FUNDAMENTALS OF DATABASES

Relational Model of Data

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References

- Nguyen Hoang Ha and Le Huu Ton, Fundamentals of Databases, Chapter 2, USTH Textbook 2024
- Jeffrey D.Ullman and Jennifer Widom, *A First Course In Database System, 3rd Edition, Chapter 2;* Prentice-Hall International

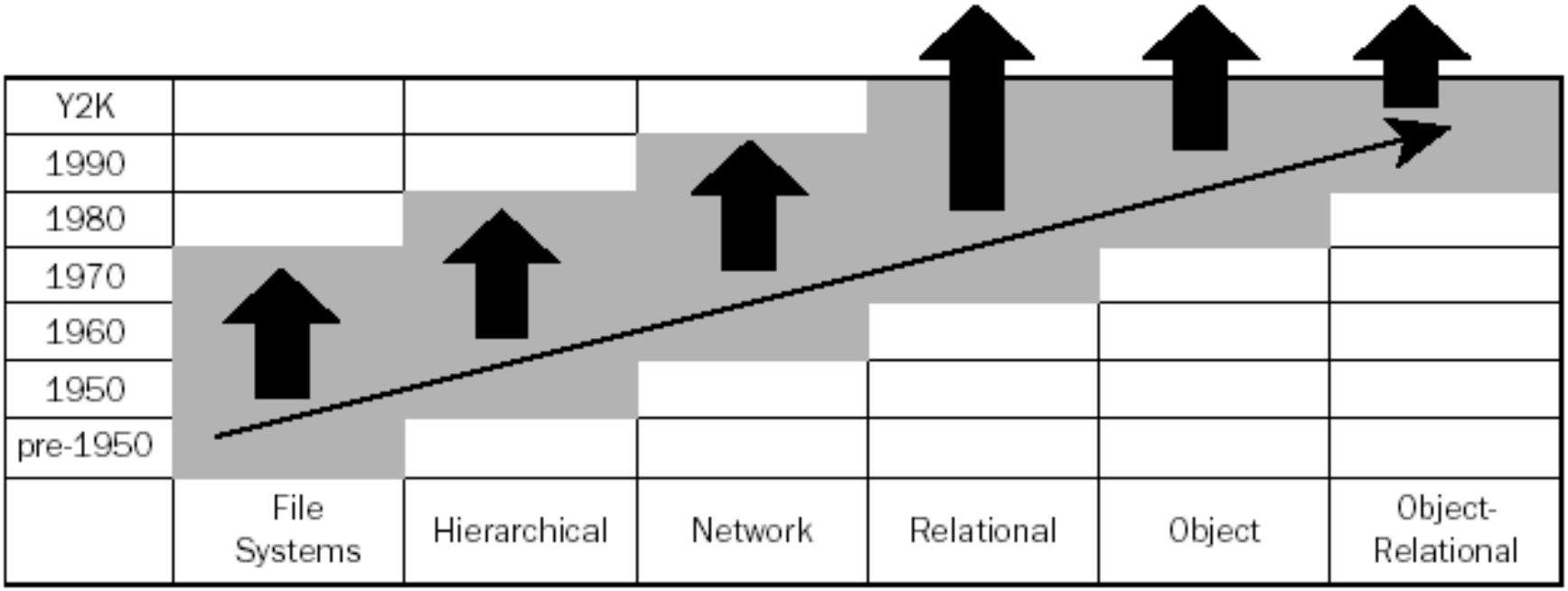
Objectives

- Understand concepts of
  - Data models
  - Relational data model
    - Structure
    - Operations with the Relational Algebra
    - Constraints

What is a Data Model?

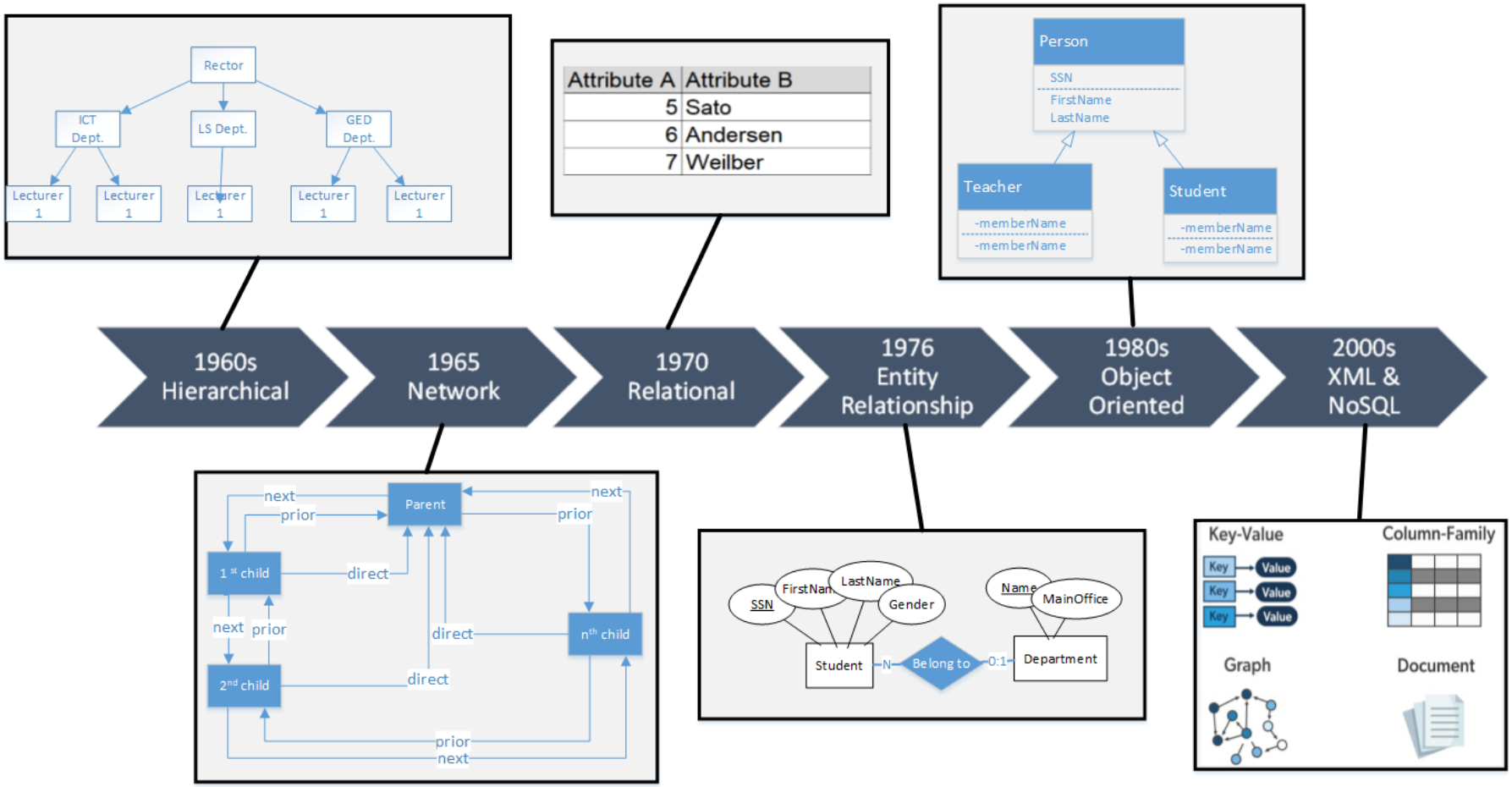
- Mathematical representation of data.
  - Relational model = tables;
  - Semi-structured model = trees/graphs.
- A model consists of:
  - Structure of data
  - Operations on data.
  - Constraints.

Data Models in history



The evolution of database modeling techniques.

Data Models in history



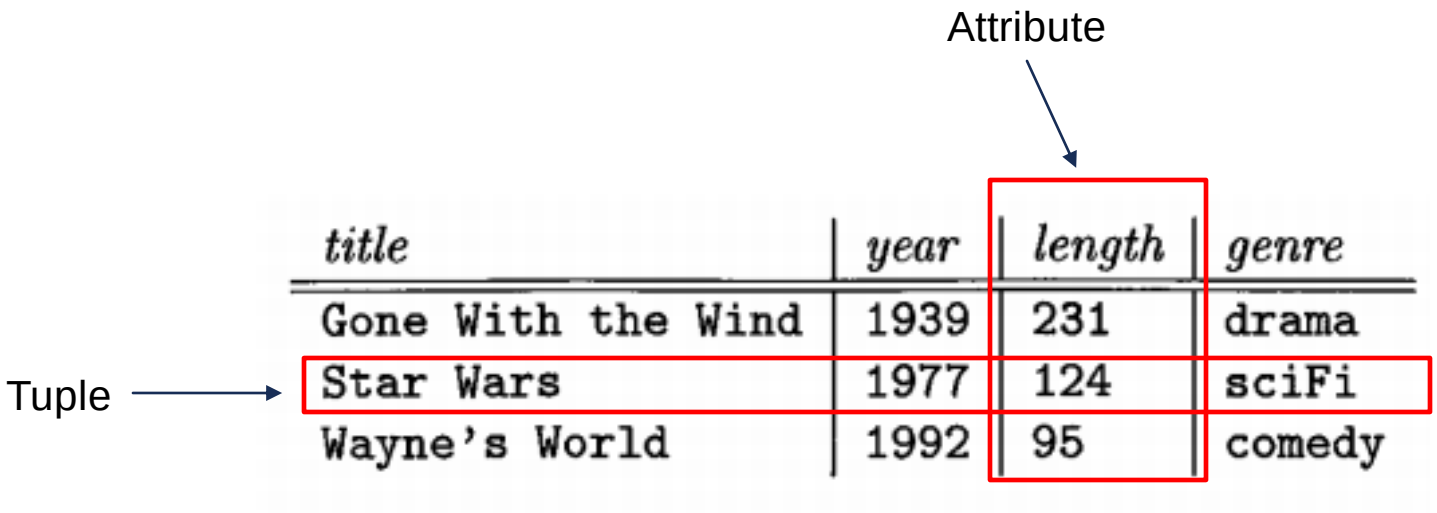
Why Relational Model?

- Very simple model.
- *Often* matches how we think about data.
- Abstract model that underlies SQL, the most important database language today.

Reference: E. F. Codd, "A relational model for large shared data banks," Comm. ACM 13:6, pp. 377-387, 1970.

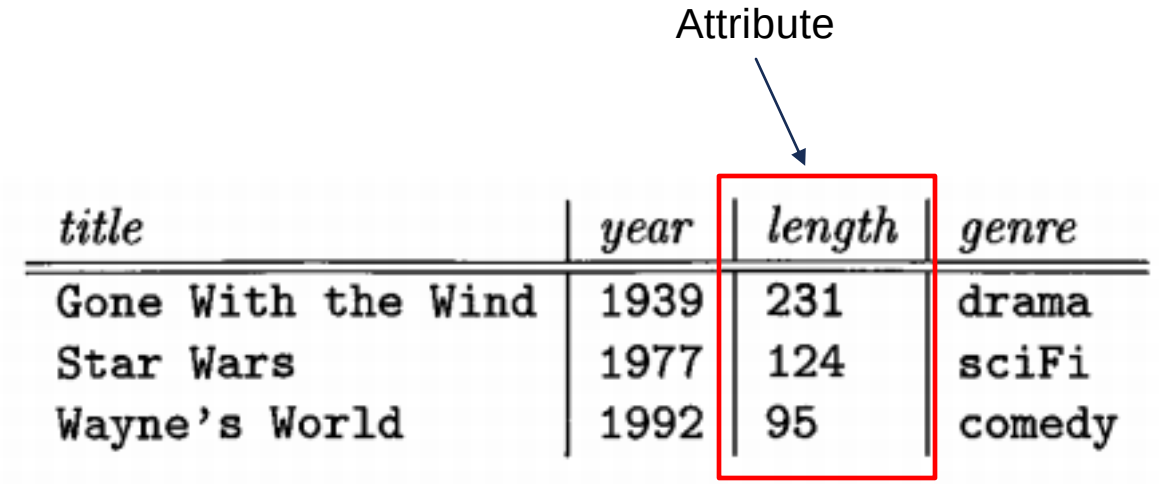
STRUCTURE OF RELATIONAL MODEL

Relational Model in brief



- The relational model represents data as a 2-dimensional table (called a relation)

Attributes



- Each column represent a property of film and also called a "**attribute**": title, year, length, genre

Domains

- The relational model requires that each component of each tuple must be atomic, that is, it must be of some elementary type such as INTEGER or STRING
- It is **not** permitted for a value to be a record structure, set, list, array or any type that can have its values broken into smaller components
- A Domain is a particular elementary type of an attribute
  - More general: a set of values for an attribute

→ What are domains of title, year, length, genre?

Tuples

Tuple →

title	*year*	*length*	*genre*
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

- A row of a relation is called a tuple (or record)
  - Eg: Each row represents a film
- When we want to write a tuple in isolation, not as part of a relation, we normally use commas to separate components
  - Eg: (Star Wars, 1977, 124, sciFi)

Equivalent representation of a relation

- Relation is a **bag** (extended set) of tuples, not a list of tuples
  - Order of tuples is not important

title	*year*	*length*	*genre*
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

=

title	*year*	*length*	*genre*
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

  - There could be identical tuples

- Question:
  - What is/are the difference(s) between a set and a list?
  - What is/are the difference(s) between a set and a bag?

Relation instances

- A relation about Movies is not static but changing over time:
  - We want to insert tuples for new Movies as these appear
  - We want to edit existing tuples if we get corrected information about a Movies
  - We want to delete a tuple from the relation
- Sometimes, we also want to add or delete attributes, this leads to the changing of the schema. So, a set of tuples for a given relation is called an instance of that relation

Relation instances example

R

A	B
1	2
3	4

R

A	B
1	3
2	5

R

A	B
3	5
4	6

Keys of relations

- A minimal set of attributes forms a key for a relation if we don't allow 2 tuples in a relation instance to have the same values in all the attributes of the key
- Eg: The key of relation
  People(<u>ID</u>, name, address)
  - (ID) is the a key
  - (ID, Name) not a key because it is not minimal
- The key selected as the main key is call Primary Key

Schemas

- Relation schema:
  - Name
  - Set of attributes.
    - Order of attributes is arbitrary. In practice we assume the order given in the relation schema
  - Other structure infor: keys…
  - Eg: Movies (Id, title, year, length, genre)
- Database schema: set of relation schema
  - Movies (Id, title, year, length, genre)
  - Producers (name, address, country)

DB Schema about Movies

```
Movies(
    title:string,
    year:integer,
    length:integer,
    genre:string,
    studioName:string,
    producerC#:integer
)
MovieStar(
    name:string,
    address:string,
    gender:char,
    birthdate:date
)
```

```
StarsIn(
    movieTitle:string,
    movieYear:integer,
    starName:string
)
MovieExec(
    name:string,
    address:string,
    cert#:integer,
    netWorth:integer
)
Studio(
    name:string,
    address:string,
    presC#:integer
)
```

OPERATIONS OF RELATIONAL MODEL

Relational Algebra (RA)

- Algebra is Mathematical system consisting of:
  - *Operands* --- variables or values from which new values can be constructed.
  - *Operators* --- symbols denoting procedures that construct new values from given values.
- Relational Algebra: an offshoot of algebra of **bags**
  - Operands:
    - Variables that stand for relations
    - Constants, which are finite relations
  - Operators work with relation(s) to create a new relation

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Notations of operators in RA

Operation	My HTML	Symbol
Projection	PROJECT	π
Selection	SELECT	σ
Renaming	RENAME	ρ
Union	UNION	$\cup$
Intersection	INTERSECTION	$\cap$
Assignment	<-	$\leftarrow$

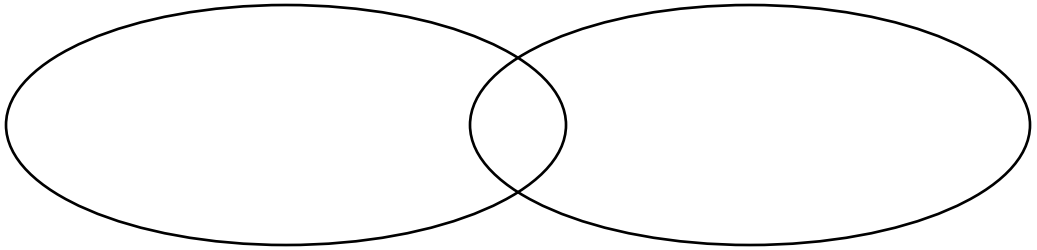
Operation	My HTML	Symbol
Cartesian product	X	$\times$
Join	JOIN	$\bowtie$
Left outer join	LEFT OUTER JOIN	$⟕$
Right outer join	RIGHT OUTER JOIN	$⟖$
Full outer join	FULL OUTER JOIN	$⟗$
Semijoin	SEMIJOIN	$\ltimes$

Primitive operations

- In any algebra, some operators are primitive and the others, being definable in terms of the primitive ones, are derived
- The six primitive operators of relational algebra are:
  - the SELECTION,
  - the PROJECTION,
  - the CARTESIAN PRODUCT (also called the *cross product* or *cross join*),
  - the SET UNION,
  - the SET DIFFERENCE, and
  - the RENAME

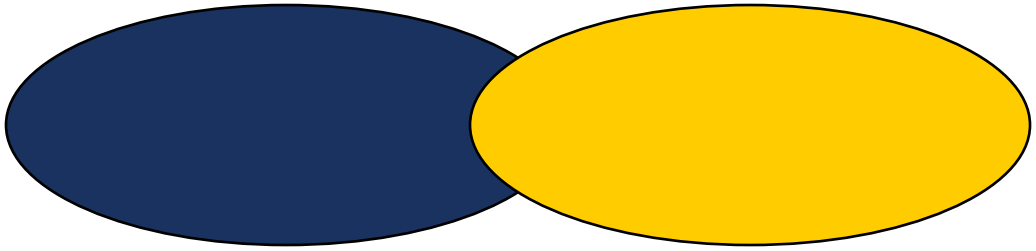
Operations on Relations - Bag Union

- Example: {1,2,1} U {1,1,2,3,1} = {1,1,1,1,1,2,2,3}



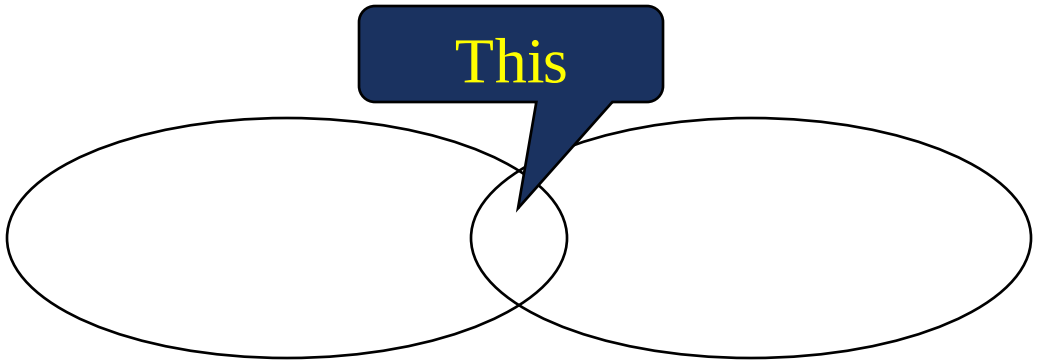
Operations on Relations - Bag Difference

- Example: {1,2,1,1} – {1,2,3} = {1,1}.



Operations on Relations - Bag Intersection

- Example: {1,2,1,1} ∩ {1,2,1,3} = {1,1,2}.



Projection

- R1 := Π_L (R2)
  - *L* is a bag of attributes from the schema of R2.
  - R1 is constructed by looking at each tuple of R2, extracting the attributes on *L*, in the order specified, and creating from those components a tuple for R1.
  - Eliminate duplicate tuples, if any.

Projection Example

Relation Sells:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Miller	3.00

Prices := $\Pi_{beer,\ price}$(Sells):

beer	price
Bud	2.50
Miller	2.75
Miller	3.00

Extended Projection

- Using the same π_L operator, we allow the L to contain arbitrary expressions involving attributes:
  - Arithmetic on attributes, e.g., A+B → C.
  - Duplicate occurrences of the same attribute.

R =

(A	B)
1	2
3	4

$\pi_{A+B->C,\ A,\ A}(R)$ =

C	A1	A2
3	1	1
7	3	3

Exercises - 1

- Suppose relation R(A,B,C) has the tuples:

R

A	B	C
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

- Compute the projection $\pi_{C,B}(R)$

Selection

R1 := σ_C (R2)

- *C* is a condition (as in "if" statements) that refers to attributes of R2.
- R1 is all those tuples of R2 that satisfy *C*.

Selection Example

Relation Sells:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Miller	3.00

JoeMenu := $\sigma_{bar="Joe's"}(Sells)$:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75

Renaming

- The ρ operator gives a new schema to a relation.
- R1 := $\rho_{R1(A1,\ldots,An)}$(R2) makes R1 be a relation with attributes A1,…,A*n* and the same tuples as R2.
- Simplified notation: R1(A1,…,A*n*) := R2.

Renaming Example

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

$R(bar, addr) := \rho_{R(bar, addr)}(Bars)$

R(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

Product

- R3 := R1 X R2
  - Pair each tuple t1 of R1 with each tuple t2 of R2.
  - Concatenation t1t2 is a tuple of R3.
  - Schema of R3 is the attributes of R1 and then R2, in order.
  - But beware attribute *A* of the same name in R1 and R2: use R1.*A* and R2.*A*.

R3 = R1 X R2

R1(

A,	B
1	2
3	4

)

R2(

B,	C
5	6
7	8
9	10

)

R3(

A,	R1.B,	R2.B,	C
1	2	5	6
1	2	7	8
1	2	9	10
3	4	5	6
3	4	7	8
3	4	9	10

)

R3 = R1 X R2

The table **E** (for **EMPLOYEE**)

enr	ename	dept
1	Bill	A
2	Sarah	C
3	John	A

The table **D** (for **DEPARTMENT**)

dnr	dname
A	Marketing
B	Sales
C	Legal

SQL	Result					Relational algebra
`select * from E, D`	enr	ename	dept	dnr	dname	E X D
	1	Bill	A	A	Marketing	
	1	Bill	A	B	Sales	
	1	Bill	A	C	Legal	
	2	Sarah	C	A	Marketing	
	2	Sarah	C	B	Sales	
	2	Sarah	C	C	Legal	
	3	John	A	A	Marketing	
	3	John	A	B	Sales	
	3	John	A	C	Legal	

Join

- Types
  - Theta join
  - Natural join
  - Inner join
  - Outer join:
    - Left outer join
    - Right outer join
    - Full outer join

Theta-Join

- R3 := R1 $\bowtie_C$ R2
  - Take the product R1 X R2.
  - Then apply σ_C to the result.
- As for σ C can be any boolean-valued condition.
  - Historic versions of this operator allowed only A θ B, where θ is =, <, etc.; hence the name "theta-join."

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Theta-Join Example

Sells(

bar,	beer,	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

)

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells $\bowtie_{\text{Sells.bar = Bars.name}}$ Bars

BarInfo(

bar,	beer,	price,	name,	addr
Joe's	Bud	2.50	Joe's	Maple St.
Joe's	Miller	2.75	Joe's	Maple St.
Sue's	Bud	2.50	Sue's	River Rd.
Sue's	Coors	3.00	Sue's	River Rd.

)

Natural Join

- A useful join variant (*natural* join) connects two relations by:
  - Equating attributes of the same name, and
  - Projecting out one copy of each pair of equated attributes.
- Denoted R3 := R1 ⋈ R2.

Eg: Natural Join

Sells(

bar,	beer,	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

)

Bars(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells ⋈ Bars

Note: Bars.name has become Bars.bar to make the natural join "work."

BarInfo(

bar,	beer,	price,	addr
Joe's	Bud	2.50	Maple St.
Joe's	Milller	2.75	Maple St.
Sue's	Bud	2.50	River Rd.
Sue's	Coors	3.00	River Rd.

)

Inner Join

The table **E** (for **EMPLOYEE**)

enr	ename	dept
1	Bill	A
2	Sarah	C
3	John	A

The table **D** (for **DEPARTMENT**)

dnr	dname
A	Marketing
B	Sales
C	Legal

SQL

```
select *
from E, D
where dept = dnr
```

Result

enr	ename	dept	dnr	dname
1	Bill	A	A	Marketing
2	Sarah	C	C	Legal
3	John	A	A	Marketing

Relational algebra

$$\textbf{SELECT}_{\text{dept} = \text{dnr}} (E \times D)$$

or, using the equivalent join operation

$$E \textbf{ JOIN}_{\text{dept} = \text{dnr}} D$$

Outer join

The table **E** (for **EMPLOYEE**)

enr	ename	dept
1	Bill	A
2	Sarah	C
3	John	A

The table **D** (for **DEPARTMENT**)

dnr	dname
A	Marketing
B	Sales
C	Legal

SQL

```
select *
from (E right outer join D on edept = dnr)
```

Result

enr	ename	dept	dnr	dname
1	Bill	A	A	Marketing
2	Sarah	C	C	Legal
3	John	A	A	Marketing
null	**null**	**null**	B	Sales

Relational algebra

$$E \text{ RIGHT OUTER JOIN}_{edept = dnr} D$$

Outer-join Example

CLASS	
CLASS_ID	**CLASS_NAME**
106	Lop 106
107	Lop 107
201	Lop 201
202	Lop 202

STUDENT		
CLASS_ID	**ID**	**NAME**
106	1	A
106	2	B
107	3	C
107	4	D
	5	E
	6	F
	7	G
	8	H

Class LEFT OUTER JOIN Student:

display all tuples from CLASS (but only tuples from STUDENT that aggree the conditions: CLASS.CLASS_ID = STUDENT.CLASS_ID)

CLASS		STUDENT		
CLASS_ID	**CLASS_NAME**	**CLASS_ID**	**ID**	**NAME**
106	Lop 106	106	1	A
106	Lop 106	106	2	B
107	Lop 107	107	3	C
107	Lop 107	107	4	D
201	Lop 201			
202	Lop 202			

Outer-join Example

CLASS	
CLASS_ID	**CLASS_NAME**
106	Lop 106
107	Lop 107
201	Lop 201
202	Lop 202

STUDENT		
CLASS_ID	**ID**	**NAME**
106	1	A
106	2	B
107	3	C
107	4	D
	5	E
	6	F
	7	G
	8	H

Class RIGHT OUTER JOIN Student

display all tuples from STUDENT (but only tuples from CLASS that aggree the conditions: CLASS.CLASS_ID = STUDENT.CLASS_ID)

CLASS		STUDENT		
CLASS_ID	**CLASS_NAME**	**CLASS_ID**	**ID**	**NAME**
106	Lop 106	106	1	A
106	Lop 106	106	2	B
107	Lop 107	107	3	C
107	Lop 107	107	4	D
			5	E
			6	F
			7	G
			8	H

Outer-join Example

CLASS	
CLASS_ID	**CLASS_NAME**
106	Lop 106
107	Lop 107
201	Lop 201
202	Lop 202

STUDENT		
CLASS_ID	**ID**	**NAME**
106	1	A
106	2	B
107	3	C
107	4	D
	5	E
	6	F
	7	G
	8	H

Class FULL OUTER JOIN Student

display all tuples from STUDENT and CLASS

CLASS		STUDENT		
CLASS_ID	**CLASS_NAME**	**CLASS_ID**	**ID**	**NAME**
106	Lop 106	106	1	A
106	Lop 106	106	2	B
107	Lop 107	107	3	C
107	Lop 107	107	4	D
			5	E
			6	F
			7	G
			8	H
201	Lop 201			
202	Lop 202			

Building Complex Expressions

- Three notations, just as in arithmetic:
  - Sequences of assignment statements.
  - Expressions with several operators.
    - Combine operators with parentheses and precedence rules.
  - Expression trees.

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Sequences of Assignments

- Create temporary relation names.
- Renaming can be implied by giving relations a list of attributes.
- Example:

  R3 := R1 $\bowtie_C$ R2 is a series of 2 operations: R3 = σ_C (R1 X R2)

  - R4 := R1 X R2
  - R3 := σ_C (R4)

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Expressions in a Single Assignment

- Example: the theta-join R3 := R1 $\bowtie_C$ R2 can be written:

  R3 := σ_C (R1 X R2)

- Precedence of relational operators:

  1. [σ, π, ρ] (highest).
  2. [x, ⋈].
  3. ∩.
  4. [∪, —]

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Expression Trees

- Leaves are operands --- either variables standing for relations or particular, constant relations.
- Interior nodes are operators, applied to their child or children.

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Example: Tree for a Query

- Using the relations Bars(name, addr) and Sells(bar, beer, price), find the names of all the bars that are either on Maple St. or sell Bud for less than $3.
- Recall: how to write the solution in:
  - Some assignments:
    - R1 = $\pi_{name}\,\sigma_{addr = \text{“Maple St.”}}$ (Bars)
    - R2 = $\rho_{R(name)}\,\pi_{bar}\;\;\sigma_{price<3 \text{ AND } beer=\text{“Bud”}}$ (Sells)
    - Result R1 U R2
  - Single (long) expression **Result** = $\pi_{name}\,\sigma_{addr = \text{“Maple St.”}}$ (Bars) $\cup$

$\rho_{R(name)}\,\pi_{bar}\;\;\sigma_{price<3 \text{ AND } beer=\text{“Bud”}}$ (Sells)

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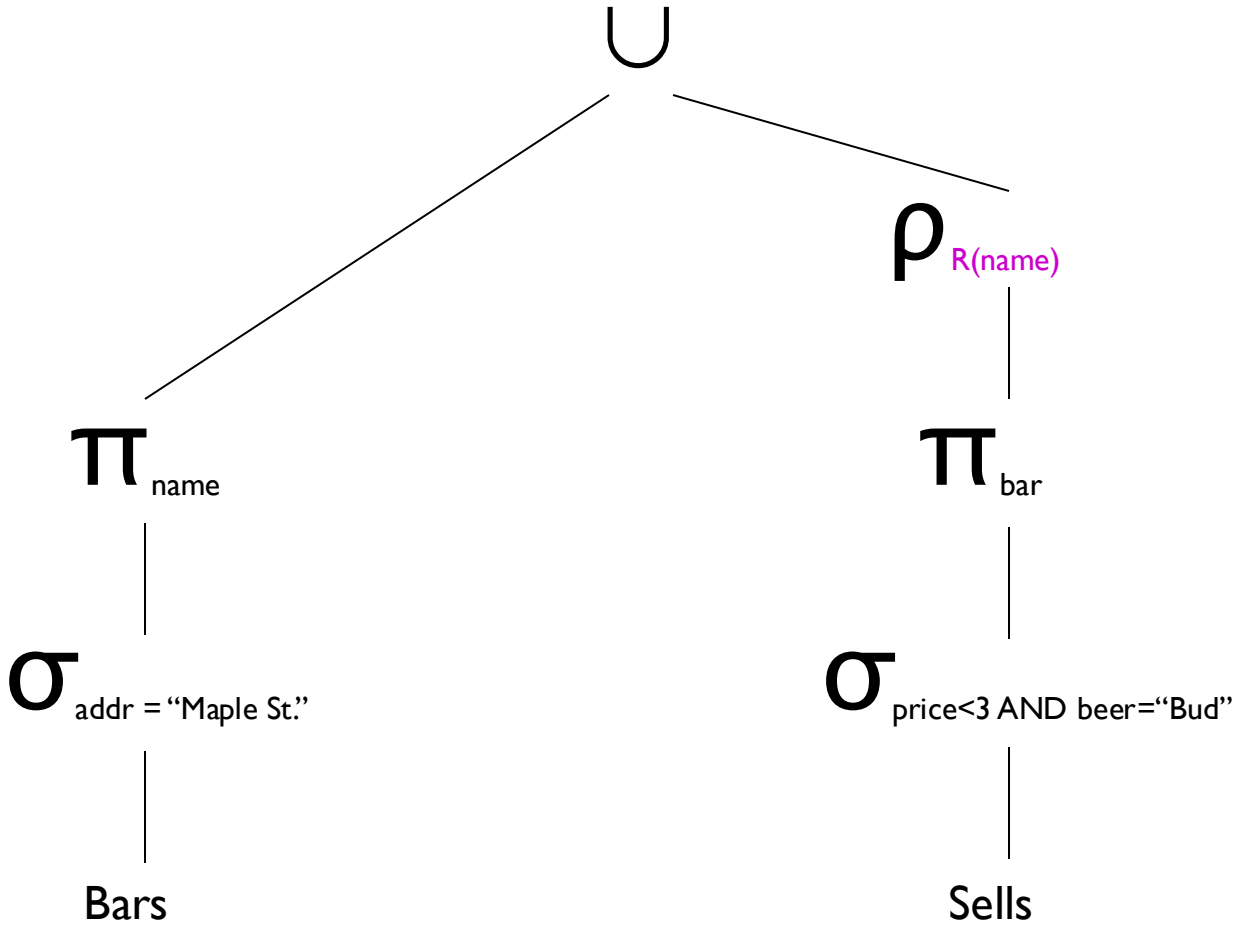
As a Tree:


∪
ρ R(name)
π name
π bar
σ addr = "Maple St."
σ price<3 AND beer="Bud"
Bars
Sells

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Example: Self-Join

- Using Sells(bar, beer, price), find the bars that sell two different beers at the same price.
- Strategy: by renaming, define a copy of Sells, called S(bar, beer1, price). The natural join of Sells and S consists of quadruples (bar, beer, beer1, price) such that the bar sells both beers at this price.

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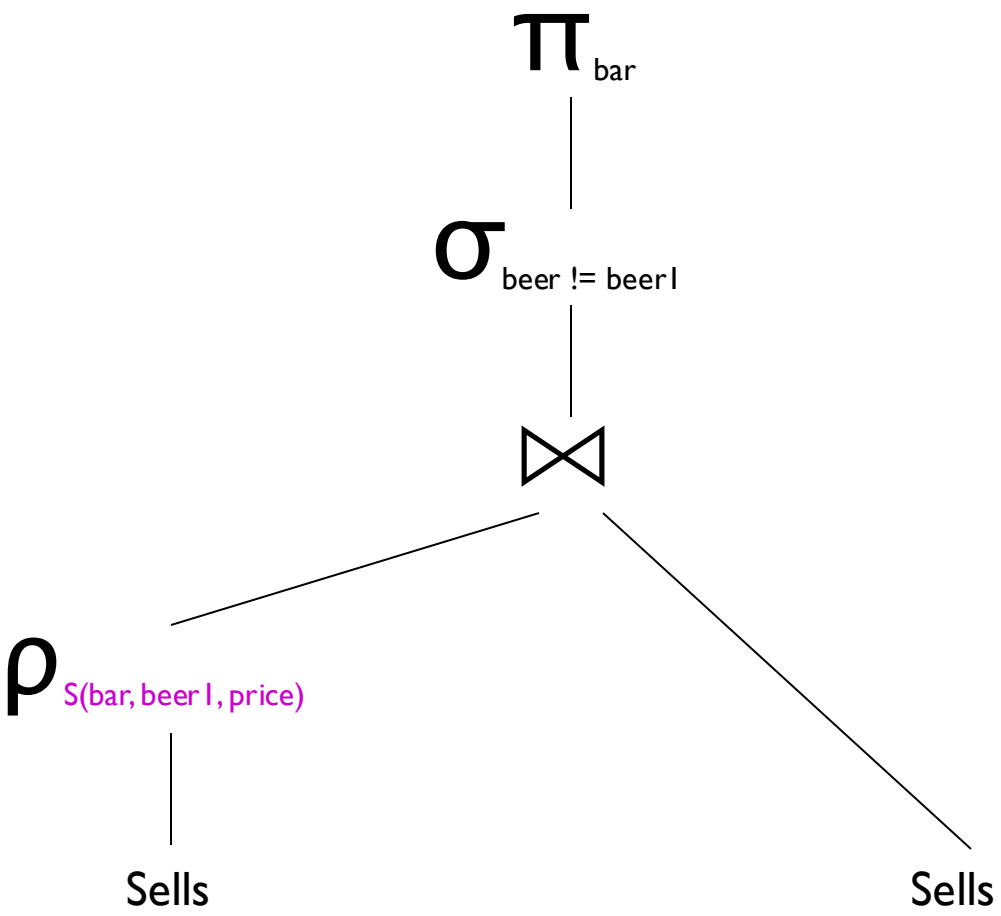
The Tree



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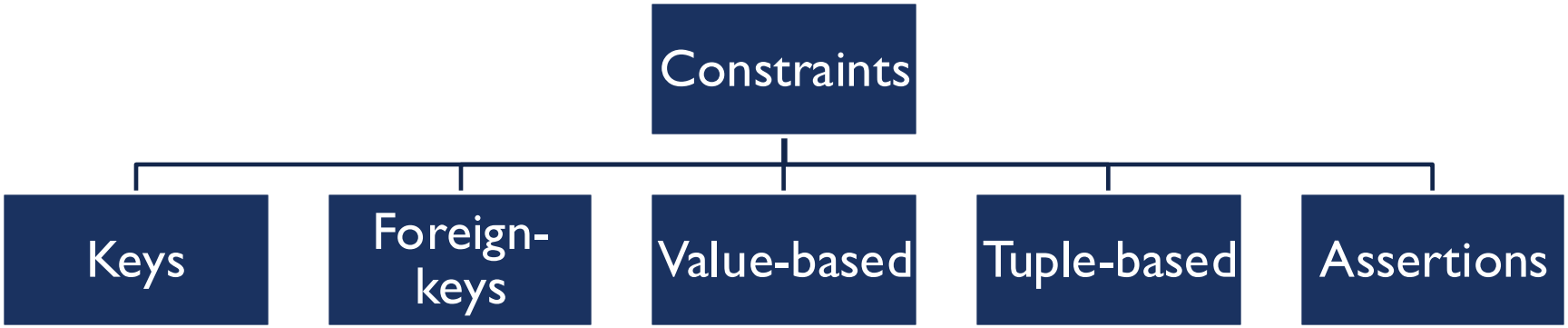
Exercise

- Consider a database schema
  - Product (maker, model, type)
  - PC (model, speed, ram, hd, price)
  - Laptop (model, speed, ram, hd, screen, price)
  - Printer (model, color, type, price)
- Write expressions in RA for following queries:
  a. What PC models have a speed of at least 3.0 → $\Pi_{model}(\sigma_{speed >= 3.0}(PC))$
  b. Which manufacturers make laptops with a hard disk of at least 100GB
  c. Find the model number and price of all products (of any type) made by manufacturer B.
  d. Find the model numbers of all color laser printers.
  e. Find those manufacturers that sell Laptops, but not PC's
  f. Find those hard-disk sizes that occur in two or more PC's
  g. Find those manufacturers of at least two different computers (PC's or laptops) with speeds of at least 2.80.
  h. Find the manufacturers of PC's with at least two different speeds

CONSTRAINS IN RELATIONAL MODEL

Introduction

- A *constraint* is a relationship among data elements that the DBMS is required to enforce.
- → mechanism that may be used to limit the values entered into a relation.



Key of a relation

- A set attributes $\{A_1, A_2, .., A_n\}$ is called a **key** of the relation R if:
  - 1. Those attributes determine all other attributes.
    → It is impossible for 2 tuples of R to agree on all of $\{A_1, A_2, .., A_n\}$
  - 2. No subset of $\{A_1, A_2, .., A_n\}$ determines all other attributes
  - →A Key must be minimal
- Example
  - Students (StudentID, FirstName, LastName, email, phone)
  - AccademicResults (StudentID, SubjectID, grade, comment)

Different key types of a relation

- Super key
  - Set of attributes (columns) to uniquely identify rows
- Key or candidate key
  - Minimal super key
- Primary key
  - One selected from candidate keys
- Alternate key
  - Candidate key other than PK
- Foreign key
  - Attribute refers to a PK of another relation
    - Students (StudentID, FirstName, LastName, email, phone)
    - AccademicResults (StudentID, SubjectID, grade, comment)
    - → AccademicResults.StudentID is the FK referring to Students.StudentID → AccademicResults.StudentID must be found in the list of values of Students.StudentID

Express Key Constraints by RA

- Suppose we have a schema $R(A_1,A_2,A_3)$
- So, we have A_1 is the key of R if:

$$\delta_{R1.A1=R2.A1 \textbf{ AND } R1.A2<>R2.A2 \textbf{ AND } R1.A3 <> R2.A3}(R \times R) = \Phi$$

Example: Express Key Constraints

- $\delta_{R1.OrderId = R2.OrderId \textbf{ AND } R1.Customer<>R2.Customer}(R \times R) = \Phi$

Order ID	Customer
10248	Vins et alcools Chevalier
10249	Toms Spezialitäten
10250	Hanari Carnes
10251	Victuailles en stock
10252	Suprêmes délices
10253	Hanari Carnes
10254	Chop-suey Chinese
10255	Richter Supermarkt
10256	Wellington Importadora
10257	HILARIÓN-Abastos
10258	Ernst Handel
10259	Centro comercial Moctezuma
10260	Ottilies Käseladen
10261	Que Delícia
10262	Rattlesnake Canyon Grocery
10263	Ernst Handel

Foreign Keys

- Values appearing in attributes of one relation must appear together in certain attributes of another relation.
- Example: in Sells(bar, beer, price), we might expect that a beer value also appears in Beers.name .

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Express Foreign Key Constraints

- Suppose we have 2 relations R and S.
  R contains an attribute A
  S contains an attribute B

- "R references to S via A and B" if: $\prod_A(R) - \prod_B(S) = \Phi$

Example: Express Foreign Key Constraints

Sells(

bar,	beer,	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

)

Bars(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

- $\prod_{Bar}(Sells) - \prod_{Bar}(Bars) = \Phi$

Express Check Constraints

- Give a schema:
  EMPLOYEE(EmpId, Name, Sex)

- If we want to specify that the only legal values for SEX attribute are ‘F’ and ‘M’, we can write:

  $SELECT_{sex <> 'F' \text{ and } sex <> 'M'} (EMPLOYEE) = \Phi$

Exercise 1

- Give a schema:
  PC (Model, Speed, RAM, HDD, Price)

- Use Relational Algebra to express following constraints:

  A PC with a processor speed less than 3.00 must not sell for more than $800

Exercise 2

- Give a schema:
  LAPTOP (Model, speed, RAM, HDD, Screen, Price)

- Use Relational Algebra to express following constraints:

  A laptop with a screen size less than 15.4 inches must have at least a 120GB hard disk or sell for less than $1000

Exercise 3

- PC (Model, Speed, RAM, HDD, Price)
- LAPTOP (Model, speed, RAM, HDD, Screen, Price)

- Use Relational Algebra to express following constraints:

  If a laptop has a larger RAM than a PC, then the laptop must also have a higher price than the PC

Exercise 4

- PC (Maker, Model)
- PRINTER (Maker, Model)

- Use Relational Algebra to express following constraints:

  No manufacturer of PC's may also make printers

Exercise 5

- PRODUCT (Maker, model)
- PC (Model)
- PRINTER (Model)

- Use Relational Algebra to express following constraints:

  No manufacturer of PC's may also make printers

Exercise 6

- PC (Model, Speed, RAM, HDD, Price)

- Use Relational Algebra to express following constraints:
  - Higher model, higher price
  - With the same model, higher Speed and RAM and HDD, higher price