

Signals &
Systems

TRAN
Hoang Tung

Objectives

Fourier
Transform

Exercises

Periodic
Signals

Homework

Continuous-time Fourier Transform (Part 1)

TRAN Hoang Tung

Information and Communication Technology (ICT) Department
University of Science and Technology of Hanoi (USTH)

The Course So Far

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Previous Lessons

- Signals and Systems
- Convolution
- Fourier Series

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Today

Fourier Transform in Continuous-time domain

periodic

CT FS

DT FS

aperiodic

CT FT

DT FT

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At the end of this lesson, you should be able to

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At the end of this lesson, you should be able to

- 1 understand that Fourier Transform is mainly for aperiodic signals

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At the end of this lesson, you should be able to

- 1 understand that Fourier Transform is mainly for aperiodic signals
- 2 find Fourier Transform of any continuous-time signals

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At the end of this lesson, you should be able to

- 1 understand that Fourier Transform is mainly for aperiodic signals
- 2 find Fourier Transform of any continuous-time signals
- 3 see the connection between Fourier Series and Fourier Transform

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2 Fourier Transform

- Review: Continuous-time Fourier Series
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Review: Continuous-time Fourier Series

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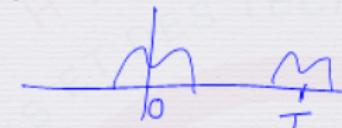
Periodic
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Homework

Fourier Pair: $\omega_0 = 2\pi/T$ $\frac{2\pi}{T}$

The **synthesis** equation

$$\underline{x(t)} = \sum_{k=-\infty}^{+\infty} a_k e^{\underline{j k \omega_0 t}}$$



The **analysis** equation

$$a_k = \frac{1}{T} \int_T^T x(t) e^{-jk\omega_0 t} dt$$

Review: Continuous-time Fourier Series

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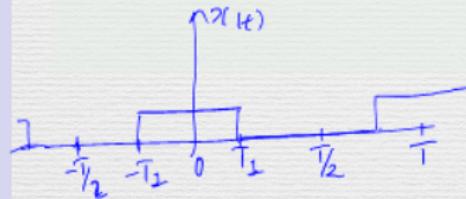
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Example

Determine the Fourier series coefficients for the periodic square wave, defined over one period as



$$x(t) = \begin{cases} 1 & \text{if } |t| < T_1 \\ 0 & \text{if } T_1 < |t| < T/2 \end{cases}$$

$$\begin{aligned} a_k &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot e^{-j k \omega_0 t} dt \\ &= \frac{1}{T} \int_{-T_1}^{T_1} e^{-j k \omega_0 t} dt \\ &= \frac{1}{T} \left[\frac{e^{-j k \omega_0 t}}{-j k \omega_0} \right]_{-T_1}^{T_1} = \frac{\sin(\pi k \frac{2T_1}{T})}{\pi k} \end{aligned}$$

Review: Continuous-time Fourier Series

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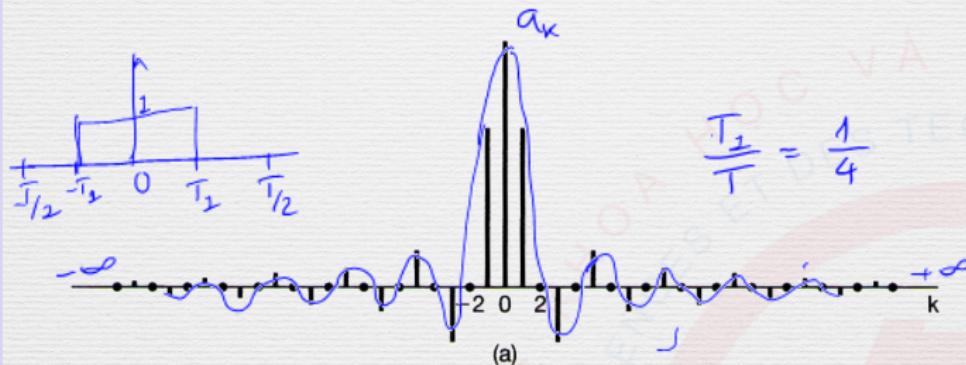
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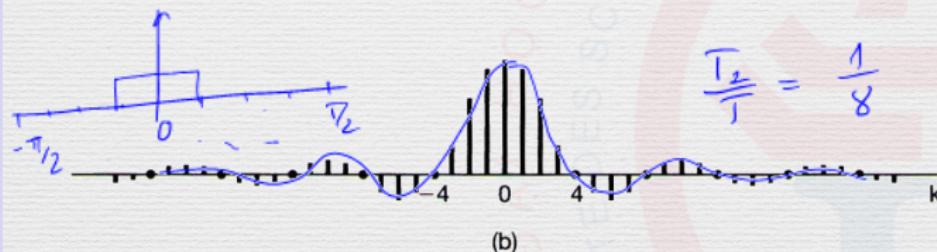
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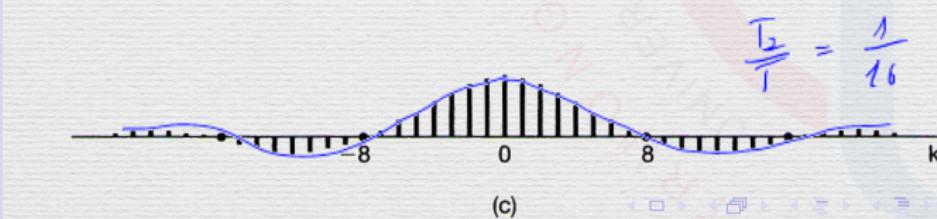
Homework



(a)



(b)



(c)

Sinc Function: $\text{sinc}(\theta) = \sin(\pi\theta)/\pi\theta$

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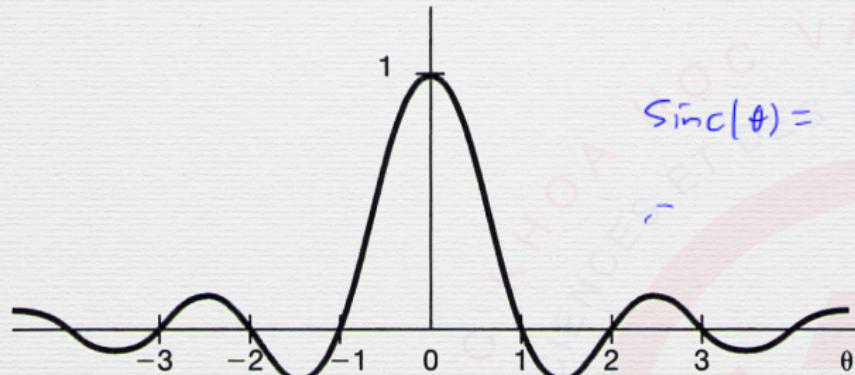
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$$\text{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}$$

The Envelope of Ta_k

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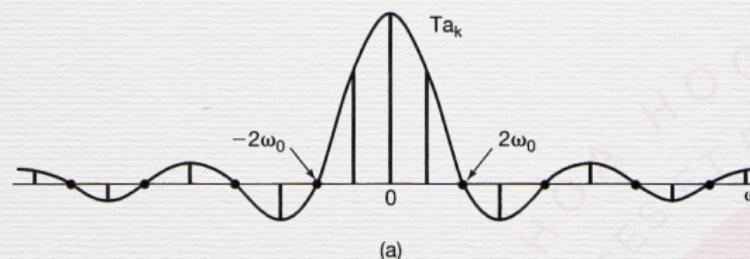
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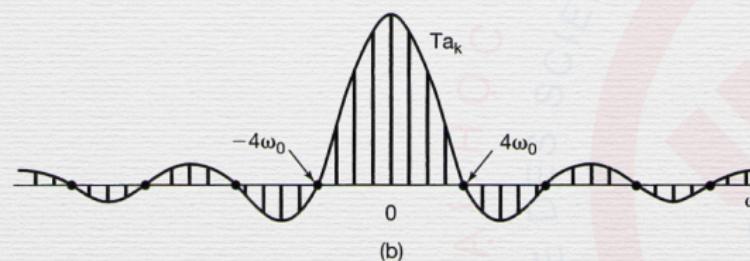
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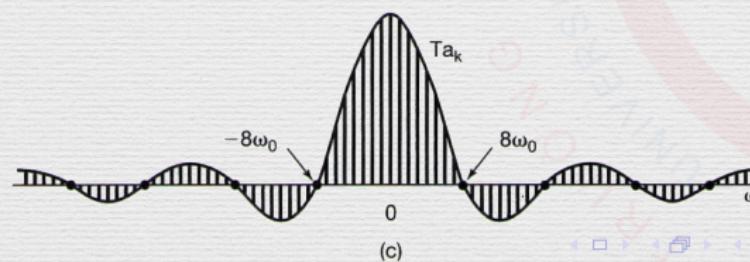
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(a)



(b)



(c)

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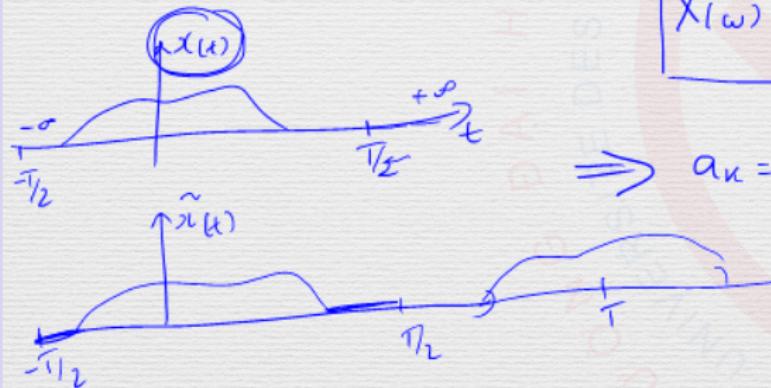
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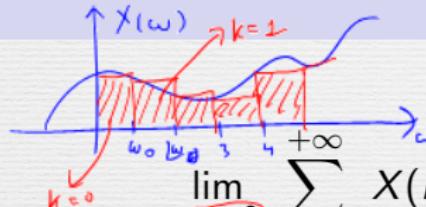
Consider a signal $x(t)$ that is of finite duration, and a periodic signal $\tilde{x}(t)$ for which $x(t)$ is one period.

$$\begin{aligned}
 T a_k &= \int_T \tilde{x}(t) e^{-jk\omega_0 t} dt = \int_{-T/2}^{T/2} \tilde{x}(t) e^{-jk\omega_0 t} dt \\
 &= \int_{-\infty}^{+\infty} x(t) e^{-jk\omega_0 t} dt = X(\omega) \Big|_{\omega=k\omega_0} \\
 &\quad \boxed{X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt}
 \end{aligned}$$



$$a_k = \frac{X(k\omega_0)}{T}$$

Definition



$$\omega_0 = \frac{2\pi}{T} \Rightarrow \frac{1}{T} = \frac{\omega_0}{2\pi}$$

$$\lim_{\omega_0 \rightarrow 0} \sum_{k=-\infty}^{+\infty} X(k\omega_0) \omega_0 = \int_{-\infty}^{+\infty} X(\omega) d\omega$$

$$\begin{aligned}\tilde{x}(t) &= \sum_{-\infty}^{+\infty} a_k \cdot e^{jk\omega_0 t} \\ &= \sum_{-\infty}^{+\infty} \frac{X(k\omega_0)}{T} \cdot e^{jk\omega_0 t} = \frac{1}{2\pi} \sum_{-\infty}^{+\infty} X(k\omega_0) \cdot e^{jk\omega_0 t} \cdot \omega_0\end{aligned}$$

$$\downarrow \omega_0 \rightarrow 0 : \tilde{x}(t) \rightarrow x(t)$$

$$\left\{ \begin{array}{l} x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) \cdot e^{j\omega t} dt \\ X(\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt \end{array} \right.$$

Fourier Transform Pair

$$x(t) = e^{-t} \cdot u(t+3)$$

Definition

$$\hookrightarrow x_{(\omega)} = ?$$

The **synthesis** equation (inverse Fourier transform)

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

The **analysis** equation (Fourier transform)

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$$



cont.
time

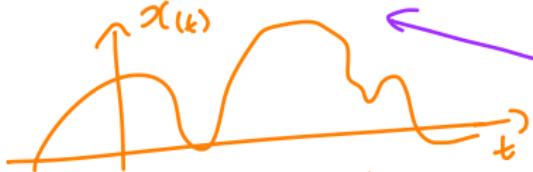
analysis

frequency

FT

synthesis

$X(\omega)$



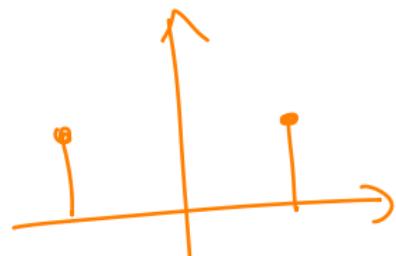
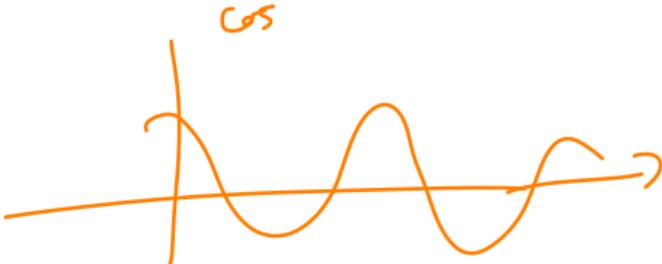
$X(\omega)$

$$\omega = 2\pi f$$

ω

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Exercise 1

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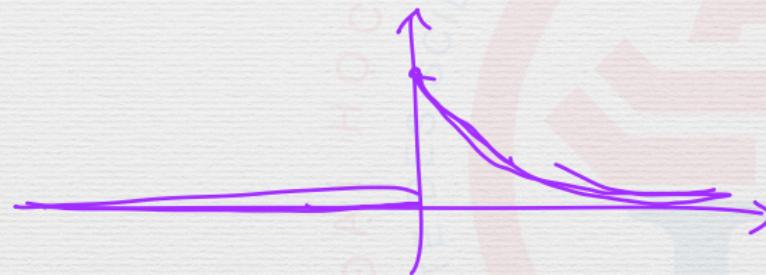
Homework

Exercise

Determine the Fourier Transform of the signal

$$x(t) = e^{-at} u(t), \text{ with } \underline{\underline{a > 0}} \rightarrow a = 1$$

$$x(t) = e^{-t} \cdot u(t)$$



$$X(\omega) = |X(\omega)| \cdot e^{j\angle X(\omega)}$$

Exercise 1

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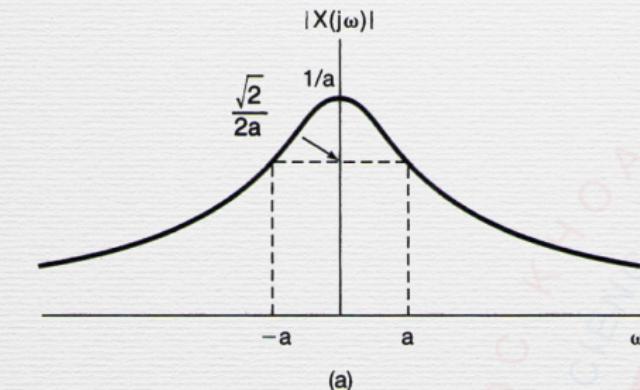
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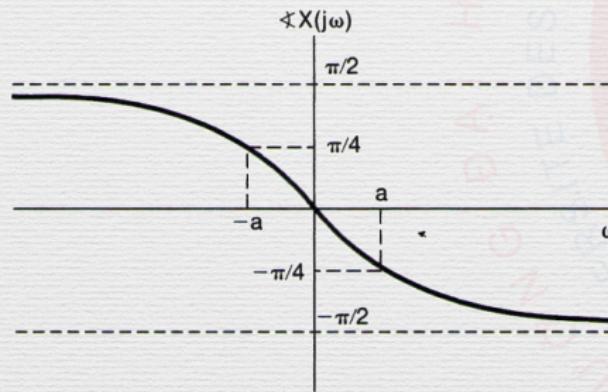
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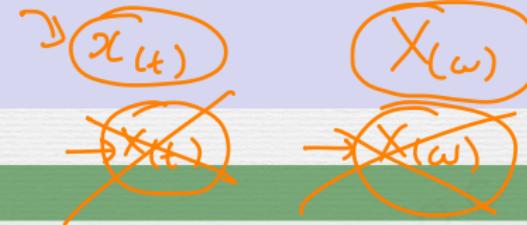


(a)



(b)

Exercise 2



Exercise

Determine the Fourier Transform of the signal

$$x(t) = e^{-a|t|}, \text{ with } a > 0$$

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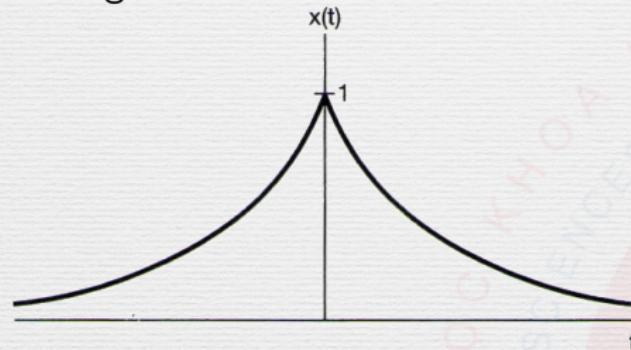
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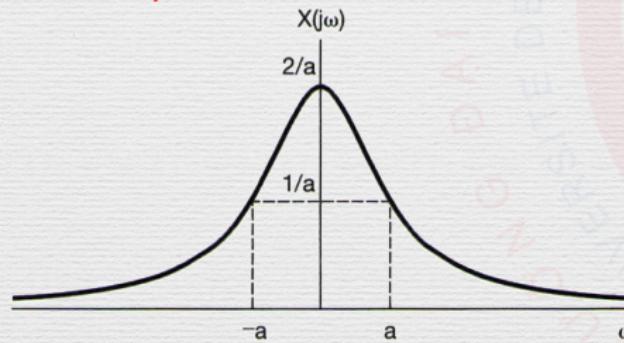
Periodic
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Homework

The signal



And its spectrum



Exercise 3

$$\delta(t) = \begin{cases} +\infty & \text{if } t=0 \\ 0 & \text{else} \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(t) dt = 1$$

Exercise

Determine the Fourier Transform of the Delta signal.



$$\delta(t) \rightarrow \Delta(\omega) = ?$$

$$\Delta(\omega) = \int_{-\infty}^{+\infty} \delta(t) \cdot e^{-j\omega t} dt = \int_{-\infty}^{+\infty} \delta(t) \cdot dt = 1$$

$\cancel{\text{if } t \neq 0:}$

$$\delta(t) = 0 \rightarrow 0 \cdot e^{-j\omega t} = 0$$

$$\boxed{\Delta(\omega) = 1 \quad \forall \omega}$$

$$\begin{aligned} & \delta(t) \cdot e^{-j\omega t} = \delta(t) \cdot 1 \\ \cancel{\text{if } t=0} \rightarrow & e^0 = 1 \rightarrow \delta(t) \cdot e^{-j\omega t} = \delta(t) \cdot 1 = \delta(t) \end{aligned}$$

Exercise 4

Euler

$$e^{j\varphi} = \cos \varphi + j \cdot \sin \varphi$$

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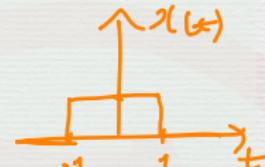
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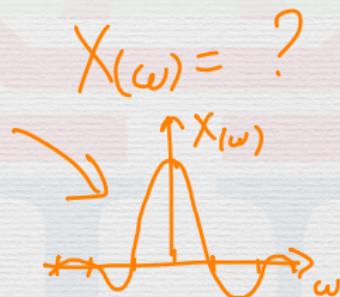
Exercise

Find the Fourier Transform of the rectangular pulse signal

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$



$$\begin{aligned} X(\omega) &= \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt \\ &= \int_{-T_1}^{T_1} e^{-j\omega t} dt = \dots \end{aligned}$$



remember:

$$\sin \varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$

Exercise 4

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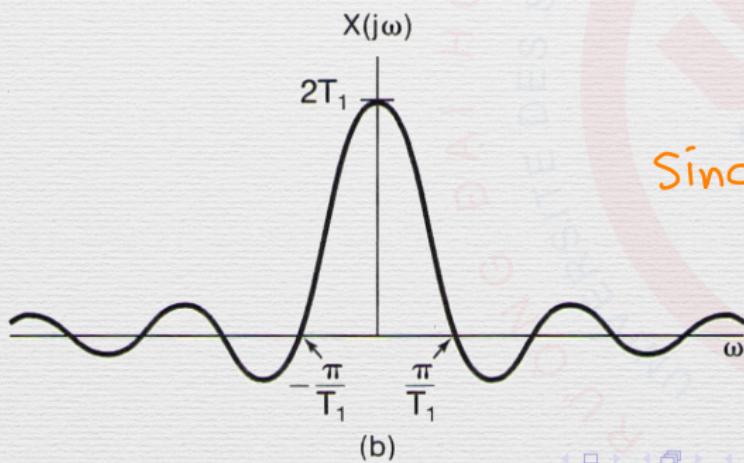
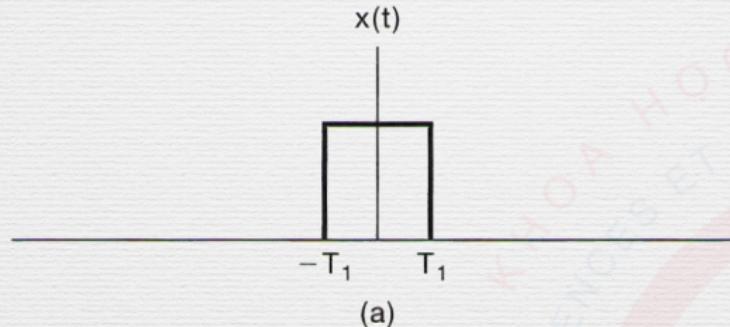
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Exercise 5

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Exercise

Determine the signal $x(t)$ whose Fourier transform is

$$X(\omega) = \begin{cases} 1, & |\omega| < \cancel{\omega_1} \\ 0, & |\omega| > \cancel{\omega_1} \end{cases}$$

Exercise 5

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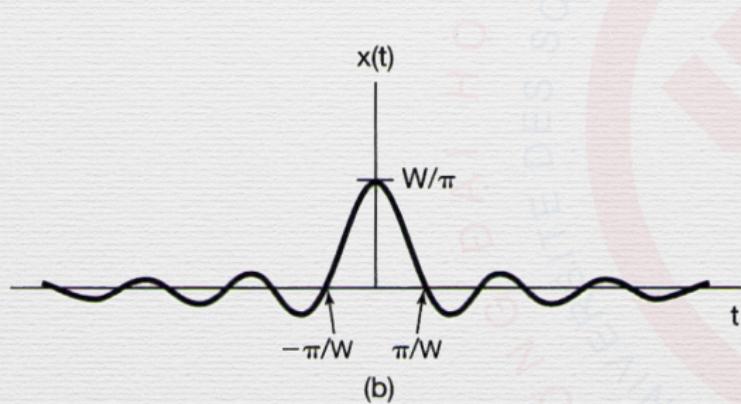
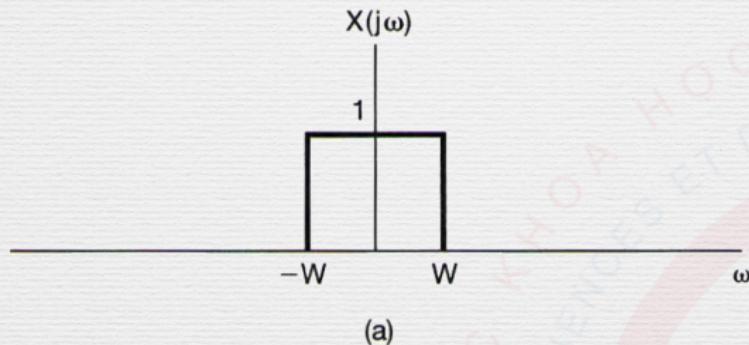
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Consider a signal with Fourier transform $X(\omega) = 2\pi\delta(\omega - \omega_0)$

what is $x(t)$? $\rightarrow e^{i\omega_0 t}$

$$X(\omega) = 2\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$\hookrightarrow x(t) = ?$

An Example

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Determine Fourier transform of $\sin(\omega_0 t)$ and $\cos(\omega_0 t)$

An Example

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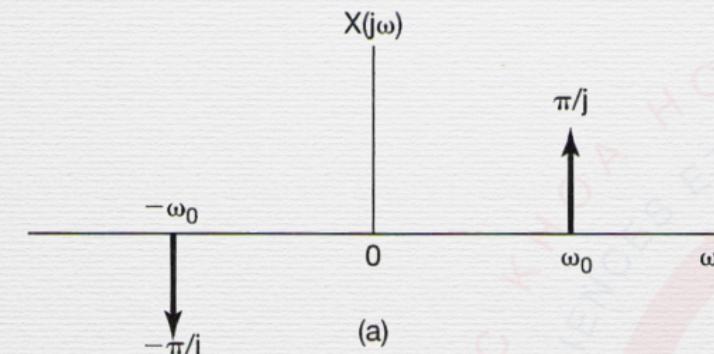
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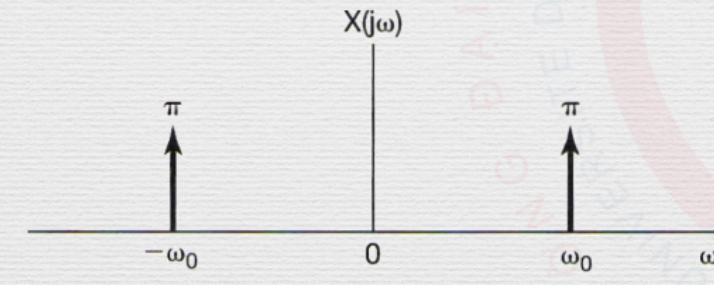
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(b)

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Continuous-time Fourier Transform Exercises

4.1, 4.3, 4.4, 4.13, 4.22