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Equilibrium-only classification of admissible universal spacetime response from SM integer structure

Michael DeMasi[1] [1]Independent Researcher, Milford, CT, USA

E-mail: demasim90@gmail.com

Abstract

We provide a constraint-based, equilibrium-only classification of *universal spacetime-facing responses* that could be compatible with the internal gauge structure of the Standard Model (SM), assuming general covariance, fixed renormalization scheme and scale, the absence of new degrees of freedom, and *unimodular invariance of SM bookkeeping data* (not unimodular gravity). At one loop, threshold decoupling organizes the gauge sector into an integer lattice whose Smith normal form certifies a unique primitive right-kernel direction. This kernel defines a unimodular-invariant internal scalar Ξ constructed from SM gauge couplings. Any admissible universal response must depend on Ξ alone and, because the kernel is defined only up to sign under admissible bookkeeping transformations, must be an *even* function of Ξ, thereby forbidding linear response about equilibrium. Imposing minimal equilibrium consistency (boundedness and C^2 regularity) yields the canonical near-equilibrium form $R'(0) = 0$, with only higher-order even structure permitted. Up to overall normalization and higher-order even terms, this identifies a single admissible universality class at the level of structural dependence. The analysis is purely classificatory and makes no claims about dynamics, cosmology, or the realization of such a response in nature. A decisive falsifier is identified: observation of any *universal near-equilibrium linear* spacetime response correlated with the SM-internal coordinate Ξ would invalidate the classification.

Keywords: general relativity, Standard Model, gauge theory, spacetime structure, renormalization group

1 Introduction

The Standard Model (SM) provides a complete and experimentally successful description of nongravitational interactions [1, 2, 3]. Gravity remains external to the SM: its overall normalization enters independently through Newton's constant. Many approaches address this separation by extending the SM field content or modifying spacetime dynamics [4, 5]. Here we take a different route. Rather than proposing new degrees of freedom or dynamical mechanisms, we pose a prior structural question:

What universal spacetime-facing responses, if any, are admissible given the internal structure of the Standard Model alone?

Our analysis is constraint-driven and classificatory. We impose a minimal set of assumptions: (i) general covariance; (ii) *unimodular invariance of SM bookkeeping data*, meaning an integer reparametrization invariance acting only on internal, scheme- and scale-level bookkeeping (not on spacetime); and (iii) the absence of new fields, sources, or tunable parameters. This unimodular ingredient is a $\mathrm{GL}(3, \mathbb{Z})$ invariance of internal bookkeeping only; it is unrelated to spacetime volume-preserving diffeomorphisms and *not* unimodular gravity.

Under these conditions the SM gauge sector exhibits a rigid integer pattern at one loop due to threshold decoupling [6, 7]. Treated canonically, this structure admits a unique primitive integer kernel on the associated lattice of gauge-coupling bookkeeping. That kernel determines a single unimodular-invariant internal scalar coordinate, denoted Ξ.

Ξ is not a spacetime field and carries no dynamics. It is an internal scalar encoding the only information invariant under all admissible redefinitions of SM gauge-sector bookkeeping. Although the explicit integer realization arises at one loop, the central parity argument below relies only on two facts: the admissible invariant subspace is one-dimensional and it has no invariant orientation. Exact integrality beyond one loop is therefore not required for the parity constraint itself.

The central result is that any universal spacetime-facing response consistent with the stated assumptions must depend on Ξ alone. Species- or sector-dependent couplings are thereby excluded by construction. Moreover, because the primitive kernel direction is defined only up to an overall sign under admissible unimodular redefinitions, any such response must be *even* in Ξ. This bookkeeping parity forbids linear response about equilibrium and enforces vanishing first derivative at equilibrium, $R'(0) = 0$.

The outcome is a classification theorem: within the stated assumptions there exists a single admissible universality class of spacetime-facing response, characterized up to overall normalization and higher-order even structure. Throughout, "equilibrium" denotes a fixed reference point in gauge-log space at a chosen renormalization scheme and scale, not a thermodynamic state. General relativity is recovered exactly at equilibrium. No dynamical claims are made regarding cosmology or modifications of GR away from equilibrium. The purpose is to delineate what is permitted—and what is forbidden—prior to any dynamical modeling.

Constraint-based results play a foundational role across theoretical physics, from anomaly cancellation and EFT consistency conditions to uniqueness and no-go theorems. The present analysis serves a similar function by establishing necessary structural conditions that any viable dynamical realization of a universal spacetime response must satisfy, independent of mechanism. Here and throughout, a "spacetime-facing response" refers solely to an admissible constitutive dependence of a universal normalization or coupling on internal Standard Model data; no spacetime fields, equations of motion, or propagating degrees of freedom are introduced or implied.

Structure of the argument. Section II states the assumptions and scope explicitly. Section III identifies the unimodular-invariant internal scalar Ξ enforced by the one-loop integer structure. Section IV derives the parity constraint on any admissible universal spacetime-facing response and formulates the classification. Section V records explicit falsifiers and abandonment criteria. The remaining sections relate the result to existing frameworks and emphasize its strictly non-dynamical, equilibrium-only character. The work is a structural consistency condition at the interface of quantum field theory and classical spacetime geometry, not a proposal for modified gravity.

Falsifier (in brief). Observation of any universal near-equilibrium linear dependence on $\delta\Xi$ would falsify this classification under the stated assumptions.

2 Assumptions and Scope

The results below follow *exclusively* from the assumptions stated here. They define the domain of validity and function as logical constraints rather than modeling choices; no additional physical hypotheses are invoked.

1. **Standard Model completeness.** The analysis is restricted to the SM gauge sector with fixed particle content. No extensions, hidden sectors, or beyond-SM degrees of freedom are introduced [1, 3].

2. **Fixed renormalization scheme and scale.** All gauge couplings are evaluated at a fixed renormalization scheme and reference scale μ. No spacetime-dependent running, backreaction, threshold motion, or dynamical renormalization effects are considered.

3. **Hypercharge normalization (conventional).** We adopt the standard GUT normalization for the abelian coupling, defining

$$g_1^2 \equiv \frac{5}{3}\, g'^2, \qquad \text{equivalently} \qquad \alpha_1 \equiv \frac{5}{3}\, \alpha_Y,$$

   where g' (and α_Y) denote the Standard Model hypercharge coupling. This convention fixes the integer normalization of the $U(1)$ column in ΔW and the electromagnetic recombination $w_{\rm em} = 3w_2 + 5w_1$. No assumption of grand unification is made.

4. **Unimodular invariance of SM bookkeeping data (not unimodular gravity).** All statements are invariant under admissible unimodular reparametrizations of *internal* SM bookkeeping variables, modeled as integer-lattice changes of basis $V \in \mathrm{GL}(3, \mathbb{Z})$. This invariance acts solely on internal representation data and is unrelated to spacetime volume-preserving diffeomorphisms, unimodular gravity, gauge symmetries, or any physical transformation of fields or coordinates.

5. **Internal coordinate, not a field.** The scalar quantity identified in this work is an internal coordinate constructed from SM gauge couplings. It is not promoted to a spacetime field and carries no kinetic term, potential, source, equation of motion, or independent degree of freedom.
6. **Universality.** Any spacetime-facing response considered is universal: it depends only on the unimodular-invariant internal scalar and does not distinguish SM representations, flavors, or species.
7. **General covariance.** Any admissible spacetime response respects general covariance. No preferred frames, background structures, or explicit symmetry breaking are introduced [8].
8. **Equilibrium-only analysis.** The classification concerns equilibrium normalization only. No claims are made regarding time evolution, relaxation, sourcing, stability, or nonequilibrium behavior.
9. **No modification of GR at equilibrium.** At equilibrium, the spacetime response reduces identically to standard general relativity. No changes to field equations, propagating degrees of freedom, or tensor structure are permitted at the equilibrium point [9].
10. **Equilibrium regularity (minimal).** In a neighborhood of equilibrium, admissible responses are assumed bounded and twice continuously differentiable (C^2) in the internal coordinate so that derivatives at equilibrium are well defined. No further analyticity or functional assumptions are imposed.

Validity and abandonment criteria. All conclusions are conditional on the assumptions above. Relaxing or violating any one of them defines a distinct problem outside the present scope. In particular, introducing new degrees of freedom; promoting the internal coordinate to a dynamical field; abandoning unimodular invariance of bookkeeping data ($V \notin \mathrm{GL}(3,\mathbb{Z})$); changing the hypercharge normalization convention; allowing spacetime-dependent running or threshold motion; departing from equilibrium normalization; loss of C^2 regularity; or loss of a one-dimensional admissible invariant subspace (if required for a given argument) voids the present classification.

3 Unimodular-Invariant Scalar Structure of the Standard Model

We now identify the internal structure that enforces the classification result. Assemble the gauge–logarithmic bookkeeping coordinates

$$\hat{\Psi} \equiv \big(\ln \hat{\alpha}_s,\ \ln \hat{\alpha}_2,\ \ln \hat{\alpha}\big), \tag{1}$$

with $\overline{\mathrm{MS}}$ couplings evaluated at a fixed reference scale μ (hats denote $\overline{\mathrm{MS}}$ quantities at that μ). These coordinates encode the relative strengths of the gauge interactions [1, 3].

One-loop integer structure (full Standard Model). At one loop, threshold decoupling in the Standard Model defines an *integer*-valued matrix $\Delta W \in \mathbb{Z}^{2\times 3}$ relating ultraviolet and infrared gauge-sector descriptions. The entries of ΔW depend only on particle content, Dynkin indices, hypercharge squares, and multiplicities, and are independent of renormalization scheme and gauge basis [6, 7].

For the full Standard Model with $N_g = 3$ fermion generations and one Higgs doublet ($N_H = 1$), choosing two independent threshold windows yields

$$\Delta W = \begin{pmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{pmatrix}. \tag{2}$$

An explicit construction of these rows from particle content, together with a worked example, is given in Appendix A.

Smith normal form and integer certificate. Viewed on the integer lattice, ΔW admits a canonical classification under unimodular equivalence: there exist unimodular matrices $U \in \mathrm{GL}(2,\mathbb{Z})$ and $V \in \mathrm{GL}(3,\mathbb{Z})$ such that

$$U\,\Delta W\,V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \end{pmatrix}. \tag{3}$$

The invariant factors $(d_1, d_2) = (1, 8)$ certify that $\mathrm{rk}(\Delta W) = 2$ and that the integer right nullspace is one-dimensional. This certificate is algorithm-independent and uniquely characterizes the integer structure of the SM threshold data [10, 11].

Primitive kernel and uniqueness. The integer right–nullspace is generated by a primitive vector

$$\chi \in \mathbb{Z}^3, \qquad \gcd(\chi_i) = 1, \tag{4}$$

unique up to an overall sign. In the basis used in the main text, a representative is

$$\chi = (16,\, 13,\, 2)^{\mathsf{T}}, \tag{5}$$

which is related by an admissible unimodular transformation to the kernel generator obtained directly from ΔW (Appendix A).

Definition (admissible transformations). Admissible redefinitions of gauge–log bookkeeping are integer linear changes of basis that preserve the one-loop threshold lattice, i.e. transformations $\hat{\Psi} \mapsto V\hat{\Psi}$ with $V \in \mathrm{GL}(3, \mathbb{Z})$. Admissible redefinitions of threshold windows correspond to integer recombinations of rows and act as left multiplication by $U \in \mathrm{GL}(2, \mathbb{Z})$. No other transformations preserve the integer threshold structure.

Lemma (windowing and basis independence). Under all admissible transformations, $\Delta W \mapsto U\,\Delta W\,V$ with $U \in \mathrm{GL}(2, \mathbb{Z})$ and $V \in \mathrm{GL}(3, \mathbb{Z})$, the Smith invariant factors and the dimension of the integer right nullspace are preserved. Consequently, the existence and one-dimensionality of the primitive kernel are invariant properties of the SM particle content. Moreover, admissible transformations act transitively on primitive kernel generators and do not endow the kernel line with a canonical orientation.

Canonical unimodular-invariant scalar. The primitive kernel direction defines a distinguished unimodular-invariant linear functional on gauge–log space. Contracting the bookkeeping coordinates with the kernel yields

$$\Xi \equiv \chi^{\mathsf{T}}\hat{\Psi}. \tag{6}$$

Under an admissible bookkeeping change $\hat{\Psi} \mapsto V\hat{\Psi}$, the kernel transforms contragrediently as $\chi \mapsto V^{-\mathsf{T}}\chi$, so that Ξ is representation-independent. Because the nullspace is one-dimensional, *any* admissible unimodular-invariant scalar constructed from the gauge sector reduces to dependence on Ξ alone.

Equilibrium reference and displacement. Fixing the reference scheme and scale μ, define

$$\Xi_{\mathrm{eq}} \equiv \chi^{\mathsf{T}}\hat{\Psi}(\mu), \qquad \delta\Xi \equiv \Xi - \Xi_{\mathrm{eq}}, \tag{7}$$

so equilibrium corresponds to the unimodular-invariant point $\delta\Xi = 0$. This notion of equilibrium is purely a bookkeeping reference at fixed μ and does not imply thermodynamic equilibrium.

Not a field; no dynamics. Ξ is not a spacetime field and carries no dynamics. It is an internal bookkeeping coordinate enforced by integer rigidity, not by symmetry breaking, variational principles, or evolution.

Robustness beyond one loop. Beyond one loop, threshold data acquire rational, scheme-dependent deformations and exact integrality of ΔW is lost. However, for sufficiently small deformations that preserve rank, the admissible invariant subspace remains one-dimensional, and no unimodular-invariant structure can select an orientation of the kernel line. Consequently, the sign ambiguity $\chi \mapsto -\chi$ persists at equilibrium, and the parity constraint derived in the next section does not rely on exact one-loop integrality.

4 Constraints on Admissible Universal Spacetime Response

Having identified the unique unimodular-invariant displacement $\delta\Xi$, we now determine the constraints it imposes on any admissible *universal spacetime-facing response*. The analysis is classificatory: we ask which functional dependences are *permitted* by the stated assumptions, independent of any model for dynamics, sourcing, relaxation, or cosmology.

Definition (universal spacetime-facing response). A universal spacetime-facing response is any constitutive normalization or coupling entering spacetime physics that (i) depends only on internal Standard-Model gauge-sector data common to all ordinary matter, (ii) introduces no new degrees of freedom, and (iii) leaves general relativity unmodified at equilibrium.

Universality and unimodular invariance. Universality requires dependence only on internal data shared by all matter. Unimodular invariance of bookkeeping further restricts admissible dependence to quantities invariant under all admissible reparameterizations of the gauge–log coordinates. Since $\delta\Xi$ is the *only* scalar invariant under admissible $GL(3,\mathbb{Z})$ transformations, any admissible universal spacetime-facing response must be expressible as

$$R = R(\delta\Xi). \tag{8}$$

Dependence on any other combination of gauge couplings would select a preferred bookkeeping representation and is therefore inadmissible within the stated scope.

Theorem (bookkeeping parity). The primitive kernel generator χ is defined only up to an overall sign. Admissible bookkeeping transformations in $GL(3,\mathbb{Z})$ therefore induce $\delta\Xi \mapsto -\delta\Xi$, corresponding to a change of internal bookkeeping conventions, not to a physical transformation. Any physical quantity depending solely on unimodular-invariant information must be invariant under this reversal.

Corollary (absence of linear response). Bookkeeping parity implies

$$R(\delta\Xi) = R(-\delta\Xi), \tag{9}$$

so R is an even function of $\delta\Xi$ and

$$R'(0) = 0. \tag{10}$$

The absence of a linear term is therefore a structural consequence of unimodular bookkeeping parity, not a modeling choice, symmetry assumption, or fine-tuning.

Equilibrium consistency and minimal regularity. Recovery of standard general relativity at equilibrium requires that R be finite and smoothly normalized at $\delta\Xi = 0$. We impose minimal regularity: boundedness and C^2 behavior in a neighborhood of equilibrium. Together with parity, this yields the near-equilibrium expansion

$$R(\delta\Xi) = R(0) + \tfrac{1}{2}R''(0)\,\delta\Xi^2 + \mathcal{O}(\delta\Xi^4), \tag{11}$$

with no implication of dynamical stabilization, relaxation, or time evolution.

Illustrative representative (non-dynamical). As a purely illustrative analytic representative of the admissible class, one may write

$$R(\delta\Xi) = R(0)\exp\!\big[c_2\,\delta\Xi^2\big], \tag{12}$$

with c_2 a constant. No preference among admissible even functions is implied; this expression is not derived from, nor asserted to encode, any underlying dynamics. It is displayed solely to make the classified parity and boundedness properties explicit.

Classification statement. *Under unimodular invariance of bookkeeping, universality, general covariance, fixed renormalization scheme and scale, boundedness, and the absence of new degrees of freedom, any admissible universal spacetime-facing response is an even, bounded function of the single internal coordinate $\delta\Xi$. Up to overall normalization and higher-order even structure, there exists a single admissible universality class at the level of structural dependence.*

Operationalization and falsifier. Although no dynamics are assumed, the classification yields a decisive structural falsifier: observation of any *universal near-equilibrium linear dependence* on $\delta\Xi$ would violate bookkeeping parity and invalidate the classification under the stated assumptions. Such a linear term could in principle manifest as a composition-independent normalization shift in precision Cavendish-type measurements or in universality tests such as clock-comparison or gravitational redshift experiments. These examples are illustrative only and do not assert that such a coupling is realized in nature.

Meaning of "universality class." Here "universality class" denotes equivalence under the stated structural constraints. It does not refer to renormalization-group universality, statistical-mechanical classification, or dynamical fixed points.

5 Observable Constraints and Falsifiers

Although the present work is classificatory and restricted to equilibrium normalization, the constraints derived above imply clear, in-principle observational consequences. These take the form of necessary consistency conditions that any universal spacetime-facing dependence compatible with the internal structure of the Standard Model must satisfy.

Parity and the near-equilibrium expansion. Because any admissible response is invariant under the bookkeeping reversal $\delta\Xi \mapsto -\delta\Xi$, linear dependence is forbidden. With equilibrium fixed at the unimodular-invariant reference point $\delta\Xi = 0$, an admissible response admits the expansion

$$R(\delta\Xi) = R(0) + \tfrac{1}{2}R''(0)\,\delta\Xi^2 + \mathcal{O}(\delta\Xi^4) = R(0)\big[1 + c_2\,\delta\Xi^2 + \mathcal{O}(\delta\Xi^4)\big]\,, \tag{13}$$

and necessarily satisfies

$$R'(0) = 0. \tag{14}$$

The absence of a linear term is not a modeling choice, symmetry assumption, or fine-tuning; it follows directly from unimodular invariance and the unavoidable sign ambiguity of the primitive kernel direction.

Equilibrium consistency (boundedness). Recovery of standard general relativity at equilibrium requires that the response be finite and smoothly normalized at $\delta\Xi = 0$. We therefore require boundedness and C^2 regularity of R in a neighborhood of $\delta\Xi = 0$. No claim is made regarding any dynamical mechanism (stabilization, relaxation, sourcing, or feedback), and no assumption is introduced beyond local regularity at the equilibrium reference point.

Structural falsifier (decisive). The falsifier is structural rather than phenomenological in detail:

> *If a universal spacetime-facing dependence exhibits a nonzero linear term near equilibrium, i.e. $R'(0) \neq 0$, then the classification is invalid under the stated assumptions.* Likewise, a universal dependence that is unbounded under admissible perturbations of $\delta\Xi$ contradicts equilibrium consistency and invalidates at least one assumption.

Operational remarks. A possible operational manifestation would be an effective normalization of a universal coupling evaluated at equilibrium (for example, a normalization factor multiplying Newton's constant), though no such identification is assumed or required here. The falsifier is independent of detailed functional form: all admissible responses share the same symmetry and parity structure near equilibrium; differences affect only overall normalization and higher-order even terms and do not alter the absence of linear response.

Interpretation. Compatibility with a strictly quadratic leading deviation is consistent with the present organizing-principle framework but does not establish any particular dynamical realization. The result provides *necessary* conditions for admissibility and identifies explicit failure modes; it is not, by itself, a predictive dynamical model.

> **Abandonment criteria.** The classification fails if any of the following are introduced: (i) new degrees of freedom; (ii) promotion of Ξ (or $\delta\Xi$) to a spacetime field; (iii) abandonment of unimodular bookkeeping invariance; (iv) departure from equilibrium normalization; or (v) loss of local C^2 regularity needed to define $R'(0)$ and $R''(0)$.

6 Relation to Existing Frameworks

It is useful to situate the present result relative to familiar approaches to gravity and spacetime response. The framework developed here is *not* a model of modified gravity and introduces no new dynamical fields, interactions, or equations of motion. Rather, it provides a constraint-level classification of admissible *universal* spacetime-facing response under explicit assumptions, with standard general relativity recovered exactly at equilibrium.

Not scalar–tensor. Although a scalar quantity Ξ appears, it is an *internal bookkeeping coordinate* constructed from Standard-Model couplings. It is not a spacetime field, carries no kinetic term or potential, induces no conformal (Jordan/Einstein-frame) rescaling, and introduces no propagating degrees of freedom. The framework therefore lies outside Brans–Dicke and related scalar–tensor extensions.

Not a modification of Einstein's equations. At equilibrium the response reduces identically to standard general relativity. Any departures from equilibrium are constrained only at the level of admissible *functional dependence* implied by the classification, not by altered field equations, sources, or propagation laws. No claims are made regarding cosmology, gravitational radiation, or strong-field dynamics.

Not unimodular gravity. The unimodular ingredient employed here is an invariance of *Standard-Model bookkeeping data* under admissible integer reparameterizations. It acts solely on internal representation data, not on spacetime geometry. It does not restrict $\det g_{\mu\nu}$ and does not correspond to a reformulation of the gravitational field equations. No equivalence with, or claim about, unimodular gravity is intended.

Not emergent or entropic gravity. No coarse-graining, thermodynamic, information-theoretic, or statistical interpretation is assumed. The organizing principle operates at the level of integer rigidity and unimodular invariance of Standard-Model threshold structure, yielding structural parity and boundedness constraints rather than collective, microscopic, or entropic mechanisms.

Orthogonal to quantum-gravity and unification programs. The present classification is logically prior to attempts to quantize gravity or to unify it with the Standard Model via additional symmetries, dimensions, or degrees of freedom. It constrains the admissible form of any universal spacetime response should a dynamical realization exist, without committing to a specific ultraviolet completion or quantization scheme.

Summary. The result does not compete with dynamical theories of gravity. It delineates a restricted universality class of spacetime-facing responses compatible with the Standard Model's internal structure and supplies a necessary consistency condition that any successful dynamical realization must satisfy, while remaining strictly equilibrium-only and non-dynamical in scope.

7 Discussion

We have established a constraint-level classification of admissible *universal* spacetime-facing response consistent with the internal structure of the Standard Model (SM). The stance is deliberately conservative: imposing unimodular invariance of SM bookkeeping, universality, general covariance, a fixed renormalization scheme and scale, boundedness, and the absence of new degrees of freedom isolates what is permitted *prior* to any dynamical, phenomenological, or cosmological modeling.

A central outcome is the identification of a unique unimodular-invariant internal scalar constructed from gauge-sector couplings. This quantity functions as an organizing coordinate, not as a spacetime field: it carries no dynamics and introduces no propagating modes. Once rigidity under admissible integer reparameterizations is enforced, universality and unavoidable bookkeeping parity restrict any admissible spacetime-facing response to even, bounded functional dependence and forbid linear response about equilibrium, $R'(0) = 0$ for $\delta\Xi \equiv \Xi - \Xi_{\rm eq}$.

The result is strictly conditional. The classification does *not* assert that spacetime must exhibit such a dependence; rather, it states that *if* a universal spacetime-facing response exists under the stated assumptions, *then* its dependence on SM data is uniquely fixed at the level of structure, up to overall normalization and higher-order even terms. Violating any assumption—introducing new degrees of freedom, abandoning unimodular invariance of bookkeeping ($V \notin GL(3, \mathbb{Z})$), promoting the internal coordinate to a spacetime field, or departing from equilibrium normalization—defines a distinct problem outside the present analysis.

Stability enters only as structural regularity. Any "self-limiting" behavior follows from parity and boundedness near equilibrium, not from relaxation, feedback, or a dynamical control mechanism. Questions of time evolution, sourcing, backreaction, or nonequilibrium behavior require additional physical input and are intentionally left open.

Possible extensions—such as promoting the internal coordinate to a dynamical variable, coupling it to stress–energy, or embedding the structure within an effective field theory—are not pursued here. Any viable extension must recover the present constraint structure in the equilibrium limit. In this sense, the classification supplies necessary consistency conditions and a decisive falsifier (the exclusion of linear response) that any more elaborate construction must satisfy, rather than proposing a competing dynamical model.

8 Conclusion

We presented a constraint-level classification of admissible *universal* spacetime-facing responses consistent with the internal structure of the SM. Imposing unimodular invariance of SM bookkeeping, universality, general covariance, a fixed renormalization scheme and scale, boundedness, and the absence of new degrees of freedom, we showed that the gauge sector furnishes a unique unimodular-invariant internal scalar coordinate. Because this coordinate is defined only up to an overall sign under admissible reparameterizations, bookkeeping parity forbids linear response about equilibrium and restricts any admissible response to even, bounded functional dependence in $\delta\Xi$.

The resulting classification identifies a single admissible universality class at the level of structural dependence—up to overall normalization and higher-order even terms—and yields a decisive falsifier: observation of any *universal, near-equilibrium linear* spacetime response correlated with the SM-internal coordinate would invalidate the classification under the stated assumptions. No claims are made regarding dynamics, cosmology, or modifications of general relativity away from equilibrium.

These results delineate a narrow and well-defined consistency class for universal spacetime-facing response. Any proposed dynamical realization or extension must reduce to the present constraint structure in the appropriate equilibrium limit; failure to do so signals that at least one stated assumption is violated and that a different theoretical problem is being addressed.

Scope, Limitations, and Interpretation

This work is a *conditional, constraint-based classification*, not a dynamical theory. Its claims have the following form: *if* the Standard Model (SM) admits any universal spacetime-facing response consistent with general covariance, unimodular invariance of internal bookkeeping, a fixed renormalization scheme and scale, bounded equilibrium behavior, and the absence of new degrees of freedom, *then* the admissible dependence of that response on SM data is uniquely constrained at the level of structure. No assertion is made that such a response must exist or is realized in nature.

The internal scalar coordinate identified here is not a spacetime field and is not endowed with dynamics, sources, or equations of motion. Accordingly, this work does not address time evolution, relaxation, backreaction, cosmology, strong-field behavior, or gravitational radiation. Any reference to “self-limiting” or bounded behavior refers solely to structural regularity implied by parity and boundedness near equilibrium, not to a dynamical control or feedback mechanism.

The classification is equilibrium-only. Departures from equilibrium, spacetime-dependent renormalization effects, or promotion of the internal coordinate to a dynamical variable fall outside the stated assumptions and define distinct problems requiring additional physical input. Likewise, introducing new degrees of freedom, abandoning unimodular invariance of bookkeeping, or modifying general relativity at equilibrium invalidates the present classification rather than extending it.

Interpreted correctly, the result functions as a necessary consistency condition and a filter on admissible model-building. Any proposed dynamical realization of a universal spacetime-facing response must reduce to the present constraint structure in the appropriate equilibrium limit; failure to do so signals that at least one stated assumption has been violated. The decisive falsifier is structural: observation of a universal near-equilibrium *linear* response correlated with the internal SM coordinate $\delta\Xi$ would rule out the classification under its stated premises.

Data Availability

All materials required to verify the integer Smith normal form (SNF) certificate and reproduce the primitive kernel χ and its admissible unimodular transport are publicly archived at Zenodo: *Standard Model Integer Kernel Certificate: Reproducibility Archive*, version 1.0 (DOI: 10.5281/zenodo.17861604). The deposit includes the SM and SI pins in `pins.json` (scheme and scale metadata, M_Z inputs, and the projector defining $\chi = (16, 13, 2)$), together with a concise `README` describing scope, expected outputs, and the procedure used to produce the integer certificate.

Code/Software Availability

A minimal, MIT-licensed Python script, `sm_integer_kernel_certifier.py` (Python $\geq$ 3.8; SymPy), computes the Smith normal form of ΔW (as used in the main text), extracts the primitive integer kernel $\chi_{\mathrm{EM}} = (-10, -18, 1)$ from the electromagnetic-recombined basis, and applies an explicit unimodular transport to obtain the canonical representative $\chi = (16, 13, 2)$. Executing `python3 sm_integer_kernel_certifier.py` produces machine-readable `results.json` and a human-readable `stdout.txt`, as documented in the `README`. Users should cite the versioned DOI above; any future updates will appear as new Zenodo versions and will not alter the results reported here.

A One-Loop Decoupling Matrix and Integer Kernel

At one loop, threshold decoupling of heavy SM fields relates ultraviolet and infrared gauge couplings through additive shifts proportional to logarithms of mass ratios. In gauge–logarithmic bookkeeping coordinates

$$\hat{\Psi} \equiv \big(\ln \hat{\alpha}_s,\ \ln \hat{\alpha}_2,\ \ln \hat{\alpha}\big), \tag{15}$$

with $\overline{\mathrm{MS}}$ couplings evaluated at a fixed reference scale μ, these threshold contributions assemble into an *integer*-valued linear map fixed entirely by particle content and charge assignments [6, 7]. Integrality follows because the one-loop coefficients depend only on Dynkin indices, hypercharge squares Y^2, and multiplicities, which are rational and become integer after a fixed normalization of the gauge basis.

Integerized species weights

To make this integrality explicit, Table 1 lists the integerized species weights used to construct the integer bookkeeping matrix $W_{\mathbb{Z}}$. For nonabelian factors, the entries $2T_3$ and $2T_2$ denote twice the Dynkin index of the representation, multiplied by the number of Weyl degrees of freedom. For the abelian factor, we adopt the integer normalizations $w_1 = 12\sum_f Y_f^2$ for Weyl fermions and $w_1 = 3\sum_s Y_s^2$ for scalars, which are the minimal common multiples rendering all SM hypercharge contributions integral. Any overall unimodular rescaling yields an equivalent Smith class and does not affect the integer kernel structure.

Table 1. Integerized species weights used to construct $W_{\mathbb{Z}}$. All entries are integers by construction.

Species	$(SU(3), SU(2), Y)$	dof	$2T_3$	$2T_2$	w_3	w_2	w_1
Q_L	$(\mathbf{3},\mathbf{2},1/6)$	$6N_g$	1	1	$6N_g$	$6N_g$	$12\sum_f Y_f^2$
u_R	$(\mathbf{3},\mathbf{1},2/3)$	$3N_g$	1	0	$3N_g$	0	$12\sum_f Y_f^2$
d_R	$(\mathbf{3},\mathbf{1},-1/3)$	$3N_g$	1	0	$3N_g$	0	$12\sum_f Y_f^2$
L_L	$(\mathbf{1},\mathbf{2},-1/2)$	$2N_g$	0	1	0	$2N_g$	$12\sum_f Y_f^2$
e_R	$(\mathbf{1},\mathbf{1},-1)$	N_g	0	0	0	0	$12\sum_f Y_f^2$
H	$(\mathbf{1},\mathbf{2},1/2)$	$2N_H$	0	1	0	$2N_H$	$3\sum_s Y_s^2$
W	adjoint $(\mathbf{1},\mathbf{3},0)$	1	0	4	0	4	0
G	adjoint $(\mathbf{8},\mathbf{1},0)$	1	6	0	6	0	0

For a given threshold window W_i, the corresponding row of the decoupling matrix ΔW is obtained by summing (w_3, w_2, w_1) over all species light in that window and taking differences between adjacent windows. Different admissible choices of windows correspond to left multiplication by unimodular integer matrices and do not affect the Smith invariants or the integer kernel.

Worked example (one generation)

As a concrete illustration, we consider a single Standard Model fermion generation ($N_g = 1$). The Higgs doublet is treated as a separate row in ΔW. Gauge-boson self-interaction terms are not included as independent rows in the decoupling matrix.

Explicit one–generation fermion-row check.

SU(3). One generation contains the colored Weyl fermions

$$Q_L : 6, \qquad u_R : 3, \qquad d_R : 3,$$

for a total of 12 colored Weyl fermions. Each Weyl fermion in the fundamental representation contributes $(2/3)\,T(\mathbf{3}) = 1/3$ to the physical one–loop coefficient, giving

$$b_3^{\mathrm{phys}} = 12 \times \frac{1}{3} = 4.$$

With the integer bookkeeping used in $W_{\mathbb{Z}}$, $w_3 \equiv 2\, b_3^{\rm phys}$, this yields

$$\sum_{1\ \rm gen} w_3 = 8.$$

SU(2). One generation contains the SU(2) doublets

$$Q_L : 6, \qquad L_L : 2,$$

for a total of 8 SU(2–doublet Weyl fermions. Each contributes $(2/3)\, T(\mathbf{2}) = 1/3$ to the physical one–loop coefficient, giving

$$b_2^{\rm phys} = 8 \times \frac{1}{3} = \frac{8}{3}.$$

With the integer bookkeeping $w_2 \equiv 3\, b_2^{\rm phys}$, we obtain

$$\sum_{1\ \rm gen} w_2 = 8.$$

U(1)$_Y$. For the abelian factor we use the integer normalization

$$w_1^{(f)} = 12 \sum Y^2 \quad \text{(Weyl fermions)},$$

where Y denotes the Standard Model hypercharge. Evaluating explicitly for one generation,

$$\begin{aligned} Q_L &: 12 \cdot 6 \cdot (1/6)^2 = 2, \\ u_R &: 12 \cdot 3 \cdot (2/3)^2 = 16, \\ d_R &: 12 \cdot 3 \cdot (1/3)^2 = 4, \\ L_L &: 12 \cdot 2 \cdot (1/2)^2 = 6, \\ e_R &: 12 \cdot 1 \cdot 1^2 = 12, \end{aligned}$$

so that

$$\sum_{1\ \rm gen} w_1 = 40.$$

Thus, a single Standard Model fermion generation contributes the integer vector

$$(w_3,\, w_2,\, w_1) = (8,\, 8,\, 40).$$

This result follows entirely from Standard Model representation data and the chosen integer bookkeeping and is independent of renormalization scheme or gauge basis.

After electroweak symmetry breaking, we adopt the conventional GUT–normalized abelian coupling $\alpha_1 \equiv g_1^2/(4\pi)$ defined by

$$g_1^2 \equiv \frac{5}{3}\, g'^2, \qquad \text{equivalently} \qquad \alpha_1 \equiv \frac{5}{3}\, \alpha_Y,$$

where g' (and α_Y) denote the Standard Model hypercharge coupling. The electroweak mixing relation then gives

$$\alpha_{\rm em}^{-1} = \alpha_2^{-1} + \alpha_Y^{-1} = \alpha_2^{-1} + \frac{5}{3}\alpha_1^{-1}.$$

At the level of the integer bookkeeping used in Table 1, this corresponds to the recombination

$$w_{\rm em} = 3w_2 + 5w_1.$$

Substituting the one–generation values yields $w_{\rm em} = 3(8) + 5(40) = 224$, producing the fermion row

$$(w_3,\, w_2,\, w_{\rm em})_F = (8,\, 8,\, 224),$$

appearing in ΔW.

Explicit Higgs-row check. Using Table 1, a single Higgs doublet $H \sim (1, 2, 1/2)$ has

$$w_3(H) = 0, \qquad w_2(H) = 1, \qquad w_1(H) = 3,$$

where $w_2(H) = 2T_2 = 1$ for an SU(2) doublet and

$$w_1(H) = 3\sum_s Y_s^2 = 3 \cdot 2 \cdot \left(\frac{1}{2}\right)^2 = 3$$

for the two complex scalar components of the doublet. In the electromagnetic-recombined normalization,

$$w_{\rm em} = 3w_2 + 5w_1,$$

so

$$w_{\rm em}(H) = 3(1) + 5(3) = 18.$$

Thus the Higgs row is

$$(w_3,\, w_2,\, w_{\rm em})_H = (0,\, 1,\, 18),$$

reproducing the second row of ΔW.

Smith normal form and primitive kernel

For the SM particle content, the one-loop decoupling matrix may therefore be written as

$$\Delta W = \begin{pmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{pmatrix}, \tag{16}$$

where rows label independent threshold windows and columns encode the gauge–logarithmic coordinates in a fixed basis.[1]

Treating ΔW on the integer lattice, its Smith normal form (SNF) is defined by

$$U\,\Delta W\,V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \end{pmatrix}, \qquad U \in \mathrm{GL}(2,\mathbb{Z}),\ V \in \mathrm{GL}(3,\mathbb{Z}), \tag{17}$$

from which the determinantal divisors follow:

$$\Delta_1 = 1, \qquad \Delta_2 = 8. \tag{18}$$

Here Δ_1 is the greatest common divisor of all entries of ΔW, and Δ_2 is the greatest common divisor of all 2×2 minors (for the present matrix, $\{8, 144, 80\} \to 8$). These invariants are algorithm-independent and constitute a canonical integer certificate of the structure.

Because $\mathrm{rk}(\Delta W) = 2$, the integer right nullspace is one-dimensional; any primitive generator is unique up to an overall sign and admissible unimodular equivalence.

Lemma (Unimodular transitivity). $\mathrm{GL}(n,\mathbb{Z})$ acts transitively on primitive integer vectors in $\mathbb{Z}^n$.

Sketch. Any primitive vector can be completed to an integer basis of $\mathbb{Z}^n$ via the extended Euclidean algorithm, defining a unimodular transformation that maps it to a standard basis vector.

Primitive kernel (displayed basis). Solving $\Delta W x = 0$ over $\mathbb{Z}$ yields

$$\chi_{\Delta W} = (-10,\, -18,\, 1)^{\mathsf{T}}, \qquad \gcd(\chi_{\Delta W,i}) = 1, \tag{19}$$

unique up to overall sign; no additional independent integer kernel directions exist.

As a consistency check, we verify that the claimed primitive kernel vector is annihilated by the electromagnetic-basis decoupling matrix:

$$\Delta W\,\chi_{\Delta W} = \begin{pmatrix} 8 & 8 & 224 \\ 0 & 1 & 18 \end{pmatrix} \begin{pmatrix} -10 \\ -18 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

This confirms that $\chi_{\Delta W} = (-10, -18, 1)^{\mathsf{T}}$ lies in the integer right kernel of ΔW.

[1]Admissible reorderings or relabelings correspond to left and right multiplication by unimodular integer matrices and do not change the Smith invariants or the integer kernel structure.

Explicit verification. One explicit admissible unimodular transformation realizing this equivalence is

$$V = \begin{pmatrix} 1 & 2 & 1 \\ -2 & -4 & -1 \\ 0 & 1 & -1 \end{pmatrix}, \qquad \det V = -1, \tag{20}$$

for which $V^{-\mathsf{T}} \chi_{\Delta W} = \chi$.

With $\chi_{\Delta W} = (-10, -18, 1)^{\mathsf{T}}$, the contragredient transport used in the main text gives

$$V^{-\mathsf{T}} \chi_{\Delta W} = \begin{pmatrix} -5 & 2 & 2 \\ -3 & 1 & 1 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} -10 \\ -18 \\ 1 \end{pmatrix} = \begin{pmatrix} 16 \\ 13 \\ 2 \end{pmatrix} \equiv \chi. \tag{21}$$

The kernel vector used in the main text,

$$\chi = (16,\ 13,\ 2)^{\mathsf{T}}, \tag{22}$$

is related to $\chi_{\Delta W}$ by an admissible unimodular change of gauge–log basis; both represent the same primitive kernel direction.

For reproducibility, note that the accompanying repository implements the equivalent transport $\chi_{\text{gauge}} = V^{-\mathsf{T}} \chi_{\Delta W} = M^{\mathsf{T}} \chi_{\Delta W}$, where $M = V^{-1}$. These forms are related by inversion and encode the same unimodular equivalence [12].

Contragredient transport and invariance of Ξ. Under an admissible unimodular redefinition of gauge–log bookkeeping,

$$\hat{\Psi} \mapsto V\, \hat{\Psi}, \qquad V \in \mathrm{GL}(3, \mathbb{Z}), \tag{23}$$

the kernel representative transforms contragrediently as

$$\chi_{\Delta W} \mapsto V^{-\mathsf{T}} \chi_{\Delta W}, \tag{24}$$

preserving primitivity and one-dimensionality of the integer kernel. Consequently, the unimodular-invariant scalar $\Xi = \chi^{\mathsf{T}} \hat{\Psi}$ is representation-independent.

Rigidity. Rank-preserving integer perturbations of ΔW generically spoil either the SNF invariants or the primitivity and uniqueness of the integer kernel, indicating that the SM matrix occupies a rigid, non-generic point in $\mathbb{Z}^{2\times 3}$.

Symbol	Meaning
$\hat{\Psi}$	Gauge–log bookkeeping coordinates $(\ln \hat{\alpha}_s, \ln \hat{\alpha}_2, \ln \hat{\alpha})$
ΔW	One-loop integer decoupling matrix
χ	Primitive integer right kernel of ΔW (unique up to sign)
Ξ	Unimodular-invariant scalar $\chi^{\mathsf{T}} \hat{\Psi}$
$\delta\Xi$	Displacement from equilibrium $\Xi - \Xi_{\text{eq}}$
U, V	Unimodular matrices in SNF ($U \in \mathrm{GL}(2, \mathbb{Z})$, $V \in \mathrm{GL}(3, \mathbb{Z})$)
Admissible	Invariant under $\mathrm{GL}(2, \mathbb{Z}) \times \mathrm{GL}(3, \mathbb{Z})$ bookkeeping

Table 2. Notation and admissible transformations.

B In-manuscript Reproducibility Snippet (Smith Certificate & Kernel Transport)

Standalone 12-line SymPy script that: (i) computes SNF(ΔW), (ii) extracts the primitive right-kernel generator $\chi_{\Delta W}$, and (iii) transports it to the canonical representative $\chi = (16, 13, 2)$. Requires: Python >= 3.8, sympy >= 1.10.

Python (SymPy), 12 lines:

```
import sympy as sp

DW = sp.Matrix([[8, 8, 224],
                [0, 1, 18]])

U_snf, D, V_snf = DW.smith_normal_form() # U_snf*DW*V_snf = D (Smith form)

chi_DW = V_snf[:, -1] # primitive right-kernel generator of DW

V_app = sp.Matrix([[ 1, 2, 1],
                   [-2, -4, -1],
                   [ 0, 1, -1]]) # unimodular basis change used in Appendix A
M = V_app.inv() # M = V_app^{-1}
chi = M.T * chi_DW # chi = V_app^{-T} chi_DW (contragredient transport)

g = sp.gcd_list([int(c) for c in chi]) # primitive normalization
chi = sp.Matrix([int(c//g) for c in chi])

print("SNF diag entries:", [D[0,0], D[1,1]])
print("chi_DW (kernel of DW):", list(chi_DW))
print("chi (canonical):", list(chi)) # expected [16, 13, 2]
```

What this verifies in one run. Running the snippet prints the invariant factors $(1, 8)$, a primitive generator $\chi_{\Delta W}$ for the integer right nullspace of ΔW, and its admissible unimodular transport to the canonical representative $\chi = (16, 13, 2)$ used in the main text. This reproduces the integer certificate and kernel transport without relying on any external files.

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