

Signals &
Systems

TRAN
Hoang Tung

Samples

Aliasing

Reconstruction

Sampling

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The Course So Far

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What we have learnt?

- Signals and Systems
- LTI systems: convolution
- Fourier Series
- Fourier Transform

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What we have learnt?

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Today

Sampling: Continuous to Discrete

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A continuous-time signal can be **completely represented** by its samples at points equally spaced in time. For examples:

- 1 moving pictures, which consist of a sequence of individual frames
- 2 printed pictures

Definition

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Attention: An infinite number of signals can generate a given set of samples.

Definition

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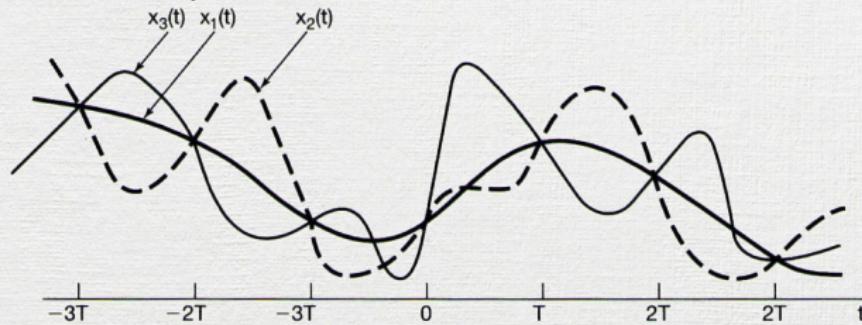
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Attention: An infinite number of signals can generate a given set of samples.



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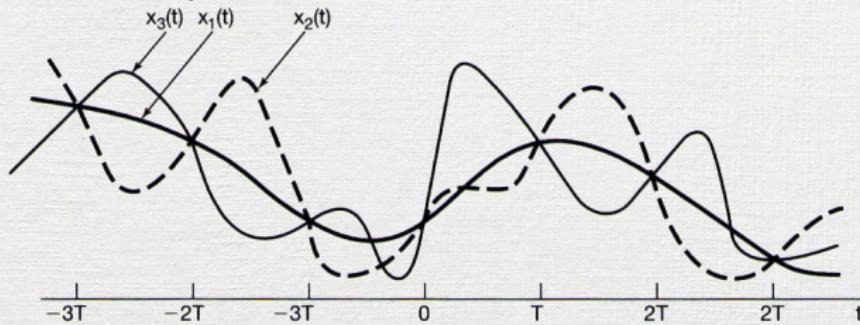
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Attention: An infinite number of signals can generate a given set of samples.



If

- a signal is band limited
- the samples are taken sufficiently close

then the samples uniquely specify the signal, and we can reconstruct it perfectly.

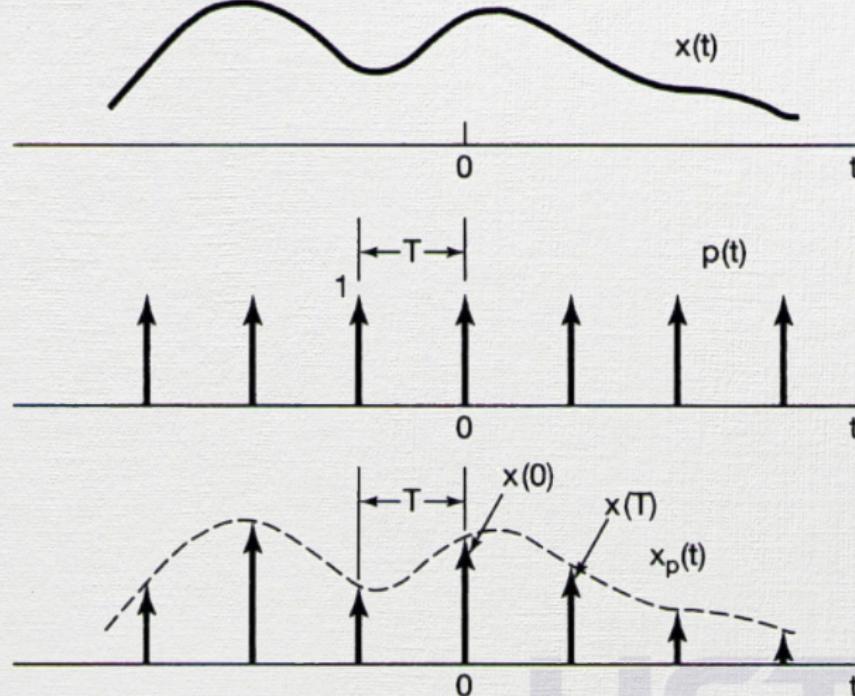
Impulse-Train Sampling

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Question: what happens in frequency domain?

Frequency Domain

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Impulse-Train Sampling

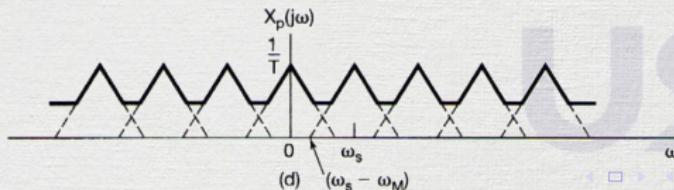
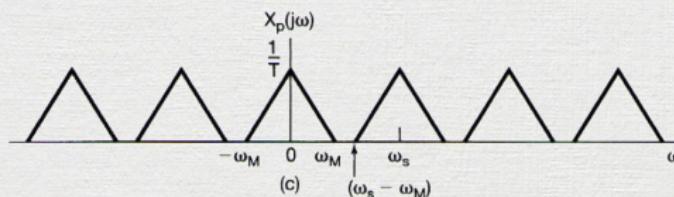
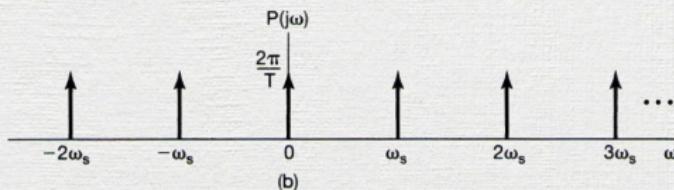
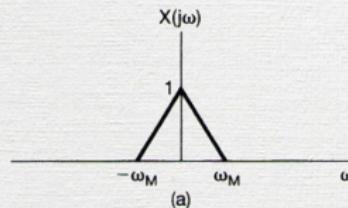
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Sampling Theorem

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$$\omega_{sampling} > 2\omega_{maximum}$$

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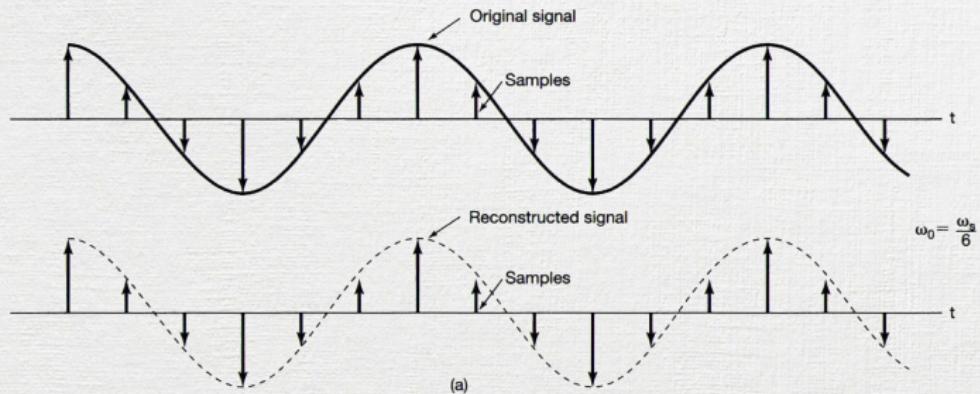
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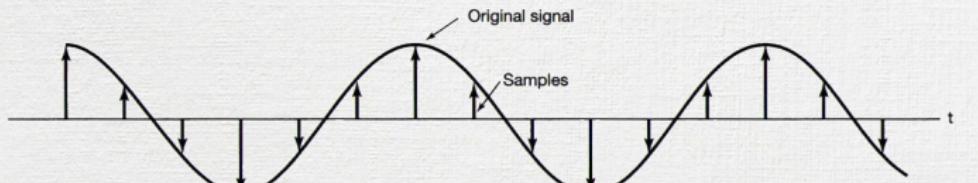
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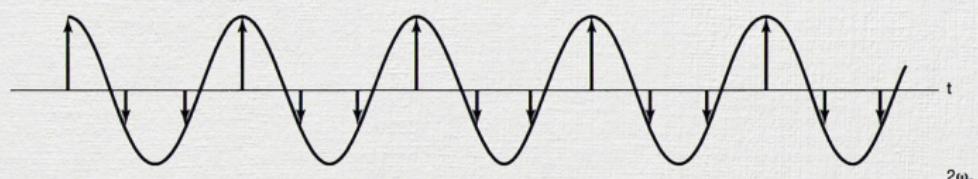
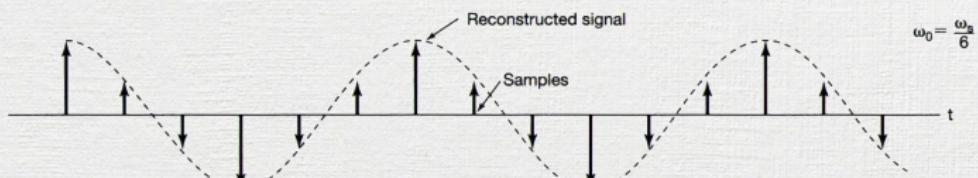
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(a)



(b)

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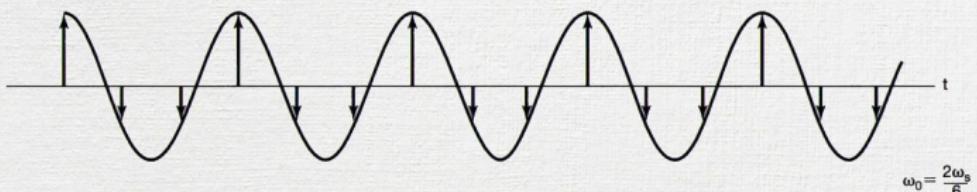
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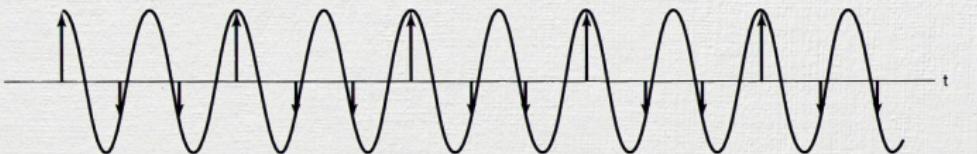
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(b)



(c)

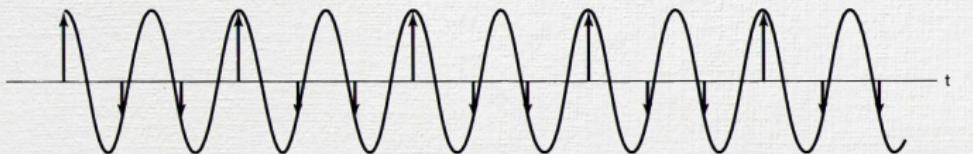
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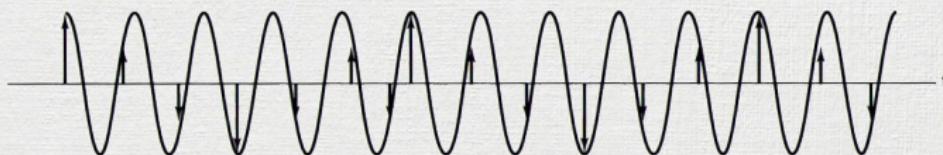
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(c)



(d)

$$\omega_0 = \frac{5\omega_s}{6}$$

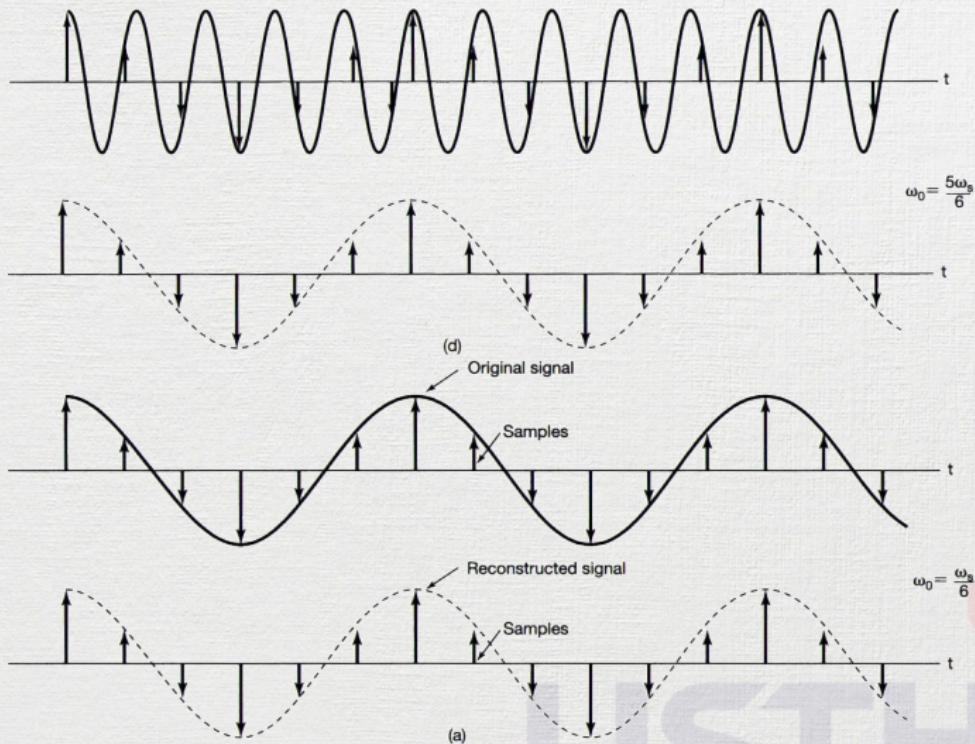
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