Signals

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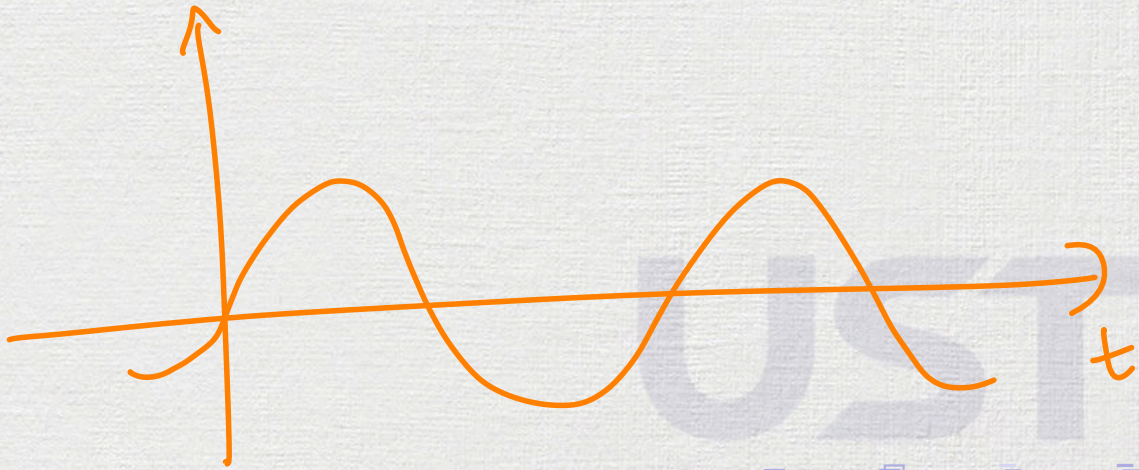
Signals and Systems: An introduction

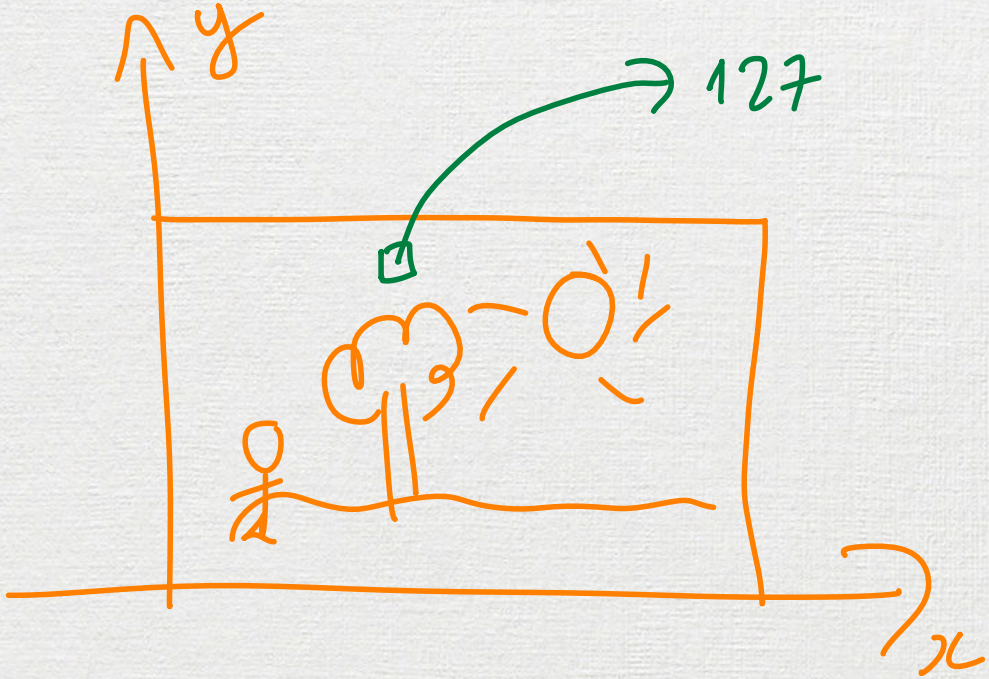
A signal is a function of one or more variables.

- 1-dimensional: a speech signal



A signal is a function of one or more variables.

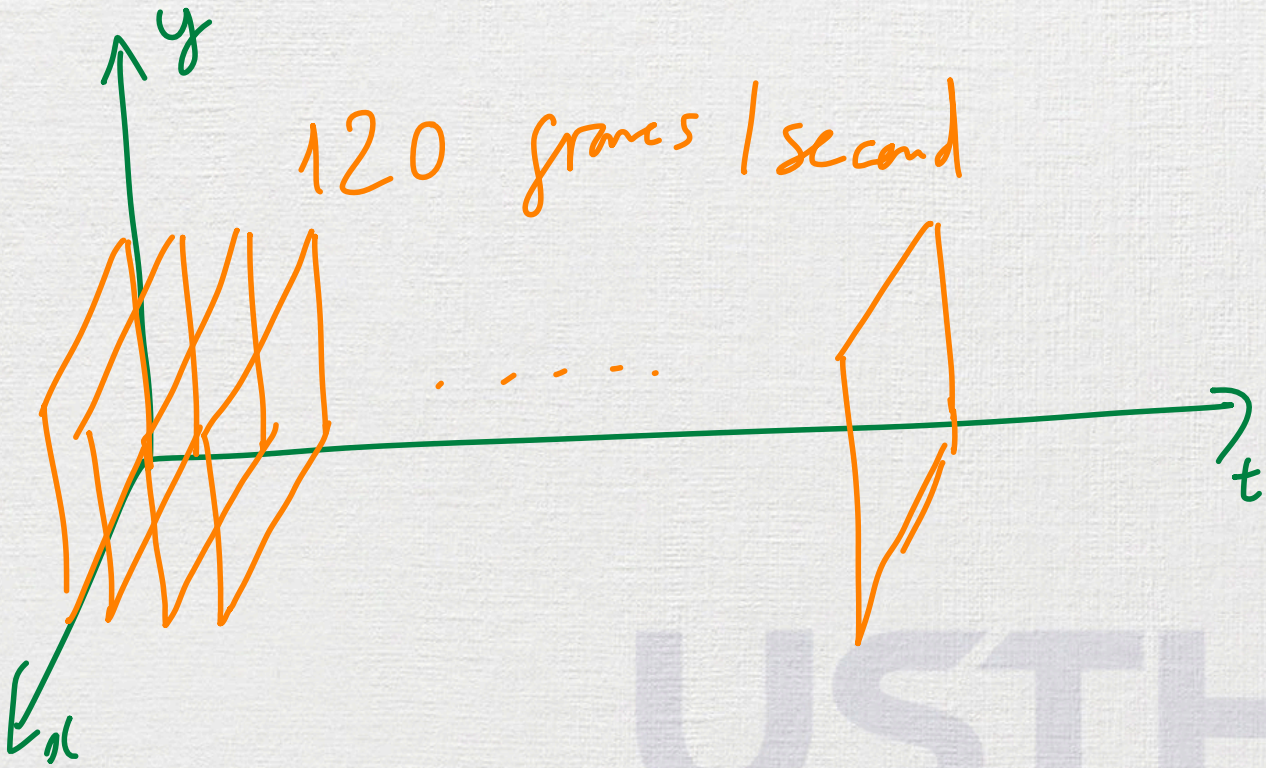
- 1-dimensional: a speech signal
- 2-dimensional: a photo



Signals and Systems: An introduction

A signal is a function of one or more variables.

- 1-dimensional: a speech signal
- 2-dimensional: a photo
- 3-dimensional: a video



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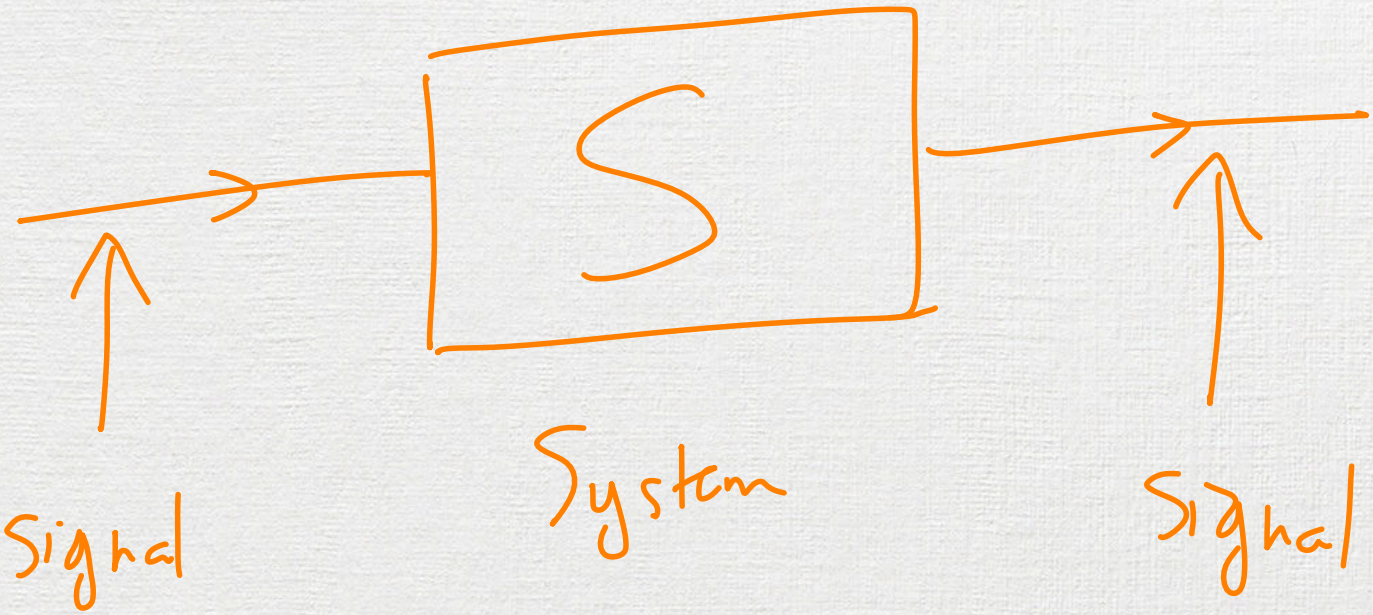

Today Objectives

At the end of this lesson, you should be able to

- Distinguish different kinds of signals: continuous, discrete, analog, digital...

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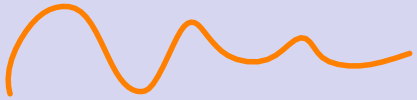
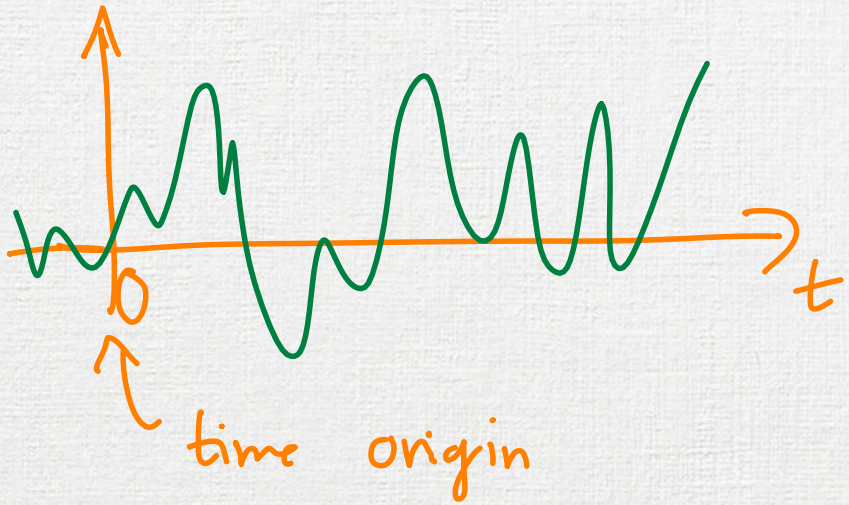
At the end of this lesson, you should be able to

- Distinguish different kinds of signals: continuous, discrete, analog, digital...
- Apply signals' transformations: time shift, time reversal, scaling

Continuous



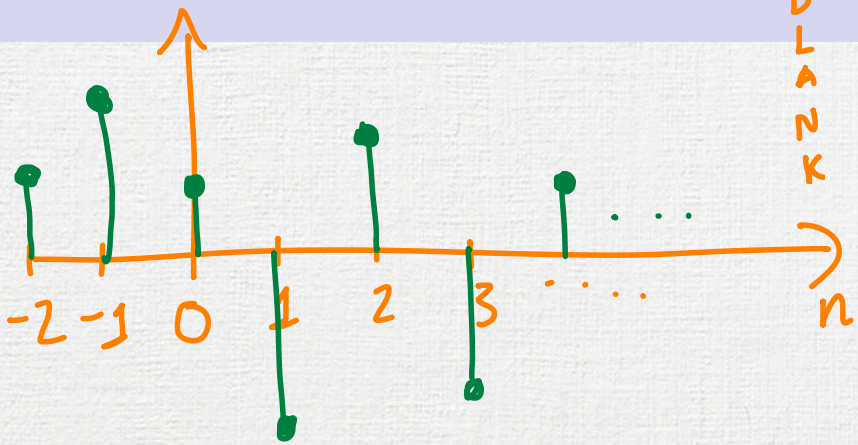
1. Graph
2. Math Formula



$$x(t) = \sin(2t + 1) \qquad t \in [-7, 170]$$

Discrete

1. Graph
2. Math Formula
3. List
4. Vector



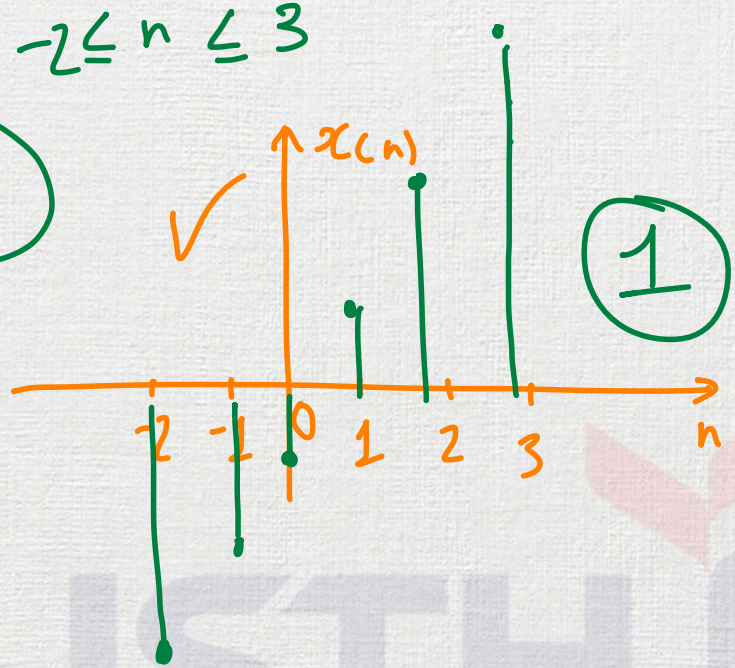
$x_{(n)} = 2n - 1$

$-2 \le n \le 3$

$x_{(-2)} = -5 \quad x_{(-1)} = -3$

$x_{(0)} = -1 \quad \ldots$

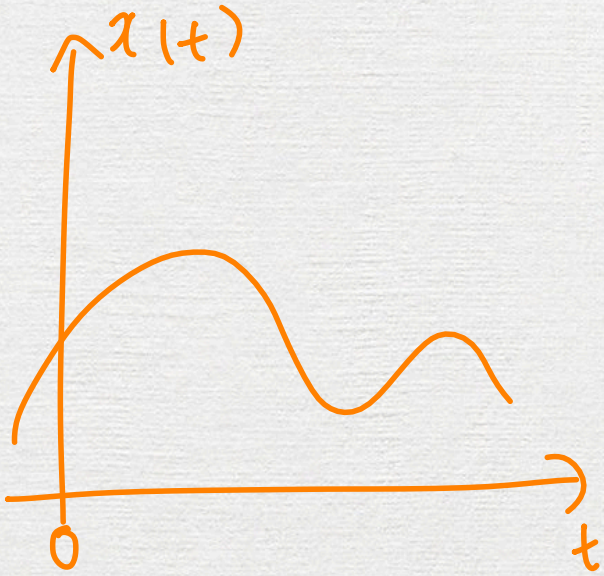
$[-5 \quad -3 \quad -1 \quad 1 \quad 3 \quad 5]$

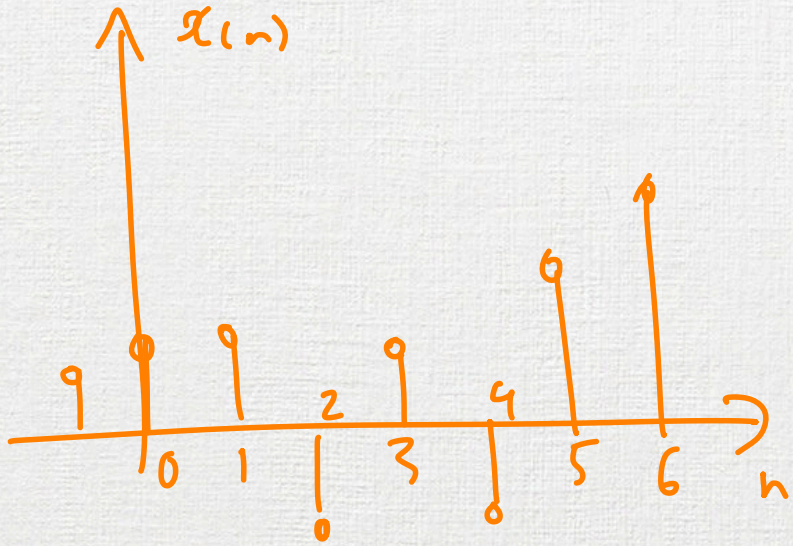


There are many ways in which signals can be classified:

1. Continuous-time and discrete-time signals
2. Analog and digital signals
3. Periodic and aperiodic signals
4. Even and odd signals
5. Energy and power signals

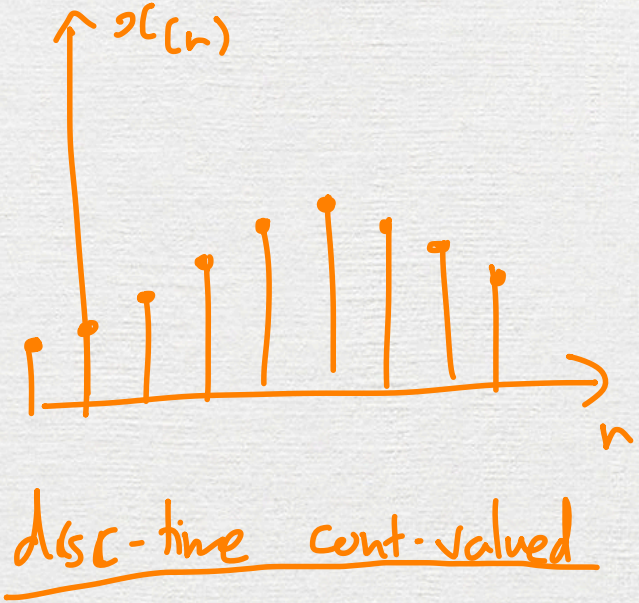
Continuous-time vs Discrete-time Signals

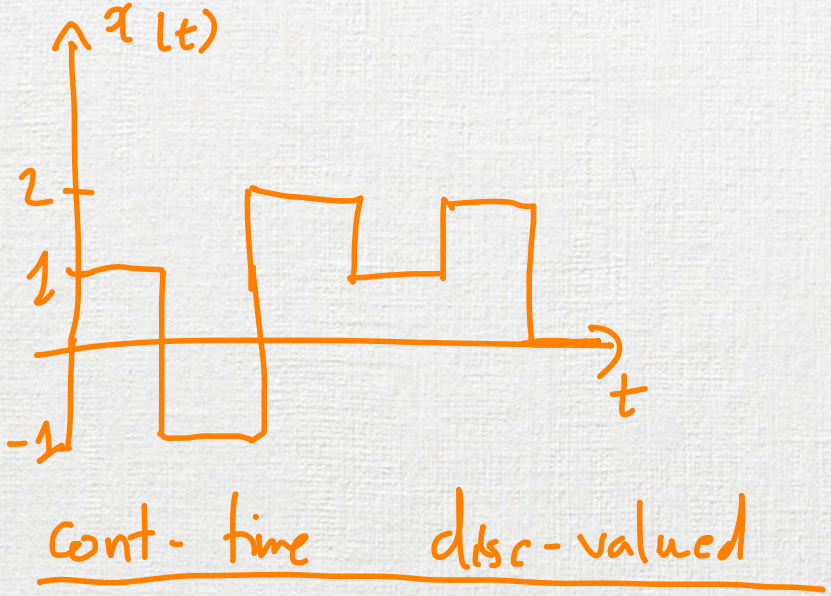




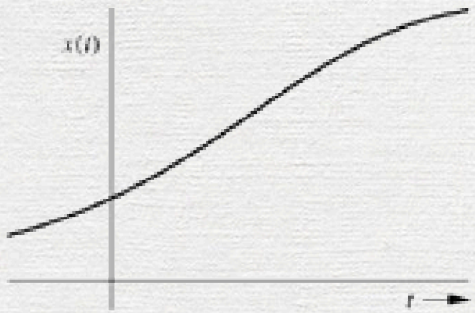
Analog vs Digital Signals

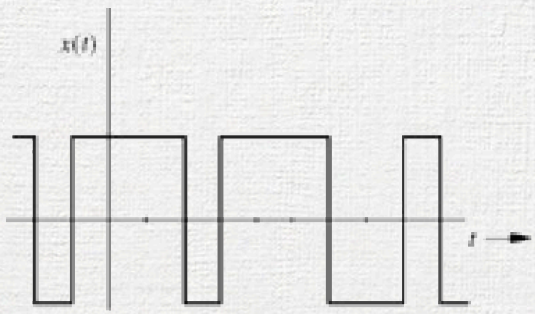
Continuous-valued vs Discrete-valued Signals



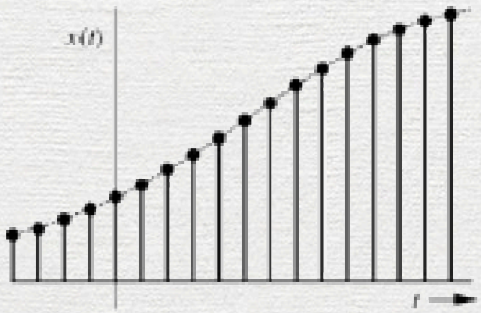


A quick summary





cont - time



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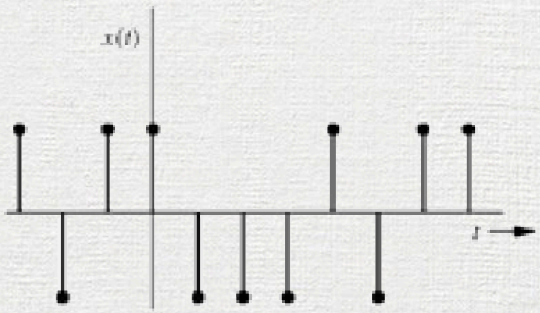

(d)

disc - time

Cont - valued

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Periodic vs Aperiodic Signals

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A signal $x(t)$ is said to be periodic if for a positive constant T_0:

$$x(t) = x(t + T_0) \text{ for all } t$$

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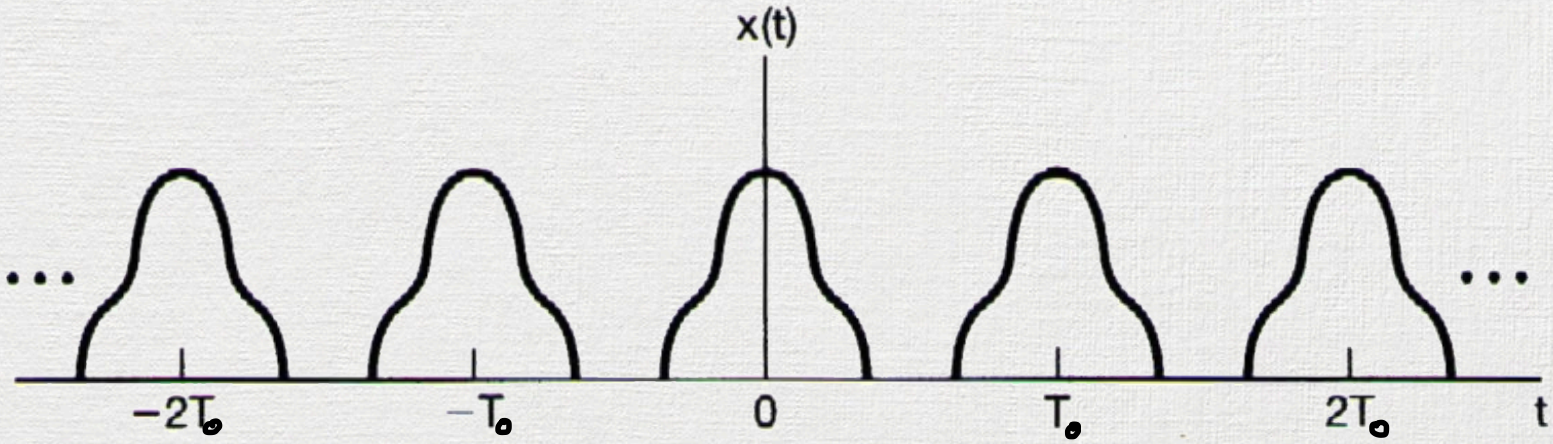

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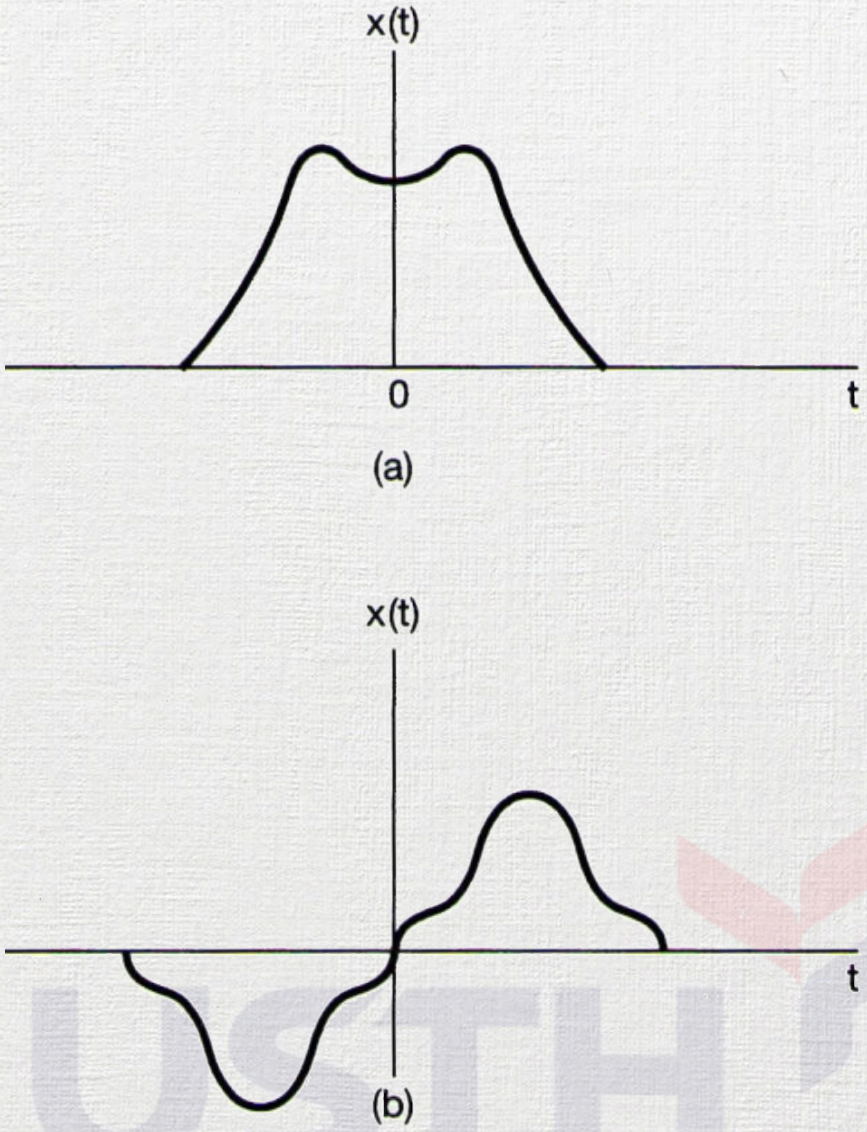

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Definition

A signal having finite energy is called an energy signal.

Power and Energy

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Definition

Power of a discrete-time signal $x[n]$ is defined as

$$P_x = \lim_{N \to +\infty} \frac{1}{2N+1} \sum_{-N}^{+N} |x[n]|^2$$

1 Objectives

2 Signals' Representation

3 Classifying signals

4 Signals' Transformations
- Time Shift
- Time Reversal
- Time Scaling

5 Homework & The Mini-Test

Time Shift Transformation

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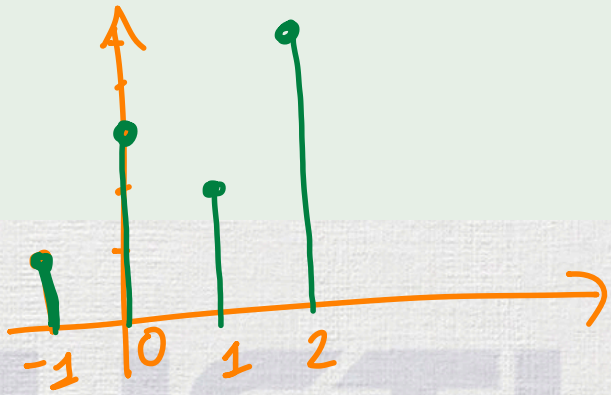

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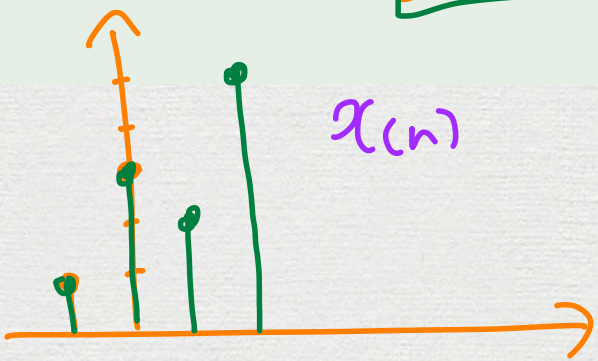
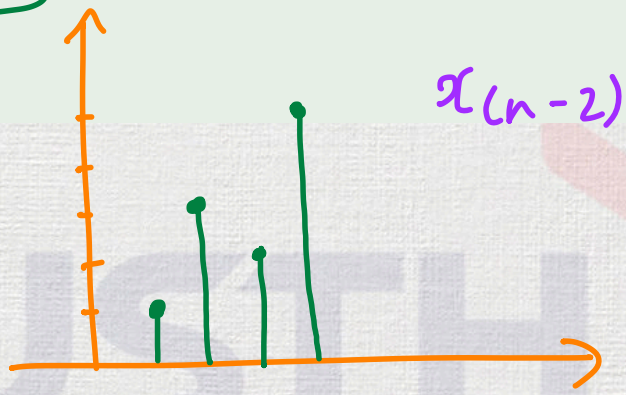
- $x(n-2)$ $\quad y_1(n) = x(n-2) \quad \forall n$





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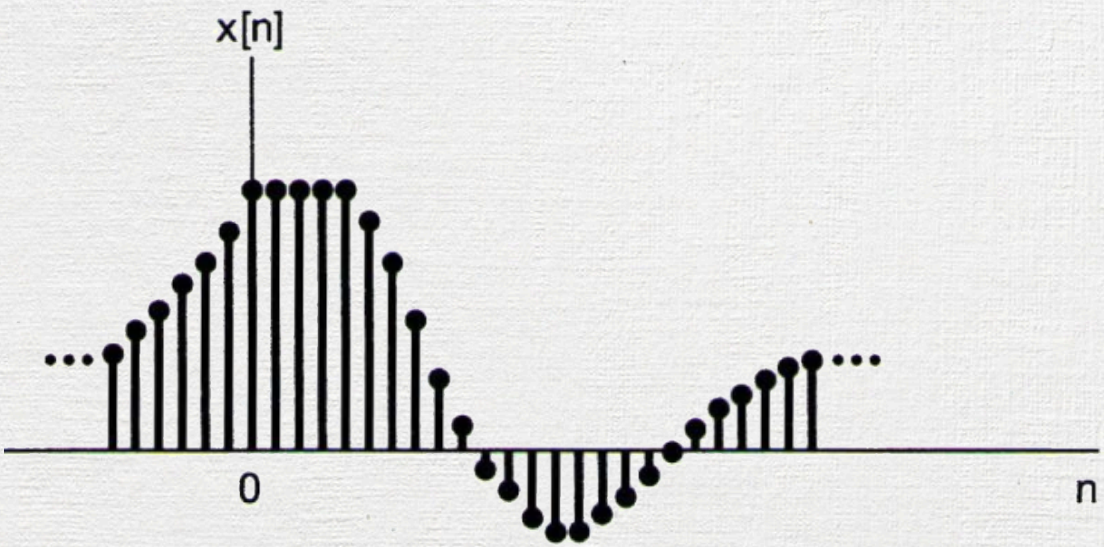
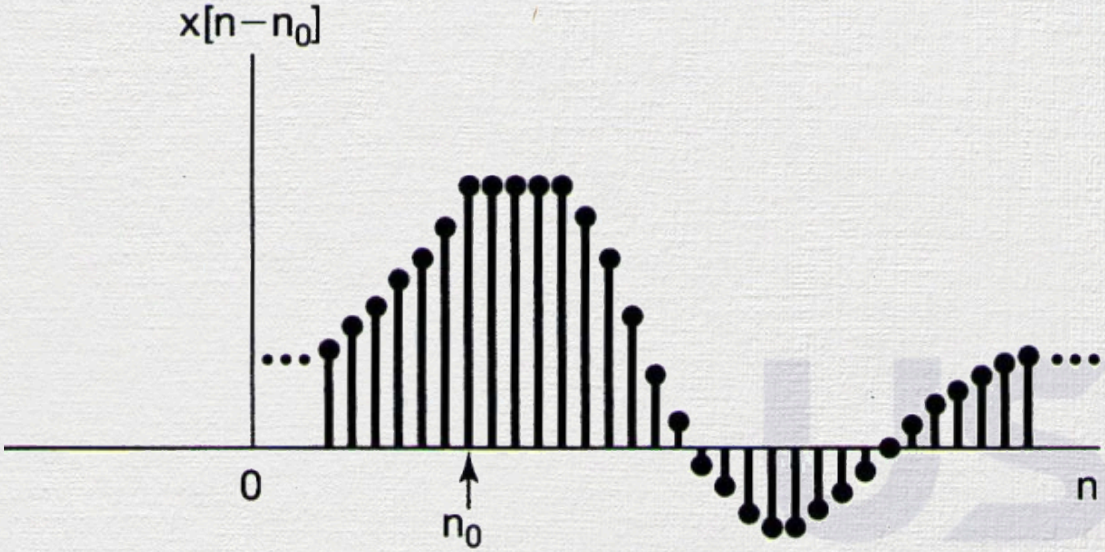
- x(n - 2)
- x(n + 3) $y_2(n) = x(n+3)$

Time Shift Summary





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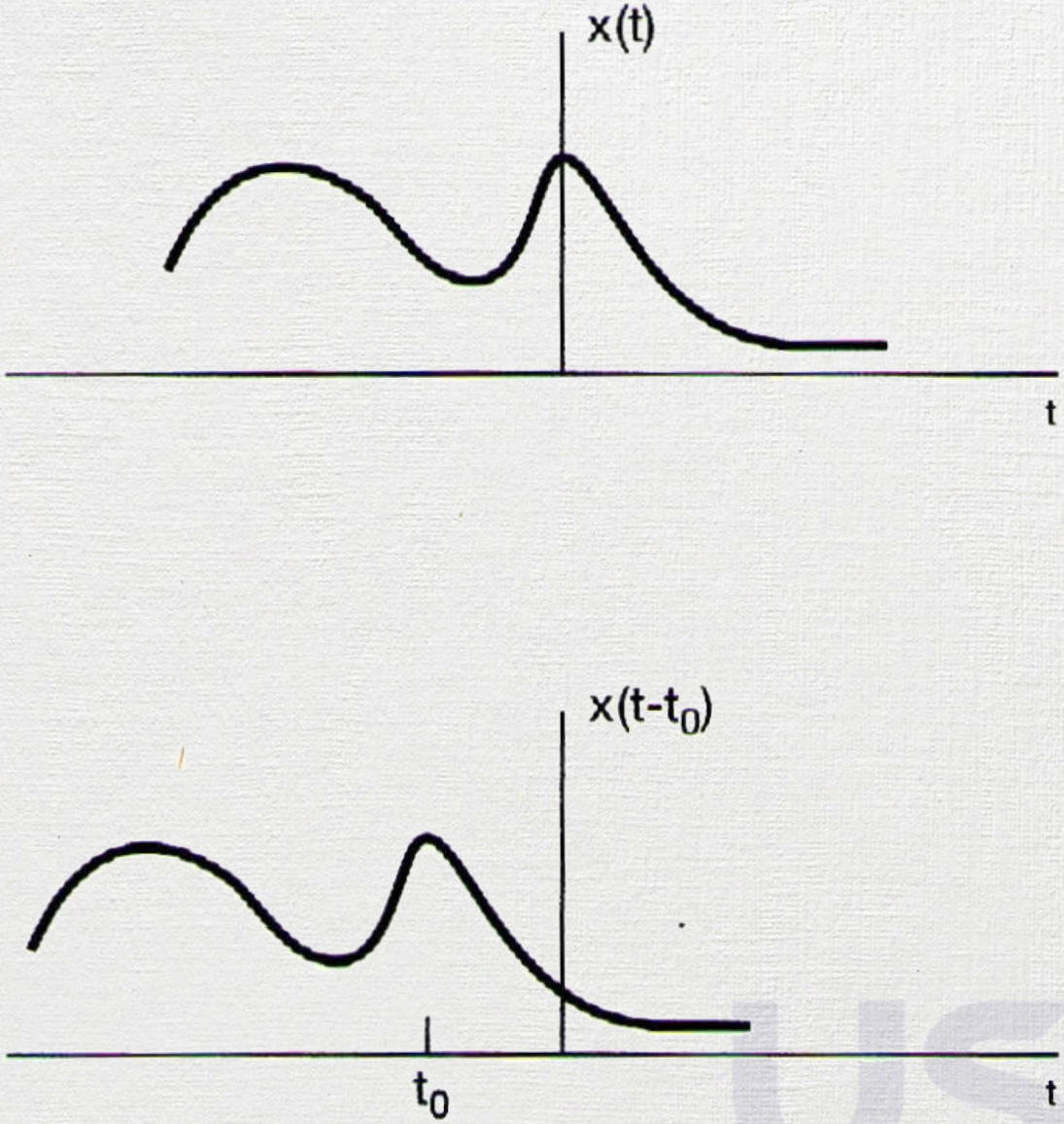

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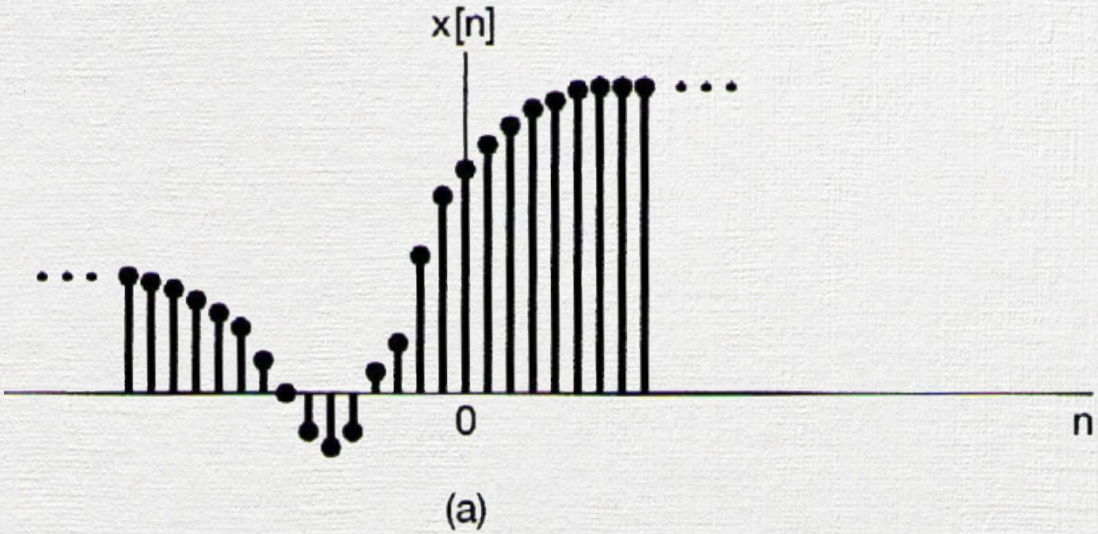
- $x(-1-n)$

Time Reversal Summary



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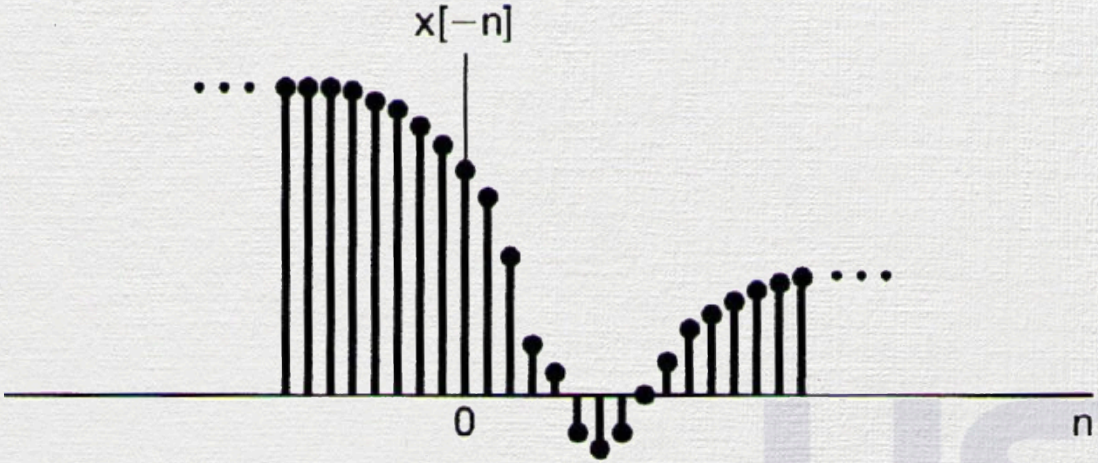
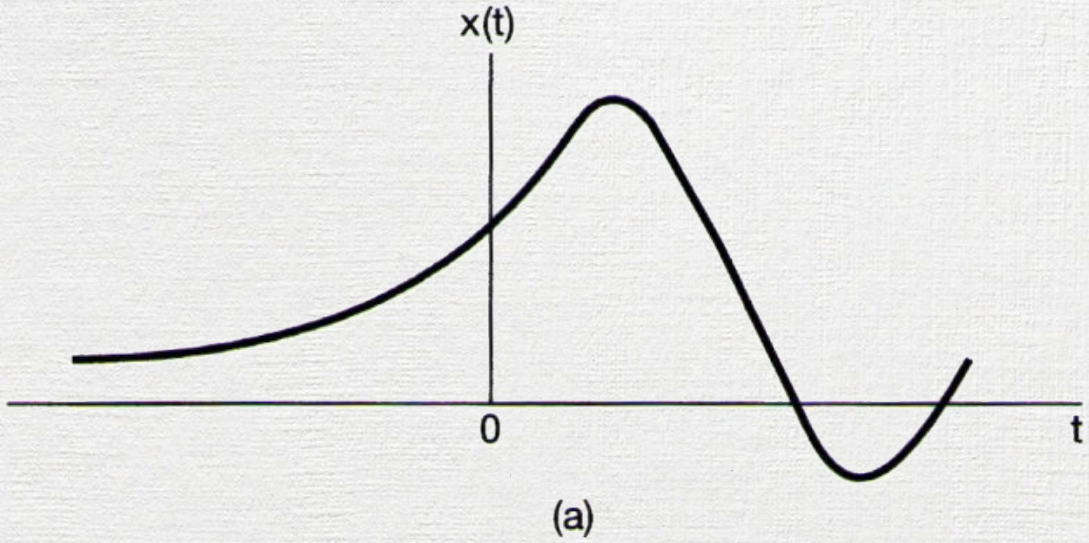


(b)

Time Reversal Summary



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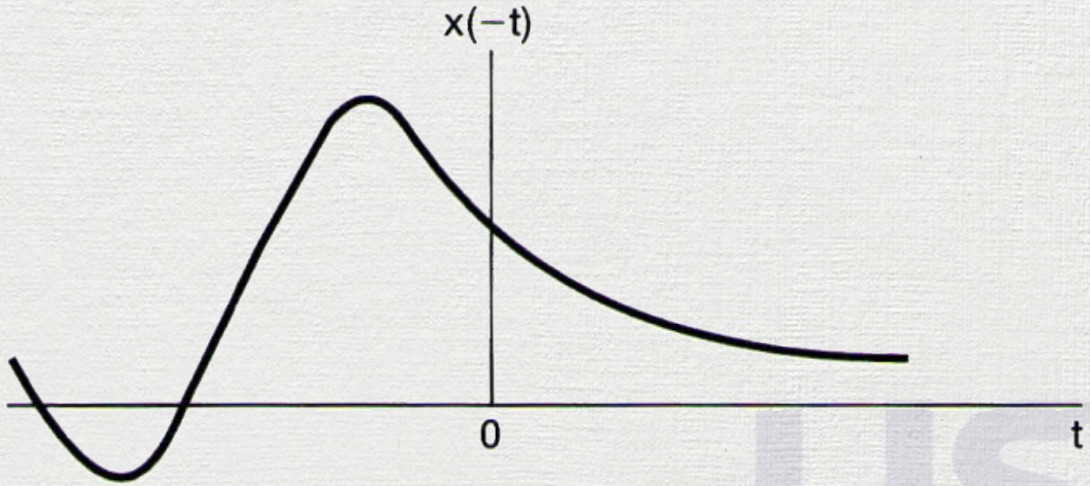

(b)

Time Scaling Transformation

Definition

Given a signal $x(t)$, the time scaling signal is defined as:

$$y(t) = x(kt)$$

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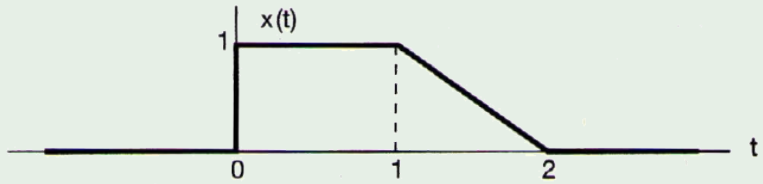
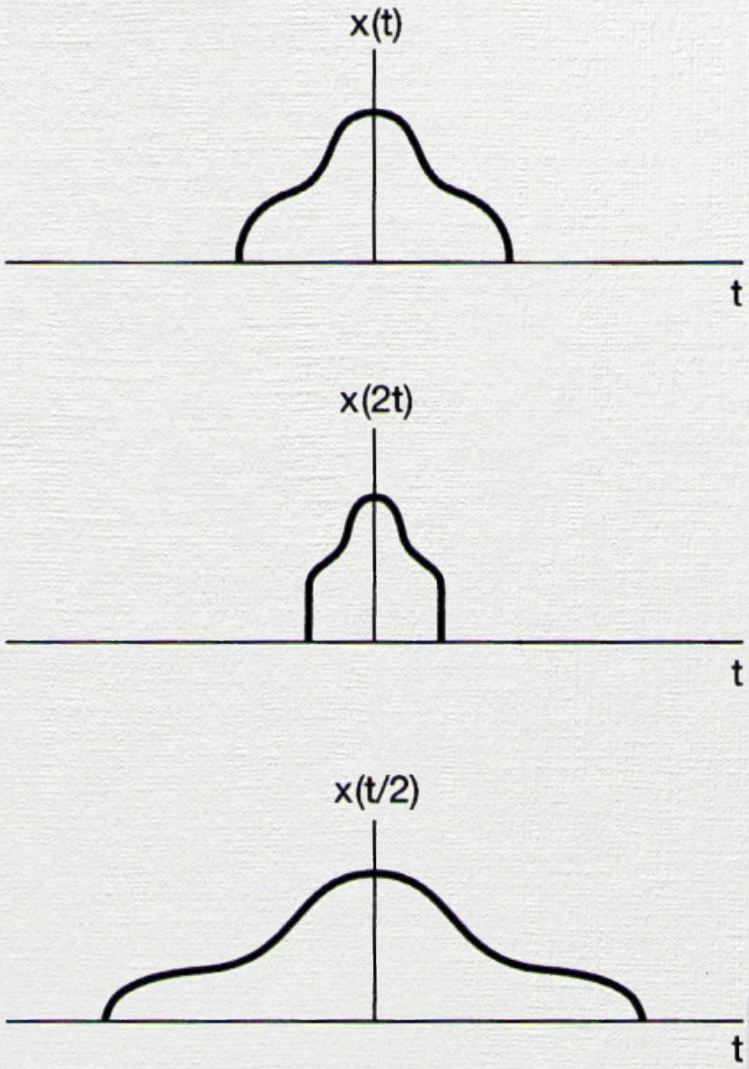
Examples

Given $x(t)$



Determine $x(\frac{3}{2}t)$

Time Scaling Summary



Homework

Signals

1.4, 1.5, 1.6, 1.21, 1.22