Systems

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Lesson Objectives

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2. determine (and justify) properties of a system:
   - Causality
   - Stability
   - Time Invariance
   - Linearity

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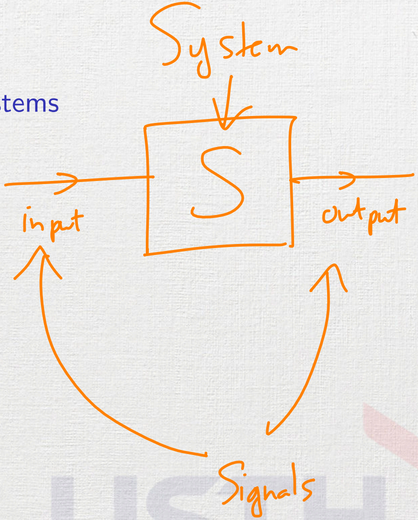
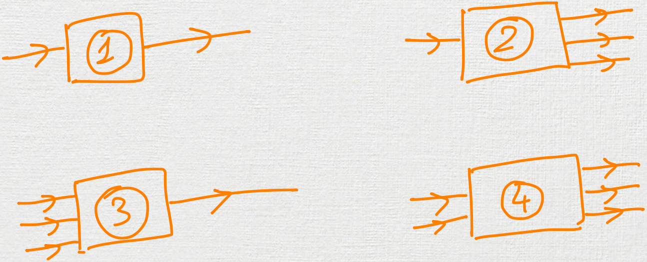
1. recognize basic system types and system interconnections
2. determine (and justify) properties of a system:
   - Causality
   - Stability
   - Time Invariance
   - Linearity
3. have some feelings about LTI systems

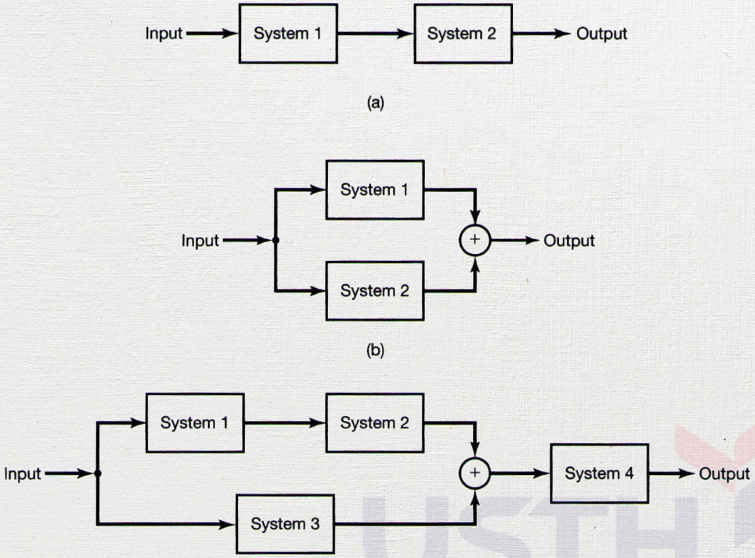

System
S
input
output
Signals

System Types

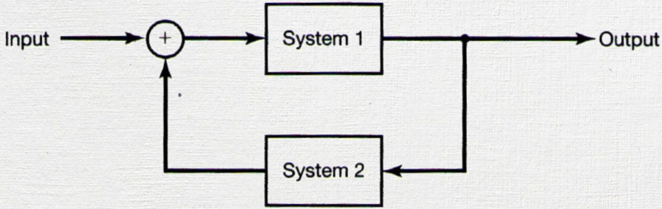
- Single-variable vs. Multiple-variable systems
  1. SISO (single-input single-output) systems
  2. SIMO (single-input multiple-output) systems
  3. MISO (multiple-input single-output) systems
  4. MIMO (multiple-input multiple-output) systems



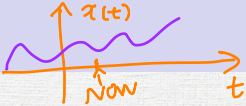
Series & Parallel



Feedback

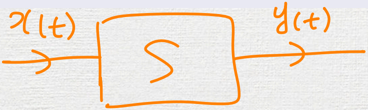


Causality



Causal Systems

Output signal depends only on the present and/or past values of the input signal.



$y(0)$ depends on $x(\alpha)$ $\alpha \le 0$

$y(0)$ doesn't depend on $x(1)$

Causality

Causal Systems

Output signal depends only on the present and/or past values of the input signal.

Examples

- $y(t) = x(t)sin(t)$

Causality

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Examples

- ~~$y(t) = x(t)sin(t)$~~
- $y[n] = x[-n]$

$y[1] = x[-1]$

$y[0] = x[0]$

$\boxed{y[-1] = x[1]}$

non causal

Causality

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Examples

- $y(t) = x(t)sin(t)$
- $y[n] = x[-n]$
- $y[n] = x[n] - x[n-1]$

causal or not ?

$\forall n_0 \quad \underbrace{y[n_0]}_{\text{now}} = \underbrace{x[n_0]}_{\text{now}} - \underbrace{x[n_0-1]}_{\text{past}}$

Stability

$\exists M : |x(t)| \leq M \quad \forall t$

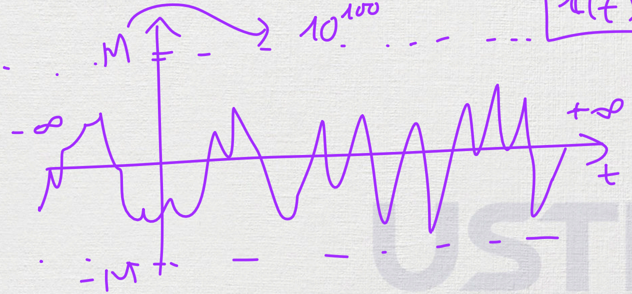
const

Stable Systems

The output signal is bounded whenever the input signal is bounded.

unbounded

$\boxed{x(t) = t}$



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Stability

S_1 is not stable!

Stable Systems

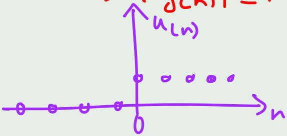
The output signal is bounded whenever the input signal is bounded. $\exists M \; |x(n)| \le M \; \forall n \Rightarrow |x(-n)| \le M \; \forall n$

Examples

- S_1: $y(t) = tx(t)$ → unstable! → $|y(n)| \le M \; \forall n$
- S_2: $y[n] = x[-n]$ → stable
- S_3: Home!

$$y[n] = \sum_{k=-\infty}^{n} u[k]$$

where $u[n]$ is the unit step



discrete

$$u(n) = \begin{cases} 1 & \text{if } n \ge 0 \\ 0 & \text{else} \end{cases}$$

Time Invariance

Time-Invariant Systems

The input-output relation does not depend on the time origin, i.e. the systems are fixed over time.

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Linearity

$x(t) \xrightarrow{\text{input}} \boxed{S} \xrightarrow{\text{output}} y(t)$

$y(t)$ is the response of S to $x(t)$

$\underbrace{3 \cdot x_1(t) + 7 x_2(t) - 2 \cdot x_3(t)}_{\text{weighted sum}}$ weight

Linear Systems

If an input consists of the weighted sum of several signals, then the output is the weighted sum of the responses of the system to each of those signals.

$3x_1(t) + 7x_2(t) \longrightarrow ? = 3y_1(t) + 7y_2(t)$

$\downarrow$

LINEAR!!!!!

Linearity

Linear Systems

If an input consists of the weighted sum of several signals, then the output is the weighted sum of the responses of the system to each of those signals.

In math language

Given a system in which:

- $y_1(t)$ is the response to an input $x_1(t)$
- $y_2(t)$ is the response to an input $x_2(t)$

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Then the system is linear if:

- The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$
  Additivity

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Then the system is linear if:

- The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$
  Additivity
- The response to $ax_1(t)$ is $ay_1(t)$, where a is a constant
  Scaling

Exercises

Examples

- $y(t) = tx(t)$

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Exercises

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- $y(t) = tx(t)$
- $y[n] = x[-n]$
- $y[n] = 2x[n] + 5$

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Homework

Systems' Exercises

1.15, 1.17, 1.18, 1.27, 1.28, 1.31, 1.42

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1.15, 1.17, 1.18, 1.27, 1.28, 1.31, 1.42

A Gentle Remind

Exam problems will not be different from homework!!