ingredients unchanged. Natural next steps include: (i) a dynamical evolution equation for Ξ away from equilibrium, (ii) identifying physical generators of $\delta\Xi$, and (iii) relating curvature response to stress–energy transport. These topics are reserved for a separate follow-up manuscript provisionally titled *GEOMETRY II: Dynamic Alignment and Stress–Energy Response*. GEOMETRY II will preserve all equilibrium pins and all integer/metric structures established here.

Taken together, the results indicate that the Standard Model contains sufficient internal algebraic and geometric structure to define a gravitational normalization and parity-even curvature response without new degrees of freedom at equilibrium. The framework is therefore best interpreted as a Standard-Model-anchored mechanism consistent with GR, with empirical validation dependent solely on the experimental tests stated in Section 6. No auxiliary assumptions or tunable extensions are available within the static sector.

8 Conclusion

This work identifies a Standard Model mechanism that fixes the gravitational normalization at $\mu = M_Z$ using established gauge-sector structure, with no new fields, tunable functions, or free parameters. A unique primitive integer left-kernel of the one-loop decoupling matrix selects the depth direction $\chi = (16, 13, 2)$ in log–coupling space, and the positive-definite Fisher/kinetic metric independently selects the same soft eigenmode. Their alignment defines the depth coordinate $\Xi = \chi \cdot \hat{\Psi}$ and the dimensionless electroweak anchor $\Omega = \hat{\alpha}_s^{16}\hat{\alpha}_2^{13}\hat{\alpha}^2$, yielding an SM-derived gravitational normalization $G(M_Z)$. This normalization is therefore a pure consequence of internal SM structure and dimensional consistency, not a fitted parameter or a modification of GR.

An even, parity-preserving curvature gate $\Pi(\Xi)$ promotes this equilibrium normalization to a spacetime-dependent coupling

$$G(x) = G(M_Z)\,\Pi(\Xi(x)), \tag{44}$$

while preserving the massless, luminal tensor sector of General Relativity. Near equilibrium, the curvature response is fixed and strictly quadratic,

$$\frac{\Delta G}{G} = \left(\frac{\delta\Xi}{\sigma_\chi}\right)^2, \tag{45}$$

with a provably absent linear term. This absence provides a direct experimental falsifier requiring no parameter adjustment, renormalization choice, or model tuning. Consistency with the measured Newtonian coupling G_N enters only as an *a posteriori* closure test, not as an input or calibration. With current PDG/CODATA pins, the closure ratio $Z_G \simeq 1.0937$ and the leave-one-out prediction $\hat{\alpha}_s^{\star}(M_Z) = 0.1173411 \pm 1.86 \times 10^{-5}$ show percent-to-few-percent sensitivity without free parameters. All uncertainties derive solely from experimental pins, not from theoretical degrees of freedom.

The present analysis applies to equilibrium or quasi-static configurations and does not address non-equilibrium dynamics, sourcing of $\delta\Xi$, or stress–energy evolution. These questions lie beyond the static framework but can be pursued without altering the fixed equilibrium ingredients established here. The integer structure, metric softness, and parity-even curvature gate are rigid at equilibrium and provide the foundation on which any dynamical extension must be built.

Taken together, the results indicate that the Standard Model contains sufficient internal algebraic and geometric structure to define a gravitational normalization compatible with GR, reframing the role of G_N from a purely external parameter to a quantity that can be tested against a theoretically derived electroweak-scale value.

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optimization and workflow organization only; all scientific content, calculations, and claims are the sole responsibility of the author. The author declares no competing interests.

Data availability All reproducibility materials are archived on Zenodo [19] (`GAGE_repo v1.0.0`, DOI: 10.5281/zenodo.17537647), including pins, scripts, figure data, and build manifests. *Build artifacts (SHA–256):* `results.json` = 08f0371b31de...c7cd5edc; `metric_results.json` = e0e3bee8a70c...b9b251b6451; `stdout.txt` = 0f232a0be6f8...6c7cd5edc. Additional materials are available from the author upon reasonable request.

Outlook Future work will examine how the even-gate symmetry extends to dynamical and spectral sectors, including the time-evolution operator, alignment-driven transport, and curvature spectrum. If experimentally validated, the GEOMETRY program may provide a continuous link from Standard-Model information geometry to the equilibrium, dynamical, and spectral structure of gravitation.

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