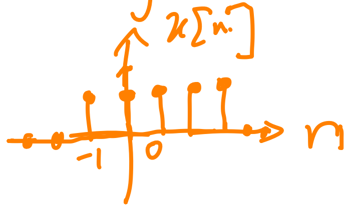
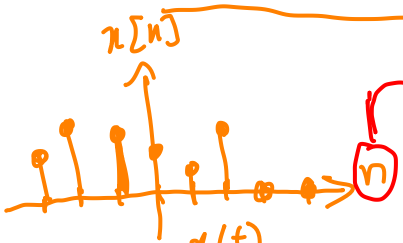
Review: Signal

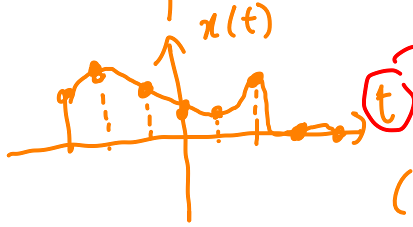
-  Diagram

- $x[n] = \begin{cases} u[n+1], & n < 4 \\ 0, & \text{else} \end{cases}$ formula

- $x[n] = [1, \underset{\uparrow}{1}, 1, 1, 1]$ vector

$\Rightarrow$ Represent Signal

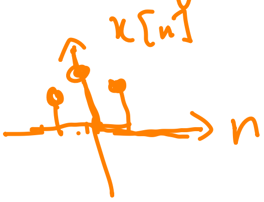
-  disc-time (time is point)

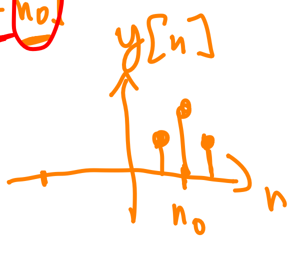
-  cont-time (time is continuous)

$\Rightarrow$ Signal Classification

- Time Shift

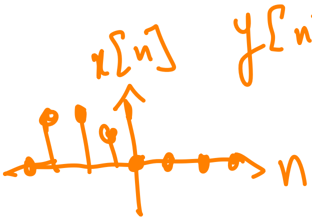
$y[n] = x[n - n_0]$

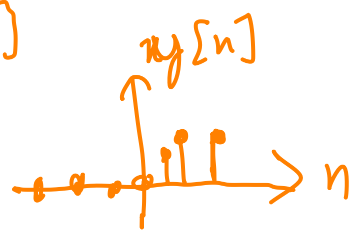




- Time Reverse:

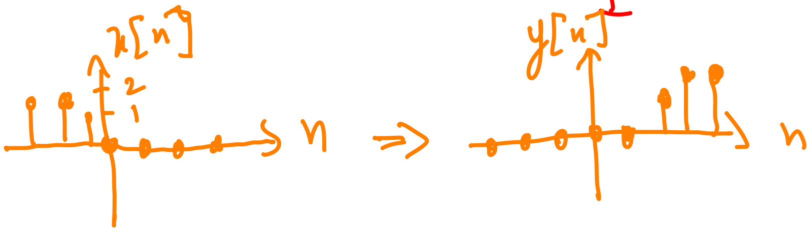
$y[n] = x[-n]$





- Combine Time Reverse + time Shift

$$y[n] = x[-n + \underbrace{n_0}_{1}]$$



$[2, 2, 1, 0]$ $\quad$ $[0, 0, 1, 2, 2]$

$\hookrightarrow y[n] = x[-n+1]$

$x[0] = 0 \rightarrow (-n+1) = 0 \Rightarrow n = 1 \Rightarrow y[1] = 0$

$x[-1] = 1 \longrightarrow n = 2 \Rightarrow y[2] = 1$

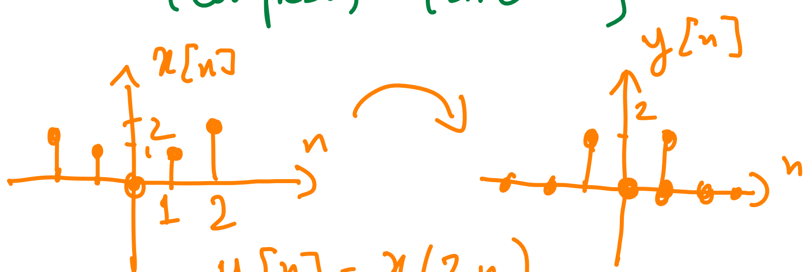
$x[-2] = 2 \longrightarrow n = 3 \Rightarrow y[3] = 2$

$x[-3] = 3 \longrightarrow n = 4 \Rightarrow y[4] = 2$

- Time Scale:

$$y[n] = x[kn] \Rightarrow (kn)$$

$k > 1$ (Compress) $\quad$ $0 < k < 1$ (stretch)



$y[n] = x(2n)$

$\hookrightarrow k > 1$

$y[1] = x[2] = 2$

$y[2] = x[4] = 0$

$y[-1] = x[-2] = 2$

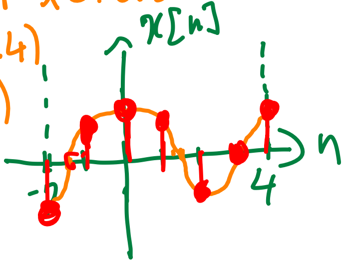
$y[2] = x[-4] = 0$

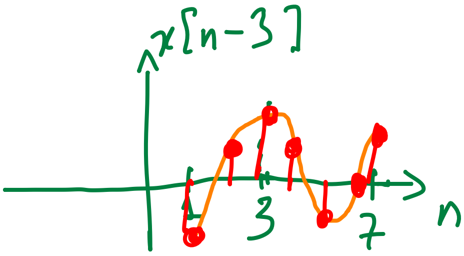
$\Rightarrow$ Signal Transform.

Exercise:

1.4)

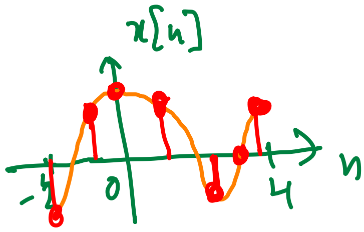
a)

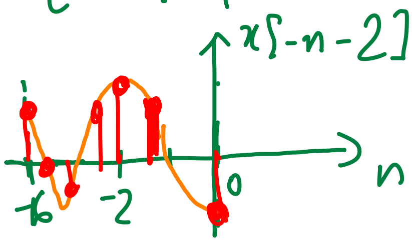




$x[n-3] = 0$, when $\begin{cases} n < 1 \\ n > 7 \end{cases}$

e)



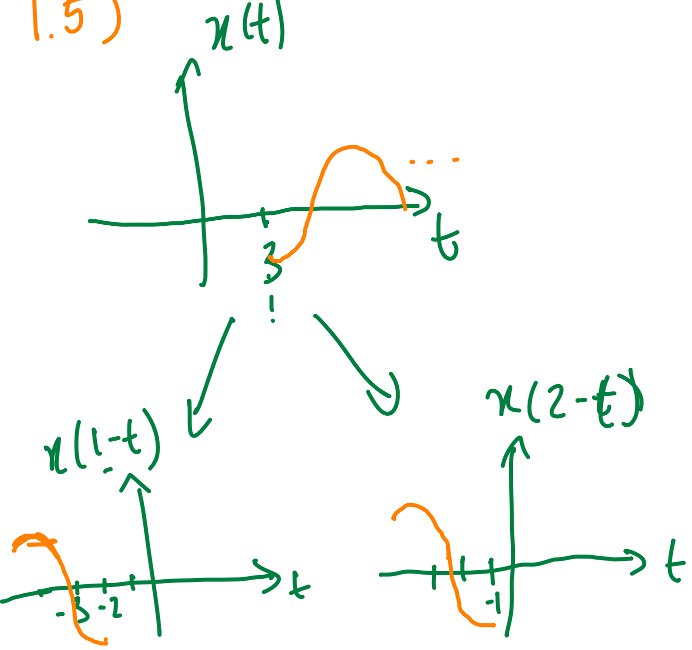


$x[-n-2] = 0$, when $\begin{cases} n < -6 \\ n > 0 \end{cases}$

$\hookrightarrow x[-n-2] = 0$, when $\begin{cases} -n-2 < -2 \\ -n-2 > 4 \end{cases}$

$\Leftrightarrow \begin{cases} n > 0 \\ n < -6 \end{cases}$

1.5)



$x(t) = 0,\ t < 3$

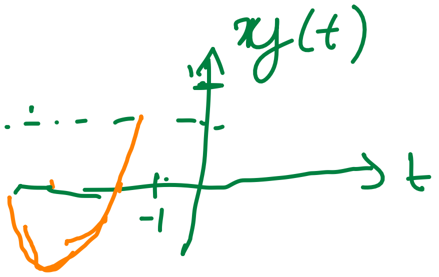
$\left.\begin{array}{l} x(1-t) = 0,\ t > -2 \\ x(2-t) = 0,\ t > -1 \end{array}\right\} \Rightarrow y(t) = 0,\ \underline{t > -1}$

$y(t) = \cos(1-t) + \cos(2-t)$

$= 2\cos\left(\frac{1-t+2-t}{2}\right)\cos\left(\frac{1-t-2+t}{2}\right)$

$= 2\cos\left(\frac{3-2t}{2}\right)\cos\left(-\frac{1}{2}\right)$

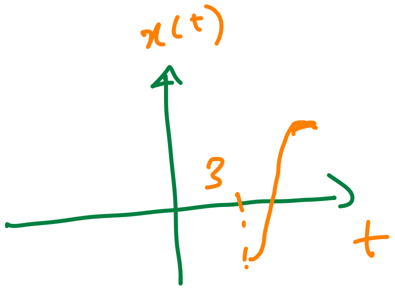
$= \underbrace{2\cos\left(-\frac{1}{2}\right)}_{A}\ \underbrace{\cos\left(-t+\frac{3}{2}\right)}_{t \leq -1}$



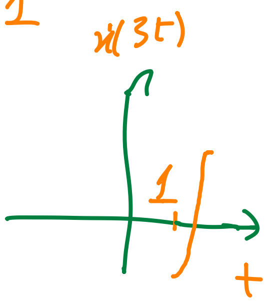
d) $x(3t)$

$x(t) = 0,\ t < 3$

$x(3t) = 0,\ 3t < 3 \iff t < 1$



$\Rightarrow$



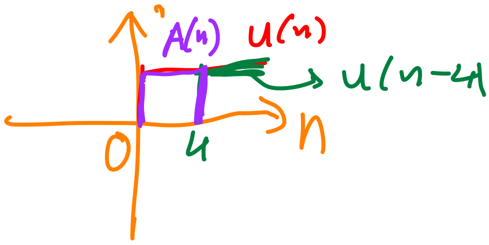
1.7)

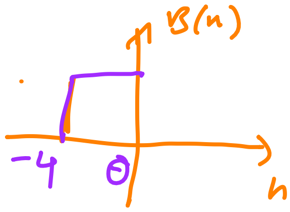
a) $Ev(x(t)) = \frac{x(t) + x(-t)}{2}$

$Ev(x(n)) = \frac{x_1(n) + x_1(-n)}{2}$

$= \underbrace{u(n) - u(n-4)}_{A(n)} + \underbrace{u(-n) - u(-n-4)}_{B(n)}$

$A(n) = 1,\ 0 < n < 4 \iff A(n) = 0 \begin{cases} n < 0 \\ n \geq 4 \end{cases}$





$$B(n) = 0, \iff \begin{cases} n > 0 \\ n \leqslant -4 \end{cases}$$

$$y(n) = \frac{A(n) + B(n)}{2} = 0 \iff \begin{cases} n \leq -4 \\ n \geqslant 4 \end{cases}$$

Review : System

- Relation between y(t) vs x(t)

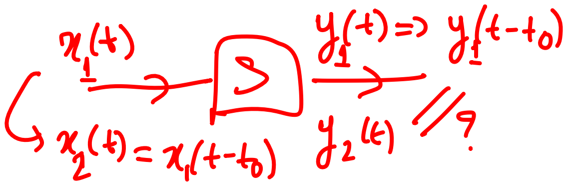
  Combination of Signal transform

- 4 properties
  - Causality
  - Stability

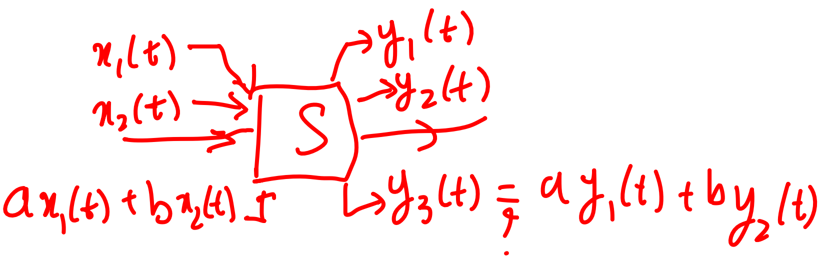
    if $|x(t)| < N \ \forall t$

    then $\exists M$ that

    $|y(t)| < M \ \forall t$
  - Time Invariance

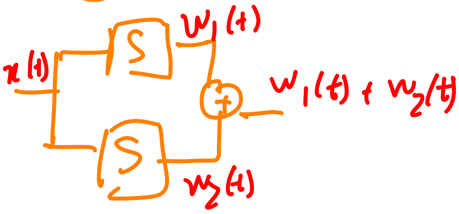


  - Linearity



- Interconnection
  - Series

  - parallel



Review: LTI System

$$\underset{\delta(t)}{x(t)} \rightarrow \boxed{LTI} \rightarrow \underset{h(t)}{y(t)}$$

- Lineality
- Time Invariance

Disc	Cont
• $\delta[n] = \begin{cases} 1 & ; n = 0 \\ 0 & , \text{else} \end{cases}$ Impulse	$\int_{-\infty}^{+\infty} \delta(t) = 1$
• $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$	• $y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$
• $h[n] = 0 \ \forall n < 0$ ↳ Causal	$h(t) = 0 \ \forall n < 0$ ↳ Causal
• $\sum_{n=-\infty}^{+\infty} \lvert h[n] \rvert < +\infty$ ↳ Stable	$\int_{-\infty}^{+\infty} \lvert h(t) \rvert\, dt < +\infty$ ↳ Stable

• Convolution Properties:

- Commutative

$$x(t) * h(t) = h(t) * x(t)$$

- Distributive

$$x(t) * (h_1(t) + h_2(t)) = x(t) * h_1(t) + x(t) * h_2(t)$$

- Associative

$$x(t) * (h_1(t) * h_2(t)) = (x(t) * h_1(t)) * h_2(t)$$

• Computation Convolution:

- Compute convolution formula directly
- Use 5 steps below:

• S1: Fix input Signal

• S2: Flip impulse Resp Signal

• S3: Align original of Fixed Signal vs Flip Signal

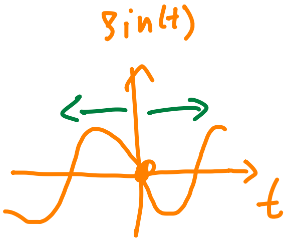
- S4: Element wise multiply 2 Signals and Sum them.
- S5: Assign value to $y[0]$ then Shift Flip Signal.

Exercise: 1.17, 2.1

(1.17)

- $y(t) = x(\sin(t))$

  ↳ not casual



$t = -\frac{\pi}{2} \Rightarrow \sin(-\frac{\pi}{2}) = -1$

$= -1.57$

$y(-1.57) = x(-1)$

- $y_1(t) = x_1(\sin(t))$

  $y_2(t) = x_2(\sin(t))$

$x_3(t) = ax_1(t) + bx_2(t)$

↳ $y_3(t) = x_3(\sin(t)) \stackrel{?}{=} ay_1(t) + by_2(t)$

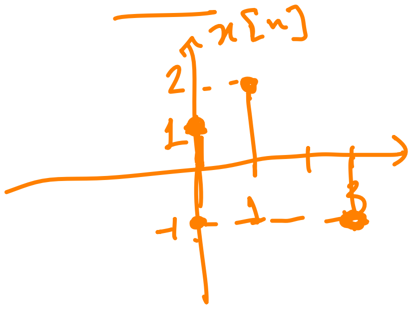
$= a x_1(\sin(t)) + b x_2(\sin(t))$

$= a y_1(t) + b y_2(t)$

$\Rightarrow$ System is linear.

2.1)

$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$



$\Rightarrow x[n] = \{\underset{\uparrow}{1}, 2, 0, -1\}$

$h[n] = 2\delta[n+1] + 2\delta[n-1] \Rightarrow h[n] = [2, \underset{\uparrow}{0}, 2]$

$y_1[n] = x[n] * h[n]$

$h'[n] = h[-n] = [2, \underset{\uparrow}{0}, 2]$

$y[0] = [\underset{\uparrow}{1}, 2, 0, -1] = 4$

$[2, \underset{\uparrow}{0}, 2]$

$y[-1] = [\underset{\uparrow}{1}, 2, 0, -1] = 2$

$[2, 0, \underset{\uparrow}{2}]$

$y[1] = [\underset{\uparrow}{1}, 2, 0, -1] = 2$

$[\underset{\uparrow}{2}, 0, 2]$

$y[2] = [\underset{\uparrow}{1}, 2, 0, -1] = 2$

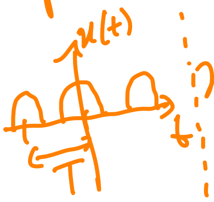
$[\underset{\uparrow}{0}, 2, 0, 2]$

$\vdots$

$y[n] = [2, \underset{\uparrow}{4}, 2, 2, 0, -2]$

Review : Fourier Series

- periodic Signal :



$x(t) = x(t + \underline{T})$

period value

Disc	Cont
• $e^{j\underset{\downarrow}{n}} = j\sin(n) + \cos(n)$ integer	• $e^{j\underset{\downarrow}{t}} = j\sin(t) + \cos(t)$ Real
• Synthesis $x[n] = \sum_{\langle N \rangle} a_k e^{jk\omega_0 n}$, $\omega_0 \Rightarrow \frac{2\pi}{N}$	• Synthesis : $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$, $\omega_0 \Rightarrow \frac{2\pi}{T}$
• Analysis $a_k = \frac{1}{N} \sum_{\langle N \rangle} x[n] e^{-jk\omega_0 t}$	• Analysis : $a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$

- $\sin(\varphi) = \dfrac{e^{j\varphi} - e^{-j\varphi}}{2j}$ $\qquad \cos(\varphi) = \dfrac{e^{j\varphi} + e^{-j\varphi}}{2}$

- Properties :

- Linear : $x(t) \overset{FS}{\longleftrightarrow} a_k$

$y(t) \overset{FS}{\longleftrightarrow} b_k$

$A\,x(t) + B\,y(t) \overset{FS}{\longleftrightarrow} A a_k + B b_k$

- Time Rev :

$$x(-t) \xleftrightarrow{FS} a_{-k}$$

- Multiply :

$$x(t) \cdot y(t) \xleftrightarrow{FS} \sum_{l=-\infty}^{+\infty} a_l b_{k-l}$$