Primitive Integer Kernel of the Standard Model Decoupling Matrix and Its Composite Gauge Coupling

Michael DeMasi[1]

[1] *Independent researcher, Milford, Connecticut 06460, USA*

(Dated: December 11, 2025)

The one–loop decoupling matrix of the Standard Model (SM) gauge sector is built entirely from fixed representation–theoretic integers. Using exact integer arithmetic, we compute its Smith normal form and find invariant factors $(1, 8)$, implying a one–dimensional primitive integer right–kernel fixed solely by the SM field content. In the gauge–log basis this kernel becomes the unique integer triplet $\chi \equiv (16, 13, 2)$, which in turn defines the composite, parameter–free gauge coupling $\Omega \equiv \alpha_s^{16}\alpha_2^{13}\alpha^2$. Evaluated at $\mu = M_Z$ with PDG inputs in the $\overline{\text{MS}}$ scheme, $\Omega(M_Z)$ yields a closure ratio $Z_G \equiv \alpha_G^{pp}/\Omega(M_Z) = 0.9143$ with the dimensionless proton–proton gravitational coupling $\alpha_G^{pp} = G_N m_p^2/(\hbar c)$. Treating $\alpha_s(M_Z)$ as unknown and enforcing $Z_G = 1$ gives a leave–one–out closure value $\alpha_s^*(M_Z) = 0.117341$, compatible with the current world average. No gravitational dynamics or model are assumed; the comparison is reported as a numerical observation. All steps are implemented with exact integer arithmetic and are fully reproducible via an accompanying open–source archive, which documents this previously unrecognized SM–internal integer invariant and its associated composite gauge coupling.

I. INTRODUCTION

The one–loop decoupling matrix of the Standard Model (SM) gauge sector is fixed entirely by representation–theoretic integers [1–3]. As a result, any structural features it possesses are automatically independent of renormalization scheme and gauge basis. Viewed as an integer matrix, it admits canonical invariants under all unimodular transformations, and these encode basis–independent relations among the gauge interactions [4, 5]. Here we isolate this purely arithmetic content by computing the Smith normal form (SNF) of the SM one–loop decoupling matrix using standard polynomial–time algorithms [4].

The SNF yields invariant factors $(1, 8)$, demonstrating that the integer right–kernel is one–dimensional and uniquely fixed by the SM field content. Transported to the gauge–log basis, this kernel takes the primitive form

$$\chi \equiv (16, 13, 2), \tag{1}$$

providing a basis–independent weighted direction in gauge–coupling space.

Exponentiating the associated projection defines the composite, parameter–free quantity

$$\Omega \equiv \alpha_s^{16}\,\alpha_2^{13}\,\alpha^2, \tag{2}$$

which depends solely on the SM gauge couplings and the integer data encoded in χ. Using PDG inputs at $\mu = M_Z$ [6], the numerical value of Ω lies close to the dimensionless proton–proton gravitational coupling $\alpha_G^{pp} = G_N m_p^2/(\hbar c)$ constructed from CODATA 2022 constants [7]. This proximity is reported strictly as a numerical observation; no gravitational assumptions are introduced.

To test the internal consistency of this SM–derived structure, we perform a leave–one–out (LOO) closure analysis in which $\alpha_s(M_Z)$ is treated as unknown and determined by enforcing the unit–closure condition

$$Z_G \equiv \frac{\alpha_G^{pp}}{\Omega(M_Z)} = 1, \tag{3}$$

while $\alpha_2(M_Z)$ and $\alpha(M_Z)$ are held at their PDG values. This procedure determines a specific closure value $\alpha_s^*(M_Z)$; future precision determinations of the strong coupling that differ significantly from $\alpha_s^*(M_Z)$ would falsify the closure relation.

The remainder of this Letter presents the integer SNF computation, constructs the composite Ω, evaluates its numerical value, and performs the LOO test. All results are fully reproducible through the accompanying Zenodo archive [8].

II. INTEGER DECOUPLING MATRIX AND SMITH NORMAL FORM

A. Field content and one–loop weights

The field content and quantum numbers follow the standard SM assignments [1, 3, 6]. To apply the Smith normal form to the one–loop decoupling matrix, we construct an integerized weight matrix $W_{\mathbb{Z}}$ in which all hypercharge contributions w_1 are rendered integral by a single overall normalization of the $U(1)_Y$ weights.[1] Using the standard one–loop coefficients, the fermionic and scalar contributions take the form

$$w_1^{(f)} = 12\sum Y^2, \qquad w_1^{(s)} = 3\sum Y^2, \tag{4}$$

[1] The relative factors 12 (fermions) and 3 (scalars) arise from the standard one–loop hypercharge coefficients—4 for Weyl fermions and 1 for scalar degrees of freedom—multiplied by the minimal global integer normalization that makes all hypercharge weights in $W_{\mathbb{Z}}$ integral. This overall scaling does not affect the primitive SNF kernel.

ensuring that every entry of $W_{\mathbb{Z}}$ (and therefore of ΔW_{EM}) is an integer. The integer normalization of the hypercharge column does not modify the primitive SNF kernel. Table I lists the light–species contributions entering $W_{\mathbb{Z}}$ on a generic renormalization window W.

TABLE I. Light species columns for $W_{\mathbb{Z}}$ on a window W. Here N_g denotes the number of fermion generations and N_H the number of Higgs doublets. Hypercharge weights are made integral using Eq. (4).

Species	$(SU(3), SU(2), Y)$	$\mathrm{dof}_{\mathrm{spec}}$	$2T_3$	$2T_2$	w_3	w_2	w_1
Q_L	$(\mathbf{3}, \mathbf{2}, 1/6)$	$6N_g$	1	1	$6N_g$	$6N_g$	$12\sum Y^2$
u_R	$(\mathbf{3}, \mathbf{1}, 2/3)$	$3N_g$	1	0	$3N_g$	0	$12\sum Y^2$
d_R	$(\mathbf{3}, \mathbf{1}, -1/3)$	$3N_g$	1	0	$3N_g$	0	$12\sum Y^2$
L_L	$(\mathbf{1}, \mathbf{2}, -1/2)$	$2N_g$	0	1	0	$2N_g$	$12\sum Y^2$
e_R	$(\mathbf{1}, \mathbf{1}, -1)$	$1N_g$	0	0	0	0	$12\sum Y^2$
H	$(\mathbf{1}, \mathbf{2}, 1/2)$	$2N_H$	0	1	0	$2N_H$	$3\sum Y^2$
W	adj. $(\mathbf{1}, \mathbf{3}, 0)$	1	0	4	0	4	0
G	adj. $(\mathbf{8}, \mathbf{1}, 0)$	1	6	0	6	0	0

Note: For Weyl fermions $w_1^{(f)} = 12\sum Y^2$ and for scalar degrees of freedom $w_1^{(s)} = 3\sum Y^2$. For the Higgs doublet $H \sim (\mathbf{1}, \mathbf{2}, 1/2)$ one has $\sum_{\mathrm{dof}} Y^2 = \frac{1}{2}$, giving $w_1(H) = 3$. All entries of $W_{\mathbb{Z}}$ (and hence ΔW_{EM}) are therefore integral.

B. Decoupling windows and the matrix ΔW

For any renormalization window $W = [\mu_{\mathrm{low}}, \mu_{\mathrm{high}}]$, the one–loop beta function of each gauge coupling receives contributions only from species lighter than μ_{high} and heavier than μ_{low}, consistent with the standard decoupling of heavy fields [9]. The integerized weight matrix $W_{\mathbb{Z}}(W)$ therefore changes only when the particle content of the window changes. A *decoupling step* $W \to W'$ induces a difference

$$\Delta W = W_{\mathbb{Z}}(W') - W_{\mathbb{Z}}(W), \tag{5}$$

which collects the integer weight jumps associated with that step.

For the electroweak–to–EM transition, the relevant 3×3 block reduces to a 2×3 system after removing rows that are identical across the window and hence contribute no change to Eq. (1). Using the weights in Table I, this yields

$$\Delta W_{\mathrm{EM}} = \begin{pmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{pmatrix}, \tag{6}$$

where the first row represents the net $SU(3)$, $SU(2)$, and integerized $U(1)_Y$ weights of one SM generation, and the second captures the corresponding Higgs and gauge–boson differences across the electroweak threshold. No tunable parameters enter this construction; it follows solely from the SM field content.

Independence of window choice. The specific electroweak windows used to obtain Eq. (6) are not unique. Any other admissible partition of the electroweak–to–EM transition, including splitting the step $W \to W'$ into multiple subwindows with the same net light particle content, corresponds to left–multiplying ΔW_{EM} by an element of $\mathrm{GL}(2, \mathbb{Z})$ and (possibly) adding or removing rows that are identical in both windows. Because the Smith normal form and its primitive right–kernel are invariant under such unimodular row operations, the invariant factors $(1, 8)$ and the generator $\chi_{\mathrm{EM}} = (-10, -18, 1)$ are independent of the particular window decomposition.

$$\Delta W_{\mathrm{EM}} \xrightarrow{\text{SNF}} \mathrm{diag}(1, 8, 0) \xrightarrow{\text{kernel}} \boxed{\chi = (16, 13, 2)}$$

FIG. 1. Smith normal form of ΔW_{EM} yields invariant factors $(1, 8)$ and a unique primitive kernel $\chi = (16, 13, 2)$ (up to overall sign).

C. Smith normal form and the primitive kernel

To extract the integer structure of Eq. (6), we compute its Smith normal form (SNF),

$$U\,\Delta W_{\mathrm{EM}}\,V = \mathrm{diag}(1,\ 8,\ 0), \tag{7}$$

where $U \in \mathrm{GL}(2, \mathbb{Z})$ and $V \in \mathrm{GL}(3, \mathbb{Z})$ are unimodular integer matrices. The invariant factors $(1, 8)$ show that ΔW_{EM} has integer rank 2 and therefore a *one–dimensional* primitive right–kernel over $\mathbb{Z}$.

Solving $\Delta W_{\mathrm{EM}}\,\chi_{\mathrm{EM}} = 0$ yields the primitive generator

$$\chi_{\mathrm{EM}} = (-10,\ -18,\ 1), \tag{8}$$

unique up to overall sign. Transporting this vector to the gauge–log basis using the corresponding unimodular change of variables gives

$$\boxed{\chi \equiv (16, 13, 2)},$$

providing a basis–independent integer direction aligned with the one–loop decoupling structure of the SM.

All SNF computations were performed with exact integer arithmetic using standard algorithms [4, 5] and are fully reproducible via the accompanying Zenodo archive [8].

III. GAUGE–LOG PROJECTION AND THE COMPOSITE Ω

Given the primitive kernel vector $\chi = (16, 13, 2)$ obtained in Sec. II, we define the associated gauge–log projection

$$\Xi = \chi \cdot \Psi = 16 \ln \alpha_s + 13 \ln \alpha_2 + 2 \ln \alpha, \tag{9}$$

where $\Psi = (\ln \alpha_s,\ \ln \alpha_2,\ \ln \alpha)$ collects the gauge couplings in logarithmic form. Because χ is integer–primitive and basis–independent under all unimodular transformations, Ξ represents the unique SM–selected linear combination of gauge logs encoded in the one–loop decoupling matrix.

Exponentiating Eq. (9) yields the composite gauge coupling

$$\Omega = e^{\Xi} = \alpha_s^{16}\,\alpha_2^{13}\,\alpha^2, \tag{10}$$

which contains no tunable parameters and is fixed entirely by the SM gauge representations. The exponents coincide with the components of the primitive kernel χ, making Ω the unique weighted–power combination of the gauge couplings dictated by the integer structure of ΔW_{EM}.

IV. NUMERICAL EVALUATION AND CLOSURE RATIO

We evaluate the composite Ω of Eq. (10) at the electroweak scale $\mu = M_Z$ using the 2024 PDG world averages for the SM gauge couplings in the $\overline{\mathrm{MS}}$ scheme [6]:

$$\alpha_s(M_Z) = 0.1180, \quad \alpha_2(M_Z) = \frac{g_2^2}{4\pi}, \quad \alpha(M_Z) = \frac{e^2}{4\pi}. \tag{11}$$

Substituting these values into Eq. (10) gives

$$\Omega(M_Z) = \alpha_s^{16}(M_Z)\,\alpha_2^{13}(M_Z)\,\alpha^2(M_Z) = 6.46 \times 10^{-39}, \tag{12}$$

in agreement with the fully reproducible computation archived in the accompanying Zenodo repository [8].

For comparison, we form the dimensionless proton–proton gravitational coupling

$$\alpha_G^{pp} = \frac{G_N m_p^2}{\hbar c}, \tag{13}$$

using the CODATA 2022 recommended constants [7], which yields

$$\alpha_G^{pp} = 5.91 \times 10^{-39}. \tag{14}$$

Following the closure ratio convention, we define

$$Z_G \equiv \frac{\alpha_G^{pp}}{\Omega(M_Z)}, \tag{15}$$

so that Eq. (3) represents the closure condition. Numerically,

$$Z_G = 0.9143. \tag{16}$$

No parameters are tuned in forming Ω; its value is fixed entirely by SM inputs and the integer kernel of Sec. II. The proximity of Z_G to unity is a numerical observation only, and no gravitational model is assumed or invoked.

V. LEAVE–ONE–OUT CLOSURE FOR THE STRONG COUPLING

The composite Ω in Eq. (10) depends on the three gauge couplings evaluated at $\mu = M_Z$. A natural leave–one–out (LOO) test is to treat the strong coupling $\alpha_s(M_Z)$ as an unknown and impose the *closure* condition Eq. (3) while keeping $\alpha_2(M_Z)$ and $\alpha(M_Z)$ fixed to their PDG values [6]. This determines a LOO value for the strong coupling,

$$\alpha_s^*(M_Z) = \left[\frac{\alpha_G^{pp}}{\alpha_2^{13}(M_Z)\,\alpha^2(M_Z)}\right]^{1/16}, \tag{17}$$

obtained directly from enforcing Eq. (3).

Using the same numerical inputs as in Sec. IV, we find

$$\alpha_s^*(M_Z) = 0.117341, \tag{18}$$

slightly below the PDG 2024 world average $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ [6] and well within current experimental uncertainties. In this LOO interpretation, the SM–determined composite Ω and the dimensionless quantity α_G^{pp} together select a specific closure value for the strong coupling; future measurements of $\alpha_s(M_Z)$ that differ significantly from $\alpha_s^*(M_Z)$ would indicate a breakdown of the closure relation (3).

The integer kernel χ itself is unaffected by this procedure, as it is fixed entirely by the SNF of ΔW_{EM}. The LOO test acts only on the numerical inputs and provides an experimentally accessible consistency check for the SM–derived composite Ω.

VI. CONCLUSIONS

We have examined the exact integer structure of the Standard Model one–loop decoupling matrix and computed its Smith normal form. The resulting invariant factors $(1, 8)$ imply a one–dimensional primitive integer kernel fixed solely by the SM field content. Transported to the gauge–log basis, this kernel yields the triplet $\chi = (16, 13, 2)$, which defines a unique SM–selected weighted–power combination of the gauge couplings.

Exponentiating the associated projection produces the composite quantity $\Omega = \alpha_s^{16}\alpha_2^{13}\alpha^2$, which contains no tunable parameters and is determined entirely by the SM gauge representations. Evaluated at $\mu = M_Z$ using PDG inputs, Ω numerically aligns with the dimensionless proton–proton gravitational coupling at the level $Z_G = \alpha_G^{pp}/\Omega(M_Z) \simeq 0.9143$. This comparison is purely numerical and does not invoke any gravitational model.

A leave–one–out closure test, in which $\alpha_s(M_Z)$ is fixed by demanding $Z_G = 1$, yields $\alpha_s^*(M_Z) = 0.117341$, well within the current PDG world–average band. In this interpretation, the SM–determined composite Ω and the measured quantity α_G^{pp} single out a specific closure value of the strong coupling; future determinations of $\alpha_s(M_Z)$ that deviate significantly from $\alpha_s^*(M_Z)$ would indicate a breakdown of the closure relation.

All calculations are performed with exact integer arithmetic and are fully reproducible using the accompanying Zenodo archive. These results identify a previously unrecognized SM–internal integer invariant and its associated composite coupling, illustrating how exact–integer methods can expose robust, scheme–independent structure in renormalization–group decoupling matrices.

APPENDIX: REPRODUCIBILITY PACKAGE

This repository provides an exact and fully reproducible computation of:

1. the Smith normal form (SNF) of the Standard Model one–loop decoupling matrix ΔW_{EM},
2. the primitive integer kernel $\chi_{\mathrm{EM}} = (-10, -18, 1)$,
3. its unimodular transport to the gauge–log basis $\chi = (16, 13, 2)$,
4. the Standard Model couplings $\alpha(M_Z)$, $\alpha_2(M_Z)$, $\alpha_s(M_Z)$,
5. the composite quantity $\Omega = \alpha_s^{16}\alpha_2^{13}\alpha^2$,
6. the dimensionless proton–proton gravitational coupling $\alpha_G^{pp} = G_N m_p^2/(\hbar c)$,
7. the closure ratio $Z_G = \alpha_G^{pp}/\Omega$ and its inverse $Z_G^{-1} = \Omega/\alpha_G^{pp}$,
8. the leave–one–out strong coupling $\alpha_s^\star(M_Z)$.

All integer algebra is performed with exact arithmetic, and all numerical quantities are computed directly from `pins.json`. No geometric modeling or additional assumptions are used [8].

Repository Contents

```
sm_integer_kernel_certifier.py
    # Main reproducibility script

pins.json
    # SM and SI input constants

results.json
    # Auto-generated machine-readable output

stdout.txt
    # Auto-generated readable summary
```

How to Run

Requires Python $\geq$ 3.8 and SymPy $\geq$ 1.9:

```
python3 sm_integer_kernel_certifier.py
```

Matrix Definition

$$\Delta W_{\mathrm{EM}} = \begin{pmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{pmatrix}.$$

Expected Output

```
SNF invariant factors: [1, 8]
Primitive kernel in (SU3,SU2,EM): (-10, -18, 1)
Transported kernel ...            (16, 13, 2)

alpha(MZ)        = 0.007815248
alpha2(MZ)       = 0.033789820
alpha_s(MZ)      = 0.118000000
Omega            = 6.459725598437e-39
alphaG_pp        = 5.906149417424e-39
Z_G              = 0.91430345
Z_G^-1           = 1.09372878
alpha_s* (LOO)   = 0.117341100
```

If used academically, please cite the Zenodo DOI associated with this repository.

[1] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, MA, 1995).
[2] P. Langacker, Rev. Mod. Phys. **81**, 1199 (2009).
[3] P. Langacker, *The Standard Model and Beyond*, 2nd ed., Series in High Energy Physics, Cosmology, and Gravitation (CRC Press, 2017).
[4] R. Kannan and A. Bachem, SIAM J. Comput. **8**, 499 (1979).
[5] M. Newman, Linear Algebra Appl. **254**, 367 (1997).
[6] S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024), and 2025 update.
[7] P. J. Mohr, D. B. Newell, B. N. Taylor, and E. Tiesinga, Rev. Mod. Phys. **97**, 025002 (2025).
[8] M. DeMasi, Standard Model Integer Kernel Certificate: Reproducibility Archive (2025), DOI: https://doi.org/10.5281/zenodo.17861604. Source repository: https://github.com/Miles-Diamecha/Standard-Model-Integer-Kernel-Certificate.
[9] T. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975).