Discrete Linear Time-Invariant (LTI) Systems

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Questions

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1 Lesson Objectives

2 Elementary Signals

3 Linear Time-Invariant Systems

4 Convolution Sum

5 Homework

Lesson Objectives

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1. understand the input-output relationship of a LTI system

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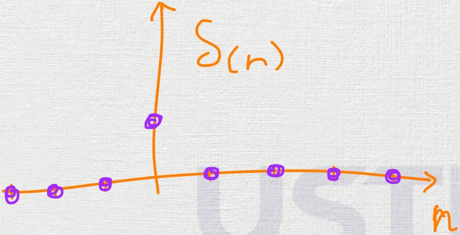
1. understand the input-output relationship of a LTI system
2. calculate convolution sum using different approaches

Other names: Dirac Delta, Impulse

Delta Signal

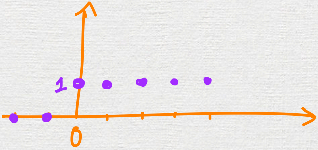
$$\delta[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$



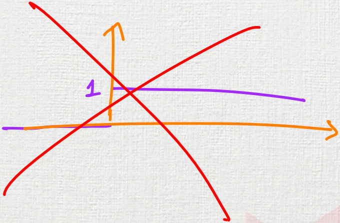
Other name: Step Signal

Unit Signal

$$u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$



Discrete



Continuous

Real Exponential Signals

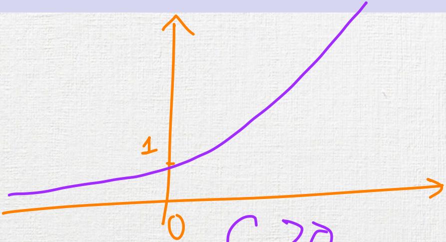
Definition

The continuous-time real exponential signal is of the form:

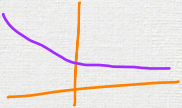
$$x(t) = Ce^{at}$$

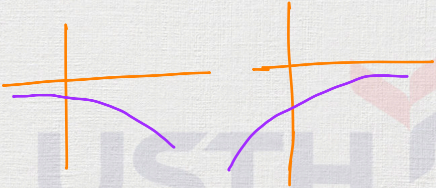
where C and a are real numbers.

$7 . e^{2t}$



$C > 0$





$C < 0$

Real Exponential Signals

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LTI

- Delta Signal
- A Simple System

4 Convolution Sum

5 Homework

Other names: Dirac Delta, unit impulse

Delta Signal

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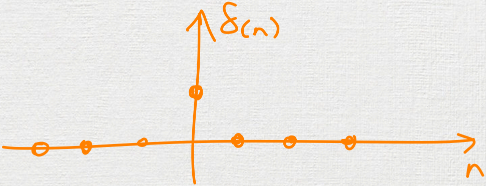

Other names: Dirac Delta, unit impulse

Delta Signal

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$

Why is this signal important?

Other names: Dirac Delta, unit impulse

Delta Signal

$$x_{(n)} = \sum_{\dots} \dots \delta(.)$$

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$

Why is this signal important?

- Delta is the bulding block of all signals

$$x_{(n)} = [1\ \underset{\uparrow}{3}\ 2] = [1\ \underset{\uparrow}{0}\ 0] = \delta(n+1)$$

$$+ [0\ \underset{\uparrow}{3}\ 0] \longrightarrow + 3.\delta(n)$$

$$+ [0\ \underset{\uparrow}{0}\ 2] \qquad + 2.\delta(n-1)$$

Other names: Dirac Delta, unit impulse

Delta Signal

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$

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- It helps to define a Linear Time-Invariant (LTI) system

Impulse Response

Delta
$\delta(n)$

A simple system

Definition

Consider a discrete-time system with input $x(n)$ and output $y(n)$ related by:

$$y(n) = x(n+1) - 2x(n)$$

A simple system

Definition

Consider a discrete-time system with input $x(n)$ and output $y(n)$ related by:

$$y(n) = x(n+1) - 2x(n)$$

Example

Determine whether the system is:

- Causal
- Stable
- Time invariant
- Linear

a LTI system: $y(n) = x(n+1) - 2x(n)$

Example

Determine $y(n)$ given:

a LTI system: $y(n) = x(n+1) - 2x(n)$

Example

Determine $y(n)$ given:

- $x(n) = \delta(n)$ $\rightarrow h(n) = \delta(n+1) - 2 . \delta(n)$

$$= [1 \quad \underset{\uparrow}{-2}]$$

a LTI system: $y(n) = x(n+1) - 2x(n)$

Example

Determine $y(n)$ given:

- $x(n) = \delta(n)$
- $x(n) = \delta(n-2)$ $\rightarrow h(n) = \delta(n-1) - 2 . \delta(n-2)$

a LTI system: $y(n) = x(n+1) - 2x(n)$

Example

Determine $y(n)$ given:

- $x(n) = \delta(n)$
- $x(n) = \delta(n-2)$
- $x(n) = [\underset{\uparrow}{1}, 1, 3]$ $\rightarrow y(n) = [1 \quad \underset{\uparrow}{-1} \quad 1 \quad -6]$

students ↓

Remind: Delta Impulse

Delta Signal

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$

Why is this signal important?

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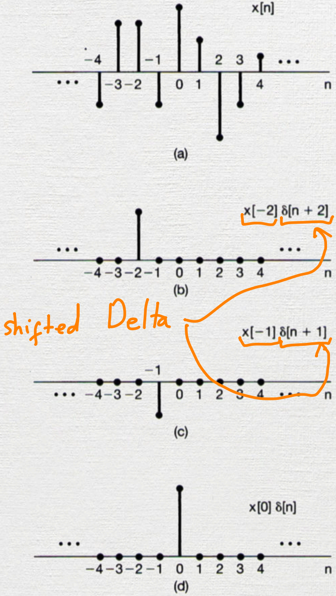
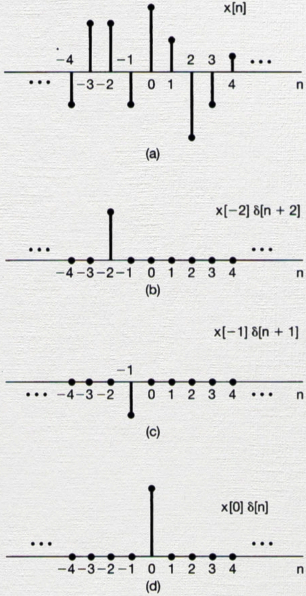
- It helps to define a Linear Time-Invariant (LTI) system

Weighted Sum of Shifted Impulses

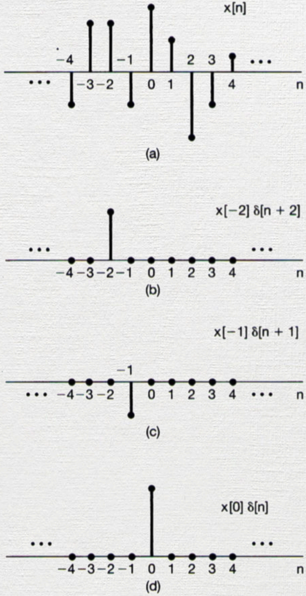


$$
\begin{aligned}
x(n) = \; & \vdots \\
& + x(-2)\cdot\delta(n+2) \\
& + x(-1)\cdot\delta(n+1) \\
& + x(0)\cdot\delta(n) \\
& + x(1)\cdot\delta(n-1) \\
& + x(2)\cdot\delta(n-2) \\
& \vdots \\
= & \sum_{k=-\infty}^{+\infty} x(k)\cdot\delta(n-k)
\end{aligned}
$$

Weighted Sum of Shifted Impulses

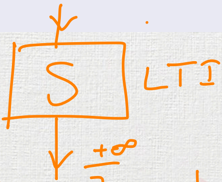


Weighted Sum of Shifted Impulses



Important

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$



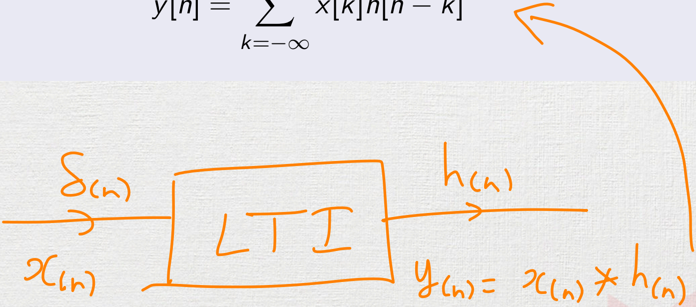
$$y(n) = \sum_{k=-\infty}^{+\infty} x(k).h(n-k)$$

$$\boxed{y(n) = x(n) * h(n)}$$

Convolution Sum

Definition

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$



Convolution Sum

Definition

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

Example

Consider a discrete LTI system:

$$y(n) = x(n+1) - 2x(n)$$

Determine $y(n)$ given $x(n) = [2, \underset{\uparrow}{1}, 3, 2]$

Direct Approach: $y(n) = x(n+1) - 2x(n)$

With $x(n) = [2, \underset{\uparrow}{1}, 3, 2]$.

Impluse #1: $y(n) = \overset{\delta}{\cancel{x}}(n+1) - 2\overset{\delta}{\cancel{x}}(n)$

With $x(n) = [2, \underset{\uparrow}{1}, 3, 2]$ and $h[n] = [1, \underset{\uparrow}{-2}]$ $\rightarrow$ Conv $y_{(n)} = x_{(n)} * h_{(n)}$

$[\underset{\uparrow}{-2} \quad 1]$

Step1: fix $x_{(n)}$

$[2 \quad \underset{\uparrow}{1} \quad 3 \quad 2]$

Step2:

$[\underset{\uparrow}{-2} \quad 1]$

step3 $\rightarrow 1.(-2) + 3.1 = 1 = y(0)$ step4

shift 1 $[\underset{\uparrow}{0} \quad -2 \quad 1] \rightarrow 3.(-2) + 2.1 = -4 = y(1$

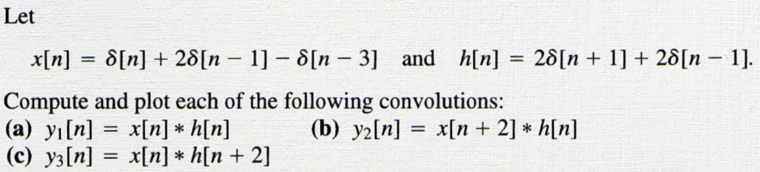
shift 2 $[\underset{\uparrow}{0} \quad 0 \quad -2 \quad 1] \rightarrow 2(-2) = -4 = y($

shift 3 $[\underset{\uparrow}{0} \quad 0 \quad 0 \quad -2 \quad 1]$

Impluse #2: $y(n) = x(n+1) - 2x(n)$

With $x(n) = [2, \underset{\uparrow}{1}, 3, 2]$ and $h[n] = [1, \underset{\uparrow}{-2}]$

Examples



Let

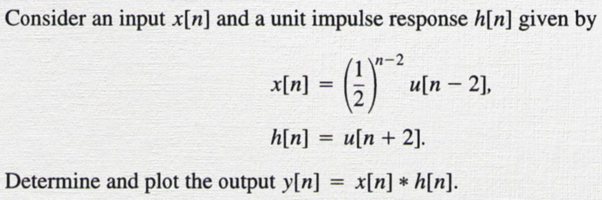
$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3] \quad \text{and} \quad h[n] = 2\delta[n+1] + 2\delta[n-1].$$

Compute and plot each of the following convolutions:

(a) $y_1[n] = x[n] * h[n]$ **(b)** $y_2[n] = x[n+2] * h[n]$

(c) $y_3[n] = x[n] * h[n+2]$

Exercises



Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2],$$

$$h[n] = u[n+2].$$

Determine and plot the output $y[n] = x[n] * h[n]$.

Homework

Discrete LTI Systems Exercises

2.3, 2.4, 2.7, 2.24