Lecture 3 Microstrip Line

1. Caculation of *RLGC* Parameters of TEM Transmission Lines

2. Parallel-plate Transmission Line

3. Microstrip Line

4. Transmission Line Calculators

5. Coding Example

1. Caculation of *RLGC* Parameters of TEM Transmission Lines

- TEM (transverse electromagnetic)

- Transverse field only: $E_z = H_z = 0$

- Number of conductors ≥ 2

- Cutoff frequency $f_c = 0$

- No dispersion (ideal case): $\beta = \dfrac{2\pi}{\lambda_g}, \ \lambda_g = \dfrac{\lambda_0}{\sqrt{\varepsilon_r}}, \alpha =$ 0 or constant

- Use of *RLGC* parameters

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \text{ (characteristic impedance)}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \text{ (propagation constant)}$$

α : attenuation constant

β : phase constant

$$V(z) = V_0^+ e^{-\gamma z} + \Gamma(0) V_0^+ e^{\gamma z}$$

$$I(z) = \frac{V_0^+ e^{-\gamma z}}{Z_0} - \frac{\Gamma(0) V_0^+ e^{\gamma z}}{Z_0}$$

$$Z_{\text{in}}(z) = \frac{V(z)}{I(z)}$$

- Low-loss line

$R << \omega L$ and $G << \omega C$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{Y_0} \text{(characteristic impedance)}$$

$$\gamma = \alpha + j\beta \text{ (propagation constant)}$$

$$\alpha = \alpha_c + \alpha_d \text{ (attenuation constant)}$$

$$\alpha_c = \frac{R}{2Z_0} \text{ (attenuation constant due to conductor loss)}$$

$$\alpha_d = \frac{G}{2Y_0} \text{ (attenuation constant due to dielectric loss)}$$

$$\beta = \frac{2\pi}{\lambda_g} = \omega\sqrt{LC} \text{ (phase propagation constant)}$$

$$\lambda_g = \frac{v_p}{f} \text{ (guided wavelength)}$$

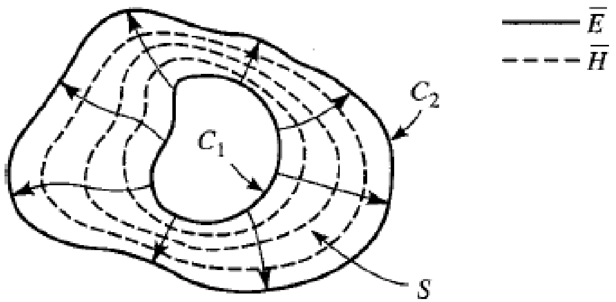
$$v_p = \frac{1}{\sqrt{LC}} \text{ (phase velocity)}$$

High-frequency formulas for *RLGC* parameter calculation

- $R \gg R_{DC}$ for $f \gg 1$

$$R = \frac{R_s}{|I_o|^2} \int_{C_1+C_2} \bar{H} \cdot \bar{H}^* dl \ \Omega/\text{m}. \qquad L = \frac{\mu}{|I_o|^2} \int_S \bar{H} \cdot \bar{H}^* ds \ \text{H/m}.$$

$$G = \frac{\omega \epsilon''}{|V_o|^2} \int_S \bar{E} \cdot \bar{E}^* ds \ \text{S/m}. \qquad C = \frac{\epsilon}{|V_o|^2} \int_S \bar{E} \cdot \bar{E}^* ds \ \text{F/m}.$$



$$G = \frac{\omega \varepsilon''}{\varepsilon} C$$

$$R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\pi f \mu}{\sigma}} \text{ (surface resistance)}$$

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}, \quad \sigma = \frac{1}{\rho} \text{ (conductivity of metal)}$$

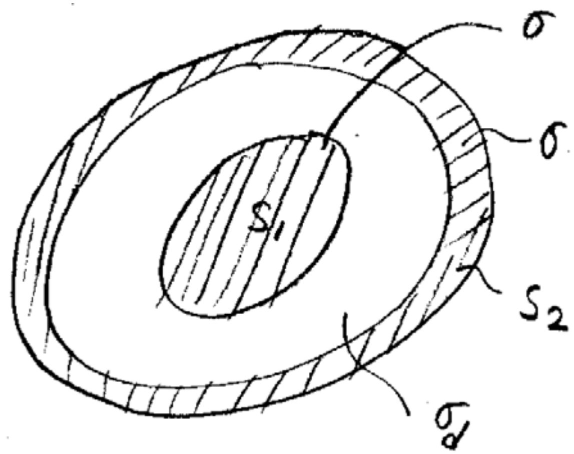
$\mu = \mu_0 \mu_r$ (permeability)

$\varepsilon'' = \varepsilon_0 \varepsilon_r''$ (imaginary part of permittivity)

$\varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 \varepsilon_r'$ (real part of permittivity)

- All-frequency formulas for *RLGC* parameter calculation

- *RLGC* parameters from DC to THz



$$R' \cong R_0 + \sqrt{f}R$$

$$L' \cong L + \frac{R}{2\pi\sqrt{f}}$$

$$G' \cong G_0 + G$$

$$C' = C$$

$R_0 = \frac{1}{\sigma S_1} + \frac{1}{\sigma S_2}$ (DC resistance), S_1 & S_2 (conductor cross sectional area)

$G_0 = \frac{\sigma_d}{\varepsilon} C$, σ_d (conductivity of medium filling the line)

- Air-filled transmission line

$$Z_{0,\text{air}} = \sqrt{\frac{L_0}{C_0}},\ L_0 C_0 = \mu_0 \varepsilon_0,\ \mu_0 \varepsilon_0 = 1/c^2,\ c = 3\times 10^8 \text{ m/s (speed of light in vacuum)}$$

$$Z_{0,\text{air}} = \frac{\sqrt{\mu_0 \varepsilon_0}}{C_0} = \frac{1}{cC_0}$$

$$Z_{0,\text{air}} = \frac{L_0}{\sqrt{\mu_0 \varepsilon_0}} = cL_0$$

$$\boxed{C_0 = \frac{1}{cZ_{0,\text{air}}}}$$

$$\boxed{L_0 = \frac{Z_{0,\text{air}}}{c}}$$

- Uniformly dielectric filled line

- $RLGC$ extraction from Z_0, ε_r, α_c, α_d

$$Z_{0,\mathrm{air}} = \sqrt{\frac{L}{C}},\ LC = \mu_0\varepsilon_0\varepsilon_r,\ \mu_0\varepsilon_0 = 1/c^2$$

$$Z_0 = \frac{Z_{0,\mathrm{air}}}{\sqrt{\varepsilon_r}} = \frac{\sqrt{\varepsilon_r}}{cC}$$

$$Z_0 = \frac{Z_{0,\mathrm{air}}}{\sqrt{\varepsilon_r}} = \frac{L_0}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}} = \frac{cL_0}{\sqrt{\varepsilon_r}}$$

$$\boxed{C = \frac{\varepsilon_r}{cZ_{0,\mathrm{air}}}},\ \boxed{L = \frac{Z_{0,\mathrm{air}}}{c} = L_0}$$

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + \frac{G}{Y_0}\right) = \alpha_c + \alpha_d \rightarrow R = 2\alpha_c Z_0,\ G = 2\alpha_d Y_0$$

$$Y_0 = 1/Z_0$$

- Partially dielectric filled line

- $RLGC$ extraction from Z_0, ε_{re}, α_c, α_d

$$Z_{0,\mathrm{air}} = \sqrt{\frac{L}{C}},\ LC = \mu_0 \varepsilon_0 \varepsilon_{re},\ \mu_0 \varepsilon_0 = 1/c^2$$

$$Z_0 = \frac{Z_{0,\mathrm{air}}}{\sqrt{\varepsilon_{re}}} = \frac{\sqrt{\varepsilon_{re}}}{cC}$$

$$Z_0 = \frac{Z_{0,\mathrm{air}}}{\sqrt{\varepsilon_{re}}} = \frac{L_0}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_{re}}} = \frac{cL_0}{\sqrt{\varepsilon_{re}}}$$

$$\boxed{C = \frac{\varepsilon_{re}}{cZ_{0,\mathrm{air}}}},\ \boxed{L = \frac{Z_{0,\mathrm{air}}}{c} = L_0}$$

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + \frac{G}{Y_0}\right) = \alpha_c + \alpha_d \rightarrow R = 2\alpha_c Z_0,\ G = 2\alpha_d Y_0$$

$$Y_0 = 1/Z_0$$

- Constants in *RLGC* calculation

$\omega = 2\pi f$

$\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m (non-magnetic dielectric materials)

$\varepsilon' = \varepsilon_0 \varepsilon_r$ (real part of complex permittivity)

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m (permittivity of vacuum)

ε_r: dielectric constant

$\varepsilon'' = \varepsilon_0 \varepsilon_r \tan\delta$ (imaginary part of complex permittivity)

$\tan\delta$: loss tangent

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}$$ (surface resistance)

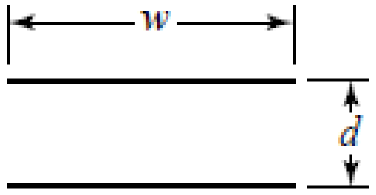
σ : conductivity (S/m), $\sigma = 5.8 \times 10^7$ S/m (copper)

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$ (m) : skin depth

2. Parallel-plate Transmission Lines

- Formula summary

PARALLEL PLATE



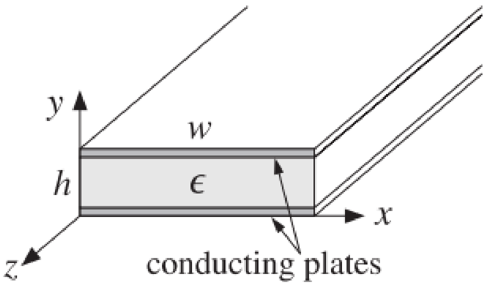
$$\frac{\mu d}{w}$$

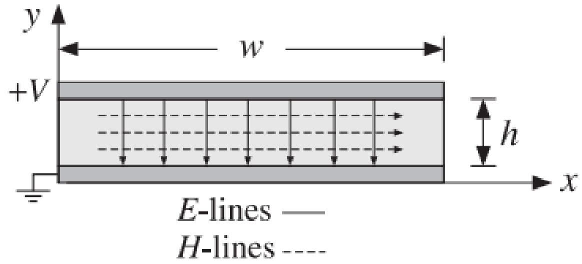
$$\frac{\epsilon' w}{d}$$

$$\frac{2R_s}{w}$$

$$\frac{\omega\epsilon'' w}{d}$$

- Formula derivation





$$V = -E_y h\,, \quad I = H_x w$$

$$Z = \frac{V}{I} = \frac{-E_y h}{H_x w} = \eta \frac{h}{w}$$

$$E_y = -\eta H_x.$$

$$P_T = \frac{1}{2\eta}|E_y|^2 (wh) = \frac{1}{2\eta}\frac{V^2}{h^2}wh = \frac{1}{2\eta}\frac{w}{h}V^2 = \frac{1}{2Z}V^2 = \frac{1}{2}ZI^2$$

$$L' = \mu\frac{h}{w}\,, \quad C' = \epsilon\frac{w}{h}$$

$$P'_{\text{loss}} = 2\frac{1}{2}R_s|H_x|^2 w = \frac{1}{w}R_s I^2$$

$$\alpha_c = \frac{P'_{\text{loss}}}{2P_T} = \frac{R'}{2Z} = \frac{R_s}{wZ} = \frac{R_s}{h\eta}$$

- Example

Find R, L, G, C of the following lines with $f = 10$ GHz, $\varepsilon_r = 2$, $\tan\delta = 0.002$,

$\sigma = 5\times107$ S/m,

1. Parallel-plate: $w = 10$ mm, $d = 2$ mm

- Example

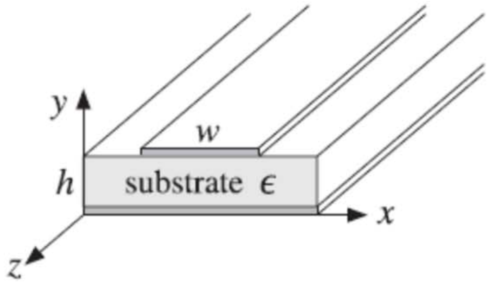
Find Z_0 and γ of the following lines with f = 10 GHz, ε_r = 2, tanδ = 0.002,

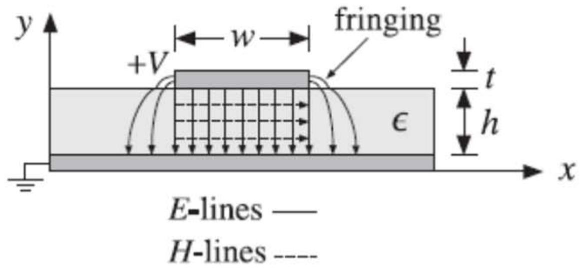
σ = 5×107 S/m,

1. Parallel-plate: w = 10 mm, d = 2 mm

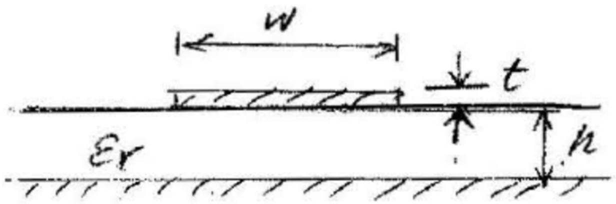
3. Microstrip Line

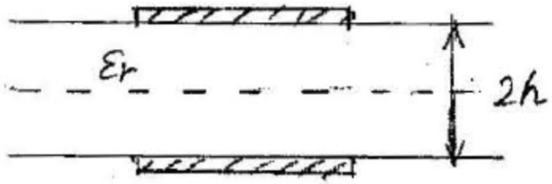
- Microstrip structure





- Microstrip vs parallel strip

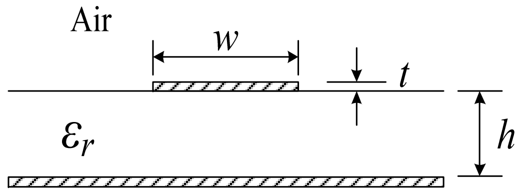




- Image theory

$$Z_{0,\text{microstrip}}(h) = \frac{1}{2} Z_{0,\text{parallel-strip}}(2h)$$

Microstrip formulas



$$u = w / h$$

$$Z_{0,\text{air}} = 30 \log\left\{1 + (4/u)\left[8/u + \sqrt{(8/u)^2 + \pi^2}\right]\right\} \text{ (air-filled microstrip line) [Wheeler(1977)]}$$

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12/u}} \text{ (effective dielectric constant) [Hammerstad(1975)]}$$

$$Z_0 = \frac{Z_{0,\text{air}}}{\sqrt{\varepsilon_{re}}} \text{ (characteristic impedance of microstrip line)}$$

$$R = \frac{2R_s}{w},\ R_s = \sqrt{\frac{\pi f \mu}{\sigma}},\ \alpha_c = \frac{R}{2Z_0} \text{ (attenuation due to conductor loss) (parallel-plate approximation)}$$

$$G = (\omega \tan\delta)\frac{w}{h},\ \alpha_d = \frac{G}{2Y_0} \text{ (attenuation due to dielectric loss) (parallel-plate approximation)}$$

$$\alpha = \alpha_c + \alpha_d \text{ (total attenuation)}$$

$$\beta = \frac{2\pi}{\lambda_g},\ \lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{re}}}$$

$$C = \frac{\varepsilon_{re}}{cZ_{0,\text{air}}},\ L = \frac{Z_{0,\text{air}}}{c} = L_0$$

- Example

Find R, L, G, C, Z_0, γ of microstrip lines with f = 10 GHz, ε_r = 2, tanδ = 0.002,

σ = 5×107 S/m,

1. h = 0.580 mm, w = 2 mm

2. h = 0.787 mm, w = 0.2 mm

3. h = 1.27 mm, w = 4 mm

4. Transmission Line Calculator

- **CST Studio Suite**

- Calculation module

- Draw transmission line structure, set up two wave ports, and do port analysis only.

2. Microstrip Analysis/Synthesis Calculator: loss included

http://mcalc.sourceforge.net/

3. RFDH

Coaxial line calculator: http://www.rfdh.com/rfdb/coaxline.htm

Microstrip calculator: http://www.rfdh.com/rfdb/msline.htm

Inverted microstrip calculator: http://www.rfdh.com/rfdb/msline_i.htm

Suspended microstrip calculator: http://www.rfdh.com/rfdb/msline_s.htm

Stripline calculator: http://www.rfdh.com/rfdb/sline.htm

Coplanar waveguide (CPW) calculator: http://www.rfdh.com/rfdb/cpw.htm

GCPW (CPW with ground) calculator: http://www.rfdh.com/rfdb/cpwg.htm

Coplnar strip (CPS) calculator: http://www.rfdh.com/rfdb/cps.htm

Slot line calculator: http://www.rfdh.com/rfdb/slotline.htm

5. Coding Example

- **Example**

Microstrip:

Given f, w, h, ε_r, tanδ, σ

Find Z_0, ε_{re} ,R, L, G, C, α_c (dB/m) , α_d (dB/m) , α (dB/m), β, λ_g

(Solution)

```
# Microwave Engingineering, Lecture 3 Python Code
# Microstrip calculation:
# Input: f, w, h, er, tand, sigma
# Output: Z0, ere, R, L, G, C, ac, ad, a, beta, lambda_g
#
from math import *
pi=3.14159265;mu=4*pi*1e-7;v=3e8;e0=8.854e-12
while True:
  f=float(input('Frequency f (Hz, 1e9) ='))
  w=float(input('Strip width w (mm, 2) ='))
  h=float(input('Substrate thickness h (mm, 1) ='))
  er=float(input('Substrate dielectric constant er (4.3) ='))
  tand=float(input('Substrate loss tangent tand (0.02)='))
  sigma=float(input('Strip/groundplane conductivity sigma (S/m, 5.8e7)='))
  u=w/h
  z0air=30*log(1+4/u*(8/u+sqrt(u**2+pi**2)))
  ere=(er+1)/2+(er-1)/2/sqrt(1+12/u)
```

5. Coding Example

```
z0=z0air/sqrt(ere)
lambda_0=3e8/f
lambda_g=lambda_0/sqrt(ere)
beta=2*pi/lambda_g
rs=sqrt(pi*f*mu/sigma)
r=2*rs/(w*1e-3)
L=z0air/v
c=ere/(v*z0air)
g=2*pi*f*e0*er*tand*w/h
ac=r/(2*z0);ad=g/(2/z0);a=ac+ad
print('  Z0 (ohm) =',z0)
print('  ere =',ere)
print('  R (Ohm/m) =',r)
print('  G (Ohm/m) =',g)
print('  L (H/m) =',L)
print('  C (F/m) =',c)
print('  ac (dB/m) =',ac*8.68)
print('  ad (dB/m) =',ad*8.68)
print('  a (dB/m) =',a*8.68)
print('  beta (rad/m) =',beta)
print('  lambda_g (m) =',lambda_g)
```

5. Coding Example

```
'''
Frequency f (Hz, 1e9) =
1e9
Strip width w (mm, 2) =
2
Substrate thickness h (mm, 1) =
1
Substrate dielectric constant er (4.3) =
4.3
Substrate loss tangent tand (0.02)=
0.02
Strip/groundplane conductivity sigma (S/m, 5.8e7)=
5.8e7
  Z0 (ohm) = 46.43003113343705
  ere = 3.2736413804652247
  R (Ohm/m) = 8.250226487396457
  G (Ohm/m) = 0.0095685874951464
  L (H/m) = 2.800227207970832e-07
  C (F/m) = 1.2989586349544232e-10
  ac (dB/m) = 0.7711815409383731
  ad (dB/m) = 1.9281309984135606
  a (dB/m) = 2.6993125393519337
  beta (rad/m) = 37.894323997365646
  lambda_g (m) = 0.16580808514849868
Frequency f (Hz, 1e9) =
'''
```

Fin

(End)